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SNARG-2D, A Two-dimensional,
Discrete-ordinate, Transport-theory
Program for the CDC-3600

by

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March 1968

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I. INTRODUCTION AND ACKNOWLEDGMENTS

SNARG-2D is based upon the DDF¹ code written jointly for the IBM 7094 and IBM 7030 in FORTRAN IV by Los Alamos Scientific Laboratory (LASL), United Nuclear Corporation, and Argonne National Laboratory (ANL) under the direction of Bengt G. Carlson and W. J. Worlton. DDF is based on the S_n discrete-ordinates method as developed by Bengt G. Carlson to solve the neutron-transport equation.^{2,3}

SNARG-2D is written for the solution of two-dimensional problems using the order $n = 2, 4, 6, 8$, or 12 , and applicable to xy , rz , and $r\theta$ geometries. The real or adjoint solution may be calculated, and homogeneous (eigenvalue) or inhomogeneous (external distributed source) problems may be solved.

Four criticality-search options are provided in addition to the flux convergence, or k_{eff} , calculation. These are:

- (1) Critical dimension, uniform, $\underline{a}$, in which the entire reactor is expanded or contracted in size to achieve criticality.
- (2) Critical dimension, nonuniform, $\underline{\delta}$, in which the size of designated regions is varied to achieve criticality.
- (3) Concentration, $\underline{c}$, in which the amount of certain designated materials in one or more specified compositions is altered to achieve criticality.
- (4) Inverse period, $\underline{\alpha}$, in which the flux is assumed separable with respect to space and time, and the time variation is given by $\exp(\alpha t)$. In this case, α is varied to achieve criticality.

A fixed k_{eff} or α value may be specified for these searches rather than the criticality value, $k_{eff} = 1$.

SNARG-2D permits the inclusion of linear anisotropic components of the scattering cross sections.

Fission-spectrum matrices are allowed as material-dependent and incident neutron energy-dependent functions.

All boundary conditions as provided for transport-theory applications in the general input specifications⁴ are available.

Cross-section data are obtained either from tapes generated by the ANL cross-section library program, XLIBIT,⁵ or from these tapes and the general input Type 700000 cards.⁴

Of great help throughout the project were discussions with B. G. Carlson, K. D. Lathrop, and W. J. Worlton, of Los Alamos Scientific Laboratory, and W. Guber of United Nuclear Corporation.

II. DESCRIPTION OF INPUT REQUIREMENTS

The input requirements for this program are as reported in the general specifications for the ANL reactor programs.⁴ The user is referred to that document (ANL-7194) for a detailed description of these specifications. This section describes the program-dependent SNARG-2D input and minor program exceptions.

A. Program-dependent Input Types

The program-dependent input deck for SNARG-2D problems will always contain a single Type 105000 or 110000 card followed by Type 120000, 130000, 16XXXX, 170000, and 180000 cards, as necessary, as shown below:

Only a single Type 105000 or 110000 card, a single Type 170000 card, and a single Type 180000 card may ever be included.

The input information to be specified on each of these card types is described in the following paragraphs.

1. Type 105000

The Type 105000 card is used for RESTART calculations in place of the Type 110000 card.

PROBLEM NUMBER						MAXOUTIT	
1	0	5	0	0	0		
1				6	7	18	19
						21	

Exactly one Type 105000 card is required in a restart input deck. The presence of this card immediately following the Type 100000 Problem Identification card implies that this problem calculation is to continue from a previous run. The restart tape generated by the previous run is assumed to be mounted as LUN43.

When initiating a restart calculation, the user may change the following:

- The maximum number of outer iterations to be performed (MAXOUTIT).
- The flux-deck and flux-tape output options.
- The elapsed time between restart dumps (RDS).

The minimum input deck for a RESTART calculation consists of a Type 100000 Problem Identification card, a Type 105000 Restart card, and an END OF PROBLEM card. If the output options are to be changed, a Type 170000 is used. To modify RDS, a Type 180000 card is included.

The problem number of the previous calculation from which this problem is to continue is specified in columns 7-18.

If a change in the value of MAXOUTIT is desired for this restart run, the increment, i.e., number of additional iterations, must be given, right-justified, in columns 19-21. In RESTART calculations, the outer-iteration count continues from the original problem. If no change is required, this field is left blank. If an incremental value is specified, it is added to the current outer-iteration count to form the new maximum; a zero value will force convergence, giving a final print and terminating the problem.

2. Type 110000

Diagram illustrating the structure of the 56-bit parameter field:

Field Name	Bit Range	Field Size (bits)
Reserved	1	1
Reserved	2	1
Reserved	3-6	4
SNORDER	7-10	4
MAXOUTIT	11-14	4
INITRMAX	15-18	4
NORMFCTR	19-30	12
NEWPARMD	31-42	12
PARAM	43-54	12
PARAMIND	55-56	2

Exactly one Type 110000 card is required in a non-RESTART input deck.

The order of the S_n calculation (SNORDER) is given right-justified in columns 7-10. If Type 120000 cards are omitted from the input deck, then SNORDER should have one of the following values: 2, 4, 6, 8, or 12.

The next four quantities on the Type 110000 card are optional and may be specified or left blank.

The following values are assumed if the respective fields are left blank:

- a. MAXOUTIT (maximum number of outer iterations) = 20
- b. INITRMAX (maximum number of inner iterations) = 25
- c. NORMFCTR (normalization factor) = 1.0
- d. NEWPARMD (parameter oscillation damper) = 1.0 for geometric searches
= 0.75 for an α calculation
= 0.50 for a concentration calculation.

If a fixed k (k_0) or α (α_0) value is desired, this value is specified in columns 43-54 (labeled PARAM). The parameter indicator (PARAMIND) is used to designate which of the two parameters is given.

If PARAMIND = 1, k_0 is given as PARAM.

If PARAMIND = 2, α_0 is given as PARAM.

If the parameter indicator (PARAMIND) is left blank, it is assumed that no parameter value (PARAM) has been given; any value given to PARAM will be ignored by the problem.

3. Type 120000

WEIGHT						ETA(η)		MU(μ)		DIREXION(θ)		DIREXION(ϕ)		CARD NUMBER								
1	2	0	0	0	0																	
1						6	7	18	19	30	31	42	43	54	55	66	67	69	70	71	72	73

The Type 120000 cards specifying weights, direction cosines, and directions are optional and may be omitted from an input deck.

If present, the Type 120000 cards are arranged in increasing card number 1, 2, 3, etc., in columns 70-72.

A single weight, a pair of cosines, and a pair of directions can be supplied on each card.

a. To assign density factors by regions

- (1) The region number is given in columns 3-6 on each card used.
- (2) The value given in columns 7-18 is assigned to all mesh spaces in that region. Columns 19-66 are ignored.

b. To assign density factors by mesh intervals

- (1) The region number must be either blank or zero on each Type 16XXXX card.
- (2) A series of values, up to five on each card, is given. These values are assigned as density factors to the mesh intervals. Values are given for all x-coordinates in increasing order of x for a given y value. The process starts at the lowest y-coordinate and continues in increasing order of y.

TWO-DIMENSIONAL MESH-SPACE
ASSIGNMENT ORDERING

- (3) The Type 16XXXX cards are ordered by increasing card number (1, 2, 3, ...) given in columns 70-72.

6. Type 170000

The Type 170000 card used to specify nonstandard I/Ø options is optional and may be omitted from an input deck.

At most, one Type 170000 card may be used. If the card is omitted, all indicators on the card are assumed to be blank.

Only columns 13 and 14 are interpreted by SNARG-2D for RESTART calculations.

The following significance is attached to the fields on this card:

<u>Column</u>	<u>Field Identification</u>	<u>Blank</u>	<u>Nonblank</u>
11	Isotope and material cross sections (composition cross sections are always printed).	Do not print.	Print.
12	Isotope cross-section decks, as altered by change cards.	Do not print.	Print.
13	The final scalar fluxes in 6XXXXX format.	Do not punch.	Punch.
14	The final scalar fluxes in 6XXXXX format (followed by an EØF).	Do not write tape.	Write tape on LUN35.
15	The initial scalar fluxes in 6XXXXX format (followed by an EØF).*	Do not read tape.	Read tape from LUN35.

*NOTE: If initial fluxes are read from tape (LUN35), the Type 600000 card must be nonblank in columns 7 and 8.

7. Type 180000

1 8 0 0 0 0						INITIAL CRITICALITY SEARCH CONVERGENCE CRITERION			NEUTRON BALANCE ERROR CRITERION			RELATIVE INNER ITERATION CONVERGENCE CRITERION			RESTART	DUMP	SPACER (RDS)											
1	8	0	0	0	0	6	7	18	19	30	31	42	43	45	46													

The Type 180000 card is optional and may be omitted from an input deck.

At most, one 180000 card may be used. If the card is omitted, all indicators on the card are assumed to be blank.

Only the RDS value is interpreted by SNARG-2D for RESTART calculations.

The initial criticality-search convergence criterion used in the CONVERGE routine is given in columns 7-18. If this field is left blank or set to zero, SNARG-2D will use the value of δ (columns 13-24) or ϵ (columns 31-42) from the Type 500000 card depending on the type problem being solved.

If the user wishes to apply the neutron-balance test in the OUTER routine he specifies that error criterion in columns 19-30. If this field is left blank or set to zero, no test is made.

Similarly, to perform the relative inner-iteration convergence test of Subroutine INNER, an error criterion must be given in columns 31-42. No test is performed if this field is blank or zero.

The three-column integer field, columns 43-45, is used to indicate the time interval, in minutes, between restart dumps. The restart tape (LUN43) will be written at the beginning of an outer iteration if RDS minutes have elapsed, either since the start of the initial iteration or since the most recent restart dump. During a problem run, each restart dump written replaces the previous one on LUN43. When this field is left blank or is equal to zero, no restart tape is written.

B. Exceptions to the General Input Specifications

The remaining cards required to specify a SNARG-2D problem are described in the general input specifications⁴ with the following exceptions:

1. Type 220000

SNARG-2D uses a modified format for the Type 220000 cards. If more than one such card is required for the same coordinate (i.e., if more than four ratios are given) *the initial ratio on the second and subsequent cards is in columns 19-30, the coordinate being repeated in columns 7-18 of these cards.*

2. Type 3XXXXX

SNARG-2D does not use the Type 3XXXXX cards.

3. Type 500000

The buckling criticality search (column 29 on the Type 500000 card) is not available in SNARG-2D.

4. Type 6XXXXX

In SNARG-2D, if column 8 on the Type 600000 card is nonblank, each value supplied on the Type 6XXXXX cards is the initial source/flux value for a mesh interval.

If the Type 6XXXXX cards are omitted, SNARG-2D assigns initial flux or fission-density values of 1.0 to fissionable regions and zero to nonfissionable regions, for all groups.

SNARG-2D uses the fission-density information supplied on Type 6XXXXX cards to form an initial fission source for the problem. If energy-dependent fission sources are specified for a problem utilizing a fission-fraction matrix or if total fission sources are specified for a problem utilizing vector fission fractions, the values are used as specified.

However, if the initial fission source given and the problem fission fractions do not match in form, SNARG-2D does the following:

a. Given a matrix fission-source guess for a vector fission-fraction problem, SNARG-2D sums the specified fission-source values over the energy groups to form a total fission source for each interval, and uses this as the initial guess.

b. Given a total fission source for a matrix fission-fraction problem, SNARG-2D expands each guess to G repeated values of "guess"/ G , where G is the total number of energy groups employed.

If spectrum averaging is requested on the Type 600000 card but the Type 6XXXXX cards are either omitted or contain group-dependent data, spectrum averaging is *not* performed.

The group numbers on mesh interval assignment Type 6XXXXX cards must either be all blank or all nonblank.

The card numbers used on mesh interval assignment Type 6XXXXX cards must be the integers 1, 2, 3, etc., for each group number.

5. Type 61XXXX

In SNARG-2D, if column 11 on the Type 600000 card is nonblank, each value supplied on the Type 6XXXXX cards is the distributed source value for a mesh interval.

SNARG-2D ignores Type 61XXXX cards for a homogeneous problem (i.e., column 10 on the Type 500000 card is blank). If a source calculation is specified on the Type 500000 card, a distributed source is indicated on the Type 600000 card, and the Type 61XXXX cards are omitted, SNARG-2D solves the problem as a reactivity calculation, assuming the external source = 0.0.

III. SCHEMATIC FLOW CHART FOR EXECUTIV2

IV. SNARG-2D FORMULA-SET DICTIONARY

Name or Variable	Definition	Calculated or Initially Used in Formula or Subroutine	Input from Card Type	FORTRAN Name
AB_j	Total reactor absorptions for group j	131		E5(K)
$\overline{AB}$	AB_j summed over all j	GROUPTOT		E5(GPSPLUS1)
α	The inverse reactor period, sec^{-1}	125		EVGUESS
α_0	Fixed inverse reactor period, sec^{-1}	125	110000	PARAM
AM_h	Interval dimension modifier for all h	110	5XXXXX	AMODIFIER(J)
AR_i	Area elements for all mesh points i	111		AREALEM(I)
AT_j	Anisotropic component of self-scattering cross section in group j , summed over the reactor	125		ANISTRIG
AY_ℓ	Axial area elements for all mesh intervals, ℓ	111		AXAREA(I)
BL_p	Neutron flow at the bottom for pass (p) A or B of inner-iteration process	145		BOTLEAK(P)
Δx_ℓ	The initial width of the ℓ th interval	110		DELTAR(I)
Δy_q	The initial width of the q th interval	110		DELTA(I)
DEV_p	The difference between the p th value and the initial value of the critical parameter divided by corresponding change in λ	MONPRINT		DEOVERDL
$DF_{\ell,q}$	Density factor for the (ℓ,q) interval	121	16XXXX	DENSFCTR(I)
DP_m	The indices of the η direction mates	104		ETAMATES(M)
DR_m	The indices of the μ direction mates	104		DIRMATES(M)
$E_{m,\ell}$	Extrapolated flux for the m th direction and ℓ th interval	99		NBAR
ϵ	Convergence-criterion factor	123	500000	EPSILON
ϵ_a	Special criticality-search convergence criterion	CONVERGE	180000	EPSA
ϵ_{II}	Relative inner-iteration convergence criterion	133	180000	EPSII
ϵ_j	Inner-iteration group-convergence criterion	123		EPSSRC
ϵ_{NB}	Neutron-balance error criterion	GROUPTOT	180000	EPSNB
ERA	Residual self-scatter (isotropic and anisotropic) error	135		E01
ES _R	Region-wise, residual self-scatter error	135		ERRSELF(IZ)
EST	The total residual self-scatter error	135		ERRSELF
η_m	The m th direction cosine in the $y(z)$ direction	104	120000	ETA(M)
EV_p	The p th value of the critical parameter	107		EVPREV
EV_p^o	The difference between the two most recent values of EV_p	107		
EV_{p-1}	The $(p-1)$ st value of the critical parameter	107		EVPREPRE
EV_0	The $(p+1)$ st value of the critical parameter	MONPRINT	500000	EVGUESS
EVM	The input critical-parameter modifier	CONVERGE	500000	EVMODFYR
$FD_{\ell,q}^0$	Fission neutrons for the (ℓ,q) th interval and p iteration	121		NEWFISS(I)
$FD_{\ell,q}^{p-1}$	Fission neutrons for the (ℓ,q) th interval and $p-1$ iteration	CONVERGE	6XXXXX	FISSIONS(I)
FG_j	Fission source for group j	122		E1(K)
$\overline{FG}$	Fission-source total (sum over all groups)	122		E1(GPSPLUS1)
$FI_{\ell,q,j}$	Fission source for interval (ℓ,q) and group j	122		
FT	Fission ratio from previous outer iteration			PREVFTOT
HL_j	Horizontal flow for group j	131		E4(K)
$\overline{HL}$	Horizontal flow summed over all groups	GROUPTOT		E4(GRPSPLUS1)
HL_p	Horizontal flow for pass (p) A or B of inner-iteration process	145		HORLEAK(P)
IT_j	Isotropic component of self-scattering cross section in group j , summed over the reactor	125		ISOTKIG

