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Argonne National Laboratory

A MANUAL OF REACTOR LABORATORY EXPERIMENTS

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Development Report

ARGONNE NATIONAL LABORATORY
9700 South Cass Avenue
Argonne, Illinois 60440

A MANUAL OF
REACTOR LABORATORY EXPERIMENTS

by

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Institute of Nuclear Science and Engineering

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PREFACE

This document contains an exposition of a basic set of formal training experiments in nuclear reactor physics, engineering, and technology; it represents the results of a development effort extending over several years and carried out by the reactor staff of the Argonne Institute of Nuclear Science and Engineering. Each experiment has been evolved for instructional purposes, has been presented to many students by the teaching staff, and has proven itself to be eminently satisfactory for this purpose.

The sustained enthusiastic reception of the 1961 edition, which was presented in the form of a record of proceedings of a Reactor Laboratory Institute for American university professors of nuclear engineering held in the Institute's laboratory, has encouraged us to present this newer edition. The basic format is now that of a laboratory manual and includes several experiments developed since the earlier edition.

Each experimental presentation included here begins with a simple description of the experiment in its theoretical and practical aspects. The specific organization related to the method of application by the Institute staff; thus, for example, it includes a set of discussion questions which become part of the briefing which precedes the experiment. An additional novel feature of this presentation is the inclusion of a typical set of results obtained by students during the performance of the experiment.

INTRODUCTION

In 1955, Argonne National Laboratory, in addition to its basic research and development functions, was charged with the task of cooperating in the implementing of an international program for peaceful uses of nuclear energy by establishing a training institute. A dual approach, involving parallel one-year programs in formal nuclear science training and in collaborating research with staff scientists and engineers, became the basis of the curriculum of International Institute of Nuclear Science and Engineering. The formal training course of study is elementary, imposes a prerequisite of a Master's degree in science or engineering or its equivalent, and is present over a two-semester period. The program consisted of a lecture series on each of nine branches of nuclear science, sequences of laboratory exercises in the nine laboratories of the Institute, and the completion of an extended special study in the area of particular interest to the trainee.

This report is a manual of the formal, one- and two-day presentation experiments offered by the Institute's Reactor Laboratory; the full series of these experiments was normally presented over the course of two semesters to those participants with adequate interest and background in this field. However, each Institute trainee-graduate, whatever his principal interest, would perform three or four of the experiments as part of his required work.

The INSE Reactor Laboratory Facilities

The nature the laboratory exercises in the reactor field that can be presented is strongly dependent upon the type of major facilities available for these training purposes. The facilities available to the student in all of the Institute's laboratories is remarkably complete; this is particularly true in the fields of neutron diffusion, neutron multiplication, and training reactors. In addition to the access to Argonne's research and developmental power reactors available to the trainee, the Institute maintains much equipment solely for its training activities in the nuclear reactor field. These include a Sigma Pile, a Standard Pile, two Subcritical Reactors, a Multiplication or Critical Assembly, two Training Reactors, and a Pulsed Neutron Generator.

The exponential assembly, which is supplied with neutrons from an external source, is perhaps the simplest of these facilities. The exponential facilities include the Sigma Pile, which is used for cross-section measurements and for the evaluation of diffusion parameters; the Standard Pile, which is used as a basic standard of flux in the determination of counter efficiencies, one type of reactor power measurement, and the determination of cross sections, is also a device in this class. A third type of exponential equipment is the Subcritical Reactor in which a fissionable isotope is added to an assembly and in which neutron multiplication is made to occur. The assembly is made up of a sample volume of the reactor under consideration,

and the critical size of the reactor is evaluated from the neutron distribution within the uranium-bearing assembly. The Institute has two natural uranium exponential assemblies, one graphite moderated and the other heavy water moderated.

Multiplication and critical facilities are also available. With these, the students have access to a critical-facility version of the original Argonaut reactor. In general, this is available for study by students of more advanced level, and the extent of its involvement at any time depends upon the special interest of the particular student concerned. Students and staff have used it diversely in studies of reactor instrumentation, Argonaut pre-critical multiplication studies, and in a study of reactor hazards evaluation.

The two training reactors available to the Institute make up the third major class of facilities used for pedagogical purposes. These reactors are specifically designed for student training, are operated by the Argonne Institute, and are devoted exclusively to instructional applications. The principal training facility available within the Institute is the well-known Argonaut reactor (Fig. 1). This reactor is now familiar to many

139-2541

Fig. 1. The Argonaut Reactor

either in the original or from contact with any of its twenty-four progeny. The reactor was designed by the Argonne Laboratory Reactor Engineering staff specifically for education purposes and is a low-power, flexible system. It has borne and continues to bear the brunt of the training load. The Argonaut is a heterogeneous, water-moderated, graphite-reflected, annular-core reactor. Although its maximum power level is 10 kW, it is held to less than 200-W operation in the interest of providing essentially continuous student access to the reactor core. It is provided with beam holes, internal and external thermal columns, and a water tank shield facility. A second critical reactor, an AGN-201, is a homogeneous, 100-mW, thermal facility fueled with enriched uranium and moderated with polyethylene. All of this reactor training equipment is grouped together (Fig. 2) in an easily accessible part of the National Laboratory which has been set aside to be used for instructional purposes alone.

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Fig. 2. Institute Reactor Physics Laboratory Equipment

The pulsed neutron generator utilizes the D-T reaction to produce high-energy neutrons in short bursts at a maximum rate of 60 per second. These neutrons are used to study the diffusion properties of thermal neutrons in light water and for fast neutron activation.

The Reactor Physics Laboratory Curriculum

This manual describes an integrated series of training experiments demonstrating principles in neutron physics, reactor physics, engineering, technology, and operation; it is organized to present a background for application of these principles to safe procedures for reactor manipulation and to provide the broad understanding necessary to the understanding of research and engineering problems.

Since, then, this experimental series was developed and has usually been presented as part of an extended matrix of Institute lecture and laboratory courses, it has been designed in part to complement these offerings. Lecture series in diffusion theory, transport theory, heat transport, control, shielding, fuel management, nuclear fuels, chemical processing, and physical metallurgy form part of the course; laboratories in reactor heat transfer, control, instrumentation, computers, chemical separation, metallurgy, radiation detection, and isotope technology are added to the reactor laboratory to complete the formal program. The usual minimum prerequisite for the Reactor Physics laboratory series has been taken to be a previous or concurrent course in Reactor Theory.

Each laboratory experiment was prepared especially to be used as part of this series; its nature is determined by the background level of the trainee and the feasibility of presentation in the limited time and with the apparatus available. The techniques and equipment used are not always necessarily the best or the most current but are selected to form a unified, broadly comprehensive nonrepetitious presentation in which each experiment is rich in new experiences and challenges for the student. Particular effort has been expended to obtain a conceptual integration, an effort which intends to include the technological skills, and auxiliary facilities used in the laboratory experiments as well.

The first group in a rather arbitrary ordering of the experiments is devoted to studies of the behavior of neutrons from fission or source energy to thermalization. Moderation and diffusion properties, diffusion length, and Fermi age are measured in a water tank facility at a reactor. Distribution of thermalized neutrons can be measured in several uranium-bearing exponential facilities in which neutron multiplication occurs and from which material buckling and critical reactor size can be inferred. In another exercise, the effective neutron temperature in two of the critical training reactors is measured by several methods.

A second series of experiments centers around reactor operation, engineering, and technology and provides the student with the opportunity to bring a reactor to critical and to operate it in both steady and transition states. The power output of a reactor is calculated from a neutron-distribution measurement, and danger coefficients and importance function are determined for materials such as neutron absorbers, sources, and scatterers within a reactor system. Measurements of temperature and void coefficients demonstrate concepts of importance in the safe engineering and operation of water-moderated reactors.

In fundamental reactor science, experimental studies display several essential concepts from neutron and reactor physics. Total neutron cross section is measured by a method involving basic neutron-beam techniques; in other exercises, measurements of isotopic neutron cross sections and absolute neutron fluxes are made by use of activation methods in the thermal, resonance, and fast regions of the neutron spectrum. The resonance escape probability and fast fission factor are evaluated in the lattice of the Argonaut reactor.

The final grouping of experiments, still in part under development, includes a presentation of some of the methods of basic research in physical science. Neutron diffraction is demonstrated and the distribution of thermal neutrons measured to familiarize the student with this useful technique. Neutron time-of-flight techniques are used to measure energy-dependent cross sections and reactor neutron spectra, and a recently developed experiment permits the student to measure prompt-neutron lifetime and short-term kinetic reactor response directly by pulsed neutron techniques.

In the nine years since the International Institute of Nuclear Science and Engineering was established, it has provided training in nuclear science including some experience in experimental reactor physics to approximately 750 international participants from 47 countries. This laboratory manual was developed and used over the course of these years for this training effort. Repeated editing and modified presentation has led to the present format, which continues to be used at the Institute and is being used or is being adapted as a text in a number of teaching institutions in America and abroad.

Since 1961 the Institute has opened its doors as well to the faculty and students of nuclear engineering from the universities of the midwest. In several special programs, faculty members of these institutions have become familiar with the laboratory curriculum in reactor physics; during this time also, over 500 students of nuclear engineering from twenty universities have received training by the Institute faculty on the Institutes reactor facilities. Argonne National Laboratory and the U. S. Atomic Energy Commission too have found the presentations useful and have sent successive engineering training groups through special programs centering about the use of these experiments and facilities.

It is not possible to compile a curriculum of this magnitude without adapting heavily from the work of many scientists whose writings have become classics in this field. It is a pleasure for the authors to acknowledge this indebtedness as well as the help of Argonne scientists and the motivation provided them by the steady stream of interested Institute and University students. Collaborative assistance with day by day experiment development and instruction has been competently provided by members of the Institute reactor laboratory staff. To this end the essential support provided by the various technical skills and sustained efforts of J. Becker, J. Concagh, R. Springer, G. Ketchmark, and V. Shoemaker must be cited. And finally our secretaries, Marjory Koch and Marilyn Spalek have brought their considerable skill and patience to the preparation of several versions of the original manuscript.

William J. Sturm

Argonne, Illinois
January 1965

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INTRODUCTION AND APPROACH TO CRITICAL; ARGONAUT REACTOR

by

D. A. Daavettila

ABSTRACT

The Argonaut reactor was first brought to criticality early in 1957. Since that time, operational experience has proven it to be of good design and well suited for both training and research purposes. The main elements of its design include a 24-in.-high reactor core contained in the annular region between 24- and 36-in.-diameter aluminum tanks. Up to 17 plate-type fuel elements (20% enriched U_3O_8 dispersed in aluminum) with a 1/4-in. space filled with water between plates, make up a fuel box. Six fuel boxes make up the simplest form of the reactor. The individual fuel boxes are separated by graphite wedges. The inner and outer reactor core reflector is graphite, whereas the upper and lower is light water. Various core geometries varying from a single slab (6 boxes) to an annular core (24 boxes) can be readily constructed.

Reactor criticality is attained and maintained with cadmium-sheet safety, shim, and control rods that move vertically along the outer side of the 36-in.-diameter tank. Primary features for reactor safety are the quick dump system for the water moderator, and the availability for sudden insertion of 3 cocked safety rods (similar to the control rods), plus the control rods. Further details of the reactor design and safety aspects are discussed in Appendix B. About 24 reactors basically similar to the Argonaut have been or are being constructed in various parts of the world.

The Argonaut is introduced to students by tour and lecture. Next, the students perform a complete reactor checkout and the approach-to-critical experiment using a control rod. This is followed by a brief study of some reactor kinetics and an experimental determination of some delayed-neutron parameters.

OUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
- IV. PROCEDURE
 - A. Reactor Checkout
 - B. Approach to Critical
 - C. Supercritical Operations
 - D. Delayed Neutrons
- V. RESULTS
- VI. ANALYSIS AND CONCLUSIONS
- VII. BIBLIOGRAPHY
- VIII. APPENDICES
 - A. Discussion Questions
 - B. Description of the Argonaut Reactor

I. INTRODUCTION

The aim of the first part of the experiment day is to acquaint the student with some of the details of the reactor design through a tour and lecture, and to introduce the operational and safety aspects of the reactor by means of a complete reactor checkout. The second part of the day is devoted to the approach-to-critical experiment using a control rod and to reactor kinetic studies.

Whenever the critical mass is uncertain or some change has been made in core geometry that might possibly affect significantly the critical mass, an approach-to-critical experiment is made to determine the critical mass. In this experiment a critical mass is assembled while insuring that more fuel is not loaded into the core than can be controlled. With no fuel in the core, the neutron population due to the source is measured. Then a small fuel addition is made and the population remeasured. From the change in population, the critical mass can be predicted according to the theory developed in the next section. More fuel additions are made, and after each addition the critical mass can be predicted with increased accuracy. The caution of the approach, which is reflected in the size and number of the fuel additions, depends on the particular problem and conditions. As the critical mass is very closely approached, the neutron population increases tremendously over the source level and becomes difficult to measure. However, by this time the critical mass can be accurately predicted, and this is the purpose of the experiment.

The initial approach with a new reactor is a check on the accuracy of the reactor calculations, and as mentioned before, ensures that more fuel (reactivity) is not loaded than can be safely controlled. One can conclude that the approach-to-critical experiment is an important experiment

in that it is the first operational step with all reactors and a necessary procedure from time to time thereafter. About 5 to 10 such experiments per year are made at the Argonaut.

Because of time limitations, the approach-to-critical experiment today will be made with the use of a poison control rod instead of fuel. Although this is of lesser practical significance, it duplicates in essence the principal concepts and procedures followed during an approach in which fuel is used. The main difference in procedure of the two approaches is the removal of poison versus the addition of fuel, but in either case, the net result as regards the reactor is the same, because both add positive reactivity primarily by increasing the thermal utilization. During the approach-to-critical experiment, it is interesting to observe the reactor kinetic behavior during subcritical operation due to changes in the control rod position and to note how this behavior changes as criticality is approached and, later, passed.

II. THEORY

If the reactor is in the subcritical condition only because of the negative reactivity of a control rod, the reactor will approach critical as the rod is removed and at some control rod position will reach criticality. In order to duplicate the fuel approach the aim here is to predict the critical control rod position - starting with the rod in the down position. Each rod removal step will introduce positive reactivity just as if a few grams of U^{235} were added, because both act primarily through the thermal utilization factor f to increase k_{eff} . The relationship between f and k_{eff} is shown by the fundamental reactor equation

$$k_{eff} = \eta \epsilon p f (\text{non-leakage probabilities}). \quad (1)$$

when $k_{eff} = 1$, the reactor is critical.

A schematic view of the reactor is shown in Fig. 1.1. Let C_0 , and C_1 , and C_2 (neutron count rates) be respectively related to Y_0 , Y_1 , and Y_2 (control rod positions), and let C_0 and Y_0 be the initial values. The

Fig. 1.1

Principal Apparatus
for the Experiment

absolute value of C_0 is not significant as it depends on detector location and associated electronics. All count rates are normalized to C_0 so that the increase in count rate is given by $M = C_i/C_0$, where M is the multiplication.

The equation relating the multiplication to k_{eff} is readily derived with the aid of Fig. 1.2, in which time unit #1 (average generation time) is the present and all others precede it. At each time unit Q neutrons were introduced. The population at time unit #1 is given by

$$MQ = Q + kQ_2 + k^2Q_3 + k^3Q_4 + \dots \quad (2)$$

Inasmuch as each Q in this equation is the same, cancellation of the Q 's gives

$$M = 1 + k + k^2 + k^3 + \dots = \frac{1}{1 - k_{eff}} \text{ for } k_{eff} < 1. \quad (3)$$

Fig. 1.2. Neutron Multiplication by Generations

Note from equation (3) that as k_{eff} approaches one, M approaches infinity, and $1/M$ approaches zero. If a plot of $1/M$ versus rod position is made and the curve extrapolated to a $1/M$ value of zero, that control rod position must be such that $k_{eff} = 1$, and therefore is the critical rod position. Or, if one was working with fuel, it would be the critical mass because the abscissa would then be in grams U^{235} or some other fuel weight units.

For this experiment, four neutron detectors will be used with two being cadmium covered to decrease their sensitivity. During the course of the experiment, the new rod position will be decided upon on the basis of data from the four detectors. The emphasis is on a slow, conservative approach to critical, just as it would be for an actual approach.

III. APPARATUS

The reactor and associated instrumentation for neutron counting are needed. At the Argonaut, BF_3 proportional counters are used.

IV. PROCEDURE

A. Reactor Checkout: To assure safe operation of the reactor, a prescribed sequence of operations must be followed to start the reactor. This sequence, which is invariantly set by electrical interlocks in the control console, is briefly as follows: (1) neutron source in, (2) the three safety rods completely withdrawn, (3) water-dump valve closed, (4) water pumped to operating level, and (5) the control rods moved to the desired positions.

Each day before the reactor is brought to critical a daily reactor checkout is performed to confirm by actual tests that all safety circuits are operating properly and that the reactor is in perfect, mechanical operating order. A standard checklist has been prepared, and each item must be checked before the reactor is operated at critical. As you perform this daily checkout, you will become quite familiar with this reactor and how it is to be operated.

B. Approach to Critical: When the checkout has been completed, the fine and coarse control rods are positioned as specified by the instructor. After waiting an appropriate length of time, record, on the provided data sheets in Section V, rod positions, temperature, and initial count rates C_a , C_b , C_c , and C_d of the four neutron detectors called A, B, C, and D.

In addition to the data record in the experiment book, keep a record on the large data board which will be available on the day of the experiment. Also, the approach-to-critical curve should be plotted on the blackboard. In this way, your group will be able to work together more effectively, and everybody will be able to follow the experiment.

Withdraw one of the control rods a fraction of the way out while maintaining extreme vigilance to preclude the possibility of the reactor becoming supercritical without your knowledge. Wait for the steady state, then record rod position, count rate (C_a) on counter A, inverse count rate relative to the initial count rate, ($C_{a,0}/C_a$), and the similar count rates and inverse count rates for each of the counters B, C, and D. Plot the new data points on the approach-to-critical curve and then estimate the most conservative critical rod position as indicated by the data.

Select the next rod position on the basis of the previous data. Proceed in this fashion, approaching criticality in a decreasing incremental fashion.

The experiment will be completed with enough time remaining to discuss any questions which you have not had answered and to summarize the results of the experiment.

C. Supercritical Operation: After attaining criticality at as low a power level as possible, remove one of the control rods to a position which will yield about a 60-sec period. Then plot power level as a function of time for about 5 min. How long does it take after the rod movement for the period to become asymptotic?

D. Delayed Neutrons: Make the reactor critical at approximately 100 W. When you are satisfied that all transients have died out, scram the reactor. Make a similog plot of the resulting flux as a function of time until the flux as indicated by the lowest range on Linear #1 is within about 50% of chart zero. Then analyze the resultant curve to determine the delayed-neutron parameters, i.e., periods and approximate relative abundances. Only the three longest periods can be resolved.

V. RESULTS

Initial Conditions:

Rod Positions: Course_____ Fine_____ Shim _____

Water Temperature: _____

$$C_{ao} \quad C_{bo} \quad C_{co} \quad C_{du}$$

Count Rates: _____

Approach to Critical:

[illegible]

VI. ANALYSIS AND CONCLUSIONS

This is left as an exercise for the students, but include curves of $1/M$ versus rod position.

VII. BIBLIOGRAPHY

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VIII. APPENDICES

A. Discussion Questions

The following propositions should be considered prior to your arrival at the Argonaut. An effort will be made to discuss them just prior to and/or during your checkout of the reactor, as time permits.

1. What is the importance to a neutron chain-reacting system of:
 - a) fission
 - b) fission in U^{235} yielding n greater than 1
 - c) the $(n,2n)$ reaction
 - d) carbon (graphite)
 - e) heat
 - f) the (γ,n) reaction
 - g) U^{238}
 - h) neutron-induced fission in U^{235}
 - i) spontaneous fission
 - j) H_2O .
2. Where do delayed neutrons come from? Are they necessary to a neutron chain reaction? What is their practical import? Assume that there are no delayed neutrons; how would you start the reactor?
3. Assuming a subcritical assembly containing a source of thermal neutrons of strength Q , establish the relationship between source multiplication of the system and its k_{eff} in the steady state.
4. Conventionally, the reciprocal of counting rate at a fixed position in the assembly is plotted versus amount of fuel or versus withdrawal of poison. Relate these variables to those in the multiplication equation.
5. When plotting inverse count rate as a function of % of rod withdrawal, for an initial approach to critical, will you use a linear, semilog, or log-log plot?
6. Do you expect the above curve to be straight, concave, or convex?
7. What do we mean by "critical" and what criterion may we use to determine that this condition exists in our assembly?
8. How does the critical control rod position vary with power level?

9. Assume a small step insertion of reactivity. What will be the reactor response after the initial transient?
10. Among the products of fission, there exist particles which are larger, and some which are more highly ionizing, than neutrons, also gamma rays and heat. In view of this, why do we select to measure the neutron population?
11. If boron trifluoride counters are being used for an initial approach to criticality, how would you expect the following to affect the results of the experiment: Their gamma sensitivity? location? neutron sensitivity? why?
12. Of what importance is the artificial neutron source to reactor startup and operation?
13. In addition to massive concrete shielding, what other precautions are taken to prevent radiation exposure?
14. What action should be taken if, during the course of this experiment, you believe the detectors do not indicate the true situation in the reactor?
15. For the initial approach to criticality, in the case of a high-power reactor designed to operate at elevated temperatures, what precautions must be taken that are not of concern at the Argonaut?
16. What difficulties do you foresee with this experiment?

B. Description of the Argonaut Reactor

Discussions of the considerations underlying the Argonaut design and operation are given, along with a description of the reactor, in the following reports:

1. Lennox, D. H., and Kelber, C. N., Summary Report on the Hazards of the Argonaut Reactor, ANL-5647 (Dec 1956).
2. Lennox, D. H., and Spinrad, B. I., Interim Report on Argonaut, ANL-5552 (March 1956).
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5. Lennox, D. H., et al., The Argonaut Reactor: A Generalized Reactor Facility for Nuclear Technology Training and Research, Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, Switzerland, 10, 265 (1958).
6. Armstrong, R. H., and Kelber, C. N., Argonaut - Argonne's Reactor for University Training, Nucleonics, 15, 62 (March 1957).

To supply details of the reactor which are pertinent to the operational procedure, the descriptions in the above reports have been amplified and modifications to the original system have been included.

Figures on the following pages show various aspects of the Argonaut. Figure 1.3 is an overall view of a model of the Argonaut, showing the facilities. Figures 1.4 and 1.5 are views of the console. Figure 1.6 is a plan view of the core and reflector, showing the possible fuel loadings. Figure 1.7 is a flow diagram of the water system. Characteristics of the system, as of the date of this report, are given in the following pages and, in some cases, represent verification of design parameters.

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Fig. 1.3. Isometric View of the Argonaut Reactor

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Fig. 1.4. Control Console of Argonaut

Fig. 1.5. Line Drawing of Control Console

Fig. 1.6. Plan Section of Core Lattice and Reflector

Fig. 1.7. Flow Diagram of the Water System

CHARACTERISTICS OF THE ARGONAUT

General

Type:	Training Reactor
Design Power:	10 kW
Normal Operating Power:	~100 W
Normal Operating Schedule:	8 hr/day, 5 days/wk.
Principal Uses of Reactor:	Education and training

Fuel

Normal Fresh Loading:	1 slab: 2.0 kg U^{235} 2 slabs: 3.6 kg U^{235} 3 in. annular: 4.0 kg U^{235}
Total Fuel Inventory:	6 kg U^{235}
Fuel-element Shape:	24 x 2.84 x 0.098-in. plates
Fuel Alloy:	39 w/o Al, 7.8 w/o $U_3^{235}O_8$, 31.2 w/o $U_3^{238}O_8$; aluminum matrix
Fuel Dimensions:	24 x 2.84 x 0.094 in.
Cladding Thickness:	0.002 in. (avg)
Cladding Material:	Aluminum
Type of Subassembly:	Stacked parallel plates
Number of Elements per Subassembly:	17
Subassembly Dimensions:	6 x 3 x 24 in.
Normal Number of Subassemblies in Core:	1 slab: 6 subassemblies 2 slabs: 12 subassemblies 3 in. annular: 24 subassemblies
Normal Arrangements of Subassemblies:	1 slab or 2 slabs in cylindrical annulus
Normal Lifetime of Standard Subassemblies:	Indefinite

Reactor

Overall Active Core Dimensions:	1 slab: Annular sector - 30 in. OD, 24 in. ID, 24 in. high, subtending a 90° angle. 2 slabs: 2 of above diametrically opposed.
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Core-containing Vessel:

2 concentric aluminum tanks, one 30 in. in diameter, 48 in. high; the other 24 in. in diameter, 48 in. high.

Material: Aluminum

Mean Operating Pressure: Atmospheric

Mean Operating

Temperature: Room temperature

Moderator:

H₂O between plates, graphite between subassemblies.

Reflector:

Vertical: 1 ft of water

Radial: 1 ft of graphite

Biological Shield:

Ordinary concrete on sides; heavy concrete top plug, masonite and steel in some experimental facilities. Present shield is sufficient for normal operating power of approximately 100 W.

Reactor Control:

- I. Control and Safety Mechanisms:
 - a. Three 7x7 in. cadmium vertical safety blades, steel cladding, motor drive.
 - b. Three 7x7 in. (or less) cadmium vertical control blades, steel cladding, motor drive.
 - c. Water moderator and reflector dumping.
 - d. Inert gas injection in core.
- II. Scram-initiating Features:
 - a. High-level, low-level, and period trips.
 - b. High-background monitors, and personnel alarm.
 - c. Experimental facilities and top shield interlocks.
 - d. Manual
- III. Startup:

Manual; automatic operation at power available.

Primary Coolant

Fluid:

Water

Circulation:

a. Direction of flow:

Upward

b. Flow induced by forced circulation.

Heat-dissipation Method:	Water-to-water heat exchanger
Average Core Heat Flux:	360 Btu/ft ² /hr at 10 kW
Ratio of Maximum to Average Heat Flux:	3
Means of Purification:	Mechanical filter and ion-exchange column.

Nuclear Data

Fuel Loading

- a. Minimum Critical Mass: 1980 g U²³⁵ for 1-slab loading.
- b. Normal Fresh Fuel Loading: 2010 g U²³⁵ for 1-slab loading.
- c. Excess k, fresh loading: 0.5%

Fluxes:

- a. Avg Thermal Flux: 2×10^{11} n/cm²/sec at 10 kW
- b. Peak Thermal Flux: 5×10^{11} n/cm²/sec at 10 kW
- c. Avg Fast Flux: 4×10^{11} n/cm²/sec at 10 kW
- d. Peak Fast Flux: 5×10^{11} n/cm²/sec at 10 kW

Reactivity Coefficients:

- a. Temperature: 1×10^{-4} k/k/°C
- b. Void: 2×10^{-3} k/k/% void

INSTRUMENTATION

Proportional BF₃ Counters

Four BF₃ counters are provided, primarily as startup instrumentation; however, sufficient range is obtained to extend to higher power levels by choice of location, amplifier gain and pulse-height discrimination, and the use of cadmium covers. All counters are located on the East face of the graphite reflector. The typical locations of the four counters designated A, B, C and D are shown in Fig. 1.8. Each is provided with a preamplifier, accessible for servicing by removal of the square top-shield plug. Two A1 D linear amplifiers on the left-hand control console rack provide pulse-height selection and linear amplification for two of the four counters, selected by a switch on each amplifier. A dual scaler on the left-hand control console rack with automatic timer may be used for registry of counts, as in

approaches to criticality. The output of either linear amplifier, but normally that of channel 2, is fed into a counting rate meter circuit and recorded by a Brown chart recorder on the left-hand control console rack. The count rate meter has a low-level trip, adjustable from 0 to 100 counts/min, and a high-level trip adjustable from 0 to 100% of full-scale reading (10^2 , 10^3 , 10^4 counts/min as selected manually). Counters B and D are cadmium-covered.

Fig. 1.8. BF_3 Proportional Counter Channels.

B^{10} Ion Chamber Linear Safety Trip Channels

Four B^{10} -lined parallel-plate ionization chambers are located in the South face of the reflector (see Figs. 1.9 and 1.10). Their sensitivity is 10^{-14} Amp/nv. Compensation for gamma radiation is provided in channels 1 and 2. Three chambers are connected to five-tube linear DC amplifiers located on the right-hand console rack (channels 1, 2, and 3). Regulated positive and negative voltage supplies are also located in the right-hand console rack. The sensitivity is adjustable by a decade step switch.

The fourth B^{10} chamber is for log level and period indication. The circuit includes a logarithmic preamplifier, located in the South trench, and log level and period meter chassis on the right-hand console rack, with a remote period meter on the front console rack and a remote chart

ALL CHASSIS IN THE SOUTH RACK EXCEPT LOG PREAMP, REMOTE PERIOD METER & THE LOG. LEVEL RECORDER

Fig. 1.9. B^{10} Ion Chamber Channels

Fig. 1.10. South Face of the Reflector (Water Tank Rolled Out), Showing the B^{10} Ion Chamber Instrumentation

recorder located at the Northeast corner of the reactor shield. A test switch, accessible from the back of the chassis, provides calibration check of the -10 and +10-second period positions.

Source Level and High Multiplication Interlock

A low-source-level interlock is provided by a B^{10} -compensated ionization chamber (10^{-14} Amp/nv) located in the outer reflector, behind Safety Rod No. 3 (see Fig. 1.11). A chassis on the left-hand control console rack contains a high-gain linear current amplifier with high- and low-level trips. This circuit insures that, during startup, the neutron flux

Fig. 1.11. B¹⁰ Ion Chamber of Multiplication and Low-source-level Interlocks

is sufficient to take all instrument readings well above noise level, and that a sufficient number of fissions are occurring to make the reactivity statistically well defined.

It is designed also to detect excessive multiplication of source neutrons arising from too much reactivity in the core and, in that event, to terminate startup. The input sensitivity is adjustable in three factor-of-two steps in order to compensate for the decay of the Sb-Be source. The multiplication factor at which the interlock acts may be selected by a top switch at the front of the chassis. This interlock will prevent startup of the reactor if the free k_{ex} built into it is larger than a present value. Its calibration, however, does not remain valid if the core geometry is changed. Furthermore, the adequate location of the detector is a determining factor in the ability of the interlock to detect unduly large reactivities. In particular, the location described here (behind Safety Rod No. 3) is adequate for a one-slab loading in the north end of the core.

Area Monitors

Three 27-liter ionization chambers with DC amplifiers are mounted in the reactor room: one on the North side of the reactor at the external thermal column; one over the fuel storage pits; and one on the South wall. The monitors detect and warn of excessive radiation level in the reactor room, interrupt power to the ventilating and heating fans to confine possible escaping airborne activity, and drop the control rods.

Each instrument has a range-selector switch permitting choice of full-scale sensitivity of 2, 20, 200 and 2000 mR/hr. A stop switch on each monitor prevents them from use above 200 mR/hr, except for the monitor over the fuel storage pits, which stops at 2000 mR/hr.

Personnel Monitors

Individuals going to the reactor top are detected by a personnel interlock consisting of a light source and photo cell. Should this occur, the control rod clutches are de-energized (see Interlock System below).

Current Source

A current source is located on the right-hand control console rack, which may be connected in place of any of the cables normally leading in from the B¹⁰ detectors. This is used to test amplifier and trip-circuit response.

Interlock System

The interlock system of Argonaut as shown in Fig. 1.12 is designed to be "fail safe," i.e., loss of line power or failure of any component of the

system will lead to a safe situation; ordinarily this is a shutdown of the reactor. A detailed, up-to-date diagram of the complete system is on file in the office in Building D-25.

Fig. 1.12. Interlock System

Control power must first be obtained through a key switch. The key is kept locked in the combination file when not needed. This key switch permits Relay K1 to be energized, provided six trip contacts shown in the upper circuit in Fig. 1.12 are all in the closed positions. If the key is removed, control power is lost. Control power will also be lost and the reactor scrammed if, at any subsequent time,

- a. pressure is lost in the nitrogen system;
- b. high-flux trips occur in safety channels 2 or 3;
- c. a high-flux trip occurs in channel 1 accompanied by a high-level trip in the counting-rate circuit (this feature permits use of the No. 1 safety channel with the Brown chart recorder near full-scale level without attendant risk of an unintentional trip);
- d. a trip occurs in the period circuit, i.e., the instrument indicates 10 sec or less;

e. any one of five manual scram buttons is depressed; scram buttons are located at either side of the front control console, in the pit near the dump valve, on a lead through the reactor top, and next to the remote log level chart.

Control power having been obtained, the neutron-source drive can be energized through the control switch and the source inserted. If control power is subsequently lost, the source will be automatically withdrawn from the reactor unless the key-operated "Source Hold" switch is in open position. This key is removed after checkout according to checkout procedure. Insertion of the source closes a limit switch which will be bypassed by contacts of Relay K5 at a later point in the startup procedure. Satisfaction of this interlock and auxiliary interlocks on the top shield plug and internal thermal column allows Relay K2 to energize the three safety rod clutches, unless low- or high-level trips are registered on the source-level interlock circuit or a low-level trip is registered on the counting-rate meter. (The auxiliary interlocks are indicated by green lights at the extreme left of the control console.)

Relay K3 may next be energized in the startup sequence, provided upper limit switches on all three safety rods are closed. A manually operated key switch completes the interlock circuit for Relay K3, allowing the dump valve to be closed by operation of the control switch. The "dump valve" key switch is bypassed by a contact of Relay K3 and may then be removed.

After the dump valve is closed, with the toggle switch on the front console at the "Pump On" position, water may be pumped into the reactor vessel by operating the control switch, provided limit switches indicate full insertion of the control rods and the water level is below a "High Level Float Interlock" in the reactor vessel. A high-multiplication trip in the source interlock circuit, which de-energizes the safety rod clutches, insures that k_{ex} in the reactor is below a preset value.

When water has reached the lower or "Operating Level Float Interlock," Relay K5 is energized. Contacts of Relay K5 perform the following functions:

a. Water is pumped automatically and control rod clutches are energized, provided the control selector switch is turned on to any of the three control rod positions.

b. The "Source-In" limit switch and high-multiplication trips are bypassed, permitting raising the reactor flux level and removal of the source without the interlock protection needed prior to this point in the startup procedure. The control rods may be raised in any sequence so long as Relay K4 remains energized. Automatic water pumping continues until the "High Level Float Interlock" is reached.

Interruption of a personnel alarm on the reactor top or indication by the Area Monitors of excessive radiation levels outside the reactor shield will de-energize Relay K4. Should either of these trips open, control rods will be dropped, causing the reactor power to fall. However, as soon as the cause for the trip is eliminated, the control rod clutches may be re-energized by depressing a reset button on the control console. This may be done in spite of a trip condition in the high-multiplication interlock.*

Admission of nitrogen to the spargers in the reactor vessel is blocked by the nitrogen valve (V2 in Fig. 1.12) which is held closed by line voltage applied through normally closed contacts of the storage tank float switch (when water is not in the reactor vessel) or by control power voltage (when water is in the reactor vessel). When control power is lost after water has been admitted to the core, nitrogen is injected until sufficient water has returned to the storage tank to close the storage tank float switch. A manually operated push button at the control console allows nitrogen to be injected at other times.

In the event of high radiation levels inside the building the "Area Monitor" interlock will disconnect power to the building vent and heater fans and drop the control rods.

*The multiplication trip circuit contains two interlocks: one is in the safety rod power circuit, the other is in the control rod power circuit. The latter is superfluous as a multiplication trip. It does, however, provide a convenient interlock to insure a full insertion of the control rods with a gamma monitor of personnel-monitor trip. Without this interlock, a slight interruption of the gamma or personnel-monitor circuits would drop the control rods a small amount. This could occur unnoticed by the operator. Without the control rod power reset button (shown in Fig. 1.12), it would be impossible to regain control rod power until the background level had dropped to startup level.

The results given in the table are plotted below. Note the critical rod position is 62.6.

The Ratio C_0/C_i versus Rod Position

INTRODUCTION AND APPROACH TO CRITICAL, AGN-201 REACTOR

by

K. C. Ruzich

ABSTRACT

This experiment is designed to familiarize the participant with the AGN-201 reactor and its operation. The experiment includes a detailed description of the reactor, performance of the daily reactor checkout, and an approach to critical through use of the fine control rod. The participant will acquire an adeptness for safe techniques of reactor operation in steady-state and transient situations, both for a subcritical and critical reactor.

The AGN-201 is a low-power thermal reactor used primarily for training purposes. The solid, 25-cm, right cylinder core is made up of a mixture of polyethylene and 20% enriched UO_2 . The critical mass of the reactor is approximately 650 g U^{235} . The core is surrounded by a graphite reflector, 20 cm thick, and is shielded by 10 cm lead and 55 cm water. Four control and safety rods containing fuel are inserted from the bottom of the core. The neutron flux is monitored by three BF_3 detectors located in the shielding water. A very useful characteristic of the system is its high analytical sensitivity.

OUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
- IV. PROCEDURE
 - A. Reactor Checkout
 - B. Approach to Critical
 - C. Period Measurements
- V. RESULTS
- VI. ANALYSIS AND CONCLUSIONS
- VII. BIBLIOGRAPHY
- VIII. APPENDICES
 - A. Discussion Questions
 - B. AGN Characteristics
 - C. Description of Argonne's AGN-201 Reactor

I. INTRODUCTION

Since the use of a nuclear reactor as an experimental machine requires a thorough knowledge of its physical makeup, the first part of this experiment will be devoted to a description of the reactor components and equipment and a visual inspection of the facility. A daily reactor checkout will then be performed to introduce the operation and safety aspects of the reactor. The reactor checkout includes a step-by-step check of the reactor conditions, the neutron-monitoring instruments, and the scram circuits.

The concepts involved in an approach to critical with the fine control rod will be brought out in the latter part of the experiment. The approach to critical will be performed with the intention of showing the participant what is occurring in the reactor core during startup. The experiment will actually simulate the last steps of a criticality experiment, which is the basic experiment performed to determine the critical mass of a new system. The final result of the experiment will yield the position of the fine control rod at which the reactor becomes a self-sustaining neutron-chain-reacting system.

II. THEORY

When a neutron source is placed near a subcritical reactor, the fissions induced in the fuel multiply the source neutrons entering the core. The multiplication is defined as the ratio of the total thermal-neutron flux due to source neutrons and fission neutrons to the flux due to the source alone. The degree of multiplication is dependent upon the magnitude of the reproduction factor of the assembly. The equation relating k_{eff} and multiplication is readily derived from the sketch below, in which it is assumed that Q neutrons are introduced each second.

Fig. 2.1. Neutron Multiplication

The population at the present time (1) is given by

$$Q + k_{\text{eff}} Q + k_{\text{eff}}^2 Q + k_{\text{eff}}^3 Q + k_{\text{eff}}^4 Q + \dots$$

The multiplication, M , of the source neutrons as previously defined equals

$$M = \frac{Q + k_{\text{eff}} Q + k_{\text{eff}}^2 Q + k_{\text{eff}}^3 Q + \dots}{Q}$$

Since this expression has been derived for a subcritical reactor, $k_{\text{eff}} < 1$, the expression for M may be rewritten after cancelling out Q as

$$M = \frac{1}{1 - k_{\text{eff}}}$$

It can be noted from this equation that as k_{eff} approaches one, the multiplication approaches infinity, and $1/M$ would approach zero.

The experimental arrangement for the approach to critical through use of the fine control rod can be seen in the diagram below. The measurement of the neutron population at the detector location can be assumed to be proportional to the thermal-neutron flux in the reactor core. With the fine control rod at the edge of the core, the reproduction factor equals $(k_{\text{eff}})_0$. Let C_0 equal the measurement of the neutron population by the detector. If the rod is moved into the core to a desired position, the reproduction factor becomes $(k_{\text{eff}})_i$ and the new neutron population measurement C_i . With all neutron measurements

Fig. 2.2. Experimental Arrangement

normalized to C_0 , the multiplication at any rod position is given by $M_i = C_i/C_0$. A linear plot of the inverse of the multiplication versus rod position can be extrapolated to a $1/M$ value of zero corresponding to the fine control rod in its critical position. The linear extrapolation is based on the assumption that the change in the reproduction factor per centimeter movement of the fine rod is constant. This assumption has been found to be true only after the rod has been moved past 8 cm of its total 24-cm travel.

When a reactor is made supercritical, $k_{\text{eff}} > 1$, the neutron population will begin to rise to higher levels. This rise in neutron level can be described as a summation of exponentials:

$$\phi(t) = \sum_{i=0}^m A_i e^{\omega_i t}$$

where the A_i 's are constants determined by the initial conditions of the reactor and the ω_i 's are the $m + 1$ roots of the fundamental inhour equation. The rate at which the flux rises is dependent upon the amount that k_{eff} is above unity. All the terms in the expression above, except the first, have negative exponents when positive reactivity is added, and their contribution to the flux decreases rapidly to zero after a short time has elapsed. This summation then reduces to

$$\phi(t) = \phi_0 e^{\omega_0 t},$$

where

$$\phi_0 = \text{flux at } t = 0;$$

$$\omega_0 = \text{the inverse of the reactor period.}$$

The positive asymptotic period $T = 1/\omega_0$ can easily be determined from a plot of neutron population versus time. The period is defined as the time it takes the neutron level to increase by a factor of e after the transient terms have died out.

Another important parameter is the doubling time. This is defined as the time it takes the neutron level to double. It is related to the period by the following expression:

$$T = \frac{t_2}{\ln 2},$$

where t_2 is the doubling time. It is possible to determine the reactivity, $(k_{\text{eff}} - 1)/k_{\text{eff}}$, of the assembly, which corresponds to a certain measured positive period through the use of the basic inhour equation. This equation,

derived from the time-dependent diffusion equation, involves the delayed-neutron parameters and the thermal-neutron lifetime of the reactor in question.

III. APPARATUS

- A. Reactor and associated instrumentation for neutron counting.
- B. External BF_3 count system to be placed in access port number four (see Fig. 2.3).

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Fig. 2.3. BF_3 Proportional Counter and Associated Electronics

IV. PROCEDURE

A. Reactor Checkout: The daily reactor checkout is performed. This is a sequence of checks to assure that the reactor equipment is functioning properly.

B. Approach to Critical: After the reactor is taken to its subcritical position (safety rods in), the fine and coarse control rods are inserted to the position specified by the instructor. After waiting an appropriate length of time, record rod positions, temperature, and the initial readings of the following instruments:

- Channel 1. Count rate meter,
- Channel 2. Logarithmic micromicroammeter,
- Channel 3. Linear micromicroammeter, and
- Channel 4. BF_3 counter (placed in access port #4).

Insert the fine control rod a fraction of the way in while maintaining extreme vigilance to preclude the possibility of the reactor becoming supercritical without your knowledge. Wait for steady state, then record rod position, temperature, and instrument readings. Determine the inverse of the multiplication relative to the initial instrument readings. Plot the inverse of the multiplication versus rod position for each instrument. Estimate the critical rod position as indicated by this plot.

Select the next rod position on the basis of the previous data. Proceed in this fashion, approaching criticality in a decreasing incremental fashion. Record your final estimated critical rod positions.

C. Positive Period Measurement: After obtaining criticality, insert one of the control rods to a position which will yield a period of about 100 sec. Determine this period by measuring the doubling time of the power level on Channels 1 and 3. After the power has risen to about 10 mW, return rods to their critical positions, and make certain that the power remains constant. Are the critical rod settings the same as you estimated? From the chart recorder which has monitored the power rise, plot the power level versus time on semilog paper. Does the period remain constant with time? Why?

V. RESULTS

The student is to make his own data sheet. Include with results the plot of inverse multiplication versus rod position and of power level versus time.

VI. ANALYSIS AND CONCLUSIONS

A brief analysis of the experiment together with significant conclusions is left as an exercise for the student.

VII. BIBLIOGRAPHY

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VIII. APPENDICES

A. Discussion Questions

The following propositions should be considered prior to arrival at the AGN-201. An effort will be made to discuss them prior to and/or during checkout of the reactor, as time permits.

1. Discuss the possibility of having a neutron-chain-reacting system with:
 - a) fission;
 - b) the $(n,2n)$ reaction;
 - c) the (γ,n) reaction.
2. What is the importance to a thermal nuclear reactor of:
 - a) fission in U^{235} yielding η greater than 1;
 - b) neutron-induced fission in U^{235} ;
 - c) a moderator;
 - d) a reflector;
 - e) U^{238} ;
 - f) heat;
 - g) spontaneous fission?
3. What are delayed neutrons? Are they necessary to a neutron chain reaction? What is their practical import? Assume that there are no delayed neutrons - what control problems exist?
4. If we have a subcritical reactor containing a neutron source, what will its response be to a small positive step insertion of reactivity which makes the reactor less subcritical? Critical? Supercritical?
5. Assuming a subcritical assembly containing a source of thermal neutrons of strength Q , establish the relationship between source multiplication of the system and its k_{eff} in the steady state.
6. Conventionally, the reciprocal of counting rate at a fixed position in the assembly is plotted versus amount of fuel or versus withdrawal of poison. Relate these variables to those in the multiplication equation.
7. How will the reciprocal of the counting rate vary with the position of a control rod containing fuel? What shape do you expect this curve to have?
8. Why are the positions of the neutron detectors important relative to the source and core?

9. What do we mean by "critical" and what criterion may we use to determine that this condition exists in our assembly?
10. How does the critical position of a control rod vary with power level?
11. Among the products of fission there exist particles which are larger, and some which are more highly ionizing, than neutrons, as well as gamma rays and heat. Why do we elect to measure the neutron population?
12. Of what importance is the artificial neutron source to reactor startup and operation? What types of sources can be used?
13. In the AGN-201 reactor, positive reactivity is added by inserting fuel control rods in the upward direction from a position below the core. In the Argonaut reactor, positive reactivity is added by removal of absorbing control rods in the upward direction to a position above the core. In both cases, why is it desirable to add reactivity in the manner described?
14. What action should be taken if during the course of this experiment the detectors do not indicate the true situation in the reactor?
15. In addition to the reactor shielding, what other precautions are taken to prevent radiation exposure?

B. AGN Characteristics

The AGN-201 is a useful low-power reactor that has the distinct advantage of small overall size, which enables it to be readily located in existing buildings. The reactor is designed for use in education, research, medical diagnosis, and industrial control. The main design objectives were to make the reactor as safe and foolproof as possible, to use a minimum critical mass, and to provide a high analytical sensitivity.

The AGN-201 is a 100-mW, homogeneous, thermal reactor. The core is made up of a series of circular polyethylene discs, 25 cm in diameter. It contains 20% enriched UO_2 and has a critical mass of approximately 650 g U^{235} . The cylindrical core is surrounded by a graphite reflector, 20 cm thick, and is shielded by 10 cm lead and 55 cm water.

The core and part of the reflector are contained in an aluminum tank which has re-entrant thimbles in its base, into which four control and safety rods are inserted. Each rod contains fuel sealed in an aluminum capsule, so that reactivity increases as rods are inserted. Both safety rods and one control rod are held in position by electromagnets for scram purposes. Each of the four rods is driven by a lead screw.

The core is divided into half-sections with a plane of separation occurring at the level of a one-inch diameter through "glory" hole. The lower section is held in place by a thermal fuse which is designed to soften at 100°C , and when it functions as specified, permits the section to drop 2 in. to the bottom of the core tank. The excess reactivity is limited to a maximum of 0.25 percent or a minimum period of about 15 sec by the amount of fuel available.

The two safety rods and the coarse control rod are each worth approximately 1.3 percent reactivity. The fine control rod, fuel loaded, is worth about 0.3 percent. The neutron flux is monitored by three detectors: two BF_3 ionization chambers and one BF_3 proportional counter, all of which are located in the water tank just outside the lead shield. The detectors are connected respectively to a logarithmic micromicroammeter, a linear micromicroammeter, and a pulse amplifier and count rate meter located on the reactor console. Each indicator is connected through a sensitrol relay to a scram circuit. Additional safety interlocks provide for reactor shutdown if the level of the shield water drops, if the reactor temperature falls below 16°C , or if an earthquake occurs. Sequential interlocks are also present to ensure that the proper operational method is followed.

C. Description of Argonne's AGN-201 Reactor

1. Photographs and Sketches

These are shown as in Figs. 2.4 through 2.8.

139-2669

Fig. 2.4. Reactor in Present Location

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Fig. 2.5. Reactor Console

Fig. 2.6
AGN-201 Reactor Unit

Fig. 2.7. AGN-201 Core Tank and Contents

Fig. 2.8
AGN-201 Control Rod Assembly

Fig. 2.9
Fine Control Rod Calibration Curve

2. Reactor Characteristics

a. General

Type	Homogeneous thermal reactor
Principal Uses	Education and Training
Maximum Operating Power	100 mW
Core	UO ₂ (20% enriched in U ²³⁵) particles homogeneously distributed in solid polyethylene plastic moderator, cylindrical in shape
Reflector	Graphite
Shield	Lead and water
Control	Two safety and two control rods, fuel loaded

b. Fuel

Fuel Material	20% enriched UO ₂
Fuel Disc	Right circular cylinder fabricated from a mixture of fuel material and polyethylene powder
	Specifications:
	UO ₂ powder
	U ²³⁵ enrichment - $19.5 \pm 0.5\%$
	Particle size - $15 \pm 10 \mu$
	Polyethylene powder
	Particle size - 100μ
	Purity - commercial
Approximate Core Fuel Disc Sizes and U ²³⁵ Content	9 in number (25.6-cm diameter) Four discs, 4 cm high (96 g each) Three discs, 2 cm high (58 g each) Two discs, 1 cm high (29 g each)
Approximate Control and Safety Rod Fuel Disc Sizes and U ²³⁵ Content	12 discs, 4.7-cm diameter, 4 cm high (3.6 g each) 4 discs, 2.3-cm diameter, 4 cm high (0.9 g each)
Total Fuel Loading	664 g
U ²³⁵ Density	0.056 g U ²³⁵ /cm ³
Normal Lifetime of Fuel Discs	Indefinite
Core Fuse	Small, right circular cylinder, 2.2 cm in diameter, 0.9 cm high, 0.40 g U ²³⁵ . Fabricated from mixture of fuel material and polystyrene powder

c. Reactor

Core-containing Vessel	Gas-tight, 65-mil (61ST) aluminum cylindrical tank, 32.2 cm in diameter and 76 cm high. Vessel has a 1-in. glory hole on the horizontal center line and appropriate control rod thimbles. Tank contains core fuel discs and graphite reflector plugs.
Core Description	9 fuel discs, 25.6 cm in diameter, 23.8 cm high, separated in half at glory-hole mid-plane by a thin aluminum baffle. Lower half of core contains appropriate holes from control and safety rod thimbles and is part of the safety fuse system.
Reflector	Heavy-density (1.7 g/cm^3) graphite, approximately 20 cm on all sides of core. Top and bottom reflector in core tank, side reflector surrounds core tank and contains four 4-in. through-holes running tangential to core tank.
Gamma Shield	10 cm of lead completely surround reflector.
Reactor Tank	$\frac{5}{16}$ -in.-wall steel tank, 95-cm diameter, and 148 cm high; contains core tank, reflector, and lead shield, and appropriate holes for control and safety rods, glory hole, and access ports. Gas-tight vessel when all seals are made. Upper portion of tank contains a removable thermal column tank.
Neutron Shield	55 cm of water surrounds the reactor tank except at its bottom face.
Water Tank	Steel tank serving as the main structural tank, $6\frac{1}{2}$ ft in diameter and contains 1,000 gal of water. Tank is supported at its bottom by the reactor skirt.
Reactor Dimensions	$6\frac{1}{2}$ ft in diameter and $9\frac{1}{2}$ ft high.
Reactor Weight	15,000 lb (less shield water).
Reactor Control	Two safety rods, 14.5 g U^{235} each. One coarse control rod, 14.5 g U^{235} . One fine control rod, 3.6 g U^{235} . All rods, with the exception of the fine control rod, which is mechanically coupled, are magnetically coupled to carriage which is

driven in a vertical direction on a lead screw by a reversible AC motor. Total travel is about 24 cm.

Experimental Facilities

7/8-in. glory hole passing through the center of core at core median plane. Four, 4-in.-diameter access ports passing through the graphite reflector tangential to the core. Thermal column tank above the core.

Neutron Source

Ra(Be) source utilizing (α, n) reaction; 10 mg of Ra mixed homogeneously with Be powder.

d. Nuclear Data

(1) Fuel Loading

(a) Approximate Critical Mass 660 g U^{235}

(b) Excess Reactivity at 20° C
with Glory Hole Empty 0.18% $\Delta k/k$

(2) Flux

Average Thermal Flux 3.0×10^6 n/cm²/sec at 100 mW

Peak Thermal Flux 4.9×10^6 n/cm²/sec at 100 mW

(3) Reactivity Worth of Reactor Components and Parameters

(a) Safety and Coarse Control Rods 1.25% $\Delta k/k$ (each)

(b) Fine Control Rod
Fuel-loaded 0.310% $\Delta k/k$
Polystyrene-loaded 0.155% $\Delta k/k$

(c) Fine Control Rod Fuel in
Glory Hole
At Core Surface 0.042% $\Delta k/k/gm$
At Core Center 0.100% $\Delta k/k/gm$

(d) Polystyrene in Glory Hole
(completely filled) 0.29% $\Delta k/k$

(e) Access Port Plugs
1 Wood Slug 0.002% $\Delta k/k$
1 Section Pb 0.015% $\Delta k/k$
1 Section c 0.194% $\Delta k/k$
Total Worth of Plugs in One
Access Port 0.422% $\Delta k/k$

- (f) Temperature Coefficient of Reactivity (Approximate) $-0.025\% \Delta k/k/^{\circ}\text{C}$
- (g) Reactor Sensitivity at Core Center, Measured with $1/v$ Absorber $-0.14\% \Delta k/k/\text{cm}^2$

(4) Pertinent Figures

- (a) Control Rod Calibration Given in Fig. 2.9.
- (b) Inhour Equation Given in Fig. 2.10.
- (c) Flux Plot Given in Fig. 2.11.

Fig. 2.10. AGN-201 Inhour Equation

Fig. 2.11. Horizontal Thermal Flux Plot at 100 mW

3. Instrumentation

The flux-monitoring devices for the AGN-201 reactor consist of one BF_3 proportional counter and two BF_3 ionization chambers. These neutron detectors are located in the shielding water tank just outside the reactor tank (as shown in Fig. 2.12). They are placed in water-tight aluminum cans and can be removed from the reactor through the manhole cover at the top of the water tank.

Fig. 2.12
Neutron-detector
Location

Channel 1

The BF_3 proportional counter, (see Fig. 2.13) which is cadmium-covered, is provided with a preamplifier mounted above the counter in the water-tight can. The output of the preamplifier is sent to a count rate meter circuit located in the reactor console. At present a Tracerlab SC-34A Precision Ratemeter is being used. This instrument provides high- and low-voltage power supplies. The high-voltage supply is continuously variable from 500 to 2500 V. Calibration of the instrument meter can be made with an internal 60-cps circuit. An aural monitor circuit, containing a 2-in. loud-speaker, is also available. A click is produced for each pulse generated by the counter. The 0-200- μ Amp read-out meter indicates the counting rate directly in counts per minute, and a switch circuit provides for indication of the high-voltage output.

Fig. 2.13. Channel 1 BF_3 Proportional Counter

Whenever the Zero-Operate-Cal switch is removed from its operate position, the output to the sensitrol relay is shorted by a switching circuit. The sensitrol relay will then drift toward zero, eventually giving a low-level trip. This condition is provided to ensure that the instrument cannot be made inoperative during the operation of the reactor. The same is true for the CPM-Meter-H.V. switch; if the switch is turned to the H.V. position, the low-level scram will occur on Channel 1. The output of the count rate is fed to a Varian recorder and the corresponding sensitrol relay of the channel.

Channel 2

The output of one of the BF_3 ionization chambers is fed to a logarithmic micromicroammeter (see Fig. 2.14) located in the reactor console. This ammeter (Keithly Model 412) is a vacuum tube electrometer

covering the range from 10^{-7} to 10^{-13} Amp without range switching. The logarithmic scale is obtained by negative feedback from output to input with a diode whose emission is limited by negative plate-to-cathode voltages. The ammeter provides an internal calibration circuit consisting of a mercury battery and appropriate resistors. The calibration is performed by adjusting potentiometers located on the instrument front panel to a corresponding calibration signal read properly on the read-out meter.