Name or Variable	Definition	Calculated or Initially Used in Formula or Subroutine	Input from Card Type	FORTRAN Name
J	Current group	OUTER		PRESGRP
J	Number of energy groups	123		NUMVGRPS
$J_{\ell,q}^H$	Horizontal component of current on (ℓ,q) interval for group j	123		HORCURNT(I)
$J_{\ell,q}^V$	Vertical component of current on (ℓ,q) interval for group j	123		VERCURNT(I)
$J_{\ell,q}^{HA}$	Horizontal component of current on (ℓ,q) interval for group j for pass A	134		PRESHCUR(I)
$J_{\ell,q}^{VA}$	Vertical component of current on (ℓ,q) interval for group j for pass A	134		PRESVCUR(I)
k_0	A specified value of k_{eff}	113	110000	PARAM
λ_p	Fission ratio for sum of total fission source and inhomogeneous source for current outer iteration, ρ , divided by analogous sum from the previous outer iteration) for current outer iteration	122		FISRATIO
λ_0	Fission ratio at the beginning of current outer iteration	GROUPTOT		LAMDOUTR
λ_p	Fission ratio corresponding to parameter EV_p	CONVERGE		LAMDPREV
λ_{p-1}	Fission ratio corresponding to parameter EV_{p-1}	CONVERGE		LAMDOLD
ℓ	Interval index for $x(r)$ axis	110		I
LC	Angular flux pass count for current outer iteration	MONPRINT		LOOPCT
LL_p	Left flow for pass A or B	145		LEFTLEAK(P)
LN_j	Net flow for group j	131		LEAKNET(K)
$\overline{LN}$	Total net flow over all groups	GROUPTOT		LEAKNET(GPSPLUS1)
LT_j	Top flow for group j	131		LEAKTOP(K)
$\overline{LT}$	Total top flow over all groups	GROUPTOT		LEAKTOP(GPSPLUS1)
LV_j	Vertical flow for group j	131		LEAKVERT(K)
$\overline{LV}$	Total vertical flow over all groups	GROUPTOT		LEAKVERT(GPSPLUS1)
m	Direction index	104		M
M	Number of directions	104		NUMDIR
μ_m	The mth direction cosine in the $x(r)$ direction	104	120000	MU(M)
$\bar{\mu}_m$	Recursive geometric element used in finite-difference solution	104		MUBAR(M)
N	S_n order	104	110000	SNORDER
NB_j	Neutron balance for group j	MONPRINT		NTRNBAL(K)
$\overline{NB}$	Total neutron balance (sum over all groups)	GROUPTOT		NTRNBAL(GPSPLUS1)
NF	Normalization factor	114	110000	NORMFCTR
NL_p	Left flow for pass (p) A or B of inner-iteration process	145		NETLEAK(P)
$\phi_{m,\ell}$	Angular flux for interval (ℓ,q)	99		ANGLRFLX(M,I)
$\phi_{\ell,q}^A$	Scalar flux from pass A for (ℓ,q) interval	134		PASSAFLX(I)
$\phi_{m,q}^{A,R}$	Angular flux at right boundary at interval q and direction m from pass A	134		RTFLUXA(M,J)
$\phi_{m,\ell}^{A,T}$	Angular flux at top boundary at interval ℓ and direction m from pass A	134		TOPFLUXA(M,I)
$\phi_{m,\ell,j}^B$	Angular flux for interval ℓ and group j along the bottom boundary of the reactor	131		BOTFLUX(I)
$\phi_{m,q,j}^L$	Angular flux for interval q and group j along the left boundary of the reactor	131		LEFTFLUX(J)
$\phi_{\ell,q,j}^T$	Scalar flux for group j on interval (ℓ,q) for the π th inner iteration	134		NEWFLUX(I)
$\phi_{m,q,j}^R$	Angular flux for interval q and group j along the right boundary of the reactor	122		RTFLUX(J)
$\phi_{\ell,q,j}^{p-1}$	Scalar flux for (ℓ,q) interval and group j and $(p-1)$ st outer iteration	121	6XXXXX	OLDFLUX(I)
$\phi_{\ell,q,j}^p$	Scalar flux for (ℓ,q) interval and group j and pth outer iteration	131	6XXXXX	NEWFLUX(I)
$\phi_{m,\ell,j}^T$	Angular flux for interval ℓ and group j along the top boundary of the reactor	122		TOPFLUX(I)
OC_j	Number of times neutron-balance error limit has been exceeded for group j	141		NBALXCS(K)
π	Inner-iteration index	134		INITERCT

Name or Variable	Definition	Calculated or Initially Used in Formula or Subroutine	Input from Card Type	FORTTRAN Name
$\pi_{\max}$	Maximum number of inner iterations for any group	134		INITRMAX
POD	Parameter oscillation damper used in criticality searches	CONVERGE	110000	NEWPARMD
q	Interval index for y (z or θ) axis	110		J
Q	Number of axial intervals	110		NUMAXINT
$Q_{\ell,q,j}$	Normalized distributed source for interval (ℓ,q) and group j	114		FIXEDSRC(I)
$Q_{\ell,q,j}^*$	Input distributed source for an inhomogeneous problem	114	61XXXX	FIXEDSRC(I)
QG_j^*	Volume summed (over reactor) of unnormalized distributed source in group j	114		E0(K)
QG_j	Normalized distributed source in group j	114		E0(K)
$\overline{QG}$	Normalized distributed source over reactor	114		E0(GPSPLUS1)
ρ	Outer-iteration count	MONPRINT		OUTRITCT
RL_j	Left neutron flow for group j	131		E7(K)
RL_p	Left neutron flow for pass (p) A or B of inner-iteration process	145		RITELEAK(P)
$\overline{RL}$	Total right flow over all groups	GROUPTOT		E7(GPSPLUS1)
RM_i	Radial modifiers for a dimension search for all i	110		RMODIFIER(I)
$S_{\ell,q,j}$	Total isotropic source for interval (ℓ,q) and group j	132		SOURCE(I)
$S_{\ell,q,j}^H$	Horizontal component for anisotropic component of source for interval (ℓ,q) and group j	124		HORISRC(I)
$S_{\ell,q,j}^V$	Vertical component for anisotropic component of source for interval (ℓ,q) and group j	124		VERTISRC(I)
SC_j	The scattering source into group j	124		E2(K)
$\overline{SC}$	Scattering source over reactor	GROUPTOT		E2(GPSPLUS1)
$SH_{\ell,q,j}$	Total anisotropic horizontal component of the source for interval (ℓ,q) and group j	132		TOHORSRC(I)
$SI_{\ell,q,j}$	Source for interval (ℓ,q) and group j	124		FIXEDSRC(I)
SS_j	Self-scattering source for group j	131		E3(K)
$\overline{SS}$	SS_j summed over all j			E3(GPSPLUS1)
ST_j	Scattering source for group j	131		E6(K)
$\overline{ST}$	ST_j summed over all j			E6(GPSPLUS1)
$SV_{\ell,q,j}$	Total anisotropic vertical component of the source for interval (ℓ,q) and group j	132		TOVERSRC(I)
$SV_{\ell,q,j}$	Total anisotropic vertical component of the source for interval (ℓ,q) and group j	134		TOVERSRC(I)
t_j	The time-absorption factor used to modify the total cross section in an α calculation for group j	125		TIMEABSR(I)
TL_p	Top neutron flow for pass (p) A or B of inner-iteration process	145		TOPLEAK(P)
TS_j	Total source for group j	124		TOTSRC
v_j	Velocity for jth group, cm/sec	125		VELOCITY(K)
$V_{\ell,q}$	Volume element for (ℓ,q) interval	111		VOLELEM(I,J)
VL_p	Vertical neutron flow for pass (p) A or B of inner-iteration process	145		VERTLEAK(P)
W_m	Weight for mth direction	104	120000	WEIGHT(M)
WD_m	Product of W_m and μ_m	104		WEIGHTMU(M)
WP_m	Product of W_m and η_m	104		WEIGHTET(M)
x_ℓ	Midpoint distance of ℓ th interval, radial direction	111		RADAVG(I)
x_i^p	Distance of outer edge of ith interval for the pth outer iteration	110		MODRADII(I)
y_h^p	Distance of top edge of hth interval for the pth outer iteration	110		MODAXII(J)
y_q	Midpoint distance of qth interval, axial direction	111		AXAVG(J)

V. SNARG-2D FORMULA SET

Before the formula set of SNARG-2D is entered, all input parameters and data are stored.

ADJØINT

If an adjoint solution has been specified, group indexing on the group-dependent input data is inverted (i.e., $1 \rightarrow J$, $2 \rightarrow J - 1$, ..., $j \rightarrow (J - j + 1)$, ..., $J \rightarrow 1$). Quantities involved include fluxes and/or sources, cross sections, velocities, and fission fractions.

BEGINSN

This subroutine checks the S_N constants (Section 104), composes and/or modifies the geometric elements (110 and 111), applies fixed reactivity value (113), computes the fixed source (114), and computes the fission densities (121).

104

This formula checks the S_N constants. (This section is entered only if the outer-iteration count, ρ , is 0.)

$WD_m = W_m \mu_m$ and $WP_m = W_m \eta_m$ are calculated for $m = 1, 2, \dots, M$.

W_m are the weights and μ_m and η_m the direction cosines specified on the 120000 cards--if omitted, code assigned weights and cosines.

$M = N(N + 4)/2$, where N is the S_N order (M is the same for all geometries).

DR_m and DP_m , the indices of the direction mates (the direction cosines opposite in sign but equal in magnitude) for the μ_m and η_m cosines, respectively, are generated.

$$\bar{\mu}_m = \frac{W_{m-1} \cdot \bar{\mu}_{m-1} - W_{m-1} \cdot \mu_{m-1} - W_m \cdot \mu_m}{W_m} \quad \text{for } W_m \neq 0 \quad \left. \vphantom{\frac{W_{m-1} \cdot \bar{\mu}_{m-1} - W_{m-1} \cdot \mu_{m-1} - W_m \cdot \mu_m}{W_m}} \right\} \text{for } m = 1, 2, \dots, M.$$

$$\bar{\mu}_m = 0 \quad \text{for } W_m = 0$$

Tests are made to ensure that

$$(1) \quad |\mu_m + \mu_{DP_m}| < 10^{-4}$$

and

$$(2) \quad \left| -1 + \sum_m W_m \right| + \left| \sum_m W_m \cdot \mu_m \right| + \left| \sum_m W_m \cdot \eta_m \right| \leq 10^{-4}.$$

107

For a concentration search, the cross sections are modified:

$$\sigma_{x,j}^{c,p} = \sigma_{x,j}^{c,p-1} + EV_p^* \cdot \sigma_{x,j}^{c'}$$

where x is the type of cross section; c indicates a composition that is varied; and c' is the pseudocomposition, i.e., the change in composition c made to achieve criticality.

$$EV_p^* = EV_p - EV_{p-1},$$

where EV_p = p th value of critical parameter.

The fission-spectrum fractions are also modified (if necessary) according to

$$\chi^{c,p}(j,k) = \frac{\nu\sigma_{f,j}^{c,p-1} \cdot \chi^{c,p-1}(j,k) + EV_p^* \nu\sigma_{f,j}^{c'} \cdot \chi^{c'}(j,k)}{\nu\sigma_{f,j}^{c,p}}.$$

110

For the initial outer iteration ($\rho = 0$), the mesh points are determined from the problem input specifications.

For subsequent outer iterations in dimension criticality-search problems, mesh points are calculated as

$$x_{i+1}^0 = x_i^0 + \Delta x_\ell (1 + EV_p \cdot RM_\ell),$$

for all mesh points, i , with $x_1^0 = X_L$;

$$y_{h+1}^0 = y_h^0 + \Delta y_q (1 + EV_p \cdot AM_q),$$

for all mesh points, h , with $y_1^0 = Y_B$.

$RM_\ell(AM_q) = x(y)$ dimension region modifier (as specified on a 5XXXXX card), for all intervals $\ell(q)$ in the corresponding region.

$$\Delta x_\ell = x_{i+1}^0 - x_i^0$$

$$\Delta y_q = y_{h+1}^0 - y_h^0.$$

111

For the initial iteration (outer-iteration count, $\rho = 0$), and subsequent outer iterations in dimension criticality-search problems, the formula computes area, volumes, and interval midpoints.

$$x_\ell = (x_{i+1} + x_i)/2 \quad \text{for all intervals, } \ell;$$

$$y_q = (y_{h+1} + y_h)/2 \quad \text{for all intervals, } q;$$

$$AR_i = \begin{cases} 1 & \text{for } x, y \\ 2\pi x_i & \text{for } r, z \\ 2\pi x_i & \text{for } r, \theta \end{cases} \left\{ \begin{array}{l} \text{radial area elements for all mesh points, } i, \\ \text{along the radial axis;} \end{array} \right.$$

$$AY_\ell = \begin{cases} \Delta x_\ell & \text{for } x, y \\ 2\pi(\Delta x_\ell)x_\ell & \text{for } r, z \\ \Delta x_\ell & \text{for } r, \theta \end{cases} \left\{ \begin{array}{l} \text{axial area elements for all mesh intervals,} \\ \ell, \text{ along the radial axis;} \end{array} \right.$$

$$V_{\ell,q} = \begin{cases} (\Delta x_\ell)(\Delta y_q) & \text{for } x, y \\ 2\pi(\Delta x_\ell)(\Delta y_q)x_\ell & \text{for } rz \text{ or } r\theta \end{cases} \left\{ \begin{array}{l} \text{volume elements for all mesh} \\ \text{intervals } (\ell, q); \end{array} \right.$$

where

$$\Delta x_\ell = x_{i+1} - x_i$$

and

$$\Delta y_q = y_{h+1} - y_h$$

for all mesh-point indices, i, h , and all mesh-interval indices, ℓ, q .

113

This formula is used only for problems where a fixed k_{eff}, k_0 , is specified on card 110000.

$$\frac{1}{k_0} \chi^n(j, k) \rightarrow \chi^n(j, k)$$

for all fissionable materials, n , with fission-spectrum fractions specified in this matrix form.

$$\frac{1}{k_0} \chi_j^v \rightarrow \chi_j^v$$

for all fissionable materials, v , with fission-spectrum fractions specified in this vector form.

114

If a distributed source is specified (column 10 on the 600000 card is blank and 61XXXX cards are present), the following calculations are performed:

$$QG_j^* = \sum_{\ell, q} V_{\ell, q} Q_{\ell, q, j}^*,$$

$$\overline{QG} = \sum_j QG_j^*,$$

$$Q_{\ell,q,j} = \frac{NF}{\overline{QG}} \cdot Q_{\ell,q,j}^*,$$

$$QG_j = \frac{NF}{\overline{QG}} \cdot QG_j^*,$$

$$NF \rightarrow \overline{QG},$$

where NF is the normalization factor, an input parameter, given on the 110000 card, and $Q_{\ell,q,j}^*$ is the distributed source from the 61XXXX cards.

121

This formula saves previously computed $\overline{FG}$ as FT, calculates $DF_{\ell,q} \cdot (v\sigma_f)_{\ell,q,j} \rightarrow (v\sigma_f)_{\ell,q,j}$, and forms

$$FD_{\ell,q,j} = \left\{ \begin{array}{ll} (v\sigma_f)_{\ell,q,j} \cdot \phi_{\ell,q,j}, & \text{real case} \\ \sum_k \chi_{\ell,q}(j,k) \phi_{\ell,q,k}, & \text{adjoint case} \end{array} \right\} \text{for } \chi \text{ fission matrix}$$

or

$$FD_{\ell,q} = \left\{ \begin{array}{ll} \sum_k (v\sigma_f)_{\ell,q,k} \phi_{\ell,q,k}, & \text{real case} \\ \sum_j \chi_{\ell,q,j} \phi_{\ell,q,j}, & \text{adjoint case} \end{array} \right\} \text{for } \chi \text{ fission vector.}$$

The scalar flux in interval (ℓ,q) , for groups j is represented as $\phi_{\ell,q,j}$.

FISCAL

This subroutine completes the calculation of the fission source (Section 122) and normalizes all necessary elements (Section 123).

122

The fission-source calculation is completed.

$$FI_{\ell,q,j} = \left\{ \begin{array}{ll} V_{\ell,q} \sum_k \chi_{\ell,q}(k,j) \cdot FD_{\ell,q,k} & \text{real case} \\ V_{\ell,q} (v\sigma_f)_{\ell,q,j} \cdot FD_{\ell,q,j} & \text{adjoint case} \end{array} \right\} \text{for a } \chi \text{ fission matrix}$$

or

$$FI_{\ell,q,j} = \left\{ \begin{array}{ll} V_{\ell,q} \chi_{\ell,q,j} FD_{\ell,q} & \text{real case} \\ V_{\ell,q} (v\sigma_f)_{\ell,q,j} FD_{\ell,q} & \text{adjoint case} \end{array} \right\} \text{for a } \chi \text{ fission vector.}$$

$$FG_j = \sum_{\ell, q} FI_{\ell, q, j}.$$

Except for the initial outer iteration, λ is computed as follows:

$$\lambda_p = \frac{\overline{QG} + \overline{FG}}{\overline{QG} + \overline{FT}},$$

where

$$\overline{FG} = \sum_j FG_j.$$

If a k_{eff} problem is being solved, the quantities FG_j , $\overline{FG}$, and $\chi_{\ell, q}(j, k)$ or $\chi_{\ell, q, j}$ (and $FD_{\ell, q}$ or $FD_{\ell, q, j}$ in the adjoint case) are divided by the λ value obtained and restored.

123

This formula normalizes the following quantities (for all problems except source calculations--column 10 of 500000 card nonblank) by multiplication by the quantity $NF/\overline{FG}$:

$$FG_j, FI_{\ell, q, j}, FD_{\ell, q, j} \text{ or } FD_{\ell, q}, \phi_{\ell, q, j}, J_{\ell, q, j}^H, J_{\ell, q, j}^V, \phi_{m, q, j}^R, \phi_{m, \ell, j}^T$$

when applicable.

$$\epsilon_J = \epsilon(\overline{FG} + \overline{QG})/J,$$

where ϵ is the convergence criterion from the 500000 card and J is the total number of groups used; ϵ is taken from columns 31-42 for criticality problems (column 12 nonblank), from 13-24 (δ) for source or flux convergence problems (column 10 or 11 nonblank).