Fig. 2.14. Channel 2 Logarithmic $\mu\mu$ ammeter

An external positive current source is also available for testing this instrument. The operate switch, when turned to current source, will feed the output of the positive current source into the output of the ammeter. The operate switch becomes a hold switch in this position. The output of the Keithly ammeter is fed to the Varian recorder, the Channel 2 sensitrol relay, an external period meter circuit, and the period scram circuit. When the Current Source-Operation-Calibrate switch is placed in the calibrate position, a switching circuit disconnects the output of the ammeter and causes a low-level scram on the Channel 2 sensitrol relay.

Channel 3

The output of the other BF_3 ionization chamber is fed to a linear micromicroammeter (see Fig. 2.15) located in the reactor console. This ammeter, Keithly Model 410, has a range from 3×10^{-13} to 10^{-3} Amp. The maximum current used during reactor operation is 1×10^{-8} Amp. The instrument is a vacuum tube electrometer designed for measuring small currents.

Fig. 2.15. Channel 3 Linear $\mu\mu$ ammeter

An external current source is used for calibration of the instrument. This is accomplished by turning the operate switch (a hold switch) to the current source position and adjusting the current source output to a desired signal. The output of this instrument is fed to its corresponding sensitrol relay and the Varian recorder.

Positive Current Source

The positive current source, located in the reactor console, is used as an external current source to calibrate the micromicroammeters and to check the performance of the scram circuits. Adjusting the current source to a desired output current is simply done by setting an appropriate voltage indicated on the read-out meter and selecting the proper resistor by which the voltage is divided.

Recorder

A Varian Model G-11 strip chart recorder is located in the reactor console. A three-position selector switch is available to record any one of the outputs of the neutron-monitoring channels of this instrument. The main recorder controls consist of a damping control, a zero-adjust control, and a calibration control for adjusting the span of the instrument.

There are four different chart speeds acquired by proper selection of two separate speed chart drive motors, high and low, and two gear ratios, high and low. The times required for the chart to move one inch for each chart speed are listed in the table below. A standby position on the chart "High-Low" switch exists. This removes power from the chart drive motors. The recorder is normally left on with this switch in the "Standby"

position when the reactor is shut down. A neutral position is available on the Chart Speed Shifter, which selects the desired gear ratio. The neutral position disengages the chart drive drum.

<u>Chart "High-Low" Switch</u>	<u>Chart Speed Shifter</u>	<u>Sec/in.</u>
Low	Low	480
Low	High	120
High	Low	30
High	High	7.5

Period Meter

A read-out period meter circuit is mounted on top of the reactor console. This instrument operates from the output of the logarithmic micro-microammeter. A simple RC differentiating network is used with a $\pm 25 \mu\text{Amp}$ readout meter. Adjustments are available for the time constant of the circuit and the calibration of the meter. The period range of the meter extends from infinity to 10 sec. An on-off switch is available for the purpose of protecting the instrument when large transients are purposely placed on the output of Channel 2.

Area Gamma Monitor

A model CD-179 gamma monitor is mounted on the top of the reactor console. This instrument is a sensitive DC current-measuring device with a trip circuit which operates a relay when the current exceeds a preset level. An audible alarm is then given. The CD-179 receives its input from a nitrogen-filled ion chamber 1C-34, which is placed near the reactor tank. This chamber has a sensitivity of 10^{-9} Amp per R/hr. The range of the gamma monitor is from 0.001 to 10 R/hr. The instrument is calibrated and tested by means of an internal calibration circuit, which is also used to set the trip level of the instrument. The normal trip level of the instrument is set in the range from 2 to 3 mR/hr.

Scram Network

The scram chassis, which is located in the reactor console, contains most of the scram circuitry in the reactor control system. The instrument panel of this chassis contains the following:

- Sensitrol relays
- Scram lights (relays and period)
- Reset buttons (relays and period)
- Interlock lights (open and closed)
- Magnet on button
- Magnet on light
- Magnet ground light (B+ fuse blown).

The main scram system, which is put to use when a high- or low-level trip in any of the sensitrol relays occurs, utilizes a negative voltage supply which cuts off the current passing through an electron tube. This method of scrambling the reactor also is used when a period trip occurs or the interlock circuit opens. Figure 2.16 below is a simplified diagram of the system in question.

Fig. 2.16. Simplified Diagram of Scram Circuit

The negative voltage is applied to the control grid of the 6L6. This voltage cuts off the plate current which is used in part to energize the safety and coarse rod magnets. The safety and coarse control rods drop out of the core to their down position. The scram relay K-4 also opens, giving an audible alarm, lighting the scram light, and turning off the "magnets on" light. As seen in the diagram, if the negative voltage supply fails, relay K-7 opens and the B+ to the 6L6 electron tube is removed, holding the reactor in a scram condition. The two other methods for scrambling the reactor, manual scram and stop button which removes

the AC power from the scram chassis and motor panel, function so as to remove the B+ voltage used by the 6L6 electron tube. The manual scram button, which is normally closed, is installed in the B+ line. When the button is pushed downward, this line is physically broken, cutting off the scram tube and scrambling the reactor. When the AC power is removed from the scram chassis by pushing the stop button, the B+ power supply is turned off and the reactor is scrambled.

After the reactor has been scrambled by any one of the previously stated conditions, except the stop button, the rod carriages of all four safety and control rods are automatically driven out to their down position. If the AC power is turned off by the stop button, the power to the drive motors is also turned off and the carriages remain in their scram position. When the AC power is turned back on, the carriages are automatically driven to their lower limits.

All the AC power to the console is supplied through a main power switch which is located inside the reactor console. Loss of the main power will, of course, scram the reactor and leave the rod carriages in their scram position.

Interlock System

Before and during the operation of the reactor, a safety interlock circuit must be satisfied to remove the reactor from a scram condition. The interlock circuit is a simple continuity circuit containing a number of switches and devices which assure that certain safety components of the system are in working order. The block diagram of Fig. 2.17 describes the interlock circuit.

Fig. 2.17. Interlock Circuit

If the interlock circuit opens, relay K-5 opens and a negative voltage is placed on the grid of the scram tube, cutting the tube off and hence scrambling the reactor.

The shielding-water switch, which consists of a water-tight micro switch and an actuator connected to a float bob, indicates whether or not the shielding water level is at the proper height in the water tank.

The earthquake switch consists of a brass ball mounted delicately on two terminal strips. If the reactor receives a physical shock, the ball will move and break the electrical contact being made.

The low-temperature switch, which has been calibrated to open at 16°C , is a simple bimetal thermal switch. As the temperature of the switch reaches this low level, the bimetal strip which makes up one side of the switch bends, breaking the interlock circuit. The bending action takes place because the two metals used in the strip have different linear coefficients of expansion.

To insure that no instrument can be made inoperative during the operation of the reactor by holding its reset button down, all reset buttons are included in the interlock circuit. Pushing one of these buttons physically breaks the continuity interlock circuit and causes a scram condition to exist. In addition, all cable connectors to the control and safety rods have two pins, which have been wired into the interlock circuit. Any connector which is not connected properly will therefore open the interlock circuit. If any of these circuits are interrupted, the "interlock open" light on the scram chassis panel will be lighted.

STUDENT RESULTSAPPROACH TO CRITICALDATA

Position of Fine Control Rod, cm	Ch 1 (cpm)	Ch 2 (Amp)	Ch 3 (Amp)	Ch 4 (cpm)	$\frac{C_0}{C_0/C_i}$
0	850 1	1.8×10^{-11} 1	1.8×10^{-11} 1	6701 1	
5	925 0.919	2.0×10^{-11} 0.900	1.95×10^{-11} 0.924	7419 0.902	
9	1050 0.810	2.3×10^{-11} 0.783	2.25×10^{-11} 0.800	8279 0.809	
12	1350 0.640	3.0×10^{-11} 0.600	2.95×10^{-11} 0.611	10330 0.648	
15	1725 0.493	4.0×10^{-11} 0.450	4.0×10^{-11} 0.450	13630 0.491	
18	2600 0.327	6.2×10^{-11} 0.291	6.2×10^{-11} 0.291	21300 0.311	
21	6700 0.127	1.7×10^{-12} 0.106	1.8×10^{-12} 0.100	57010 0.117	

Estimated critical rod setting = 22.9 + zero off-set 0.09
= (22.99)

Actual critical rod setting for the fine rod was found to be 22.83.

Positive period measurement and k_{ex} determination.

Coarse Rod 24.50 cm doubling time = 43.6 sec
Fine Rod 22.82

$$\phi = \phi_0 e^{t/T}$$

$$T = \frac{t_2(\text{doubling time})}{\ln 2} = \frac{t_2}{0.693} = 63 \text{ sec.}$$

From the plot of the inhour equations relating the stable reactor period to k_{ex} , $k_{ex} = 0.10\% \Delta k/k$.

Fig. S2.1

CONTROL ROD CALIBRATION, ARGONAUT REACTOR

by

L. L. Lawyer

ABSTRACT

It is the purpose of this experiment to calibrate an Argonaut control rod in terms of reactivity versus rod position by means of the reactor kinetic equations and to observe other examples of kinetic reactor behavior. Both the "positive-period" and "rod-drop" methods are utilized and an intercomparison of the two methods is made.

OUTLINE

- I. INTRODUCTION
- II. THEORY
 - A. General
 - B. Derivation
 - C. Positive Reactivities
 - D. Negative Reactivities
- III. APPARATUS
- IV. PROCEDURE
- V. RESULTS
- VI. ANALYSIS OF RESULTS AND CONCLUSIONS
- VII. BIBLIOGRAPHY
- VIII. APPENDIX
 - Discussion Questions

I. INTRODUCTION

When attempting to determine quantitatively the relative worth of various reactor parameters, it is convenient, if not absolutely necessary, that the effect of a variation in those parameters be assigned some standard unit of effect for the reactor. The effect most commonly used is that on reactivity, which can be expressed in any of several units. An additional convenience is obtained by using a control element as a calibrated secondary standard.

The reactivity equivalent of a control rod may be obtained, at least roughly by calculation, but ultimately the values must be checked by direct measurement. In an earlier experiment we studied the subcritical operation of the reactor and determined criteria for criticality. Using this knowledge, we want in this experiment to investigate the reactor's kinetic behavior as it deviates from criticality, both for positive reactivities ($k > 1$) and negative reactivities ($k < 1$), and, by means of the reactor response to control rod movements, to calibrate the control rod in terms of reactivity.

II. THEORY

A. General: In the following section, it will be our purpose to establish those relationships which govern the response of the reactor flux to simple transients in reactivity in order that we may measure the magnitude of those transients. We require a quantitative relationship between flux and reactivity. From our knowledge of the fission process and of reactors, we would imagine that this relationship would also be time- and space-dependent, but we can essentially eliminate these dependences by fixing time and space, i.e., by considering only one value of time and space.

The theory necessary to this experiment is more thoroughly treated in Reference 6. The following comments are designed to present these derivations in an abbreviated and less rigorous fashion.

We assume a very general relationship between flux and reactivity, and then find the limitations on and the exact function of the reactivity. In order to find these limitations, we begin by establishing the diffusion equation for thermal neutrons in a nonequilibrium system. We next modify that diffusion equation for the effect of delayed neutrons and simplify. This gives us a differential equation involving flux and delayed-neutron-precursor density as space and time variables. Next, we write an equation for the net rate of formation of the delayed-neutron precursors. This gives us two linear first-order differential equations so that we may restrict the reactivity and time functions which we originally assumed. Then, by imposing initial conditions, we can eliminate the delayed-neutron-precursor density from the modified diffusion equation. Finally, we simplify the modified diffusion equation and arrange it in the standard "characteristic inhour equation" form, which relates our original function of reactivity to some of the nuclear properties of the materials present in the reactor.

B. Derivation: As mentioned above, we begin by assuming our flux versus reactivity equation in a general way. We expect the flux ϕ to be an exponential function involving reactivity ρ and time in addition to the nuclear properties of the reactor.

Let us assume the equation to be of the form

$$\begin{aligned} \phi = & f_0(\rho)e^{g_0(\rho)h_0(t)} + f_1(\rho)e^{g_1(\rho)h_1(t)} + \dots + f_i(\rho)e^{g_i(\rho)h_i(t)} \\ & + \dots + f_n(\rho)e^{g_n(\rho)h_n(t)} \end{aligned} \quad (1)$$

We must find the functions f_i , g_i , and h_i ; $i = 1, 2, \dots, n$.

To determine these functions, we write the diffusion equation for thermal neutrons in a nonequilibrium system:

$$D\nabla^2\phi - \Sigma_a\phi + S = \frac{1}{v} \frac{\partial\phi}{\partial t}, \quad (2)$$

where $D\nabla^2\phi$ is the neutron leakage per cm^3 per second, $\Sigma_a\phi$ is the number of thermal neutrons per cm^3 per second being absorbed, and

$$\frac{1}{v} \frac{\partial\phi}{\partial t} = \frac{\partial n}{\partial t}; \quad \phi = nv.$$

Further, S is the thermal-neutron source term due to fissions:

$$S = k \Sigma_a \phi e^{-B^2 \tau}, \quad (3)$$

where k is the infinite multiplication constant and $e^{-B^2 \tau}$ is the nonleakage probability during slowing down.

Since we are considering a critical reactor, we may assume that there is no primary source present. However, since we are interested in the effects of delayed neutrons, we must consider our source term to be composed of two parts: the source of prompt fission neutrons and the source of delayed fission neutrons. If we let β represent the fraction of the total number of fission neutrons that are delayed and β_i the fraction that are associated with a single time of delay (a single precursor), then $(1 - \beta)$ is the fraction of fission neutrons that are emitted promptly. We can then write our prompt source term as before, except that it must be decreased by a factor $(1 - \beta)$. An additional source term must be included for the delay-neutron source. The rate of formation of delayed neutrons of the i^{th} kind is equal to the rate of decay of its precursor, which is $\lambda_i C_i$, where λ_i is its decay constant and C_i its density. Thus, the total

rate of formation of delayed neutrons will be $\sum_{i=1}^m \lambda_i C_i$. The thermal-

neutron source term due to delayed neutrons is just this rate of formation times the resonance escape probability times the nonleakage probability. Then

$S = \text{prompt source term} + \text{delayed source term}$

$$= (1 - \beta) k \Sigma_a \phi e^{-B^2 \tau} + p e^{-B^2 \tau} \sum_{i=1}^m \lambda_i C_i. \quad (4)$$

Our modified diffusion equation is then

$$D \nabla^2 \phi - \Sigma_a \phi + \left[(1 - \beta) k \Sigma_a \phi e^{-B^2 \tau} + p e^{-B^2 \tau} \sum_{i=1}^m \lambda_i C_i \right] = \frac{1}{v} \frac{\partial \phi}{\partial t}, \quad (5)$$

where ϕ and C_i are functions of both space and time.

If the space and time variables are separable (which requires that reactivity be changed in a step fashion), $\nabla^2 \phi$ may be replaced by $-B^2 \phi$ in Eq. (5). The result simplifies to the differential equation

$$[k_{\text{eff}}(1 - \beta) - 1] \phi + \frac{p k_{\text{eff}}}{k \Sigma_a} \sum_{i=1}^m \lambda_i C_i = \ell \frac{d\phi}{dt}, \quad (6)$$

obtained by dividing through by Σ_a , substituting L^2 for D/Σ_a , dividing through by $1 + L^2 B^2$, and introducing the definitions

$$k_{\text{eff}} = \frac{k e^{-B^2 \tau}}{1 + L^2 B^2}$$

and

$$\ell = \frac{\ell_0}{1 + L^2 B^2},$$

where

ℓ_0 = neutron lifetime in an infinite medium;

ℓ = neutron lifetime in a finite medium.

Eq. (6) is a differential equation involving flux and delayed-neutron-precursor density as space and time variables.

In the next step the delayed-neutron-precursor density is eliminated by writing an additional differential equation for the net rate of formation of the delayed-neutron precursors and simultaneously imposing initial conditions on these two differential equations.

Since the overall rate of production of fission neutrons, including both prompt and delayed, is $(k/p)\Sigma_a\phi$, the rate of formation in the fission process of the precursors of the i^{th} group is $\beta_i(k/p)\Sigma_a\phi$. Also, since C_i is the concentration of these precursors, their rate of radioactive decay is $\lambda_i C_i$ (where $-\lambda_i C_i$ is the rate of formation). Then the net rate of formation of the delayed-neutron precursors of the i^{th} group is

$$\frac{\partial C_i}{\partial t} = \beta_i \frac{k}{p} \Sigma_a \phi - \lambda_i C_i, \quad (7)$$

where again C_i and ϕ are functions of both time and space.

The solution of the differential Eqs. (6) and (7) will be of the form

$$\phi = \phi_0 e^{\omega t}; \quad C_i = C_{i0} e^{\omega t}, \quad (8)$$

which restricts the functional relationship originally assumed in (1) in that

$$h_i(t) = t.$$

By substituting the relationships (8) into Eq. (7) we may evaluate C_{i0} in terms of ϕ_0 , ω , and the reactor constants. Substitution of this result together with Eq. (8) into (6) gives

$$k_{\text{eff}}(1 - \beta) - 1 + k_{\text{eff}} \sum_{i=1}^m \frac{\lambda_i \beta_i}{\omega + \lambda_i} = \ell \omega. \quad (9)$$

By means of the definitions

$$k_{ex} \equiv k_{eff} - 1$$

and

$$\rho \equiv \frac{k_{ex}}{k_{eff}} = \frac{k_{eff} - 1}{k_{eff}},$$

Eq. (9) reduces to the "characteristic inhour" equation

$$\rho = \frac{\ell\omega}{\ell\omega + 1} + \frac{1}{\ell\omega + 1} \sum_{i=1}^m \frac{\omega\beta_i}{\omega + \lambda_i}, \quad (10)$$

which is an algebraic equation of degree $m + 1$ in ω , so that there are $m + 1$ values of ω corresponding to each value of the reactivity ρ . Thus, we see that in Eq. (1) $g_i(\rho)$ is just the parameter ω_i and that m refers to the number of delayed-neutron-precursor groups. A plot of Eq. (10) is given as Fig. 2 in Reference 2. Equation (1) may now be rewritten in the form,

$$\frac{\phi(t)}{\phi(0)} = \sum_{j=1}^7 A_j e^{\omega_j t}. \quad (11)$$

where

$$A_j = (1 - \rho) \frac{\ell + \sum_{i=1}^6 \frac{\beta_i}{(\omega_j + \beta_i)}}{\ell(1 - \rho) + \sum_{i=1}^6 \frac{\beta_i \lambda_i}{(\omega_j + \lambda_i)^2}} = \frac{f_i(\rho)}{\phi_0}. \quad (12)$$

Here ϕ_0 is the flux just before a step change in reactivity and $\phi(t)$ is the time-dependent flux after the step change.

C. Positive Reactivities: If the reactivity is positive of the $m + 1$ roots of Eq. (10) m are negative and one is positive. We also note that each of the negative ω 's numerically is of the same order as one of the decay constants, λ_i . Therefore, as the time t after the step change increases, the contributions of all terms beyond the first on the right-hand side of Eq. (11) decrease rapidly to zero, because of the negative ω values, the first term increases with time because of its positive ω value. Hence, after a very short time, Eq. (11) reduces to

$$\frac{\phi(t)}{\phi(0)} = A_0 e^{\omega_0 t}.$$

We may now define the stable reactor period or the asymptotic reactor period, T , as

$$T \equiv 1/\omega_0,$$

where T is the reactor period after the lapse of sufficient time for the contributions of the transient terms to damp out. A plot of this period versus the reactivity required to produce it for the Argonaut is given in Fig. 3.6.

D. Negative Reactivities: Again referring to Eq. (10), if ρ is negative, all $m + 1$ values of ω are negative. Equation (11) will then consist of $m + 1$ terms all of which have negative exponents. The flux ratio will thus decrease with time. The first term, with the smallest numerical value of ω , namely, ω_0 , will decrease at a slower rate than the others. Therefore, for any input of reactivity (negative), the reactor will finally yield a negative stable period equal to $1/\omega_0$. Figures 3.4 and 3.5 give the numerical solution in graphical form of Eq. (11) for negative reactivities.

III. APPARATUS

The control element to be calibrated is one of the Argonaut control rods normally used for control of the reactor (see Figs. 3.1 and 3.2). Data sheets are provided as part of Section V. Working models of the necessary curves are included as Figs. 3.3, 3.4 and 3.5. These curves were taken directly from Reference 2. No additional equipment is necessary.

IV. PROCEDURE

Figure 5 of Reference 2 can be used to determine the negative reactivity input required to produce a given flux response as a function of time. Use of these curves presupposes that the reactor is in a "one-slab loading" condition and that the assembly is critical. If these conditions are fulfilled, one need merely cause the control rod being calibrated to be inserted fully and rapidly into the assembly by means of the rod-drop pushbutton on the reactor console and to read the flux response of the system from detection instruments and strip-chart recorders provided (see Fig. 3.6). This procedure will be iterated with successively larger reactivity inputs, noting the portion of the rod travel for each input, until finally the entire rod travel is used as an input. The reactivity values thus obtained can be plotted versus the position from which the rod began to fall to obtain an integral calibration or "worth" curve.

111-2425

Fig. 3.1. Control Rod

111-2415

Fig. 3.2. Control Rod Drive Mechanisms

Fig. 3.3
Flux Ratio $\phi(t)/\phi(0)$ Versus Negative
Reactivity Step ρ Dollars at Time t
Seconds after Step (0 to 0.35 \$ Range)

Fig. 3.4
Flux Ratio $\phi(t)/\phi(0)$
Versus Negative Reactivity Step ρ Dollars
at Time t Seconds
after Step (0-4.1 \$ Range)

Fig. 3.5. Positive Reactivity ρ Dollars Versus Positive Asymptotic Period t Seconds

139-1786

Fig. 3.6. Control Console of Argonaut

An alternative method of calibrating the rod is as follows:

The reactor is made critical with the rod to be calibrated in its most fully inserted position. The rod is then withdrawn a known amount and, after waiting for the flux to be represented by the dominant term (~ 90 sec for most practical values of reactivity), the e-folding time of the reactor flux is measured. A control rod other than the one being calibrated is then used to return the reactor to criticality at a low flux. The rod being calibrated is then again withdrawn a known amount and the above procedure repeated. The reactivity equivalents of the periods can then be found from the computed curve (Fig. 3.6).

It should be noted that, while calibrating the control rod under consideration, the control rod used to compensate is also in effect being calibrated.

By means of the available systems for insertions of positive and negative reactivities, the reactor response to various types of transients may be observed and qualitatively analyzed. A typical transient is discussed in response to Question 5 of the Appendix.

V. RESULTS

The following data sheet will serve as a guide and assist in the collecting of the data. Additional data sheets, if required, will be available at the time of the experiment.

ROD DROP DATA

Chart zero _____ Shim rod _____ Coarse rod _____ Fine rod _____ to _____

Time after
drop (sec) ϕ gross ϕ net ϕ_t/ϕ_0 $\rho(\%)$

0

 ρ_{av} _____ \$ = _____ %

10

20

30

40

50

Chart zero _____ Shim rod _____ Coarse rod _____ Fine rod _____ to _____

Time after
drop (sec) ϕ gross ϕ net ϕ_t/ϕ_0 $\rho(\%)$

0

 ρ_{av} _____ \$ = _____ %

10

20

30

40

50

Chart zero _____ Shim rod _____ Coarse rod _____ Fine rod _____ to _____

Time after
drop (sec) ϕ gross ϕ net ϕ_t/ϕ_0 $\rho(\%)$

0

 ρ_{av} _____ \$ = _____ %

10

20

30

40

50

Chart zero _____ Shim rod _____ Coarse rod _____ Fine rod _____ to _____

Time after
drop (sec) ϕ gross ϕ net ϕ_t/ϕ_0 $\rho(\%)$

0

 ρ_{av} _____ \$ = _____ %

10

20

30

40

50

VI. ANALYSIS AND CONCLUSIONS

The integral control rod calibration curve should be qualitatively compared with the curve predicted in Question 4 of the Appendix. In addition, the values of reactivity obtained from the "period-measurement" method should be quantitatively compared with those obtained by the "rod-drop" method.

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VIII. APPENDIXDiscussion Questions

The following propositions should be considered prior to arrival at the Argonaut. An effort will be made to discuss them just prior to and/or during checkout of the reactor, as time permits.

1. What are the basic variables involved in calibration of a control rod?
2. Define the following:
 - a. multiplication factor;
 - b. reactivity;
 - c. inhour;
 - d. $\% \Delta k/k$;
 - e. $\$$ (Dollars).
3. Of what use is a calibration curve for a control rod?
4. The change in reactivity due to the insertion of an absorber at any point in a reactor may be taken to be proportional to the square of the thermal-neutron flux at that point. On this basis, derive the general shape of a predicted integral control rod calibration curve of reactivity versus the extent of withdrawal of the control rod. Assume the flux to have a cosine distribution in the direction of rod motion and that the flux perturbation due to the rod is not severe.
5. Assume the Argonaut to be critical with the source out. At time zero, a step insertion of $-0.1\% \Delta k/k$ is made and at time t the reactor is returned to critical by inserting $+0.1\% \Delta k/k$. Make a rough sketch of ϕ as a function of time.
6. Would you expect the control rod calibration to be a function of the power level at which it is measured?
7. Discuss the "shadowing" or "interaction" between control rods.
8. What considerations dictate the design of control rods? Of safety rods? Shim rods?
9. Name as many methods as you can conceive of controlling a reactor.

STUDENT RESULTS

CONTROL ROD CALIBRATION

ROD DROP DATA

Chart zero 4.5 Shim rod in Coarse rod 67 Fine rod 15 to 0

$$\phi_{\text{net}} = \phi_{\text{gross}} - \text{chart zero} = \phi_{\text{gross}} - 4.5$$

Time after drop (sec)	ϕ_{gross}	ϕ_{net}	ϕ_t/ϕ_0	$\rho(\$)$	
0	90.0	85.5	1.		$\rho_{\text{av}} \underline{0.03 \$} = \underline{0.0225\%}$
10	83.0	78.5	0.918	0.03	
20	80.0	75.5	0.883	0.03	
30	78.0	73.5	0.860	0.03	
40	75.7	71.2	0.833	0.03	
50	73.8	69.3	0.810	0.03	

Chart zero 4.5 Shim rod in Coarse rod 62 Fine rod 25 to 0

Time after drop (sec)	ϕ_{gross}	ϕ_{net}	ϕ_t/ϕ_0	$\rho(\$)$	
0	90.0	85.5	1.		$\rho_{\text{av}} \underline{0.092 \$} = \underline{0.069\%}$
10	72.0	67.5	0.789	0.090	
20	66.5	62.0	0.725	0.087	
30	62.5	58.0	0.678	0.085	
40	55.0	50.5	0.590	0.100	
50	52.0	47.5	0.556	0.097	

Chart zero 4.5 Shim rod in Coarse rod 56 Fine rod 35 to 0

Time after drop (sec)	ϕ_{gross}	ϕ_{net}	ϕ_t/ϕ_0	$\rho(\$)$	
0	90.0	85.5	1.		$\rho_{\text{av}} \underline{0.161 \$} = \underline{0.121\%}$
10	62.0	57.5	0.673	0.157	
20	54.0	49.5	0.579	0.160	
30	48.5	44.0	0.515	0.160	
40	43.7	39.5	0.459	0.162	
50	39.5	35.0	0.409	0.167	

Chart zero 4.5 Shim rod in Coarse rod 46.5 Fine rod 50 to 0

Time after drop (sec)	ϕ_{gross}	ϕ_{net}	ϕ_t/ϕ_0	$\rho(\$)$	
0	90.0	85.5	1.		$\rho_{\text{av}} \underline{0.30 \$} = \underline{0.225\%}$
10	48.0	43.5	0.509	0.30	
20	40.0	35.5	0.415	0.30	
30	34.0	29.5	0.345	0.30	
40	30.0	25.5	0.298	0.30	
50	26.7	22.2	0.260	0.30	

CONTROL ROD CALIBRATIONROD DROP DATAChart zero 4.5 Shim rod in Coarse rod 36.4 Fine rod 70 to 0

Time after drop (sec)	ϕ_{gross}	ϕ_{net}	ϕ_t/ϕ_0	$\rho(\$)$	
0	90.0	85.5	1.		$\rho_{av} \underline{0.44 \$} = \underline{0.33\%}$
10	40.0	35.5	0.415	0.44	
20	32.0	27.5	0.322	0.44	
30	26.5	22.0	0.257	0.44	
40	23.0	18.5	0.216	0.44	
50	20.0	15.5	0.181	0.44	

Chart zero 4.5 Shim rod in Coarse rod 33 Fine rod 85 to 0

Time after drop (sec)	ϕ_{gross}	ϕ_{net}	ϕ_t/ϕ_0	$\rho(\$)$	
0	90.0	85.5	1.		$\rho_{av} \underline{0.492 \$} = \underline{0.369\%}$
10	38.0	33.5	0.392	0.48	
20	29.5	25.0	0.292	0.49	
30	24.5	20.0	0.234	0.49	
40	20.5	16.0	0.187	0.50	
50	18.0	13.5	0.158	0.50	

Chart zero 4.5 Shim rod in Coarse rod 32 Fine rod 100 to 0

Time after drop (sec)	ϕ_{gross}	ϕ_{net}	ϕ_t/ϕ_0	$\rho(\$)$	
0	90.0	85.5	1.		$\rho_{av} \underline{0.496 \$} = \underline{0.371\%}$
10	37.5	33.0	0.386	0.49	
20	29.5	25.0	0.292	0.49	
30	24.5	20.0	0.234	0.50	
40	21.0	16.5	0.190	0.50	
50	18.0	13.5	0.158	0.50	

Chart zero 4.5 Shim rod in Fine rod 100 Coarse rod 32 to 0

Time after drop (sec)	ϕ_{gross}	ϕ_{net}	ϕ_t/ϕ_0	$\rho(\$)$	
0	90.0	85.5	1.		$\rho_{av} \underline{0.266 \$} = \underline{0.199\%}$
10	50.0	45.5	0.532	0.28	
20	42.0	37.5	0.439	0.27	
30	37.0	32.5	0.380	0.26	
40	33.0	28.5	0.333	0.26	
50	29.5	25.0	0.292	0.26	

Positive Period Measurement

	<u>Fine Rod</u>	<u>Coarse Rod</u>
Reactor critical at	16	67
Reactor supercritical at	16	100
Resulting e-folding time	$T = 68 \text{ sec}$ $\Delta\rho = 0.12 \$$

Fig. S3.1

CONTROL ROD CALIBRATION; AGN-201 REACTOR

by

K. C. Ruzich and William F. Gilbert

ABSTRACT

In this experiment the basic concepts of control rod calibration of a nuclear reactor are considered. Reactivity changes are measured by observing positive asymptotic periods of the reactor. These asymptotic periods are related to reactivity by the reactor kinetic equations and the usual inhour equation. Several examples of kinetic behavior in response to changes in reactivity are qualitatively observed so that operating skill and safe attitudes pertaining to reactor operation may be developed.

OUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
- IV. PROCEDURE
 - A. Calibration of Fine Control Rod
 - B. Study of Kinetic Behavior
 - C. Inter-Calibration of Control Rods
- V. RESULTS
- VI. ANALYSIS AND CONCLUSIONS
- VII. BIBLIOGRAPHY
- VIII. APPENDICES
 - A. Discussion Questions
 - B. AGN-201 Inhour Equation

I. INTRODUCTION

This experiment offers a technique, widely used, by which control rods can be calibrated. The calibration is actually a measurement of the reactivity worth of a rod at any of its positions in the reactor. The knowledge of the reactivity worths of control elements is essential for both safety and practical purposes. Since control rods are used for startup, steady-state operation, and shutdown of the reactor, their individual ability to perform these tasks must be known accurately before a reactor is put into operation. In some cases, it is desirable to know only the total worth of a rod (such as a safety rod), in other cases the worth at any rod position (shim and control rods). The practical importance of obtaining a complete rod calibration is primarily to detect reactivity changes that occur in the reactor. A change in the critical settings of the control rods will indicate such a reactivity change. Normal reactivity changes can occur from effects such as temperature, fuel burnup, fission product poisoning, and poison burnup. The magnitude of the change can be determined from the calibration of the rods and the critical rod settings.

The fine control rod of the AGN-201 reactor will be calibrated in this experiment. The technique used to measure reactivity changes at different rod positions in the core will be measurements of positive asymptotic periods. The reactivity of the assembly corresponding to a measured stable period will be determined from the basic inhour equation, which is derived below.

II. THEORY

The derivation of the inhour equation, which relates period measurements to reactivity, involves the use of the basic diffusion equation:

$$D\nabla^2\phi(r,t) - \Sigma_a\phi(r,t) + S = \frac{1}{v} \frac{\partial\phi(r,t)}{\partial t}, \quad (1)$$

that is,

leakage - absorption + production = time rate of change of neutron density,

where

D = diffusion coefficient in the core,

Σ_a = macroscopic absorption cross section in the core,

$\phi(r,t)$ = neutron flux at r ,

and S is the source term which includes prompt and delayed neutrons. The number of neutrons absorbed per cm^3 per second is $\Sigma_a\phi$. The total number of fission neutrons is $(k/p)\Sigma_a\phi$. If β_i is the fraction of neutrons belonging to the i th group, $(\beta_i k/p)\Sigma_a\phi$ is the rate of formation of the nuclei which are the precursors of those neutrons. Each precursor emits a delayed neutron when the fission fragment decays. Since the precursors are radioactive, they can be expected to obey the well-known radioactive law

$$\frac{dC_i}{dt} = -\lambda_i C_i + \left(\beta_i \frac{k}{p}\right) \Sigma_a \phi, \quad (2)$$

where C_i is the precursor concentration and λ_i the decay constant of group i . The rate of formation of delayed neutrons in any group equals the rate of decay of the precursor, $\lambda_i C_i$. Therefore, the rate at which delayed neutrons are supplied to the reactor is

$$S_d = \sum_{i=1}^m \lambda_i C_i \text{ (per cm}^3 \text{ per second).}$$

The quantity $(1-\beta)(k/p)\Sigma_a\phi$, where

$$\beta = \sum_{i=1}^m \beta_i,$$

is the total number of prompt neutrons due to fission. If we assume that the delayed and prompt neutrons have the same fast nonleakage probability ($\mathcal{L}_F$), the source term becomes

$$S = S_p + S_d = \left[(1 - \beta)(k/p) \Sigma_a \phi + \sum_{i=1}^m \lambda_i C_i \right] p \mathcal{L}_F \quad (3)$$

Substituting Eq. (3) in (1) and dividing through by Σ_a , the diffusion equation becomes

$$L^2 \nabla^2 \phi(r, t) + \left[(1 - \beta)k \mathcal{L}_F - 1 \right] \phi(r, t) + \frac{p \mathcal{L}_F}{\Sigma_a} \sum_{i=1}^m \lambda_i C_i(r, t) = \ell_0 \frac{\partial \phi(r, t)}{\partial t} \quad (4)$$

where $\ell_0 = 1/v \Sigma_a$ is the thermal neutron lifetime in an infinite assembly and $L^2 = D/\Sigma_a$ is the square of the thermal diffusion length. It can be shown that if a reactor, operating in the steady state, acquires a small step change in the reproduction factor at a time t taken as zero, the space and time variables of Eq. (4) are separable. The spatial flux distribution is represented by the well-known wave equation

$$\nabla^2 \phi(r, t) + B^2 \phi(r, t) = 0, \quad (5)$$

where ∇^2 has no effect on the time dependence of the equation and B^2 is the geometric buckling with the flux being zero at the extrapolated boundary of the system.

The two differential Eqs. (2) and (4) are now solved by assuming that the time-dependent solutions are of the forms

$$\phi(r, t) = \phi_0 e^{t\omega} \quad (6)$$

and

$$C_i(r, t) = C_{i0} e^{t\omega}, \quad (7)$$

where the subscript zero indicates values at time $t = 0$. Substitution of these expressions into Eq. (2) yields

$$C_{i0} = \frac{(k/p) \beta_i \Sigma_a \phi_0}{\omega + \lambda_i}. \quad (8)$$

Introduction of (5), (6), (7), and (8) into Eq. (4) yields

$$(L^2 B^2 + 1) + (1 - \beta)k \mathcal{L}_F + k \mathcal{L}_F \sum_{i=1}^m \frac{\lambda_i \beta_i}{\omega + \lambda_i} = \ell_0 \omega.$$

Multiplying through by $1/1 + L^2 B^2$, the slow nonleakage probability ($\mathcal{L}_S$), gives,

$$\frac{(1-\beta)k_{\text{eff}}}{1 + L^2 B^2} - 1 + \frac{k_{\text{eff}}}{1 + L^2 B^2} \sum_{i=1}^m \frac{\lambda_i \beta_i}{\omega + \lambda_i} = \frac{\ell_0 \omega}{1 + L^2 B^2}.$$

Since $k_{\text{eff}} = k_{\text{S}} k_{\text{F}}$ and the lifetime for thermal neutron in a finite medium is

$$\ell = \frac{\ell_0}{1 + L^2 B^2},$$

the above expression can be written as

$$(1-\beta)k_{\text{eff}} - 1 + k_{\text{eff}} \sum_{i=1}^m \frac{\lambda_i \beta_i}{\omega + \lambda_i} = \ell \omega.$$

Rearrangement of terms gives

$$(k_{\text{eff}} - 1) + k_{\text{eff}} \sum_{i=1}^m \left[\frac{\lambda_i \beta_i}{\omega + \lambda_i} - \beta_i \right] = \ell \omega$$

or

$$k_{\text{eff}} - 1 = k_{\text{eff}} \sum_{i=1}^m \frac{\omega \beta_i}{\omega + \lambda_i} + \ell \omega. \quad (9)$$

If reactivity is defined as

$$\rho = \frac{k_{\text{eff}} - 1}{k_{\text{eff}}},$$

Equation (9) becomes

$$\rho = \frac{\ell \omega}{k_{\text{eff}}} + \sum_{i=1}^m \frac{\omega \beta_i}{\omega + \lambda_i}. \quad (10)$$

For each value of ρ this algebraic equation will yield $m + 1$ corresponding values of ω . A plot of these parameters as a function of ρ can be found in Reference 1. The expression for the flux (6) can be expressed as a summation of exponentials

$$\phi = \sum_{i=0}^m A_i e^{\omega_i t}.$$

When ρ is positive, all the ω_i are negative except the first. After a short period of time, these negative terms die out and the expression for the flux reduces to the first term only:

$$\phi = \phi_0 e^{\omega_0 t}$$

where $A_0 = \phi_0$ is the flux at time $t = 0$.

The quantity ω_0 is the reciprocal of the stable asymptotic period T . If a measurement of this stable period is made when a reactor is supercritical, the reactivity corresponding to the period can be determined from Eq. (10). Substituting $\omega = 1/T$ into the basic inhour equation gives

$$\rho = \frac{\ell}{T k_{\text{eff}}} + \sum_{i=1}^m \frac{\beta_i}{1 + \lambda_i T}$$

Knowledge of the thermal-neutron lifetime and the delayed-neutron parameters of the reactor in question will enable a plot of reactivity versus stable period to be made. This plot for the AGN-201 reactor is made in Appendix B.

Calibration of the fine control rod of the AGN-201 reactor involves determining the reactivity worth of the rod at different positions in the reactor. When the rod is in its down position, which is completely out of the reactor core, its worth is taken as zero. If the rod is moved from this position to a location in the core when the reactor is in a critical state, the subsequent power rise will indicate the reactivity change. The resulting

Fig. 4.1. Reactor Core

plot of reactivity versus rod position will have a certain shape which can easily be determined from theoretical considerations. According to perturbation theory, for a thermal bare-reactor model the reactivity change due to the insertion of a given amount of fuel at a certain location in the reactor core is proportional to the square of the flux at that location.

The reactor is schematically indicated in Fig. 4.1. If the fuel-containing control rod is moved from the bottom to the top of the core, the total reactivity change will be $\Delta k/k$. The reactivity change due to an incremental change in rod position (dz) can then be expressed as

$$\rho = \frac{\Delta k}{k} \frac{\phi^2(z) dz}{\int_{-H/2}^{H/2} \phi^2(z) dz}$$

If we may assume that the radial flux is constant and that $\phi(z) \sim \cos(\pi z/H)$, the reactivity worth of the partially inserted rod is

$$\rho = \frac{\Delta k}{k} \frac{\int_{-H/2}^z \cos^2(\pi z/H) dz}{\int_{-H/2}^{H/2} \cos^2(\pi z/H) dz}$$

Use of the trigonometric identity $\cos^2 x = 1/2(1 + \cos 2x)$ gives

$$\rho = \frac{\Delta k}{k} \left[\left(\frac{z}{H} + \frac{1}{2} \right) + \frac{1}{2\pi} \sin \frac{2\pi z}{H} \right].$$

A plot of the expected worth curve for a fuel rod is contained in Fig. 4.2. The dotted lines represent the straight line and sine wave portions of the derived expression.

Fig. 4.2
Calibration Curve

III. APPARATUS

- A. AGN-201 Reactor and associated instrumentation for neutron counting (see Fig. 4.3).
- B. Stop watches, for period measurements.

139-3413

Fig. 4.3. BF_3 Counter and Associated Electronics

IV. PROCEDURE

The daily reactor checkout is first performed.

A. Calibration of Fine Control Rod

1. With the fine control rod all the way out, make the reactor slightly supercritical by adjusting the coarse control rod. Measurement of the positive stable reactor period will be made by monitoring the rise of the neutron flux level and determining the time taken for the reactor power to increase by a factor of e . Record the asymptotic period of the reactor and the control rod positions.
2. Reduce the reactor power by withdrawing the coarse control rod and then insert the fine control rod a few centimeters. Bring the coarse control rod to the same position recorded in procedure 1 and record the asymptotic period of the reactor and the control rod positions.

3. Repeat procedure 2 for several centimeters beyond the previous position. Continue until the fine control rod is fully inserted.
4. For each asymptotic period observed the reactivity of the reactor may be found from a chart of reactivity versus period which will be available. The net reactivity worth of the fine control rod at any position is found by subtracting the initial reactivity found in procedure 1 from the total reactivity determined at the fine control rod position.
5. Plot the curve of reactivity versus rod position for the fine control rod. Try to explain any deviation from the "S" shape usually associated with such calibration curves.

B. Study of Kinetic Behavior

Under the direction of the instructor make positive and negative reactivity changes in the reactor and observe the resulting kinetic response of the reactor. One set of changes might be similar to that proposed in Question 5 in the question discussion section of the experiment.

C. Intercalibration of Control Rods (if time permits)

1. With the fine control rod out, make the reactor critical by adjusting the coarse control rod. Record the control rod positions.
2. With the fine control rod fully inserted again, make the reactor critical by adjusting the coarse control rod. Record the control rod positions.
3. From the data obtained in Sections A and B and using the fine control rod calibration previously obtained, estimate the total worth of the coarse control rod. (Assume the shape of the coarse control rod calibration curve to be similar to that of the fine control rod.)

V. RESULTS

The student is to make his own data sheet. Include with the results a calibration curve for the fine control rod.

VI. ANALYSIS AND CONCLUSIONS

A brief analysis of the experiment together with significant conclusions is left as an exercise for the student.

VII. BIBLIOGRAPHY

1. S. Glasstone and M. C. Edlund, The Elements of Nuclear Reactor Theory, D. Van Nostrand Co., Inc. (1952).
2. R. L. Murray, Nuclear Reactor Physics, Prentice-Hall, Inc. (1957).

VIII. APPENDICES

A. Discussion Questions

The following propositions should be considered prior to arrival at the AGN-201. An effort will be made to discuss them just prior to and/or during checkout of the reactor, as time permits.

1. What basic variables are involved in calibration of a control rod?
2. Define the following: a) multiplication factor; b) reactivity; c) inhour; d) $\% \Delta k/k$; e) $\$(Dollars)$.
3. Of what use is a control rod calibration curve?
4. The change in reactivity due to the insertion of fuel at any point in a reactor may be taken to be proportional to the square of the thermal-neutron flux at that point. On this basis, derive the general shape of a predicted integral control rod calibration curve of reactivity versus the extent of insertion of the control rod. Assume the flux to have a cosine distribution in the direction of rod motion and that the flux perturbation due to the rod is not severe.
5. Assume the AGN-201 to be critical with the source out. At time zero, a step insertion of $-0.1\% \Delta k/k$ is made and at time t the reactor is returned to critical by inserting $+0.1\% \Delta k/k$. Make a rough sketch of ϕ as a function of time.
6. What is meant by the asymptotic period of a nuclear reactor?
7. What effect would a neutron source in the reactor have on the observed reactor period?
8. Discuss the terms "shadowing" or "interaction" between control rods. We mean that the worth of the fine rod over a given interval of its travel is a function of the coarse rod position and vice versa.
9. What considerations influence the design of control rods? Of safety rods? Shim rods?
10. What methods can be used for controlling a nuclear reactor?

B. AGN-201 Control Rod Calibration

AGN-201 Inhour Equation

The basic inhour equation derived from the time-dependent diffusion equation is

$$\rho = \frac{\ell}{T k_{\text{eff}}} + \beta_{\text{eff}} \sum_{i=1}^6 \frac{a_i}{1 + \lambda_i T}$$

where

ρ = reactivity;

ℓ = prompt-neutron lifetime;

T = stable asymptotic period;

a_i = relative abundance of the i th group of delayed neutrons;

λ_i = decay constant of the i th group;

β_{eff} = delayed neutron fraction.

For the AGN-201 reactor,

$$\ell = 10^{-4} \text{ sec}; \quad \beta_{\text{eff}} \sum_{i=1}^6 a_i = 0.00800$$

i	a_i	$\lambda_i \text{ (sec}^{-1}\text{)}$
1	0.033	0.0124
2	0.219	0.0305
3	0.196	0.111
4	0.395	0.301
5	0.115	1.13
6	0.042	3.00

The computed curve is plotted in the Figs. 4.4 and 4.5.

Fig. 4.4
AGN-201 Inhour Equation
(0-1400 $\mu\rho$ range)

Fig. 4.5
AGN-201 Inhour Equation
(0-140 $\mu\rho$ range)

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STUDENT RESULTS

Data for Fine Control Rod Calibration (Rod is Loaded with Polystyrene)

FR	CR		Period	$\Delta k/k$ $\times 10^{-6}$	Period Calculation
0	22.77		∞	0	$\phi(t) = \phi_0 e^{t/T}$
3.00	22.77		1660	61.8	$T = \frac{t_2 - t_1}{\ln(\phi_2/\phi_1)}$ <p>For FR at 3.00 CR at 22.77 From Power Run Curve</p> $T = \frac{300 \text{ sec}}{\ln(1.9/1.0)}$ <p>$T = 1660 \text{ sec}$</p>
6.00	22.77		452	213	
9.00	22.77		205	437	
12.00	22.77		115	709	
15.00	22.77		76.0	970	
18.00	22.77		54.5	1230	
21.00	22.77		41.2	1490	
24.00	22.77		32.5	1730	
24.40	20.24		∞	0	
24.40	21.24		113	717	
24.40	22.24		43.0	1450	
24.40	23.24		23.1	2110

Fig. S4.1. Fine Control Rod Calibration
(Polystyrene Loaded)

Fig. S4.2. Power Runs with Reactor Supercritical by the
Reactivity Worth Corresponding to the Fine
Rod Position

Fig. S4.3. Power Runs with the Reactor Supercritical by 1, 2, and 3 cm of Coarse Control Rod

Fig. S4.4. Coarse Control Rod Reactivity Worth

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CONTROL ROD WORTHS BY PULSED NEUTRON
TECHNIQUES, AGN-201 REACTOR

by

K. C. Ruzich

ABSTRACT

This experiment is designed to determine the reactivity worth of the control rods by a pulsed neutron technique. A burst of neutrons is injected into the reactor, and the decay rate of the resultant neutron flux is measured. The decay rate measured is that of the prompt fission neutrons and is proportional to the prompt critical reactivity of the reactor. Measurements will be made with the reactor in subcritical conditions and at delayed critical. The decay rate at delayed critical yields the constant of proportionality between the decay rate of the neutron flux and the reactivity in dollars. This constant is equal to the ratio of the effective delayed-neutron fraction to the prompt-neutron lifetime.

OUTLINE

- I. INTRODUCTION
 - II. THEORY
 - III. APPARATUS
 - IV. PROCEDURE
 - V. RESULTS
 - VI. ANALYSIS AND CONCLUSIONS
 - VII. BIBLIOGRAPHY
 - VIII. APPENDIX
- Discussion Questions

I. INTRODUCTION

A very important consideration in connection with the operation of a reactor is the ability of the system to be controlled safely. The use of control rods to adjust a reactor core to criticality at different power levels and to safely shut the chain reaction off is widely used in the reactor field.

Control rods are usually designated as safety, shim, or control (regulating) rods. The safety rods shut down the reactor and are normally interlocked in a given position until they are needed to scram the reactor. Shim rods are used for coarse adjustment to criticality. Control (regulating) rods are used to control the reactor at a steady power level and to safely increase or decrease the power level.

The ability of the rods to perform their respective operations properly is very important from a safety standpoint. The speed of insertion or removal of the rods and their reactivity worths must therefore be known accurately. Rods and their drive mechanisms are of course designed to perform safely. Mechanical considerations will accurately provide information as to the speed at which rods can be moved in the system. Their reactivity worth is therefore the primary factor which must be measured accurately before and during operation of the reactor.

The three methods for measuring reactivity in a nuclear reactor are the inverse multiplication method, the positive-period and rod-drop methods based on reactor kinetics equations, and the pulsed neutron technique. This experiment will acquaint the participant with the latter technique. The pulsed neutron technique has two advantages over the others. The first is that large negative reactivities can be measured with good accuracy. Secondly, the method obtains its own calibration at delayed critical, relating the prompt-neutron decay constant to the reactivity of the system.

II. THEORY

The initial viewpoint in this section will be to give a qualitative explanation of what occurs in a subcritical reactor when fast neutrons are injected. A simple derivation of the relationship between the decay of the resulting thermal-neutron population and the reactivity of the system will be made purely from physical considerations. Finally, consideration will be made regarding two-group diffusion theory to describe the proposed experiment.

Fast neutrons are injected into the subcritical reactor from a position of symmetry outside the reactor core. The total number of neutrons in the burst will be emitted over a period of a few microseconds. The fast neutrons diffuse into the core and are slowed down primarily by making elastic collisions with the atoms in their flight paths. The amount of time it takes the neutrons to be slowed down to the thermal region will depend on the energy of the neutrons in question and on the slowing-down power of the reactor.

The resulting thermal-neutron population will rise to some maximum value. The time of rise depends on the average slowing-down time of the source neutrons and the time width of the neutron burst. This statement is true only if the slowing time of the medium is smaller than the diffusion time (thermal-neutron lifetime).

The resulting thermal neutrons will diffuse through the core and eventually be absorbed or escape from the system. Those neutrons absorbed in the fuel will cause fissions and produce more fast fission neutrons. The fission neutrons in question are prompt neutrons, for the delayed neutrons are now stored in their respective precursors whose probability of decay is quite small over the time scale of the experiment. The fission neutrons now suffer the same fate as the source neutrons; they are thermalized and then absorbed or lost from the system. The thermal-neutron population in the reactor will decrease with time since the reactor is subcritical and because there is a negligible number of delayed neutrons entering the system. The population will decay exponentially, and the decay rate will be related to the reactivity of the system measured from prompt critical.

Figure 5.1 describes the time behavior of the neutron population after a burst of neutrons is injected into the reactor. Region one is that of the initial buildup of the neutrons in the reactor as the source neutrons are slowed down. Region two, which is the main region of interest, is that of the exponential decay of the resultant neutron population. This decay is due to prompt neutrons; hence the generation time to be considered is the prompt-neutron lifetime. In region three the neutron population has reached

Fig. 5.1. Decay of a Neutron Burst

neutrons have changed noticeably. Therefore successive generations of the neutron population will have a constant ratio corresponding only to the prompt-neutron criticality factor

$$k_p = (1 - \beta)k_d,$$

where

β = the effective delayed-neutron fraction;

k_d = the delayed-neutron criticality factor.

The fractional change in the reactor neutron population in successive generations will be

$$\frac{\Delta n}{n} = (1 - \beta)k_d - 1.$$

Upon introducing the time element of interest, ℓ_p , the prompt-neutron lifetime, the fractional change per unit time, which is constant, can be expressed as

$$\frac{dn/n}{dt} = \frac{(1 - \beta)k_d - 1}{\ell_p};$$

hence,

$$n(t) \sim e^{-\frac{1}{\ell_p} [1 - (1 - \beta)k_d] t}$$

a relatively low level, and the delayed neutrons entering the system are the pace setters of the decay. The decay of the neutron population in region three will depend primarily on the mean lifetime of the delayed neutrons (~ 12 sec) and on the reactivity of the system, whereas the decay time in region two is of the order of a few milliseconds.

Returning now to region two, it can be easily shown that the expected decay of the neutron population will be exponential. In the time interval of interest the decay effects observed are completed before decay effects due to delayed

The decay constant of the neutron population after the reactor has been pulsed will be designated as ω :

$$\omega = [1 - (1 - \beta)k_d]/\ell_p;$$

or, rewriting,

$$\omega = \frac{\beta}{\ell_p} k_d \left[\frac{1 - k_d}{\beta k_d} + 1 \right].$$

Since $\rho = (k_d - 1)/k_d$ and $(1 - k_d)/\beta k_d = -\rho/\beta = R$ dollars of negative reactivity, and since in our work k_d will always be close to one, $\omega = (\beta/\ell_p)(1 + R)$. If the reactor is pulsed at delayed critical, the resulting measurement of the decay constant will determine the β/ℓ_p ratio ($R = 0$). This measurement will serve as the calibration point relating the decay constants directly to the negative reactivity of the reactor in a subcritical condition.

$$R = \frac{\omega}{\omega_c} - 1,$$

where ω_c equals β/ℓ .

The observed decay of the neutron population will actually be represented by an exponential plus a nearly constant background due to delayed neutron effects:

$$n(t) = Ce^{-\omega t} + B$$

and

$$dn/dt = -\omega Ce^{-\omega t} = -\omega(n - B).$$

It is of interest to compare this equation with the basic reactor kinetics equation:

$$\begin{aligned} \frac{dn}{dt} &= \frac{\rho - \beta}{\ell(1 - \rho)} n + \Sigma\lambda C \\ &= \frac{\Delta k - \beta k}{\ell} n + \Sigma\lambda C \\ &= \frac{(1 - \beta)k - 1}{\ell} n + \Sigma\lambda C. \end{aligned}$$

Equating the coefficients of n in the two equations again yields

$$\omega = \frac{1 - (1 - \beta)k}{\ell}$$

Diffusion theory can be used to theoretically describe the kinetic behavior of a multiplying system following a burst of neutrons (References 3 and 4). The reactor model chosen is normally a two-energy-group, one-delayed-neutron-fraction, homogeneous bare system. The derivation of the diffusion equations for this model yields a time-dependent equation for the neutron density. This equation has the same form as that previously described for the physical model when considering times long compared with the slowing-down time of the fast neutrons originally injected.

The reactivity measurement which is made is shown to be highly sensitive to any harmonic effects present in the theoretical model chosen. The harmonic effects arise in the expression for the spacial flux distribution in accordance with eigenfunctions of the buckling equation (wave equation). The measured reactivity, that reactivity described by the physical model, is shown to be that of the fundamental mode.

The experiment requires that the decay-constant measurement must be made only after all the higher modes have become nonexistent. This condition can be satisfied most easily when the reactor is near criticality. However, as the reactor is made more and more subcritical, the higher modes of the neutron density decay at slower and slower rates, hence interfering with a decay measurement. An analysis of the harmonic effects on the decay of the neutron population can be made by measuring the decay of the neutron burst at different positions in the reactor.

It should be pointed out that the model chosen to represent the burst decay in the reactor is overly simplified. This is true because the reactor is reflected with graphite; hence the lifetime of the system will be longer than estimated by bare reactor theory. Since the AGN-201 has a small hydrogenous core, the primary leakage from the system will be fast neutrons. Fast neutrons that diffuse into the reflector are moderated, and some return to the core. These neutrons will have much longer lifetimes than neutrons from the same generation that remained in the core. A more correct model of the reactor would be a two-group, two-region system, taking into account the graphite reflector.