MØNPRINT

This subroutine edits and prints the following information for each outer iteration (the first monitor line for a "restart problem" will be the last underlined monitor line for the problem for which this is a restart. See Type 105000 card for more information):

Outer-iteration count, angular-flux count for this outer iteration, the total neutron balance, ϵ , the ratio $(EV_p - EV_0)/(\lambda_p - \lambda_0^{conv})$, λ_p ,

$$\sum_{\ell, q} V_{\ell, q} |FD_{\ell, q}^0 - \lambda_{p-1} \cdot FD_{\ell, q}^{0-1}|$$

or

$$\sum_{\ell,q} V_{\ell,q} \sum_j |FD_{\ell,q,j}^0 - \lambda_{p-1} \cdot FD_{\ell,q,j}^{0-1}|,$$

$$EV_p,$$

and the machine time required for this outer iteration. EV_0 is the initial parameter guess from the Type 500000 card, and λ_0^{conv} is the converged value of λ corresponding to that guess.

ØUTER

This subroutine calculates the total source for each group, j , computes the angular and scalar fluxes for group j by calling Subroutine INNER, and determines the neutron balance and fission source for each group, j .

124

This formula computes the isotropic and anisotropic group source, the group scattering source, and the group-convergence criterion to be used in the inner-iteration scheme.

$$V_{\ell,q} DF_{\ell,q} \sigma_{\ell,q}^{(0)}(k \rightarrow j) \rightarrow \sigma_{\ell,q}^{(0)}(k \rightarrow j),$$

$$SI_{\ell,q,j} = \sum_{\substack{k \\ k \neq j}} \sigma_{\ell,q}^{(0)}(k \rightarrow j) \phi_{\ell,q,k} + FI_{\ell,q,j} + V_{\ell,q} Q_{\ell,q,j}$$

for the isotropic component;

$$V_{\ell,q} DF_{\ell,q} \sigma_{\ell,q}^{(1)}(k \rightarrow j) \rightarrow \sigma_{\ell,q}^{(1)}(k \rightarrow j),$$

$$S_{\ell,q,j}^H = 3 \sum_{\substack{k \\ k \neq j}} \sigma_{\ell,q}^{(1)}(k \rightarrow j) J_{\ell,q,j}^H$$

and

$$S_{\ell,q,j}^V = 3 \sum_{\substack{k \\ k \neq j}} \sigma_{\ell,q}^{(1)}(k \rightarrow j) J_{\ell,q,j}^V$$

for the anisotropic components;

$$SC_j = \sum_{\ell,q} (SI_{\ell,q,j} - FI_{\ell,q,j} - V_{\ell,q} Q_{\ell,q,j});$$

$$TS_j = \sum_{\ell,q} SI_{\ell,q,j}.$$

125

This formula computes the necessary velocity adjustments for inverse reactor period α and/or fixed α_0 problems and calculates the self-scatter terms (isotropic and anisotropic).

$$t_j = \begin{cases} \frac{EV_p + \alpha_0}{v_j} & \text{for } \alpha \text{ calculation with fixed } \alpha_0, \\ EV_p/v_j & \text{for } \alpha \text{ calculation,} \\ \alpha_0/v_j & \text{for fixed } \alpha_0. \end{cases}$$

$$IT_j = \sum_{\ell, q} \sigma_{\ell, q}^{(0)}(j \rightarrow j),$$

$$AT_j = \sum_{\ell, q} \sigma_{\ell, q}^{(1)}(j \rightarrow j).$$

131

This formula calls Subroutine INNER, which solves the S_N difference equations. If

$$\sum_{\ell, q} SI_{\ell, q, j} \neq 0, \quad \phi_{\ell, q, j}^{0-1} \rightarrow \phi_{\ell, q, j}^0$$

for all ℓ, q , and INNRL00P is called. If

$$\sum_{\ell, q} SI_{\ell, q, j} = 0,$$

INNER is bypassed and the following are set to zero for group j :

- (1) $\phi_{\ell, q, j}^0, \phi_{\ell, q, j}^{0-1}, J_{\ell, q, j}^H, J_{\ell, q, j}^V$, for all ℓ, q .
- (2) $\phi_{m, \ell, j}^B, \phi_{m, \ell, j}^T$ for all m, ℓ ; and $\phi_{m, q, j}^L, \phi_{m, q, j}^R$.
- (3) $SS_j, HL_j, AB_j, ST_j, RL_j, LV_j, LT_j, LN_j, NB_j$.

141

This formula calculates statistics for group j and computes the up-scattering error if $TS_j \neq 0$.

$$SS_j = \sum_{\ell, q} v_{\ell, q} \sigma_{\ell, q}^{(0)}(j \rightarrow j) \phi_{\ell, q, j}^0$$

$$AB_j = \sum_{\ell, q} v_{\ell, q} \sigma_{a_{\ell, q, j}} \phi_{\ell, q, j}^0$$

$$ST_j = \sum_{\ell, q} V_{\ell, q} \left(\sigma_{t_{\ell, q, j}} - \sigma_{a_{\ell, q, j}} - \sigma_{\ell, q}^{(0)}(j \rightarrow j) \right) \phi_{\ell, q, j}^0$$

$$NB_j = |TS_j - NL_j - AB_j - ST_j| / TS_j$$

If $|NB_j| > \epsilon_j$, then $OC_j + 1 \rightarrow OC_j$,

$$FD_{\ell, q, j}^0 = \begin{cases} (\nu\sigma_f)_{\ell, q, j} \cdot \phi_{\ell, q, j}^0, & \text{real case} \\ \sum_k \chi_{\ell, q}(j, k) \phi_{\ell, q, k}^0, & \text{adjoint case} \end{cases} \text{ for } \chi \text{ fission matrix;}$$

$$FD_{\ell, q}^0 = \begin{cases} \sum_k (\nu\sigma_f)_{\ell, q, k} \phi_{\ell, k}^0, & \text{real case} \\ \sum_j \chi_{\ell, q, j} \phi_{\ell, j}^0, & \text{adjoint case} \end{cases} \text{ for } \chi \text{ fission vector.}$$

$$E_{up} = \sum_{\ell, q} E_j(\ell, q),$$

$$AE_{up} = \sum_{\ell, q} |E_j(\ell, q)|,$$

the upscatter error measures, where

$$E_j(\ell, q) = V_{\ell, q} \left(\phi_{\ell, q, j}^{0-1} - \phi_{\ell, q, j}^0 \right) \sum_g \sigma_{\ell, q}(j + g \rightarrow j); \quad j + 1 \rightarrow j.$$

If $j \leq J$, return is made to Section 124.

INNER

This subroutine calculates the angular and scalar fluxes and currents for group j for inner iteration π .

132

This formula calculates the self-scattering contribution to the source term and defines the flow for pass A of iteration π .

$$S_{\ell, q, j} = SI_{\ell, q, j} + V_{\ell, q} \phi_{\ell, q, j}^{\pi} \sigma_{\ell, q}^{(0)}(j \rightarrow j) \quad \text{for all } \ell, q.$$

$$SH_{\ell, q, j} = S_{\ell, q, j}^H + 3V_{\ell, q} J_{\ell, q, j}^H \sigma_{\ell, q}^{(1)}(j \rightarrow j) \quad \text{for all } \ell, q.$$

$$SV_{\ell, q, j} = S_{\ell, q, j}^V + 3V_{\ell, q} J_{\ell, q, j}^V \sigma_{\ell, q}^{(1)}(j \rightarrow j) \quad \text{for all } \ell, q.$$

Source fixup:

If $F \cdot |SH_{\ell,q,j}| > S_{\ell,q,j}$, then $SH_{\ell,q,j} = \text{sign}(SH_{\ell,q,j}) \cdot \left| \frac{S_{\ell,q,j}}{F} \right|$

If $F \cdot |SV_{\ell,q,j}| > S_{\ell,q,j}$, then $SV_{\ell,q,j} = \text{sign}(SV_{\ell,q,j}) \cdot \left| \frac{S_{\ell,q,j}}{F} \right|$

where $F = \sqrt{2}$. $0 \rightarrow \phi_{\ell,q,j}^{\pi}$ for all ℓ, q . If there is anisotropic scattering,
 $0 \rightarrow J_{\ell,q,j}^H, J_{\ell,q,j}^V$ for all ℓ, q .

142

This formula establishes values for the top boundary fluxes.

Free top boundary: For all m such that $\eta_m < 0$, $\phi_{m,\ell,j}^T = \phi_{m,\ell,j}^B = 0$
 for all ℓ .

Reflective top: For all m such that $\eta_m < 0$, $\phi_{m,\ell,j}^T = \phi_{m,\ell,j}^B = \phi_{DP_m,\ell,j}^T$
 for all ℓ .

Periodic top: For all m such that $\eta_m < 0$, $\phi_{m,\ell,j}^B = \phi_{m,\ell,j}^T$ for all ℓ .

White top: For all m such that $\eta_m < 0$,

$$\phi_{m,\ell,j}^T = \phi_{m,\ell,j}^B = \frac{\sum_{\substack{m \\ \eta_m > 0}} w_m \eta_m \phi_{m,\ell,j}^T}{\sum_m w_m \eta_m}$$

99

This formula solves the S_N finite-difference equations in the downward direction, i.e., by making sweeps along the reactor from top to bottom for $m = 1, m/2$; $q = Q, 1$. If the right boundary is white, then

$$\phi_{m,q}^R = \phi_{m,q}^L = \frac{\sum_{\substack{m \\ \mu_m > 0}} w_m \mu_m \phi_{m,q}^R}{\sum_m w_m \mu_m}$$

for m such that $\mu_m \leq 0$. At the right boundary ($\mu_m < 0$):

Free: $\phi_{m,q}^L = \phi_{m,q}^R = 0$

Reflective: $\phi_{m,q}^L = \phi_{m,q}^R = \phi_{DR_m,q}^R$

Periodic: $\phi_{m,q}^L = \phi_{m,q}^R$

At the left boundary ($\mu_m \geq 0$):

Free: $\phi_{m,q}^L = \phi_{m,q}^R = 0$

Reflective: $\phi_{m,q}^R = \phi_{m,q}^L = \phi_{DRm,q}^L$

Periodic: $\phi_{m,q}^L = \phi_{m,q}^R$

$0 \rightarrow \phi_{\ell,m}$ for all m, ℓ when $m = 1$ ($\phi_{\ell,m}$ is the angular flux for all intervals ℓ , directions m and axial interval q). For this m, q and for all ℓ ($q = 1, Q$ for $\mu_m > 0$; $q = Q, 1$ for $\mu_m < 0$), $\eta_m < 0$, $\mu_m < 0$:

$$E_{m,\ell} = \frac{S_{\ell,q,j} + (\Delta y_q) \bar{\mu}_m (AR_{i+1} - AR_i) \phi_{m-1,\ell} + \mu_m SH_{\ell,q,j} + \eta_m SV_{\ell,q,j} + (\Delta y_q) |\mu_m (AR_{i+1} + AR_i)| \phi_{m,q}^L + 2|\eta_m| \cdot AY_{\ell} \phi_{m,\ell}^B}{(\Delta y_q) |\bar{\mu}_m (AR_{i+1} - AR_i)| + 2|\eta_m| \cdot AY_{\ell} + (\Delta y_q) \bar{\mu}_m (AR_{i+1} - AR_i) + V_{\ell,q} \sigma t_{\ell,q,j} + t_j}$$

$$\phi_{m,q}^L = 2E_{m,\ell} - \phi_{m,q}^L$$

$$\phi_{m,\ell}^B = 2E_{m,\ell} - \phi_{m,\ell}^B$$

For $\eta_m > 0$, $\mu_m < 0$:

$$E_{m,\ell} = \frac{S_{\ell,q,j} + (\Delta y_q) \bar{\mu}_m (AR_{i+1} - AR_i) \phi_{m-1,\ell} + \mu_m SH_{\ell,q,j} + \eta_m SV_{\ell,q,j} + (\Delta y_q) |\mu_m (AR_{i+1} + AR_i)| \phi_{m,q}^L + 2|\eta_m| \cdot AY_{\ell} \phi_{m,\ell}^T}{(\Delta y_q) |\bar{\mu}_m (AR_{i+1} - AR_i)| + 2|\eta_m| \cdot AY_{\ell} + (\Delta y_q) \bar{\mu}_m (AR_{i+1} - AR_i) + V_{\ell,q} \sigma t_{\ell,q,j} + t_j}$$

$$\phi_{m,q}^L = 2E_{m,\ell} - \phi_{m,q}^L$$

$$\phi_{m,\ell}^T = 2E_{m,\ell} - \phi_{m,\ell}^T$$

For $\eta_m < 0$, $\mu_m > 0$:

$$E_{m,\ell} = \frac{S_{\ell,q,j} + (\Delta y_q) \bar{\mu}_m (AR_{i+1} - AR_i) \phi_{m-1,\ell} + \mu_m SH_{\ell,q,j} + \eta_m SV_{\ell,q,j} + (\Delta y_q) |(AR_{i+1} + AR_i)| \phi_{m,q}^R + 2|\eta_m| \cdot AY_{\ell} \phi_{m,\ell}^B}{(\Delta y_q) |\bar{\mu}_m (AR_{i+1} - AR_i)| + 2|\eta_m| \cdot AY_{\ell} + (\Delta y_q) \bar{\mu}_m (AR_{i+1} - AR_i) + V_{\ell,q} \sigma t_{\ell,q,j} + t_j}$$

$$\phi_{m,q}^R = 2E_{m,\ell} - \phi_{m,q}^R$$

$$\phi_{m,\ell}^B = 2E_{m,\ell} - \phi_{m,\ell}^B$$

For $\eta_m > 0$, $\mu_m > 0$:

$$E_{m,\ell} = \frac{S_{\ell,q,j} + (\Delta y_q) \bar{\mu}_m (AR_{i+1} - AR_i) \phi_{m-1,\ell} + \mu_m SH_{\ell,q,j} + \eta_m SV_{\ell,q,j} + (\Delta y_q) |(AR_{i+1} + AR_i)| \phi_{m,q}^R + 2|\eta_m| \cdot AY_{\ell} \phi_{m,\ell}^T}{(\Delta y_q) |\bar{\mu}_m (AR_{i+1} - AR_i)| + 2|\eta_m| \cdot AY_{\ell} + (\Delta y_q) \bar{\mu}_m (AR_{i+1} - AR_i) + V_{\ell,q} \sigma t_{\ell,q,j} + t_j}$$

$$\phi_{m,q}^R = 2E_{m,\ell} - \phi_{m,q}^R$$

$$\phi_{m,\ell}^T = 2E_{m,\ell} - \phi_{m,\ell}^T$$

$$\phi_{m,\ell} = \begin{cases} E_{m,\ell} & \text{for } W_m = 0 \\ 2E_{m,\ell} - \phi_{m-1,\ell} & \text{for } W_m \neq 0 \end{cases}$$

$$\phi_{\ell,q,j}^\pi = \phi_{\ell,q,j}^{\pi-1} + W_m$$

$$J_{\ell,q,j}^{H,\pi} = J_{\ell,q,j}^{H,\pi-1} + W_m \mu_m E_{m,\ell}$$

$$J_{\ell,q,j}^{V,\pi} = J_{\ell,q,j}^{V,\pi-1} + W_m \eta_m E_{m,\ell}$$

Formula 99 is entered again to sweep the mesh in the upward direction, i.e., from bottom to top for $m = M/2 + 1, M$; $q = 1, Q$. Formula 144 is used to establish the bottom boundary fluxes before entering 99.

144

This formula establishes values for the bottom boundary fluxes (for all m such that $\eta_m > 0$).

Free: $\phi_{m,\ell}^T = \phi_{m,\ell}^B = 0.$

Reflective: $\phi_{m,\ell}^T = \phi_{m,\ell}^B = \phi_{DPm,\ell}^B.$

Periodic: $\phi_{m,\ell}^B = \phi_{m,\ell}^T.$

White: $\phi_{m,\ell}^B = \phi_{m,\ell}^T = \frac{\sum_m W_m \eta_m \phi_{m,\ell}^B}{\sum_m W_m \eta_m}.$

145

This formula forms the leakage terms and performs adjustments necessary to define proper boundary conditions. (The subscript p indicates either pass A(1) or pass B(2).)

Right leakage: $RL_p = AR_{xR} \sum_q \Delta y_q \sum_m W_m \mu_m \phi_{m,q}^R$

Left leakage: $LL_p = AR_{xL} \sum_q \Delta y_q \sum_m W_m \mu_m \phi_{m,q}^L$

$$\text{Top leakage: } TL_p = \sum_{\ell} AY_{\ell} \sum_m W_m \eta_m \phi_{m,\ell}^T$$

$$\text{Bottom leakage: } BL_p = \sum_{\ell} AY_{\ell} \sum_m W_m \eta_m \phi_{m,\ell}^B$$

$$\text{Horizontal leakage: } HL_p = RL_p - LL_p$$

$$\text{Vertical leakage: } VL_p = TL_p - BL_p$$

$$\text{Net leakage: } NL_p = HL_p + VL_p$$

For a periodic right boundary, for $\mu_m < 0$: $\phi_{m,q}^R = \phi_{m,q}^L$ for all q , and m such that $\mu_m < 0$. For a periodic top boundary, $\phi_{m,\ell}^T = \phi_{m,\ell}^B$ for all ℓ , and m such that $\eta_m < 0$.

134

This formula calculates the pass B components of the flux. This extra iteration is performed with a zero source and is necessary until the horizontal and vertical leakages are sufficiently small. Pass B is entered if either the right boundary is not free and $|HL_1| > \epsilon_J/\sqrt{L}$, or the top boundary is not free and $|VL_1| > \epsilon_J/\sqrt{Q}$. (Otherwise control is transferred to Formula 135.)

$$\left. \begin{aligned} \phi_{\ell,q}^A &= \phi_{\ell,q,j}^{\pi} \\ \phi_{\ell,q,j}^{\pi} &= 0 \\ S_{\ell,q,j} &= 0 \\ JH_{\ell,q,j}^A &= JH_{\ell,q,j}^H \\ Jv_{\ell,q,j}^A &= Jv_{\ell,q,j}^V \\ JH_{\ell,q,j}^H &= 0 \\ Jv_{\ell,q,j}^V &= 0 \\ SH_{\ell,q,j} &= 0 \\ SV_{\ell,q,j} &= 0 \end{aligned} \right\} \text{for all intervals, } \ell, q.$$

$$\phi_{m,q}^{A,R} = \phi_{m,q}^R \quad \text{for all } m, q.$$

$$\phi_{m,\ell}^{A,T} = \phi_{m,\ell}^T \quad \text{for all } m, \ell.$$

Control is now transferred to Formula 142 to establish top boundary fluxes, to Formula 99 to sweep the mesh from top to bottom for $m = 1, m/2$, to Formula 144 to establish bottom boundary fluxes, and to Formula 99 to sweep the mesh from the bottom to the top. Formula 145 is entered to compute the pass B leakages.

The scale factor, JR, is computed to scale the pass B fluxes before adding them to the pass A fluxes.

$$JR = \begin{cases} -HL_1/HL_2 & \text{if RBND} \neq 0 \\ -VL_1/VL_2 & \text{if TBND} \neq 0 \\ -NL_1/NL_2 & \text{if RBND} \neq 0, \text{ TBND} \neq 0 \end{cases}$$

$$\left. \begin{aligned} \phi_{\ell,q,j}^{\pi} &= \phi_{\ell,q,j}^{\pi,B} \cdot (JR) + \phi_{\ell,q,j}^{\pi,A} \\ J_{\ell,q,j}^H &= J_{\ell,q,j}^{H,B} \cdot (JR) + J_{\ell,q,j}^{H,A} \\ J_{\ell,q,j}^V &= J_{\ell,q,j}^{V,B} \cdot (JR) + J_{\ell,q,j}^{V,A} \end{aligned} \right\} \text{for all } \ell, q$$

$$\phi_{m,q}^R = \phi_{m,q}^{R,B} \cdot (JR) + \phi_{m,q}^{A,R} \quad \text{for all } m, q.$$

$$\phi_{m,\ell}^T = \phi_{m,\ell}^{R,B} \cdot (JR) + \phi_{m,\ell}^{B,R} \quad \text{for all } m, \ell$$

$$LL_1 = LL_1 + LL_2 \cdot (JR)$$

$$RL_1 = RL_1 + RL_2 \cdot (JR)$$

$$HL_1 = HL_1 + HL_2 \cdot (JR)$$

$$BL_1 = BL_1 + BL_2 \cdot (JR)$$

$$TL_1 = TL_1 + TL_2 \cdot (JR)$$

$$VL_1 = VL_1 + VL_2 \cdot (JR)$$

$$NL_1 = NL_1 + NL_2 \cdot (JR)$$

135

This formula computes the inner-iteration scale factor and performs the scaling.

$$EST = \sum_{\ell,q} V_{\ell,q} \sigma_{\ell,q}(j \rightarrow j) (\phi_{\ell,q}^{\pi} - \phi_{\ell,q}^{\pi-1}),$$

the residual self-scatter error, total.

$$ES_R = \sum_{\ell,q \in R} V_{\ell,q} \sigma_{\ell,q}(j \rightarrow j) (\phi_{\ell,q}^{\pi} - \phi_{\ell,q}^{\pi-1}),$$

the region-wise, residual self-scatter error.

$$ERA = \sum_{\ell, q} V_{\ell, q} |\phi_{\ell, q}^{\pi} - \phi_{\ell, q}^{\pi-1}| \left(|\sigma_{\ell, q}^{(0)}(j \rightarrow j)| + |\sigma_{\ell, q}^{(1)}(j \rightarrow j)| \right)$$

$$SF = TS_j / (TS_j - EST)$$

$$\phi_{\ell, q}^{\pi} \cdot SF \rightarrow \phi_{\ell, q}^{\pi} \quad \text{for all } \ell, q.$$

$$\phi_{m, j}^R \cdot SF \rightarrow \phi_{m, j}^R \quad \text{for all } m, j.$$

$$\phi_{m, \ell}^T \cdot SF \rightarrow \phi_{m, \ell}^T \quad \text{for all } m, \ell.$$

$LL_1, RL_1, HL_1, TL_1, BL_1, VL_1, NL_1$ are also multiplied by SF .

$$J_{\ell, q, j}^H \cdot SF \rightarrow J_{\ell, q, j}^H \quad \text{for all } \ell, q.$$

$$J_{\ell, q, j}^V \cdot SF \rightarrow J_{\ell, q, j}^V \quad \text{for all } \ell, q.$$

133

This formula checks for inner-loop convergence for group j . The loop count LC is advanced by 1: $LC + 1 \rightarrow LC$. Inner iteration is terminated for group j if either:

$$\left. \begin{array}{l} (1) \quad \pi > \pi_{\max} \\ (2) \quad IT_j + AT_j = 0 \end{array} \right\} \text{Return is made to Formula 141 of Subroutine } \emptyset\text{UTER.}$$

Inner-iteration convergence is accomplished for group j if all of these conditions are satisfied:

$$(3) \quad |HL_1| \leq \epsilon_J / \sqrt{L} \quad (\text{only if right boundary is not free}).$$

$$(4) \quad |VL_1| \leq \epsilon_J / \sqrt{Q} \quad (\text{only if top boundary is not free}).$$

$$(5) \quad |EST| \leq \epsilon_J$$

$$(6) \quad ERA \leq \epsilon_J \sqrt{LQ}$$

$$(7) \quad \text{Max for all } \ell, q \left\{ 1 - \phi_{\ell, q, j}^{\pi-1} / \phi_{\ell, q, j}^{\pi} \right\} \leq \epsilon_{II} \quad (\text{only if } \epsilon_{II} > 0).$$

If conditions 3-7 are satisfied, then return is made to Formula 141 of Subroutine $\emptyset\text{UTER}$; otherwise, control is transferred to Formula 132 to begin the next iteration for group j .

GRØUPTØT

This subroutine performs summaries over all groups at the end of an outer iteration.

$$\lambda_p \rightarrow \lambda_0$$

$$\overline{SC} = \sum_{j=1}^J SC_j \quad \overline{AB} = \sum_{j=1}^J AB_j \quad \overline{SS} = \sum_{j=1}^J SS_j$$

$$\overline{ST} = \sum_{j=1}^J ST_j \quad \overline{HL} = \sum_{j=1}^J HL_j \quad \overline{RL} = \sum_{j=1}^J RL_j$$

$$\overline{LV} = \sum_{j=1}^J LV_j \quad \overline{LT} = \sum_{j=1}^J LT_j \quad \overline{LN} = \sum_{j=1}^J LN_j$$

$$\overline{NB} = \frac{\overline{QG} + \overline{FG} + \overline{SC} - \overline{LN} - \overline{AB} - \overline{ST}}{\overline{QG} + \overline{FG} + \overline{SC}}$$

An error is noted and the next case is started if $\overline{QG} + \overline{FG} + \overline{SC} = 0$. If ϵ_{NB} is specified (columns 19 through 30 on Type 180000 card are nonblank) then, each time $\overline{NB} > \epsilon_{NB}$, a line of asterisks (*) is printed. After the last outer iteration, the following elements are printed with the column headings as indicated. Each element is printed by group followed finally by a summary over all groups.

<u>Column Heading</u>	<u>Elements Printed</u>
G	Group number, j.
E0	QG _j
E1	FG _j
E2	SC _j
E3	SS _j
E4	HL _j
E5	AB _j
E6	ST _j
E7	RL _j
LEAK. VERT.	LV _j
TØP LEAK.	LT _j
NET LEAK.	LN _j
NEUT. BAL.	NB _j

CØNVERGE

This subroutine analyzes problem convergence for all problem types in two sections. Section I makes preliminary convergence tests independent of problem type and defines some convergence-test factors for Section II. Section II makes convergence tests only for criticality searches.

Section I

$$V = \varepsilon \cdot G \cdot (\overline{QG} + \overline{FG}).$$

Convergence Criteria

Step 1. For all calculations: no upscattering--return to final print, upscattering--calculate scale-factor

$$E = \frac{\overline{QG} + \overline{FG}}{\overline{QG} + \overline{FG} + E_{up}}. \quad (1.1)$$

Scale $\phi_{\ell,q}^{\rho-1}$, $\phi_{m,q}^R$, $\phi_{m,\ell}^T$, and in an anisotropic case, $J_{\ell,q}^H$, $J_{\ell,q}^V$ by E . Test to see if $E_{up} \leq V$. If yes, test to see if $|E_{up}| \leq V \cdot \sqrt{LQ}$. If yes, proceed to Step 2 (if $FG \neq 0$) to check fission convergence. If no, proceed to Step 3.

Step 2. For calculations with fission: compute

$$FC = \sum_{\ell,q} V_{\ell,q} \sum_j |FD_{\ell,q,j}^{\rho} - FD_{\ell,q,j}^{\rho-1}|. \quad (2.1)$$

Test to see if $|1 - \lambda_{\rho}| \leq \varepsilon$ and $FC \leq V \cdot \sqrt{LQ}$. If both are satisfied, the convergence trigger, CT , is set to 1, and flow proceeds to Step 3. If either or both are not satisfied, flow proceeds to Step 3.

Step 3. $0 \rightarrow FD_{\ell,q,j}^{\rho-1}$ for all ℓ,q,j , $0 \rightarrow E_{up}$, $|E_{up}|$. If $CT = 1$, proceed to final print. If $CT \neq 1$, exit to resume iteration process in a source calculation, or change critical parameters in a criticality search in Section II, or determine new value of $k_{eff}^{(\rho)} = k_{eff}^{(\rho-1)} \cdot \lambda_{\rho}$, $EV_1 = \lambda_1$, and resume iteration process.

Section II

Step 1. Several outer iterations are attempted until $|\lambda_{\rho} - \lambda_{\rho-1}| \leq \varepsilon$.

$$\text{Set } \lambda_{\rho} \rightarrow \lambda_p, EV_0 \rightarrow EV_p \text{ and } \begin{cases} EV_0 + EVM \rightarrow EV_0 & \text{if } \lambda_{\rho} \geq 1 \\ EV_0 - EVM \rightarrow EV_0 & \text{if } \lambda_{\rho} < 1 \end{cases} \quad (1.1)$$

Two pairs, (λ_p, EV_p) , (λ_{ρ}, EV_0) , are now available for future interpolation.

Step 2. Only two previous generations of λ and EV are available. Outer iterations continue until $|\lambda_p - \lambda_{p-1}| \leq \epsilon$. Compute

$$\frac{dEV}{d\lambda} = \frac{EV_p - EV_0}{\lambda_p - \lambda_0} \quad (2.1)$$

Test to see if $|1 - \lambda_p| \leq \lambda_L$. If yes, set flag CV = 1, $0 \rightarrow \lambda_p$, $0 \rightarrow \lambda_{p-1}$ and proceed to Step 3; henceforth only linear interpolation will be used via Step 5 after the next outer iteration. If no, test to see if $|1 - \lambda_p| \leq \lambda_{up}$. If yes, set $\lambda_p \rightarrow \lambda_{p-1}$, $\lambda_0 \rightarrow \lambda_p$, $EV_p \rightarrow EV_{p-1}$, $EV_0 \rightarrow EV_p$ and proceed to Step 3. If no, set (sign E, λ_{up}) $\rightarrow$ E, $\lambda_p \rightarrow \lambda_{p-1}$, $\lambda_0 \rightarrow \lambda_p$, $EV_p \rightarrow EV_{p-1}$, $EV_0 \rightarrow EV_p$, and proceed to Step 3.

Step 3. Compute

$$EV_0 + (P\phi D) \cdot \left(\frac{dEV}{d\lambda} \right) \cdot E \rightarrow EV_0. \quad (3.1)$$

Test to see if λ_p and λ_{p-1} both lie on the same side of 1.0, i.e., if $(\lambda_{p-1} - 1)/(\lambda_p - 1) \geq 0$. If yes, return to iterate further. If no,

$$EV_0 = \begin{cases} \text{as computed in (3.1) if } EV_0 \text{ lies on the interval } (EV_p, EV_{p-1}). \\ (EV_p + EV_{p-1})/2 \text{ if } EV_0 \text{ lies outside of } (EV_p, EV_{p-1}). \end{cases}$$

Return to iterate further.

Step 4. Iteration continues until $|\lambda_p - \lambda_{p-1}| \leq \epsilon_a$. (It is assumed here that λ_{p-1} , $\lambda_p \neq 0$, CV = 0.) Compute the coefficients A, B and C of the quadratic equation

$$A + B(EV) + C(EV)^2 = (\lambda - 1) \quad (4.1)$$

using the three pairs, $(\lambda_{p-1}, EV_{p-1})$, (λ_p, EV_p) , (λ_0, EV_0) , in the fitting process. Compute

$$R = B^2 - 4AC, \quad (4.2)$$

the discriminant of Eq. 4.1 when $\lambda = 1$, i.e.,

$$A + B(EV) + C(EV)^2 = 0. \quad (4.3)$$

If complex roots in Eq. 4.3, return to Step 2, starting with Eq. 2.1, and proceed through Steps 2 and 3, using the linear interpolation found therein. If real roots in Eq. 4.3, test to see if $|1 - \lambda_p| \leq \lambda_L$. If it is, set CV = 1, $0 \rightarrow \lambda_p$, $0 \rightarrow \lambda_{p-1}$ and proceed through Step 3; henceforth only linear interpolation will be used via Step 5 after the next outer iteration. If it is not, compute

$$\frac{dEV}{d\lambda} = \frac{1}{B + 2C(EV_0)} \quad (4.4)$$

and set $\lambda_p \rightarrow \lambda_{p-1}$, $\lambda_0 \rightarrow \lambda_p$, $EV_p \rightarrow EV_{p-1}$, $EV_0 \rightarrow EV_p$, and compute

$$EV_0 = \frac{-B \pm \sqrt{B^2 - 4AC}}{2C}.$$

(The value of EV_0 closer to EV_p is used.) Step 3 is now entered, but Eq. 3.1 is bypassed.

Step 5. Enter this step when $CV = 1$. This step provides linear interpolation with a fixed slope during the final stages of the problem. If the convergence trigger has been set ($CT \neq 0$), return to the final print. If $CT = 0$, test to see if $|1 - \lambda_0| \leq \epsilon$. If yes, return to final print. If no, proceed through Step 3 and return to the next outer iteration.

BALANCES

This subroutine is entered only after the final iteration and is called by Formula EXECTIV2. It prints the following quantities for each group (and a final summary over all groups) for each region R:

- (1) Fixed Source: $\sum_{\ell, q \in R} V_{\ell, q} Q_{\ell, q, j}$
- (2) Fission Neutrons: $\sum_{\ell, q \in R} FI_{\ell, q, j}$
- (3) Scatter Source: $\sum_{\ell, q \in R} \sum_k \sigma_{\ell, q}^{(0)}(k \rightarrow j) \phi_{\ell, q, k}$
- (4) Production: (1) + (2) + (3)
- (5) Scatter Removal: $\sum_{\ell, q \in R} \sum_k \sigma_{\ell, q}^{(0)}(j \rightarrow k) \phi_{\ell, q, j}$
- (6) Absorption Neutrons: $\sum_{\ell, q \in R} V_{\ell, q} (\sigma_{c\ell, 1, j} + \sigma_{f\ell, q, j}) \phi_{\ell, q, j}$
- (7) Removal: (5) + (6)
- (8) Leakage: (4) - (7)
- (9) Flux Integral: $\sum_{\ell, q \in R} V_{\ell, q} \phi_{\ell, q, j}$
- (10) Total Density: (9) $\div v_j$
- (11) Fission Integral: $\sum_{\ell, q \in R} V_{\ell, q} \nu \sigma_{f, \ell, q, j} \phi_{\ell, q, j}$

After all regions have been edited, a reactor summary (total for all groups) is printed.

FINPRINT

Enter this subroutine at completion of the final iteration to perform necessary editing for printing and/or punching of the following:

1. Final spatial configuration in a critical-dimension search.
2. Final composition cross sections and specifications for a concentration-search calculation.
3. Fission neutrons by interval (and by group if the fission fraction spectra are incident group dependent).
4. Final scalar fluxes by group and by mesh interval.
5. Final monitor line (see MØNPRINT).
6. A table giving the inner iteration excess counts accumulated for the final outer iteration and the neutron balance excess counter OC_j as generated in Formula ØUTER (141).
7. Total iteration time and total problem time in milliseconds and in minutes and seconds.
8. The number of tape reads, tape writes, and tape rewinds, and the total time (in milliseconds) spent in reading, writing, and rewinding individually.
9. A final flux deck in Type 6XXXXX card format or a final flux tape in Type 6XXXXX format, depending on the contents of columns 13 and 14 of the Type 170000 card.