III. APPARATUS

The apparatus used in this experiment consists of the following:

- a. AGN-201 reactor
- b. Kaman pulse neutron generator
- c. TMC 256-channel analyzer with 212 pulse neutron logic
- d. AID linear amplifier and preamplifier
- e. BF_3 detector.

A detailed description of the AGN-201 reactor can be found in ANL-6542 (AGN-201 Hazard Report). The Operating Manual for the AGN-201 Reactor describes the general operation of the reactor. The operations necessary for this experiment will be described in Section IV. Procedure.

The main components of the reactor in this experiment are the control rods. All four control rods are fuel bearing (with a fuel composition same as that of the core). The fine control rod (of 2.3-cm diameter) contains 3.6 g U^{235} . The two safety and coarse control rods are 4.7 cm in diameter, and each contains about 14.4 g U^{235} . The critical mass of the reactor is about 662 g U^{235} . The positions of the control rods in the core are shown in Fig. 5.2. Each rod when inserted fully into the core extends from the bottom of the core to a position about 4.0 cm above the horizontal midplane of the core.

The Kaman pulse neutron generator provides a burst of 14 MeV neutrons of the order of 10^7 n/burst. The time duration of the burst is about 4 μ sec. The reaction utilized to obtain the 14-MeV neutrons is $H^3(H^2, n)He^4$. This reaction is highly exoergic, $Q = 17.6$ MeV, and the cross section for the reaction as a function of deuterium energy is shown in Fig. 5.3.

Fig. 5.2. Control Rod Positions in the Reactor Core

Fig. 5.3. Variation of the Cross Section for the Reaction $H^3(d,n)He^4$ with the Energy of the Deuteron

The Kaman "Pulsatron" (see Fig. 5.4) consists of: the main evacuated accelerating tube (VN tube) which contains the deuterium ion source, accelerating electrodes, tritium target, and an ion pump; a high-voltage transformer and the externally associated electronics; power supplies, and trigger circuits. Figure 5.5 is a schematic diagram of the important portions of the Pulsatron.

139-2860

Fig. 5.4. Pulse Neutron Generator

Fig. 5.5. Simplified Schematic of the Pulsatron

The basic operation of the Pulsatron consists of the following: At the lower end of VN tube is shown a stack of titanium (deuterium) and mica washers. The discharge electrode is at the lower end of this stack of washers, when a pulse of approximately 6 kV is applied to the electrode, the mica washers are not capable of holding this voltage and a spark discharge is initiated across each of the washers. The arc causes deuterium and titanium atoms to be dislodged from the titanium washers. As the atoms move into the discharge stream, they become ionized. A certain number of the ions will, due to their ionization energy, diffuse out of the end of the lower shield past the grid. At the same time the 6-kV pulse is applied to the discharge electrode, it is applied to the primary of the main pulse transformer. This transformer has a step-up ratio such that 100 kV is applied to the tritium target and its corona shield. As soon as the ions diffuse past the grid, they reach an electric field of about 50 kV/cm and are accelerated into the target. The grid is used to straighten the electric field lines at the end of the corona shield and hence diffuses the ion beam, assuring that it spreads evenly over the target area. The 100-keV deuterium ions then strike the target and produce 14-MeV neutrons whose emission pattern is essentially isotropic. The 6-kV pulse is supplied by the discharge of the energy storage capacitor when the thyatron switch is closed. The thyatron switch is triggered by a smaller thyatron. The small thyatron is, in turn, driven by a conventional Eccles-Jordan multi-vibrator or a similar external input. The main power supply is a simple half-wave rectifier utilizing two solid-state diodes in order to provide the required voltage.

The same high voltage is used to supply voltage for a Pfenning ion pump, which consists of a stainless steel housing lined with titanium plates. Within this housing is a titanium electrode resembling an egg crate structure. A permanent magnet is placed around the pump. The high voltage applied to the center electrode of the pump will cause the small number of free electrons in the pump to be accelerated toward it. The electrons will follow long spiral paths due to the effect of the magnetic field. These long paths raise the probability that the electrons will strike gas molecules as they move toward the electrode. A certain portion of these molecules will be ionized, and the field will cause them to be accelerated toward either the center electrode or the titanium plates. The gas ions will become occluded in the titanium and will no longer contribute to the pressure in the tube. The pump maintains the tube pressure at approximately 10^{-6} mm Hg. The capacity of the pump is about 1 liter/sec for hydrogen.

The VN tube and the main pulse transformer are packaged in a stainless can, 4 in. in diameter and 18 in. high. The can is filled with standard transformer oil so that severe dielectric interfaces between the transformer and tube are avoided. The control unit of the Pulsatron contains the power supplies, energy-storage capacitor, and the thyatron tube

and associated electrical circuits. The controls on this unit consist of a variable high-voltage adjustment, a trigger switch for internal, external, or single pulsing, and a rate control (1-10 pulses/sec) for automatic triggering.

The TMC 256-channel analyzer (see Fig. 5.6) with the pulsed neutron logic 212 is used as the time analyzer to collect data at different times after the system receives a neutron burst. This analyzer has 256 channels with a variable channel length from 10 to 2560 μsec . The operation of the system is quite simple. A positive trigger pulse is applied to the analyzer. The first channel then counts background for a time equal to the channel length setting times a background ratio ($\times 2^n$, $n = 1$ to 8). The data collected in this channel is stored in the memory. It should be noted that after each analysis channel there is a 10- μsec storage time required to store the data collected in the memory and to prepare the arithmetic scaler for the additional counts to be made in the next channel. A 10-V trigger pulse is generated at the end of the background channel. This pulse is fed to the Pulsatron and triggers the thyatron switch. Fast neutrons are now injected into the system. A delay time equal to 10 μsec plus the analysis channel length multiplied by the delay time ratio ($\times 2^n$, $n = 1$ to 8) now exists. Following this delay, the other 255 channels collect data for the analysis time chosen, with a 10- μsec storage time between each channel.

The detector used in this experiment is a simple BF_3 proportional counter (see Fig. 5.7), connected to a preamp whose output is in turn connected to an AID amplifier (see Fig. 5.8). The output pulses of the AID have a width of less than 2 μsec and hence allow counting rates of 0.02 count/ μsec with a dead-time loss of only 4%. The positive output pulses are fed to the 256-channel analyzer, where they are counted in an analyzer channel according to the time which the pulse occurred after the burst of neutrons.

IV. PROCEDURE

The apparatus for the experiment is set up according to Fig. 5.8. The following procedure will be used to measure the negative reactivity associated with a subcritical reactor condition. This procedure can be used for each of the following measurements:

1. coarse control rod calibration (reactivity worth of rod at different positions in the reactor);
2. total reactivity worth of the fine control rod;
3. shutdown-reactivity measurement.

The reactor is first checked out in accordance with the daily reactor checkout procedure. The reactor is then brought to criticality at a high power level (~ 50 mW) following the normal startup procedure. A positive-period measurement will be made with the rods fully inserted as

139-2853

Fig. 5.6. 256 Channel Time Analyzer

139-3413

Fig. 5.7. BF_3 Counter and Associated Electronics

Fig. 5.8. Diagram of Experimental Apparatus

the reactor is brought to full power. This measurement in conjunction with the basic inhour equation will enable a determination of the excess reactivity of the system to be made. The critical positions of the control rod are then found, first with the fine rod fully inserted and, secondly, with the coarse rod fully inserted. The two sets of critical rod positions will be used when the reactor is pulsed at delayed critical. It is important to ensure that there will be no reactivity changes in the system other than

those made by changes of controls and safety-rod positions (as for example, by temperature changes or detector position changes).

The reactor is now brought to a subcritical condition by removing the fine and coarse control rods. The power level is allowed to drift to zero. All low-level trips on the reactor should be removed as the power is approaching zero. The period trip is also removed. Once the power has essentially reached a zero level, the reactor is ready to be pulsed with the neutron source. Before pulsing the system at delayed critical (rod settings were previously determined above), the reactor is pulsed with the fine and coarse rods removed. Pulses are first injected one at a time to ensure that the equipment is operating satisfactorily, both with respect to safety and performance. The rate of the pulses is then increased slowly until a value of one per second is reached. The reactor is now pulsed at this rate throughout the entire experiment. A trial run will be made with the reactor in this condition. Enough data should be collected to ensure that the equipment is operating properly and the resultant data represent the decay of the prompt neutrons in the reactor.

With the trial run completed, the reactor can be placed in any subcritical condition and pulsed. The data are then collected (see Data Sheet), pulsing the reactor enough times to give good statistical accuracy. The channel length used during the pulsing will depend on the subcriticality of the system. Normal channel lengths will range from 20 to 320 μsec . The

DATA SHEET

Reactor Period

$$k_{ex} =$$

Reactor Temperature

Critical rod settings:

FR (IN) _____

FR

CR _____

CR (IN) _____

[illegible]

data collected will then be printed out on tape. The number of counts in a channel is then plotted on semilog paper versus the channel. The time between channels is just the channel analysis length plus $10 \mu\text{sec}$. The resultant plot will yield a straight-line portion; the decay constant will be calculated from this segment. The decay constant will be proportional to the reactivity of the system measured from prompt critical. As the reactor is brought closer to delayed critical, a delayed-neutron background will appear in the collected data. This background should be subtracted from the total delay curve before the decay constant is calculated.

After a few subcritical runs have been made, the reactor can be brought to delayed critical and pulsed. Care must be taken at this point, since the delayed-neutron background will yield a steady-state neutron level in the reactor. The background will steadily increase each time more neutrons are injected into the system. After enough data have been taken, the pulsing is stopped, and the reactor power level should level off and remain at steady state, since the reactor is in a delayed critical condition. The decay constant calculated for this run will yield the constant of proportionality between the decay constant and the negative reactivity (in dollars) of the system. This constant is equal to $\beta_{\text{eff}}/\ell_p$ as pointed out in Section II.

The measurements which will be made during this experiment will depend on the experimental time allowed. The coarse control rod can first be calibrated, and then measurements of the worths of other rods and shutdown reactivity can be made if time allows.

V. RESULTS

The results of this experiment will include the following:

- a. Coarse control rod calibration. A plot of the reactivity worth of the coarse control rod versus rod position will be made.
- b. Fine control rod worth. A total reactivity worth for this rod will be determined.
- c. Shutdown reactivity. Shutdown reactivity will be determined.
- d. Reactor prompt-neutron lifetime. The lifetime shall be determined from the delayed-critical decay constant measurement, assuming $\beta = 0.0080$.

The reactivity calculations will be determined from the measured decay constant of the corresponding reactor condition using the previously derived equation

$$R = \frac{\omega}{\omega_c} - 1.$$

VI. ANALYSIS AND CONCLUSIONS

A brief analysis of the experiment by the student will be made. Possible sources of error in the reactivity measurements should be considered.

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VIII. APPENDIXDiscussion Questions

The following propositions should be considered prior to arrival at the AGN-201. An effort will be made to discuss them just prior to and/or during checkout of the reactor, as time permits.

1. What basic variables are involved in calibration of a control rod?
2. Define the following: a) multiplication factor; b) reactivity; c) inhour; d) $\% \Delta k/k$; e) β ; f) shutdown reactivity; g) delayed critical; h) prompt critical.
3. Of what use is a control rod calibration curve?
4. The change in reactivity due to the insertion of fuel at any point in a reactor may be taken to be proportional to the square of the thermal-neutron flux at that point. On this basis, derive the general shape of a predicted integral control rod calibration curve of reactivity versus the extent of insertion of the control rod. Assume the flux to have a cosine distribution in the direction of rod motion and that the flux perturbation due to the rod is not severe.
5. Briefly describe the three basic methods for measuring reactivity.
6. From physical considerations, derive the simple time-dependent equation describing the decay of the resultant thermal-neutron density in a subcritical reactor after fast neutrons have been injected into the system. Assume at any instant n neutrons/cm³ are present in the system; k_d and ℓ_p are the delayed criticality factor and prompt neutron lifetime, respectively.
7. What is meant by the asymptotic period of a nuclear reactor?
8. What effect would a neutron source in the reactor have on the observed reactor period?
9. Assume the AGN-201 to be critical with the source out. At time zero, a step insertion of $-0.1\% \Delta k/k$ is made, and at time t the reactor is returned to critical by inserting $+0.1\% \Delta k/k$. Make a rough sketch of ϕ as a function of time.
10. Discuss the terms "shadowing" or "interaction" between control rods. We mean that the worth of the fine rod over a given interval of its travel is a function of the coarse rod position, and vice versa.
11. What considerations influence the design of control rods? Of safety rods? Shim rods?

STUDENT RESULTSDATA SHEETReactor Period 35.3 $k_{ex} =$ 1550 $\mu\rho$ Reactor Temperature 20.1

Critical rod settings:

FR (IN) 2497FR 1677CR 2221CR (IN) 2448

Run	FR	CR	SR (#1)	SR (#2)	Reactor Temperature	Comments
1	1677	Out	In	In	20.1	Subcritical
2	Out	2221	In	In	20.1	Subcritical
3	2497	2221	In	In	20.1	Delayed Critical
4	1677	2448	In	In	20.1	Delayed Critical
5	Out	Out	Out	Out	20.1	Shutdown
6	Out	Out	In	Out	20.1	Subcritical
7	Out	Out	In	In	20.1	Subcritical

Sample CalculationRun 1

$$\omega = \frac{\ln \frac{238}{1400}}{(26-8)(330)(10^{-6})} = \frac{\ln 0.17}{5.94 \times 10^{-3}} = 298 \text{ 1/s}$$

Runs 4 & 5

$$\text{yield } \omega_c = 122 \text{ 1/s}$$

The negative reactivity of the system during Run 1 was,

$$R = \frac{\omega}{\omega_c} - 1 = \frac{298}{122} - 1 = \$1.44.$$

The results for each run follows in the table below

Run	ω	$R(-\Delta k/k\beta)$	Reactor Condition
1	298 1/s	\$1.44	Subcritical by CR
2	173 1/s	0.42	Subcritical by FR
3	120 1/s	0	Delayed Critical
4	124 1/s		Delayed Critical
5	574 1/s	3.70	Subcritical by All Rods, $-k_{ex}$
6	462 1/s	2.79	Subcritical by SR #2, CR, FR, $-k_{ex}$
7	310 1/s	1.54	Subcritical by CR, FR, $-k_{ex}$

The reactivity worth of the control rods and the shutdown reactivity are obtained by subtracting the appropriate negative reactivities of specific runs listed in the table,

CR Worth	\$1.44	1.15% $\Delta k/k$
FR Worth	0.42	0.336% $\Delta k/k$
Shutdown ρ	3.89 ($R_5 + k_{ex}$)	3.11% $\Delta k/k$
SR #1 Worth	0.91	0.728% $\Delta k/k$
SR #2	1.25	1.00% $\Delta k/k$

Fig. S5.1
Decay of Neutron
Population

Fig. S5.2
Decay of Neutron
Population

Fig. S5.3
Decay of Neutron
Population

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POWER CALIBRATION, ARGONAUT REACTOR

by

L. L. Lawyer

ABSTRACT

A method for determining the power output of a low-power thermal reactor is given. The method depends upon the evaluation of the average neutron flux in the reactor combined with a knowledge of the total amount of U^{235} in the core and its fission cross section. The most important corrections are evaluated and the precision of the results estimated. Finally, a correlation between electronic instruments, peak absolute flux, and power output for the reactor are given.

OUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
- IV. PROCEDURE
 - A. General
 - B. Flux Measurements
- V. RESULTS
- VI. ANALYSIS AND CONCLUSIONS
- VII. BIBLIOGRAPHY
- VIII. APPENDICES
 - A. Discussion Questions
 - B. Indium Decay Table

I. INTRODUCTION

The power output of power reactors can be calculated by calorimetric methods, steam flow, etc. For low-power reactors, these methods in general are not suitable.

A more fundamental approach to the problem, and one which adapts well to all chain-reacting systems, including subcriticals, is the method described in this experiment.

This method can be considered, in the simplest case, merely as a measurement of the neutron flux present for a known quantity of fuel with a known cross section; or for those experimentalists with a more advanced background, a more detailed consideration can be given to the energy spectrum of these neutrons, to the neutron-velocity dependence of the fissionable isotope, and to the spatial distribution of both the neutrons and the fissionable atoms.

Regardless of the level of the approach, consideration must be given to the absolute determination of neutron flux. The level of approach and available time here also determine the amount of detail.

The utility of the results will, of course, be determined by the amount of detail in which these areas are covered. A rather good power calibration, however, can be made even in the least detailed case.

II. THEORY

The fission rate in a small section of the reactor, assuming the fuel and moderator to be homogeneous, is given by the relation

$$R_f = \bar{\phi}_{th} \sigma_f N, \quad (1)$$

where

$\bar{\phi}_{th}$ = the average thermal-neutron flux in the section under consideration;

σ_f = the cross section of U^{235} for fission by neutrons of energy 0.025 eV;

N = number of U^{235} atoms in the section.

This fission rate may then be converted to an energy rate by assuming that each fission gives rise to $1/k$ Watt-sec of energy. The power developed in the small section can then be computed from the relation

$$P = R_f/k \text{ (in W).}$$

III. APPARATUS

The spatial distribution of neutron flux in the reactor may be determined with the use of indium foils and foil holders (see Fig. 6.1). For determining the energy distribution of the neutrons, sections of a fuel plate, cadmium covers, and standard gold foils are used (see Fig. 6.1). The only additional equipment required are gas-flow end-window counters, and a counting system for which an absolute calibration is known (see Figs. 6.2 and 6.3). The experimental arrangement is shown in Fig. 6.4.

139-2421

Fig. 6.1. Foils and Covers

139-3424

Fig. 6.2. Gas-flow Proportional Counter

139-2420

Fig. 6.3. End Window G-M Counter

139-2418

Fig. 6.4. Experimental Arrangement

IV. PROCEDURE

For a one-region loading of fuel in the Argonaut reactor, the small sections may to a very good approximation be taken as the entire core region, unless a very severe flux tilt is indicated or unless a major departure from a uniform loading of fuel is known to exist. For a two-slab loading, each region or "slab" should be evaluated independently.

All of the experimental measurements are determinations of neutron flux. With the exception of the last (absolute flux determination) they will all be applied as corrections to an approximate power-level determination. Depending on the factors previously discussed, some or all of the following classes of neutron flux measurements will be made.

a) Spatial neutron flux distribution: In order to obtain the average flux required for the power evaluation, three traverses will be made of the neutron flux in each of the sections of the reactor core to be considered. These distributions should be as nearly as possible, orthogonal at a single point near the center of the region. Identical indium foils will be used for the distribution measurements and the data normalized to this common control point.

b) Flux depression: The flux distribution as measured above will be correct only if the indium foils are identical and are uniformly distributed throughout the section of the core being investigated. This condition should be very easy to approximate in practice; however, in order to determine the magnitude of any error involved, it may be desirable to determine the amount of depression due to the presence of the finite indium foils during the irradiation. The upper limit of this error can be determined by irradiating identically two gold foils with and without the indium foils in place.

c) Cadmium-ratio measurements: In order to determine the relative contribution of thermal and epithermal neutrons to the reactor fissions, a cadmium-ratio measurement of a section of Argonaut fuel plate can be made. A cadmium-ratio measurement should also be made of standard gold foils identical with those used for the absolute flux determination in order to determine the fractional gold foil activation in the thermal and epithermal flux regions.

d) Absolute flux measurement: The absolute flux determination will be made by irradiating a standard gold foil of known thickness and weight at the common central point of the three orthogonal distributions referred to in Section IV (a). The foil will then be counted absolutely in an end-window gas flow counter in which the absolute number of disintegrations per unit time has been derived from an observed counting rate from a standard foil irradiated to a known nvt. After correction for saturation, activation, decay, foil weight, and counter dead time, the absolute neutron

flux can be computed. The value of n_v thus obtained, corrected for its gold-cadmium ratio, represents the flux of thermal neutrons at the common central point as seen by gold.

Data should be recorded on the Data Sheet (q.v.).

DATA SHEET

Name _____ Irradiation Period _____

Date _____ Reactor Power _____

Background _____

1	2	3	4	5	6	7	8	9	10
Foil Loc.	Activity	Time	$e^{-\lambda t}$	Foil Weight (gm)	4 x 5	$1 - e^{-\lambda t}$	6 x 7	Activity- background	Saturated Activity per gm

V. RESULTS

By means of Eq. (1) and application of the necessary corrections from Section IV, the reactor power output can be calculated. A sample calculation is included in the Student Results.

VI. ANALYSIS OF RESULTS AND CONCLUSIONS

Consideration should be given to the list of assumptions presented in Item 13 of Appendix A. Some attention should be given to the precision of results and, finally, a correlation between reactor power, peak thermal flux, and ion-chamber current should be presented.

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VIII. APPENDICES

A. Discussion Questions

The following propositions should be considered prior to arrival at the Argonaut. A discussion of them will be considered a part of this experiment.

1. The measurement of the reactor power output will involve computation of the fission rate in the reactor core. What other method or methods might be used to measure the power output of this or another reactor?
2. For the purpose of this experiment, we will consider that a fission rate of 3.1×10^{10} fissions/sec is equivalent to 1 W. Analyze this in terms of MeV/fission.
3. Analyze the total MeV/fission derived in (2) above in terms of the amount of energy carried off by the various particles and photons, i.e., fission fragments, gamma rays, etc.
4. Assuming the fuel distribution in the reactor to be homogeneous and the distribution of neutron flux to be known, how may we arrive at the average neutron flux?
5. Why is it desirable to use ideally thin foils in measuring neutron fluxes?
6. To measure the flux distributions in the reactor core it has been suggested that we use indium foils. Since the indium foils are of finite thickness, they might produce a flux depression on each other. Is it necessary to correct for this flux depression?
7. Outline the steps involved in measurement of absolute neutron flux by means of a standard pile. How would you measure the neutron yield of a Ra-Be source?
8. In addition to irradiation of indium foils for measuring flux distributions in (6) above, it is desired that we irradiate a gold foil for absolute flux calibration at one point in the spectrums. Is it now necessary to correct for flux depression?
9. We propose to measure the cadmium ratio of gold and fuel plate during this experiment. Discuss the need for and give the direction of this correction. How does the absorption of epithermal neutrons by cadmium affect the cadmium ratio?

10. Define and discuss disadvantage factor.
11. What is beta-ray self-shielding in foils and how may its effect be eliminated?
12. Estimate the effect of the energy contribution due to fission in U^{238} .
13. The following is a list of assumptions made in a power calibration of an Argonaut-type reactor. Explain and determine the validity of each assumption and examine the list for completeness as applied to our proposed experiment.

The spatial energy spectrum of neutrons throughout the reactor core is assumed to be constant.

Control rod perturbations are assumed to be constant. This implies that the power measurement is more accurate for k_{ex} of 0.52%, and one can expect small variations for different one-slab loadings.

It has been assumed that the interaction between the indium foils did not depend on their center-to-center placement distances. This varied from 0.5 in. in the radial case to several inches in the vertical case.

It is assumed that the variables are separable in the determination of the relative flux distribution. This assumption was used in evaluating the average neutron flux.

The fission product decay spectrum and time parameters were assumed to be independent of the energy of the neutron causing the fission.

No hardening of the Maxwellian component of the reactor flux was assumed to be produced by the selective absorption of low-energy neutrons in fission, control rod, and other capture processes. Of course, such hardening exists, but since it affects both the standard gold activation and U^{235} fission, the direction of changes are opposite and tend to cancel each other.

B. Indium Decay Table

$T_{1/2} = 54.31$

$\tau = 78.35 \text{ min}$

t (min)	$e^{t/\tau}$	$1 - e^{-t/\tau}$	t (min)	$e^{t/\tau}$	$t - e^{-t/\tau}$
0	1.0000	0.0000	51	1.9171	0.4784
1	1.0128	0.0127	52	1.9417	0.4850
2	1.0258	0.0252	53	1.9669	0.4916
3	1.0390	0.0376	54	1.9920	0.4980
4	1.0524	0.0498	55	2.0177	0.5044
5	1.0658	0.0618	56	2.0437	0.5107
6	1.0795	0.0737	57	2.0699	0.5169
7	1.0934	0.0855	58	2.0964	0.5230
8	1.1075	0.0971	59	2.1235	0.5291
9	1.1217	0.1085	60	2.1505	0.5350
10	1.1361	0.1198			
11	1.1507	0.1310	61	2.1781	0.5409
12	1.1655	0.1420	62	2.2065	0.5468
13	1.1804	0.1529	63	2.2346	0.5525
14	1.1956	0.1636	64	2.2634	0.5582
15	1.2109	0.1742	65	2.2925	0.5638
16	1.2265	0.1847	66	2.3218	0.5693
17	1.2422	0.1950	67	2.3518	0.5748
18	1.2583	0.2053	68	2.3820	0.5802
19	1.2743	0.2153	69	2.4125	0.5855
20	1.2908	0.2253	70	2.4431	0.5908
21	1.3073	0.2351	71	2.4746	0.5959
22	1.3241	0.2448	72	2.5068	0.6012
23	1.3412	0.2544	73	2.5387	0.6061
24	1.3583	0.2638	74	2.5713	0.6111
25	1.3758	0.2732	75	2.6041	0.6161
26	1.3935	0.2824	76	2.6378	0.6209
27	1.4114	0.2915	77	2.6716	0.6257
28	1.4295	0.3005	78	2.7063	0.6305
29	1.4480	0.3094	79	2.7412	0.6352
30	1.4664	0.3181	80	2.7762	0.6398
31	1.4854	0.3268	81	2.8121	0.6444
32	1.5044	0.3353	82	2.8481	0.6489
33	1.5236	0.3437	83	2.8843	0.6533
34	1.5434	0.3521	84	2.9214	0.6577
35	1.5632	0.3603	85	2.9594	0.6621
36	1.5832	0.3684	86	2.9967	0.6663
37	1.6035	0.3764	87	3.0358	0.6706
38	1.6241	0.3843	88	3.0750	0.6748
39	1.6450	0.3921	89	3.1142	0.6789
40	1.6661	0.3998	90	3.1535	0.6829
41	1.6874	0.4074	91	3.1948	0.6870
42	1.7091	0.4149	92	3.2351	0.6909
43	1.7313	0.4224	93	3.2776	0.6949
44	1.7534	0.4297	94	3.3189	0.6987
45	1.7758	0.4369	95	3.3613	0.7026
46	1.7988	0.4441	96	3.4048	0.7063
47	1.8218	0.4511	97	3.4482	0.7100
48	1.8453	0.4581	98	3.4927	0.7137
49	1.8691	0.4650	99	3.5385	0.7174
50	1.8928	0.4717	100	3.5829	0.7209

STUDENT RESULTSINDIUM FOIL DATA

Name

Irradiation Period 1 hour

Date

Reactor Power ≈ 1 Watt

Standard Counts on #1 - 85984

Background #1 Counter 28 c/m

Standard Counts on #2 - 86809

#2 Counter 42 c/m

Factor - 1.0096

1 Foil Loc.	2 Activity	3 Background Corrected	4 Time	5 Decay Corrected ($\pm e^{-\lambda t}$)	6 Foil Weight	7 Counter Normalization	8 Activity per Unit Weight	9	10 Normalized Activity per Unit Weight
R2	830430	830388	30	1220460	0.1401		871130		1.164
R4	706068	706026	28	1011380	0.1357		745299		0.996
R6	734821	734779	26	1025880	0.1400		732770		0.979
R8	925771	925729	10	1052530	0.1406		748587		1.000
R10	836719	836677	24	1138550	0.1423		800100		1.069
R12	845010	844968	22	1120690	0.1364		821621		1.098
R14	836686	836644	20	1081520	0.1402		771409		1.030
R16	896006	895964	18	1128840	0.1428		790500		1.056
B13	575267	575239	18	724756	0.1366	731714	535662		0.716
B14	722391	722363	20	933792	0.1407	942756	670047		0.895
B15	925771	925729	10	1052530	0.1406		748587		1.000
B16	867282	867254	22	1150250	0.1402	1161292	828311		1.106
B17	704959	704931	26	984211	0.1437	993659	691482		0.924
B18	505833	505805	28	724566	0.1373	731516	532787		0.712
V1	438358	438305	16	538233	0.1418		379568		0.507
V2	671984	671979	14	804274	0.1409		570810		0.763
V3	856207	856154	12	998733	0.1408		709319		0.948
V4	925771	925718	10	1052514	0.1406		748587		1.000
V5	984616	984563	4	1036436	0.1432		723768		0.967
V6	808563	808510	2	829539	0.1402		591680		0.790
V7	557035	556982	0	556982	0.1410		395020		0.528

SAMPLE CALCULATIONS DATA SHEETFoil Positions

Bare Au	B15	R8	V4
Cd-Au-Cd	B16	R8	V4
Bare U (#4)	B14	R8	V4
Cd-U-Cd (#3)	B17	R8	V4

Background Counts on U Foils

U #3	3354 c/min
U #4	3316 c/min

Counts after Irradiation

#3	22,091 c/min	Time 15:57:00
#4	175,293 c/min	Time 16:01:00
Bare Au	8,385 c/min	Time 15:42:00
Cd-Au-Cd	3,545 c/min	Time 15:45:00

Calibration Factor for Absolute Counter

$$11.70 \frac{\text{dpm}}{\text{cpm}}$$
Relative Fluxes Normalized to the Common Point

$$K_B = 0.903$$

$$K_V = 0.838 \quad \bar{\phi} = K_B K_V K_R = 0.756$$

$$K_R = 0.999$$
Absolute Thermal-neutron Flux Measurement

Bare Au: $C_0 = 8.3 \times 10^3 \text{ c/min}$ Foil weight = 50 mg

Cd-covered Au: $C_0 = 3.5 \times 10^3 \text{ c/min}$ Foil weight = 50 mg

$$C_{0\text{Thermal}} = C_{0\text{Bare}} - C_{0\text{Cd}} = 4.8 \times 10^3 \text{ c/min}$$

$$A_{\text{th}} = K C_{0\text{th}} = \sigma_{\text{act}} \phi_{\text{th}} N_T (1 - e^{-\lambda T}) e^{-\lambda t} 60$$

$$N_T = \text{number of atoms in foil} = \frac{N_0 W}{A} = \frac{0.6023 \times 10^{24} \times 0.05}{197.2}$$

where

t = decay time

T = irradiation time

σ_{act} = 96 b

ϕ_{th} = neutron flux

$$\phi_{th} = \frac{K C_{0th}}{\sigma_{act} N_T (1 - e^{-\lambda T}) e^{-\lambda t(0.60)}} = 6.1 \times 10^6 \text{ n/cm}^2/\text{sec.}$$

Power Calibration

$$P = \frac{\phi_{th} \sigma_{f235} N_{235} \left(\frac{CR}{CR - 1} \right)_u}{3.1 \times 10^{10} \frac{\text{fiss}}{\text{Watt-sec.}}}$$

where

$\bar{\phi}_{th}$ = average thermal flux in the fuel

$$= \phi_{th} \times \bar{\phi} \times \frac{1}{\text{disadvantage factor}}$$

$$= 6.1 \times 10^6 \times 0.756 \times \frac{1}{1.07} = 4.32 \times 10^6$$

$$\sigma_{f235} = 570 \times 10^{-24} \text{ cm}^2$$

$$N_{235} = \frac{N_0 W}{A} = \frac{0.6023 \times 10^{-24} \times 1.8 \times 10^3}{235} = 4.6 \times 10^{24}$$

$W = 1843 \text{ g. of } U^{235} \text{ in core}$

$$\left(\frac{CR}{CR - 1} \right)_u = 1.1$$

then

$$P = 0.4 \text{ Watt.}$$

Instrument Readings.

Log	1×10^{-8} Amps
Linear #1	0.6×10^{-8} Amps
Linear #2	0.18×10^{-8} Amps
Linear #3	0.04×10^{-9} Amps

Fig. S6.1
Foil Locations for
Power Calibration

Fig. S6.2

Fig. S6.3

Fig. S6.4

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POWER CALIBRATION, AGN-201 REACTOR

by

K. C. Ruzich

ABSTRACT

The method used to calibrate the neutron-flux-monitoring instruments of the AGN-201 reactor to a given power level will be presented in this experiment. The experiment involves a measurement of the average thermal-neutron flux at a certain power level. This measurement is then used to calculate the fission rate occurring in the reactor knowing the fuel mass, fission cross section, and the core volume. Since each fission occurring in the reactor can be assumed to release some average amount of energy, the fission-rate calculation can be easily converted to the power level of the reactor. The resulting power level will then correspond to the readings indicated by the flux-monitoring instruments during the flux measurement. The accuracy and the corrections involved in the calibration will be brought out during the course of the experiment.

ACKNOWLEDGEMENT

This experiment was prepared and performed by Samuel Helf and Dirceu L. Coutinho, Institute participants, as a student project. Their work is contained in an unpublished INSE report.

OUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
- IV. PROCEDURE
 - A. Dysprosium Wire and Standard Gold-foil Irradiation
 - B. Standard Cadmium-covered Gold-foil Irradiations
- V. RESULTS
- VI. ANALYSIS AND CONCLUSIONS
- VII. BIBLIOGRAPHY
- VIII. APPENDIX
 - Discussion Questions

I. INTRODUCTION

In high-power reactors, it is possible to determine a given power level by calorimetric methods, that is, by measurements of core and coolant temperatures, coolant flow rates, etc. Low-power reactors do not have this advantage since the heat generated during normal operation is small. Therefore, the approach used to measure a given power level of the AGN-201 reactor will not involve the product of nuclear fission (that is, heat), but the initiator of the reaction, namely, neutrons. Each fission which occurs in the reactor core releases an average energy of 201 MeV. The rate at which energy is being released, the power level, is proportional to the fission rate. A measurement of the fission rate will therefore constitute a determination of power level.

The method involved will employ the use of activation detectors to measure the neutron flux in the reactor core, and hence the fission rate. Dysprosium-aluminum wire has been chosen to measure the thermal-neutron spatial distribution. Dysprosium has a high cross section for thermal-neutron activation and a convenient half-life of 140 min. Thermal-neutron flux distributions will be made through the glory hole on both sides of core center and along the fine control rod when it is fully inserted. An absolute thermal-flux measurement will be made at the core center with a standard gold foil. This foil will be counted on an end-window GM counter whose efficiency for the standard gold foil has been determined from a previous standard pile irradiation. The results from the two activation detectors will yield the average thermal flux in the reactor core as described in Section II. A cadmium-ratio measurement of AGN-201 fuel can also be made in order to determine the fraction of the total fission rate due to epithermal neutrons.

II. THEORY

If the reactor may be assumed to be an all-thermal homogeneous model, the following expression describes the total fission rate in the steady state:

$$R_f(\text{fissions/sec}) = \Sigma_f \int_{\text{vol}} \phi dV \quad (1)$$

where

ϕ = the thermal-neutron flux, that is, the total neutron density times the most probable value of the thermal-neutron velocity distribution at standard temperature;

Σ_f = macroscopic fission cross section of the fuel at the most probable velocity corrected, if necessary, for a non- $1/v$ behavior.

The spatial distribution of the neutron flux in the core depends on the geometry of the system. If the average thermal-neutron flux can be determined, Eq. (1) will simplify to

$$R_f = \Sigma_f \bar{\phi} V_{\text{core}}$$

with the spatial average of the thermal flux being given by

$$\bar{\phi} = \frac{\int_V \phi dV}{\int_V dV}$$

The power level of the reactor corresponding to the average thermal flux is then equal to the total fission rate divided by the number of fissions per second required to produce one Watt. For a reactor utilizing U^{235} as fuel,

$$\text{Power} = \frac{R_f}{3.1 \times 10^{10}} = \frac{\Sigma_f \bar{\phi} V}{3.1 \times 10^{10}} \text{ (Watts)}. \quad (2)$$

A good approximation of the average thermal flux can be found from a measurement of the thermal-flux spatial distribution along the three lattice axes and a determination of the absolute thermal flux at the core center. When it is possible, the core center should represent a common point for all three distribution measurements for normalization purposes. The average flux will be expressed as

$$\bar{\phi} = \phi_0 K_x K_y K_z,$$

where ϕ_0 is the absolute thermal flux at the core center, and K_x , K_y , and K_z are the numerically averaged fluxes in the indicated direction divided by the flux corresponding to the core center in the same direction. For the AGN-201 reactor, which has a cylindrical core, $K_x K_y = K_{\phi r}$;

$$\bar{\phi} = \phi_0 K_{\phi r} K_z. \quad (3)$$

The factor $K_{\phi r}$ is determined from a volume-weighted radial flux distribution.

Although Eq. (2) has been derived for a simplified model, its accuracy when used for the AGN-201 reactor power calibration will depend primarily on the average thermal-flux measurement and to a lesser extent on a correction made for nonthermal fission. The energy distribution of the neutrons in the core actually covers a wide range, from fission energies down to thermal. Before the neutrons are slowed down into the thermal region, some will be captured in U^{235} and U^{238} , and hence cause additional fissions. The fission in U^{238} is caused only by neutrons of high energy.

The threshold reaction contributions to the total fission rate can be assumed small for the AGN-201 reactor, since its moderator-to-uranium volume ratio is appreciable and its fuel is enriched with the U^{235} isotope. Very fast fission is normally accounted for in the four-factor formula by the factor ϵ , the number of neutrons produced by all fissions divided by the number produced by thermal fission. In the AGN-201, nonthermal fission is predominately resonance fission, since U^{235} has finite fission cross sections at all energies. The amount of epithermal fission can be determined by a simple cadmium-ratio measurement of AGN-201-type fuel. The fission product activity of a bare and cadmium-covered fuel sample can be counted on a proportional counter after two similar irradiations in the reactor core. Their ratio will yield the amount of nonthermal fission to total fission after proper corrections for differences of sample weight, irradiation times, and power level have been made. The final expression for power level then becomes

$$P = \frac{\Sigma_f \bar{\phi} V}{3.1 \times 10^{10}} \left[\frac{CR}{CR-1} \right]_{\text{fuel}} \quad (\text{Watts}), \quad (4)$$

where CR is the fuel-sample cadmium ratio.

The average spatial thermal flux is actually the total neutron density times 2200 m/sec. This is true because the absolute thermal flux is measured with $1/v$ -absorber, using its cross section at this standard velocity. The macroscopic fission cross section of U^{235} is that for the same velocity. Actually, since this cross section does not have a $1/v$ -dependence, a correction can be made to make it an equivalent $1/v$ -cross section, which will give the correct fission rate in Eq. (4). This correction is based on a Maxwellian neutron distribution and is a function of the neutron temperature. The temperature of the neutrons in the AGN-201 core is approximately 350°K, which corresponds to a correction factor of 0.98.

The absolute flux measurements at the core center with a standard gold foil requires the knowledge of fundamental activation analysis. The activity of the foil immediately after the irradiation can be expressed as

$$A_0 \text{ (dis/sec)} = N_T \sigma_{\text{act}} \phi k (1 - e^{-\lambda t}), \quad (5)$$

where

N_T = total number of detector atoms;

σ_{act} = thermal-neutron activation cross section at 2200 m/sec;

ϕ = neutron flux (nv₂₂₀₀);

k = thermal-flux depression, self-shielding factor;

λ = detector decay constant;

t = irradiation time.

If the detector is counted with a system whose inverse efficiency, E (dis/sec/count/sec), is known at a time t_c after irradiation, the detector activity becomes

$$EC = A_0 e^{-\lambda t_c}, \quad (6)$$

where C is the counts/sec at t_c .

Solution of (5) and (6) for the neutron flux gives

$$\phi = \frac{EC e^{\lambda t_c}}{N_T \sigma_{\text{act}} k (1 - e^{-\lambda t})}. \quad (7)$$

This neutron flux is the proper flux to be used in Eq. (4) only if the activity is caused by thermal neutrons alone. Since the cross section of gold has a strong resonance-activation peak, a correction must be made to separate the resonance activation from the thermal activation. This can be done by making a cadmium-ratio measurement with the standard gold foil. If CR is the measured value, the thermal activity of the detector becomes

$$A_{0\text{th}} = A_0 \left[\frac{CR-1}{CR} \right]_{\text{Au Foil}}.$$

Hence, the flux in (7) must be multiplied by the factor in the brackets to yield the correct thermal flux.

III. APPARATUS

- A. AGN-201 reactor.
- B. Dysprosium-aluminum wire for flux distribution measurements.
- C. End window GM counter and wire-scanning equipment (see Figs. 7.1 and 7.2).
- D. Standard gold foil and cadmium covers.
- E. Standard end window GM counter (see Fig. 7.3).

139-3415

Fig. 7 1. GM Tube with Shield and Wire Scanning Apparatus

139-3423

Fig. 7.2. Dy-Al Wire Scanning Apparatus and GM Counter

139-2420

Fig. 7.3. Standard End Window GM Counter

IV. PROCEDURE

A. Dysprosium Wire and Standard Gold-foil Irradiation:

Two dysprosium-aluminum wires of the proper length are placed in the reactor core. One wire will be positioned in the glory hole for a radial flux traverse, the other in the fine control rod for a partial vertical traverse. The glory hole wire will be placed in the center of a 7/8-in.-diameter polystyrene rod and positioned so as to traverse the core region completely. Care must be taken to mark the wire positions at the center and edges of the core. The second wire is attached to a piece of polystyrene which is placed in the fine control rod fuel capsule. The top of the polystyrene rod will serve as the reference for the wire position in the core. A standard gold foil will be placed at the core center for an absolute flux measurement. The foil will be contained in a small aluminum container and shall be mounted in the polystyrene rod.

The reactor is started up following the proper procedures. The fine control rod should be moved and left completely inserted in the core. A measurement of the reactor period, after source removal should be made and recorded on the data sheet. Once full power has been reached, the reactor should be made critical by adjustment of the coarse control rod. The wires and foil are to be irradiated for approximately one hour at full power. The control rod settings, the readings on each of the three flux-monitoring instruments, and the irradiation times should be recorded on the data sheet.

After the reactor is scrammed, the wires and foil are removed from the core and taken to the south counting room. During their removal, survey instruments should be used to monitor the irradiation levels of the detectors. After 20 min has elapsed from scram time, the wires will be counted on the GM wire scanner to obtain their respective relative flux traverses. The wire should be scanned in 1/2- or 1-in. segments. The decay of wire activity during the counting period must be taken into consideration with the initial counting time taken as a reference. A plot of the corrected counting rates versus core position should be made for each wire. For convenience, the plot can be normalized to unity at the core center. The vertical plot will yield the flux distribution in that direction for the lower part of the core; it will be assumed that the upper part has the same distribution. The error involved in this assumption will not be large. For a more detailed description of the vertical flux distribution, see the unpublished report in the Appendix. From these two plots, the average flux relative to the core center can be numerically determined, yielding K_r and K_z in Eq. (3) of Section II.

The standard gold foil will be counted with the standardized GM end window counter. The efficiency of this counter has been measured for the gold foil from previous standard pile irradiations. The counter

background and dead time must be taken into consideration to obtain the correct counting rate. The times involved and the counting rates should be recorded on the data sheet.

B. Cadmium-covered Standard Gold-foil Irradiations

The second irradiation of the experiment will be started immediately after the first is completed. A standard gold foil, cadmium covered, is placed at the core center in the polystyrene rod. The procedure for this irradiation is identical with that of the first irradiation. Care must be taken to bring the reactor to the same power level on approximately the same period. After the hour irradiation, the reactor is scrammed, and the foil is taken to the counting room and counted on the standardized GM end window counter.

AGN-201 REACTOR POWER CALIBRATION

DATA SHEET

Run	Start-up Period	Critical Settings		Time Full Power is Reached	Channel 1	Channel 2	Channel 3	Scram Time
		FR	CR					
1								
2								

Wire Segment	Start Time	Counting Rate	Corrected Counting Rate	Wire Segment	Start Time	Counting Rate	Corrected Counting Rate

Gold Foil Weight	Scram Time	Counting Time	cpm	Corrected cpm
Al				
Cd				

V. RESULTS

The results of the experiment will include the power calibration, plots of the vertical and radial flux distributions, a determination of the average flux in the core, the absolute flux at the core center, and a cadmium-ratio measurement with the standard gold foil. The calculations necessary to obtain these results involve the equations in Section II. The fuel-cadmium ratio has been found to equal 19.7. The flux depression and self shielding

of the standard gold foil can be determined from ANL-5800, page 487. This correction is quite small and can be neglected if desired.

VI. ANALYSIS AND CONCLUSIONS

A brief analysis of the experiment together with significant conclusions is left as an exercise for the participant.

VII. BIBLIOGRAPHY

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VIII. APPENDIXDiscussion Questions

1. The measurement of the reactor power output involves computation of the fission rate in the reactor core. What other method or methods might be used to measure the power output of this or another reactor?
2. For the purpose of this experiment, consider that a fission rate of 3.1×10^{10} fissions/sec is equivalent to 1 W. Analyze this in terms of MeV per fission.
3. Analyze the total MeV/fission derived in (2) above in terms of the amount of energy carried off by the various particles and photons, i.e., fission fragments, gamma rays, etc.
4. Assuming the fuel distribution in the reactor to be homogeneous and the distribution of neutron flux to be known, how may we arrive at the average neutron flux?
5. Why is it desirable to use ideally thin foils in measuring neutron flux?
6. Outline the steps involved in using a standard pile to calibrate a counting system which will be used for absolute neutron-flux measurements with standard foils.
7. What is beta-ray self-shielding in foils, and how may its effect be eliminated?
8. We propose to measure the cadmium ratio of gold and a fuel sample during this experiment. Discuss the need and give the direction of this correction. How does the absorption of epithermal neutrons by cadmium affect the cadmium ratio?
9. What factors affect the energy contribution due to fission in U^{238} ?
10. The final result of the experiment yields a given power level which corresponds to certain readings on the flux-monitoring instruments. How may this calibration be used to determine the power level at other instrument readings?

Wire Scanning GM Counter
Resolving Time: 210 μ sec

9-5-61

RADIAL FLUX TRAVERSE OF AGN-201

Wire Pos.	Wait Time	Counts	Dead Time* Correction	Bkgd.	Net Counts	$e^{\lambda t_w}$	Correct Counts	A/A ₀
0	0	3,488	+43	24	3,507	-	3,507	0.2739
$\frac{1}{2}$	2	5,276	97	24	5,349	1.010	5,402	0.4220
1	3:30	6,377	143	24	6,496	1.0175	7,610	0.5944
$1\frac{1}{2}$	5	7,413	193	24	7,582	1.025	7,772	0.6071
2	7:30	8,214	237	24	8,437	1.038	8,158	0.6841
$2\frac{1}{2}$	9:00	9,111	290	24	9,425	1.0460	9,859	0.7701
3	10:30	9,733	332	24	10,010	1.0535	10,546	0.8238
$3\frac{1}{2}$	12	10,458	385	24	10,819	1.062	11,490	0.8975
4	15:30	10,859	413	24	11,248	1.080	12,148	0.9989
$4\frac{1}{2}$	17:00	11,047	427	24	11,450	1.088	12,450	0.9731
5	18:30	11,066	427	24	11,469	1.0965	12,576	0.9823
$5\frac{1}{2}$	20:00	11,120	434	24	11,530	1.105	12,741	0.9952
6	21:30	11,095	431	24	11,502	1.1130	12,802*	1
$6\frac{1}{2}$	23:00	10,503	427	24	10,906	1.127	12,291	0.9601
7	25:00	7,818	337	24	10,131	1.133	11,478	0.8966
$7\frac{1}{2}$	26:30	9,583	322	24	9,881	1.141	11,274	0.8806
8	28:00	8,971	283	24	9,230	1.150	10,615	0.8292
$8\frac{1}{2}$	29:30	8,290	241	24	8,507	1.158	9,851	0.7695
9	31:00	7,332	188	24	7,496	1.167	8,748	0.6833
$9\frac{1}{2}$	32:30	6,440	145	24	6,561	1.1755	7,712	0.6024
10	34:00	5,332	96	24	5,404	1.184	6,398	0.4998
$10\frac{1}{2}$	35:30	4,261	64	24	4,301	1.193	5,131	0.4008
11	37:00	2,770	24	24	2,770	1.202	3,330	0.2601

*Resolving time of system - 210 μ sec.

AXIAL FLUX TRAVERSE

Length of Wire = 9-15/16

Wire Pos.	Wait Time	Counts	Dead Time Correction	Bkgd.	Net Counts	$e^{\lambda t_w}$	Correct Counts	A/7000
0	0	1,477	8	24	1,461	-	1,461	0.209
1	4	1,966	13	24	1,955	1.020	1,994	0.285
2	6	2,854	17	24	2,247	1.030	2,314	0.331
3	8	2,793	26	24	2,795	1.041	2,910	0.416
3.5	10	2,923	29	24	2,928	1.051	3,077	0.439
4	12	3,368	40	24	3,384	1.062	3,594	0.513
4.5	14	4,198	62	24	4,236	1.072	4,541	0.648
5	16	4,781	80	24	4,837	1.083	5,238	0.748
5.5	18	4,924	85	24	4,985	1.094	5,454	0.780
6	20	5,453	99	24	5,528	1.105	6,108	0.873
6.5	22	5,654	112	24	5,742	1.116	6,408	0.915
7.0	24	5,852	120	24	5,948	1.127	6,703	0.958
7.5	26	5,848	119	24	5,943	1.138	6,763	0.966
8.0	28	5,958	124	24	6,058	1.150	6,967*	0.996
8.5	30	5,672	113	24	5,761	1.161	6,689	0.956
9.0	32	5,767	117	24	5,860	1.173	6,874	0.983
9.5	34	5,676	113	24	5,765	1.184	6,826	0.976
10.0	36	181	-	24	-	-	-	-

Top of
Core

Max. Flux. Det.

Av. Foils	Weight	Irradiation Time	cpm - Bkgd.
Bare	48.5	60 min.	6,127
Cd Cov.	48.6	60 min.	3,079

DATA

STUDENT RESULTS

Wire Position	Core Position	$A/A_0 = \phi/\phi_0$	$r \times \phi/\phi_0$	$\Gamma \times \phi/\phi_0$ Normalized
0	5.5	0.274	1.507	0.623
1/2	5	0.422	2.110	0.872
1	4.5	0.594	2.673	1.105
1-1/2	4	0.607	2.428	1.003
2	3.5	0.684	2.394	0.989
2-1/2	3	0.770	2.310	0.955
3	2.5	0.824	2.060	0.851
3-1/2	2	0.898	1.796	0.742
4	1.5	0.946	1.419	0.586
4-1/2	1	0.973	0.973	0.402
5	0.5	0.982	0.196	0.081
5-1/2	0	0.995	0	0
6	0.5	1.000	0.200	0.083
6-1/2	1	0.960	0.960	0.397
7	1.5	0.897	1.346	0.556
7-1/2	2	0.881	1.762	0.728
8	2.5	0.829	2.073	0.857
8-1/2	3	0.765	2.295	0.948
9	3.5	0.683	2.391	0.988
9-1/2	4	0.603	2.412	0.997
10	4.5	0.500	2.250	0.930
10-1/2	5	0.401	2.005	0.829
11	5.5	0.260	1.430	0.591

Fig. S7.1

Fig. S7.2

Fig. S7.3. Relative $\phi_{\text{(total)}}$ in Annular Rings

Power Calibration Calculations

Maximum Neutron Flux at Core Center

$$\phi_0 = \frac{E C_{th}}{N \sigma_{act} k}$$

where

$$E = 6.87 \pm 11.6\% \frac{\text{disintegrations}}{\text{unit count}}$$

C_{th} = saturated counts per sec per gm of Au due to thermal neutron activation only at the end of the irradiations.

N = number of Au^{197} atoms per gm

σ_{act} = microscopic activation cross section for the $Au^{197} (n, \gamma) Au^{198}$

$$N = \frac{6.023 \times 10^{23}}{197} = 3.06 \times 10^{21} \text{ atoms/gm}$$

$$\sigma_{act} = 98 \times 10^{-24} \text{ cm}^2/\text{atom}$$

k = self shielding factor $\cong 0.96$

C_{th} = saturated cps of bare foil per gm minus cps of the cadmium covered foil per gm at the end of their respective irradiations.

$$C_{th} = \frac{102.1 e^{\lambda(10)}}{48.5 \times 10^{-3} \text{ gm} (1 - e^{-\lambda(60)})} - \frac{51.3 \text{ cps } e^{\lambda(20)}}{48.6 \times 10^{-3} \text{ gm} (1 - e^{-\lambda(60)})}$$

where, the bare foil is irradiated for sixty minutes and counted ten minutes later; and the Cd covered foil is irradiated for sixty minutes and counted twenty minutes later.

$$C_{th} = 1.985 \times 10^5 - 0.994 \times 10^5 = 0.991 \times 10^5 \text{ cps/gm}$$

$$\phi_{th} = \frac{(11.7)(0.991 \times 10^5)}{(3.06 \times 10^{21})(98 \times 10^{-24})(0.96)} = 4.04 \times 10^6 \frac{11}{\text{cm}^2 \text{ sec}}$$

Average Neutron Flux in the Core.

The average flux in the core is

$$\bar{\phi} = K_{\phi r} K_2 \phi_0$$

$K_{\phi r}$ and K_2 are obtained by graphical integration of the proper flux plots.

The total relative flux in an annular ring equals

$$\frac{\phi(r)}{\phi_0} 2\pi r dr dz$$

Average ϕ_T in an annular ring is 0.705 of the maximum rings value. Since there are R annular rings the total ϕ in all rings is

$$\int_0^R 0.705 \left[2\pi \left(\frac{\phi(r_{\max})}{\phi_0} r_{\max} \right) dr dz \right]$$

where

$$r_{\max} = 10.16 \text{ cm}$$

$$\frac{\phi_{r_{\max}}}{\phi_0} = 0.605$$

The average ϕ , based on unity at the center, in a radial plane with a height dz is therefore:

$$K_{r\phi} = \frac{0.705 \times 2\pi \left(\frac{\phi(r_{\max})}{\phi_0} r_{\max} \right) r \, dz}{\pi R^2 \, dz}$$

$$= 0.681$$

$$R = 12.7 \text{ cm}$$

$$K_2 = 0.745$$

$$\bar{\phi} = \phi_0 K_{r\phi} K_2$$

$$\bar{\phi} = 4.04 \times 10^6 (0.681)(0.745)$$

$$\bar{\phi} = 2.05 \times 10^6$$

Power Level Calculation

$$P (\text{Watts}) = \frac{\Sigma_F \bar{\phi} \text{ Volume}}{3.1 \times 10^{10}} \left[\frac{CR}{CR-1} \right]_{\text{fuel}}$$

$$\Sigma_F \text{ Volume} = \sigma_F N$$

where

σ_F = microscopic fission cross section

N = total number of U^{235} atoms in the core

$$\sigma_F N = 582 \times 10^{-29} \text{ cm}^2 (660 \text{ gms}) \frac{(6.023 \times 10^{23})}{235} = 985 \text{ cm}^2$$

$$P = \frac{985 (2.05 \times 10^6)}{3.1 \times 10^{10}} \left[\frac{19.7}{18.7} \right]$$

$$= 0.0686 \text{ Watts}$$

The reactor instruments indicated the following:

Channel 1	190,000 cpm	}	68.6 milliwatt
Channel 2	2.2×10^{-9} Amp		
Channel 3	1.2×10^{-9} Amp		

therefore

Channel 1	278,000 cpm	}	100 milliwatt
Channel 2	3.2×10^{-9} Amp		
Channel 3	1.75×10^{-9} Amp		

VOID COEFFICIENT OF REACTIVITY, ARGONAUT REACTOR

by

D. A. Daavettila

ABSTRACT

A critical reactor will become either sub- or supercritical when a void is introduced into the core. The magnitude and sign of the reactivity effect due to the void is known as the void coefficient of reactivity, and it is a complex function of core design, void location, and void volume. The void coefficient is an important parameter in the hazards evaluation of a broad class of reactors and also in the operation of certain reactors, such as the boiling water type.

In this experiment, void coefficient measurements are made in the Argonaut reactor as a function of the void location within the core, and the average void coefficient of the core is estimated from the measured data. During the pre-experiment briefing session, the discussions will review quite generally the details and problems of void coefficient measurements.