GØØNDUMP

This subroutine writes a restart dump tape immediately after an outer iteration has terminated if at least RDS minutes (contained in columns 43-45 of Type 180000 card) have elapsed since either the beginning of the problem or the last restart dump. No restart dump tape is formed if either (1) the Type 180000 card is absent from the input deck, or (2) the contents of columns 43-45 are blank or zero on the Type 180000 card.

GØØNREAD

This subroutine reads a restart dump tape previously formed by GØØNDUMP. See writeup of Type 105000 for discussion of how to use this facility.

VI. MATHEMATICAL MODEL

Here we briefly summarize the mathematical basis of the SNARG-2D program.

If $\vec{x}$ is a position vector, $\vec{\Omega}$ a point on the unit direction sphere associated with the point $\vec{x}$, and v_j is the neutron speed in energy group j , then the equation for the angular flux in group j , $\phi_j(t, \vec{x}, \vec{\Omega})$ is

$$\frac{1}{v_j} \frac{\partial \bar{\phi}_j}{\partial t} + \vec{\Omega} \cdot \vec{\nabla} \bar{\phi}_j + \sigma_j(\vec{x}) \bar{\phi}_j = \bar{S}_j(t, \vec{x}, \vec{\Omega}), \quad (1)$$

where $\sigma_j(\vec{x})$ is the total cross section and $\bar{S}_j$ the source for group j .

Before proceeding with the finite difference form of Eq. 1, we consider the time dependence. It is assumed that the angular flux and the source are separable in the same way into the product of a time-independent function of position and direction and a time-dependent function $e^{\alpha t}$. Equation 1 becomes

$$\vec{\Omega} \cdot \vec{\nabla} \phi_j + \left(\sigma_j(\vec{x}) + \frac{\alpha}{v_j} \right) \phi_j = S_j(\vec{x}, \vec{\Omega}),$$

where

$$\phi_j = \phi_j(\vec{x}, \vec{\Omega}). \quad (2)$$

For the time-independent case, $\alpha = 0$.

The angular variable Ω in Eq. 1 is discretized into $\{\vec{\Omega}_m\}$, and integration is performed by using weights W_m and direction cosines μ_m and η_m which satisfy the relations

$$\sum_{m=1}^M W_m = 1, \quad (3)$$

$$\sum_{m=1}^M W_m \mu_m \eta_m = 0, \quad (4)$$

and

$$\sum_{m=1}^M W_m \mu_m^2 = \sum_{m=1}^M W_m \eta_m^2 = \frac{1}{3}, \quad (5)$$

where M is the total number of directions on the octant of the unit sphere.

For a more detailed discussion of the construction of these " S_n constants" see Chapter VI, Section A, of ANL-7221.⁶

A. Difference Equations

Standard techniques may be used to write a finite-difference form for Eq. 2. However, in doing so, we must maintain neutron conservation.^{7,8}

We follow the approach taken in Refs. 2 and 3. A neutron-balance equation is written for an arbitrary cell enclosing the point $(\vec{x}, \vec{\Omega}_m)$, which, as the cell size shrinks, becomes Eq. 2 and is equivalent to the difference equation in Ref. 8.

Let i be the index specifying the mesh structure along the x axis, let j be the index specifying the mesh structure along the y axis, and let m be the direction index and k the group index. Here x and y stand for the space coordinates in x - y , r - z , and r - θ geometry. The cell face area perpendicular to x at i is denoted by A_i ; the area element related to the y dimension is denoted by B_j .

The balance equation for any of these geometries for group j is then

$$\begin{aligned} & \frac{\mu_m}{V_{i+(1/2),j+(1/2)}} (A_{i+1} \phi_{m,i+1,j+(1/2),k} - A_i \phi_{m,i,j+(1/2),k}) \\ & + \frac{\eta_m}{V_{i+(1/2),j+(1/2)}} (B_{j+1} \phi_{m,i+(1/2),j+1,k} - B_j \phi_{m,i+(1/2),j,k}) \\ & + \frac{1}{V_{i+(1/2),j+(1/2)}} \frac{(A_{i+1} - A_i)}{W_m} \beta_{m+(1/2)} \phi_{m+(1/2),i+(1/2),j+(1/2),k} - \beta_{m-(1/2)} \phi_{m-(1/2),i+(1/2),j+(1/2),k} \\ & + \sigma_{i+(1/2),j+(1/2),k} \phi_{m,i+(1/2),j+(1/2),k} = S_{m,i+(1/2),j+(1/2),k} \end{aligned}$$

with

$$\begin{aligned} \phi_{m,i+(1/2),j+(1/2),k} &= \frac{1}{2} (\phi_{m,i+1,j+(1/2),k} + \phi_{m,i,j+(1/2),k}) \\ &= \frac{1}{2} (\phi_{m,i+(1/2),j+1,k} + \phi_{m,i+(1/2),j,k}), \end{aligned} \quad (7)$$

where the index $i + 1/2(j + 1/2)$ denotes the midpoint of the interval between $x_i(y_j)$ and $x_{i+1}(y_{j+1})$.

The first two terms in Eq. 6 represent flow across the four cell faces. The terms with coefficients β represent ray-to-ray transfer through streaming. The last term on the left-hand side represents the neutrons released by sources. The right-hand side represents the neutrons removed by collisions. $V_{i+(1/2),j+(1/2)}$ is the geometric volume of the cell.

The third term, the angular redistribution term, occurs only in curved geometries (rz or $r\theta$) and accounts for the directional derivatives that occur in the transport equation.^{7,8}

The β 's satisfy the relation

$$\beta_{m+(1/2)} - \beta_{m-(1/2)} = W_m \mu_m. \quad (8)$$

For neutron conservation, the first and last in the sequence of β 's must be zero.

Defining the curvature factor, $\bar{\mu}_m$, as

$$\bar{\mu}_m = \frac{2\beta_{m+(1/2)}}{W_m} + \mu_m, \quad (9)$$

and using Eq. 8 to eliminate the β 's, we obtain the recurrence relation for $\bar{\mu}_m$ used in Formula 104 of Subroutine BEGINSN of the SNARG-2D formula set.

Equation 6 put into a recursive form is the basic equation for the determination of the angular flux when the boundary conditions and starting values are given. Consider

$$\phi_{m,\ell,q,k} = \phi_{m,i+(1/2),j+(1/2),k}$$

and

$$S_{m,\ell,q,k} = S_{m,i+(1/2),j+(1/2),k},$$

and requiring

$$\phi_{m,\ell,q,k} = \frac{1}{2} \left(\phi_{m+(1/2),i+(1/2),j+(1/2),k} + \phi_{m-(1/2),i+(1/2),j+(1/2),k} \right), \quad (10)$$

we obtain, by the use of Eqs. 7-10, the four equations for $E_{m,\ell}$ in Formula 132 of Subroutine INNER (where the group subscript is j instead of k). These are the "angular-flux equations."

With the correspondences,

$$V_{\ell,q} S_{m,\ell,q,k} = \mu_m S H_{\ell,q,k} + \eta_m S V_{\ell,q,k} + S_{\ell,q,k},$$

$$\phi_{m,\ell,q,k} = \left\{ \begin{array}{l} E_{m,\ell,q,k} \\ 2E_{m,\ell,q,k} - \phi_{m-1,\ell,q,k} \end{array} \right\},$$

and

$$V_{\ell,q} \sigma_{\ell,q,k} + t_j = \sigma t_{\ell,q,k}.$$

Equations 7 and 9 and the four equations for $E_{m,\ell}$ as defined in INNER are the basic equations used in the SNARG-2D angular-flux pass.

B. Description of Source Components

In general, the source term in Eq. 6 may contain a fission-source term, a scattering-source term, and an external source. S may be isotropic or linearly anisotropic.

The scalar flux is defined as

$$\phi_{\ell,q,k} = \sum_m W_m \phi_{m,\ell,q,k}, \quad (11)$$

and the current components are

$$J_{\ell,q,k}^H = \sum_m W_m \mu_m \phi_{m,\ell,q,k}, \quad (12)$$

the horizontal component, and

$$J_{\ell,q,k}^V = \sum_m W_m \eta_m \phi_{m,\ell,q,k}, \quad (13)$$

the vertical component.

Denoting the isotropic component of $V_{\ell,q}S_{m,\ell,q,k}$ by $SI_{\ell,q,k}$ plus a self-scatter term, and the anisotropic source components by $SH_{\ell,q,k}$ and $SV_{\ell,q,k}$, we have

$$SI_{\ell,q,k} = \sum_{\substack{j \\ j \neq k}} \sigma_{\ell,q}^{(0)}(j \rightarrow k) \phi_{\ell,q,j} + FI_{\ell,q,k} + V_{\ell,q}Q_{\ell,q,k}, \quad (14)$$

$$SH_{\ell,q,k} = \sum_{\substack{j \\ j \neq k}} 3\sigma_{\ell,q}^{(1)}(j \rightarrow k) J_{\ell,q,j}^H + 3\sigma_{\ell,q}^{(1)}(k \rightarrow k) J_{\ell,q,k}^H, \quad (15)$$

and

$$SV_{\ell,q,k} = \sum_{\substack{j \\ j \neq k}} 3\sigma_{\ell,q}^{(1)}(j \rightarrow k) J_{\ell,q,j}^V + 3\sigma_{\ell,q}^{(1)}(k \rightarrow k) J_{\ell,q,k}^V, \quad (16)$$

where the upper index on σ corresponds to the Legendre component of the transfer cross section from $j \rightarrow k$, and $FI_{\ell,q,k}$ is the fission source from Formula 122 of Subroutine FISCAL.

C. Boundary Conditions

Several types of boundary conditions may be applied to the solution of the angular flux equations in Subroutine INNER. Formulas 99, 142, 144, and 145 of Subroutine INNER contain these conditions. The free-boundary condition sets the angular flux for all incoming directions at the given boundary equal to zero. The periodic-boundary condition requires equal angular flux at both boundaries for all directions. The reflective-boundary condition equates the angular flux at the given boundary, for all μ_m or η_m equal in absolute value.

The reflective-boundary condition is used primarily for cell problems. However, for some cells in cylindrical geometry, consisting of, say, a central fuel region surrounded by a nonfuel region, the reflective-boundary condition gives rise to the Newmarch effect;¹⁰ i.e., the flux tends to build up excessively in the vicinity of the outer boundary. A more proper boundary condition for such cases¹¹ would be a linear combination of the reflective condition, which gives complete correlation between ingoing and outgoing angular fluxes, and a condition in which the incoming flux is uncorrelated in angle (isotropic return) with the outgoing angular flux. This latter condition is called the "white-boundary" condition.¹¹ Formulas 99, 142, and 144 contain provisions for a white-boundary condition at the right, top, and bottom boundaries.

D. Convergence Considerations

To make the computation of the angular flux stable with respect to round-off errors and speed convergence, the directional-evaluation rule and a scaled iteration procedure^{2,3,7} are used.

The directional-evaluation rule requires the computation to follow neutron travel in direction and energy.

The computation is started with the highest energy group with a sweep from top to bottom for each q and for $m = 1$ to $m/2$. For each q , the calculation proceeds along the inward μ_m directions until the inner boundary is met; then the outward fluxes are obtained (cf. Formula 99 of INNER). Then an upward sweep is made for each q and for $m = m/2 + 1$ to m sweeping backward and forward as above. The procedure is continued through successive energy groups.

The scaled iteration procedure is discussed below within the context of the inner iterations.

When the source and boundary values in the angular-flux equations are specified, either from initial guessed values or from a previous estimate of the fluxes, the angular flux is determined for all directions and points in an energy group. This process is termed an inner iteration. Let π stand for the inner-iteration count, and F^π the value of the fission source for this iteration. From Eq. 6, upon multiplying by W_m and summing with respect to m , we obtain the balance equation.

$$A_{i+1}J_{i+1,q,k}^{H,\pi} - A_iJ_{i,q,k}^{H,\pi} + B_j(J_{\ell,j+1,k}^{V,\pi} - J_{\ell,j,k}^{V,\pi}) + \sigma_{t\ell,q,k}\phi_{\ell,q,k}^\pi \\ = SI_{\ell,q,k} + \sigma_{\ell,q}^{(0)}(k \rightarrow k)\phi_{\ell,q,k}^{\pi-1} \quad (17)$$

Equation 17 would be exact if a previous value of $\phi_{\ell,q,k}$ were not used in the self-scattering term. Multiplying Eq. 17 by a factor f and rearranging terms, we obtain

$$f \left[A_{i+1}J_{i+1,q,k}^{H,\pi} - A_iJ_{i,q,k}^{H,\pi} + B_j(J_{\ell,j+1,k}^{V,\pi} - J_{\ell,j,k}^{V,\pi}) + \sigma_{t\ell,q,k}\phi_{\ell,q,k}^\pi \right] = SI_{\ell,q,k} \\ + f\sigma_{\ell,q}^{(0)}(k \rightarrow k)\phi_{\ell,q,k}^\pi + f\sigma_{\ell,q}^{(0)}(k \rightarrow k)(\phi_{\ell,q,k}^{\pi-1} - \phi_{\ell,q,k}^\pi) + (f-1)SI_{\ell,q,k} \quad (18)$$

Equation 18 would be an exact balance statement for the flux level $f\phi$ due to fixed source $SI_{\ell,q,k}$, if we could choose f so that the remaining three terms on the right side of Eq. 18 sum to zero.

The single factor f used to satisfy the above in the average over all the space points is the inner-iteration scaling factor computed as

$$f = \frac{\sum_{\ell,q} SI_{\ell,q,k}}{\sum_{\ell,q} \left[SI_{\ell,q,k} - \sigma_{\ell,q}^{(0)}(k \rightarrow k)(\phi_{\ell,q,k}^\pi - \phi_{\ell,q,k}^{\pi-1}) \right]}$$

in Formula 135 of INNER.

All relevant quantities are multiplied by f , and the source is updated before proceeding to the next inner iteration in that group. These iterations are continued, in general, until

$$\sum_{\ell, q} \sigma_{\ell, q}^{(0)}(j \rightarrow j) |\phi_{\ell, q, j}^{\pi} - \phi_{\ell, q, j}^{\pi-}| \leq \epsilon_J$$

and/or the other tests listed in Formula 133 are met. The term ϵ_J is a convergence criterion computed from a given input-convergence criterion in Formula 123 of Subroutine FISCAL.

If the boundary conditions are of the implicit type (e.g., like the reflective condition), the requirements that the difference in the self-scattering source for two successive inner iterations satisfies the convergence criterion, and that the angular flux satisfies the prescribed boundary conditions, may cause the number of inner iterations to be large. A method used to accelerate the rate of convergence is to perform two angular-flux passes. The angular flux from the first pass is used as the source for the second pass, all other source components being set to zero. Then a linear combination of the angular fluxes from the two passes (Formula 134 of INNER) is used as the angular flux. The convergence of the inner iterations is tested, and the calculation proceeds.

The inner iterations as described are performed for all the energy groups. Inner-iteration convergence for all the groups defines an outer iteration.

For an inhomogeneous problem ($Q \neq 0$) with neither fissions nor upscattering, one outer iteration completes the problem. For an inhomogeneous problem with fission and/or upscattering, the total source is recomputed after each outer iteration ρ , and a new cycle of calculations is started. Convergence is achieved when the tests described in Subroutine CONVERGE are satisfied.

If no external source is present (a homogeneous problem), we may compute the k_{eff} that corresponds to the system, or we may want to determine a parameter in the system that corresponds to a given k_{eff} . The latter type of calculation is a criticality search, usually $k_{\text{eff}} = 1.0$, and the term "eigenvalue" ($= \text{EV}$) is used for the critical parameter.

In the k_{eff} calculation, after each outer iteration, we compute the total source and the ratio (λ_ρ) of the total fission source of this ρ th outer iteration to the total fission source (not normalized) of the $(\rho - 1)$ th iteration, and divide the fission source by λ_ρ ; then we continue a new cycle of iterations. The product of the successive λ_ρ 's converges to k_{eff} ; the convergence criteria are in Subroutine CONVERGE.

In the criticality searches, we obtain a sequence of converged outer iterations, and each of these corresponds to a pair (EV, λ) . The process is as follows: Given an initial specified EV, we determine the corresponding converged λ . With this λ and previous pairs (EV, λ) , we construct a new value for EV in Subroutine CONVERGE. A new cycle of calculations is started with the changed EV resulting in a corresponding λ . This is continued until the desired $\lambda(k_{\text{eff}})$ is obtained, and thus the final EV is obtained.

VII. LIMITATIONS

Use of density-factor data (Type 16XXXX cards) and fission-spectrum matrices has not been implemented in the SNARG-2D program although the information is processed.

A negative-flux fix-up procedure is not included.

Upscattering cross sections are not handled correctly.

APPENDIX A--SNARG-2D S_n Constants

For the completely symmetric set of weights and directions, each member of the triplet (μ, η, ξ) corresponding to the direction $\vec{\Omega}$ is chosen from the fundamental set of directions and associated weights, as given in Appendix A of ANL-7221.⁶

In xy geometry and cylindrical $r\theta$ geometry, η and μ are as shown below for $n = 2, 4, 6, 8$, and 12. The same values are used for rz geometry with the understanding that ξ is the variable instead of η .