OUTLINE

- I. INTRODUCTION
 - II. THEORY
 - III. APPARATUS
 - IV. PROCEDURE
 - V. RESULTS
 - VI. ANALYSIS AND CONCLUSIONS
 - VII. BIBLIOGRAPHY
 - VIII. APPENDIX
- Discussion Questions

I. INTRODUCTION

In the hazards evaluation of a broad class of reactors, it is important to know what is the reactivity effect introduced by a void in the core. This effect is described as being either positive or negative, where negative refers to a reactivity decrease due to the void. The magnitude of the reactivity effect is usually expressed in units of percent Δk_{eff} per one percent void volume. The sign and magnitude of the reactivity effect due to the void is known as the void coefficient of reactivity.

Knowledge of the void coefficient is also important during the initial design and everyday operation of reactors, such as the boiling water reactor. For those reactors in which the void coefficient will be an important consideration in either the hazards or operation analysis, the coefficient is first calculated by theory and perhaps checked by experiment in a critical facility. When the reactor is completed, void coefficient measurements are often early on the agenda.

The overall purpose of the Argonaut experiment is to present some of the details and problems involved in void coefficient measurements. For example, what is the reference volume upon which to base the percent void volume? Furthermore, how much void, and of what size and shape, should be used in the measurements? Some additional problems are to decide where and how the void should be introduced, and also to choose the best of several methods of measuring the reactivity due to the void.

The answers to the above problems will depend on the particular reactor design and whether a certain local or a uniform void coefficient is being sought. During the pre-experiment briefing session, these general problems will be reviewed by group discussion.

The Argonaut void coefficient experiment itself will be a measurement of the void coefficient as a function of void location in the reactor core. The experimental apparatus consists mainly of at least four Lucite void

holders, a vacuum system to remove the water from the void holders, a water reservoir from which to fill the void holders, a solenoid-valve manifold to select a particular void holder, and finally a switch box to open and close the selected solenoid valves remotely. With this equipment, the void location can be shifted without the necessity of shutting down the reactor. However, it is necessary to shut down the reactor to change the location of the void holder. Normally, the experiment is made up of two reactor runs, that is, one void-holder-relocation setup is required.

After all experimental data are obtained, a post-experiment summary session will be held to review methods to estimate the average core void coefficient from the data and to complete all calculations and graphs.

II. THEORY

The nature of the relationship of reactivity to void can be developed from a consideration of the effect of void on the parameters in the two-group criticality equation

$$k_{\text{eff}} = \frac{\eta \epsilon p f e^{-\tau B^2}}{1 + L^2 B^2}, \quad (1)$$

where

η = neutrons born per thermal neutron captured in the uranium,

ϵ = fast fission factor,

p = resonance escape probability,

f = thermal utilization,

τ = neutron age,

L^2 = thermal diffusion area, and

B^2 = the geometric buckling.

The quantity $e^{-\tau B^2}$ is the fast nonleakage probability, and $1/(1 + L^2 B^2)$ is the thermal nonleakage probability. When the reactor is critical, k_{eff} is equal to one. Introducing void into the core will change the values of the parameters in Eq. (1) so that, most likely, k_{eff} will not equal one, and as a result the reactor power will begin to change with time. The rate of change of power level (reactor period) is a function of the reactivity worth of the void. From a measurement of the reactor period, the reactivity can be determined from an available calculated curve. Since the void volume is known, one can calculate the void coefficient from

$$\text{Void Coefficient} = \frac{k_{\text{eff}} - 1}{k_{\text{eff}}} \bigg/ \text{one percent void} \quad (2)$$

Of course, the interpretation of this void coefficient value depends on the details of the void location, size, etc.

Now to consider the effect of the void on each of the parameters given in Eq. (1). One effect of the void will be a reduction of the nonleakage probabilities, because the reduced moderator density decreases the macroscopic cross sections for both scattering and absorption, which increases both τ and L^2 . They vary with density in a homogeneous system as

$$\tau + L^2 = \tau_0 \left[\frac{d_0}{d} \right]^2 + L_0^2 \left[\frac{d_0}{d} \right]^2, \quad (3)$$

where d is the density.

Because flux peaking in the moderator is reduced when void is present, both moderator neutron capture and the disadvantage factor are reduced, which results in an increase in f :

$$f = \frac{\Sigma_f}{\Sigma_f + (\bar{\phi}_m / \bar{\phi}_f) \Sigma_m}, \quad (4)$$

where $\bar{\phi}_m / \bar{\phi}_f$ is the disadvantage factor, and subscripts m and f refer to moderator and fuel. If the reactor is overmoderated, the removal of moderator will be to the reactor a removal of poison, which is a positive reactivity contribution.

In general p decreases when void is introduced because the resonance-energy neutrons have a greater probability of reaching a fuel plate.

As U^{238} only fissions for neutrons with energy in excess of about 1 MeV, only a few moderator collisions are necessary to reduce the neutron energy below the threshold. A void between fuel plates increases the probability of a neutron colliding with a U^{238} atom while it yet is above the threshold; hence ϵ would tend to increase.

The effect on η is negligible. With hardening of the neutron spectrum, ν increases, but the ratio of U^{238} to U^{235} captures also increases.

The overall effect of void will depend on which factors dominate, and in most cases by suitable reactor design the overall effect of void is to reduce k_{eff} .

III. APPARATUS

- A. Argonaut reactor
- B. Lucite void holders (see Fig. 8.4)
- C. Water reservoir to fill the void holders
- D. Vacuum system to empty the void holders
- E. Solenoid-valve manifold by which a particular void holder can be selected and either emptied or filled
- F. An electric switch box, in order that the solenoid-valve manifold can be remotely controlled

IV. PROCEDURE

After the pre-experiment briefing session is completed, the void holders will be installed within the chosen fuel box or boxes. Sketch into Fig. 8.1 the void locations, and include in the sketch the void identifying number, void volume, and also the dummy fuel element locations.

When the fuel box is ready for loading into the reactor core, one of the end plates should be chosen as a reference zero line, and then the distance to each void measured, in centimeters, as shown in Fig. 8.2.

Fig. 8.1. Void Location

Fig. 8.2. Arrangement for Measuring Distance to Void Location

These data will be useful in the construction of the graph of void coefficient versus void location.

The fuel box will then be loaded into the reactor core, and the rest of the apparatus connected and thoroughly tested for proper operation. A sketch of the complete setup is shown in Fig. 8.3 and a photograph in Fig. 8.4.

The void holders should all be filled with water before the reactor cover is installed. Recheck that the voids are full before the cover is installed.

Now the reactor safety checkout should be completed and the reactor brought to criticality at a power level of about 0.5 W. The critical control rod positions should be carefully determined, and the positions entered into the data sheet (see Table 8.1). It is a good idea to keep a complete record of the critical control rod positions, because these data may prove useful in a number of ways. For example, if the control rod calibration experiment has preceded this experiment, you could make use of your rod calibration curve to check on the proper magnitude of the new reactivity determinations. Additional information is entered in Table 8.2.

With the reactor critical and all void holders filled with water, empty a void holder while watching reactor power behavior. From the power level behavior, what can you conclude about the void coefficient? Assume that the power level is decreasing; then adjustments are to be made in the fine control rod setting to make the reactor critical at a power setting of about 20 or 30 on linear power level recorder #1. Enter the new fine rod critical position on your data sheet.

Fig. 8.3. Void Coefficient Apparatus

139-3350

Fig. 8.4. Experimental Apparatus

Table 8.1

EXPERIMENTAL DATA

Refer to Fig. 8.1 for location details. Include the moderator temperature in the column titled Run Number.

Run Number	Operation	Critical Control Rod Position	Period	Reactivity

Table 8.2

SUPPLEMENTARY DATA

Number of fuel plates in core = _____.

Number of dummy plates in core = _____.

Volume of a dummy or fuel plate is 107.416 cm³.

Volume of a graphite wedge is 2253.07 cm³.

Outer tank diameter is 36 in. and the

Inner tank diameter is 24 in.

Reactor temperature coefficient is 0.01% Δk per °C.

At an arbitrarily chosen time zero, mark the recorder chart and close the switch to fill the empty void holder. Shortly after the void holder is full (~30 sec) the transient periods will disappear, and a steady asymptotic period will be established. From the power level versus time data given on the power level recorder chart, the reactor period can be determined in several ways; perhaps the most instructive is to plot the power level versus time data on semilog paper. What does the straight-line portion of the graph indicate? From this graph, measure the period. A calculated curve is available to convert the measured asymptotic period to reactivity.

After the power level has increased by about a decade, check the power increase with the fine control rod and cause the power to decrease. Now the next void holder can be emptied and the reactor adjusted to criticality at the low power level; the measurement cycle is repeated. This sequence is continued until all the void holders have been used and measurements made. The students performing the experiment should rotate the various duties so that each student will become familiar with every part.

It is best if the data analysis keeps pace with the experiment, that is, after each void holder test somebody should analyze the data and plot the reactivity worth of the void on the blackboard. In this way, apparently bad points can be spotted and rechecked. When all are happy with the results, the reactor is scrammed.

After a brief (~30 min) delay to allow for decay of the core radioactivity level, the fuel box or boxes containing the voids can be removed and the void holders relocated for another reactor run. Usually two runs can be fitted into the experiment day.

If the reactor power increases when the water is removed from the void holder, what do you consider the proper measuring procedure to be?

When all the experiment data have been accumulated, the reactor will be scrammed and the group should reassemble in the briefing room for a summary session. During this session an attempt will be made to clear up all confusing data points and questions, and each student should complete all calculations and graphs.

V. RESULTS

Some of the end results sought are the void coefficient versus void location graph and an estimation of the average void worth in the core. The latter result can be checked against the number given in the Argonaut Hazards Report, ANL-5647.

From your experimental data, make a plot of the void coefficient as a function of core location.

Making use of your experimental data, and, if necessary, additional data from the instructor, estimate the core average void coefficient.

VI. ANALYSIS AND CONCLUSIONS

This is an exercise for the student. Include in your discussion the nature or relationship of the compensation for void by the removal of a control rod. Which factor of Eq. (1) is most directly a function of rod position?

Discuss how the magnitude of the void coefficient that you have measured is related to the average coefficient.

VII. BIBLIOGRAPHY

1. S. Glasstone and M. C. Edlund, The Elements of Nuclear Reactor Theory, D. Van Nostrand Co., Inc., New York (1952).

VIII. APPENDIX

A. Discussion Questions

The following propositions should be considered prior to arrival at the Argonaut. An effort will be made to discuss them just prior to and/or during checkout of the reactor, as time permits.

1. What is a reactor void coefficient, and of what use is a knowledge of the void coefficient of reactivity?
2. Discuss the effect of the void on resonance escape, thermal utilization, fast fission factor, η , neutron age, and diffusion length.
3. In an overmoderated system, the void coefficient is found to be a function of the distance from the nearest fuel. Do you think the void coefficient might be zero at some point and of opposite sign on either side of this point? Why?
4. Compare the microscopic cross sections for absorption and scattering for hydrogen and for oxygen. Which is the predominant absorber? Discuss the moderating efficiency of hydrogen and oxygen.
5. In measuring the void coefficient of reactivity for the Argonaut, we will insert a void of the order of 25 cm³ and thus measure a heterogeneous "bulk" void coefficient. For a reactor run at constant temperature, what other types of void coefficient might be of interest in reactors in general?
6. Discuss the necessary relationship between temperature coefficient and void coefficient in a water-moderated system, i.e., what physical variable do they both affect?
7. Explain neutron Doppler effect in a heterogeneous, water-moderated, slightly enriched system, and its relationship to temperature and void coefficients of reactivity.
8. Might a reactor design incorporate overmoderation? Why?
9. How will you determine the overall reactivity effect of the insertion of the voids in this experiment. What objections might there be to use of the control rod calibration curve obtained during the control rod calibration experiment?
10. A commercially available resinous compound named "Plexiglas" is used in this experiment as the device which contains the void (air space). Strictly from a consideration of nuclear properties, is this material to be preferred to a metal void container?
11. Does the simulation of void by air space introduce a significant error in this experiment? If so, estimate the magnitude of such an error.

STUDENT RESULTS

VOID COEFFICIENT EXPERIMENT

I. Reactor Run

This experiment was performed in fuel box 16 of a one-slab, north quadrant, core such as is shown in Fig. 8.1. The reactor core contained 101 plates, including both fuel and dummies, and the power level was 0.2 W.

II. Void Locations

The numbers given below the voids in Fig. S8.1 are the distances in centimeters from the outside edge of fuel plate 17 to the center of the void volume. The void volumes in cubic centimeters are $V_1 = 140$, $V_2 = V_3 = 142$, and $V_4 = 145$.

Fig. S8.1. Void Locations in the Fuel Box

III. Measured Results

The measured periods are given in Table S8.1 for the operation of filling the void in the initially critical reactor, and the results are plotted in Fig. S8.2. The raw measured data for the determination of a typical period is given in Table S8.2, and the corresponding plot and calculation for the period are presented in Fig. S8.3.

Table S8.1

MEASURED RESULTS

Run Number	Operation	Critical Control Rod Position	Period/sec	Positive Reactivity, % $\Delta k/k$	% $\Delta k/k$ per cm^3 of Void ($\times 10^3$)
1	Fill V_1		85	0.078	-0.56
1	Fill V_2		42.5	0.128	-0.90
1	Fill V_3		28.6	0.164	-1.16
1	Fill V_4		27.5	0.166	-1.14
2	Fill V_1		31.0	0.158	-1.13
2	Fill V_2		39.4	0.132	-0.93
2	Fill V_3		59.8	0.100	-0.71
2	Empty V_4		562	0.015	+0.14

Figure S8.2
Void Worth Versus
Void Location

Table S8.2
RAW PERIOD DATA FOR RUN 1, V_2

Time, sec =	0	10	20	30	40	50	60	70
ϕ_{gross}	20.0	26.4	33.7	41.5	51.5	63.7	80.0	98.0
ϕ_{net}	16.0	22.4	29.7	37.5	47.5	59.7	76.0	94.0

Chart Zero = 4.0

Figure S8.3
Typical Period Measurement

IV. Calculation of the Core Water Volume

In order to convert the measured reactivity worth of a cubic centimeter of void into worth per percent void volume, the core water volume must be calculated. The core height is 24 in., and it is built in the annular region between a 24-in.-OD and 36-in.-ID tank. The total annular volume is

$$\frac{\pi}{4}(36^2 - 24^2) = 13572 \text{ in.}^3 = 222404 \text{ cm}^3.$$

The core volume is

$$\frac{1}{4}(222404 - 2253) = 53348 \text{ cm}^3;$$

where the volume 2253 cm^3 is subtracted because the core contains 5 instead of 6 wedges. The core water volume is given by

$$53348 - 5(2253) - 101(107.3) = 31234 \text{ cm}^3,$$

where 101 is the number of fuel plus dummy plates and 107.3 cm^3 is the volume of a single plate.

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TEMPERATURE COEFFICIENT OF REACTIVITY,
ARGONAUT REACTOR

by

D. A. Daavettila

ABSTRACT

Reactors are subject to temperature fluctuations for a variety of reasons, which include changes in ambient temperature and reactor power. These temperature fluctuations cause reactivity disturbances through changes in the physical and nuclear properties of the core. The relationship between temperature and reactivity (that is, the temperature coefficient) is usually stated as the magnitude and sign of the reactivity change due to a unit change in temperature.

In this experiment the overall temperature coefficient of the Argonaut reactor is measured. The basic procedure is to change the temperature of the water moderator, and from the movements of a calibrated control rod required to maintain criticality, the reactivity worth of the temperature change is determined.

OUTLINE

- I. INTRODUCTION
- II. THEORY
 - A. General
 - B. Density Coefficient
 - C. Nuclear Coefficient
- III. APPARATUS
- IV. PROCEDURE
- V. RESULTS
- VI. ANALYSIS AND CONCLUSIONS
- VII. BIBLIOGRAPHY
- VIII. APPENDIX
 - A. Discussion Questions

I. INTRODUCTION

In nuclear reactors temperature changes are inherent because the fission event releases energy, and most of this energy quickly appears as heat in the fuel. Since some of the physical and nuclear properties of the core are temperature dependent, there exists an interaction between the reactor temperature and the reactor multiplication. The magnitude and sign of the change in multiplication (reactivity) caused by a unit increase in temperature is known as the temperature coefficient of reactivity. It is pointed out that the overall temperature coefficient is the sum total of a large number of local temperature coefficients of the fuel, moderator, reflector, and coolant; a particular local coefficient is not necessarily of the same sign as the overall coefficient.

The operation of a reactor in the practical sense requires a small temperature coefficient in order that criticality can be easily maintained. Usually in addition to its small magnitude, the overall reactor temperature coefficient is designed to be negative to increase stability and to enhance the safety of the reactor.

In this experiment the overall reactor temperature coefficient is measured. This is done by heating the water moderator with electrical heaters and monitoring the water temperature. The reactivity due to the temperature change is determined from the measured movements of the calibrated control rod that are required to maintain criticality.

II. THEORY

A. General: The qualitative relationship between temperature and reactivity will be developed from a consideration of the effect of temperature on the individual parameters of the two-group criticality equation:

$$k_{\text{eff}} = \eta \epsilon p f \left[\frac{1}{1 + L^2 B^2} \right] e^{-\tau B^2}, \quad (1)$$

where

η = number of fast neutrons born per thermal neutron captured in the fuel,

ϵ (fast fission factor) = number of fast neutrons slowing down past the U^{238} fast fission threshold per fast neutron from U^{235} fission,

p (resonance escape probability) = probability of a fast neutron escaping nonfission capture while slowing down to thermal energies,

$e^{-\tau B^2}$ = fast nonleakage probability ($\mathcal{L}_1$), where τ is the neutron age and B^2 the geometric buckling, and

$\frac{1}{1 + L^2 B^2}$ = thermal nonleakage probability ($\mathcal{L}_2$), where L is the thermal diffusion length.

The reactor is critical when $k_{\text{eff}} = 1$, that is, on the average, one of the neutrons born in a thermal fission causes another fission. Then the neutron population is independent of time and is said to be self-sustaining. If $k_{\text{eff}} \neq 1$, the population will continuously change, increasing if $k_{\text{eff}} > 1$ and otherwise decreasing. The rate of change of population is a function of the reactivity (ρ), defined as

$$\rho = \frac{(k_{\text{eff}} - 1)}{k_{\text{eff}}}. \quad (2)$$

The temperature coefficient of reactivity is given by

$$\frac{d\rho}{dT}, \quad (3)$$

where T is usually in $^{\circ}\text{C}$.

Usually as a safety feature and for stability reasons, the overall reactivity temperature coefficient is designed to be negative. In this core the temperature of a critical reactor increases, k_{eff} becomes less than one and

the reactor is subcritical at the new temperature. On the other hand, if $d\rho/dT$ were positive, and as the fission process releases heat, the reactor would be unstable and become increasingly more supercritical as the temperature increases.

The effects of temperature on the factors of Eq. (1) arise mainly from the neutron-energy dependence of the microscopic absorption (and to a lesser extent scattering) cross sections, and from the variation of the macroscopic cross sections due to variations in core density. Also, the effect of volume changes on the geometrical buckling can be an important effect. With increasing temperature, some factors increase, some decrease, and some hardly change, so that overall coefficient depends on which factors dominate.

In the following discussion the temperature coefficient is considered separable into a density and a nuclear coefficient.

B. Density Coefficient: A decrease in density decreases the macroscopic cross sections, which results in an increase of the mean free paths for absorption and scattering. The result is that the thermal diffusion length (L) and neutron age (τ) increase. Because of the increase in L and τ , the thermal and fast nonleakage probabilities are reduced. From Eq. (1) note that the reduced nonleakage probabilities decrease k_{eff} , which means that the reactivity effect is negative. A partial compensation for the effect of an increased L and τ on the nonleakage if the core volume expands is a decrease in B^2 .

Another effect of a density decrease which can be very important in overmoderated liquid- or gas-cooled heterogeneous reactors is the decrease of poison in the core due to the expansion of the coolant-moderator.

C. Nuclear Coefficient: As the core temperature increases, the average neutron energy increases. With increasing neutron energy, the microscopic absorption cross section (excluding resonances) and the scattering cross section decrease. For most materials the absorption cross section varies as $1/v$ for neutron-nucleus interaction energies below that of the first resonance.

In the following discussion, the nuclear temperature dependence of τ and ϵ will be considered to be negligible, because they are functions of high-energy cross sections which are hardly temperature-dependent.

With increasing neutron energy beyond 0.025 eV, η decreases for natural or slightly enriched uranium reactors because, whereas U^{238} is a $1/v$ -absorber, U^{235} is less than $1/v$. Hence, as the temperature increases, U^{238} will nonfission capture an increasingly larger fraction of thermal neutrons. However, the change in η is so small, and it is usually considered to be temperature independent.

Because of Doppler broadening of the U^{238} resonance peaks with increasing temperature p decreases. The reason the resonance peaks broaden (Doppler broadening) with increased temperature is that the microscopic cross sections actually depend on the relative motion of the neutron and the nucleus. At room or moderate temperatures the nucleus can be considered at rest, and then only neutrons in a very narrow energy range will see the large resonance cross section. On the other hand, at higher temperatures a nonresonant-energy neutron may strike a nucleus whose velocity is such that the interaction energy corresponds to a resonance peak. It is also true that resonance-energy neutrons may interact with nuclei whose velocities are such that the interaction energy does not lie within a peak. The overall Doppler effect of temperature is to make the peaks broader and lower.

The thermal utilization is temperature independent if all cross sections vary as $1/v$, but U^{235} is not $1/v$, and so a slight decrease of f with increasing temperature is expected if the rest of the materials are $1/v$. In the case of heterogeneous reactors, the decrease in uranium absorption with increasing temperature will decrease the disadvantage factors, and this effect can be quite large and can even result in a positive fuel temperature coefficient.

The decrease in microscopic cross sections will increase L^2 , and hence decrease the thermal nonleakage probability.

III. APPARATUS

- A. Argonaut reactor:
- B. Heaters to heat moderator (see Fig. 9.1).
- C. Temperature-sensing devices. (See Figs. 9.2 and 9.3.)

The temperature-sensing devices consist of Plexiglas frames in which is strung a piece of nickel wire, 0.0089 in. in diameter and approximately twice the length of a fuel box. One of these devices will be placed in each of the fuel boxes. The individual nickel wires will then be connected in series by means of a piece of large-diameter, low-resistance wire. These elements will then indicate a rather good average core temperature. An impedance bridge will be used to measure the resistance of the entire assembly. This resistance can then be converted to a temperature by means of a calibration curve. If a calibration curve is not available, the students will utilize equipment available at the Argonaut and calibrate the resistance of these devices against a mercury standard thermometer.

Fig. 9.1

Flow Diagram of
Heater Arrangement

For heating: Open valves
A, B, F, G, H, I and J

Fig. 9.2
Plexiglas Holder for
Resistance Wires

139-2429

Fig. 9.3. Temperature Coefficient Apparatus

IV. PROCEDURE

The chronological order of events will be somewhat as follows:

1. question-discussion period;
2. calibration of temperature-sensing devices;
3. placement of the experimental apparatus;
4. reactor checkout and determination of criticality;
5. adjustment of reactor temperature and the determination of the resultant reactivity changes;
6. experiment summary and computation of the reactivity temperature coefficient.

The reactor will be checked out and taken to critical by the students. During this time the heat exchanger may be run to stabilize the temperature at some value below ambient, thereby providing a wider range of temperature. Final criticality should be at about $1/2$ W, since this flux level is high enough to give relatively noise-free operation of the linear counting chassis and yet produce a reasonably small increase in the residual radioactivity after shutdown.

The reactor should be maintained at criticality during the experiment. This may be accomplished by adjustment of the control rods. The emphasis here is on the critical situation rather than on the maintenance of a particular power level.

Two 2-kW heaters provided in the "pit" will be used to increase the temperature of the reactor coolant. Water is circulated through the heaters and the dump tank, and also between the dump tank and the reactor vessel. The heaters operate automatically to maintain a constant temperature determined by a control system located in the temperature-control rack near the pit. By this method the temperature of the reactor coolant will be raised to and maintained for a period of time at a few points between the temperature of the tap water (starting temperature) and 120°F , the upper limit being set by the necessity of preventing, as far as possible, the formation of condensate on control rods and other electrical appliances.

After selecting a particular temperature at the temperature-control rack, record data until the temperature stabilizes at the newly selected value. These data will consist of reactor core average temperature, reactor overflow temperature, control rod position, $\Delta\rho$, and time. The transient states will last a time of the order of $1\frac{1}{2}$ hr, changes in average reactor temperatures becoming evident about 20 min after the heaters come on. This step will then be repeated beginning with a change in demand temperature at the temperature-control rack until reaching the upper limit temperature.

The reactivity temperature coefficient can then be calculated from the foregoing data for each of the several increments and for the overall change.

V. RESULTS

A record of such data as temperature, control rod position, and time should be kept on a form such as given in Table 9.1. Choose the individual column headings for your convenience. From the raw data, make a plot of reactivity versus temperature. Include in the results the control rod calibration curve used to calculate the reactivity changes.

Table 9.1

MEASURED EXPERIMENTAL DATA

Date: _____

Fuel Loading: _____

VI. ANALYSIS AND CONCLUSIONS

This is left as an exercise for the student. Include a discussion of the shape of the reactivity-temperature curve shape.

VII. BIBLIOGRAPHY

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2. S. Glasstone and M. C. Edlund, The Elements of Nuclear Reactor Theory, D. Van Nostrand Co., Inc., New York (1952) pp. 339-344.

VIII. APPENDIXDiscussion Questions

1. What is "nuclear coefficient," "density temperature coefficient"?
2. Discuss each of these temperature coefficients (nuclear and density) in the following regions in the Argonaut reactor:
 - a. moderator;
 - b. reflector;
 - c. fuel;
 - d. control rods.
3. Do the fission product poisons affect any of the above coefficients? If so, how? Might this effect be appreciable?
4. In considering the density temperature coefficient, what is the qualitative relationship between L^2 , τ , and density?
5. In reactors in which light water acts almost exclusively as coolant while the neutron moderation takes place in another medium, the temperature coefficient may be positive. Explain.
6. For the initial approach to criticality of a high-power reactor designed to operate at elevated temperatures, what precautions must be taken that are not of concern to us at the Argonaut?
7. Explain neutron Doppler effect in a heterogeneous, water-moderated, slightly enriched system, and its relationship to the temperature coefficient of reactivity.
8. In a solid homogeneous fuel moderator reactor such as Triga, would you expect the reactivity temperature coefficient to be more or less negative than that of the Argonaut? Why?
9. Qualitatively discuss the effect of temperature changes (in the region of room temperature) on the factors of the four-factor formula.

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STUDENT RESULTS

TEMPERATURE COEFFICIENT EXPERIMENT

I. Reactor Run

This experiment was performed in a one-slab, north quadrant core of six fuel boxes. In the experiment the temperature coefficient was determined by measuring the total change in reactivity when heated water is circulated through the core. The experiment was performed in both conditions of temperature: dynamic and static. In dynamic conditions the temperature is not in equilibrium and it rises with time; under static conditions the temperature is kept steady.

II. Data and Calculations

1. Time (min)

Time was observed from the clock in the rack beside the control console. The time for each reading of data and the time of putting on the heater were recorded.

2. R (ohms)

The resistance of the nickel wire and the leads was read off from the impedance bridge. The resistance of the leads was found to be 0.14Ω before the experiment began, and this value was subtracted from the observed total resistance.

3. Moderator temperature in core ($^{\circ}\text{C}$)

The moderator temperature was obtained from the net resistance of the nickel wire. A calibration curve of temperature versus the resistance of the nickel wire had previously been made with the same nickel wire interconnecting the six individual elements as when mounted in the reactor.

4. Outlet temperature T ($^{\circ}\text{F}$)

This was taken from the indicator on the control console as a matter of interest. This is the temperature of the water leaving the reactor vessel and this differs somewhat from the temperature obtained from the resistance bridge.

5. Fine control rod position

The reactor was initially made critical by positioning the coarse rod, leaving the fine rod to be manipulated during the experiment. Each fine rod position was recorded.

6. $\rho(\%)$

This was obtained for a particular control rod position from the most recent fine rod calibration curve that had been made for this specific one-slab loading.

Procedure

With the necessary apparatus in place inside the reactor vessel, and the top shielding in place, the reactor was started up and made critical at a power level of about 0.4 W. When criticality was reached, it was then maintained at the same flux by manipulating the fine control rod. The secondary pump was set in operation to insure isothermal water. Data (see Table S9.1) were taken every 5 min (later on, 10 min) and entered into data sheets. When an equilibrium temperature was observed at 13:10, heater No. 1 was put on to increase moderator temperature. Similarly heater No. 2 was put on at 14:20.

Table S9.1

MEASURED EXPERIMENTAL DATA

Date: _____

Fuel Loading: _____

1-1932

Time	Temp, °C	Fine Rod Position	Reactivity, %	Time	Temp, °C	Fine Rod Position	Reactivity, %
11:15	14.4	30	.009	13:42	20.4	42.0	.129
11:22	14.4	30.4	.013	13:49	20.7	42.7	.136
11:27	14.6	30.5	.014	13:53	21.2	43.4	.142
11:32	14.6	30.7	.016	14:00	21.6	44.3	.151
11:37	15.0	31.1	.020	14:07	21.6	44.3	.151
11:42	15.6	32.3	.032	14:15	21.6	44.3	.151
11:48	16.0	33.1	.040	14:23	21.5	44.2	.150
11:51	16.2	34.1	.050	14:35	21.5	44.3	.151
11:54	17.0	35.1	.060	14:42	21.7	44.6	.154
12:00	17.3	35.9	.068	14:48	22.2	45.5	.163
12:06	17.7	36.4	.073	14:56	22.7	46.8	.176
12:14	18.0	37.1	.080	15:03	23.3	47.5	.183
12:20	18.3	37.7	.086	15:10	23.7	48.8	.196
12:33	18.4	37.9	.088	15:17	24.2	49.7	.205
12:39	18.7	38.4	.093	15:25	24.7	50.4	.212
12:48	19.0	39.1	.100	15:31	25.1	51.4	.222
12:58	19.2	39.4	.103	15:38	25.2	51.5	.223
13:06	19.3	39.6	.105	15:46	25.3	51.8	.226
13:15	19.5	40.1	.110	15:55	25.7	52.6	.234
13:22	19.7	40.4	.113	16:03	25.9	53.0	.238
13:30	19.8	40.6	.115	16:08	26.3	53.8	.246
13:35	20.0	41.1	.120	16:15	26.3	53.8	.246

Final Calculation

1. A curve of temperature ($^{\circ}\text{C}$) versus time is plotted in Fig. S9.1. This temperature-time curve showed steady or equilibrium states at temperatures of 19.3°C , 21.5°C , 26.3°C plus the initial point of equilibrium at 14.3°C . These four points were selected for calculation of the temperature coefficient.

Fig. S9.1. Argonaut Temperature

Fig. S9.2 Temperature Coefficient of Reactivity

2. In Fig. S9.2 the reactivity change is plotted versus moderator temperature. The four indicated equilibrium points were so chosen that the highest reactivity value for the same temperature should represent the equilibrium state the best. The four equilibrium points form an almost straight line. Hence, the slope of the straight line, i.e., the value of α , is found as

$$\alpha = \frac{0.155 - 0.028}{27 - 14} = -0.00977\% \Delta k/^{\circ}\text{C}.$$

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ABSORPTION CROSS-SECTION MEASUREMENTS BY THE
DANGER COEFFICIENT METHOD

by

K. C. Ruzich

ABSTRACT

This experiment is designed to acquaint the participant with the use of the danger coefficient method to measure the absorption cross sections of certain elements. The experiment consists of a measurement of the sensitivity of the AGN-201 reactor at the core center with a standard $1/v$ -absorber and subsequent cross-section measurements of a selected group of materials in terms of the reactor sensitivity. Reactivity will be determined by measurement of positive periods with and without the material in the reactor core. The excess reactivity of the supercritical reactor is related directly to the positive period measurement through the basic inhour equation. The results of the experiment will be compared with the known thermal cross sections and a complete analysis of any discrepancies will be made.

OUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
- IV. PROCEDURE
 - A. Daily Reactor Checkout
 - B. Reactor Sensitivity Measurement
 - C. Cross Section Measurements of the Selected Materials
- V. RESULTS
- VI. ANALYSIS AND CONCLUSIONS
- VII. BIBLIOGRAPHY
- VIII. APPENDICES
 - A. Discussion Questions
 - B. Plots of Inhour Equation

I. INTRODUCTION

The use of a nuclear reactor to measure the absorption cross sections of certain elements was formulated early in the history of reactors. During the late forties, many elements with unknown or doubtful thermal-neutron cross sections were being studied for possible use in nuclear reactors. There was also a definite need to measure the purity of materials which were to be used as moderators, reflectors, etc.

A critical reactor itself, in some cases, is highly sensitive to the loss of neutrons when an absorbing material is placed in its core. The material is "dangerous" pertaining to the sustention of the neutron chain reaction. The neutron-absorbing material reduces the value of the reproduction factor k by decreasing the thermal utilization. If the material's effect on the moderating and scattering properties of the reactor can be assumed to be small, a measurement of the change in k will indicate the material's ability to absorb neutrons.

The sensitivity of a reactor, which is defined as the change in the reproduction factor occurring when one square centimeter of absorber cross-sectional area is placed at a certain position in the reactor core, can be estimated by theoretical considerations. The use of the basic reactor diffusion equations to describe the change in k caused by small changes in reactor parameters is formulated in perturbation theory. Experimentally, the sensitivity of the reactor can be measured in terms of a standard absorber. Boron, whose absorption cross section has a $1/v$ dependence, will be used for this measurement. Its cross section will be taken as the value at 2200 m/sec. The subsequent cross-section measurements of $1/v$ -absorbers by the danger coefficient method will yield their respective values at the same velocity. For an absorber which does not

have a $1/v$ -dependence, the measured cross section will be for that of an equivalent $1/v$ -absorber which produces the same effect in the reactor.

Many factors affect the results of this experiment. For each material, every atom should have the same chance of absorbing a neutron. No self-shielding or flux depression should be present. Materials whose cross sections exhibit low-level absorption peaks in the resonance region cannot be used unless a correction can be made for resonance absorption. The effect of scattering and moderation on the reproduction factor must be assumed negligible. Any change in reactivity due to temperature or pressure fluctuation during the course of the experiment is of course undesirable.

II. THEORY

When an absorber is placed in a reactor at a certain location, the number of neutrons absorbed depends upon the magnitude of the material's cross section over the neutron-energy range and the neutron flux-energy distribution. The effect of the loss of neutrons on the reproduction factor, the reactivity change, is dependent on the fractional number of neutrons lost and the importance of these neutrons relative to keeping the reactor in a critical state. An absorber placed at the edge of the reactor core will not have as great an effect on k as it would if it were placed in the core center. This is true because the absorption rate would be lower, and the daughters of the neutrons on the core edge have a smaller nonleakage probability than those produced by neutrons at the core center. The maximum sensitivity of a reactor for an absorbing material can be expected to be located at the core center where the neutron flux is the highest.

The danger coefficient method for absorption cross-section measurements is actually a comparison of the absorption abilities of a standard $1/v$ -absorber, boron, and a material whose cross section is to be measured. By making the comparison of these two materials at a single location in the reactor core, it is possible to ensure that the neutrons absorbed by the materials are of the same importance to the neutron chain reaction. Assuming the simplified model of a bare thermal reactor and one-group diffusion theory, the following expression, given by perturbation theory, describes the reactivity change due to the insertion of a small absorber:

$$\rho = \frac{N_T \sigma_a \phi_0 \psi_0^*}{\int \nu \Sigma_F \phi \psi^* dv} \quad (1)$$

where

σ_a = absorber microscopic absorption cross section

N_T = total number of absorber atoms

ϕ_0 = thermal neutron flux at the absorber location

ψ_0^* = importance of neutrons located at the absorber location

ν = number of neutrons per fission

Σ_F = core macroscopic fission cross section

ϕ = thermal neutron flux at volume element dv

ψ^* = importance of neutrons located in volume element dv

The denominator is integrated over the core volume. For the one-group model ψ^* is everywhere proportional to ϕ ; hence Eq. (1) can be rewritten

$$\rho = \frac{N_T \sigma_a \phi_0^2}{\nu \Sigma_F \int_{\text{vol}} \phi^2 dv} = \frac{N_T \sigma_a \phi_0^2}{\nu \Sigma_F \overline{\phi^2} [\text{Vol}]_{\text{Reactor}}}$$

with $\nu \Sigma_F$ a constant for homogeneous reactor and

$$\overline{\phi^2} \int_{\text{vol}} dv = \int_{\text{vol}} \phi^2 dv.$$

The ϕ^2 weighting term involves the reaction rates and the spatial importance of the neutrons in question. From this expression it can be seen that the sensitivity of a reactor is inversely proportional to the reactor volume. A reactivity comparison of a standard absorber and a material of unknown cross section will yield a ratio of the total absorption area of the absorbers ($N_T \sigma_a$) providing the neutron-flux spatial distribution remains constant. Flux depression in the vicinity of the absorber could be undesirable since this might change the distribution.

The neutron flux expressed in Eq. (1) as the thermal flux is actually a much more complicated function, with a strong energy dependence. Neutrons are produced at fission energies and then slowed down into the thermal region. A material which exhibits a $1/v$ -cross section actually absorbs not only thermal neutrons, but neutrons in the resonance region as well. The amount of resonance absorption will depend on the magnitude of the resonance flux which fortunately is much less than the thermal flux in a thermal reactor. The ratio of the resonance absorption to the total absorption should be less than 5% for a $1/v$ -absorber. A large number of elements exhibit cross sections with a $1/v$ -dependence through the thermal region and low-level resonance-absorption peaks in the resonance region. Materials of this type are capable of absorbing a great number of resonance neutrons. A danger coefficient measurement comparing this type of a material with boron necessitates the use of a

correction factor to eliminate the resonance absorption. If the material can be assumed to be a $1/v$ -absorber throughout the neutron-energy range, no corrections are necessary since the neutrons in the resonance region affect both the sample and the standard boron identically. The cross section for boron is taken as the value at 2200 m/sec. The result of the danger coefficient measurement of a $1/v$ -material will be the absorption cross section at the same velocity. If the sample does not have a $1/v$ -dependence, the cross section obtained will be that of an equivalent $1/v$ -absorber which produces the same effect in the reactor.

The effect of the absorbing material on the moderating and scattering properties of the reactor must be taken into consideration if a material does not have a large absorption cross section. According to perturbation theory, the scattering effect of a sample, which changes the leakage properties of the core, is weighed against the gradient of the flux in the reactor core. The gradient of the flux at the core center is equal to zero; hence a danger coefficient measurement at this location eliminates the influence of scattering on reactivity. The moderating ability of the sample will depend on its slowing down power ($\xi \Sigma_s$). The reproduction factor will be altered through a change in the resonance escape probability of the reactor. If the slowing down power of the sample is small compared with its absorption cross section, the sample moderating effect on reactivity can be assumed small for a danger coefficient measurement.

The reactivity changes that occur in the reactor when an absorber is placed at its center will be measured in terms of asymptotic positive periods. The stable period of a supercritical reactor is related to the reactivity through the basic inhour equation. This expression involves the prompt neutron lifetime and the delayed neutron parameters (see Appendix B).

III. APPARATUS

- A. AGN-201 reactor and console.
- B. External BF_3 proportional counting system (see Fig. 10.1):
 - a. BF_3 cylindrical counter (of one-inch diameter);
 - b. preamplifier;
 - c. AID amplifier;
 - d. MPC scaling unit.
- C. Sample materials which are homogeneously distributed in $3/4$ -in.-diameter, cylindrical, Lucite discs (see Fig. 10.2):
 - a. boron (as H_3BO_3);
 - b. copper;
 - c. cobalt;
 - d. cadmium (as CdCO_3);
 - e. sodium chloride;
 - f. Lucite.

139-3413

Fig. 10.1. BF_3 Counter and Associated Elect

Fig. 10.2

Sample Materials and
Positioning Rods

139-3414

IV. PROCEDURE

A. A daily reactor checkout will be performed to assure that the reactor equipment is functioning properly.

B. A standard boron sample A is placed at the core center in the glory hole. Following the proper procedures, the reactor is brought to a near critical position at about 5 mW. The source is then driven out of the reactor. By inserting the control rods, the reactor is made supercritical. The rods should be positioned to yield a positive asymptotic period of about 1500 sec. This can be estimated from the chart recorder monitoring channel three.

The exact stable period of the reactor is determined by the external BF_3 counting system. The BF_3 counter is placed at a desired location in access port number four. The counts per fraction of a minute are then taken periodically. A plot of the counting rate versus time is made on semilog paper. The plotting should commence when the first count is taken, this will enable the participant to determine whether or not the reactor is rising on a stable period. The stable reactor period will be determined from the semilog plot and recorded along with the control rod position and temperature on the data sheet.

The coarse control rod is then lowered 5 cm to a position which renders the reactor very subcritical. After the power level has dropped below 5 mW, the boron standard is removed. Its removal must be performed cautiously to lessen the chance of scrambling the reactor. The reactor operator should pay close attention to the console instruments and inform the participants changing the sample of any sudden changes in the instrument readings.

A Lucite blank is then placed at the center of the glory hole in place of the boron sample A. Caution is again stressed in this operation, since positive reactivity is being inserted. The coarse control rod is then raised to its previously recorded position. Once the reactor has reached its stable period, a period measurement should be made with the external BF_3 counter as described above. The reactor period should, of course, be shorter since the boron absorber has been removed from the core. Record control rod settings, period and temperatures.

The reactor is once again brought to a subcritical position and boron sample B is inserted into the glory hole. The procedure for measuring the period is then the same as it was for the Lucite blank and boron sample A. The reactivity of the reactor for each run can be evaluated from the basic inhour equation, which relates the reactor period to the reactivity. A plot of this equation can be found in Appendix B. The reactivity worths of the boron samples is determined by subtracting the

reactivity for a sample run from that of a blank run. If we know the cross section of boron at 2200 m/sec, the sensitivity of the reactor, $\Delta k/k/\text{cm}^2$, at the core center can be calculated.

C. The materials chosen for the cross-section measurements are cobalt, sodium chloride, cadmium carbonate, and copper. Each Lucite sample contains a known amount of material. The experimental procedure for measuring the worths of the samples is identical with that of the boron standard. The reactor period with the samples on the core center line are measured in the following order:

1. Cadmium Carbonate	4. Sodium Chloride
2. Cobalt	5. Copper
3. Lucite Blank	6. Lucite Blank

DATA SHEET

Sample	Sample Weight	Lucite Weight	Control Rod Positions		Temp, °C	Period
			FR	CR		
Boron A						
Lucite						
Boron B						
CdCO ₃						
Cobalt						
Lucite						
NaCl						
Copper						
Lucite						

The excess reactivity for each run is determined from the plot of the basic inhour equation. The reactivity worth of each sample is found by subtracting the reactivity of the sample run from that of the blank Lucite. Two blank runs are made to ensure that no changes have occurred which might effect the core excess reactivity. The control rods for each of the six runs must of course be at the same positions. Once the worth of each sample is determined, its cross section can easily be calculated knowing the amount of material present in the sample and the sensitivity of the reactor.

V. RESULTS

The results of the experiment can be placed in the tables below:

Reactor Sensitivity (Measured with $1/v$ Boron Standard)

$$\text{Boron: } \sigma_a = 755 \times 10^{-24} \text{ cm}^2$$

Boron Standard	$N_T \sigma$	Period	Reactor $\Delta k/k$ ($\mu\rho$)	Standard Worth ($\mu\rho$)	Reactor Sensitivity ($\mu\rho/\text{cm}^2$)
Standard A					
Lucite					
Standard B					

Average Sensitivity _____

Absorption Cross-section Measurements

Sample	Number of Atoms or Molecules	Period	Reactor $\Delta k/k$ ($\mu\rho$)	Sample Worth ($\mu\rho$)	cm^2 of Absorber	σ_{2200}
Cadmium						
Cobalt						
Lucite						
NaCl						
Copper						
Lucite						

VI. ANALYSIS OF RESULTS AND CONCLUSIONS

This section is to be completed by the participant in his experimental writeup. The cross sections obtained for the samples should be compared with known values. Any differences should be analyzed thoroughly, and some conclusions should be stated as to why the errors existed.

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VIII. APPENDICES

A. Discussion Questions

The following questions, which are directly related to the experiment on hand, should be considered prior to arrival at the AGN-201. An effort will be made to discuss them before the performance of the experiment.

1. Describe the danger coefficient method for absorption cross-section measurements. What basic variables are involved in the experiment?
2. What is meant by the sensitivity of the reactor? Why is it a function of the core volume?
3. Which factor in the four-factor formulas is changed when an absorber is placed in a critical reactor, and why?
4. How and why will the location of the absorber in the reactor core influence the magnitude of the change in the reproduction factor?
5. Besides absorption, what other properties of the sample will change the reactivity? How can these effects be kept small?
6. How will the reactivity worth of an absorber be measured in this experiment? What basic equation will be utilized in its measurement?
7. The standard chosen for the measurement of sensitivity is boron. Why was this choice made? What is its absorption reaction rate independent of?
8. How will the absorption of neutrons in the resonance region effect a cross-section measurement? When will a correction for this absorption have to be made? How can this correction be determined?
9. How will the following affect the results of the experiment:
 - a) Temperature changes;
 - b) Flux depression and self-shielding of the sample;
 - c) Mounting the samples in a Lucite disc?
 How can their effects on reactivity be kept small?
10. What other methods can be used for absorption cross-section measurements?

B. Plots of Inhour Equation

Fig. 10.3

AGN-201
Inhour Equation
(0-140 $\mu\rho$ range)

Fig. 10.4

AGN-201
Inhour Equation
(0-1400 $\mu\rho$ range)

STUDENT RESULTSDATA SHEET

Sample	Sample Weight (g)	Lucite Weight (g)	Control Rod Positions		Temp, °C	Period
			FR	CR		
Boron A (6-1)	0.0155	3.069	22.80	22.85	20.1	
	Molecular Weight $H_3BO_3 = 61.84$					
Lucite (3-4)	-	3.075	22.80	22.85	20.1	
Boron B (6-5)	0.031	3.033	22.80	22.85	20.1	
CdCO ₃ (7-1)	0.0163	3.069	22.80	22.85	20.1	
	Molecular Weight $CdCO_3 = 172.42$					
Cobalt (B-1)	0.454	3.042	22.80	22.85	20.1	
	Atomic Weight $Co = 58.94$					
Lucite (3-4)	-	3.075	22.80	22.85	20.1	
NaCl (8-1)	0.421	2.807	22.80	22.85	20.1	
	Molecular Weight $NaCl = 58.4$					
Copper (C-1)	3.47	2.635	22.80	22.85	20.1	
	Atomic Weight $Cu = 63.54$					

CALCULATIONSNumber of atoms in each sample

$$N = \frac{(\text{Weight of sample material})(\text{Avegado's number})}{(\text{Molecular or Atomic Weight})}$$

$$\times (\text{Number of atoms of sample/molecule})$$

Boron atoms, Sample 6-1 (H_3BO_3)

$$N = \frac{(0.0155 \text{ g})(6.023 \times 10^{23} \text{ mol/g-atom}) \times 1}{61.81 \text{ g/g-atom}}$$

$$= 1.51 \times 10^{20} \text{ atoms}$$

Cobalt atoms, Sample B-1

$$N = \frac{(0.454 \text{ g})(6.023 \times 10^{23} \text{ atoms/g-atom})}{(58.94 \text{ g/g-atom})}$$

$$= 4.65 \times 10^{21} \text{ atoms}$$

Reactor Period Calculation

Boron Sample (6-5)

Time (min)	Counts per 0.3 min
0	15447
1	16477
2	17530
3	18604
4	19668
5	20627
6	21681
7	23152
8	24494
9	25833
10	27504

$$C(t_2) = C(t_0)e^{t/T}$$

$$T = \frac{\Delta t}{\ln \frac{C(t_2)}{C(t_0)}}$$

$$T = \frac{600}{\ln \frac{27400}{15600}}$$

$$T = \frac{600}{0.565} = 1060 \text{ sec}$$

Lucite Sample (3-4)

Time (min)	Counts per 0.2 min
0	2731
1	3611
2	4593
3	6239
4	8192
5	10535
6	13834
7	18433

$$T = \frac{\Delta t}{\ln \frac{C(t_2)}{C(t_0)}}$$

$$T = \frac{480}{\ln \frac{24100}{271}} = \frac{480}{2.18}$$

$$T = 222 \text{ sec}$$

Fig. S10.1
Supercritical Power Runs.

The reactivity for each run is determined from the basic inhour equation. The reactivity worth of each sample is simply the difference in the reactivity of the Lucite blank and the sample runs.

Boron 6-5 $T = 1060 \text{ sec}$

$$\Delta k/k = 88 \mu\rho$$

Lucite 3-4 $T = 222 \text{ sec}$

$$\Delta k/k = 389 \mu\rho$$

Cobalt B-1 $T = 780 \text{ sec}$

$$\Delta k/k = 118 \mu\rho$$

The worth of (6-5)

$$\Delta k/k(3-4) - \frac{\Delta k}{k}(\text{sample}) = \text{sample worth}$$

$$389 - 88 = 301 \mu\rho$$

and for B-1

$$389 - 118 = 271 \mu\rho$$

Reactor sensitivity

$$\frac{\text{worth of the boron sample}}{N_T \sigma_a, \text{ cm}^2 \text{ of } 1/\nu\text{-absorber in the sample}}$$

For 6-5,

$$\frac{301 \mu\rho}{0.228 \text{ cm}} = 1320 \mu\rho/\text{cm}^2.$$

and 6-1,

$$\frac{167 \mu\rho}{0.114 \text{ cm}} = 1460 \mu\rho/\text{cm}^2.$$

Reactor sensitivity at the center of the core is $1390 \mu\rho/\text{cm}^2$.

Total absorption cross-sectional area is the sample

$$\frac{\text{worth of sample}}{\text{reactor sensitivity}}$$

for B-1,

$$\frac{271 \mu\rho}{1390 \mu\rho/\text{cm}^2} = 0.195 \text{ cm}^2.$$

Absorption cross section per atom =

$$\frac{\text{total absorption cross-sectional area}}{\text{total number of sample atoms}}$$

For B-1,

$$\frac{0.195 \text{ cm}^2}{4.65 \times 10^{21} \text{ atoms}} = 41.9 \times 10^{-24} \text{ cm}^2/\text{atom}.$$

Reactor Sensitivity (Measured With $1/v$ Boron Standard)

$$\text{Boron } \sigma_a = 755 \times 10^{-24} \text{ cm}^2$$

Boron Standard	$N_T \sigma$	Period	Reactor $\Delta k/k$ ($\mu\rho$)	Standard Worth ($\mu\rho$)	Reactor Sensitivity ($\mu\rho/\text{cm}^2$)
Standard A	0.228	1060	88	301	-1320
Lucite	-	222	389	-	-
Standard B	0.114	405	222	167	-1460

Average Sensitivity -1390

Absorption Cross-section Measurements

Sample	Number of Atoms or Molecules	Period, sec	Reactor $\Delta k/k$ ($\mu\rho$)	Sample Worth ($\mu\rho$)	cm^2 of Absorber	$\sigma_{2200, b}$ Exp	$\sigma_{2200, b}$ from BNL 325
Cadmium	5.68×10^{19}	998	93	296	0.213	3750	2550 x 1.3
Cobalt	4.65×10^{21}	780	118	271	0.195	42.1	38.0
Lucite	-	222	389	-	-	-	-
NaCl	4.33×10^{21}	521	175	214	0.154	35.4	34.4
Copper	3.29×10^{22}	448	201	188	0.135	4.11	3.85

REACTIVITY IMPORTANCE FUNCTION FOR
NEUTRON ABSORBERS, AGN-201 REACTOR

by

William F. Gilbert

ABSTRACT

The effect of a thermal-neutron absorber on the reactivity of a nuclear reactor is found experimentally as a function of the absorber position in the reactor. The relative thermal flux at each location of the absorber is determined by counting a wire, made of dysprosium dispersed in aluminum, which is irradiated in the reactor. The experimentally determined absorber importance function is compared with the importance function based on one-group perturbation theory, and a neutron importance in the reactor as a function of position is determined by comparison with the results of a two-group perturbation theory.

OUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
- IV. PROCEDURE
- V. RESULTS
- VI. ANALYSIS AND CONCLUSIONS
- VII. BIBLIOGRAPHY
- VIII. APPENDICES
 - A. Discussion Questions
 - B. Inhour-equation Plots
 - C. Dysprosium Decay Correction Factors

I. INTRODUCTION

The effects of various materials on the reactivity of a nuclear reactor are dependent not only upon the nuclear properties of the materials, but are also a function of the positions of the materials in the reactor. A compilation of the relative reactivity effect of a given material at various positions in a reactor is known as the reactivity importance function for that material. A general knowledge of importance functions is essential to reactor designers and operators, for importance functions allow the prediction of reactivity effects of various materials placed at given positions in a reactor.

A knowledge of the importance functions for materials is basic. Only the importance function for a neutron-absorbing material is considered in this experiment, and the experimental results obtained will be compared with those predicted by perturbation theory.

II. THEORY

The change in reactivity produced when a neutron-absorbing sample is added to a critical reactor system is derived in reactor theory as

$$\Delta \rho = \frac{\Delta k}{k} = \frac{\int_{\text{sample}} \phi \Sigma_a \phi^+ dV}{k \int_{\text{reactor}} \phi \Sigma_a \phi^+ dV} = \frac{\int_{\text{sample}} \phi \Sigma_a \phi^+ dV}{\int_{\text{reactor}} \phi \nu \Sigma_f \phi^+ dV}, \quad (1)$$

where $\phi \Sigma_a$ is the neutron-absorption rate per unit volume; ϕ^+ is the importance of neutrons at each specific location; $\phi \nu \Sigma_f$ is the neutron-production rate per unit volume. If the neutron importance could be

considered as constant throughout the reactor, this expression for the reactivity change becomes simply the fraction of total neutron absorptions in the reactor system occurring in the sample.

The neutron importance in the reactor is, however, a strong function of position in the reactor, usually being maximum near the center of the reactor and approaching zero at some point near the edge. This is largely because of neutron leakage from the reactor and, since neutrons nearer the edge have a greater chance of leaking from the system, their importance to the system is reduced.

If a physically small sample is used, the neutron flux and its importance throughout the sample can be considered constant and removed from the integral in the numerator of Eq. (1). If the effect of the sample on k can be assumed to be small and the neutron distribution remains nearly constant, the denominator can be considered unchanged by the addition of the sample and therefore constant. Equation (1) then becomes

$$\Delta\rho = \frac{\phi_s \phi_s^+ \int_{\text{sample}} \Sigma_a dV}{K}, \quad (2)$$

where the subscript s refers to values of the parameters at the location of the sample and K is a constant. For a given sample the integral $\int \Sigma_a dV$ is constant, and therefore the reactivity change due to the sample is found to be

$$\Delta\rho = C \phi_s \phi_s^+, \quad (3)$$

where C is a constant.

The reactivity effect of the sample is therefore found to be proportional to the product of the neutron flux and the neutron importance at the location of the sample. The product is known as the importance function for an absorber placed in the reactor.

Equation (3) used out of context would lead us to believe that changing the neutron flux at the sample by any means, such as changing the power of the reactor, would cause the reactivity effect of the sample to change also. Reference to Eq. (1), however, indicates that the numerator and denominator increase equally when the power changes, since the shape of the flux distribution is independent of power level. Therefore, the flux in Eq. (3) must be considered as a function of position only and not as an absolute value.

In reactor perturbation theory, the neutron importance ϕ^+ is the adjoint flux obtained by interchanging rows and columns in the neutron-flux matrix operator and solving. The resulting solution is orthogonal to the flux.

In one-group perturbation theory the neutron flux is self-adjoint, that is, the neutron importance has the same variation with position in the reactor as does the flux. For this case, the importance function becomes the square of the flux or, for an absorber placed in the reactor,

$$\Delta\rho = C'\phi^2, \quad (4)$$

where C' is a constant. This approximation is often used to obtain a rough estimate of the worth of an absorber placed at different positions in a reactor since the shape of the flux is usually reasonably well-known.

Two-group perturbation theory predicts a neutron importance having a somewhat different variation with position than does the neutron flux.

III. APPARATUS

1. Reactor.
2. End-window GM wire counter for beta counting.
3. Instrumentation for period measurement if the reactor instrumentation is not used.
4. Dysprosium-in-aluminum wire for thermal-flux plot;
5. Suitable absorber sample.