1. S_2 Weights and Direction Cosines

Weights	Direction Cosines	
	η	μ
0.00000000	-0.57735027	-0.81649658
0.25000000	-0.57735027	-0.57735027
0.25000000	-0.57735027	0.57735027
0.00000000	0.57735027	-0.81649658
0.25000000	0.57735027	-0.57735027
0.25000000	0.57735027	0.57735027

2. S_4 Weights and Direction Cosines

Weights	Direction Cosines	
	η	μ
0.00000000	-0.33333333	-0.94280903
0.08333333	-0.33333333	-0.88191710
0.08333333	-0.33333333	-0.33333333
0.08333333	-0.33333333	0.33333333
0.08333333	-0.33333333	0.88191710
0.00000000	-0.88191710	-0.47140451
0.08333333	-0.88191710	-0.33333333
0.08333333	-0.88191710	0.33333333
0.00000000	0.33333333	-0.94280903
0.08333333	0.33333333	-0.88191710
0.08333333	0.33333333	-0.33333333
0.08333333	0.33333333	0.33333333
0.08333333	0.33333333	0.88191710
0.00000000	0.88191710	-0.47140451
0.08333333	0.88191710	-0.33333333
0.08333333	0.88191710	0.33333333

3. S_6 Weights and Direction Cosines

Weights	Direction Cosines	
	η	μ
0.00000000	-0.25819889	-0.96609180
0.04021531	-0.25819889	-0.93094934
0.04311802	-0.25819889	-0.68313005

3. S₆ Weights and Direction Cosines (Contd.)

Weights	Direction Cosines	
	η	μ
0.04021531	-0.25819889	-0.25819889
0.04021531	-0.25819889	0.25819889
0.04311802	-0.25819889	0.68313005
0.04021531	-0.25819889	0.93094934
0.00000000	-0.68313005	-0.73029674
0.04311802	-0.68313005	-0.68313005
0.04311802	-0.68313005	-0.25819889
0.04311802	-0.68313005	0.25819889
0.04311802	-0.68313005	0.68313005
0.00000000	-0.93094934	-0.36514839
0.04021532	-0.93094934	-0.25819889
0.04021532	-0.93094934	0.25819889
0.00000000	0.25819889	-0.96609180
0.04021531	0.25819889	-0.93094934
0.04311802	0.25819889	-0.68313005
0.04021531	0.25819889	-0.25819889
0.04021531	0.25819889	0.25819889
0.04311802	0.25819889	0.68313005
0.04021531	0.25819889	0.93094934
0.00000000	0.68313005	-0.73029674
0.04311802	0.68313005	-0.68313005
0.04311802	0.68313005	-0.25819889
0.04311802	0.68313005	0.25819889
0.04311802	0.68313005	0.68313005
0.00000000	0.93094934	-0.36514839
0.04021532	0.93094934	-0.25819889
0.04021532	0.93094934	0.25819889

4. S₈ Weights and Direction Cosines

Weights	Direction Cosines	
	η	μ
0.00000000	-0.21821789	-0.97590007
0.02665019	-0.21821789	-0.95118973
0.02529318	-0.21821789	-0.78679579
0.02529318	-0.21821789	-0.57735027
0.02665019	-0.21821789	-0.21821789
0.02665019	-0.21821789	0.21821789
0.02529318	-0.21821789	0.57735027
0.02529318	-0.21821789	0.78679579
0.02665019	-0.21821789	0.95118973
0.00000000	-0.57735027	-0.81649658
0.02529318	-0.57735027	-0.78679579
0.01829035	-0.57735027	-0.57735027
0.02529318	-0.57735027	-0.21821789
0.02529318	-0.57735027	0.21821789
0.01829035	-0.57735027	0.57735027

4. S₈ Weights and Direction Cosines (Contd.)

Weights	Direction Cosines	
	η	μ
0.02529318	-0.57735027	0.78679579
0.00000000	-0.78679579	-0.61721340
0.02529318	-0.78679579	-0.57735027
0.02529318	-0.78679579	-0.21821789
0.02520318	-0.78679579	0.21821789
0.02529318	-0.78679579	0.57735027
0.00000000	-0.95118973	-0.30860670
0.02665019	-0.95118973	-0.21821789
0.02665019	-0.95118973	0.21821789
0.00000000	0.21821789	-0.97590007
0.02665019	0.21821789	-0.95118973
0.02529318	0.21821789	-0.78679579
0.02529318	0.21821789	-0.57735027
0.02665019	0.21821789	-0.21821789
0.02665019	0.21821789	0.21821789
0.02529318	0.21821789	0.57735027
0.02529318	0.21821789	0.78679579
0.02665019	0.21821789	0.95118973
0.00000000	0.57735027	-0.81649658
0.02529318	0.57735027	-0.78679579
0.01829035	0.57735027	-0.57735027
0.02529318	0.57735027	-0.21821789
0.02529318	0.57735027	0.21821789
0.01829035	0.57735027	0.57735027
0.02529318	0.57735027	0.78679579
0.00000000	0.78679579	-0.61721340
0.02529318	0.78679579	-0.57735027
0.02529318	0.78679579	-0.21821789
0.02529318	0.78679579	0.21821789
0.02529318	0.78679579	0.57735027
0.00000000	0.95118973	-0.30860670
0.02665019	0.95118973	-0.21821789
0.02665019	0.95118973	0.21821789

5. S₁₂ Weights and Direction Cosines

Weights	Direction Cosines	
	η	μ
0.00000000	-0.17407766	-0.98473193
0.01594215	-0.17407766	-0.96922337
0.01422496	-0.17407766	-0.87038828
0.01134400	-0.17407766	-0.75878691
0.01134400	-0.17407766	-0.62764591
0.01422496	-0.17407766	-0.46056619
0.01594214	-0.17407766	-0.17407766
0.01594214	-0.17407766	0.17407766
0.01422496	-0.17407766	0.46056619

5. S_{12} Weights and Direction Cosines (Contd.)

Weights	Direction Cosines	
	η	μ
0.01134400	-0.17407766	0.62764591
0.01134400	-0.17407766	0.75878691
0.01422496	-0.17407766	0.87038828
0.01594215	-0.17407766	0.96922337
0.00000000	-0.46056619	-0.88762536
0.01422496	-0.46056619	-0.87038828
0.00868436	-0.46056619	-0.75878691
0.00756891	-0.46056619	-0.62764591
0.00868436	-0.46056619	-0.46056619
0.01422496	-0.46056619	-0.17407766
0.01422496	-0.46056619	0.17407766
0.00868436	-0.46056619	0.46056619
0.00756891	-0.46056619	0.62764591
0.00868436	-0.46056619	0.75878691
0.01422496	-0.46056619	0.87038828
0.00000000	-0.62764591	-0.77849895
0.01134400	-0.62764591	-0.75878691
0.00756891	-0.62764591	-0.62764591
0.00756891	-0.62764591	-0.46056619
0.01134400	-0.62764591	-0.17407766
0.01134400	-0.62764591	0.17407766
0.00756891	-0.62764591	0.46056619
0.00756891	-0.62764591	0.62764591
0.01134400	-0.62764591	0.75878691
0.00000000	-0.75878691	-0.65133895
0.01134400	-0.75878691	-0.62764591
0.00868436	-0.75878691	-0.46056619
0.01134400	-0.75878691	-0.17407766
0.01134400	-0.75878691	0.17407766
0.00868436	-0.75878691	0.46056619
0.01134400	-0.75878691	0.62764591
0.00000000	-0.87038828	-0.49236596
0.01422496	-0.87038828	-0.46056619
0.01422496	-0.87038828	-0.17407766
0.01422496	-0.87038828	0.17407766
0.01422496	-0.87038828	0.46056619
0.00000000	-0.96922337	-0.24618298
0.01594214	-0.96922337	-0.17407766
0.01594214	-0.96922337	0.17407766
0.00000000	0.17407766	-0.98473193
0.01594215	0.17407766	-0.96922337
0.01422496	0.17407766	-0.87038828
0.01134400	0.17407766	-0.75878691
0.01134400	0.17407766	-0.62764591
0.01422496	0.17407766	-0.46056619
0.01594214	0.17407766	-0.17407766
0.01594214	0.17407766	0.17407766
0.01422496	0.17407766	0.46056619

5. S_{12} Weights and Direction Cosines (Contd.)

Weights	Direction Cosines	
	η	μ
0.01134400	0.17407766	0.62764591
0.01134400	0.17407766	0.75878691
0.01422496	0.17407766	0.87038828
0.01594215	0.17407766	0.96922337
0.00000000	0.46056619	-0.88762536
0.01422496	0.46056619	-0.87038828
0.00868436	0.46056619	-0.75878691
0.00756891	0.46056619	-0.62764591
0.00868436	0.46056619	-0.46056619
0.01422496	0.46056619	-0.17407766
0.01422496	0.46056619	0.17407766
0.00868436	0.46056619	0.46056619
0.00756891	0.46056619	0.62764591
0.00868436	0.46056619	0.75878691
0.01422496	0.46056619	0.87038828
0.00000000	0.62764591	-0.77849895
0.01134400	0.62764591	-0.75878691
0.00756891	0.62764591	-0.62764591
0.00756891	0.62764591	-0.46056619
0.01134400	0.62764591	-0.17407766
0.01134400	0.62764591	0.17407766
0.00756891	0.62764591	0.46056619
0.00756891	0.62764591	0.62764591
0.01134400	0.62764591	0.75878691
0.00000000	0.75878691	-0.65133895
0.01134400	0.75878691	-0.62764591
0.00868436	0.75878691	-0.46056619
0.01134400	0.75878691	-0.17407766
0.01134400	0.75878691	0.17407766
0.00868436	0.75878691	0.46056619
0.01134400	0.75878691	0.62764591
0.00000000	0.87038828	-0.49236596
0.01422496	0.87038828	-0.46056619
0.01422496	0.87038828	-0.17407766
0.01422496	0.87038828	0.17407766
0.01422496	0.87038828	0.46056619
0.00000000	0.96922337	-0.24618298
0.01594214	0.96922337	-0.17407766
0.01594214	0.96922337	0.17407766

APPENDIX B--SAMPLE PROBLEM

1. Listing of Problem Input Deck

100000200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

110000 4 25 20

130000NUM1 NUM2

170000 XX

200000 0.0 25.0 X 0.0 25.0

2000020.0 25.0 0.0 25.0

2000010.0 6.0 0.0 6.0

210000 13.0 15.0 16.0 X

210000 17.0 110.0 116.0 X

210000 125.0 X

210000 13.00 15.0 16.0 Y

210000 17.0 110.0 116.0 Y

210000 125.0 Y

300000 X X X X

400000000000 DDF01

400000 1NUM1 .94 NUM2 .06

400000 2NUM1 .007 NUM2 .993

410000 1 1 .048

410000 2 2 .048

420000 1 1

420000 2 2

500000 X .000001

600000

700000NUM1 9

DDF01NUM1 X5 3 11.8625 0 0

700000NUM2 9

DDF01NUM2 X5 3 12.3 0 0

END OF PROBLEM

2. Printed Results

SEQUENCE,2110
 JOB,T100000,205432,10,MIDDLE DESK
 SCOPE 6.2112 ANL SYS 1C-17B
 +SPECIAL INSTRUCTIONS
 +MCUNT,L40340
 +MOUNT,L19513
 EQUIP,9=61
 EQUIP,10=60
 EQUIP,29=(SNARG2CCVERLAY,44),RO,SV
 EQUIP,44=(SNARG XLIBIT DEBUG,8),RO,SV
 LOADMAIN,29,70,99999,7,
 PRESENT CLOCK READING 1246 - 37
 EXECUTION STARTED AT 1246 -37

AT 1246 - 32

ON 02/27/68

200004.000C1 K,R-Z,.000001,7X7,ANISO. 7 CARD.

THE FOLLOWING IS THE MERGED DECK OF CROSS SECTION DATA CARDS FOR ISOTOPE NUM2

DDF01NUM2 + X-SEC. SET FOR DCF TEST PROBLEM, MAT. 2.
 DDF01NUM2 X0 0 4.00000+000 0.00000+000
 DDF01NUM2 X0 1 3.00000+000 2.87682-001 2.22000+001
 DDF01NUM2 X0 2 2.00000+000 6.93147-001 1.25000+001
 DDF01NUM2 X0 3 1.00000+000 1.38629+000 5.60000+000
 DDF01NUM2 1 1 7.49064-002 4.45000+000-7.50000-001 1.35000+000 1.00000+000
 DDF01NUM2 1 2 6.66667-002 5.00000+000 1.00000-001 5.00000-002 1.00000+000
 DDF01NUM2 1 3 3.50877-002 9.50000+000 3.00000-001 0.00000+000 0.00000+000
 DDF01NUM2 2 0 1 5.60000-001 3.41000-001 9.90000-002
 DDF01NUM2 5 1 0 1.00000+000 0 2
 DDF01NUM2 5 2 0 1.00000+000 0 1
 DDF01NUM2 5 3 0 9.20000+000 0 0
 DDF01NUM2 X5 3 12.3 0 0
 DDF01NUM2 6 1 10 1.90000+000 1.50000+000 4.50000-001
 DDF01NUM2 6 2 10 4.25000+000 6.00000-001
 *

200004.000C1 K,R-Z,.000001,7X7,ANISO. 7 CARD.

THE FOLLOWING IS THE MERGED DECK OF CROSS SECTION DATA CARDS FOR ISOTOPE NUM1

DDF01NUM1 + X-SEC. SET FOR DDF TEST PROBLEM, MAT. 1.
 DDF01NUM1 X0 0 4.00000+000 0.00000+000
 DDF01NUM1 X0 1 3.00000+000 2.87682-001 2.22000+001
 DDF01NUM1 X0 2 2.00000+000 6.93147-001 1.25000+001
 DDF01NUM1 X0 3 1.00000+000 1.38629+000 5.60000+000
 DDF01NUM1 1 1 7.32601-002 4.55000+000-1.90000+000 3.25000+000 1.00000+000
 DDF01NUM1 1 2 6.41026-002 5.20000+000-1.62500+000 3.07500+000 1.00000+000
 DDF01NUM1 1 3 3.50877-002 9.50000+000-2.20000+000 4.25000+000 1.00000+000
 DDF01NUM1 2 0 1 5.60000-001 3.41000-001 9.90000-002
 DDF01NUM1 5 1 0 1.00000+000 0 2
 DDF01NUM1 5 2 0 1.00000+000 0 1
 DDF01NUM1 5 3 0 7.45000+000 0 0
 DDF01NUM1 X5 3 11.8625 0 0
 DDF01NUM1 6 1 10 1.80000+000 1.05000+000 3.50000-001
 DDF01NUM1 6 2 10 3.30000+000 4.50000-001
 *

20C004.000C1 K,R-7,.000001,7X7,ANISO. 7 CARD.

THIS IS A REGULAR SOLUTION TO A REACTIVITY PROBLEM WITH ANISOTROPIC SCATTERING.

AN RZ GEOMETRY WITH AN S(4) ORDER WAS USED.

THE LEFT BOUNDARY IS REFLECTIVE AT X = 0.000000+000.

THE RIGHT BOUNDARY IS FREE AT X = 2.500000+001.

THE BOTTOM BOUNDARY IS REFLECTIVE AT Y = 0.000000+000.

THE TOP BOUNDARY IS FREE AT Y = 2.500000+001.

THE INITIAL EIGENVALUE IS 0.000000+000.

THE EIGENVALUE MODIFIER IS 0.000000+000.

THE PARAMETER OSCILLATION DAMPER IS 1.000000+000.

THE NORMALIZATION FACTOR IS 1.000000+000.

THE CONSTANT FACTOR IN THE VARIOUS CONVERGENCE CRITERIA IS 1.000000-006.

THE INITIAL CRITICALITY SEARCH CONVERGENCE CRITERION IS 1.000000-006.

THE NEUTRON BALANCE ERROR CRITERION IS 0.000000+000.

THE RELATIVE INNER ITERATION CONVERGENCE CRITERION IS 0.000000+000.

THE MAXIMUM NUMBER OF INNER ITERATIONS IS 20.

THE MAXIMUM NUMBER OF OUTER ITERATIONS IS 25.

THE CROSS SECTION SET I.C. IS CDF01 AND THE SET HAS 3 VELOCITY GROUP(S). THE MAXIMUM UPSCATTERING IS 0 VELOCITY GROUP(S), AND THE MAXIMUM DOWNSCATTERING IS 2 VELOCITY GROUP(S).

200004.000C1 K,R-2,.000001,7X7,ANISO. 7 CARD.

THE REACTOR WAS DIVIDED INTO 2 REGION(S), 7 RADIAL(X) INTERVAL(S) AND 7 AXIAL(Y) INTERVAL(S) AS SHOWN IN THE FOLLOWING TABLE.

THE FIRST ROW(X =) GIVES THE RADIAL INTERVAL NUMBER(S). THE FIRST COLUMN(Y) GIVES THE AXIAL INTERVAL NUMBER(S). THE RECTANGULAR ARRAY OF NUMBERS STARTING WITH THE SECOND ROW AND SECOND COLUMN ARE THE REGION NUMBERS CORRESPONDING TO THE CELLS GIVEN BY THE X AND Y INTERVAL NUMBERS.

X =	1	2	3	4	5	6	7
Y							
7	2	2	2	2	2	2	2
6	2	2	2	2	2	2	2
5	2	2	2	2	2	2	2
4	2	2	2	2	2	2	2
3	1	1	1	2	2	2	2
2	1	1	1	2	2	2	2
1	1	1	1	2	2	2	2

200004.000C1 K,R-2,.000001,7X7,ANISO. 7 CARD.