IV. PROCEDURE

The dysprosium-in-aluminum wire, supported on an aluminum tube (see Fig. 11.1), is placed in the glory hole through the core of the reactor and irradiated to produce sufficient activity for counting. The wire is counted over half its length in an end-window GM wire counter (see Figs. 11.2 and 11.3) to determine the thermal flux distribution through half the core region. Correction for the decay time must be made, but since only a relative flux plot is desired, no correction for the irradiation time need be made. For convenience, the resulting thermal-flux plot may be normalized to unity at the center of the core.

139-3412

Fig. 11.1. B_4C Absorber and Al-Dy Wire Mounted on Aluminum Positioning Rods

139-3415

Fig. 11.2. GM Tube with Shield and Wire Scanning Apparatus

Fig. 11.3

Dy-Al Wire Scanning Apparatus and GM Counter

139-3423

The absorber sample is a 1/4-in.-wide band of B_4C at the center of a long aluminum tube. The tube is inserted through the glory hole and the sample positioned at the center of the reactor core. The reactor is then made critical and the control rod positions recorded. The sample is then moved to various positions measured from the center of the core and the reactor period measured at each position. The absorber sample should be moved through the same half of the core through which the flux was previously measured. The reactor period is determined by observing the increase in count rate with time as indicated by a $B^{10}F_3$ counter (see Fig. 11.4) placed in one of the reactor access ports. Each time the sample is to be moved, a control rod is adjusted to make the reactor very subcritical and then brought back to its original position for the period measurement.

The reactivity of the reactor for each period measured is determined by referring to the plots of the reactor inhour equation. All reactivity values determined must be subtracted from the maximum reactivity found with the absorber removed from the effective reactor volume in order to refer the reactivity effect of the absorber to zero when the absorber is out of the reactor system. The reactivity values thus found are then divided by the maximum reactivity value to normalize the curve to unity at the center of the core. The resulting curve is the relative importance function of an absorber at various points in the reactor.

The normalized thermal-flux-squared curve is obtained by squaring values obtained from the smooth curve drawn through the flux data points. The neutron importance is obtained by dividing the importance function values found from the importance function curve by the thermal-flux values

found from the thermal-flux curve. The normalized thermal flux, normalized thermal-flux squared, the normalized importance function, and normalized neutron importance should be plotted on the same graph so that their shapes may be easily compared.

139-3413

Fig. 11.4. BF_3 Counter and Associated Electronics

V. RESULTS

Results will consist of the normalized plots of thermal flux, thermal-flux squared, importance function, and neutron importance as a function of position in the reactor.

VI. ANALYSIS AND CONCLUSIONS

The experimental importance function should be compared with that predicted by one-group theory. The importance of errors in the various measurements made should be considered.

VII. BIBLIOGRAPHY

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VIII. APPENDICES

A. Discussion Questions

The following should be carefully considered by the student prior to the time when the experiment is scheduled.

1. The effect of inserting an absorber into a reactor is to change k_{∞} . Which factor in the four-factor formula is changed and why?
2. What factors determine the amount of absorber which must be inserted at a particular position in a reactor to obtain a given change in reactivity?
3. Why can the activity induced in dysprosium be assumed to be due to only the thermal-neutron flux in the reactor?
4. Why is the center of the reactor chosen as the initial position of the absorber in this experiment?
5. Would a temperature change in the reactor during the course of the experiment have any effect on the results?
6. One method of mounting the small amount of absorber needed for this experiment would be to mix the absorber with Lucite powder and mold the mixture into a solid sample. What effect would this have on the results obtained? How might we eliminate this effect?
7. Must flux depression and self-shielding in the sample be considered in evaluating the results of this experiment?
8. In this experiment, would you expect to obtain the same results with a $1/v$ -absorber such as boron as you would with a non- $1/v$ -absorber such as cadmium?
9. What methods could be used to evaluate the reactivity changes produced by the absorber? Why is the removal of absorber to produce a positive period more desirable than the insertion of absorber to produce a negative period?
10. For a reactor rising in power on an asymptotic period the flux variation with time is given as $\phi_2 = \phi_1 e^{t/\tau}$, where τ is the period of the reactor. Derive an expression for the period in terms of the flux levels measured at two times separated by an interval Δt .

B. Inhour-equation Plots

Fig. 11.5

AGN-201 Inhour Equation (0-140 $\mu\rho$ range)

Fig. 11.6
AGN-201 Inhour Equation (0-1400 $\mu\rho$ range)

C. Dysprosium Decay Correction Factors

Time (min)	$e^{\lambda t}$	Time (min)	$e^{\lambda t}$	Time (min)	$e^{\lambda t}$
1	1.005	31	1.167	61	1.355
2	1.010	32	1.173	62	1.362
3	1.015	33	1.178	63	1.368
4	1.020	34	1.184	64	1.375
5	1.025	35	1.190	65	1.382
6	1.030	36	1.196	66	1.389
7	1.035	37	1.202	67	1.396
8	1.041	38	1.208	68	1.403
9	1.046	39	1.214	69	1.410
10	1.051	40	1.220	70	1.417
11	1.056	41	1.227	71	1.424
12	1.062	42	1.233	72	1.431
13	1.067	43	1.239	73	1.438
14	1.072	44	1.245	74	1.446
15	1.077	45	1.251	75	1.453
16	1.083	46	1.257	76	1.460
17	1.088	47	1.264	77	1.467
18	1.094	48	1.270	78	1.475
19	1.099	49	1.276	79	1.482
20	1.105	50	1.283	80	1.489
21	1.110	51	1.289	81	1.497
22	1.116	52	1.295	82	1.504
23	1.121	53	1.302	83	1.511
24	1.127	54	1.309	84	1.519
25	1.133	55	1.315	85	1.527
26	1.138	56	1.322	86	1.535
27	1.144	57	1.328	87	1.542
28	1.150	58	1.335	88	1.550
29	1.155	59	1.342	89	1.558
30	1.161	60	1.348	90	1.565

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STUDENT RESULTSWIRE COUNTING DATA

Time, min	Distance from Core Centerline, in.	Counts/2 min, $\times 10^{-3}$ (background correction)	Decay Correction $e^{\lambda t}$	$Ce^{\lambda t} \times 10^{-3}$	$C'/C_0 = \phi/\phi_0$
t	d	C	$e^{\lambda t}$	C'	C'/C_0
0	0	8.27	1.00	8.27	1.00
3	0.5	8.03	1.015	8.15	0.986
6	1.0	7.88	1.030	8.12	0.982
9	1.5	7.60	1.046	7.95	0.962
12	2.0	7.25	1.062	7.70	0.931
15	2.5	6.80	1.077	7.32	0.886
18	3.0	6.11	1.094	6.68	0.808
21	3.5	5.45	1.110	6.05	0.732
24	4.0	4.92	1.127	5.55	0.671
27	4.5	4.42	1.144	4.83	0.585
30	5.0	3.60	1.161	4.19	0.507
33	5.5	3.24	1.178	3.82	0.462
36	6.0	2.72	1.196	3.26	0.394
39	6.5	2.38	1.214	2.89	0.350
42	7.0	2.31	1.233	2.85	0.345
45	7.5	2.04	1.251	2.55	0.308
48	8.0	1.97	1.270	2.50	0.302
51	8.5	1.86	1.289	2.40	0.290

TYPICAL PERIOD DATA AND CALCULATION FOR d = 2.0 in.

Time, min	Counts/0.4 min, $\times 10^{-3}$	
t	C	C/C_0
0	12.4	1.00
1	13.4	1.08
2	14.6	1.18
3	15.8	1.27
4	17.2	1.38
5	18.5	1.49
6	20.1	1.62
7	21.7	1.75
8	23.6	1.90
9	25.6	2.06
10	27.7	2.23

From graph:

When

$$t = 2 \text{ min, } C/C_0 = 1.175$$

$$t = 12 \text{ min, } C/C_0 = 2.63$$

$$\phi_2/\phi_1 = \frac{2.63}{1.175} = 2.24$$

$$\tau = \frac{\Delta t}{\ln(\phi_2/\phi_1)} = \frac{10 \times 60}{\ln 2.24}$$

$$\tau = \frac{600}{0.806} = 745 \text{ sec}$$

IMPORTANCE FUNCTION DATA

Absorber Distance from Core Centerline, in. d	Reactor Period, sec τ	Reactivity, microrhos ρ	$\rho_0 - \rho$	Importance Function $\frac{\rho_0 - \rho}{\rho_0}$ I.F.
1.	2020	48	602	0.927
1.5	1165	81	569	0.876
2.0	745	125	525	0.808
2.5	451	200	450	0.693
3.0	345	254	396	0.610
3.5	254	344	306	0.471
4.0	210	408	242	0.372
4.5	165	502	148	0.228
5.0	144	562	88	0.135
6.0	131	608	42	0.065
9.0	120	650 = ρ_0	0	0

 $(\phi/\phi_0)^2$ AND ϕ^+ CALCULATIONS

Distance from Core Centerline, in. d	(from graph) ϕ/ϕ_0	$(\phi/\phi_0)^2$	Importance Function (from graph) I.F.	$\phi^+ = \frac{\text{I.F.}}{\phi/\phi_0}$ ϕ^+
0	1.00	1.00	1.00	1.00
0.5	0.997	0.994		
1.0	0.984	0.968	0.940	0.955
1.5	0.964	0.929	0.880	0.913
2.0	0.930	0.865	0.800	0.860
2.5	0.882	0.778	0.712	0.807
3.0	0.820	0.672	0.600	0.732
3.5	0.746	0.557	0.482	0.646
4.0	0.670	0.449	0.360	0.537
4.5	0.584	0.341	0.230	0.394
5.0	0.505	0.255	0.140	0.277
5.5	0.440	0.194	0.088	0.200
6.0	0.388	0.151	0.062	0.160
6.5	0.353	0.125		
7.0	0.330	0.109		
7.5	0.312	0.097		
8.0	0.300	0.090

Fig. S11.1. Measurement of Reactor Period

Fig. S11.2. Comparison of ϕ^+ and ϕ

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RESONANCE ESCAPE PROBABILITY,
ARGONAUT REACTOR*

by

D. A. Daavettila

ABSTRACT

In a uranium-fueled thermal reactor, the neutrons born in a fission event have energies from 10 MeV to thermal, with the majority having energies ranging from 1 to 3 MeV. In order to greatly increase the probability that these neutrons cause another fission event, the fast neutrons must slow down; during this slowing down process, some will leak from the reactor and some will be captured. The resonance escape probability p is a measure of the neutron probability to escape nonfission capture while slowing down. Because of the relatively huge resonance peaks of U^{238} , this capture will take place essentially completely in U^{238} , yield radioactive Np^{239} as a decay product; therefore, the probability of escaping U^{238} capture (p^{28}) is equivalent to p .

In this experiment the resonance escape probability in the core is determined through a foil measurement of the U^{238} cadmium ratio. A number of core parameters, such as thermal utilization and nonleakage probabilities, are required in this determination, and are calculated by the student with the aid of Appendix B. Two techniques for resolving the Np^{239} activity from the measured foil activity, which is due in part to U^{235} and U^{238} fissions, are given in the Appendices.

*W. L. Grant (South Africa) assisted in the development of this experiment.

OUTLINE

- I. INTRODUCTION
- II. THEORY
 - A. Derivation of Equations
 - B. Foil Description and Purpose
 - C. True Np^{239} Activity
- III. APPARATUS
- IV. PROCEDURE
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- VIII. APPENDICES
 - A. Discussion Questions
 - B. Evaluation of Constants
 - C. Measurement of Np^{239} Activity in Irradiated (Metal) Uranium Foils
 - D. Measurement of Np^{239} Activity in Thin Uranium Composition Foils
 - E. Calculation of Resonance Escape Probability
 - F. Effective Resonance Integral versus Foil Thickness

I. INTRODUCTION

The infinite multiplication factor k_{∞} is a basic reactor constant: it is the ratio of the average number of neutrons in one generation to that of the preceding generation in an infinite reactor core. For a finite reactor core an effective multiplication factor k_{eff} is given in terms of k_{∞} as follows:

$$k_{\text{eff}} = k_{\infty}(\text{total nonleakage probability}).$$

When k_{eff} is equal to one, the reactor is critical.

On the average, in a critical reactor, one of the neutrons born in a fission event must cause another fission. From a consideration of this fission chain or cycle, the following important reactor (four-factor) formula is readily developed:

$$k_{\infty} = \eta \epsilon p f,$$

where η is the average number of fast neutrons born per thermal neutron captured in the fuel, ϵ (the fast-fission factor) is the average number of neutrons slowing down past the U^{238} fast-fission threshold of about 1.1 MeV per fast neutron from a U^{235} fission, p (the resonance escape probability) is the probability of these fast neutrons escaping nonfission capture while

slowing down to thermal energy, and f (the thermal utilization) is that fraction of thermal neutrons absorbed that are absorbed in the fuel, which completes the cycle.

In uranium-fueled thermal reactors, the nonfission capture of neutrons during the slowing down process occurs primarily in U^{238} . Its cross section versus neutron-energy response has huge resonance peaks in the energy region between 200 and 6 eV, and by comparison the capture in other core materials is negligible.

The numerical value of p , which is always less than one, tends to increase with fuel enrichment, fuel lumping, and moderator-to-fuel ratio. Enrichment increases p by reducing the amount of U^{238} . Because uranium is a poor moderator, fuel lumping reduces the resonance flux in the inner volume of the lump through self-shielding. Increasing the moderator-to-fuel ratio with proper core geometry increases the probability of a neutron slowing down through the resonance region while being in the moderator.

In this experimental determination of p , the actual measured quantity is the Np^{239} activity (from U^{238} neutron capture) of a bare and a cadmium-covered uranium foil. From the cadmium ratio, the resonance escape probability is calculated according to the theory and equations developed in the next section. However, it is a significant fact that the U^{238} nonfission cadmium ratio is an important number in itself, because theoretical reactor physicists often prefer to check theory by experiment through a comparison of a calculated versus an experimental U^{238} cadmium ratio instead of by a comparison of p values.

II. THEORY

A. Derivation of Equations: From the sketch in Fig. 12.1 of the slowing down scheme of fast neutrons in the reactor core, the relationship of p to the U^{238} nonfission cadmium ratio will be derived; for a more complete discussion, see 1 and 3 of the bibliography.

From the relative activity of the bare and cadmium-covered U^{238} foils located within the fuel, the ratio of neutron captures in the resonance and thermal-energy regions can be determined. The U^{238} foils are activated under neutron bombardment because U^{238} is converted to Pu^{239} according to the decay scheme

Fig. 12.1

Slowing Down of Fast Neutrons in Core

A ratio " ρ_C^{28} " is defined as follows:

$$\rho_C^{28} = \frac{U^{238} \text{ epi-cadmium captures}}{U^{238} \text{ sub-cadmium captures}}$$

This ratio " ρ_C^{28} " is also expressed as follows in terms of the cadmium ratio of the U²³⁸ foils:

$$\rho_C^{28} = \frac{A_c}{A_b - A_c} = \frac{1}{CR - 1}$$

where A_c and A_b are the activities of the bare and covered foils and CR is the cadmium ratio.

Referring back to the slowing down picture, observe that for ρ_C^{28}

$$\rho_C^{28} = \frac{(1-p) U L_0 L_1}{U L_0 L_1 L_2 L_3 pf F}$$

Hence,

$$\frac{1-p}{L_2 L_3 pf F} = \rho_C^{28}$$

from which

$$p = \frac{1}{1 + \mathcal{L}_2 \mathcal{L}_3 f R \rho_c^{28}} \quad (1)$$

where

f = thermal utilization;

$$R = \frac{\Sigma_a^{238}}{\Sigma_a^{238} + \Sigma_a^{235}};$$

$\mathcal{L}_2$ = nonleakage probability between resonance and thermal regions;

$\mathcal{L}_3$ = thermal nonleakage probability.

In Eq. (1), the only quantity that is experimentally determined is the cadmium ratio. The others are calculated. Obviously the value of p will also depend on the accuracy of these calculated quantities.

In BNL-486, p. 17, a more exact equation for p is given after similar derivation as presented for Eq. (1). In the Argonaut core the equations give nearly the same answer. From BNL-486,

$$p = \frac{1 + \frac{(\epsilon - 1)\alpha_{28}}{\epsilon(\nu_{28} - 1 - \alpha_{28}) \mathcal{L}_1}}{1 + fF \mathcal{L}_2 \mathcal{L}_3 \left[\frac{1}{CR - 1} \right] \left[\frac{1 - \delta(CR - 1)}{1 + \delta} \right]} \quad (2)$$

The new quantities are $\mathcal{L}_1$ (fast nonleakage probability), α_{28} (capture to fission ratio above fast fission threshold), ϵ (fast-fission factor), and δ , which is defined by the equation

$$\delta = \frac{1/\nu\text{-capture of cadmium-covered } U^{238} \text{ foil}}{\text{sub-cadmium capture of } U^{238} \text{ foil}}$$

The term $\frac{1 - \delta(CR - 1)}{1 + \delta}$ therefore corrects for the effect of the $1/\nu$ -capture in U^{238} in the epi-cadmium range because $1/\nu$ -capture is considered in this derivation to be thermal absorption. From experimental work in the Argonaut core (according to methods given in BNL-486, pp. 32 and 33), the value of δ has been experimentally determined to be between 0.040 and 0.062. The $\mathcal{L}_2$ quantities of Eqs. (1) and (2) are not identical, because they differ in their upper energy cutoffs, which are about 200 and 6 eV, respectively.

B. Foil Description and Purpose: In Fig. 12.2, the proper foil location within a fuel element for a measurement of p is given. Because of time limitations and the core radioactivity level, it is very likely that the foil setup for this experiment will be somewhat different, so that several different possibilities of foil placement are also given in Fig. 2.

Fig. 12.2
Details of Foil Location

As may be seen from Fig. 12.2, a foil is located in a water space instead of within a fuel element. This approximation is very good for most neutron-material interactions, since the Argonaut core can well be considered to be homogeneous, but in the case of U^{238} captures, special considerations are involved to make the approximation good and to measure the proper cadmium ratio. Those considerations which affect the design of the foil package will now be discussed.

First, in regard to location 1, the fuel sections are included within the cadmium cover to eliminate neutron streaming and to insure that the resonance flux seen by the thin uranium foil is that seen by the nearby fuel. Otherwise, the thin-foil Np^{239} activation would in part be a function of its own effective resonance integral which is characteristic of its thickness. The aluminum foils are fission fragment barriers.

For location 2, the effort is to choose a foil of thickness t such that its effective resonance integral is equal to that calculated for the fuel elements. In Appendix E, the calculation of the effective resonance integral is outlined, and a curve of the integral versus metal foil thickness is given.

At the third location, Argonaut thin-fuel foils or thin depleted uranium oxide-aluminum foils of the Argonaut composition are jacketed by fuel sections. The thickness t is made equal to that of the fuel element again in an effort to match effective resonance integrals. Foils can also be placed on the outside of the fuel sections to check interior versus exterior activation.

The advantage of using depleted uranium for the uranium foils will be pointed out in the following section.

C. True Np²³⁹ Activity: The U²³⁸ nonfission capture cadmium ratio is measured by determining the relative Np²³⁹ activity in the bare and cadmium-covered uranium foils with a scintillation counter and a gamma-ray spectrometer. Np²³⁹ in its decay to Pu²³⁹ results in a 103-keV photopeak which is an unresolved combination of 106-keV Pu²³⁹ γ rays and 98-keV Pu²³⁹ X rays. The gamma-ray spectrometer is used to resolve the 103-keV photopeak out of the gross activity. However, both the U²³⁵ and U²³⁸ fission products and the natural uranium decay contribute to the measured activity in the 103-keV region, and this complicates the measurement of Np²³⁹ activity. The methods available for making the activity corrections will now be discussed.

The background and natural decay correction is made by obtaining the 103-keV activity of each foil before irradiation or of identical reference foils and subtracting this activity from the value after irradiation. The significance of the correction is reduced by long, high-power reactor runs. A satisfactory run is 20 W for 0.5 hr.

When highly depleted uranium foils are used for the measurement of the cadmium ratio, the correction for fission product activity is insignificant, and it will be neglected. For foils of natural or enriched uranium, the fission contribution may not be negligible. There are two techniques to make the fission correction, depending on which foil (metal or oxide) is being corrected.

The technique described in detail in Appendix C for thin metal foils is based on an analysis of the relative 103-keV activity of two foils differing in U²³⁸- to U²³⁵-atom ratios. A pair of foils is exposed bare, and another pair is cadmium covered. The foils must be of equal thickness and of known uranium weight. It is the necessity of the known uranium weight which makes this method inapplicable to foils containing oxide.

For the very thin foils containing oxide, the correction technique is given in Appendix D. The correction is based on a direct measurement of the fission contribution to the neptunium peak. The measurement is made by comparing the pure fission activity of an aluminum catcher foil which was activated by recoil fission fragments from a source uranium metal foil with the combined neptunium and fission activity of the oxide foil. From the catcher foil activity, the ratio of fission activity in the neptunium peak region to that above 600 keV is determined, and as all γ activity above 600 keV arises from fission products, a simple proportion equation yields the fission contribution to the neptunium peak. For metal foils or thick composition foils, an additional experiment for attenuation versus foil thickness is needed.

III. APPARATUS

- A. Reactor.
- B. Two highly depleted uranium foils, each about $1/4$ sq in. in area and 5 mils in thickness.
- C. Aluminum shield foils, 1 mil thick.
- D. Foil holders designed to keep foils positioned in the core; the design of these holders may be as simple as a Lucite rod with the foils taped to it.
- E. Counting system that includes a NaI scintillation counter, multi-channel analyzer, amplifier, and digital printer (see Fig. 12.3).

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Fig. 12.3. Counting System

IV. PROCEDURE

On the day of the experiment, the choice of foils will be reviewed, and their number and location decided. If foils other than the highly depleted uranium are used, the fission correction is to be made according to the specific procedure given in the appropriate Appendix. The procedure given in this section assumes that only depleted uranium foils are to be used.

The foils will be irradiated in the reactor core for 0.5 hr at 20 W. After the reactor scram, at least 3 hr will be allowed to pass before the

foils are removed from the core and the counting started, so as to enable the U^{239} to decay practically completely to Np^{239} . Part of this waiting period will be used to calculate the quantities required in Eq. (1).

After the foils are removed from the core, they will be separated onto individual paper envelopes for ease in handling. Before foil counting begins, the gamma-ray-spectrometer, which includes a 256-channel pulse-height analyzer, is to be set up to observe the energy range from about 40 to 300 keV. This region is calibrated in terms of energy versus channel number by counting a U^{235} source which has photopeaks at 95, 143 and 185 keV. A thorough discussion of the counting apparatus will be given on the day of the experiment.

Foil counting amounts to measuring the gamma spectrum of each foil in the calibrated low-energy region. After each foil has been counted once, the counting cycle should be repeated two or three times to improve statistics and to obtain data to check (and if necessary to correct) for decay of activity.

A typical gamma spectrum observed for a uranium foil is shown in Fig. 12.4. The activity recorded as due to Np^{239} is the cross-hatched area minus the activity before irradiation.

Fig. 12.4

Typical Gamma Spectrum from
Irradiated Uranium Foil

After foil counting is complete, there will be a brief summary session in which all calculations will be completed. The experimental value of p will be determined and compared with that calculated in Appendix E.

V. RESULTS

To keep your results in order, use should be made of Tables 12.1 and 12.2.

Table 12.1

FOIL DATA

Foil	Weight	Activity before Irradiation	Activity after Irradiation	Net Activity due to Irradiation

- NOTES: 1. U^{235} standard photopeaks in channels _____, _____, and _____.
2. The reference foil R is used to provide data to correct the activities before and after irradiation to the same standard in case the spectrometer drifts between the two sessions of foil counting.

Table 12.2

CALCULATION OF p

$$p = \frac{1}{1 + \mathcal{L}_2 \mathcal{L}_3 f R \rho_C^{28}}$$

where $f =$ _____

$\mathcal{L}_2 =$ _____

$R =$ _____

$\mathcal{L}_3 =$ _____

and $f R \mathcal{L}_2 \mathcal{L}_3 =$ _____

$$CR = \frac{\text{Bare-foil Activity}}{\text{Covered-foil Activity}} = \frac{\text{_____}}{\text{_____}} = \underline{\underline{\text{_____}}}$$

$$\rho_C^{28} = \underline{\underline{\text{_____}}}$$

$$p = \frac{1}{1 + (\text{_____})(\text{_____})} =$$

$p = \underline{\hspace{2cm}}$

VI. ANALYSIS AND CONCLUSIONS

This is left as an exercise for the student.

VII. BIBLIOGRAPHY

Resonance escape probability, being a fundamental reactor parameter, has been extensively investigated, and the following list is not one iota of the total literature. It is intended to provide a start for those interested in further reading peculiar to this technique of measuring p , and also the source of the constants.

1. First International Conference on the Peaceful Uses of Atomic Energy, Vol. 5, Physics of Reactor Design, Geneva, Switzerland (1955); see especially pages 58 and 211.
2. Proceedings of the Brookhaven Conference on Resonance Absorption of Neutrons in Nuclear Reactors, BNL-433 (Sept 1956).
3. H. Kouts and R. Sher, Experimental Studies of Slightly Enriched Uranium, Water Moderated Lattices, BNL-486 (Sept 1957).
4. S. Stein, Resonance Capture in Heterogeneous Systems, WAPD-139 (Nov 1955).
5. D. P. Moon, Two-dimensional Two-group Calculation of the Argonaut One-slab Loading, ANL-6154 (April 1960).
6. S. E. Liverhant, Nuclear Reactor Physics, Wiley, 1960, pages 215-224.

VIII. APPENDICESA. Discussion Questions

1. Consider a plot of the cross section versus energy for U^{238} , and discuss the following aspects of the plot: resonance region, $1/v$ -absorption, non- $1/v$ factor, and thermal cross section.
2. Compare the epi-cadmium, nonfission absorption cross sections of U^{238} , U^{235} , Al, C, and H_2O at a few energies from 0.4 eV to 2 MeV. Generalize to rank them in order. If possible, look up their resonance integrals.
3. The atom or molecular ratios of the core constituents are $U^{235} = 1$, $U^{238} = 4$, Al = 116, $H_2O = 203$, C = 181, and O = 13(U_3O_8). On the basis of your answer to question 1, what can you conclude about U^{238} absorption to total epi-cadmium absorption?
4. Define the quantity ν . How is it related to η of $k = \eta \epsilon p f$? Is ν dependent on neutron energy?
5. In a four-factor formulation of k_{eff} for an enriched reactor, how is the epi-thermal U^{235} fission and nonfission absorption taken into account?
6. Give a work definition of ϵ , f , and F .
7. $\mathcal{L}_1$, $\mathcal{L}_2$ (equal $e^{-\tau B^2}$) and $\mathcal{L}_3$ [equal $1/(1 + L^2 B^2)$] are nonleakage probabilities. In general, how will they vary with core size? Core density? How is B^2 related to core dimensions? Explain how and why a reflector affects the nonleakage probability.
8. In general, would you expect "p" to be larger in homogeneous or heterogeneous reactors, and why?

B. Evaluation of Constants

In this Appendix the evaluations of f , R , $\mathcal{L}_1$, $\mathcal{L}_2$, and $\mathcal{L}_3$ are outlined.

Tables 12.3 and 12.4, the equation for L^2 , and the values of B^2 , τ_1 , and τ_2 have been taken completely from the work of D. P. Moon in ANL-6154 (Bibliography entry #5). Although this work was based on a one-slab loading, the materials constants will hardly change for a two-slab loading because the fuel-to-moderator ratio and geometry are the same. The calculations are based on the assumption that the core is homogeneous.

Table 12.3

ATOMS PER cm^3 IN THE CORE

NH_2O	$= 0.019449 \times 10^{24} \text{ molecules/cm}^3$
N_G	$= 0.016944 \times 10^{24} \text{ atoms/cm}^3$
N_{Al}	$= 0.012375 \times 10^{24} - 0.0008020 \times 10^{24} W_{\text{U}^{235}} \text{ atoms/cm}^3$
$N_{\text{U}^{235}}$	$= 0.000048045 \times 10^{24} W_{\text{U}^{235}} \text{ atoms/cm}^3$
$N_{\text{U}^{238}}$	$= 0.00019215 \times 10^{24} W_{\text{U}^{235}} \text{ atoms/cm}^3$
$N_{\text{O}^{16}}$	$= 0.00064036 \times 10^{24} W_{\text{U}^{235}} \text{ atoms/cm}^3$
$W_{\text{U}^{235}}$	$= \text{kilograms of } \text{U}^{235} \text{ in reactor core}$

Table 12.4

MICROSCOPIC THERMAL CROSS SECTIONS

Material	σ_{a2200}	σ_{f2200}	f	$\bar{\sigma}_a$ 49°C 37°C	$\bar{\sigma}_f$ 49°C 37°C	$\bar{\sigma}_s$
H_2O	0.664	0	1	0.562 0.573	0	-
C	-	0	1	0.00675	0	4.8
Al	0.230	0	1	0.195 0.199	0	1.4
O^{16}	$<0.2 \times 10^{-3}$	0	1	≈ 0	0	4.2
U^{238}	2.71	$<0.5 \times 10^{-3}$	1.02	2.34	≈ 0	8.3
U^{235}	694	582	0.975	572 583	480 489	10

Values needed for the evaluation of f should be entered into Table 12.5. The actual core volume cancels out of the calculation of f . Assume the following disadvantage factors (taken from ANL-6154), $\phi_H/\phi_f = 1.06$, and $\phi_g/\phi_f = 1.10$:

$$f = \frac{\Sigma_a^{235} + \Sigma_a^{238}}{\Sigma_a^{235} + \Sigma_a^{238} + \Sigma_a^c(1.10) + \Sigma_a^{H_2O}(1.06) + \Sigma_a^{Al}}$$

$$f = \underline{\hspace{10em}} =$$

$$f = \underline{\hspace{10em}}$$

Table 12.5

EVALUATION OF f

Kilograms of U^{235} in core ($W_{U^{235}}$)

Material	$\frac{\text{Atoms}}{\text{cm}^3}$	$\bar{\sigma}_a$	$\Sigma_a \times 10^{-3}$
H ₂ O (molecules)			
Graphite			
Aluminum			
U^{235}			
U^{238}			
Oxygen			

Evaluation of R

$$R = \frac{\Sigma_a^{238}}{\Sigma_a^{235} + \Sigma_a^{238}} =$$

$$R = \underline{\hspace{10em}}$$

Evaluation of $\mathcal{L}_2$, and $\mathcal{L}_3$

$$\tau_2 \text{ from } 6 \text{ eV to } 0.181 \text{ eV} = 1.5 \text{ cm}^2$$

$$B^2 = 0.0099 \text{ cm}^2$$

$$\mathfrak{L}_2 = e^{-\tau_2 B^2}$$

$$\mathfrak{L}_2 = \underline{\underline{\hspace{2cm}}}$$

$$L^2 = \frac{1}{0.064973 + 0.11955 W_{U^{235}}} =$$

$$\mathfrak{L}_3 = \frac{1}{1 + L^2 B^2} = \frac{1}{1 +}$$

$$\mathfrak{L}_3 = \underline{\underline{\hspace{2cm}}}$$

C. Determination of True Np^{239} Activity in Metal Foils

Introduction

When uranium foils are irradiated, a number of neutron-induced reactions occur. In the case of measuring resonance escape probability by the cadmium-ratio method only the Np^{239} activity is significant, yet other activities primarily due to U^{235} fission will be present, having initial or Compton scattered rays within the 104-keV range. The greater the U^{235} content within the foil, the larger will be the fission product contribution to the neptunium peak. A two-foil irradiation method for resolving the neptunium activity from the measured activity in the 104-keV range is presented.

Theory

Let foils 1 and 2 be of different U^{235} enrichment but irradiated under the same flux conditions. Let N be the number of atoms of a particular isotope in a foil. Let suffixes 9, 5, and 8 denote Np^{239} , U^{235} , and U^{238} . Let "a" denote the specific activity measured by the counting mechanism in the neptunium resonance peak, i.e., 104 keV. Thus, ${}_1N_9$ will denote the number of Np^{239} atoms in foil 1, and ${}_1N_9a_9$ will denote the total activity measured by the counter due to decay of Np^{239} in foil 1. Similarly, let a_5 and a_8 denote the specific activities measured by the counter due to fission per atom of U^{235} and U^{238} in the neptunium resonance peak.

If "A" denotes the measured activity, the measured activities in foil 1 and 2 are

$$\left. \begin{aligned} A_1 &= {}_1N_9a_9 + {}_1N_5a_5 + {}_1N_8a_8; \\ A_2 &= {}_2N_9a_9 + {}_2N_5a_5 + {}_2N_8a_8. \end{aligned} \right\} \quad (1)$$

Let the ratio of fast fission per atom U^{238} to thermal fission per atom U^{235} be δ for the particular spectrum involved. Then, in Eq. (1) we may substitute the following:

$$a_8 = \delta a_5, \quad (2)$$

assuming the fission products to have the same specific activity in the neptunium resonance region. Eq. (1) then becomes

$$\left. \begin{aligned} A_1 &= {}_1N_9a_9 + {}_1N_5a_5 + {}_1N_8\delta a_5; \\ A_2 &= {}_2N_9a_9 + {}_2N_5a_5 + {}_2N_8\delta a_5. \end{aligned} \right\} \quad (3)$$

Furthermore, because the flux intensity and spectrum are the same for the two foils concerned, the same amount of neptunium will be produced per U^{238} atom. We may thus say

$$\frac{{}_1N_9}{{}_2N_9} = \frac{{}_1N_8}{{}_2N_8} \quad \text{or} \quad {}_1N_9 = {}_2N_9 \frac{{}_1N_8}{{}_2N_8} \quad (4)$$

Substituting this in Eq. (3) yields

$$\left. \begin{aligned} A_1 + {}_2N_9 \left(\frac{{}_1N_8}{{}_2N_8} \right) a_9 + {}_1N_5 a_5 + {}_1N_8 \delta a_5; \\ A_2 = {}_2N_9 a_9 + {}_2N_5 a_5 + {}_2N_8 \delta a_5, \end{aligned} \right\} \quad (5)$$

or

$$\left. \begin{aligned} A_1 &= {}_2N_9 \left(\frac{{}_1N_8}{{}_2N_8} \right) a_9 + a_5 ({}_1N_5 + {}_1N_8 \delta); \\ A_2 &= {}_2N_9 a_9 + a_5 ({}_2N_5 + {}_2N_8 \delta). \end{aligned} \right\} \quad (5a)$$

Let

$$C_1 = {}_1N_5 \left[1 + \frac{{}_1N_8}{{}_1N_5} \delta \right]; \quad C_2 = {}_2N_5 \left[1 + \frac{{}_2N_8}{{}_2N_5} \delta \right]. \quad (6)$$

The value of δ can be readily obtained from the measurement of the fast-fission factor. Aluminum catcher foils in contact with uranium foils of two different enrichments will have different gross fission product activations, so that from a plot of gross activity versus enrichment the following factor "F" is determined from the equation

$$F_1 = \frac{{}_1N_8 a_8}{{}_1N_5 a_5}. \quad (7)$$

(For greatest accuracy ${}_1N_8 a_8$ and ${}_1N_5 a_5$ should really be measured in the energy spectrum of interest, i.e., the 104-keV range in this case.) Substituting $a_8 = \delta a_5$ in Eq. (7) yields

$$\delta = \frac{{}_1N_5 F}{{}_1N_8}.$$

Substitution of this value of δ into Eqs. (6) yields after simplification

$$C_1 = {}_1N_5 \left[1 + F_1 \right]; \quad C_2 = {}_2N_5 \left[1 + \frac{{}_2N_8}{{}_1N_8} \times \frac{{}_1N_5}{{}_2N_5} F_1 \right].$$

Rewrite the equations expressing the foil activities as follows:

$$A_1 = {}_2N_9 \left(\frac{{}_1N_8}{{}_2N_8} \right) a_9 + a_5 C_1;$$

$$A_2 = {}_2N_9 a_9 + a_5 C_2,$$

or

$$C_2 A_1 = {}_2N_9 a_9 \left(\frac{{}_1N_8}{{}_2N_8} \right) C_2 + a_5 C_1 C_2;$$

$$C_1 A_2 = {}_2N_9 a_9 C_1 + a_5 C_1 C_2;$$

$$C_2 A_1 = C_1 A_2 = {}_2N_9 a_9 \left[\left(\frac{{}_1N_8}{{}_2N_8} \right) C_2 - C_1 \right].$$

Hence the neptunium activity in foil No. 2 is

$${}_2Np = {}_2N_9 a_9 = \frac{C_2 A_1 - C_1 A_2}{\frac{{}_1N_8}{{}_2N_8} C_2 - C_1} \quad (7)$$

or

$${}_2Np = {}_2N_9 a_9 = \frac{\frac{C_2}{C_1} A_1 - A_2}{\frac{{}_1N_8}{{}_2N_8} \frac{C_2}{C_1} - 1}, \quad (7a)$$

where the value of ${}_2Np$ is the true Np^{239} activity in foil No. 2.

D. Measurement of Np^{239} Activity in Thin Uranium Composition Foils

True Np^{239} Activity. To determine the magnitude of the contribution of fission product activity to the Np^{239} peak, the special foil package of Fig. 12.5 is included for the irradiation. The enriched uranium metal foil is a source of recoil fission fragments which are captured by the aluminum catcher foil. In principle, the source uranium foil (1.46 a/o U^{235}) should match the enrichment of the fuel foils (20 a/o U^{235}) in order to match $\text{U}^{235}/\text{U}^{238}$ fission ratios, because the isotope fission-yield curves for U^{235} and U^{238} are slightly different. To correct for the mismatch in enrichment, the special foil package will be irradiated in the external thermal column to reduce to the desired degree the U^{238} fissions.

Fig. 12.5
Special Foil Package

At some time t after the reactor scram, the gamma spectra of the catcher foils and the fuel foils are observed and typically could be pictured as shown in Fig. 12.6. Referring to Fig. 12.6, what is to be determined is f_u , the fission activity in the Np^{239} peak. The quantities F_c and F_u are the measured integral fission activities above E_c , and f_c is

Fig. 12.6. Gamma Spectra

the fission activity in the neptunium peak region as seen by the catcher foil. The following equation is written subject to qualifications discussed later:

$$\frac{f_u}{F_u} = \frac{f_c}{F_c}, \quad (15a)$$

which can be rewritten as

$$f_u = \frac{f_c}{F_c} F_u. \quad (15b)$$

In Eq. (15b), the unknown f_u is given in terms of directly measureable quantities. The ratio of f_c/F_c is time-dependent, and substituting $r(t)$ for this ratio in Eq. (15b) yields

$$f_u = r(t)F_u. \quad (16)$$

With the magnitude of f_u thus known (refer back to Fig. 12.2), the true Np^{239} activity of a fuel foil at time t can be written as

$$A^{\text{Np}} = N' - rF_u, \quad (17)$$

where N' is the gross measured activity in the Np^{239} peak region (N) minus the background and natural decay activities.

The two basic assumptions behind Eq. (17) will be discussed. The first assumption is that the light and heavy fission fragments have equal ranges, and hence are equally well captured in the aluminum catcher foil. This is necessary in order for the fuel and catcher foils to have identical fission product activity gamma spectra. On the basis of experimental evidence, the effect of the difference in range is at least small. The second question is whether the thickness of the fuel foil modifies the $r(t)$ value measured from the 1-mil-thick aluminum foil, because of Bremsstrahlung radiation and the uranium scattering higher energy gammas into the Np^{239} peak. The significance of this question can be resolved by a simple attenuation versus shield-thickness counting experiment, and can be made negligible by use of thin fuel foils.

E. Calculation of Resonance Escape Probability

The theory is briefly developed, and an outline form given for a simple calculation of the resonance escape probability.

A differential portion of the resonance region is shown in the following sketch together with a few neutron paths. Let

Σ_0 = probability of a scattering collision

Σ_u = probability of an absorption collision

and neglect

scattering due to uranium

absorption due to moderator.

After a collision the chance or probability that the neutron exists is given by

$$\frac{\Sigma_s}{\Sigma_s + \Sigma_u}$$

After c collisions the probability that the neutron still exists is given by

$$\left(\frac{\Sigma_s}{\Sigma_s + \Sigma_u} \right)^c,$$

or in another form by

$$\left(1 - \frac{\Sigma_u}{\Sigma_s + \Sigma_u} \right)^c.$$

If c is large or if $\Sigma_u/(\Sigma_s + \Sigma_u)$ is small,

$$\left(1 - \frac{\Sigma_u}{\Sigma_s + \Sigma_u}\right)^c \approx e^{-\frac{\Sigma_u c}{\Sigma_s + \Sigma_u}}$$

Define c as the average number of collisions a neutron makes in the energy interval $d(\ln E)$:

$$c = \frac{d(\ln E)}{\xi}$$

Then the probability that a neutron will escape capture while slowing down through $d(\ln E)$ is given by

$$dp = e^{-\frac{\Sigma_u d(\ln E)}{\xi(\Sigma_s + \Sigma_u)}}$$

The probability that a neutron escapes capture in the resonance region while slowing down is given by the product of its probabilities of escaping capture in each $d(\ln E)$:

$$e^a e^b e^c \dots = e^{a+b+c+\dots};$$

hence

$$p = dp_1 dp_2 dp_3 \dots = e^{-\frac{\Sigma_{u1} d(\ln E)_1}{\xi(\Sigma_{s1} + \Sigma_{u1})} - \frac{\Sigma_{u2} d(\ln E)_2}{\xi(\Sigma_{s2} + \Sigma_{u2})} \dots}$$

Upon replacing the infinite sum by an integral, we obtain

$$p = e^{-\int \frac{\Sigma_u d(\ln E)}{\xi(\Sigma_s + \Sigma_u)}}$$

Now it is assumed that as scattering cross sections are only slowly varying functions of energy, an average $\xi \Sigma_s$ for the resonance region can be chosen. Then $\xi \Sigma_s$ can be removed from the integral to give

$$p = e^{-\frac{1}{\xi \Sigma_s} \int \frac{\Sigma_u d(\ln E)}{1 + \frac{\Sigma_u}{\Sigma_s}}}$$

Inasmuch as $\Sigma_u = N_u \sigma_u$,

$$p = e^{-\frac{N_u}{\xi \Sigma_s} \int \left(\frac{\sigma_u}{1 + \frac{N_u \sigma_u}{\Sigma_s}} \right) \frac{dE}{E}}$$

The integral is called the effective resonance integral, I_{eff} :

$$p = e^{-\frac{N_0 I_{\text{eff}}}{\xi \Sigma_s}}$$

After much experimental study, an empirical formula for I_{eff} has been determined:

$$I_{\text{eff}} = 3.85 \left(\frac{\Sigma_s}{N_u} \right)^{0.415}$$

It will be observed that I_{eff} depends on the total macroscopic scattering cross section of the mixture per U^{238} atom and includes the scattering due to the fuel. The limits of applicability of the formula are given as 10 to 1000 b, although Fig. 7.16 of Ref. 6 yields values of I_{eff} for Σ_s/N_0 up to 3000 b.

For a mixture of moderator materials an average ξ is determined from

$$\xi = \frac{\sum \xi_i \Sigma_{si}}{\sum \Sigma_{si}}$$

and the mixture Σ_s from

$$\Sigma_s = \Sigma_{s1} + \Sigma_{s2} + \Sigma_{s3} + \dots$$

CONSTANTS*

(From Appendix 2)

Atoms per $\text{cm}^3 \times 10^{-22}$

$N_{U^{238}}$	=	N_G	=
$N_{U^{235}}$	=	N_{Al}	=
N_{H_2O} (molecules)	=	$N_0(U_3O_8)$	=

Cross Sections

	σ_s	ξ
H ₂ O*	44.2	0.916
C	4.8	0.16
Al	1.4	0.07
O(U ₃ O ₈)	4.2	0.12
U ²³⁸	8.3	0.0084
U ²³⁵	8.3	0.0084

CALCULATIONSI_{eff}

$$I_{\text{eff}} = 3.85 \left(\frac{\Sigma_s}{N_u} \right)^{0.415} = 3.85 \left(\frac{\quad}{\quad} \right)^{0.415}$$

$$I_{\text{eff}} = \quad \text{barns}$$

$$\text{From Fig. 7.16 of Ref. 6, } I_{\text{eff}} = \quad \text{barns}$$

 ξ

$$\bar{\xi} = \frac{\sum \xi_i \Sigma_{si}}{\sum \Sigma_{si}} =$$

$$\bar{\xi} = \quad$$

 Σ_s

$$\Sigma_s = \Sigma_{s1} + \Sigma_{s2} + \dots =$$

$$\Sigma_s = \quad$$

p

$$p = e^{-\frac{N_u I_{\text{eff}}}{\xi \Sigma_s}} = e^{-\frac{\quad}{\quad}} =$$

$$p = \quad$$

$$*\sigma_s(\text{H}_2\text{O}) = 2\sigma_s(\text{H}) + 1\sigma_s(\text{o}) = 2(20) + 1(4.2) = 44.2$$

$$\xi(\text{H}_2\text{O}) = \frac{\xi(\text{H})\sigma_s(\text{H})N(\text{H}) + \xi(\text{o})\sigma_s(\text{o})N(\text{o})}{\sigma_s(\text{H})N_{\text{H}} + \sigma_s(\text{o})N(\text{o})} = \frac{(1)(20)(2) + (.12)(4.2)(1)}{44.2} = 0.916$$

F. Effective Resonance Integral Versus Foil Thickness

Fig. 12.7. Effective Resonance Capture Integral for U^{238} as a Function of Foil Thickness (Isotropic Flux in Cavity)

M = total mass in gm of U^{238} in the foil

S = total surface area including edges

Fig. 12.8. Effective Resonance Integral Versus Metal Foil Thickness (Derived from Fig. 12.7 Assuming Uranium Density of 18.9 gm/cm^3)

STUDENT RESULTS

RESONANCE ESCAPE PROBABILITY

I. Foil Location and Irradiation

The reactor loading was a one-slab, north quadrant core of six fuel boxes containing 1.93 kg of U^{235} . The foils, located on Lucite foil holders and placed in the water space between fuel plates 8 and 9, were irradiated for 10 min at a power level of 100 W. Details of foil location are given in Fig. S12.1, where all the foils are of highly depleted uranium metal (383 ppm U^{235}) and 4 mils thick. The even-numbered foils had 20-mil-thick cadmium covers.

Fig. S12.1
Foil Holder and Location Detail

II. Calibration of Gamma Spectrometer

The settings of the gamma-ray spectrometer for this experiment were 740 on the high-voltage potentiometer, 120 for the base line, and 5 for amplifier gain. For these settings the U^{235} standard foil had photopeaks centered in channels 36, 65.5, and 89.5, and these are known to correspond to the gamma energies of 95, 145, and 185 keV. The plot of energy versus channel number is shown in Fig. S12.2, from which we see that the 103-keV photopeak of Np^{239} is expected to be centered in channel 43.5.

Fig. S12.2
Calibration of Gamma-ray Spectrometer

III. Measured Foil Activity

The foils were counted before the irradiation for 4 min each, and the data of counts versus channel number were obtained in the region from channel 0 to channel 70. The exact channel region of interest about the expected 103-keV photopeak location is not determined until the foils are counted after the irradiation.

After the irradiation, one of the cadmium-covered foils was counted first. A plot of counts versus channel number made to determine the channel extent of the Np^{239} , 103-keV photopeak is shown in Fig. S12.3.

Fig. S12.3
 Np^{239} Photopeak

The measured foil activities before and after the irradiation are summarized in Table S12.1 together with the foil weights.

Table S12.1

FOIL DATA

Foil	Weight, mg	Activity (counts/2 min) before Irradiation	Activity (counts/2 min) 6 hr after Irradiation	Net Activity due to Irradiation	Normalized Net Activity (counts/2 min/mg)
1	60.6	1140	11150	10010	165.2
2	62.7	1100	8600	7500	119.6
3	46.2	865	13100	12235	264.8
4	50.3	865	9850	8985	178.6

From Table S12.1 the cadmium ratios were found to be

$$\text{CdR}_{1,2} = \frac{165.2}{119.6} = 1.38; \quad \text{CdR}_{3,4} = \frac{264.8}{178.6} = 1.48.$$

The core average U^{238} cadmium ratio was taken to be 1.43.

IV. Evaluation of p

The experimental p equation is given below; the evaluation of the various parameters is presented in the next section.

$$p = \frac{1}{1 + fF \mathcal{L}_2 \mathcal{L}_3 \left(\frac{1}{CR-1} \right)}$$

where

$$f = 0.7961; \quad \mathcal{L}_2 = 0.985;$$

$$F = 0.01609; \quad \mathcal{L}_3 = 0.968,$$

$$fF \mathcal{L}_2 \mathcal{L}_3 = 0.0122.$$

Further,

$$CR = \frac{\text{Bare Foil Activity}}{\text{Covered Foil Activity}} = 1.43,$$

so that

$$\frac{1}{CR - 1} = 2.33.$$

Accordingly,

$$p = \frac{1}{1 + (0.0122)(2.33)} = \frac{1}{1 + 0.0284} \\ = 0.972.$$

V. Evaluation of the Constants in the p Equation

We start by determining the number of atoms per cubic centimeter in the assumed homogeneous core. Results are shown in Table S12.2.

Table S12.2

ATOMS/cm³ IN THE CORE

$$\begin{aligned} \text{NH}_2\text{O} &= 0.019449 \times 10^{24} \text{ molecules/cm}^3 \text{ (only in this case are molecules and not atoms found)} \\ \text{NG} &= 0.016944 \times 10^{24} \\ \text{NAl} &= 0.012375 \times 10^{24} - 0.0008020 \times 10^{24} W_{\text{U}^{235}} [0.012375 - (0.0008020 \times 1.932)] \times 10^{24} \\ \text{N}_{\text{U}^{235}} &= 0.000048045 \times 10^{24} W_{\text{U}^{235}} = 0.000048045 (1.932) \times 10^{24} \\ \text{N}_{\text{U}^{238}} &= 0.00019215 \times 10^{24} W_{\text{U}^{235}} = 0.00019215 (1.932) \times 10^{24} \\ \text{NO}^{16} &= 0.00064036 \times 10^{24} W_{\text{U}^{235}} = 0.00064036 (1.932) \times 10^{24} \\ W_{\text{U}^{235}} &= \text{kg U}^{235} \text{ in reactor core} = 1.932 \end{aligned}$$

The evaluation of f is shown in Table S12.3

Table S12.3

EVALUATION OF f

Material	$\frac{\text{Atoms}}{\text{cm}^3} \times 10^{-24}$	$\bar{\sigma}_a \times 10^{24}$	$\Sigma_a \times 10^3$
H ₂ O (molecules)	0.019449	0.562	10.93
Graphite	0.01694	0.00672	0.114
Aluminum	0.010826	0.195	2.111
U ²³⁵	0.000092822	572	53.09
U ²³⁸	0.00037123	2.34	0.868
Oxygen (U ₃ O ₈)	0.0012372	0	-

The actual core volume cancels out of the calculation of f . Assume the following disadvantage factors due to ANL-6154 $\phi_H/\phi_f = 1.06$ and $\phi_g/\phi_f = 1.10$.

$$f = \frac{\Sigma_a^{235} + \Sigma_a^{238}}{\Sigma_a^{235} + \Sigma_a^{238} + \Sigma_a^c(1.10) + \Sigma_{\text{H}_2\text{O}}(1.06) + \Sigma_{\text{Al}}}$$

$$f = \frac{53.09 + 0.87}{53.09 + 0.87 + 0.13 + 11.59 + 2.11} = \frac{53.96}{67.78}$$

$$f = \underline{\underline{0.796}}$$

Evaluation of F

$$F = \frac{\Sigma_a^{238}}{\Sigma_a^{235} + \Sigma_a^{238}} = \frac{0.868}{53.094 + 0.868} = \frac{0.868}{53.926}$$

$$F = \underline{\underline{0.01609}}$$

Evaluation of $\mathcal{L}_2$, and $\mathcal{L}_3$

$$\tau_2 \text{ from } 6 \text{ keV to } 0.181 \text{ keV} = 1.54 \text{ cm}^2$$

$$B^2 = 0.0099 \text{ cm}^2$$

$$\mathcal{L}_2 = e^{-\tau_2 B^2} = e^{-1.54(0.0099)} = e^{-0.0152}$$

$$\mathcal{L}_2 = \underline{\underline{0.985}}$$

$$L^2 = \frac{1}{0.064973 + 0.11955W_{U^{235}}} = \frac{1}{0.064973 + 0.11955(1.932)} = \frac{1}{0.29594}$$

$$\mathcal{L}_3 = \frac{1}{1 + L^2 B^2} = \frac{1}{1 + 0.03345}$$

$$\mathcal{L}_3 = \underline{\underline{0.968}}$$

$$p = \frac{1}{1 + 0.0122 \left(\frac{1}{1.43 - 1} \right)} = \frac{1}{1 + 0.0122(2.33)} = \frac{1}{1.0284}$$

$$p_{\text{exp}} = \underline{\underline{\underline{0.972}}}$$

VI. Theoretical Calculation of p

As is described in Appendix E, p can be shown by theory to be given by

$$p = e^{-\frac{N_u I_{\text{eff}}}{\xi \Sigma_s}}$$

where I_{eff} is the effective resonance integral which is equal to

$$I_{\text{eff}} = 3.85 \left(\frac{\Sigma_s}{N_u} \right)^{0.415}$$

Taking the values from Table S12.1 of this section.

$$I_{\text{eff}} = 3.85 \left(\frac{860 + 81.3 + 15.1 + 0.7 + 3.1 + 5.2}{0.00037123 \times 10^{24}} \times 10^{-3} \right)^{0.415}$$

$$I_{\text{eff}} = 3.85 \left(\frac{965.4 \times 10^{-3}}{0.00037123 \times 10^{24}} \right)^{0.415}$$

$$I_{\text{eff}} = 3.85 (2600 \times 10^{-24})^{0.415}$$

$$I_{\text{eff}} = 3.85 \left(2600 \frac{\text{barns}}{U^{238} \text{ atom}} \right)^{0.415} = 100 \text{ barns}$$

VII. Evaluation of $\xi \Sigma_s$

$$\bar{\xi} = \frac{\sum_i \xi_i \Sigma_i}{\sum_i \Sigma_{si}} = \frac{0.916(0.860) + 0.16(0.081) + 0.12(0.0052)}{0.9654} = \frac{0.800}{0.9554}$$

Therefore

$$p = e^{-\frac{(0.000371 \times 10^{+24})(100 \times 10^{-24})}{\frac{0.800}{0.965}(0.965)}} = e^{-0.046} = 0.954$$

$$\underline{\underline{p_{\text{theory}} = 0.954}}$$

FAST FISSION FACTOR, ARGONAUT REACTOR*

by

D. A. Daavettila

ADSTRACT

Uranium-238 will fission appreciably only for neutrons with energies above one MeV, but, as most of the neutrons are born in fission with energies above one MeV, some of these fast neutrons will initiate U^{238} fissions. The U^{238} fissions result in an increased number of neutrons slowing down in the core, and their contribution to the reactor multiplication is considered as the fast fission factor ϵ , where ϵ is defined as the number of neutrons slowing down past the U^{238} fission threshold per fast neutron from a U^{235} fission.

In this experiment, the value of ϵ is determined from a measurement of the U^{238} to U^{235} fission ratio. The basic procedure is to irradiate in the core two uranium foils with different U^{238} to U^{235} atom ratios, and then to measure the fission activity of the foils. A mathematical analysis of the characteristic fission activities yields the U^{238} to U^{235} fission ratio, and from this ratio ϵ is interpreted according to a simple theory.

* W. L. Grant (S. Africa) assisted in the development of the experiment.

OUTLINE

- I. INTRODUCTION
- II. THEORY
 - A. Derivation of the Experimental Equation
 - B. Measurement of Fission Activity
 - C. Evaluation of δ^{28}
- III. APPARATUS
- IV. PROCEDURE
- V. RESULTS
- VI. ANALYSIS AND CONCLUSION
- VII. BIBLIOGRAPHY
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 - A. Discussion Questions
 - B. Measurement of the Fast Fission Factor Using a Depleted and Enriched Uranium Metal Foil
 - C. $P(t)$ versus Time after Shutdown

I. INTRODUCTION

The fissioning of U^{238} atoms by fast neutrons contributes to the neutron multiplication of uranium-fueled reactors, but the significance of the contribution strongly depends on both fuel enrichment and reactor design. The measure of the contribution is given by the fast fission factor ϵ in the four-factor reactor formula:

$$k_{\infty} = \eta \epsilon p f, \quad (1)$$

which gives the ratio of neutrons absorbed in one generation to that absorbed in the preceding generation for an infinite reactor lattice. In the equation, k_{∞} is the infinite multiplication factor, η is the number of fast neutrons born per thermal neutron captured in the fuel, ϵ is the number of neutrons slowing down past the U^{238} fast fission threshold per fast neutron from a U^{235} fission, p is the probability of a fast neutron escaping capture while slowing down, and f is the fraction of thermal neutrons absorbed that are absorbed in the fuel. Calculations of critical reactor size often start with calculations of the four factors. For a finite reactor lattice, the actual multiplication factor, k_{eff} , is less than k_{∞} because of neutron leakage, that is, $k_{eff} = k_{\infty}$ multiplied by the total nonleakage probability. When k_{eff} is equal to one, the reactor is critical.

Fast fission is a bonus because some of the neutrons causing fast fission in the U^{238} would have been lost through leakage and capture while slowing down. The value of ϵ for natural uranium reactors is nominally 1.03, and for certain reactor designs it may be as much as 1.10, but for highly enriched reactors like the Argonaut, it is essentially one.

It is interesting to consider further the inclusion of the U^{238} fast fissions as a distinct factor ϵ in the four-factor formula, while the epithermal U^{235} fissions are often not mentioned. Yet, the fission cross section of U^{235} is always larger than that of U^{238} , and above the U^{238} threshold the cross-section ratio is about 2.5 to 1. The reason is that the early reactors were fueled with natural uranium, and consequently had such large U^{238} to U^{235} atom ratios that the macroscopic effect of U^{235} fissions above the U^{238} threshold was relatively small. Furthermore, by reactor design the resonance-region effects were minimized to the end result that in regard to fissions, there were only two significant regions: fast U^{238} fission and thermal U^{235} fissions. Hence the small, but in certain cases very significant, effect of fast fissions was considered to be due to U^{238} alone.

As the fuel enrichment is increased, it becomes more difficult to fit the four-factor-formula analysis to the evaluation of k_{∞} , because the separation of neutron events into the characteristic energy groups is not clear, due to the increasing significance of U^{235} epithermal fissions and captures. However, since all $1/v$ -processes can be considered thermal events, only the resonance (non- $1/v$) U^{235} events are usually not explicitly taken into account in the four factors. Yet, in thermal reactors of high enrichment the contribution due to resonance U^{235} fissions may exceed that due to U^{238} , but then both are small relative to the thermal ($1/v$) U^{235} fissions. In order to keep ϵ defined as the U^{238} fast fission effect alone, which is a common definition and so defined for this experiment, the resonance U^{235} fissions when considered can be included in a modified resonance escape probability (see Bibliography Entry Number 1).

The fast fission factor tends to be larger in a heterogeneous reactor than in a homogeneous reactor, because in a homogeneous reactor the fast neutrons immediately have a good chance of being scattered below the threshold as a result of collisions with moderator atoms. On the other hand, elastic scattering collisions with fuel nuclei generally leave the neutron above the threshold, and even if inelastic collisions do not, the net result is that neutrons within a fuel lump exist above the threshold for relatively more collisions. Besides, they have a high probability that their next collision will be with a U^{238} nucleus; hence, they have an increased probability of causing a U^{238} fission. A representative set of U^{238} cross sections in Fermis for threshold-energy neutrons is the following:

$$\sigma_f = 0.29, \sigma_i = 2.47, \sigma_e = 1.5, \text{ and } \sigma_c = 0.04.$$

The actual measured number in this experiment is the U^{238} to U^{235} fission ratio known as δ^{28} . This is an important number in itself, because theoreticians often prefer to check theory by experiment through a comparison of δ^{28} values instead of ϵ values, since δ^{28} is the basic measured number, and from it a theoretical interpretation is required to obtain ϵ . In the next section, the theory and analysis to obtain ϵ from δ^{28} is derived.

II. THEORY*

A. Derivation of the Experimental Equation: The equation for the experimental determination of ϵ is simple in form and obtained from the following analysis starting with a fast neutron from a U^{235} fission birth. Let n be the probability of fast fission, r the probability of nonfission capture. Hence, $1 - n - r$ is the probability that neither of the preceding events occur, neglecting the fast neutron leakage and the probability product of nr , since n and r are small. The fast fission factor according to the definition and from Fig. 13.1 is

$$\begin{aligned}\epsilon &= (1 - n - r) + n\nu^{28}(1 - n - r) + (n\nu^{28})^2(1 - n - r) + \dots \\ &= (1 - n - r)[1 + n\nu^{28} + (n\nu^{28})^2 + \dots] \\ &= \frac{1 - n - r}{1 - n\nu^{28}}.\end{aligned}\tag{2}$$

Fig. 13.1. Fast Neutron Slowing Down
above U^{238} Threshold

The U^{238} fissions per neutron from thermal fission is given by the sum

$$n + n^2\nu^{28} + n^3(\nu^{28})^2 + n^4(\nu^{28})^3 + \dots,$$

which can be rewritten as

$$\frac{n}{1 - n\nu^{28}}.$$

The fast neutrons resulting from a U^{235} fission is the value of ν^{25} , and, inversely, $1/\nu^{25}$ is the number of U^{235} fissions per neutron from fission. A factor δ^{28} is defined by the following equation:

* Edited from Bibliography Entry Number 3.

$$\delta^{28} = \frac{U^{238} \text{ fissions}}{U^{235} \text{ fissions}} = \frac{\frac{n}{1 - n\nu^{28}}}{\frac{1}{\nu^{25}}} = \frac{n\nu^{25}}{1 - n\nu^{28}} \quad (3)$$

As fast leakage is neglected,

$$r = \alpha^{28} n, \quad (4)$$

where α^{28} is the capture-to-fission ratio above the threshold averaged over the fission-neutron spectrum.

Subtracting 1 from each side of Eq. (2), the following equation is obtained:

$$\epsilon - 1 = \frac{1 - n - r}{1 - n\nu^{28}} - 1 = \frac{1 - n - r - 1 + n\nu^{28}}{1 - n\nu^{28}} = \frac{n\nu^{28} - n - r}{1 - n\nu^{28}} \quad (5)$$

If Eq. (4) is multiplied by $n\nu^{25}/n\nu^{25}$, and use made of Eqs. (2) and (3), the equation for ϵ is obtained:

$$\epsilon = 1 + (\delta^{28}/\nu^{25})(\nu^{28} - 1 - \alpha^{28}). \quad (6)$$

The values of ν^{25} , ν^{28} , and α^{28} are taken to be 2.50, 2.65, and 0.107, respectively. The aim of the experiment is to measure δ^{28} and then to calculate ϵ from Eq. (6). Because the nonfission capture above the U^{238} fission threshold can be considered to be included in the measurement of resonance escape probability, α^{28} is more often omitted from equations such as (6) in which case ϵ is given by

$$\epsilon = 1 + [(\nu^{28} - 1)/\nu^{25}] \delta^{28}. \quad (6')$$

The basic procedure for the measurement of δ^{28} is to measure the fission activity of, or due to, two uranium-bearing foils of different U^{238} to U^{235} atom ratios. Then a mathematical analysis of these two foil fission activities yields the value of δ^{28} . In the following section, two methods of measuring fission activity will be described.

B. Measurement of Fission Activity: The main problem in the determination of fission activity is to separate the fission product activity from the gross measured activity, which is in part due to U^{238} and U^{235} nonfission neutron captures and natural radioactive decay. In the subsequent discussion, F_D and F_E will identify the net fission activity of the relatively depleted foil and the relatively enriched uranium foil, respectively.

The first techniques to be discussed to effect the separation of fission from nonfission activity is the catcher-foil method. This method is based on the physical fact that the highly energetic fission fragments have a range of up to 3 cm in air or of about 10^{-3} cm in aluminum. Therefore, if aluminum is tightly pressed against uranium, some fission fragments from the surface and near-surface fission events will be captured in the aluminum, but essentially no U^{236} or U^{239} nuclei will leave the uranium foil. The activation of the catcher-foil package in a neutron flux is shown in Fig. 13.2.

Fig. 13.2. Induced Activity of Catcher-foil Package

Activity of DU and EU due to	Activity of D and E due to
(1) U^{238} and U^{235} fission fragments	(1) U^{238} and U^{235} fission fragments
(2) U^{238} and U^{235} nonfission capture	(2) Neutron-induced transmutations
(3) Natural radioactive decay	(3) Counter background.
(4) Counter background	
Activity of B due to	
(1) Neutron-induced transmutation	
(2) Counter background	

From a review of Fig. 13.2 it follows that the characteristic activities F_D and F_E are found by counting foils D and E, and subtracting from the experimental values the activity of B. Because the activities of D, E, and B are dependent on the elapsed time since the reactor shutdown, a plot of activity versus time is first made for each foil, and then the respective activities are read at the same time ordinate.

An advantage of the catcher-foil method is that a simple proportional or Geiger-Mueller counter can be used for the foil counting. The disadvantages arise from the short range of the fission fragments, which requires that the surfaces of foils EU and DU must be clean and equally oxidized or deoxidized. Furthermore, considerable care must be exercised in the assembly and disassembly of the foil package to prevent foil damage or displacement, as the effective exposed surface areas must be known.

A second technique to separate the fission activity is a so-called bias method. The counting equipment requirements now include a gamma scintillation counter, and at least a single-channel pulse-height analyzer, but preferably a multichannel analyzer. With this method, the characteristic fission activities can be measured directly from foils DU and EU. These foils must be of known weight, but only nominally equal thickness, surface area, and conditions of surface oxidation.

The separation of activities is accomplished by setting a low-energy gamma bias, E_c , above the significant gamma photopeaks arising from nonfission capture. Of course, this also eliminates the fission activity below E_c , but this is of no concern. The point is that an activity due entirely to fission products can be measured by taking the integral activity above E_c after making a small correction for the activity contributed by the natural radioactive decay of uranium and the background. This is usually accomplished by counting the foils prior to the irradiation.

The bias cutoff E_c could be as low as 500 keV, but for a reason developed in the next section, a bias of 1.2 MeV is chosen. Figure 13.3 presents some details of the photopeaks in the gamma spectrum of a uranium foil.

Fig. 13.3. Gamma Spectrum of Uranium Foil

C. Evaluation of δ^{28}

As the value of δ^{28} is a function of spatial location within the core, the average core value of δ^{28} would have to be interpreted from a series of measurements. In the following discussion, the measured δ^{28} is considered as though it were the core average and the proper value with which to determine ϵ from Eqs. (6) or (6').

With F_D and F_E known at time t after the reactor scram, there are two methods of analysis presented by which to determine δ^{28} . The first, which is commonly used in research measurements, is based on the measured foil fission activity ratio $\gamma(t)$ defined as

$$\gamma(t) = \frac{\text{fission activity of the depleted foil at time } t}{\text{fission activity of the enriched foil at time } t} = \frac{F_D}{F_E} \quad (7)$$

The relationship of δ^{28} to $\gamma(t)$ is given by Eq. (8), whose purely algebraic derivation is given in Appendix B:

$$\delta_E^{28} = \frac{\gamma(t) - \frac{N_D^{235}}{N_E^{235}}}{\frac{N_D^{238}}{N_E^{238}} - \gamma(t)} \times P(t) \quad (8)$$

In the equation, N stands for the number of atoms in the foil, and $P(t)$ is a time-dependent parameter defined as the measured fission activity per U^{235} fission divided by the measured fission activity per U^{238} fission. The value of $P(t)$ has been measured for a bias of 1.2 MeV and for a time interval after the reactor shutdown of about 30 to 140 min. In order to make use of the literature measurements of $P(t)$, a bias of 1.2 MeV is chosen for this experiment. A curve suitable for this experiment is given in Appendix C.

In Eq. (8) the subscript E on δ^{28} is necessary, because the measured value is a function of the enrichment of foil E . The Argonaut fuel value is obtained from δ_E^{28} according to Eq. (9), where the subscript f refers to the fuel:

$$\delta^{28} = \delta_E^{28} \times \frac{N_E^{235}}{N_E^{238}} \times \frac{N_f^{238}}{N_f^{235}} \quad (9)$$

Another method of obtaining δ^{28} from the fission activities F_D and F_E is based on a plot of these activities as a function of their respective foil atom percent U^{235} . Extrapolation of this curve to zero atom percent yields the fission activity due to U^{238} fissions alone, F_D^8 . Such a plot is shown as Fig. 13.4.

Fig. 13.4
F versus Atom % U^{235}

The extrapolation to zero may be done graphically or by a similar-triangle analysis. More than two foils may be employed to increase the accuracy.

About one hour after the irradiation run ends, the foils will be removed from the core. During the one-hour delay, the radioactivity level in the core will fall to a permissible level for the foil removal. After the foils are removed from their holders and separated into individual paper envelopes, they will be taken to the counting room.

Before counting the foil set, the operation of the gamma-ray spectrometer will be thoroughly reviewed. The instrument is to be calibrated to set the value of E_c at 1.2 MeV, by counting the standard gamma sources, Cs^{137} and Co^{60} , and observing the channel location of their photo-peaks; then a plot of energy versus channel number is constructed as shown in Fig. 13.7, and the channel equivalent of 1.2 MeV determined.

Fig. 13.7. Spectrometer Calibration

The channel location of 1.2 MeV is dependent on the photomultiplier high voltage, amplifier gain, and base-line setting. One or more of these can be varied to fix the 1.2-MeV point in about channel 50. This leaves some 200 channels to display the fission spectrum of interest. With a little more effort, the 1.2-MeV point can be located in channel zero, but no real advantage occurs.

The foil-counting session amounts to measuring the integral fission activity above E_c for each foil. Because the foil activity is decaying with time, the foil set is to be recounted a number of times to obtain data to correct for the foil decay. The decay correction is made by plotting the activity of each foil as a function of time since the reactor shutdown on semilog paper. Then a single time ordinate is chosen, and all the foil activities are read at this time from their respective curves.

After the counting is completed, there will be a summary session in which all calculations are to be completed. Then the experiment will be briefly reviewed and concluded.

V. RESULTS

Table 13.2

SPECTROMETER CALIBRATION

High-voltage potentiometer setting _____ Amplifier gain _____
 Base-line potentiometer setting _____
 Cs¹³⁷ photopeak: 0.662-MeV Ch _____
 Co⁶⁰ photopeaks: 1.17-MeV Ch _____; 1.33 MeV Ch _____
 Therefore, $E_c = 1.2$ MeV is in Ch _____.

Starting at time t_0 after the scram, the foil activities above E_c referred to as "F:" will be measured. Recount the foil set at least three times to yield the data to correct for activity decay during the counting session. Enter the gross measured activities in Table 13.3.

Table 13.3

GROSS MEASURED ACTIVITY

Each count _____ min long. $t_0 =$ _____ min after scram.

Foil	Time	F	Foil	Time	F	Foil	Time	F

On semilog paper plot the measured gross activity as a function of time since t . Choose a certain time ordinate, and read the activities of all the foils at this time. Enter the results in Table 13.4.

Table 13.4

ACTIVITY DUE TO IRRADIATION

Foil activity read at _____ min after scram.

Foil	Gross Measured Activity (cpm)	Pure Background (cpm)	Net Measured Activity (cpm)	Weight- normalized Net Activity (cpm/mg)	Normalized Net Activity before Irradiation (cpm/mg)	Normalized Net Activity due to Irradiation (cpm/mg)
Symbol	F	B	F'	F''	F _B	F*

On the basis of the normalized net activity due to the irradiation, choose either of the two methods of analysis and calculate δ^{28} .

Summary of Results

$$\delta^{28} = \text{_____} P(t) \text{ taken as } \text{_____}.$$

$$\epsilon = 1 + \frac{\nu^{28} - 1 - \alpha^{28}}{\nu^{25}} \delta^{28}$$

$\epsilon =$

VI. ANALYSIS AND CONCLUSION

This section is left as the main exercise for the student. Include with your discussion the estimated precision of the results, and discuss the correctness of the extrapolation of the natural uranium ϵ value to the core ϵ value that was presented in the Theory section.

VII. BIBLIOGRAPHY

1. H. Kouts and R. Sher, Experimental Studies of Slightly Enriched Uranium, Water Moderated Lattices, BNL 486 (Sept 1957).
2. A. H. Futch, Jr., Fast Fission Effect in Lattices of Natural Uranium and Heavy Water, Nuc. Sci. Eng. 5, 61-67 (1959).
3. H. Kouts, et al., Exponential Experiments with Slightly Enriched Uranium Rods in Ordinary Water, Proc. Intl. Conf. on the Peaceful Uses of Atomic Energy, 5, 183 (1955).
4. S. Glasstone and M. C. Edlund, The Elements of Nuclear Reactor Theory (D. Van Nostrand Co., Inc., New York, 1952, pp. 276-279).

VIII. APPENDICES

A. Discussion Questions

The following questions will be discussed as part of the general experiment discussion. The student is urged to contribute more questions and general comments to the discussion.

1. What is the important practical significance of the nonfission capture in U^{238} ?
2. The nonfission capture of U^{238} is the significant contribution to one of the four factors of the four-factor formula. Which one, and what is the nature of the contribution?
3. If the Argonaut fuel was natural uranium, would you expect it to go critical?
4. What is the general shape of the fission-yield curves for U^{235} and U^{238} ?
5. In general, how does $\sigma_f U^{238}$ compare with $\sigma_f U^{235}$? Does fast fission occur in U^{235} ? What is spontaneous fission?
6. In general, how does the " ϵ " of a homogeneous core compare to the " ϵ " of a heterogeneous core? Explain.
7. Explain why ϵ approaches one as the fuel enrichment increases.
8. The U^{235} - U^{238} fission product activity ratio is not equal to the fission rate ratio. What is the reason for this difference?

B. The Measurement of the Fast-fission Factor Using a Depleted and an Enriched Uranium Metal Foil*

1. The measureable foil activity ratio $\gamma(t)$ is defined as

$$\gamma(t) = \frac{\text{Depleted U Foil Fission Activity}}{\text{Enriched U Foil Fission Activity}}$$

$$= \frac{a_8 N_D \sigma_f^8 + a_5 N_D \sigma_f^5}{a_8 N_E \sigma_f^8 + a_5 N_E \sigma_f^5}, \quad (1)$$

where a_8 and a_5 are the specific activities of the U^{238} and U^{235} fission products and are time-dependent, i.e., $a_8 = \text{cpm}/U^{238}$ fission, and where

$$\sigma_f^8 = \int_0^\infty \sigma_f^8(E) \phi(E) dE; \quad \sigma_f^5 = \int_0^\infty \sigma_f^5(E) \phi(E) dE.$$

From the measured $\gamma(t)$ we seek the ratio of U^{238} fissions to U^{235} fissions in the enriched U foil, which is defined as follows:

$$\delta = \frac{U^{238} \text{ fissions}}{U^{235} \text{ fissions}} = \frac{\sigma_f^8 N_E^8}{\sigma_f^5 N_E^5}. \quad (2)$$

We develop the experimental equation for δ by algebraic manipulations from the measured $\gamma(t)$.

2. Divide through Eq. (1) by a_8 which yields

$$\gamma(t) = \frac{N_D \sigma_f^8 + \frac{a_5}{a_8} N_D \sigma_f^5}{N_E \sigma_f^8 + \frac{a_5}{a_8} N_E \sigma_f^5}.$$

Now let

$$P = a_5/a_8.$$

Then

$$\gamma(t) = \frac{N_D \sigma_f^8 + P N_D \sigma_f^5}{N_E \sigma_f^8 + P N_E \sigma_f^5}. \quad (3)$$

* (1) Nuclear Science and Engineering, pp. 403-427, April, 1958.

(2) Measurements of U^{238}/U^{235} Fission Ratio for Borax-V Cores, ANL Internal Memo, Quincey L. Baird.

3. Divide through Eq. (3) by $\sigma_f^8 N_E^5$, which yields

$$\gamma(t) = \frac{\frac{N_D^8}{N_E^5} + \frac{PN_D^5 \sigma_f^5}{N_E^5 \sigma_f^8}}{\frac{N_E^8}{N_E^5} + P \frac{\sigma_f^5}{\sigma_f^8}} \quad (4)$$

4. Multiply $\gamma(t)$ by the right-hand denominator of Eq. (4).

$$\gamma(t) \left[\frac{N_E^8}{N_E^5} + P \frac{\sigma_f^5}{\sigma_f^8} \right] = \frac{N_D^8}{N_E^5} + \frac{PN_D^5 \sigma_f^5}{N_E^5 \sigma_f^8}$$

Simplify the above equation as follows:

$$\gamma \left(\frac{N_E^8}{N_E^5} \right) + \gamma P \frac{\sigma_f^5}{\sigma_f^8} = \frac{N_D^8}{N_E^5} + P \frac{\sigma_f^5}{\sigma_f^8} \frac{N_D^5}{N_E^5}$$

Then

$$\gamma \left(\frac{N_E^8}{N_E^5} \right) - \frac{N_D^8}{N_E^5} = P \frac{\sigma_f^5}{\sigma_f^8} \left(\frac{N_D^5}{N_E^5} - \gamma \right)$$

From the above equation we write the following

$$\frac{\sigma_f^5}{\sigma_f^8} = \frac{\gamma \left(\frac{N_E^8}{N_E^5} \right) - \frac{N_D^8}{N_E^5}}{P \left(\frac{N_D^5}{N_E^5} - \gamma \right)}$$

or, inversely,

$$\frac{\sigma_f^8}{\sigma_f^5} = \frac{P \left(\frac{N_D^5}{N_E^5} - \gamma \right)}{\gamma \left(\frac{N_E^8}{N_E^5} \right) - \frac{N_D^8}{N_E^5}} \quad (5)$$

Now write Eq. (5) over a common denominator N_E^5 to obtain

$$\frac{\sigma_f^8}{\sigma_f^5} = \frac{P(N_D^5 - \gamma N_E^5)}{\gamma N_E^8 - N_D^8} \quad (6)$$

Multiply Eq. (6) by -1:

$$\frac{\sigma_f^8}{\sigma_f^5} = \frac{P\gamma N_E^5 - P N_D^5}{N_D^8 - \gamma N_E^8} \quad (7)$$

5. Multiply each side of Eq. (7) by (N_E^8/N_E^5)

$$\begin{aligned} \frac{\sigma_f^8 N_E^8}{\sigma_f^5 N_E^5} &= \frac{P(\gamma N_E^5 - N_D^5)}{N_D^8 - \gamma N_E^8} \frac{N_E^8}{N_E^5} \\ &= \frac{P\left(\gamma - \frac{N_D^5}{N_E^5}\right)}{\frac{N_D^8}{N_E^8} - \gamma} \end{aligned} \quad (8)$$

Now the left-hand side of Eq. (8) is the desired δ [see Eq. (2)], and the right-hand side is a function of the measured γ and the known atom ratios, PLUS the as-yet unknown values for P (see Appendix C).

We can rewrite Eq. (8) as

$$\delta_E = \frac{P\left(\gamma - \frac{N_D^5}{N_E^5}\right)}{\frac{N_D^8}{N_E^8} - \gamma} \quad (9)$$

To obtain δ for the fuel composition, we multiply the δ_E value from Eq. (9) by the ratio

$$\frac{N_{\text{fuel}}^8 N_E^5}{N_{\text{fuel}}^5 N_E^8}$$

and finally obtain

$$\epsilon = 1 + \frac{U^{238} - 1 - \alpha^{238}}{U^{235}} \delta_{(\text{fuel})}^{28} \quad (10)$$

C. $P(t)$ versus Time after Shutdown

From A. H. Futch, Jr., Fast Fission Effect in Lattices of Natural Uranium and Heavy Water, Nucl. Sci. Eng. 5, 61-67 (1959).

Fig. 13.8. Fast Fission in U-D₂O Lattices

STUDENT RESULTS

FAST-FISSION EXPERIMENT

I. Foil Location and Irradiation

The reactor loading was a one-slab, north-quadrant, core containing 1.932 kg of U^{235} . The irradiation run was at a power level of 50 W for 30 min. The foils were located on a Lucite foil holder which was placed between fuel plates 8 and 9 of box 16 (see Fig. S13.1). The enriched uranium was 1.468 a/o U^{235} , and the depleted uranium was 0.0383 a/o. Both foils were 5 mils thick.

Fig. S13.1

Foil Location Details

II. Spectrometer Calibration

The details of the spectrometer calibration are as follows.

SPECTROMETER CALIBRATION DATA

High-voltage potentiometer setting 740 Amplifier gain 2
 Base-line potentiometer setting 120
 Cs^{137} photopeak: 0.662 MeV, Ch 20
 Co^{60} photopeaks: 1.17 MeV, Ch 56; 1.33 MeV, Ch 67.5,
 Therefore, $E_c = 1.2$ MeV is in Ch 58.

Fig. S13.2

Spectrometer Calibration Curve

III. Measured Foil Activity

The foil activity is the integral fission activity above 1.2 MeV or the integral from channel 58 to channel 256. Before the irradiation, the foil activities including the pure background were #1 = 102, #2 = 49, #3 = 164, and #4 = 53 cpm. The foil activities in cpm after the irradiation are given in Table S13.1.

Table S13.1

GROSS MEASURED ACTIVITY

Each count 1 min long. $t_0 = \underline{70}$ min after scram.

Foil	Time	F	Foil	Time	F	Foil	Time	F
1	0	31072	1	9.5	26577	1	18.0	24032
2	3	4278	2	11.75	3748	2	20.0	3285
3	5	53394	3	14.0	46467	3	22.0	42511
4	7.25	5340	4	16.0	4772	4	24.0	4240

The foil activities are plotted in Fig. S13.3, and the respective activities read at the 10 minute ordinate (80 min after the scram). The normalized net foil activities calculated are given in Table S13.2.

Fig. S13.3. Measured Foil Activities after the Irradiation

From Fig. 13.4 it follows that the U^{238} fission contribution in foil E, F_E^8 , is given by

$$F_E^8 = \frac{100 - \text{atom } \% U^{235} \text{ in foil E}}{100} \times F_D^8 \quad (10)$$

and that

$$F_E^5 = F_E - F_E^8. \quad (11)$$

Hence, a fission activity ratio $\delta_E^{28'}$ can be defined as

$$\delta_E^{28'} = \frac{U^{238} \text{ fission activity in foil E}}{U^{235} \text{ fission activity in foil E}} = \frac{F_E^8}{F_E^5}. \quad (12)$$

According to the definition of $P(t)$, it follows that

$$\delta_E^{28} = \delta_E^{28'} \times P(t), \quad (13)$$

and by Eq. (9), the core value of δ^{28} is obtained.

III. APPARATUS

- A. Reactor
- B. Aluminum foil - 1 mil thick
- C. At least two uranium foils - 2 to 5 mils thick - with different U^{238} to U^{235} atom ratios
- D. Foil holder to position foils in core
- E. Counting system including NaI scintillation counter, 256-multichannel analyzer, amplifier, and digital printer, (see Fig. 13.5).

139-3318

Fig. 13.5. Counting System

IV. PROCEDURE

It may be that only a single foil pair will be irradiated to determine δ^{28} ; however, more foil pairs could be included in the experiment to increase the accuracy of the determination of average values of δ^{28} , and space is so provided in the tables of Part V. The foil selection, number, and location will be reviewed on the experiment day.

Two small, 5-mil-thick, uranium metal foils of different U^{238} to U^{235} atom ratios will be irradiated in the core at equivalent flux positions for 0.5 hr at 20 W. The proper location of the foils would be within a fuel element, as shown in Fig. 13.6a; however, because of the core radio-activity level and time limitations, the foils will be positioned as in Fig. 13.6b. The effective difference between the two positions is believed to be small.

Fig. 13.6. Detailed Foil Location

The foils, located in the water space, will be taped onto a Lucite foil holder. A string will be attached to the foil holder so that a quick pull will remove the foils from the core.

In Table 13.1, enter the foil data, and in the space below the table sketch in the foil locations. The activity before irradiation will either be measured by you or given, depending on the time schedule.

Table 13.1

FOIL DATA

Foil	Weight, mg	U^{235} , a/o	Activity before Irradiation	Foil	Weight, mg	U^{235} , a/o	Activity before Irradiation

Table S13.2

ACTIVITY DUE TO IRRADIATION

Foil activity read at 80 min after scram.

Foil	Activity after, cpm	Activity before, cpm	Activity Net, cpm	Foil Weight, mg	Normalized Foil Activity, cpm/mg
1	26500	102	26400	161.1	163.9
2	3840	49	3790	304.2	12.46
3	49600	164	49435	192.3	257.1
4	5160	53	5110	302.7	16.88

IV. Determination of δ^{28}

The analysis for δ^{28} will only be shown for foils #3 and #4; analysis of foils #1 and #2 would be exactly similar. Equation (8) is reproduced below:

$$\delta_E^{28} = \frac{\gamma(t) - \frac{N_D^{235}}{N_E^{235}}}{\frac{N_D^{238}}{N_E^{238}} - \gamma(t)} P(t).$$

where

$$\gamma(t) = \frac{F_D}{F_E} = \frac{16.88}{257.1} = 0.06565;$$

$$P(80 \text{ min}) = 1.204.$$

Because the foil activities have been normalized to a weight of 1 mg, the atom ratios are given by the ratios of the atom percents:

$$\frac{N_D^{235}}{N_E^{235}} = \frac{0.0383}{1.468} = 0.02608;$$

$$\frac{N_D^{238}}{N_E^{238}} = \frac{100 - 0.0383}{100 - 1.468} = \frac{99.9617}{98.5320} = 1.0145.$$

Substitution of the above values into the equation for δ_E^{28} yields

$$\delta_E^{28} = \frac{0.06565 - 0.02608}{1.0145 - 0.6565} = \frac{0.03957}{0.94885} = 0.0417 \cdot P(t)$$

$$= 0.0417 \times 1.204 = 0.0502.$$

By Eq. (9),

$$\delta_{\text{fuel}}^{28} = \delta_E^{28} \times \frac{N_E^{235}}{N_E^{238}} \times \frac{N_f^{238}}{N_f^{235}} = 0.0502 \times \frac{1}{68.1} \times \frac{4}{1}$$

$$= 0.0030$$

The same analysis for foils 1 and 2 yields $\delta_{\text{fuel}}^{28} = 0.0037$. The average $\delta_{\text{fuel}}^{28}$ is taken to be 0.0034, and the core value of ϵ is evaluated as follows.

Summary of Results

$$\delta_{\text{fuel}}^{28} = \underline{0.0034} \cdot P(t) \text{ taken as } \underline{1.204}.$$

$$\epsilon = 1 + \frac{\nu^{28} - 1 - \alpha^{28}}{\nu^{25}} \delta^{28} = 1 + \frac{2.65 - 1.00 - 0.107}{2.50} (0.0034)$$

$$= 1 + (0.617)(0.0034)$$

$\epsilon = 1.0021$

CONVERSION RATIO, ARGONAUT REACTOR

by

D. A. Daavettila

ABSTRACT

The conversion ratio is defined as the rate at which new fuel is created divided by the rate at which the reactor fuel is consumed through fission and radiative capture. The two materials usually considered as the raw material for conversion are U^{238} and Th^{232} . Both isotopes, which are the most abundant forms of these elements in nature, fission appreciably only for high-energy neutrons. Through radiative capture they are converted to Pu^{239} and U^{233} , respectively, which fission for thermal neutrons.

As the Argonaut is a uranium-fueled reactor, the overall aim of this experiment is to determine the ratio of Pu^{239} atoms formed relative to U^{235} atoms destroyed. The foil technique for making the main measurement and the necessary data corrections are presented.

OUTLINE

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- III. APPARATUS
- IV. EXPERIMENTAL PROCEDURE
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 - 1. General
 - 2. True Np^{239} Activity
 - 3. True U^{235} Fission Product Activity
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 - A. Raw Measured Foil Activity
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 - B. Fuel Production Isotope Chain

I. INTRODUCTION

The conversion ratio can be defined as the rate at which new reactor fuel is created divided by the rate at which the reactor fuel is consumed through fission and radiative capture. The magnitude of this ratio is important in power and converter reactor economics because the new fuel is worth money, and the fuel has potential use whether or not it is suitable in practice for refueling the source reactor. The two isotopes usually considered as the raw or fertile materials for conversion are U^{238} and Th^{232} . These two isotopes fission appreciably only for high-energy neutrons, and neither could support a fission chain reaction. However, through radiative capture events, they are converted into the fissile isotopes Pu^{239} and U^{233} , respectively, both of which have large thermal-fission cross sections, and both can be arranged so as to maintain a chain reaction.

The only isotope available in significant amounts in nature that will support a fission chain reaction is U^{235} . Since U^{238} and Th^{232} make up 99.3 and essentially 100 percent of these elements, the importance of the conversion of fertile to fissile isotopes is obvious, as it results in a 100-fold or more expansion of the available fuel supplies for nuclear power. In addition, as many reactor designs call for small cores and enriched fuel, conversion yields another benefit. This benefit is that the separation of Pu^{239} and U^{233} from their parent materials is less difficult than the separation of U^{235} from U^{238} , because a separation involving a difference of elements is easier than isotopic separation. As uranium is more abundant than thorium and somewhat more easily prepared for reactor use, it is unfortunate that plutonium is an extremely difficult material to handle and fabricate relative to uranium or thorium, even though all three elements are toxic, radioactive, and pyrophoric.

The Argonaut is a uranium-fueled reactor. The overall aim of this experiment is to determine the ratio of Pu^{239} atoms formed relative to the U^{235} atoms destroyed. The primary measurement is the determination of Np^{239} (activity due to U^{238} captures) and the U^{235} fission product activity of three Argonaut fuel foils. Two are irradiated in the core, with one being cadmium covered, and the other is in the graphite thermal column. Additional foils are also included for the irradiation to provide data to correct the measured fuel foil activities to yield explicitly the Np^{239} and U^{235} fission product activities.

II. THEORY

The conversion ratio at a given location x in the reactor core, or better the differential conversion ratio $d(\gamma)$ is defined as

$$d(\gamma) = \frac{Pu^{239} \text{ atoms created in region } x}{U^{235} \text{ atoms destroyed in region } x}, \quad (1)$$

which can be rewritten in terms of flux ϕ , cross section σ , and the number of atoms, N , as

$$d(\gamma) = \frac{N^8 \int_0^\infty \sigma_c^8(E) \phi(E) dE}{N^5 \int_0^\infty \sigma_f^5(E) \phi(E) dE} \frac{1}{1 + \alpha^5} \quad (2)$$

where the superscripts 8 and 5 refer to U^{238} and U^{235} , respectively, and the α^5 is the U^{235} capture to fission ratio.

From a consideration of Eq. (2), one can observe that $d(\gamma)$ is a function of the neutron spectrum, and since the spectrum is a function of core location, it follows that $d(\gamma)$ will depend on core location. The overall core conversion ratio, γ , could be determined from a series of measurements of $d(\gamma)$ as a function of core location, or calculated from one known $d(\gamma)$ and the flux distribution.

Another point to be made is that the conversion ratio is affected by burnup, and hence is time dependent for those reactors in which burnup is significant. For such reactors, one usually considers the initial conversion ratio, that is, for the initial cold reactor loading.

The cross sections are considered to be averaged over the neutron spectrum, so that

$$N^8 \int_0^\infty \sigma_c^8(E) \phi(E) dE = N^8 \bar{\sigma}_c^8 \phi \quad (3a)$$

and

$$N^5 \int_0^\infty \sigma_f^5(E) \phi(E) dE = N^5 \bar{\sigma}_f^5 \phi, \quad (3b)$$

where

$$\bar{\sigma} = \frac{\int_0^\infty \sigma(E) \phi(E) dE}{\int_0^\infty \phi(E) dE} \quad (3c)$$

Substituting the results of the above equations into Eq. (2), the following equation is obtained:

$$d(\gamma) = \frac{N^8 \bar{\sigma}_c^8}{N^5 \bar{\sigma}_f^5} \frac{1}{1 + \alpha^5}. \quad (4)$$

The N^8/N^5 atom ratio is known for the fuel and the value of α^5 is taken from the literature. This leaves the cross-section ratio $\bar{\sigma}_c^8/\bar{\sigma}_f^5$ to be experimentally measured.

To measure the cross-section ratio, two identical uranium fuel foils are irradiated, one in the core and the other in the thermal column.

(For the time being the cadmium-covered foil mentioned in the introduction section will be forgotten, because its purpose is to provide a thermal activity by cadmium difference, and hence it is identical in purpose with the thermal column foil.) Figure 14.1 describes the activation of the two foils during the irradiation and it is the reference for the derivation of the experimental equation, which will relate the cross-section ratio to measurable quantities.

Fig. 14.1. Foil Activation

From the notation of Fig. 14.1, the core-to-thermal column ratios of fission and capture are written as follows:

$$\frac{\phi \bar{\sigma}_f^5}{\phi_T \sigma_{fT}^5}; \quad (5a)$$

$$\frac{\phi \bar{\sigma}_c^8}{\phi_T \sigma_{cT}^8}, \quad (5b)$$

where the subscript T stands for thermal. Multiplication of ratio (5a) by σ_{fT}^5 and of ratio (5b) by σ_{cT}^8 yields the following:

$$\frac{\phi \bar{\sigma}_f^5}{\phi_T \sigma_{fT}^5} \sigma_{fT}^5 = \frac{\phi}{\phi_T} \bar{\sigma}_f^5; \quad (6a)$$

$$\frac{\phi \bar{\sigma}_c^8}{\phi_T \sigma_{cT}^8} \sigma_{cT}^8 = \frac{\phi}{\phi_T} \bar{\sigma}_c^8. \quad (6b)$$

Division of Eq. (6b) by (6a) and transfer of $\bar{\sigma}_c^8$ and $\bar{\sigma}_f^5$ to the left-hand side yields

$$\frac{\bar{\sigma}_c^8}{\bar{\sigma}_f^5} = \frac{\phi \bar{\sigma}_c^8 / \phi_T \sigma_{cT}^8}{\phi \bar{\sigma}_f^5 / \phi_T \sigma_{fT}^5} \frac{\sigma_{cT}^8}{\sigma_{fT}^5} \quad (7)$$

Equation (7) gives the desired cross-section ratio.

The measurement of the right-hand side of (7) is discussed. The capture of a neutron by U^{238} yields radioactive Np^{239} as a member of the decay chain. The Np^{239} activity ratio of the core- and thermal-column-irradiated foils is equal to the U^{238} capture ratio* as given below:

$$\frac{\phi \bar{\sigma}_c^8}{\phi_T \sigma_{cT}^8} = \frac{A_{\text{core foil}}^{Np}}{A_{\text{column foil}}^{Np}}, \quad (8)$$

where A^{Np} is the measured Np^{239} activity (cpm).

When a U^{235} atom fissions, radioactive isotopes are created. With the assumption that the measured activity of the fission products per thermal fission is equal to that for a fast fission, the following ratio and equality is defined:

$$\frac{\phi \bar{\sigma}_f^5}{\phi_T \sigma_{fT}^5} = \frac{A_{\text{core foil}}^{FP}}{A_{\text{column foil}}^{FP}}, \quad (9)$$

where A^{FP} is the measured U^{235} fission activity (cpm). Substitution of (8) and (9) into Eq. (7) yields Eq. (10), which presents the desired cross-section ratio in terms of measureable quantities and the literature values of the thermal cross sections:

$$\frac{\bar{\sigma}_c^8}{\bar{\sigma}_f^5} = \frac{A_{\text{core}}^{Np} / A_{\text{column}}^{Np}}{A_{\text{core}}^{FP} / A_{\text{column}}^{FP}} \frac{\sigma_{cT}^8}{\sigma_{fT}^5} \quad (10)$$

To simplify Eq. (10) further, let R stand for the measured activity ratio, so that Eq. (10) can be rewritten as

$$\frac{\bar{\sigma}_c^8}{\bar{\sigma}_f^5} = R \frac{\sigma_{cT}^8}{\sigma_{fT}^5} \quad (11)$$

Now, substitution of Eq. (11) into Eq. (4) results in the derivation of the final experimental equation:

$$d(\gamma) = R \frac{\sigma_{cT}^8}{\sigma_{fT}^5} \frac{N^8}{N^5} \frac{1}{1 + \alpha^5} \quad (12)$$

where R is the quantity to be experimentally determined.

III. APPARATUS

1. Reactor
2. Five foils of the reactor fuel composition
3. Aluminum foil, about 1 mil thick
4. Two enriched and one depleted uranium metal foils
5. Cadmium foil covers
6. Counting system (see Fig. 14.2) including NaI scintillation counter, amplifier, 256-multichannel analyzer, and digital printer.

139-3318

Fig. 14.2. Counting System

VI. EXPERIMENTAL PROCEDURE

A. Principles

1. General. The overall experimental purpose is to measure the activity ratio R required in Eq. (12):

$$R = \frac{A_{\text{core}}^{\text{Np}} / A_{\text{column}}^{\text{Np}}}{A_{\text{core}}^{\text{FP}} / A_{\text{column}}^{\text{FP}}}. \quad (13)$$

The raw data from which A^{Np} and A^{FP} are to be determined is the measured gamma spectra of the irradiated core and thermal column fuel foils at time "t" after the scram. One such measured spectrum is shown in Fig. 14.3, where, the letters N and F refer to the gross measured activities in the two cross-hatched regions. The main Np^{239} activity is at 103 keV, although there are smaller peaks at 228 and 278 keV, and a relatively insignificant one at 506 keV, only the 103-keV peak is of interest for this experiment. The lower energy cut off for the F region should at least be above 300 keV to eliminate what otherwise would be a significant nonfission activity contribution to F due to Np^{239} .

Fig. 14.3. Measured Gamma Spectrum

From Fig. 14.3, it is clear that the desired activities A^{Np} and A^{FP} are not explicitly presented, and that to determine them the complicating contributions must be eliminated or measured. The contribution of background and natural decay activity is easily eliminated by subtracting the pre-irradiation spectrum from the post-irradiation spectrum. The other two corrections are more difficult and complicate the measurement of conversion ratio. To determine them, special foils have to be irradiated and counted. In the following section these two corrections are discussed.

2. True Np^{239} Activity. To determine the magnitude of the contribution of the fission product activity to the Np^{239} peak, the foil package depicted in Fig. 14.4 is included for the irradiation. The enriched uranium metal foil is a source of recoil fission fragments which are captured by the aluminum catcher foil. In principle, the source uranium

foil (1.46 a/o U^{235}) should match the enrichment of the fuel foils (20 a/o U^{235}) in order to match U^{235}/U^{238} fission ratios, because the isotope fission yield curves for U^{235} and U^{238} are slightly different. To correct for the mismatch in enrichment, the special foil package will be irradiated in the external thermal column to reduce to the desired degree the U^{238} fissions.

Fig. 14.4

Special Foil Package

At some time "t" after the reactor scram, the gamma spectra of the catcher foils and the fuel foils are observed and typically could be pictured as shown in Fig. 14.5. What is to be determined is f_u , the fission activity in the Np^{239} peak. The quantities F_c and F_u are measured integral fission activities above E_c , and f_c is the fission activity in the neptunium peak region as seen by the catcher foil. The following equation is written subject to qualifications discussed later:

$$f_u/F_u = f_c/F_c, \quad (14)$$

which can be rewritten as

$$f_u = (f_c/F_c)F_u. \quad (15)$$

In Eq. (15), the unknown f_u is given in terms of directly measurable quantities. The ratio of f_c/F_c is time-dependent. Substitution of $r(t)$

Fig. 14.5. Gamma Spectra

for this ratio in Eq. (15) yields

$$f_u = r(t)F_u. \quad (16)$$

With the magnitude of f_u thus known (refer back to Fig. 14.3), the true Np^{239} activity of a fuel foil at time "t" can be written as

$$A^{\text{Np}} = N' - rF_u, \quad (17)$$

where N' is the gross measured activity in the Np^{239} peak region (N) minus the activity of background and natural decay. Now the two basic assumptions behind Eq. (17) will be discussed.

The first assumption is that the light and heavy fission fragments have equal ranges, and hence are equally well captured in the aluminum catcher foil. This is necessary in order for the fuel and catcher foils to have identical fission product activity gamma spectra. On the basis of experimental evidence, the effect of the difference in range is at least small, and it will not be considered further. The second question is whether the thickness of the fuel foil modifies the $r(t)$ value measured from the 1-mil-thick aluminum foil, because of Bremsstrahlung radiation and the uranium scattering higher-energy gammas into the Np^{239} peak. The significance of this question can be resolved by a simple attenuation versus shield thickness counting experiment, and can be made negligible by using thin fuel foils.

3. True U^{235} Fission Product Activity. Because the Argonaut fuel is 20% enriched, the fission product activity due to U^{238} fissions can be completely neglected. However, for the sake of completeness, a technique for separating U^{235} fission activity from U^{238} fission activity will be discussed.

Two small, thin (5-mil) uranium metal foils of differing U^{238} to U^{235} atom ratios will be irradiated in the core at flux-equivalent positions to that of the fuel foils. The fission product activity F of these two foils will be measured. Let the subscripts D and E refer to the relatively depleted and enriched foils, respectively. The activities F_D and F_E are plotted against the source uranium foil U^{235} content in atom percent U^{235} , as in Fig. 14.6

Fig. 14.6
Fission Activity versus U^{235} Content

The straight line is extrapolated to zero U^{235} content, which is the fission activity due to U^{238} fissions alone, F_0^8 . Taking into account that the U^{238} surface atom density is less in enriched uranium than in pure U^{238} - a factor of 0.986 for 1.44% enriched - $0.986 F_0^8$ is subtracted from F_E to determine the activity due to U^{235} fissions, F_E^5 . Then if the reactor fuel enrichment was 1.44%, only the fraction

$$b_E = \frac{U^{235} \text{ fission activity}}{\text{total fission activity}} = \frac{F_E^5}{F_E^5 + F_E^8} \quad (18)$$

of the measured F_E would be entered into Eq. (14) as A^{FP} . The fraction b_E can be written in the following form:

$$b_E = \frac{1}{1 + \frac{F_E^8}{F_E^5}} \quad (19)$$

As the Argonaut fuel U^{238}/U^{235} atom ratio is 4:1 instead of 68:1 as in 1.44% enriched uranium, the factor b for the reactor fuel can be shown to be given by

$$b_f = \frac{1}{1 + \frac{F_E^8}{17F_E^5}} \quad (20)$$

and hence

$$A^{FP} = b_f F(\text{fuel foil}). \quad (21)$$

B. Detailed Procedure:

1. Foil Description. The detailed foil layout for the irradiation run together with a brief description of the purpose of each foil is given in Fig. 14.8. In Table 14.1 foil data such as weights and dimensions are given.

The core location of the fuel foils will likely be at the core center; however, a different position or indeed more than one could be investigated. Also, the conversion ratio could be measured using depleted and enriched uranium metal foils, and the result compared with that measured using fuel foils. Such decisions will be made on the experiment day and the details noted in Fig. 14.8. The considerations involved in the choice of foils will be discussed as discussion question Number 1 of Appendix A.

Table 14.1
FOIL DETAILS

DESCRIPTION

Foils - 1, 2, 3, 4, and 5 are 5 mil thick, 1/2 in. diameter, Argonaut fuel.

Foils - A1 and A2 are made of 1-mil-thick aluminum.

Foil - EU is 1.44% enriched uranium metal, 2 mil thick.

Foil - DU is depleted uranium metal, 383 ppm U^{235} , 5 mil thick.

Foil - FFS is a 3 x 2-in. piece of 0.144% enriched uranium metal - 2 mil thick.

RELATIVE WEIGHTS

1 _____, 2 _____, 3 _____, 4 _____,
5 _____, EU _____, DU _____,
A1 _____ in.², A2 _____ in.², FFS _____

Whether or not the core fuel foils will be contained within an Argonaut fuel plate, which would be proper, will depend on the time schedule and the level of core radioactivity. The foils can be simply and readily irradiated in the water space between plates. The difference between the two choices can be made small, because from the neutron's point of view the core is approximately homogeneous. In Fig. 14.7 the proper location of the fuel foil in the core is compared with the probable foil location for this experiment.

Fig. 14.7. Fuel Foil Location in Core

The aluminum foils are fission-fragment barriers. The thick fuel section is included in the cadmium-covered package located within the fuel element to reduce neutron streaming and to insure that the resonance neutron spectrum entering the thin fuel foil is the same as seen by the rest of the fuel. In the water-space location of the fuel foils the fuel sections serve a similar function. The effort is to match roughly the effective U^{238} resonance integral of the fuel element by making the dimension T equal to that of the fuel elements; 2-mil-thick uranium metal shields can be used to match the effective resonance integral of the foil package to that of the fuel. Other thin fuel foils are placed on the outside of the fuel sections or shields to check interior versus exterior activation.

2. Irradiation Run. The foil irradiation run will nominally be at a power of 30 W for 30 min. After the reactor scram, the foils will be allowed to decay for about 4 hr before counting, to reduce the significance of the decay of fission activity during the counting session. It is better to wait 24 hr before counting the foils, but then an irradiation run of 100 W for 30 min is necessary for good counting statistics.

3. Foil Counting. The foils will be counted with a 2 x 2-in. NaI crystal scintillation counting system, and the gamma spectrum obtained with a 256-channel analyzer. The aim of the foil counting is to obtain for each foil (see Fig. 14.3) the measured gross activity in the Np^{239} region, "N," and the measured fission activity above E_C , that is, "F," at the same time. Instead of observing both N and F activities simultaneously, as shown in Fig. 14.3, they will be measured separately to enable the energy abscissa to be expanded. After determining the N activity of each foil, the spectrometer-analyzer system will be reset to measure the F region. Since the foil activities will decay a measureable amount during the counting session, data to correct for the foil decay is to be obtained by recycling the foils through the N and F counting cycle a number of times. The foil activities are then plotted as a function of time on semilog paper, and at a single time ordinate all activities read from their respective curves.

The system settings to use to determine N and F are obtained from a system calibration. A U^{235} gamma-ray source peaks at 95, 143, and 185 keV, and is used to calibrate the low-energy region in terms of energy versus channel number. The actual channel interval containing the main Np^{239} peak, "N," is determined by observing the post-irradiation spectrum of foil DU of Fig. 14.8 because of its essentially pure Np^{239} activity. The F region is calibrated by working with both Co^{60} and Cs^{137} gamma sources. Once the slope of gamma energy versus channel number is known, the low-energy activity is cut out by raising the base-line setting on the 256-channel analyzer to E_C (about 600 keV). The actual value is arbitrary as long as it is above the significant Np^{239} peaks.

FOIL GROUP	IRRADIATED IN CORE	IRRADIATED IN THERMAL COLUMN
I PRIMARY CONVERSION RATIO MEASUREMENT	<p>TO DETERMINE N AND F IN CORE</p> <p>20 MILL CADMIUM</p> <p>SHIELD FOILS TO MATCH U^{238} RESONANCE INTEGRALS</p> <p>A THERMAL ACTIVITY IS DETERMINED BY SUBTRACTING ACTIVITY OF CF FROM BF.</p>	<p>TO DETERMINE N AND F IN THERMAL FLUX</p> <p>BACK 20" IN J-10</p>
II DATA CORRECTION MEASUREMENTS	<p>THIS PAIR CAN BE USED AS THE BARE SHIELDS NOTED ABOVE</p> <p>(1) U^{238} FISSION ACTIVITY CORRECTION</p> <p>(2) DU IS USED TO LOCATE THE Np^{239} PEAK AND TO DEFINE THE N INTERVAL</p>	<p>(1) TO CHECK EPITHERMAL ACTIVATION OF 5</p> <p>20 MILL Cd.</p> <p>(2) EVALUATION OF $r(t)$</p> <p>FISSION FRAGMENT SOURCE FOIL (FFS)</p>
III SPECIAL FOILS FOR THE EXTENTION OF THE MEASUREMENTS		

Fig. 14.8. Foil Layout and Purpose

4. Summary Session. After the foil counting is completed, there will be a summary session during which the experimental conversion ratio will be evaluated from the measured foil activities according to the outline given in the next section. This experimental value will then be compared with the results obtained by two different, but elementary, theoretical estimates which are outlined in Section VI.

V. RESULTS

A. Raw Measured Foil Activity: The basic counting and calibration data are to be entered in Table 14.2 and the raw measured foil activities before the irradiation in Table 14.3. The post-irradiation activities are to be entered in Table 14.4.

Table 14.2

COUNTING AND CALIBRATION DATA

- (1) Zero time for counting is _____ min after the reactor scram.
- (2) Each count is _____ min long.
- (3) N Region; integral channels _____ to and including _____.
 - (a) Calibration U^{235} gamma peaks energy versus channel number
95 keV _____, 143 keV _____, 185 keV _____.
 - (b) High-voltage pot setting _____, baseline setting _____.
- (4) F Region; integral channels _____ to and including _____.
 - (a) Calibrate Cs^{137} standard 662 keV channel _____.
 - (b) Calibrate Co^{60} standard 1170 keV _____, 1330 keV _____.
 - (c) High-voltage pot setting _____, baseline setting _____.
 - (d) E_c = _____ keV.

Table 14.3

ACTIVITY BEFORE IRRADIATION

Foil	N	F	Foil	N	F

Table 14.4

RAW MEASURED FOIL ACTIVITY

Foil	Time	N	Foil	Time	F

B. Calculations

1. Net Activity due to the Irradiation. To correct for the time dependence of the foil activities, plot them on semilog paper as functions of time since the reactor scram. Choose a single time ordinate and read from the respective curve the raw measured N and F activity of each foil. Enter these values into Table 14.5.

Table 14.5

RAW MEASURED FOIL ACTIVITY AT

____ Hours after Shutdown

Foil	N	F	Foil	N	F

Not all the measured N or F activity is due to the irradiation, but is due in part to background and to the activity generated by the radioactive decay of uranium. Since the experiment rests on the analysis of the foil activities due to the irradiation, these must be explicitly determined and are referred to as N' and F'. The simplest method to obtain N' and F' from N and F is to measure the N and F activity of each foil before the irradiation, and then to subtract this measured pre-irradiation activity from the post-irradiation activity. Enter the net foil activities into Table 14.6:

Table 14.6

NET FOIL ACTIVITY AT

____ Hours after Shutdown

Foil	N'	F'	Foil	N'	F'

For ease in calculation, normalize the net activities of the fuel foils to the weight of foil 1. The normalized net foil activities will be referred to as N* and F*, and are to be entered into Table 14.7.

Table 14.7

NORMALIZED NET ACTIVITY (cpm)

At a Mean Count Time of ____ Hours after Scram.

Foil	N*	F*	Foil	N*	F*

2. Calculations of r. The activity ratio r is the fission activity in the Np^{239} region divided by the integral fission activity above E_C . Previous measurements have shown that the ratio r for bare and cadmium-covered foils in the core are essentially equal, and this assumption can be made for this experiment. Calculate r from data in Table 14.7:

$$r_1 = \frac{N^*}{F^*} = \frac{N^* A_1 - N^* A_2}{F^* A_1 - F^* A_2}$$

at time = _____ hr after scram.

3. Calculation of b_f . That fraction of the measured fission activity that is U^{235} fission activity is called b_f . As was pointed out before, it could be taken as 1.0; however, to calculate b_f by Eq. (20), first plot the activities of F_D and F_E versus U^{235} content of the foils as in Fig. 14.6. With respective U^{235} contents of 0.038 and 1.458 a/o U^{235} , it can be shown by similar triangles that $F_0^8 = 1.027 F_D - 0.0270 F_E$. Hence,

$$F_0^8 = \underline{\hspace{2cm}}$$

$$F_E^8 = 0.986 F_0^8 \underline{\hspace{2cm}}$$

$$F_E^5 = F_E - F_E^8 = \underline{\hspace{2cm}}$$

and by Eq. (20)

$$b_f = \frac{1}{1 + \frac{F_E^8}{17F_E^5}} \frac{1}{1 + \frac{\hspace{1cm}}{\hspace{1cm}}}$$

$$b_f = \underline{\hspace{2cm}}$$

4. Measured Np^{239} Activity. With the values of b_f and r known, the values of $A^{Np} = N^* - rF^*$ and $A^{FP} = b_f F^*$ can be found. Enter these in Table 14.8.