TABLE OF INITIAL RADII AND INITIAL AXII

X MESH POINT NUMBER	INITIAL RADII	Y MESH POINT NUMBER	INITIAL AXII
1	0.000000+000	1	0.000000+000
2	3.000000+000	2	3.000000+000
3	5.000000+000	3	5.000000+000
4	6.000000+000	4	6.000000+000
5	7.000000+000	5	7.000000+000
6	1.000000+001	6	1.000000+001
7	1.600000+001	7	1.600000+001
8	2.500000+001	8	2.500000+001

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

TABLE OF SN CONSTANTS

DIRECTION NUMBER	ANGLE(THETA)	ANGLE(PHI)	WEIGHTS	DIRECTION COSINES(ETA)	DIRECTION COSINES(MU)
1	0.000000+000	0.000000+000	0.000000+000	-3.333333-001	-9.428090-001
2	0.000000+000	0.000000+000	8.333333-002	-3.333333-001	-8.819171-001
3	0.000000+000	0.000000+000	8.333333-002	-3.333333-001	-3.333333-001
4	0.000000+000	0.000000+000	8.333333-002	-3.333333-001	3.333333-001
5	0.000000+000	0.000000+000	8.333333-002	-3.333333-001	8.819171-001
6	0.000000+000	0.000000+000	0.000000+000	-8.819171-001	-4.714045-001
7	0.000000+000	0.000000+000	8.333333-002	-8.819171-001	-3.333333-001
8	0.000000+000	0.000000+000	8.333333-002	-8.819171-001	3.333333-001
9	9.000000+001	9.000000+001	0.000000+000	3.333333-001	-9.428090-001
10	9.000000+001	9.000000+001	8.333333-002	3.333333-001	-8.819171-001
11	9.000000+001	9.000000+001	8.333333-002	3.333333-001	-3.333333-001
12	9.000000+001	9.000000+001	8.333333-002	3.333333-001	3.333333-001
13	9.000000+001	9.000000+001	8.333333-002	3.333333-001	8.819171-001
14	9.000000+001	9.000000+001	0.000000+000	8.819171-001	-4.714045-001
15	9.000000+001	9.000000+001	8.333333-002	8.819171-001	-3.333333-001
16	9.000000+001	9.000000+001	8.333333-002	8.819171-001	3.333333-001

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

INPUT FISSION DENSITY TABLE

NO FISSION DENSITIES WERE PROVIDED FOR THIS PROBLEM.

THE CODE SUPPLIED INITIAL VALUES OF 1.0 IN FISSIONABLE REGIONS AND 0.0 IN NON-FISSIONABLE REGIONS, FOR ALL GROUPS.

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

TABLE OF VELOCITIES

GROUP NUMBER	VELOCITY
1	2.220000+001
2	1.250000+001
3	5.600000+000

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

MATERIAL CROSS SECTION SPECIFICATIONS

MATERIAL NUMBER 1 = 9.400000-001 * NUM1
+ 6.000000-002 * NUM2

MATERIAL NUMBER 2 = 7.000000-003 * NUM1
+ 9.930000-001 * NUM2

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

COMPOSITION CROSS SECTION SPECIFICATIONS

COMPOSITION NUMBER 1 = 4.800000-002 * MATERIAL NUMBER 1

COMPOSITION NUMBER 2 = 4.800000-002 * MATERIAL NUMBER 2

COMPOSITION NUMBER 1 IS IN REGION(S) 1.

COMPOSITION NUMBER 2 IS IN REGION(S) 2.

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

ISOTOPE CROSS SECTIONS

ISOTOPE NAME IS NUM2

ENERGY GROUP	FISSION CROSS SECTION	NU TIMES FISSION CROSS SECTION	ABSORPTION CROSS SECTION	TOTAL CROSS SECTION	SELF-SCATTERING CROSS SECTION
1	1.35000+000	1.35000+000	6.00000-001	4.45000+000	1.90000+000
2	5.00000-002	5.00000-002	1.50000-001	5.00000+000	4.25000+000
3	0.00000+000	0.00000+000	3.00000-001	9.50000+000	9.20000+000

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

SCATTERING CROSS SECTIONS FOR NEUTRONS DOWNSCATTERED FROM ENERGY GROUP (J - DELTA) INTO ENERGY GROUP J

DELTA	J=1	J=2	J=3
1	0.00000+000	1.50000+000	6.00000-001
2	0.00000+000	0.00000+000	4.50000-001

ANISOTROPIC SCATTERING CROSS SECTIONS

ENERGY GROUPS	ANISOTROPIC SELF-SCATTERING
1	0.00000+000
2	0.00000+000
3	2.30000+000

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

ANISOTROPIC SCATTERING CROSS SECTIONS
SCATTERING CROSS SECTIONS FOR NEUTRONS DOWNSCATTERED FROM ENERGY GROUP (J - DELTA) INTO ENERGY GROUP J

DELTA	J=1	J=2	J=3
1	0.00000+000	0.00000+000	0.00000+000
2	0.00000+000	0.00000+000	0.00000+000

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

CHI FISSION FRACTIONS ARE INDEPENDENT OF THE INCIDENT NEUTRON ENERGY.
K IS THE INDEX ON THE ENERGY GROUP INTO WHICH NEUTRONS ARE BORN

K	CHI FISSION FRACTIONS
1	5.60000-001
2	3.41000-001
3	9.90000-002

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

ISOTOPE NAME IS NUM1

ENERGY GROUP	FISSION CROSS SECTION	NU TIMES FISSION CROSS SECTION	ABSORPTION CROSS SECTION	TOTAL CROSS SECTION	SELF-SCATTERING CROSS SECTION
1	3.25000+000	3.25000+000	1.35000+000	4.55000+000	1.80000+000
2	3.07500+000	3.07500+000	1.45000+000	5.20000+000	3.30000+000
3	4.25000+000	4.25000+000	2.05000+000	9.50000+000	7.45000+000

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

SCATTERING CROSS SECTIONS FOR NEUTRONS DOWNSCATTERED FROM ENERGY GROUP (J - DELTA) INTO ENERGY GROUP J

DELTA	J=1	J=2	J=3
1	0.00000+000	1.05000+000	4.50000-001
2	0.00000+000	0.00000+000	3.50000-001

ANISOTROPIC SCATTERING CROSS SECTIONS

ENERGY GROUPS	ANISOTROPIC SELF-SCATTERING
1	0.00000+000
2	0.00000+000
3	1.86250+000

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

ANISOTROPIC SCATTERING CROSS SECTIONS
SCATTERING CROSS SECTIONS FOR NEUTRONS DOWNSCATTERED FROM ENERGY GROUP (J - DELTA) INTO ENERGY GROUP J

DELTA	J=1	J=2	J=3
1	0.00000+000	0.00000+000	0.00000+000
2	0.00000+000	0.00000+000	0.00000+000

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

CHI FISSION FRACTIONS ARE INDEPENDENT OF THE INCIDENT NEUTRON ENERGY.
K IS THE INDEX ON THE ENERGY GROUP INTO WHICH NEUTRONS ARE BORN

K CHI FISSION FRACTIONS

1	5.60000-001
2	3.41000-001
3	9.90000-002

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

MATERIAL CROSS SECTIONS

MATERIAL NUMBER 1

ENERGY GROUP	FISSION CROSS SECTION	NU TIMES FISSION CROSS SECTION	ABSORPTION CROSS SECTION	TOTAL CROSS SECTION	SELF-SCATTERING CROSS SECTION
1	3.13600+000	3.13600+000	1.30500+000	4.54400+000	1.80600+000
2	2.89350+000	2.89350+000	1.37200+000	5.18800+000	3.35700+000
3	3.99500+000	3.99500+000	1.94500+000	9.50000+000	7.55500+000

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

SCATTERING CROSS SECTIONS FOR NEUTRONS DOWNSCATTERED FROM ENERGY GROUP (J - DELTA) INTO ENERGY GROUP J

DELTA	J=1	J=2	J=3
1	0.00000+000	1.07700+000	4.59000-001
2	0.00000+000	0.00000+000	3.56000-001

ANISOTROPIC SCATTERING CROSS SECTIONS

ENERGY GROUPS	ANISOTROPIC SELF-SCATTERING
1	0.00000+000
2	0.00000+000
3	1.88875+000

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

ANISOTROPIC SCATTERING CROSS SECTIONS
SCATTERING CROSS SECTIONS FOR NEUTRONS DOWNSCATTERED FROM ENERGY GROUP (J - DELTA) INTO ENERGY GROUP J

DELTA	J=1	J=2	J=3
1	0.00000+000	0.00000+000	0.00000+000
2	0.00000+000	0.00000+000	0.00000+000

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

CHI FISSION FRACTIONS ARE INDEPENDENT OF THE INCIDENT NEUTRON ENERGY.
K IS THE INDEX ON THE ENERGY GROUP INTO WHICH NEUTRONS ARE BORN

K CHI FISSION FRACTIONS

1	5.60000-001
2	3.41000-001
3	9.90000-002

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

MATERIAL NUMBER 2

ENERGY GROUP	FISSION CROSS SECTION	NU TIMES FISSION CROSS SECTION	ABSORPTION CROSS SECTION	TOTAL CROSS SECTION	SELF-SCATTERING CROSS SECTION
1	1.36330+000	1.36330+000	6.05250-001	4.45070+000	1.89930+000
2	7.11750-002	7.11750-002	1.59100-001	5.00140+000	4.24335+000
3	2.97500-002	2.97500-002	3.12250-001	9.50000+000	9.18775+000

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

SCATTERING CROSS SECTIONS FOR NEUTRONS DOWNSCATTERED FROM ENERGY GROUP (J - DELTA) INTO ENERGY GROUP J

DELTA	J=1	J=2	J=3
1	0.00000+000	1.49685+000	5.98950-001
2	0.00000+000	0.00000+000	4.49300-001

ANISOTROPIC SCATTERING CROSS SECTIONS

ENERGY GROUPS	ANISOTROPIC SELF-SCATTERING
1	0.00000+000
2	0.00000+000
3	2.29694+000

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

ANISOTROPIC SCATTERING CROSS SECTIONS

SCATTERING CROSS SECTIONS FOR NEUTRONS DOWNSCATTERED FROM ENERGY GROUP (J - DELTA) INTO ENERGY GROUP J

DELTA	J=1	J=2	J=3
1	0.00000+000	0.00000+000	0.00000+000
2	0.00000+000	0.00000+000	0.00000+000

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

CHI FISSION FRACTIONS ARE INDEPENDENT OF THE INCIDENT NEUTRON ENERGY.
K IS THE INDEX ON THE ENERGY GROUP INTO WHICH NEUTRONS ARE BORN

K	CHI FISSION FRACTIONS
1	5.60000-001
2	3.41000-001
3	9.90000-002

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

COMPOSITION CROSS SECTIONS

COMPOSITION NUMBER 1

ENERGY GROUP	FISSION CROSS SECTION	NU TIMES FISSION CROSS SECTION	ABSORPTION CROSS SECTION	TOTAL CROSS SECTION	SELF-SCATTERING CROSS SECTION
1	1.50528-001	1.50528-001	6.26400-002	2.18112-001	8.66880-002
2	1.38888-001	1.38888-001	6.58560-002	2.49024-001	1.61136-001
3	1.91760-001	1.91760-001	9.33600-002	4.56000-001	3.62640-001

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

SCATTERING CROSS SECTIONS FOR NEUTRONS DOWNSCATTERED FROM ENERGY GROUP (J - DELTA) INTO ENERGY GROUP J

DELTA	J=1	J=2	J=3
1	0.00000+000	5.16960-002	2.20320-002
2	0.00000+000	0.00000+000	1.70880-002

ANISOTROPIC SCATTERING CROSS SECTIONS

ENERGY GROUPS	ANISOTROPIC SELF-SCATTERING
1	0.00000+000
2	0.00000+000
3	9.06600-002

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

ANISOTROPIC SCATTERING CROSS SECTIONS
SCATTERING CROSS SECTIONS FOR NEUTRONS DOWNSCATTERED FROM ENERGY GROUP (J - DELTA) INTO ENERGY GROUP J

DELTA	J=1	J=2	J=3
1	0.00000+000	0.00000+000	0.00000+000
2	0.00000+000	0.00000+000	0.00000+000

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

CHI FISSION FRACTIONS ARE INDEPENDENT OF THE INCIDENT NEUTRON ENERGY.
K IS THE INDEX ON THE ENERGY GROUP INTO WHICH NEUTRONS ARE BORN

K	CHI FISSION FRACTIONS
1	5.60000-001
2	3.41000-001
3	9.90000-002

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

COMPOSITION NUMBER 2

ENERGY GROUP	FISSION CROSS SECTION	NU TIMES FISSION CROSS SECTION	ABSORPTION CROSS SECTION	TOTAL CROSS SECTION	SELF-SCATTERING CROSS SECTION
1	6.54384-002	6.54384-002	2.90520-002	2.13634-001	9.11664-002
2	3.41640-003	3.41640-003	7.63680-003	2.40067-001	2.03681-001
3	1.42800-003	1.42800-003	1.49880-002	4.56000-001	4.41012-001

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

SCATTERING CROSS SECTIONS FOR NEUTRONS DOWNSCATTERED FROM ENERGY GROUP (J - DELTA) INTO ENERGY GROUP J

DELTA	J=1	J=2	J=3
1	0.00000+000	7.18488-002	2.87496-002
2	0.00000+000	0.00000+000	2.15664-002

ANISOTROPIC SCATTERING CROSS SECTIONS

ENERGY GROUPS	ANISOTROPIC SELF-SCATTERING
1	0.00000+000
2	0.00000+000
3	1.10253-001

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

ANISOTROPIC SCATTERING CROSS SECTIONS
SCATTERING CROSS SECTIONS FOR NEUTRONS DOWNSCATTERED FROM ENERGY GROUP (J - DELTA) INTO ENERGY GROUP J

DELTA	J=1	J=2	J=3
1	0.00000+000	0.00000+000	0.00000+000
2	0.00000+000	0.00000+000	0.00000+000

200004.000C1 K,R-Z,.0000C1,7X7,ANISO. 7 CARD.

CHI FISSION FRACTIONS ARE INDEPENDENT OF THE INCIDENT NEUTRON ENERGY.
K IS THE INDEX ON THE ENERGY GROUP INTO WHICH NEUTRONS ARE BORN

K CHI FISSION FRACTIONS
1 5.600CC-001
2 3.410CC-001
3 9.900CC-002

200004.000C1 K,R-Z,.0000C1,7X7,ANISO. 7 CARD.

2483 LCCATIONS HAVE BEEN USED BY THIS PROBLEM FOR VARIABLE STORAGE. 25517 LOCATIONS REMAIN UNUSED.

I	W0	M5	M7	W1	W2	M3	M4	M6
1	0.0000000C+000	-3.3333330-C01	-9.42809030-C01	0.00000000+000	0.00000000+000	0.00000000+000	9	5
2	8.3333330C-002	-3.3333330-C01	-8.81617100-C01	-7.34930887-002	-2.77777764-002	8.81917100-001	10	5
3	8.3333330C-002	-3.3333330-C01	-3.3333330-001	-2.77777764-002	-2.77777764-002	2.09716753+000	11	4
4	8.3333330C-002	-3.3333330-001	3.3333330-001	2.77777764-002	-2.77777764-002	2.09716753+000	12	3
5	8.3333330C-002	-3.3333330-C01	8.81617100-C01	7.34930887-002	-2.77777764-002	8.81917100-001	13	2
6	0.0000000C+000	-8.81917100-C01	-4.71404510-C01	0.00000000+000	0.00000000+000	0.00000000+000	14	8
7	8.3333330C-002	-8.81917100-001	-3.3333330-001	-2.77777764-002	-7.34930887-002	3.3333330-001	15	8
8	8.3333330C-002	-8.81917100-001	3.3333330-001	2.77777764-002	-7.34930887-002	3.3333330-001	16	7
9	0.0000000C+000	3.3333330-001	-9.42809030-C01	0.00000000+000	0.00000000+000	0.00000000+000	1	13
10	8.3333330C-002	3.3333330-001	-8.81617100-C01	-7.34930887-002	2.77777764-002	8.81917100-001	2	13
11	8.3333330C-002	3.3333330-001	-3.3333330-001	-2.77777764-002	2.77777764-002	2.09716753+000	3	12
12	8.3333330C-002	3.3333330-001	3.3333330-001	2.77777764-002	2.77777764-002	2.09716753+000	4	11
13	8.3333330C-002	3.3333330-001	8.81617100-C01	7.34930887-002	2.77777764-002	8.81917100-001	5	10
14	0.0000000C+000	8.81917100-C01	-4.71404510-C01	0.00000000+000	0.00000000+000	0.00000000+000	6	16
15	8.3333330C-002	8.81917100-001	-3.3333330-001	-2.77777764-002	7.34930887-002	3.3333330-001	7	16
16	8.3333330C-002	8.81917100-001	3.3333330-001	2.77777764-002	7.34930887-002	3.3333330-001	8	15

200004.000C1 K,R-Z,.0000C1,7X7,ANISO. 7 CARD.