Table 14.8

DETERMINATION OF A^{Np} AND A^{FP}

Foil	N^*	F^*	rF^*	A^{Np}	A^{FP}

C. Experimental Value of the Conversion Ratio: According to Eq. (13)

$$d(\gamma) = R \frac{\sigma_{cT}^8}{\sigma_{fT}^5} \frac{N^8}{N^5} \frac{1}{1 + \alpha^5},$$

where

$$R = \frac{A_{\text{core}}^{\text{Np}} / A_{\text{thermal}}^{\text{Np}}}{A_{\text{core}}^{\text{FP}} / A_{\text{thermal}}^{\text{FP}}}$$

Two sets of foils are used to evaluate the activity ratio R. First, using the bare and covered core and taking the foil activities from Table 14.7,

$$R_1 = \frac{A_1^{\text{Np}} / A_1^{\text{Np}} - A_3^{\text{Np}}}{A_1^{\text{FP}} / A_1^{\text{FP}} - A_3^{\text{FP}}}$$

$$= \frac{\quad}{\quad};$$

$$R_2 = \frac{A_2^{\text{Np}} / A_2^{\text{Np}} - A_4^{\text{Np}}}{A_2^{\text{FP}} / A_2^{\text{FP}} - A_4^{\text{FP}}}$$

$$= \frac{\quad}{\quad}.$$

From the core and thermal column foils,

$$R_1 = \frac{A_1^{\text{Np}} / A_5^{\text{Np}}}{A_1^{\text{FP}} / A_5^{\text{FP}}}$$

$$= \frac{\quad}{\quad};$$

$$R_2 = \frac{A_2^{\text{Np}} / A_5^{\text{Np}}}{A_2^{\text{FP}} / A_5^{\text{FP}}}$$

$$= \frac{\quad}{\quad}.$$

The thermal cross sections of U^{238} and U^{235} averaged over a Maxwellian are $\sigma_{cT}^8 = 2.37$ b and $\sigma_{fT}^5 = 485$ b.* Although the capture-to-fission ratio α^5 is a function of neutron energy, the value is quite constant, 0.190 at 1 eV. As some 93% of the fissions occur below 0.4 eV, the value of α^5 is taken as the world average (for 2200 m/s) of 0.189. The Argonaut N^8/N^5 ratio is 4:1.

From Eq. (13), the average experimental values of the conversion ratio at the point of measurement are

$d(\gamma)_1 =$ _____
$d(\gamma)_2 =$ _____

VI. ANALYSIS AND CONCLUSIONS

A. Estimated Conversion Ratio #1:** This method is derived from an analysis of the neutron cycle starting with a neutron entering the resonance region, although one could start anywhere in the cycle. The situation is described in Fig. 14.9.

Fig. 14.9. Sketch of Neutron Slowing Down

From Fig. 14.9, the U^{238} capture to U^{235} destroyed ratio which is the overall core conversion ratio, CR, is calculable as

*D. P. Moon, Two-dimensional Two-group Calculation of the Argonaut One-slab Loading, ANL-6154 (April 1960).

**R. L. Murray, Introduction to Nuclear Engineering, 2nd Ed., Prentice-Hall, Inc., New York (1961), p. 114.

$$CR = \frac{(1-p) + pL_T f \frac{\Sigma_a^8}{\Sigma_a^U}}{pL_T f \frac{\Sigma_a^5}{\Sigma_a^U}} \quad (24)$$

Upon making use of the following familiar equations:

$$(a) \quad 1 = \eta \epsilon p f L_T L_1 \text{ or } L_T = 1/\eta \epsilon p f L_1;$$

$$(b) \quad \eta_{235} \Sigma_a^5 = \eta \Sigma_a^U \text{ from } \eta = \Sigma_f^5/\Sigma_a^U, \quad \eta_5 = \Sigma_f^5 \nu / \Sigma_a^5$$

we can rewrite Eq. (16) as

$$\gamma = \eta_5 \epsilon L_1 (1-p) + \left(\Sigma_a^8 / \Sigma_a^5 \right). \quad (25)$$

Substituting the approximate values for the Argonaut core into Eq. (25): $\eta_5 = 2.07$, $\epsilon = 1.002$, $L_1 = 0.65$, $p = 0.99 \pm 0.005$, $\Sigma_a^8 = 0.00447$, and $\Sigma_a^5 = 0.2730$, one obtains for the core conversion ratio the following value:

$$\gamma = \underline{\underline{\hspace{2cm}}}$$

B. Estimated Conversion Ratio #2: This method only estimates the differential or local conversion ratio for those core positions where the U^{238} cadmium ratio is known. Given the U^{238} cadmium ratio, the resonance to thermal flux ratio can be calculated. If, in addition, the U^{238} effective resonance integral, σ_R^8 , is calculated* (~ 100 b), and the thermal Maxwellian averaged cross sections are known ($U^{238} = \sim 2.34$ b and $U^{235} = 485$ b), the conversion ratio is given by

$$d(\gamma) = \frac{N_8 \sigma_C^8 \phi_T + N_8 \sigma_R^8 \phi_R}{N_5 \sigma_f^5 \phi_T (1.07)} \frac{1}{1 + \alpha^5} \quad (26)$$

where the factor 1.07 takes into account the experimental fast to thermal U^{235} fission ratio.

The U^{238} cadmium ratio determined in the experiment was _____, and the estimated $d(\gamma)$ from Eq. (26) is _____.

C. Summary and Sources of Error. As part of your conclusion, discuss the sources of error in the evaluation of r and in the rest of the experiment. Propose, where possible, ways to eliminate or reduce the significance of an error. On the basis of the experiment and the rough theoretical estimates, conclude the numerical bounds containing the conversion ratio, its most probable value, and explain your reasoning.

VII. BIBLIOGRAPHY

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VIII. APPENDICES

A. Discussion Questions

(Concerning Uranium-fueled Reactors)

1. What considerations are involved in the nature and dimensions of the foils used to measure the conversion ratio?
2. Why is fuel reprocessing necessary for power reactors, when U^{235} fissions and U^{238} either fast fissions or is converted by neutron capture to Pu^{239} ?
3. Again, if U^{238} either fissions or is converted to Pu^{239} , can a pure U^{238} -fueled reactor be made? Could a Pu^{239} -fueled reactor be made? Could a combined Pu^{239} and U^{238} loading go critical?
4. What happens to the U^{238} to Pu^{239} decay chain if either the U^{239} or Np^{239} captures a neutron? How are Pu^{240} and Pu^{242} formed in a reactor? As Pu^{241} has a large fission cross section (see Appendix B), why are we more interested in Pu^{239} ?
5. What is the significance of the names "breeder and converter" in regard to reactors? Which reactor, thermal or fast, is most favorably viewed as a breeder and why?
6. Why is the Pu^{239} formation from U^{238} important in the very long-range view of man's energy needs?
7. How does the Pu^{239} formation enter into reactor economic studies?
8. How is the price of Pu^{239} reflected in the costs of electric power generation?
9. What are some of the problems in the use of Pu^{239} as a reactor fuel?
10. What is the cheapest source of Pu^{239} ? (Hint: a certain portion of a certain class of reactors).
11. What considerations are involved in the decision to process a certain core fuel element for its Pu^{239} ?
12. In keeping with the main reactor purpose, what design parameters could be varied to maximize the conversion ratio?

B. Fuel Production Isotope Chain

Fig. 14.10

STUDENT RESULTS

CONVERSION RATIO EXPERIMENT

I. Foil Location and Irradiation

The reactor loading was a one-slab, north-quadrant core of six fuel boxes, and contained 1.932 kg of U^{235} . The irradiation was at a power level of 100 W for 30 min. The core foils were located on a Lucite foil holder which was placed between fuel plates 8 and 9 of box 16, and the thermal column foils were in the J-8 stringer. The details are presented in Fig. S14.1 and Table S14.1.

Fig. S14.1. Details of Foil Location

Table S14.1

FOIL DETAILS

DESCRIPTION

- Foils - 1, 2, 3, 4, and 5 are 5 mil thick, 1/2 in. in diameter, and Argonaut fuel.
- Foils - A1 and A2 are made of 1-mil-thick aluminum.
- Foil - EU is 1.44% enriched uranium metal, 1 x 2 in. and 5 mil thick.
- Foils - The shields foils are depleted uranium metal, 0.4 w/o U^{235} , 2 mil thick, except one bare one which is 1.45 w/o U^{235} .

RELATIVE WEIGHTS

1 100.0 , 2 99.10 , 3 102.7 , 4 100.9 ,
5 97.60 , A1 4 , A2 4 in.².

II. Spectrometer Calibration

There are two regions of interest; the first (N) about the 103-keV photopeak of Np^{239} , and the other (F) is the integral fission activity above 600 keV. Details are given in Fig. S14.2 and Table S14.2.

Fig. S14.2. Spectrometer Calibration Curves

Table S14.2

SPECTROMETER CALIBRATION

N Region; integral channels 32 to and including 45.

(a) Calibration U^{235} gamma peaks energy versus channel number
95 keV 33.5, 143 keV 61.5, 185 keV 84.

(b) High-voltage pot setting 740, baseline setting 120, gain 5.

F Region; integral channels to and including .

(a) Calibrate Cs^{137} standard 662 keV channel 19.7.

(b) Calibrate Co^{60} standard 1170 keV 55.8, 1330 keV 66.5.

(c) High-voltage pot setting 740, baseline setting 120, gain 2.

(d) $E_c = \text{Ch14}$ (600 keV).

III. Measured Foil Activity

The foil activities were measured before and 32 hr after the irradiation for both the N and F regions. However, the exact channel width of the N region was determined after the irradiation by counting the cadmium-covered depleted uranium shield foil because of its nearly pure Np^{239} activity. A plot of counts per channel versus channel number is shown in Fig. S14.3.

Fig. S14.3
Np²³⁹ Photopeak

In Table S14.3 the measured activities before and 32 hr after the irradiation are summarized.

Table S14.3

FOIL ACTIVITIES (cpm)

Foil	Region	Before	After	Net	Normalized
1	N	501	9161	8660	8660
	F	60	9464	9404	9404
2	N	493	7476	6983	7046
	F	60	9448	9388	9473
3	N	509	4444	3935	3833
	F	60	660	600	584
4	N	497	2982	2485	2463
	F	60	640	580	575
5	N	485	742	257	249
	F	58	554	496	481
A1	N	-	2168*	2088	2088
	F	-	6464*	5882	5882
A2	N	-	80*	-	-
	F	-	582*	-	-

*5 min counts

IV. Evaluation of the Conversion RatioCalculation of A^{Np}

From the data in Table S14.3

$$r = \frac{2168 - 80}{6464 - 582} = \frac{2088}{5882} = 0.355.$$

Therefore, since $A^{Np} = N' - rF$,

$$A_1^{Np} = 8660 - 0.355 (9404) = 8660 - 3338 = 5322$$

$$A_2^{Np} = 7046 - 0.355 (9473) = 7046 - 3362 = 3684$$

$$A_3^{Np} = 3833 - 0.355 (584) = 3833 - 207 = 3626$$

$$A_4^{Np} = 2463 - 0.355 (575) = 2463 - 204 = 2259$$

$$A_5^{Np} = 249 - 0.355 (481) = 249 - 171 = 78$$

Calculation of A^{FP}

The value of A^{FP} could well be taken as the normalized value of F , but for completeness the calculation of A^{FP} is made according to the equation

$$A^{FP} = b_f F,$$

where

$$b_f = 1 + \frac{F_E^8}{17F_E^5}$$

The two uranium metal shield foils surrounding foil 2 were of different U^{238} to U^{235} atom ratios: foil E, 1.468 a/o U^{235} , and 164.1 mg; foil D, 0.398 a/o U^{235} , and 163.9 mg. Thirty-two hours after the scram, their net fission activities were $F_E = 37.10$ and $F_D = 11.64$ cpm/mg.

Fig. S14.4

Determination of U^{238}
to U^{235} Fission Ratio

From Fig. S14.4 b_f is calculated. By graphical or mathematical extrapolation to zero atom percent, f_0^8 is found to be 2.17. Therefore, $F_E^8 = 2.14$, $F_E^5 = 34.96$, and

$$b_f = \frac{1}{1 + \frac{2.14}{17(34.96)}} = \frac{1}{1 + \frac{2.14}{594}} = \frac{1}{1 + 0.0036} = 0.996,$$

which, as expected, is essentially 1.000, and therefore A^{FP} is taken as the normalized F in Table S14.3.

Calculation of R

The value of R is given by

$$R = \frac{A_{\text{core}}^{\text{Np}}/A_{\text{thermal}}^{\text{Np}}}{A_{\text{core}}^{\text{FP}}/A_{\text{thermal}}^{\text{FP}}}.$$

A first determination is by use of the cadmium difference method to obtain the thermal activity.

From the outer foils 1 and 3,

$$R_{13} = \frac{A_1^{\text{Np}}/(A_1^{\text{Np}} - A_3^{\text{Np}})}{A_1^{\text{FP}}/(A_1^{\text{FP}} - A_3^{\text{FP}})} = \frac{5322/(5322 - 3626)}{9404/(9404 - 584)} = \frac{5322/1966}{9404/8820} = \frac{3.138}{1.066} = 2.94.$$

From the inner foils 2 and 4,

$$R_{24} = \frac{A_2^{\text{Np}}/(A_2^{\text{Np}} - A_4^{\text{Np}})}{A_2^{\text{FP}}/(A_2^{\text{FP}} - A_4^{\text{FP}})} = \frac{3684/(3684 - 2259)}{9473/(9473 - 575)} = \frac{3684/1425}{9473/8898} = \frac{2.586}{1.059} = 2.44.$$

Although the activity of foil 5 is too small to be statistically good, the corresponding values of R are

$$R_{15} = \frac{A_1^{\text{Np}}/A_5^{\text{Np}}}{A_1^{\text{FP}}/A_5^{\text{FP}}} = \frac{5322/78}{9404/481} = \frac{68.23}{19.55} = 3.49$$

and

$$R_{25} = \frac{A_2^{Np}/A_5^{Np}}{A_1^{FP}/A_5^{FP}} = \frac{3684/78}{9473/481} = \frac{47.23}{19.69} = 2.40.$$

The average value of R is determined by referring back to the details of foil location.

For lack of better theory, the average is simply defined as

$$R_{avg} = \frac{R_{outer} + R_{inner} + R_{outer}}{3} = \frac{2.94 + 2.44 + 2.94}{3} = 2.77.$$

Calculation of the Conversion Ratio

The conversion ratio γ is defined by

$$d(\gamma) = R \frac{\sigma_{cT}^8}{\sigma_f^5} \frac{N^8}{N^5} \frac{1}{1 + \alpha^5}$$

where $\sigma_{cT}^8 = 2.37$ b, $\sigma_f^5 = 485$ b, $N^8 = 4$, $N^5 = 1$, and $\alpha^5 = 0.189$. Therefore,

$$d(\gamma) = R \frac{2.37}{485} \frac{4}{1} \frac{1}{1.189} = 1.64 R\%.$$

Since R_{avg} has been found to be 2.77,

$d(\gamma) = 4.5\%.$

U²³⁵ DELAYED-NEUTRON PARAMETERS

by

D. A. Daavettila

ABSTRACT

Through use of a "rabbit" facility at the Argonaut reactor, the delayed-neutron parameters of U²³⁵ are measured. A small sample of U²³⁵ is irradiated to equilibrium in a neutron flux and subsequently rapidly removed to a shielded counting pig. By means of a neutron counter and a time analyzer, the neutron emission versus time is determined. From these data, at least four of the five so-called groups can be resolved and characterized in terms of their half-lives and relative abundances.

The pre-experiment briefing session will include group discussions of such parameters as β and β_{eff} , and of the effect of delayed neutrons on reactor control.

ACKNOWLEDGEMENT

H. Muller (Austria) and L. S. Zingoni (Brazil) assisted in the development of this experiment.

OUTLINE

- I. INTRODUCTION
- II. DELAYED NEUTRONS
- III. THEORY OF MEASUREMENT
- IV. APPARATUS
 - A. Rabbit Facility
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I. INTRODUCTION

The fission fragments created in the fission of a nucleus of high atomic number have neutron-to-proton ratios that are too large for the nuclides to be stable. One possible mode of radioactive decay for these nuclides is the ejection of a neutron. Those neutrons emitted in less than 10^{-12} sec of the fission event are known as prompt neutrons, and they make up more than 99 percent of the total number of neutrons emitted. The rest of the neutrons are known as delayed neutrons, because they will be emitted over a time interval measured in minutes. These delayed neutrons, although a small fraction of total emitted, are of very great importance in reactor control.

This experiment is a measurement of the thermal-fission delayed-neutron parameters of U^{235} . The parameters are determined from an analysis of the neutron counts versus time observed when a U^{235} sample is irradiated in the rabbit facility and ejected to a remote neutron-counting system. The pre-experiment briefing session will include a general discussion of delayed neutrons, the rabbit facility design, and the theory and procedure of the measurement.

II. DELAYED NEUTRONS¹

It can be assumed that the decay of the delayed-neutron activity with time can be represented by a linear superposition of exponential decay periods, and this general exponential decay problem was studied by Keepin at Los Alamos.² The problem was linearized by Taylor's series

¹Edited from a discussion prepared by L. S. Zingoni (Brazil) for Bibliography Entry 1.

²G. R. Keepin, Neutron Data for Reactor Kinetics, Nucleonics, p. 150, Aug 1962.

expansion and coded for iterative least-squares analysis with an IBM-704 computer. After several calculations, he observed that only six exponential periods were necessary and sufficient for an optimum least-squares fit. If five or fewer periods were used, satisfactory convergence was not obtained, and the errors were large. If seven periods were used, the calculated abundance of the seventh period was essentially zero. The calculated errors indicated that the two groups with the longest half-lives are best determined from long irradiations, whereas prompt-burst irradiations gave the most accurate results for the shorter groups.

The mechanism of delayed-neutron emission has been discussed by Bohr and Wheeler.³ They suggested that certain fission products may have β -decay energies in excess of the neutron-binding energies of their daughter nuclides. In some fraction of such decays the β transition may proceed to a state of the daughter nucleus whose excitation energy is greater than the neutron-binding energy; and from such a state, neutron emission is energetically possible. Mayer⁴ pointed out that the two chemically identified delayed-neutron emitters are nuclei with one neutron in excess of a closed shell, for which neutron the binding energy is expected to be anomalously low, and the probability of neutron emission correspondingly high.

Bohr and Wheeler's theory is illustrated in Fig. 15.1. The possibility of the daughter nucleus being left in an excited state, which then decays by gamma emission, is shown. This is an unlikely event, since the states of the final (closed neutron shell) nucleus should be widely spaced. The neutrons will range in energy from zero to $E_\beta - B$, where E_β is the maximum β -decay energy and B is the neutron binding energy, since nucleus recoils may be neglected.

Fig. 15.1

Schematic Representation of
Delayed-neutron Emission

³N. Bohr and J. A. Wheeler, Phys. Rev. 56, 426 (1939).

⁴M. G. Mayer, Phys. Rev. 74, 235 (1948).

Theoretical predictions of the contributions of the fission products to the various delayed-neutron groups have been given by Pappas⁵ and by Keepin,² and are extensions of the Bohr-Wheeler theory of delayed-neutron emission. They predict that all delayed fission neutrons of significant yield should be found in two Z versus A regions (totalling about 30 nuclides) just above the closed shells containing 50 and 82 neutrons. Also, the preponderance of delayed-neutron activity is expected to come from isotopes of bromine and iodine. These general predictions have been confirmed by the halogen diffusion experiments of Cowan and Orth,⁶ which indicate that 80% or more of the delayed-neutron precursors do indeed come from bromine and iodine fission products. In addition to the well-known 55-sec Br⁸² and 24-sec I¹³⁷ precursors, several precursors of shorter half-life have been identified recently by Stehney and Perlow.⁷ They used a gas-flow technique developed specifically for isolating short-lived fission-product activities.

A summary of the identified and possible delayed-neutron precursors is given in Table 15.1.

Table 15.1

DELAYED-NEUTRON PRECURSORS IN U²³⁵ FISSION

Group	Half-life (sec)	Precursor	Radiochemical and Cf ²⁵² Yield Data
1	54.5	Br ⁸⁷	0.00058
2	24.4	I ¹³⁷	0.00220
	16.3	Br ⁸⁸	0.00157
3	6.3	I ¹³⁸	0.00103
	4.4	Br ⁽⁸⁹⁾	0.00298
	~6	Rb ^(93,94)	~0.00001
4	2.0	I ¹²⁹	0.00084
	(1.6-2.4)	(Cs, Sb or Te)	0.00206
	1.6	Br ⁽⁹⁰⁻⁹³⁾	0.00236
	~1.5	Kr ⁹³	~0.00007
5	~0.5	(I ¹⁴⁰ + Kr?)	-
6	~0.2	(Br, Rb, As?)	-

⁵A. C. Pappas and G. Rudstan, Nuclear Phys. 21, 353 (1960).

⁶G. A. Cowan and C. J. Orth, Proc. Second UN Intl. Conf. on the Peaceful Uses of Atomic Energy, Geneva, Switzerland, 7, 328 (1958).

⁷G. J. Perlow and A. F. Stehney, Phys. Rev. 107, (1957).

III. THEORY OF MEASUREMENT

A small sample⁸ of the fissile material is irradiated in a neutron flux⁹ to saturation of the delayed-neutron groups (which takes ~ 6 min). Then the sample is suddenly ejected to a remote, well-shielded neutron-counting system, and the delayed-neutron emission as a function of time is determined.

The neutron counts versus time are analyzed to obtain the delayed-neutron groups (periods and relative abundances) as follows: The neutron counts less background are plotted as a function of time on semilog paper. A typical curve is shown in Fig. 15.2. The period of the longest group is apparent from the straight-line portion of the graph.

Fig. 15.2. Curve of Neutron Counts versus Time

The straight line is extrapolated back to zero time. The counts A_0 thereby obtained is then subtracted from the measured curve. The resulting difference curve is plotted again on semilog paper to pick off the next longest period by again observing the straight-line portion of the curve. This procedure continues until no difference curve exists. The intercepts of the straight-line extrapolations with the zero-time ordinate yield zero-time count rates (A_0) which are directly proportional to the relative abundances.

The rabbit facility, which was designed and built for quickly inserting and removing the sample from the neutron flux, and the counting apparatus are described in the next section.

⁸The sample must be small enough to avoid significant neutron multiplication in the counting pig.

⁹The measured delayed-neutron parameters are dependent on the neutron energy spectrum of the irradiation flux.

IV. APPARATUS

A. Rabbit Facility (see Figs. 15.3 and 15.6): The rabbit transport pipe is a 1-in.-ID aluminum tube which is made in two parts. The first, about 82 in. long, is an integral part of an aluminum container which just replaces the normal J-8 stringer (see Fig. 15.7 below). The aluminum container is filled with wax for neutron moderation, both for purposes of providing a thermal flux for the irradiation and for fast neutron shielding. Installation or removal of the rabbit facility was easy, because the aluminum container is readily interchangeable for the J-8 stringer. The second part, 96 in. long, of the rabbit pipe is readily joined to the first part and leads to the counting pig which contains a 40-cm-pressure BF_3 counter. At the outer end of this length of pipe is the rabbit loading access port (see Fig. 15.8 below).

For the rabbit transport in the pipe, a vacuum system consisting of a vacuum pump and a reservoir with solenoid valves is used (see Fig. 15.8). The manipulations of the solenoid valve necessary to evacuate the pipe and move the rabbit are given in Table 15.2. For rabbit delivery, a vacuum of 25 lb is produced ahead of the rabbit, and then normal air is admitted behind the rabbit. The pressure differential pushes the rabbit from the irradiation to the counting position or vice versa.

From the data analysis, it is necessary to fix a zero-time, that is, an effective irradiation time and point in order that a number of individual runs can be superimposed and also to have the point at which to determine relative abundances. To fix the zero time, a trigger switch (see Fig. 15.4) for the multichannel analyzer was built and installed 8 in. from the rabbit irradiation position. The 8-in. position was chosen because at this point the irradiation flux had dropped to one-half the value existing at the irradiation position. The switch closes the trigger circuit when the rabbit, on its way out, moves the hand which is caught by a magnet. The hand-magnet connection prevents the second disc of the rabbit from retriggering the analyzer. The Teflon rabbit design is shown in Fig. 15.5.

To stop the rabbit in the proper position for counting, which is in the center of the counting pig, a wooden stick with a small piece of plastic foam as a bumper is placed in the rabbit pipe through the access port.

Fig. 15.3. Rabbit Facility

Fig. 15.4. Trigger Switch

Fig. 15.5. Rabbit

Fig. 15.6. Rabbit Facility

Fig. 15.7. Rabbit Stringer

Fig. 15.8. Vacuum System

Table 15.2

RABBIT VALVE OPERATIONS

VALVE	TO EVACUATE RABBIT PIPE	RABBIT MOVING IN	RABBIT MOVING OUT
A	CLOSE	CLOSE	OPEN
B1	CLOSE	OPEN	CLOSE
B2	CLOSE	CLOSE	CLOSE
C1	OPEN	OPEN	CLOSE
C3	CLOSE	CLOSE	OPEN

B. Counting System: The counting system is arranged to yield a time display of the neutron counts relative to the trigger zero time.¹⁰ To obtain the time display, a TMC 256-channel analyzer was modified such that a choice of a wide variety of time intervals per channel is available.¹¹ The channel time length depends on the channel length selector switch position on the MCA and the frequency of the low-frequency generator. The counting system is shown schematically in Fig. 15.9 and in a photograph in Fig. 15.10.

Consider Fig. 15.9. The low-frequency generator is used to modify the time interval per channel of the TMC 256-channel analyzer. The sine-square wave generator serves as the trigger signal generator for the TMC. The trigger switch closes or opens the trigger circuit. The multichannel analyzer with the data recorder separates out the neutron counts as a function of time. The data-output printer gives the counts as a function of channel number, where the channel number is a function of time, according to the time interval per channel settings.

The MPC unit serves as the high-voltage source to the BF_3 counter, and low-gain amplifier, and as a pulse inverter. The MPC switch is a convenient method of suddenly removing or restoring the signal input to the multichannel analyzer¹² for the time-versus-channel number calibration.

¹⁰Trigger zero time is set by the rabbit activating the trigger switch as it moves from the irradiation to the counting position.

¹¹From 10 μs to 2000 sec.

¹²Switch moved from amplifier to GM position interrupted the signal.

Fig. 15.9. Block Diagram of Counting System

Fig. 15.10. Counting Apparatus

V. PROCEDURE

The procedure given is that followed for the trial measurement and includes, where instructive for general information, the actual values.

1. Choice of Sample and Reactor Power

The sample is a _____ gram piece of _____.

The reactor power is set at _____ Watts.

2. Settings of Counting System

- a. Low-frequency generator: frequency - _____;
range - _____; amplitude - _____; wave shape - square.

The frequency of the low-frequency generator should be adjusted such that the approximate channel length is 2 sec for the MCA switch position of 1280 μ s.

- b. Trigger-signal generator: frequency - _____;
range - _____; amplitude - maximum; wave shape - square.

- c. Multichannel analyzer (MCA): background ratio - 2;
discriminator - 3; delay-time ratio - 2.

- d. BF_3 Counting System (MPC): high voltage - _____;
amplifier gain - _____; see Step 3 for discussion.

3. BF_3 Counter Plateau

With the high voltage set at 2300 V, the amplifier gain is turned down until the output neutron pulses are about 5 V or less, which is the proper pulse height input for MCA. Test with a pure gamma source to determine whether the gamma background is negligible.

A Ra-Be neutron source is used to find the counter plateau, using the MCA as the counts-per-unit-time detector. For this purpose, the range of the low-frequency generator is set at 10 and the channel length selector switch adjusted to yield a nominal channel length of 20 sec. The analyzer is then manually triggered and counting allowed to proceed for about 20 channels. The average counts per channel is determined as a function of the high voltage to the BF_3 counter. Enter the data obtained in Table 15.3.

Table 15.3

BF₃ COUNTER PLATEAU DATA

High Voltage	Counts per Channel

From the data above, the high voltage to use is chosen to be _____ V.

4. Time versus Channel-number Calibration

The purpose of this calibration is to obtain for each channel length a curve of the elapsed time since trigger versus the channel number. This curve is used to convert the raw data output of counts versus channel number to counts versus time, and to determine the average elapsed time per channel.

The calibration is accomplished by placing a Ra-Be neutron source next to the counting pig and then manually triggering the MCA while simultaneously starting a stop watch. After a known elapsed time interval, the pulse input to the MCA is blocked by switching the input selector switch on the MPC from the amplifier to the GM position. After another known time interval, the pulse input to the MCA is returned. This switching back and forth is continued until the 256-channel run is completed. The observed data output is sketched below for a 30-sec switching cycle. Very often the input to MCA will be returned or removed midway in a particular channel. The fraction of the channel is determined by dividing its accumulated counts by the average counts per channel. Three runs are made for each of the channel length settings, and the data are entered in Table 15.4 of channel number versus time.

The curves of time versus channel number are to be plotted from the above data. Then, given any channel number, the time elapsed from zero can be read directly from these curves. To find the average time interval per channel, divide the time it takes to count 100 channels by 100.

5. Data Accumulation

The sample is irradiated for 6 min (to saturation) and then ejected to the counting pig. When the 256 channels of data are obtained, the sample is returned for irradiation. This is repeated five times for each of the three channel lengths. For channel lengths of $1/2$ sec or less, only a readout of the first 50 channels is necessary, because the time region beyond 50 is covered best with a 2-sec-channel length.

6. Background Measurements

For each of the three channel-length settings, the background is measured as the sum of five individual runs so that it can be used directly in the data analysis.

7. Data Analysis

The data corresponding to the longest time per channel is plotted on semilog paper in terms of counts per channel versus channel number. In the region of poor counting statistics, after about 100 sec, as many as seven channels can be summed and the average counts plotted in the mean channel. This simplifies the drawing of the decay curve because there are fewer points to consider. As statistics improve during the analysis involving lower channel numbers, instead of averaging groups of seven channels, groups of five and later three channels can be averaged. For the low channel numbers plot the data channel by channel. Analyze the curve as shown in Fig. 15.2.

The channel region of interest for each of the shorter channel time lengths is from zero to the time at which the analysis of the next longest group indicates an inadequate number of datum points. There is a problem in how to translate the known delayed-neutron decay data to the analysis using the shorter channel length. A solution is given by means of the following example.

Suppose Groups 1, 2, and 3 have been determined from the analysis of the 2-sec per channel data, and Group 4 is being sought in the $1/2$ -sec per channel data. The $1/2$ -sec per channel data are plotted from 0 to channel 40. From the time versus channel number calibration curve for the $1/2$ -sec channel length, the elapsed time to channel 40 is determined. Now, referring to the 2-sec channel length calibration curve, the corresponding channel for this time, t_{40} , is determined. From the activity versus time curve for the 2-sec channel length, the counts at time zero and time t_{40} are read for Groups 1, 2, and 3. These counts are divided by four and plotted at channel zero and channel 40 on the $1/2$ -sec activity plot. A straight line between these points is then drawn.

The half-life is first determined in numbers of channels and is then converted to a time interval by the time versus channel number calibration curves, or by the following equation.

$$\text{Time interval} = \text{channel interval} \times \frac{\text{average elapsed time}}{\text{channel}}$$

VI. RESULTS

In order to keep track of the progress of the experiment, complete Tables 15.5, 15.6, and 15.7.

Table 15.5

DATA FOR IRRADIATION RUN

Run	Irradiation		Counting Channel 0 to	Nominal Channel Length
	Time In	Time Out		

Table 15.6

SUMMARY OF THE IRRADIATION RUNS

Run Number	Number of Runs Being Summed	Average Elapsed Time per Channel	Length of Irradiation	Remarks
A				
B				
C				
D				

Plot the important parts of the counts versus channel number data, and on these curves carry out the analysis to determine the half-lives and relative abundances. Include these plots as part of your report.

Table 15.7

SUMMARY OF MEASURED RESULTS

Group	$T_{1/2}$ (sec) Measured	β_i^* Measured	$T_{1/2}$ (sec) Literature	β_i Literature
1			55.72 ± 1.28	0.052 ± 0.005
2			22.72 ± 0.71	0.346 ± 0.018
3			6.22 ± 0.23	0.310 ± 0.036
4			2.30 ± 0.09	0.624 ± 0.026
5			0.61 ± 0.083	0.182 ± 0.015
6			0.230 ± 0.025	0.066 ± 0.008

* β_i is the absolute yield in percent.

VII. ANALYSIS AND CONCLUSIONS

This is left as an exercise for the students.

VIII. BIBLIOGRAPHY

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IX. APPENDIXDiscussion Questions

1. Compare the delayed versus prompt neutrons in regard to origin, birth energy, and time of emission.
2. Define in words the parameters λ and C as found in the reactor kinetics equation.
3. Define the parameter β . How does it differ for the various fissile isotopes?
4. Explain how β differs from β_{eff} , and which of the two is important in reactor kinetics calculations?
5. If the thermal lifetime in a U^{235} reactor is 2×10^{-4} sec, what is the average time between generations when delayed neutrons are considered?
6. Compare the power rise versus time for the two cases of Question No. 5 for a $k_{\text{eff}} = 1.002$.
7. Explain why delayed neutrons are important in reactor control. Explain the term "prompt critical."
8. Why, in the measurements of β , must the sample be small?

STUDENT RESULTS

I. Reactor Run

The delayed-neutron parameters for U^{235} thermal fissions were measured with a 1.2-g sample at 93 percent enriched uranium metal foil. The reactor loading was a 6-fuel box, north-quadrant core, containing 1.93 kg U^{235} . The reactor power level was 50 W, and the rabbit stringer was in the J-8 position.

II. Calibration of Counting System

1. Settings of Counting System

- a. Low-frequency generator: frequency - 1100 cycles/sec; amplitude - 85%; wave shape - square.
- b. Trigger-signal generator: frequency - 360,000 cycle/sec; amplitude - maximum; wave shape - square.
- c. Multichannel analyzer (MCA); background ratio - 2; delay time ratio - 2; discriminator - 3.
- d. BF_3 Counting System (MPC); high voltage - 2350 V; amplifier gain - $\times 10$; see step 2 for discussion.

For these settings, a stop watch check of the total time to count 256 channels showed that the approximate channel time lengths were

Setting of Channel Length Selector Switch, μs	Channel Length, sec
2560	2
640	$1/2$
160	$1/8$

2. BF_3 Counter Plateau

With the high voltage set at 2300 V, the amplifier gain was turned down until the output neutron pulses were about 5 V or less, which was the proper pulse-height input for MCA. Tests with a pure gamma source showed that the gamma background was negligible.

A Ra-Be neutron source was used to find the counter plateau using the MCA as the (counts per unit time) detector. For this purpose, the range of frequency generator I was set at 10 and the channel length selector at 2560 to yield a nominal channel length of 20 sec. The analyzer was then manually triggered and counting proceeded for about 10 channels. The average counts per channel were determined as a function of the high voltage to the BF₃ counter. The data obtained is given below.

High Voltage, V	Counts per Channel
2200	1030
2300	1343
2350	1388
2400	1425
2450	1536

From the data above, the high voltage to use was chosen to be 2350 V.

3. Calibration for Time versus Channel Number

The calibration was accomplished by placing a Ra-Be neutron source next to the counting pig and then manually triggering the MCA while simultaneously starting a stop watch. After a known elapsed time interval, the pulse input to the MCA was blocked by switching the input selector switch on the MPC from the amplifier to the GM position. After another known time interval the pulse input to the MCA was returned.

Often the input to MCA was returned or removed midway in a particular channel, and the fraction of the channel that counted was determined by dividing its counts by the average counts per channel. Three runs were made for each of the three channel length settings. The data are given below in terms of channel number versus time measured in seconds.

Data for the Time versus Channel Calibration

2560- μ s channel selector (~ 2 sec/channel)

Run No.	Time, sec									
	30	60	90	120	150	180	210	240	270	300
	Channel Number									
1	14.3	28.9	43.3	57.8	72.5	86.4	101.1	115.6	130.5	144.4
2	13.8	28.9	43.5	57.8	72.7	86.9	101.2	115.5	130.0	144.5
3	14.4	28.9	43.4	47.8	72.4	86.6	101.1	X	129.9	144.3

Data for the Time versus Channel Calibration

640- μ s channel selector ($\sim 1/2$ sec/channel)

Run No.	Time, sec					
	20	40	60	80	100	120
	Channel Number					
1	38.0	76.8	113.0	151.8	190.0	228.3
2	38.0	76.5	114.3	151.8	190.0	228.0
3	38.0	75.6	114.0	151.6	190.0	228.0

160- μ s channel selector ($\sim 1/8$ sec/channel)

Run No.	Time, sec			Total run time, sec
	15	20	30	
	Channel Number			
1	-	143.0	217.0	35.5
2	106.0	-	214.5	35.8
3	102.0	-	210.8	36.4

Curves at time versus channel number were plotted from the above data. Then, given any channel number, the elapsed time from zero can be read directly from these curves. Figure S15.1 is a typical plot.

Fig. S15.1. Calibration Curve for Time versus Channel Number

5. Data Accumulation

The data runs were then started. The sample was irradiated for 6 min (to saturation) and then ejected to the counting pig. When the 256 channels of data were obtained, the sample was returned for irradiation. This was repeated 5 times for each of the 3 channel lengths. The raw data is presented in Tables S15.2, S15.3, and S15.4 in tabular form, and part of the raw data is plotted in Figs. S15.2 and S15.3.

6. Background Measurements

For each of the three channel length settings, the background was measured as the sum of five individual runs so that it could be used directly in the data analysis.

7. Data Analysis

Starting with the 2-sec per channel data, it was plotted on semilog paper in terms of counts per channel versus channel number. In the region of poor counting statistics, after about 100 sec, as many as 7 channels were summed and the average counts plotted in the mean channel. This simplified the drawing of the decay curve, because there were fewer points to consider. As statistics improved during the analysis involving lower channel numbers, groups of 5, and later 3, channels were averaged instead of 7.

In summary, the three longest groups can best be determined from a study of the 2-sec per channel data. The next group is determined from the $1/2$ -sec per channel data in the region of channels 0 to 35, and the fifth group from the first 30 channels of the $1/8$ -sec per channel data. In each case the half-life was first determined in numbers of channels, and then converted to a time interval by referring to the respective time versus channel number calibration curve, such as Fig. S15.1.

III. Results

The final results of the measurement are given in Table S15.1 together with the literature values. Figures S15.2 and S15.3 show sample curves to show the experimentally measured decay curve and the extraction of the first two groups. In practice this curve plotting would be done on larger paper. Tables S15.2, S15.3, and S15.4 present the raw measured data of neutron counts versus channel number.

Fig. S15.2. Determination of Group 1

Fig. S15.3. Determination of Group 2

Table S15.1

SUMMARY OF THE MEASURED RESULTS

Group	$T_{1/2}$ (sec) Measured	B_i Measured	$T_{1/2}$ (sec) Literature	B_i Literature
1	56.5	0.052	55.72 ± 1.28	0.052 ± 0.005
2	22.5	0.29	22.72 ± 0.71	0.346 ± 0.018
3	5.9	0.31	6.22 ± 0.23	0.310 ± 0.036
4	2.6	0.46	2.30 ± 0.09	0.624 ± 0.026
5	0.55	0.28	0.61 ± 0.083	0.182 ± 0.015
6			0.230 ± 0.025	0.056 ± 0.008

Table S15.2

2 SECOND/CHANNEL

Channel No.	Counts	Channel No.	Counts	Channel No.	Counts
001	31962	043	02184	085	00509
002	10867	044	02187	086	00482
003	57522	045	-	087	00403
004	24641	046	02021	088	00449
005	21377	047	01945	089	00468
006	19501	048	01832	090	00453
007	17530	049	01831	091	00394
008	16006	050	01716	092	00416
009	14711	051	01600	093	00404
010	13703	052	01579	094	00395
011	12625	053	01483	095	00366
012	11993	054	01480	096	00413
013	11140	055	01397	097	00358
014	10458	056	01344	098	00362
015	10026	057	01262	099	00354
016	09317	058	01255	100	00322
017	08728	059	01193	101	00317
018	08289	060	01129	102	00349
019	07773	061	01122	103	00312
020	07283	062	01108	104	00315
021	06954	063	01055	105	00280
022	06672	064	01023	106	00307
023	06165	065	00926	107	00283
024	05864	066	00934	108	00277
025	05612	067	00902	109	00250
026	05301	068	00866	110	00270
027	05039	069	00847	111	00242
028	04721	070	00824	112	00249
029	04451	071	00793	113	00251
030	04254	072	00712	114	00271
031	04007	073	00750	115	00236
032	03909	074	00733	116	00227
033	03674	075	00690	117	00235
034	03488	076	00666	118	00226
035	03408	077	00627	119	00192
036	03190	078	00636	120	00203
037	02933	079	00605	121	00213
038	02931	080	00543	122	00187
039	02783	081	00540	123	00186
040	02607	082	00564	124	00190
041	02476	083	00515	125	00194
042	02397	084	00530	126	00177

Table S15.2 (Cont'd.)

2 SECOND/CHANNEL

Channel No.	Counts	Channel No.	Counts	Channel No.	Counts
127	00178	170	00084	214	00051
128	00179	171	00099	215	00067
129	00166	172	00090	216	00063
130	00165	173	00073	217	00044
131	00164	174	00072	218	00065
132	00169	175	00075	219	00059
133	00153	176	00083	220	00066
134	00181	177	00058	221	00054
135	00157	178	00080	222	00059
136	00145	180	00090	223	00045
137	00149	181	00084	224	00045
138	00155	182	00079	225	00066
139	00142	183	00066	226	00056
140	00118	184	00072	227	00055
141	00148	185	00081	228	00054
142	00132	186	00064	229	00059
143	00125	187	00088	230	00060
144	00128	188	00064	231	00062
145	00131	189	00071	232	00053
146	00130	190	00075	233	00046
147	00116	191	00060	234	00043
148	00126	192	00075	235	00043
149	00107	193	00071	236	00049
150	00113	194	00071	237	00049
151	00130	195	00073	238	00053
152	00115	196	00041	239	00046
153	00099	197	00065	240	00039
154	00115	198	00062	241	00052
155	00102	199	00070	242	00050
156	00105	200	00071	243	00059
157	00111	201	00071	244	00055
158	00110	202	00048	245	00054
159	00103	203	00069	246	00049
160	00093	204	00063	247	00055
161	00095	205	00063	248	00047
162	00089	206	00071	249	00045
163	00095	207	00058	250	00052
164	00110	208	00044	251	00034
165	00101	209	00063	252	00059
166	00108	210	00054	253	00048
167	00086	211	00064	254	00055
168	00084	212	00062	255	00042
169	00091	213	00056	256	00046

Table S15.3

1/2 SECOND/CHANNEL

Channel No.	Counts	Channel No.	Counts	Channel No.	Counts
001	00284	017	06660	034	04209
002	05414	018	06477	036	03992
003	07355	019	06111	037	03902
004	12663	020	05923	038	03808
005	11938	021	05745	039	03821
006	10876	022	05658	040	03694
007	10352	023	05446	041	03613
008	09928	024	05288	042	03530
009	09315	025	05007	043	03473
010	08794	026	04975	044	03484
011	08508	027	04923	045	03343
012	07997	028	04704	046	03195
013	07744	029	04603	047	03206
014	07294	030	04525	048	03120
015	07002	031	04409	049	01851
016	06807	032	04281	050	02988

Table S15.4

1/8 SECOND/CHANNEL

Channel No.	Counts	Channel No.	Counts	Channel No.	Counts
001	00003	018	03204	034	02514
002	00523	019	03233	035	02351
003	00861	020	03090	036	02301
004	01284	021	02965	037	02438
005	01799	022	02997	038	02317
007	03664	023	02954	039	01821
008	03892	024	02918	040	02341
009	03993	025	02754	041	02216
010	03812	026	02862	042	02166
011	03751	027	02768	043	02181
012	03700	028	02646	044	02111
013	03615	029	02619	045	02122
014	03523	030	02630	046	02095
015	03468	031	02583	047	02090
016	03469	032	02551	048	02016
017	03271	033	02410		

β/ℓ BY PILE NOISE ANALYSIS, ARGONAUT REACTOR

by

L. L. Lawyer and S. G. Kaufman

ABSTRACT

This experiment is a measurement of β/ℓ for the Argonaut reactor. The measurements comprise the harmonic analysis of the noise component of a signal derived from a high-efficiency ion chamber exposed to the reactor flux. The spectrum of the noise can be used to yield β/ℓ through a simple application of the autocorrelation theorem.

ACKNOWLEDGMENT

C. E. Cohn prepared and performed the original pile noise experiment on the Argonaut reactor.

OUTLINE

- I. INTRODUCTION
- II. THEORY
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by Pile Noise Analysis by C. E. Cohn

I. INTRODUCTION

The quantities β (the fractional contribution of delayed neutrons to the total) and ℓ (the neutron lifetime) are of prime importance in reactor control because of their effect on the dynamic properties of a reactor. The measurements of either β or ℓ present considerable difficulty; however, the ratio β/ℓ may be determined through a relatively simple measurement. This quantity has been measured by three general methods: (1) study of the frequency dependence of the reactor transfer function; (2) study of a reactor's time response to a pulse of neutrons; (3) study of correlation between individual fission events.

The random processes and resulting power fluctuations, herein termed "pile noise," which are associated with the propagation of neutron chains in multiplying media were first discussed by Feynman, DeHoffmann, and Serber at Los Alamos and by Courant and Wallace at Chalk River.⁽⁵⁾⁽⁶⁾ Since that time, a number of experimenters have investigated pile noise. Recently, simple and successful measurements have been made by Cohn;⁽⁷⁾ our experiment will follow his work.

II. THEORY

A. In a critical reactor, the neutron population is exactly sustained by the contributions of fission neutrons, which appear promptly upon fission, plus the delayed neutrons which are emitted by the fission fragments and appear second or minutes after the fission event. This condition of neutron balance is called "delayed critical," denoting the fact that prompt and delayed neutrons are needed to balance neutron losses from absorption and leakage and thus to sustain the chain reaction. A "prompt critical" condition is one in which the neutron economy is balanced by the contribution of the prompt neutrons alone, without the need of the delayed neutrons.

Figure 16.1 shows a typical fluctuation of the neutron population after an initial neutron has caused a fission event and has thus, instantaneously increased the neutron population to, say, three neutrons. Note that the time scale is very short so that delayed neutrons from this fission event have not had time to make their appearance. The reactor, since it is delayed critical so that the delayed-neutron contribution is necessary to sustain the neutron population, will show a behavior which is characteristic of a subcritical pile, until the delayed neutrons appear. Note also that the origin of the figure is shifted in such a way that the average neutron population is omitted. The figure represents one, typical, time development of the fluctuation of the neutron population, triggered by an initial fission event and dying out for need of delayed neutrons.

Fig. 16.1

Neutron Population as a Function of Time

A single neutron chain cannot be described mathematically; however, the average properties of a very large number of such chains can be expressed in terms of the statistical concepts of k_{eff} , ℓ , and n , where k_{eff} is the ratio of the number of neutrons present in any generation to the number present in the preceding generation, including delayed neutrons, and n the neutron density.

The description of the statistical properties of neutron chains such as shown in Fig. 16.1 is particularly simple, since it need not take into account the delayed-neutron contribution, and may thus be written⁽⁸⁾ as

$$\dot{n} = (k_{\text{ex}}/\ell) n. \quad (1)$$

In a delayed-critical pile, the prompt neutrons fall short of sustaining the neutron population by β , the fractional contribution from the delayed neutrons. Thus, in this case,

$$k_{\text{ex}} = -\beta; \quad n(t) = n_0 e^{-\frac{\beta}{\ell}t} \quad (2)$$

It is thus apparent that the average of many neutron chains of the type shown in Fig. 16.1 would define the exponential decay shown as the smooth, dashed curve.

B. We may express the difference between the signal output $V(t)$ from a detector exposed to pile flux at some time t and the average output $\bar{V}$ over a time interval T as a Fourier expansion:

$$\phi(t) \equiv V(t) - \bar{V} = \sum_{n=1}^{\infty} a_n \cos \omega_n t + b_n \sin \omega_n t; \quad 0 \leq t \leq T. \quad (3)$$

The coefficients a_n and b_n are accessible to measurement through harmonic analysis, using tuned circuits of known frequency and response, of the time-dependent component $\phi(t)$ of the signal derived from the ion chamber inside the reactor.

We now find it convenient to define the autocorrelation function of $\phi(t)$ as

$$\psi(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \phi(t) \phi(t+\tau) dt. \quad (4)$$

Now $\psi(\tau)$ depends on the product of the value of ϕ at time t by the value of ϕ at a time τ seconds earlier. If there is absolutely no correlation between the value of the function at t and at $t + \tau$ (e.g., in the case of random noise), then $\psi(\tau)$ is zero, unless τ is zero; i.e., $\psi(\tau) \sim \delta(\tau)$.

If $\phi(t)$ is a sinusoidal function, $\psi(\tau)$ represents a measure of the power dissipation in an AC circuit where τ is the phase shift between current and voltage. Thus, if τ is zero, the power dissipation is maximized as in a purely resistive circuit where current and voltage are in phase. On the other hand, if τ corresponds to a 90° phase shift, the power dissipation is minimized as in a purely inductive or capacitive circuit and $\psi(\tau)$ vanishes.

It has been shown that

$$(a_n^2 + b_n^2) \sim \int_0^{\infty} \psi(\tau) \cos 2\pi f_n(\tau) d\tau, \quad (5)$$

where a_n and b_n are amplitudes of the frequencies f_n found in the harmonic analysis of a function ϕ and where $\psi(\tau)$ is the autocorrelation of $\phi(t)$. In the specific situation which we discuss here, ϕ is the pile noise, containing a random component and also a coherent component made up of fission chains of the type shown in Fig. 16.1 depending on the time through $\phi(t)_{\text{coherent}} \sim e^{-(\beta t/\ell)}$.

Upon substituting this ϕ into the relations (4) and (5), we obtain a frequency dependence of the power spectrum of the form

$$(a_n^2 + b_n^2) \sim A + \frac{B}{(2\pi f)^2 + (\beta/\ell)^2} \quad (6)$$

It is this relation which we propose to employ to find the exponent β/ℓ once the $a_n + b_n$ have been measured for many values of n .

III. APPARATUS

The instrumentation consists of five units: a detector, a pre-amplifier, an intermediate amplifier, an analyzer, and a recorder. Auxiliary equipment includes a well-charged storage battery, an oscilloscope, and a reactor. Some of the following optional equipment will be necessary: scissors, an analytical balance, and/or a planimeter.

The detector is an ion chamber which is heavily loaded with boron enriched in B^{10} in order to have a large capture cross section for thermal neutrons. This detector feeds the pre-amplifier and receives its high voltage from an external shielded storage battery.

The equipment is shown in Fig. 16.2. A schematic diagram of the pre-amplifier is shown in Fig. 16.3. The pre-amplifier uses a 12J7GT operated as a cathode follower with reduced plate and filament voltages to reduce grid currents. The DC component of the voltage produced across the grid resistor by the ion chamber current can be bucked out by a negative bias controlled by the "pre-amp bias" control on the front panel of the intermediate amplifier, to maintain the 12J7GT at its correct operating point. This should be about 90-100 μA .

The intermediate amplifier (see Fig. 16.4) has on its front panel, in addition to the bias control, a meter for reading the pre-amp cathode current. This amplifier consists of a 5693 and a 6J5GT, which amplify the signals from the pre-amp to the level required by the analyzer. The signal on the cathode of the 6J5 is brought out to a connector on the front panel labeled "scope A" to allow for examination of the signal with an oscilloscope.

The pile-noise analyzer (see Fig. 16.5) has, in order to allow for a wide range of pile fluxes, an attenuator with six steps which change the gain by factors of five. The attenuator feeds a 6SJ7 voltage amplifier, which in turn feeds a bandpass filter. This is a simple parallel resonant circuit consisting of a high-Q audio choke across which either of 18 condensers may be connected to give a range of 18 resonant frequencies.

Fig. 16.2. Experimental Apparatus

Fig. 16.3. Pre-amplifier

Fig. 16.4. Intermediate Amplifier

Fig. 16.5. Pile-noise Analyzer

A resistor is inserted in series to prevent loading of the tuned circuit by the preceding stage, and is made approximately 10 times the resonant impedance of the tuned circuit for each setting.

The output of the filter feeds a linear feedback-type vacuum tube voltmeter, consisting of the 6SF5 and one-half of the 6SL7. The meter on the front panel indicates the output of this circuit. A shunt in the meter circuit provides a signal for a 10-mV recorder. This signal is brought out to an AN connector on the back of the chassis. The "TIME CONSTANT" switch on the front panel places a large condenser across the meter circuit to damp the fluctuations in the output. The other section of the 6SL7 is a cathode follower which feeds the SCOPE "B" terminal on the front panel to allow observation of the filtered noise components with an oscilloscope.

The setup should be connected to a good Earth ground at one and only one point to prevent hum pickup. Grounding at more than one point should be avoided, because it can set up ground loops which can introduce a considerable amount of hum. When the setup is complete, it should be turned on and an oscilloscope connected to the SCOPE "A" terminal to check for hum pickup. This can be done by synchronizing the oscilloscope sweep with the line frequency and looking for any signals which remain stationary. The total sweep time for this test should be around 50 or 100 ms. If everything is alright, the predominant signal that should appear is the noise from the input resistor.

IV. PROCEDURE

The experiment is performed in two stages. The first stage is the calibration of the equipment with reactor "white" noise; the second stage is the reactor pile-noise run. The operating procedure for the two stages is identical, the only difference being in the exposure of the chamber. The particular placement of the chamber for each stage will be prescribed by the instructor.

Set the FREQUENCY SELECTOR switch to the first position. Set the GAIN switch to the highest position that will allow the voltmeter to remain on scale at all times with the TIME CONSTANT in. Note that this circuit also requires some waiting time to come to equilibrium after each adjustment, in this case about 30 sec. When the proper settings are obtained, the recorder may be started and a trace taken. The trace should be taken for about 3 min, this being sufficient to average out most of the random fluctuations in noise component amplitude. Following this, all of the other FREQUENCY SELECTOR settings may be covered in a similar manner.

Data Sheets for β/l by Reactor Noise Analysis

Pile Noise Run

Channel	Frequency	Gain	Area	Length	$\frac{I_0 \text{ Area}}{\text{Gain} \times \text{Length}}$

White Noise Run

Channel	Frequency	Gain	Area	Length	$\frac{I_0 \text{ Area}}{\text{Gain} \times \text{Length}}$	$\frac{I}{I_0}$	$\left(\frac{I}{I_0}\right)^2$

When the calibration is complete, the reactor pile-noise run may be done. When you are through with the equipment, be sure to turn off all units to avoid premature battery exhaustion. The on-off switch for the intermediate amplifier and pre-amplifier is on the intermediate amplifier front panel; that for the analyzer is on the back of the analyzer chassis.

After completion of the runs, the recorder traces for each frequency setting must be averaged. This may be done by the use of a planimeter, the cut and weight system, or as noted in the accompanying paper "Determination of Reactor Kinetic Parameters by Pile Noise Analysis," by C. E. Cohn. Division of the averaged reading for the reactor run by that for the calibration run at each FREQUENCY SELECTOR setting (allowing for differences in GAIN setting for the two runs) and squaring the ratio will yield the mean-square noise amplitude in arbitrary units for each of these frequencies. By use of the frequencies corresponding to each FREQUENCY SELECTOR setting of the analyzer, the reactor noise spectrum can be directly plotted.

V. RESULTS

Determination of β/ℓ : As noted earlier and in the accompanying paper by Dr. Cohn, the square modulus of the transfer function is proportional to $1/(\alpha^2 + \omega^2)$, where $\alpha = \beta/\ell$. Therefore, our observed noise spectrum (normalized mean square amplitude versus ω) can be represented by the expression $A + (B/\alpha^2 + \omega^2)$, where A is the white noise component and $B/(\alpha^2 + \omega^2)$ represents the pile-noise component.

VI. ANALYSIS OF RESULTS AND CONCLUSIONS

The data may be fitted and α determined as outlined by Dr. Cohn in Appendix B. However, in the interest of time, an approximate value for α can be obtained as indicated by the following analysis.