TABLE OF VOLUME ELEMENTS

Y IN- TERVAL NUMBER	AVERAGE AXII	AVERAGE RADIUS = 1.500000+000	AVERAGE RADIUS = 4.00000C+000	AVERAGE RADIUS = 5.500000+000	AVERAGE RADIUS = 6.500000+000	AVERAGE RADIUS = 8.500000+000	AVERAGE RADIUS = 1.300000+001	AVERAGE RADIUS = 2.050000+001
7	2.05000C+001	2.544688+002	4.52389C+002	3.110174+002	3.675660+002	1.441990+003	4.410792+003	1.043322+004
6	1.300000+001	1.696459+002	3.015926+002	2.073449+002	2.450440+002	9.513265+002	2.940528+003	6.955480+003
5	8.500000+000	8.482293+001	1.507963+002	1.036725+002	1.225220+002	4.806633+002	1.470264+003	3.477740+003
4	6.50000C+000	2.827431+001	5.026544+001	3.455749+001	4.084067+001	1.602211+002	4.900880+002	1.159247+003
3	5.50000C+000	2.827431+001	5.026544+001	3.455749+001	4.084067+001	1.602211+002	4.900880+002	1.159247+003
2	4.000000+000	5.654862+001	1.005309+002	6.911498+001	8.168134+001	3.204422+002	9.801761+002	2.318493+003
1	1.500000+000	8.482293+001	1.507963+002	1.036725+002	1.225220+002	4.806633+002	1.470264+003	3.477740+003

200004.000C1 K,R-Z,.0000C1,7X7,ANISO. 7 CARD.

MONITOR PRINT

OUTER ITERATION COUNT	LOCP COUNT	TOTAL NEUTRON BALANCE	EPSILON	EQ	LAMBDA	FISSION CHANGE	EIGEN- VALUE	TIME IN MILLISECONDS
0	0	0.000000+000	1.0C0000-006	0.000000+000	0.000000+000	0.000000+000	0.000000+000	1.854700+005
1	50	2.559804-007	1.0C0000-006	0.000000+000	3.641395-001	7.562233-001	3.641395-001	2.900000+004
2	51	2.774066-007	1.0C0000-006	0.000000+000	1.477720+000	2.420386-001	5.380963-001	3.159200+004
3	51	2.961348-007	1.0C0000-006	0.000000+000	1.441690+000	3.495647-001	7.757677-001	2.940600+004
4	50	3.071386-007	1.0C0000-006	0.000000+000	1.220665+000	2.540333-001	9.469527-001	2.898100+004
5	50	3.114775-007	1.0C0000-006	0.000000+000	1.077200+000	1.084779-001	1.020057+000	2.897000+004
6	49	3.125785-007	1.0C0000-006	0.000000+000	1.023954+000	3.617015-002	1.044491+000	2.851100+004
7	47	3.128013-007	1.0C0000-006	0.000000+000	1.007276+000	1.121467-002	1.052091+000	2.752300+004
8	44	3.127879-007	1.0C0000-006	0.000000+000	1.002234+000	3.456505-003	1.054442+000	2.609500+004
9	40	3.131257-007	1.0C0000-006	0.000000+000	1.000700+000	1.081413-003	1.055180+000	2.408900+004
10	36	3.129579-007	1.0C0000-006	0.000000+000	1.000224+000	3.451752-004	1.055417+000	2.220800+004
11	33	3.12874C-007	1.0C0000-006	0.000000+000	1.000073+000	1.124631-004	1.055494+000	2.075500+004
12	25	3.128742-007	1.0C0000-006	0.000000+000	1.000024+000	3.705736-005	1.055519+000	1.683500+004
13	15	3.129080-007	1.0C0000-006	0.000000+000	1.000008+000	1.196660-005	1.055527+000	1.192900+004
14	7	3.129164-007	1.0C0000-006	0.000000+000	1.000002+000	3.634620-006	1.055530+000	7.941000+003
15	4	3.132285-007	1.0C0000-006	0.000000+000	1.000001+000	1.682958-006	1.055531+000	6.553000+003

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

NEUTRON BALANCE FOR GROUP NUMBER 1

REGION NUMBER	FIXED SOURCE	FISSION NEUTRONS	SCATTER SOURCE	PRODUCTION	SCATTER REMOVAL	ABSORPTION NEUTRONS	REMOVAL	LEAKAGE
1	0.000000+000	4.315459-001	1.463400-001	5.778858-001	2.624558-001	1.057440-001	3.681997-001	2.096861-001
2	0.000000+000	1.284541-001	2.487455-001	3.771997-001	5.036269-001	7.926775-002	5.828946-001	-2.056950-001
REGION SUMMARY	0.000000+000	5.600000-001	3.950855-001	9.550855-001	7.660826-001	1.850117-001	9.510944-001	3.991120-003
REGION NUMBER	FLUX INTEGRAL	TOTAL DENSITY	FISSION INTEGRAL					
1	1.688122+000	7.604155-002	2.541097-001					
2	2.728478+000	1.229044-001	1.785472-001					
REGION SUMMARY	4.416601+000	1.989460-001	4.326569-001					

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

NEUTRON BALANCE FOR GROUP NUMBER 2

REGION NUMBER	FIXED SOURCE	FISSION NEUTRONS	SCATTER SOURCE	PRODUCTION	SCATTER REMOVAL	ABSORPTION NEUTRONS	REMOVAL	LEAKAGE
1	0.000000+000	2.627806-001	4.318324-001	6.946130-001	3.916751-001	1.408224-001	5.324975-001	1.621155-001
2	0.000000+000	7.821939-002	2.373397+000	2.451616+000	2.484694+000	8.163781-002	2.566331+000	-1.147153-001
REGION SUMMARY	0.000000+000	3.410000-001	2.805229+000	3.146229+000	2.876369+000	2.224602-001	3.098829+000	4.740019-002
REGION NUMBER	FLUX INTEGRAL	TOTAL DENSITY	FISSION INTEGRAL					
1	2.138338+000	1.710671-001	2.969895-001					
2	1.069005+001	8.552044-001	3.652150-002					
REGION SUMMARY	1.282839+001	1.026271+000	3.335110-001					

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

NEUTRON BALANCE FOR GROUP NUMBER 3

REGION NUMBER	FIXED SOURCE	FISSION NEUTRONS	SCATTER SOURCE	PRODUCTION	SCATTER REMOVAL	ABSORPTION NEUTRONS	REMOVAL	LEAKAGE
1	0.000000+000	7.629114-002	5.720205-001	6.483116-001	4.960620-001	1.277089-001	6.237709-001	2.454077-002
2	0.000000+000	2.270886-002	8.720581+000	8.743290+000	8.354403+000	2.839283-001	8.638331+000	1.049588-001
REGION SUMMARY	0.000000+000	9.900000-002	9.292601+000	9.391601+000	8.850465+000	4.116372-001	9.262102+000	1.294996-001
GROUP SUMMARY	0.000000+000	1.000000+000	1.249292+001	1.349292+001	1.249292+001	8.191091-001	1.331203+001	1.808909-001
REGION NUMBER	FLUX INTEGRAL	TOTAL DENSITY	FISSION INTEGRAL					
1	1.367919+000	2.442712-001	2.623121-001					
2	1.894371+001	3.382805+000	2.705162-002					
REGION SUMMARY	2.031163+001	3.627076+000	2.893637-001					
GROUP SUMMARY	3.755662+001	4.852294+000	1.055532+000					

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

G	EC	E1	E2	E3	E4	E5	E6	E7
1	0.00000+000	5.60000-001	-1.45519-011	3.95085-001	1.90923-003	1.85012-001	3.70997-001	1.90923-003
2	0.00000+000	3.41000-001	2.83307-001	2.52192+000	3.13292-002	2.22460-001	3.54447-001	3.13292-002
3	0.00000+000	9.90000-002	4.42137-001	8.85046+000	9.04434-002	4.11637-001	-4.65661-010	9.04434-002
4	0.00000+000	1.00000+000	7.25444-001	1.17675+001	1.23682-001	8.19109-001	7.25444-001	1.23682-001
G	LEAK. VERT.	TCP. LEAK.	NET LEAK.	NEUT. BAL.				
1	2.08185-003	2.08185-003	3.99108-003	6.83160-008				
2	1.60708-002	1.60708-002	4.74001-002	2.01366-007				
3	3.90558-002	3.90558-002	1.29459-001	6.94954-007				
4	5.72085-002	5.72085-002	1.80850-001	3.12950-007				

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

FINAL PRINT

FINAL FLUX PRINT BY GROUP

GROUP NUMBER 1

Y INTERVAL NUMBER	AVERAGE AXII	X INTERVAL NUMBER 1 AVERAGE RADII = 1.500000+000	X INTERVAL NUMBER 2 AVERAGE RADII = 4.000000+000	X INTERVAL NUMBER 3 AVERAGE RADII = 5.500000+000	X INTERVAL NUMBER 4 AVERAGE RADII = 6.500000+000	X INTERVAL NUMBER 5 AVERAGE RADII = 8.500000+000	X INTERVAL NUMBER 6 AVERAGE RADII = 1.300000+001	X INTERVAL NUMBER 7 AVERAGE RADII = 2.050000+001
7	2.050000+001	2.265497-005	2.737790-005	2.750847-005	2.689381-005	2.363430-005	1.202012-005	6.580737-007
6	1.300000+001	1.910971-004	1.572731-004	1.366159-004	1.239022-004	9.430931-005	4.050013-005	9.637051-006
5	8.500000+000	6.651766-004	5.553852-004	4.388998-004	3.761673-004	2.472220-004	8.670770-005	1.751199-005
4	6.500000+000	1.443939-003	1.158728-003	8.543603-004	6.496831-004	3.644119-004	1.096605-004	1.940485-005
3	5.500000+000	2.180516-003	1.762295-003	1.266776-003	8.216157-004	4.110656-004	1.181265-004	2.027243-005
2	4.000000+000	3.017717-003	2.424353-003	1.708006-003	1.074123-003	4.989022-004	1.322108-004	2.103561-005
1	1.500000+000	3.618558-003	2.926327-003	2.059135-003	1.309444-003	6.148328-004	1.587921-004	1.584851-005

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

FINAL FLUX PRINT BY GROUP

GROUP NUMBER 2

Y INTERVAL NUMBER	AVERAGE AXII	X INTERVAL NUMBER 1 AVERAGE RADII = 1.500000+000	X INTERVAL NUMBER 2 AVERAGE RADII = 4.000000+000	X INTERVAL NUMBER 3 AVERAGE RADII = 5.500000+000	X INTERVAL NUMBER 4 AVERAGE RADII = 6.500000+000	X INTERVAL NUMBER 5 AVERAGE RADII = 8.500000+000	X INTERVAL NUMBER 6 AVERAGE RADII = 1.300000+001	X INTERVAL NUMBER 7 AVERAGE RADII = 2.050000+001
7	2.050000+001	1.696839-004	1.641348-004	1.560581-004	1.486024-004	1.301208-004	8.553140-005	3.023183-005
6	1.300000+001	7.025494-004	6.306574-004	5.672588-004	5.213188-004	4.249941-004	2.469434-004	7.903317-005
5	8.500000+000	1.583694-003	1.370676-003	1.175071-003	1.040895-003	7.902703-004	4.073372-004	1.149151-004
4	6.500000+000	2.365190-003	2.016028-003	1.668706-003	1.420731-003	1.020228-003	4.902964-004	1.290981-004
3	5.500000+000	2.929959-003	2.481400-003	2.012461-003	1.640261-003	1.135148-003	5.280470-004	1.349517-004
2	4.000000+000	3.622236-003	3.043316-003	2.426910-003	1.937028-003	1.296801-003	5.782975-004	1.408670-004
1	1.500000+000	4.219578-003	3.540623-003	2.805720-003	2.228071-003	1.472235-003	6.345094-004	1.435061-004

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

FINAL FLUX PRINT BY GROUP

GROUP NUMBER 3

Y INTERVAL NUMBER	AVERAGE AXII	X INTERVAL NUMBER 1 AVERAGE RADII = 1.500000+000	X INTERVAL NUMBER 2 AVERAGE RADII = 4.000000+000	X INTERVAL NUMBER 3 AVERAGE RADII = 5.500000+000	X INTERVAL NUMBER 4 AVERAGE RADII = 6.500000+000	X INTERVAL NUMBER 5 AVERAGE RADII = 8.500000+000	X INTERVAL NUMBER 6 AVERAGE RADII = 1.300000+001	X INTERVAL NUMBER 7 AVERAGE RADII = 2.050000+001
7	2.050000+001	3.788640-004	3.622588-004	3.452053-004	3.307368-004	2.955433-004	2.137465-004	8.925213-005
6	1.300000+001	1.045548-003	9.827232-004	9.196035-004	8.700077-004	7.577297-004	5.217169-004	2.061195-004
5	8.500000+000	1.629905-003	1.511805-003	1.397571-003	1.307708-003	1.118631-003	7.406361-004	2.783138-004
4	6.500000+000	1.895665-003	1.746046-003	1.613133-003	1.515802-003	1.295852-003	8.428022-004	3.087410-004
3	5.500000+000	1.581725-003	1.814136-003	1.677282-003	1.607361-003	1.378789-003	8.873097-004	3.212537-004
2	4.000000+000	2.141219-003	1.950191-003	1.799359-003	1.727362-003	1.481308-003	9.425188-004	3.353424-004
1	1.500000+000	2.344668-003	2.124026-003	1.949010-003	1.860078-003	1.584332-003	9.957344-004	3.487067-004

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

FISSION NEUTRON DENSITY

Y INTERVAL NUMBER	AVERAGE AXII	X INTERVAL NUMBER 1 AVERAGE RADII = 1.500000+000	X INTERVAL NUMBER 2 AVERAGE RADII = 4.000000+000	X INTERVAL NUMBER 3 AVERAGE RADII = 5.500000+000	X INTERVAL NUMBER 4 AVERAGE RADII = 6.500000+000	X INTERVAL NUMBER 5 AVERAGE RADII = 8.500000+000	X INTERVAL NUMBER 6 AVERAGE RADII = 1.300000+001	X INTERVAL NUMBER 7 AVERAGE RADII = 2.050000+001
7	2.050000+001	2.603231-006	2.869621-006	2.826220-006	2.739865-006	2.413171-006	1.384017-006	2.737994-007
6	1.300000+001	1.639832-005	1.384961-005	1.219110-005	1.113137-005	8.705439-006	4.238933-006	1.194981-006
5	8.500000+000	5.322928-005	4.318515-005	3.473115-005	3.003931-005	2.047510-005	8.123268-006	1.935985-006
4	6.500000+000	1.652765-004	8.520625-005	6.391249-005	4.953258-005	2.918251-005	1.005457-005	2.151755-006
3	5.500000+000	3.115180-003	9.577900-004	7.918275-004	6.166431-005	3.274651-005	1.080111-005	2.246395-006
2	4.000000+000	1.367936-003	1.161582-003	9.392164-004	7.937320-005	3.919306-005	1.197327-005	2.336664-006
1	1.500000+000	1.580357-003	1.339548-003	1.073381-003	9.595611-005	4.752584-005	1.398075-005	2.025328-006
FISS. NEUT. SUMMARY =		4.240980-003	3.604030-003	2.918085-003	3.304367-004	1.802416-004	6.055592-005	1.216491-005

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

Y IN- TERVAL NUMBER	FISSION NEUTRON SUMMARY
7	1.510992-005
6	6.770975-005
5	1.917192-004
4	3.453167-004
3	2.972256-003
2	3.601611-003
1	4.152773-003

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

FINAL MONITOR PRINT

OUTER ITERATION COUNT	LOCP COUNT	TOTAL NEUTRON BALANCE	EPSILON	EQ	LAMBDA	FISSION CHANGE	EIGEN- VALUE	TIME IN MILLISECONDS
16	3	3.129502-007	1.000000-006	0.000000+000	1.000001+000	9.935854-007	1.055531+000	5.968300+004

A=THE NUMBER OF TIMES THE IIM HAS BEEN EXCEEDED FOR EACH GROUP.

C=THE NUMBER OF TIMES THE NEUTRON BALANCE HAS EXCEEDED THE INNER LOOP CONVERGENCE CRITERION FOR EACH GROUP.

GROUP NUMBER	A	C
1	0	0
2	6	0
3	11	16

200004.00001 K,R-Z,.000001,7X7,ANISO. 7 CARD.

TOTAL ITERATION TIME = 4.000710+005 MILLISECOND(S) = 6 MINUTE(S) AND 40 SECOND(S).

TOTAL PROBLEM TIME = 5.855410+005 MILLISECOND(S) = 9 MINUTE(S) AND 45 SECOND(S).

99 WRITE REQUEST(S) TOOK A TOTAL OF 4.884600+004 MILLISECOND(S).

198 READ REQUEST(S) TOOK A TOTAL OF 4.277800+004 MILLISECOND(S).

149 REWIND REQUEST(S) TOOK A TOTAL OF 2.010300+004 MILLISECOND(S).

THE TOTAL TAPE MOTION TIME = 1 MINUTE(S) AND 51 SECOND(S).

RUN COMPLETED.

PRINT REQUESTS 694

END JOB SEQUENCE 2110 DATE 02/27/68 TIME 1256 - 34 ELAPSED TIME 00 HRS 10 MIN 01 SEC

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8. B. G. Carlson and K. D. Lathrop, *Transport Theory--The Method of Discrete Ordinates*, Los Alamos Scientific Laboratory Report, LA-3251-MS (April 1, 1965).
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11. J. R. Askew and R. J. Brissenden, *Some Improvements in the Discrete Ordinate Method of B. G. Carlson for Solving the Neutron Transport Equation*, UKAEA Report, AEEW-R161 (Aug 1963).