The expression $A + (B/\alpha^2 + \omega^2)$ at high frequency should asymptotically approach the value A and therefore, the white noise component can be determined. As we approach zero frequency, it will approach $A + (B/\alpha^2)$ and by subtracting the previously determined A we can determine B/α^2 in terms of the normalized mean-square amplitude. Finally, when $\alpha = \omega$, our expression takes the form $A + (B/2\alpha^2)$, for which the pile-noise component $B/2\alpha^2$ is one-half its value at zero frequency. By reading this value of ω from our plot and equating it to α , we can thus determine β/ℓ .

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VIII. APPENDICES

A. Discussion Questions

The following propositions should be considered prior to arrival at the Argonaut. These will be discussed in a group discussion just prior to or during the experiment. It should be stressed that these propositions are meant as a guide to informative discussion, and, although they are tailored to a particular procedure, it is intended that the student be allowed maximum flexibility in his choice of experimental procedure.

1. Compare the two reactor conditions "delayed critical" and "prompt critical."
2. Defend the statement "On a microsecond time scale, a delay critical reactor is subcritical."
3. Justify that the expression $dn/dt = nk_{ex}/l$ describes correctly the change of neutron density n with time t on the microsecond time scale. Here k_{ex} is the excess prompt reactivity and l is the neutron lifetime.
4. Show from all of this that the neutron population of a delayed-critical reactor decays on the microsecond time scale according to $\exp(-\beta t/l)$.
5. Use this result to make a picture for yourself of the noise component on top of the reactor flux. Compare this type of noise with the noise which you would expect to find superimposed on the current passing through a vacuum tube.
6. An important function in the description of noise is the autocorrelation function

$$\psi(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \phi(t) \phi(t + \tau) dt.$$

If $\phi(t)$ is a sinusoidal function, can you picture for yourself how the autocorrelation function of $\phi(t)$ will vary with τ ?

7. If we interpret $\phi(t)$ as the reactor flux, time-dependent through the noise component, do we have an experimental way to approach and to measure the quantities a_n^2 and b_n^2 ?
8. Do we know the form of the autocorrelation function $\psi(\tau)$ for reactor noise? (See 6 above).

9. Summarize: How may we measure the coefficient a_n and b_n and determine the ratio β/l ?
10. Inspect the circuit diagram and try to draw a block diagram. Show the importance of each block.
11. Since the circuits comprise amplifying stages and resonating circuits, it is to be expected that the overall circuit response will be different for different frequencies. Can you suggest a method of determining this frequency dependence?
12. What is the difference between exposing a low-efficiency counter to a high flux and exposing a high-efficiency counter to a low flux? How do you expect the noise spectrum to differ?

B. Determination of Reactor Kinetic Parameters by Pile Noise Analysis

**Determination of Reactor Kinetic Parameters by Pile Noise
Analysis**

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Determination of Reactor Kinetic Parameters by Pile Noise Analysis*

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An experimental investigation was made of the statistical fluctuations in neutron intensity which occur in a nuclear reactor. An ion chamber was exposed to reactor flux, and the fluctuations in its output current were analyzed in a tunable bandpass filter to get the frequency spectrum of these fluctuations, which has the shape of the square modulus of the transfer function. Results are presented of some measurements made on various low-power experimental reactors at Argonne National Laboratory. For reactors with prompt neutron lifetime between 15 and 70 μ sec, the quantity l/β was determined within 5 per cent or better from a least squares fit to the transfer function thus measured.

INTRODUCTION

Since a nuclear reactor is a statistical system, it will show fluctuations in neutron intensity. These fluctuations, or "pile noise," are not commonly considered of interest in themselves, but only as interference to other experiments. However, since the nature of the pile noise depends strongly on important reactor parameters, its study can enable the determination of quantities less easily accessible by other means. In particular, Moore (1) points out that the noise spectrum of such a system, that is, the mean square noise amplitude per unit band width, is proportional to the square modulus of the transfer function or to the Fourier cosine transform of the autocorrelation function. Thus, observation of the noise spectrum of a reactor could yield information about the shape of its transfer function. To test this technique, pile noise analyses were done on various low-power experimental reactors at Argonne National Laboratory. Since these reactors operate at such a low level that power effects on reactivity do not appear, the shape of the low-frequency portions of their transfer functions would depend only on fairly well-known delayed neutron parameters, and thus would be of little interest. However, the high-frequency rolloff portion of the transfer function is strongly dependent on the quotient of the effective delayed neutron fraction over the prompt neutron

lifetime. Since this quantity was of interest to the experimental programs of some of the reactors involved, it was decided to measure it.

APPARATUS AND METHODS

The measurements were made by exposing an ionization chamber to the reactor flux. The fluctuations in the current signal from this chamber were analyzed in a tunable bandpass filter to obtain the frequency spectrum. These fluctuations are made up of two portions. The first is the pile noise, and the second is the white noise resulting mainly from the statistics of neutron collection in the chamber. The amplitude of the signal resulting from the pile noise is proportional both to the square root of the power level of the reactor, and to the fraction of reactor neutrons absorbed in the chamber. The amplitude of the white noise, however, is proportional to the square root of the total number of neutrons per second entering the chamber. Thus, it is proportional to the square root of the reactor power and to the square root of the fraction of neutrons absorbed. For a good measurement it is necessary to have a high ratio of pile noise to white noise, which thus requires that the chamber absorb a sufficient fraction of the neutrons in the reactor. To achieve this, the chamber must be heavily loaded with boron and be placed in a high flux region in the reactor. The chamber used in these experiments was a parallel-circular-plate type and was loaded with about 4 gm of boron 10, in the form of both boron

* Work performed under the auspices of the U. S. Atomic Energy Commission.

coating on the plates and BF_3 gas filling. In the reactors tested, this chamber removed a few tenths of a per cent of reactivity.

A schematic of the electronics used in these measurements is shown in Fig. 1. The output of the chamber was taken to a cathode follower preamplifier using a 12J7GT tube operated with reduced filament and plate voltages to minimize grid current. A battery and potentiometer were provided to insert a bucking voltage to the grid circuit to oppose the voltage drop across the grid resistor produced by the dc chamber current and thus to hold the tube at the correct operating point. The output of the preamplifier fed the intermediate amplifier, which in turn fed the analyzer. The 6SJ7 voltage amplifier stage in the analyzer drove the bandpass filter, consisting of the components L , C , and R shown inside

type linear vacuum tube voltmeter. In theory, since it is the mean square amplitude that is of interest, a square law voltmeter should be used. However, the reading of a square law voltmeter and the square of the reading of a linear voltmeter, when averaged, will always be proportional on a Gaussian signal. The linear voltmeter thus gives the spectrum correctly, and is much more convenient than the square law circuit. The indicating meter had a large condenser which could be switched across it to provide a time constant of about 1 sec, and a shunt to provide a signal for a 10 mv Brown strip chart recorder. To eliminate hum, all supply voltages for the equipment were obtained from dry batteries, except for the filaments of all tubes but the preamplifier, which were powered by a storage battery. Other precautions

FIG. 1. Schematic diagram of pile noise analyzer.

the dotted line. Different combinations of L and C could be switched in to obtain a wide range of resonant frequencies—from 9.4 to 438 cycles per second. The low frequency limit was determined by the characteristics of available components, while the high frequency limit was selected to be around that point where the pile noise in the reactors tested became small compared with the white noise due to the rolloff of the transfer function. The inductances L were high Q audio chokes of 3 and 20 henries, while the condensers C ranged from 0.05 to 12 μf . For each combination, R was chosen to be about ten times the resonant impedance of the tuned circuit in order to prevent loading by the preceding stage. The Q of the tuned circuit varied from 10 to 50 for different settings.

The amplitude of the frequency components selected by the filter was measured by a feedback-

to avoid hum included the grounding of the setup at one point only to avoid ground loops.

Noise spectra obtained with this equipment had to be corrected for its frequency response, which was dependent upon many factors, including the resonance curve of the filter, the frequency response of the amplifier, and the collection properties of the chamber. To allow for all these factors, the system was calibrated with white noise in the form of a 40-curie Co^{60} source. With the chamber exposed to the source, readings were taken at all the filter settings to be used in subsequent runs. Then, by dividing readings obtained from a reactor by readings obtained from the Co^{60} source at corresponding frequencies, the reactor noise spectrum was directly obtained. Care was taken to establish that the chamber voltage ($67\frac{1}{2}$ volts) was sufficient to give saturation at the chamber currents used, since it was

found that lack of saturation in the chamber could give rise to odd spectrum shapes.

Since the amplitude of each frequency component of the noise was itself fluctuating, it was necessary to average it over a period of time to obtain an accurate reading. (According to Van der Ziel (2), the fractional standard deviation of a noise measurement, n , is given by the equation

$$\delta n/n = \sqrt{K/BT} \quad (1)$$

where B is the filter bandwidth, T the averaging time, and K a constant dependent on the dynamic characteristics of the averaging mechanism.) Thus, during each run, voltmeter output was recorded for several minutes at each frequency setting. The recorder trace was read at about $1\frac{1}{2}$ sec intervals and the points were averaged to give the reading.

The shape of the noise spectrum observed with a given setup should be independent of reactor power level, as pointed out previously. In practice, though, there are upper and lower limits to the power that may be used. It must be sufficiently high so that the pile noise is large compared with the background noise of the apparatus, but must not be so large that the chamber behaves improperly or the signal overloads the amplifiers. Also, in order to get an accurate value of the lifetime, the power level should be high enough so that the reactor is truly critical and not merely multiplying some extraneous or inherent source.

In the frequency range used, delayed neutron effects on the transfer function are negligible. This allows the zero power transfer function $ZP(\omega)$ to be represented by the equation

$$ZP(\omega) = (1 - \beta)/(\beta + j\omega l)$$

where β is the effective delayed neutron fraction, l is the prompt neutron lifetime, and ω is the angular frequency. The square modulus of the transfer function then becomes proportional to the function

$$1/(\alpha^2 + \omega^2) \quad \text{where } \alpha = \beta/l.$$

The observed noise spectrum can then be represented by the expression $A + B/(\alpha^2 + \omega^2)$, where the first term is the white noise component and the second term is the pile noise component. This is the function which it was desired to fit to the experimental data.

The fitting was done by minimizing the least squares sum

$$S = \sum_i \left(n_i^2 - A - \frac{B}{\alpha^2 + \omega_i^2} \right)^2$$

where n_i^2 is the normalized mean amplitude squared observed at the frequency ω_i . The weight for each

point was taken as unity, since the presence of other errors such as reactor and instrument drifts tended to mask the statistical errors of Eq. (1) and to make the standard errors of all points about the same. Differentiating S with respect to A , B , and α and setting the derivatives equal to zero gave three equations in the three unknowns. Straightforward elimination of A and B yielded an equation for α of the form

$$F(\alpha, n_i^2, \omega_i) = 0$$

This was solved for α by the Newton-Raphson method. Then, working backwards, A and B were determined, allowing S to be calculated. The standard error on α was obtained using the method of Deming (3), in which the square of the standard error is proportional to S . The calculations were done on an IBM-704 computer.

EXPERIMENTAL RESULTS

Figure 2 shows the noise spectrum obtained in a run on the internal exponential ZPR-V (4). This was done with a fast core loading of 5 to 1 U^{238} to U^{235} enrichment ratio and a 1-in. natural uranium blanket. The chamber was placed in an instrument tube in the thermal section. For this case, α was $180.4 \pm 8.5 \text{ sec}^{-1}$ with a corresponding lifetime value of $38.8 \pm 1.8 \text{ } \mu\text{sec}$. This agrees within experimental error with several other measurements done on this facility, including a $1/v$ poison measurement which yielded $39.0 \pm 3.0 \text{ } \mu\text{sec}$; a measurement using the counting statistics method of deHoffmann which yielded 39.2 to 42.8 μsec ; and a measurement using Rossi alpha techniques which yielded $36.8 \pm 4.0 \text{ } \mu\text{sec}$. These values are all based on an effective de-

FIG. 2. Pile noise spectrum from ZPR-V.

FIG. 3. Pile noise spectrum from ZPR-IV'.

FIG. 5. Pile noise spectrum from Argonaut.

FIG. 4. Pile noise spectrum from the coupled critical experiment done in ZPR-III.

FIG. 6. Pile noise spectrum from ZPR-VII.

layed neutron fraction of 0.007. Figure 3 shows the spectrum obtained from the thermal source reactor ZPR-IV' (5). For this run, the chamber was placed in an instrument tube in an empty fuel element position in the middle of the core. Here, the value of α obtained was $100.6 \pm 2.2 \text{ sec}^{-1}$. Figure 4 shows the noise spectrum obtained from the coupled fast thermal critical experiment (6) in the Argonne Fast Critical Facility, ZPR-III. The value of α obtained was $446 \pm 21 \text{ sec}^{-1}$. With an assumed effective delayed neutron fraction of 0.0074, the lifetime was $16.5 \pm 0.8 \mu\text{sec}$. This agrees with a value of $16.5 \pm 1.7 \mu\text{sec}$ obtained from Rossi alpha measurements.

Figure 5 shows the spectrum obtained from the Argonaut low-power research reactor (7), which

yields an α of $45.3 \pm 6.3 \text{ sec}^{-1}$. Assuming an effective delayed neutron fraction of 0.007, a lifetime of $154 \pm 21 \mu\text{sec}$ was obtained. It can be seen that here α is not as well determined as it was in the previous cases. The reason for this is that the part of the transfer function observed in this run is largely on the 6 decibel per octave tail. Since this portion of the transfer function is identical for all transfer functions with rolloffs below a certain frequency, the ability to discriminate between such functions is limited. This is seen even more strongly in Fig. 6, which is the noise spectrum obtained for the heavy water critical ZPR-VII (8), which gave an α of $20.4 \pm 10.2 \text{ sec}^{-1}$ and a lifetime of 252–755 μsec , assuming an effective delayed neutron fraction of 0.0077. Better results might have been obtained in

both these cases if the range of the analyzer had extended to lower frequencies.

CONCLUSIONS

Pile noise analysis is a convenient method for obtaining the shape of reactor transfer functions, and for determining reactor prompt neutron lifetimes. The method is especially useful at frequencies which are inconveniently high for other methods, and requires much simpler equipment than many existing techniques. Other possible applications of this technique include the observation of the shape of the transfer function of power reactors in the study of instabilities, and the determination of prompt neutron lifetime in fast reactors. The latter application would require a detector with high sensitivity to fast neutrons, and the former a chamber that would operate well at the high fluxes involved.

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STUDENT RESULTS

DATA

Reactor Noise					White Noise				
Channel	$f_m(\text{sec}^{-1})$	Gain	Area inch	$\frac{I}{\text{Area}}$ Gain-inch	Gain	Weight (gm)	Length inches	$\frac{I_0}{\text{Weight}}$ Length	$\left(\frac{I}{I_0}\right)^2$
1	2.3	25	9.51	0.38	125	4.582	15	0.306	1.54
2	2.78	"	9.46	0.378	"	2.809	9	0.312	1.47
3	3.85	"	10.18	0.407	"	2.741	9	0.304	1.79
4	5.9	"	7.86	0.314	"	2.773	9	0.308	1.04
5	8.1	"	8.31	0.332	"	2.860	9	0.318	1.09
6	11.0	"	4.06	0.162	"	2.093	9.66	0.216	0.56
7	17.0	"	3.87	0.155	"	2.669	11.3	0.236	0.43
8	25	"	2.64	0.1055	"	2.801	13	0.2155	0.24
9	35	"	0.85	0.034	"	1.366	12	0.114	0.089
10	54	125	4.81	0.0385	"	1.154	10	0.1154	0.111
11	72	"	6.66	0.0533	"	2.166	11	0.197	0.073
12	93.5	"	4.65	0.0372	"	1.630	11.3	0.144	0.066
13	170	"	3.92	0.0314	"	1.241	11	0.1127	0.077
14	230	"	5.52	0.0441	"	1.844	11	0.1675	0.069
15	310	"	6.16	0.0492	"	2.841	11	0.2580	0.036
16	380	"	6.40	0.0515	"	2.996	11	0.2725	0.035
17	435	"	5.90	0.0471	"	1.933	10	0.1933	0.059
18	530	"	6.20	0.0497	"	1.854	10	0.1854	0.072

Fig. S16.1. β/l by Reactor Noise Analysis

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TIME AND SPACE DEPENDENCE OF NEUTRON FLUX
USING PULSED NEUTRON TECHNIQUES

by

W. F. Gilbert and M. Petrovic

ABSTRACT

The experiment consists essentially of irradiating a sample of water with a very short burst of fast neutrons and then measuring the decay of the resulting thermal-neutron flux at different positions in the sample. From these data the decay constants of the fundamental and higher modes are observed and calculated.

The decay constant can be used for evaluation of diffusion parameter: Σ_a , L , and D of a sample.

Characteristics and operation of a pulsed-neutron generator are described.

OUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
- IV. PROCEDURE
 - A. General
 - B. Flux Decay Measurement
- V. RESULTS
- VI. ANALYSIS AND CONCLUSIONS
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- VIII. APPENDICES
 - A. Discussion Questions
 - B. Characteristic Operating Instructions for Atomic Laboratories, Inc., Model 102 Generator

I. INTRODUCTION

High-intensity pulsed-neutron sources provide a powerful tool in nuclear physics generally, and in reactor physics in particular. Basically, the pulse technique consists of observing the transient behavior of neutron intensity following a burst of neutrons injected into the medium under study. Using pulsed-neutron techniques, one can determine neutron spectra, flux densities, capture and scattering cross sections, diffusion constants, neutron lifetime, neutron age, and reactivity.

Introduction to pulsed-neutron techniques, pulsed-neutron generators, and measurements will be the subject of this experiment.

Since much information is obtained from use of the decay constant of the fundamental mode in the first part of the experiment, we will obtain the decay rates of the spatial modes of the neutron-flux distribution and evaluate waiting time required to have only the fundamental mode present.

II. THEORY

Neutron-flux distribution in large geometries may be adequately described with application of one-velocity diffusion theory:

$$D\nabla^2\phi(x,y,z,t) - \Sigma_a\phi(x,y,z,t) = \frac{1}{v}\frac{\partial}{\partial t}\phi(x,y,z,t).$$

This equation has a general solution

$$\phi(x,y,z,t) = \sum_{lmn}^{\infty} A_{lmn}(0) R_{lmn}(x,y,z) e^{-(\Sigma_a v + DvB_{lmn}^2)t}.$$

For the geometry we will use in this experiment, $R_{lmn}(x, y, z)$ can be approximated by

$$R_{lmn} \approx \cos \frac{l\pi x}{a+2\epsilon} \cos \frac{m\pi y}{b+2\epsilon} \sin \frac{n\pi z}{c+2\epsilon}$$

and

$$B_{lmn}^2 = \left(\frac{l\pi}{a+2\epsilon} \right)^2 + \left(\frac{m\pi}{b+2\epsilon} \right)^2 + \left(\frac{n\pi}{c+2\epsilon} \right)^2,$$

where A_{lmn} is initial flux distribution and ϵ is the extrapolation distance.

From the general solution, it can be seen that the time-dependent part is a sum of the exponentials with variable B_{lmn}^2 :

$$\phi(t) = \phi(0) \sum_{lmn}^{\infty} e^{-(\Sigma_a v + Dv B_{lmn}^2)t}$$

Measured values of $\phi(t)$ can be used for evaluation of the diffusion constants in the exponential. Since B_{lmn} can have very many values, calculation can be very difficult because of the number of exponential terms. Now the question arises: what can be done to minimize the number of terms?

The pulsed-neutron source is a source of fast neutrons, and it is a point source in comparison with the dimension of the sample.

With the point source and a sample shaped as a cube, physical arrangement can be made to minimize the number of terms. If the source is symmetrical in one plane, there will be no even terms in that plane. Thus, if the source is on z axis, there will be no even terms on the x and y axes. If the distance between the source and various parts of a planar sample is different, the distribution of incident source neutrons in the xy plane will vary due to this geometrical effect. Roughly calculating the inverse square distance (see Fig. 17.1), one obtains

$$\frac{\phi(y)}{\phi(0)} = \frac{(kb)^2}{(kb)^2 + y^2}.$$

The distance from the source is given as a function of the width of the moderator sample. The distribution is calculated for several distances and plotted in Fig. 17.1. The dotted line is the cosine distribution, the expected first mode, and so the distribution will be closest to the dotted line for k between 0.2 and 0.4. We will choose $k = 0.3$ for the experiment. The expected initial distribution is shown in Fig. 17.2.

Fig. 17.1

Distribution of Source Neutrons
in the xy Plane

Fig. 17.2. Three-dimensional Flux Distribution

With such an arrangement the amplitude of higher modes in the x and y directions will be very small, so the terms with ℓ and m higher than one can be neglected. With that assumption the problem becomes one-dimensional $B_{lmn}^2 \rightarrow B_{11n}^2$, and the number of terms which must be considered is very much reduced.

The amplitude of the various terms decay at different rates. Thus, if the time-dependent parameter in the exponent is α_{lmn} , then

$$\alpha_{lmn} = \Sigma_a v + DvB_{lmn}^2.$$

In this equation, for our rectangular system,

$$B_{lmn}^2 = \left(\frac{\pi}{a+2\epsilon}\right)^2 + \left(\frac{\pi}{b+2\epsilon}\right)^2 + \left(\frac{n\pi}{c+2\epsilon}\right)^2,$$

or

$$B_{lmn}^2 = B_a^2 + B_b^2 + n^2 B_c^2.$$

For a water tank of square cross section, such as ours, the buckling is,

$$B_{lmn}^2 = (n^2 - 1) B_c^2 + B_{l11}^2;$$

similarly, the time constant, in terms of that for the fundamental mode, is given by

$$\alpha_{lmn} = \alpha_{l11} + (n^2 - 1)DvB_c^2.$$

From the last equation, it can be seen that the decay of higher modes is faster, varying as $(n^2 - 1)$, and also that the decay depends on the value of B_c^2 for the sample.

When the samples are small, B_c^2 is large, so higher modes decay much faster than the fundamental. In such a case after a reasonable time, the higher modes are negligible, and so practically only one term is present. The calculation thus becomes simple.

When the sample is large, B_c^2 becomes very small; then the higher-mode decay is comparable with that of the first one. Under these circumstances, higher modes can still be present during the time when the data are statistically valid. Harmonic analysis of the time and space decay is then necessary to obtain the decay constant of the fundamental mode alone. A Fourier harmonic analysis of a space-time neutron-flux distribution can be performed by the following method:

If $f(x)$ is an odd function, as it is in this experiment, interpolation by sine functions is appropriate and simpler.

At any instant of time for measured points $f(z_1), f(z_2), \dots, f(z_n)$:

$$\begin{array}{l} f(z_1) = b_1 \sin z_1 + b_2 \sin 2z_1 + \dots b_n \sin nz_1 \\ \text{"} \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{"} \\ \text{"} \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{"} \\ f(z_n) = b_1 \sin z_n + b_2 \sin 2z_n + \dots b_n \sin nz_n \end{array}$$

where b_k is the amplitude of different modes.

In this experiment the amplitudes of the modes higher than five are negligible. We will measure the neutron flux at five points on the z axis and thus be able to calculate the amplitude of the fundamental and four higher modes.

The coefficients (amplitudes b_k) by use of least-squares approximations are (Fourier Theory, Harmonic Analysis Interpolation by Sine Functions)

$$b_k = \frac{2}{n} \sum_{\alpha=1}^{n-1} f(z_\alpha) \sin \frac{\alpha}{n} k\pi$$

- k - is the number of the mode,
- α - is the number of the measured point,
- n - is the number of intervals,
- α/n - is a fraction of the total distance between the extrapolated zeros in the neutron flux;

or in matrix form for five uniformly distributed points,

$$b_k = \frac{2}{6} \begin{vmatrix} f(z_1), f(z_2), f(z_3), f(z_4), f(z_5) \\ \sin 1\left(1\frac{\pi}{6}\right), \sin 1\left(2\frac{\pi}{6}\right), \sin 1\left(3\frac{\pi}{6}\right), \sin 1\left(4\frac{\pi}{6}\right), \sin 1\left(5\frac{\pi}{6}\right) \\ \sin 2\left(1\frac{\pi}{6}\right), \sin 2\left(2\frac{\pi}{6}\right), " " " " " \\ " " " " " " " " \\ " " " " " " " " \\ \sin 5\left(1\frac{\pi}{6}\right), " " " " " \end{vmatrix}$$

i.e.,

$$b_k = \frac{1}{3} \begin{vmatrix} f(z_1), f(z_2), \dots, f(z_5) \\ 0.5 & 0.866 & 1 & 0.866 & 0.5 \\ 0.866 & 0.866 & 0 & -0.866 & -0.866 \\ 1 & 0 & -1 & 0 & 1 \\ 0.866 & -0.866 & 0 & 0.866 & 0.866 \\ 0.5 & -0.866 & 1 & -0.866 & 0.5 \end{vmatrix}$$

III. APPARATUS

As a pulsed-neutron source, we will use the Scientific Engineering Laboratory Pulsed Neutron Generator.

BF₃ detector, amplifier

TMC analyzer

Neutron monitor channel

Water tank.

The pulsed-neutron generator consists principally of an ion source for the production of deuterons in a pulsed beam, and focusing and accelerating electrodes to focus and accelerate this beam to strike the target. The target contains deuterium or tritium, so we obtain neutrons from the reaction $H^2 + H^2 \rightarrow He^3 + n + 3.3 \text{ MeV}$ or $H^2 + H^3 \rightarrow He^4 + n + 17.6 \text{ MeV}$. (Characteristics and operating instructions of the neutron source are given in the appendices.)

The medium under study will be water in the cubical aluminum tank, 9 x 9 x 9 in.

The neutron flux in the specimen will be measured by a BF₃ proportional counter, 3/4 in. in diameter and 1 in. long; it can be considered approximately as a point detector.

The time-analyzing electronic system is a TMC 256-channel analyzer.

To normalize measurement at various points to the total neutron yield, we use another neutron-detecting system. It consists of a long BF₃ proportional counter located in a paraffin block and connected to an amplifier and scaler.

A block scheme of the apparatus is shown in Fig. 17.3.

Fig. 17.3

Block Diagram at the
Two Counting Channels

IV. PROCEDURE

A. General

1. The pulsed-neutron generator will be put into operation according to the instructions given in the appendices.
2. The level and temperature of the water in the tank will be measured. The tank will be located symmetrically with respect to the target and at a distance of 0.3 b (2.7 in.).
3. The background of both detector systems will be measured (target voltage of accelerator at zero).

B. Flux Decay Measurement: Since the amplitude of the modes higher than the fifth are small, we will measure the first five. Since the flux at the extrapolation distance is assumed to be zero, five points in the system will be sufficient to evaluate the amplitude of these modes.

The geometrical arrangement for the measurement is shown in Fig. 17.4. The decay curves will be measured for the points therein indicated, and printed data will be obtained from the TMC analyzer. With each measurement a monitoring channel count will be recorded for normalization of the data. To have good statistics through three decades, we will measure the decay for each point until the maximum capacity of the memory in the analyzer is reached.

V. RESULTS

Using printed data from the TMC 256-channel analyzer, decay curves can be plotted on semilog paper. We can choose 5 to 8 equidistant times after the burst of neutrons and get the normalized relative flux. Using the matrix shown in Section II, we can calculate the amplitude of the different modes. Having all the amplitudes for time t ($\sim 200 \mu\text{sec}$ after the burst) and all equidistant time intervals (200, 400, 600, 800, 1000, 1200, 1400, 1600 μsec), the decay constants of different modes can be calculated or graphically obtained.

In Fig. 17.5 are presented the data measured with pulsed-neutron techniques. The pulsed length was 200 μsec ; the measurements were started 75 μsec later and taken at intervals of 250 μsec . In Fig. 17.5, the dotted line represents the fundamental mode, and it can be easily seen that after 1275 μsec the fundamental mode alone is present.

Fig. 17.4. Counter Locations

Fig. 17.5. z -axis Flux Distribution

VI. ANALYSIS AND CONCLUSION

Having the decay constants for different modes of the neutron-flux distribution in the xy plane, we can see when the higher modes become negligible with respect to the fundamental one (less than 5%), i.e., when the distribution in the z direction can be assumed to be one-dimensional.

From the data in the z direction, when the distribution is one-dimensional, we can obtain the time after which only the fundamental mode is present, and also the decay constant of the fundamental mode. If the time for the higher modes to decay is long so that we cannot measure the fundamental decay alone for want of intensity, the decay of the fundamental mode can be obtained through this analysis. This is the case for a large geometry.

With the equipment used in this experiment, pulsed-neutron generator AL-201, BF_3 counter, TMC analyzer, and the ambient natural background present, we can obtain the value of α for $B^2 > 0.1 \text{ cm}^{-2}$ directly and for $B^2 < 0.1 \text{ cm}^{-2}$ through harmonic analysis.

Fig. 17.6. Experimental Electronic Equipment

Fig. 17.7. Detail View of Pulsed-neutron Source

Fig. 17.8. Pulsed-neutron Source

139-3419

Fig. 17.9. Experimental Apparatus

Fig. 17.10. Multi-channel Time Analyzer

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VIII. APPENDICESA. Discussion Questions

1. How can we avoid the effect of the second mode in determining the decay constant of a water sample?
2. Can we avoid the effect of the second and third modes together in determining the decay constant?
3. What will be the initial rate of decay of neutrons in the sample close to the source and far from the source, in comparison with the rate of decay of the fundamental?
4. What is the use of the monitor channel, and what precautions have to be taken?
5. What difference in the initial flux distribution can we observe when using deuterium or tritium targets?
6. Can we use the decay constants of the higher modes to plot an $f(B^2)$ curve in order to obtain the diffusion constants?
7. What determines the allowable upper and lower limit of the yield of neutrons per burst?
8. What are the sources of background in these measurements, and how can they be guarded against?
9. What kind of disturbances are caused by the counter present in the sample?

B. Operation Instructions for Atomic Laboratories, Inc., Model 102 Pulsed Neutron Generator

1. Introduction

Deuterium gas is ionized in the ion source (see Fig. 17.11) and extracted from the source by the use of an electric field formed by the probe electrode. The ions are focused and accelerated toward the target. From the reaction $D(d, n)He^3$ or $T(d, n)He^4$, depending upon the target material, the neutrons are produced. The ionization is produced by pulsing the arc voltage. The duration and repetition rate of the arc voltage pulses determine the duration and repetition rate of the neutron bursts.

2. Ion Source

The cross section of the ion source is shown as part of Fig. 17.11. When the anode voltage rises, some electrons released from out of the cathodes enter into the anode space. On the way to the other cathode, an electron may lose some energy in the process of ionizing the gas present. That electron will not have enough energy to reach the other cathode and will be reflected back into the anode. Electrons will oscillate between cathodes until they lose all their energy. Since in the ionizing of one atom an electron loses about 35 eV, some twenty new ions will appear inside the anode with an arc voltage of 800 V. Positive ions will hit the cathode and produce secondary electrons. This process will rapidly build up the amount of ions until some other process will keep the ion density on a certain level. The upper level of ion density is normally set by a resistance in the anode circuit. The ion density is up to saturation value within 5 μ sec after the start of the arc voltage pulse. The arc voltage then drops to about 2/3 of its initial value.

In Fig. 17.12, typical ion-source conditions are shown through the arc voltage pulse as seen on an oscilloscope.

An axial magnetic field in the range of a few hundred gauss (produced at currents of around 1 Amp), a gas pressure of the order of 100 μ , and an anode voltage of 600-1000 V are normal operating parameters. The magnetic field limits the ion path to a diameter of about 3/16 in. The ion current out of the ion source is about 4 mA and depends on the probe voltage.

In Fig. 17.13 is shown ion current versus probe voltage.

The ratio of atomic ions to molecular ions, d^+/d_2^+ , increases with increasing arc current. The best operation in this respect is therefore obtained by running as large an anode current as possible consistent with arc stability.

To regenerate the cathode, i.e., to oxidize it, it is necessary to run the ion source for about 15 min with air as a cover gas. Twenty-five microns of air pressure is convenient with the same arc voltage and magnetic field.

Fig. 17.11. Diagram of Pulsed-neutron Source

Fig. 17.12
Typical arc
Voltage Patterns Fig. 17.13. Effect of the Probe Voltage
on the Beam Current

3. Focusing System

The ions are extracted from the ion source by the probe electrode field and must strike the target to be effective.

The probe voltage will be changed according to the ion current desired. The ions have to be spread over the target to prevent the burning of the target by a concentrated beam of ions. To meet these two needs independently, a focusing electrode is built into the accelerator system.

The values of target, focus, and probe voltage needed in order to have a beam spot on the target 2 cm in diameter is given in Fig. 17.14. An up-to-date curve will be on power supply panel.

Fig. 17.14

Typical Focus Characteristic

4. Targets

Few reactions can produce neutrons with a deuteron as an incident particle. The most convenient for the energy range up to 100 kV are $H^2(d, n)He^3$ and $H^3(d, n)He^4$. Cross-section data for these two reactions are given in Figs. 17.15 and Fig. 17.16. An H^2 target can be made out of heavy ice or deuterium gas taken up in some metal as a backing. If the incident particles are deuterons, then the source particles injected in a metal will serve as a target. The target life is then unlimited.

Preparation of the tritium targets with recommendations for using are attached. With the pulsed-neutron generator operating at a 100 kV target voltage and proper conditions of the ion source, with $H^2(d, n)He^3$ there are 10^4 n/burst; with $H^3(d, n)He^4$, there are 10^6 n/burst. The efficiency of tritium targets decreases because of loss of tritium and occlusion upon the surface of contaminating materials. Efficiency drops one-half after 1 mA-hr of operation. (AL-201 has 40μ A current as average; 4 mA in a pulse.)

Targets, used for neutron production in particle accelerators are fabricated by depositing evaporated zirconium on a platinum metal backing, then impregnating the zirconium with tritium.

Fig. 17.15. Variation of the Cross Section for the Reaction $\text{H}^3(\text{d},\text{n})\text{He}^4$ with the Energy of the Deuteron

Fig. 17.16. Total Cross Section of the Reaction $\text{H}^2(\text{d},\text{n})\text{He}^3$ as a Function of the Energy of the Deuteron

Backing. Standard backing is a platinum disk, 0.25 mm (10 mils) thick and 3.2 cm in diameter. Any available platinum up to 3 mm thick may be used. If the thickness is over 0.25 mm, price will be adjusted to include additional platinum. Backing materials thinner than 0.25 mm tend to warp and are therefore usually unsatisfactory. Targets can be made from 0.1 to 3.2 cm in diameter.

Results have been consistently good only from targets with platinum backing. Other backing materials required by the customer will be used only after coating with a layer of evaporated platinum ($\sim 60 \mu\text{g}/\text{cm}^2$).

Fabrication. Standard targets are fabricated by depositing 500 μg of pure, commercially available zirconium metal per square centimeter on one whole surface of the platinum back. On request the zirconium layer can be as thin as 20 $\mu\text{g}/\text{cm}^2$, or thicker than 500 $\mu\text{g}/\text{cm}^2$, at \$25.00 for each additional 500 $\mu\text{g}/\text{cm}^2$ or fraction thereof.

Zirconium thickness will be within $\pm 10\%$ of that requested, and determined within $\pm 1 \mu\text{g}/\text{cm}^2$.

The diameter of the zirconium layer can be made as small as 1 cm. Targets with such layers are not recommended by ORNL, because they do not perform as well as those on which zirconium completely covers one surface.

Impregnation. The standard minimum H^3 -Zr atomic ratio for impregnation is 1:1, slightly over 310 mC (0.1 cc at STP) of tritium per mg of zirconium. Targets not meeting this specification are rejected; however, each target is impregnated with the maximum amount of tritium.

The amount of tritium taken up in a target containing over 4 mg of zirconium is determined within $\pm 10\%$, by standard gas techniques checked against ion chamber measurements. The amount in a target containing less than 4 mg of zirconium is measured within $\pm 20\%$ by the ion chamber method only.

General Information

Neutrons produced from a very thin target (40 μ g or less of zirconium per square centimeter) are nearly monoenergetic.

Targets should not be set aside unused for extended periods (several months) due to target degradation; therefore it is suggested that targets be ordered only for immediate use.

Delivery of standard targets can usually be made within four weeks. Shipment is made in a disposable container.

Requests for tritium-zirconium targets of sizes and on backing metals not specified above will be considered on a special-order basis.

Recommended Directions for Using Tritium-Zirconium Targets (Courtesy of High Voltage Laboratory, ORNL Physics Division)

1. Employ forced cooling of the target. This may consist of an air-water jet, or of water cooling where it can be tolerated.

2. Start with a defocused beam at least 1 cm^2 in area and a current of approximately $1 \mu\text{A}$.

3. At some current density, target loss will become serious. This density must be learned by experimentation. Until such a guide has been established, it must be assumed that $10 \mu\text{A}/\text{cm}^2$ is the maximum allowable current.

5. Vacuum

The pulsed-neutron generator can be operated with the pressure less than 5×10^{-6} mm Hg. A 5-in., diffusion pump and forepump, however, evacuate the system to 5×10^{-6} mm Hg.

Since the ion-source cathode and target surface can lose efficiency if some organic vapor is present in the system, all seals are rubber gaskets without grease. A liquid nitrogen trap is mounted between the accelerator and vacuum system to decrease the vapor pressure.

The vacuum system is shown in Fig. 17.17.

Fig. 17.17. Diagram of Vacuum System

6. Operation Procedure

The pulsed-neutron generator units are connected electrically as it is shown in Fig. 17.18.

Fig. 17.18. Equipment Interconnections

Condensed Operating Instructions

Pumpdown:

- a. Check to see that the vent hose is connected, cooling water is flowing to the diffusion pump, leak to HIVAC is closed, and palladium leak input line is pressurized to 2 psi.
- b. Check to see that the forepump is running and that forepressure is 30 μ or less.
- c. Leave headgate valve closed and throw forepump valve to finish position. When the forepressure comes down to 100 μ , fill both traps with liquid nitrogen and close the diffusion pump breaker. After one hour, pressure reading on ion gauge power supply should read on 10 scale.
- d. Throw forepump valve to rough position. As soon as forepressure reaches 100 μ , throw forepump valve to finish position. Turn off ion gauge emission current and then throw headgate valve to open position. After one hour, high vacuum should read on 30 scale. After verifying this, leave selector switch on 100 scale.

Neutron Production:

- a. Check to see that polarity correct light is lit and that all high-voltage power-supply variacs are turned all the way down. Close the main power breaker. Check to see that the interlock complete light is lit.
- b. Raise the palladium leak heater voltage until the ion-source thermocouple reads about 100 μ . Set magnet current to about 1 Amp.
- c. Turn on arc pulser and check to see that the selector switch is on the "Int-Osc" position, unless an external signal generator is being used. Raise arc power-supply voltage and adjust all the preceding parameters for maximum arc stability.
- d. Set probe voltage and focus voltage according to the ion current and target voltage planned for operation (see Figs. 17.4 and 17.5).

Shutdown:

- a. Turn all high-voltage power-supply variacs all the way down. Open main power breaker on control rack.
- b. Close headgate valve; open diffusion-pump breaker and push OFF button on ion-gauge power supply.
- c. After 5 min, close forepump valve but leave forepump running.

Radiation Hazard:

At one meter from the target, the fast-neutron flux from the $H^2(d,n)He^3$ reaction is less than the present tolerance dose rate. However, from the $H^3(d,n)He^4$ reaction, the fast-neutron flux may considerably exceed the present tolerance dose rate up to a 5-m distance.

Bibliography:

1. T. D. Gow, High Intensity Pulsed Ion Source, Review of Scientific Instruments Vol. 24, p. 606 (1953).
2. L. Ruby, Operating Manual for Atomic Laboratories, Inc. Pulsed Neutron Generator AL-201, Berkeley (1958).

STUDENT RESULTSDATA SHEET

(Decay of Neutron Intensity at Different Counter Positions)

Channel	Counter Position							
	time, μs	Z_1, X_3	Z_2	Z_3	Z_4	Z_5	X_1, X_5	X_2, X_4
2	180	51,683	54,206	40,917	26,973	15,224	36,585	49,920
3	230	44,787	50,632	38,924	25,856	14,286	29,944	41,735
4	280	31,684	38,603	31,089	20,779	11,134	20,811	30,227
5	330	21,152	27,678	22,811	15,395	8,303	13,234	19,991
6	380	14,376	19,999	17,095	11,802	6,138	8,760	13,452
7	430	10,119	14,275	12,845	8,873	4,633	6,101	9,280
8	480	6,846	9,996	9,110	6,620	3,465	3,949	6,330
9	530	4,661	6,912	6,723	4,745	2,442	2,658	4,275
10	580	3,181	4,960	4,866	3,439	1,851	1,817	2,951
11	630	2,281	3,452	3,511	2,534	1,320	1,285	2,066
12	680	1,513	2,339	2,519	1,833	1,043	892	1,461
13	730	1,007	1,590	1,753	1,365	740	597	895
14	780	754	1,177	1,275	924	542	429	641
15	830	489	760	823	724	407	319	457
16	880	339	595	598	514	291	201	337
17	930	244	393	452	355	214	149	233
18	980	192	282	352	260	170	106	137
19	1030	122	178	219	164	120	61	117
20	1080	84	152	152	156	82	40	70

Calculationsat $t = 230 \mu s$

$$f(Z_1) = 44,787$$

$$f(Z_2) = 50,632$$

$$f(Z_3) = 38,924$$

$$f(Z_4) = 25,856$$

$$f(Z_5) = 14,286$$

$$b_1 = \frac{1}{3} [0.5 \times 44,787 + 0.866 \times 50,632 + 1 \times 38,924 + 0.866 \times 25,856 + 0.5 \times 14,286]$$

$$= \frac{1}{3} [22,394 + 43,847 + 38,924 + 22,394 + 7,143]$$

$$b_1 = \frac{134,702}{3} = \underline{44,901}$$

$$b_2 = \frac{1}{3} [0.866 \times 44,787 + 0.866 \times 50,632 + 0 - 0.866 \times 25,856 - 0.866 \times 14,286]$$

$$= \frac{1}{3} [38,785 + 43,847 - 22,391 - 12,372]$$

$$= \frac{47,869}{3} = \underline{15,956}$$

$$b_3 = \frac{1}{3} [1 \times 44,787 + 0 - 1 \times 38,924 + 0 + 1 \times 14,286]$$

$$= \frac{1}{3} [44,787 - 38,924 + 14,286] = \frac{20,149}{3} = \underline{6,716}$$

$$b_4 = \frac{1}{3} [0.866 \times 44,787 - 0.866 \times 50,632 + 0 + 0.866 \times 25,856 - 0.866 \times 14,286]$$

$$= \frac{1}{3} [38,785 - 43,847 + 22,391 - 12,372]$$

$$= \frac{5,957}{3} = \underline{1,986}$$

$$\begin{aligned}
 b_5 &= \frac{1}{3} [0.5 \times 44,787 - 0.866 \times 50,632 + 38,924 \\
 &\quad - 0.866 \times 25,856 + 0.5 \times 14,286] \\
 &= \frac{1}{3} [22,394 - 43,847 + 38,924 - 22,391 + 7,143] \\
 &= \frac{2,223}{3} = \underline{741}
 \end{aligned}$$

$$t = 280 \mu s$$

$$\begin{array}{lll}
 f(Z_1) = 31,684 & 0.5 f(Z_1) = 15,842 & 0.866 f(Z_1) = 27,438 \\
 f(Z_2) = 38,603 & 0.5 f(Z_2) = 19,302 & 0.866 f(Z_2) = 33,430 \\
 f(Z_3) = 31,089 & & \\
 f(Z_4) = 20,779 & 0.5 f(Z_4) = 10,390 & 0.866 f(Z_4) = 17,995 \\
 f(Z_5) = 11,134 & 0.5 f(Z_5) = 5,567 & 0.866 f(Z_5) = 9,642
 \end{array}$$

$$\begin{aligned}
 b_1 &= \frac{1}{3} [15,842 + 33,430 + 31,089 + 17,995 + 5,567] \\
 &= \frac{103,923}{3} = \underline{34,641}
 \end{aligned}$$

$$\begin{aligned}
 b_2 &= \frac{1}{3} [27,438 + 33,430 - 17,995 - 9,642] \\
 &= \frac{33,231}{3} = \underline{11,077}
 \end{aligned}$$

$$\begin{aligned}
 b_3 &= \frac{1}{3} [31,684 - 31,089 + 11,134] \\
 &= \frac{11,729}{3} = \underline{3,910}
 \end{aligned}$$

$$\begin{aligned}
 b_4 &= \frac{1}{3} [27,438 - 33,430 + 17,995 - 9,642] \\
 &= \frac{2,361}{3} = \underline{787}
 \end{aligned}$$

$$\begin{aligned}
 b_5 &= \frac{1}{3} [15,842 - 33,430 + 31,089 - 17,995 + 5,567] \\
 &= \frac{1,073}{3} = \underline{358}
 \end{aligned}$$

$$t = 330 \mu s$$

$$\begin{array}{lll} f(Z_1) = 21,152 & 0.5 f(Z_1) = 10,576 & 0.866 f(Z_1) = 18,318 \\ f(Z_2) = 27,678 & 0.5 f(Z_2) = 13,839 & 0.866 f(Z_2) = 23,969 \\ f(Z_3) = 22,811 & & \\ f(Z_4) = 15,395 & 0.5 f(Z_4) = 7,698 & 0.866 f(Z_4) = 13,332 \\ f(Z_5) = 8,303 & 0.5 f(Z_5) = 4,152 & 0.866 f(Z_5) = 7,190 \end{array}$$

$$b_1 = \frac{1}{3} [10,576 + 23,969 + 22,811 + 13,332 + 4,152]$$

$$= \frac{74,840}{3} = 24,947$$

$$b_2 = \frac{1}{3} [18,318 + 23,969 - 13,332 - 7,190]$$

$$= \frac{21,765}{3} = 7,255$$

$$b_3 = \frac{1}{3} [21,152 - 22,811 + 8,303]$$

$$= \frac{6,644}{3} = 2,215$$

$$b_4 = \frac{1}{3} [18,318 - 23,967 + 13,332 - 7,190]$$

$$= \frac{491}{3} = 164$$

$$b_5 = \frac{1}{3} [10,576 - 23,969 + 22,811 - 13,332 + 4,152]$$

$$= \frac{238}{3} = 79$$

$$t = 380 \mu s$$

$$\begin{array}{lll} f(Z_1) = 14,376 & 0.5 f(Z_1) = 7,188 & 0.866 f(Z_1) = 12,450 \\ f(Z_2) = 19,999 & 0.5 f(Z_2) = 10,000 & 0.866 f(Z_2) = 17,319 \\ f(Z_3) = 17,095 & & \\ f(Z_4) = 11,802 & 0.5 f(Z_4) = 5,901 & 0.866 f(Z_4) = 10,221 \\ f(Z_5) = 6,138 & 0.5 f(Z_5) = 3,069 & 0.866 f(Z_5) = 5,316 \end{array}$$

$$b_1 = \frac{1}{3} [7,188 + 17,319 + 17,095 + 10,221 + 3,069]$$

$$= \frac{54,892}{3} = \underline{18,297}$$

$$b_2 = \frac{1}{3} [12,450 + 17,319 - 10,221 - 5,316]$$

$$= \frac{14,232}{3} = \underline{4,744}$$

$$b_3 = \frac{1}{3} [14,376 - 17,095 + 6,138]$$

$$= \frac{3,419}{3} = \underline{1,140}$$

$$t = \underline{480 \mu s}$$

$f(Z_1) = 6,846$	$0.5 f(Z_1) = 3,423$	$0.866 f(Z_1) = 5,929$
$f(Z_2) = 9,996$	$0.5 f(Z_2) = 4,998$	$0.866 f(Z_2) = 8,657$
$f(Z_3) = 9,110$		
$f(Z_4) = 6,620$	$0.5 f(Z_4) = 3,310$	$0.866 f(Z_4) = 5,733$
$f(Z_5) = 3,465$	$0.5 f(Z_5) = 1,733$	$0.866 f(Z_5) = 3,001$

$$b_1 = \frac{1}{3} [3,423 + 8,657 + 9,110 + 5,733 + 1,733]$$

$$= \frac{28,656}{3} = \underline{9,552}$$

$$b_2 = \frac{1}{3} [5,929 + 8,657 - 5,733 - 3,001]$$

$$= \frac{5,852}{3} = \underline{1,951}$$

$$b_3 = \frac{1}{3} [6,846 - 9,110 + 3,465]$$

$$= \frac{1,201}{3} = \underline{400}$$

$$t = 580 \mu s$$

$$\begin{array}{lll} f(Z_1) = 3,181 & 0.5 f(Z_1) = 1,591 & 0.866 f(Z_1) = 2,755 \\ f(Z_2) = 4,960 & 0.5 f(Z_2) = 2,480 & 0.866 f(Z_2) = 4,295 \\ f(Z_3) = 4,866 & & \\ f(Z_4) = 3,439 & 0.5 f(Z_4) = 1,720 & 0.866 f(Z_4) = 2,978 \\ f(Z_5) = 1,851 & 0.5 f(Z_5) = 926 & 0.866 f(Z_5) = 1,603 \end{array}$$

$$b_1 = \frac{1}{3} [1,591 + 4,295 + 4,866 + 2,978 + 926]$$

$$= \frac{14,656}{3} = \underline{4,885}$$

$$b_2 = \frac{1}{3} [2,755 + 4,295 - 2,978 - 1,603]$$

$$= \frac{2,469}{3} = \underline{823}$$

$$b_3 = \frac{1}{3} [3,181 - 4,866 + 1,851]$$

$$= \frac{166}{3} = \underline{55}$$

Buckling Calculations

Sample 22.9 x 22.9 x 22.9 cm

2t = 0.7 cm

1, 1, 1 mode

$$B_{111}^2 = \left(\frac{\pi}{23.6}\right)^2 + \left(\frac{\pi}{23.6}\right)^2 + \left(\frac{\pi}{23.6}\right)^2 = 3\left(\frac{\pi}{23.6}\right)^2$$

$$B_{111}^2 = 3(0.1330)^2 = 3 \times 1.770(10^{-2}) = 0.05310 \text{ cm}^{-2}$$

1, 1, 2 mode

$$B_{112}^2 = \left(\frac{\pi}{23.6}\right)^2 + \left(\frac{\pi}{23.6}\right)^2 + \left(\frac{2\pi}{23.6}\right)^2 = 6\left(\frac{\pi}{23.6}\right)^2$$

$$B_{112}^2 = 6 \times 1.770(10^{-2}) = 0.1062 \text{ cm}^{-2}$$

1, 1, 3 mode

$$B_{113}^2 = \left(\frac{\pi}{23.6}\right)^2 + \left(\frac{\pi}{23.6}\right)^2 + \left(\frac{3\pi}{23.6}\right)^2 = 11\left(\frac{\pi}{23.6}\right)^2$$

$$B_{113}^2 = 11 \times 1.770(10^{-2}) = 0.1947 \text{ cm}^{-2}$$

Fig. S17.1

Relative Flux as a Function
of Time and Space

Fig. S17.2
Relative Flux as a Function
of Time and Position

Fig. S17.3
Decay of Individual Modes

Fig. S17.4
Decay Constant versus Buckling

MATERIAL BUCKLING MEASUREMENT IN A
D₂O, NATURAL URANIUM EXPONENTIAL ASSEMBLY

by

K. C. Ruzich

ABSTRACT

The material buckling of the heavy water, natural uranium subcritical assembly, with a fixed composition and lattice spacing, is determined by measuring the spatial distribution of the neutron flux. The assembly is in a steady-state condition, multiplying neutrons which originate from a neutron source properly placed near the assembly. The buckling determination yields the minimum critical volume of a neutron-chain reactor with the same composition and structure as the exponential assembly.

ACKNOWLEDGMENT

This experiment is a revision of an early institute experiment prepared and written by A. B. Smith.

OUTLINE

- I. INTRODUCTION
- II. THEORY
 - A. General Theory
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I. INTRODUCTION

The exponential experiment historically has been used to study optimum fuel arrangements for heterogeneous nuclear reactors. The assembly which is studied cannot sustain a neutron-chain reaction because of its small size. The leakage rate of neutrons from the system is too large. Since the assembly is subcritical, neutrons must be supplied to the system from an external neutron source to establish a steady-state neutron spatial distribution. The assembly may be constructed in either rectangular or cylindrical geometry. The neutron source is placed below the structure, and the source neutrons pass through moderating material before entering the assembly. This arrangement gives rise to the condition that the neutron source can be represented by a finite plane source of thermal neutrons located at the bottom face of the assembly. The neutron distribution in the subcritical system in the direction perpendicular to the plane source is described as an exponential. The measurement of the decay constant of this exponential will yield a parameter related to the critical volume of the assembly. The parameter involved is called the material buckling of the assembly, and is a function of the composition and lattice arrangement of fuel and moderator in the system.

The assembly used in this experiment is a natural uranium, heavy water system. The cylindrical tank containing the heavy water stands on a graphite pedestal. The neutron source (Sb-Be) lies below the pedestal. High-energy neutrons are moderated to thermal velocities in the carbon pedestal before entering the assembly. The natural uranium fuel in metallic

form is contained in one-inch-diameter aluminum rods, positioned in the D₂O tank in a 4.9-in. square lattice array. Spatial neutron-distribution measurements are made in the vertical and radial directions by traversing mechanisms with BF₃ detectors.

II. THEORY

A. General Theory: The model chosen to describe the neutron behavior in the assembly is taken from basic diffusion theory. The production rate of neutrons in a homogeneous volume element minus the leakage rate from the element and the absorption rate occurring within the elements must be equal to the change in neutron density with time. Hence, the mono-energetic diffusion equation may be written as

$$\left. \begin{array}{l} D \nabla^2 \phi(r,t) - \Sigma_a \phi(r,t) + k \Sigma_a \phi(r,t) + S(r) = \\ \frac{\text{neutrons leaking out}}{\text{cm}^3\text{-sec}} - \frac{\text{neutrons absorbed}}{\text{cm}^3\text{-sec}} + \frac{\text{neutrons produced}}{\text{cm}^3\text{-sec}} \\ \frac{1}{v} \frac{\partial \phi(r,t)}{\partial t} \\ \frac{\text{time rate change of neutrons}}{\text{cm}^3} \end{array} \right\} \quad (1)$$

where

D = diffusion coefficient of medium

∇^2 = Laplacian operator

$\phi(r,t)$ = thermal neutron flux at position r and time t

Σ_a = macroscopic absorption cross section of medium

k = infinite reproduction factor of medium

$S(r)$ = external source contribution at r

v = mono-velocity of thermal neutrons

The spatial distribution of $\phi(r,t)$ can be experimentally determined. This measurement can then be compared with solutions of equation (1) to obtain the characteristic parameters of the system.

In order to solve equation (1), we must separate the variables in space and time. We assume

$$\phi(r,t) = \Psi(r) T(t).$$

Since the homogeneous time-independent portion of (1) is the Laplacian

$$\nabla^2 \Psi(r) + B^2 \Psi(r) = 0, \quad (2)$$

the general solution to equation (2) will be a linear combination of all possible solutions:

$$\phi(r,t) = \sum_n A_n \Psi_n(r) T_n(t). \quad (3)$$

The source term is then expanded in a series of orthogonal functions:

$$S(r) = \sum_n S_n \Psi_n(r) \quad (4)$$

where Ψ_n are eigenfunctions of equation (2) corresponding to eigenvalues B_n . Inserting equations (4) and (3) into equation (1) and integrating, one obtains

$$\phi(r,t) = \sum_n \left[A_n \Psi_n(r) e^{\left[(k_{\text{eff}})_n - 1 \right] (t/\ell_n)} + \frac{S_n \Psi_n}{\Sigma_a (1 + L^2 B_n^2) [1 - (k_{\text{eff}})_n]} \right], \quad (5)$$

where

$$(k_{\text{eff}})_n = k / (1 + L^2 B_n^2) = \text{effective reproduction factor for } n\text{th mode.}$$

$$\frac{1}{1 + L^2 B_n^2} \text{ is the nonleakage probability of this mode.}$$

$$L^2 = D / \Sigma_a = \text{diffusion length}$$

$$\ell_n = \ell_0 / (1 + L^2 B_n^2) = \text{effective neutron lifetime for } n\text{th mode}$$

$$\ell_0 = 1 / v \Sigma_a = \text{the neutron lifetime in an infinite medium.}$$

The solution for $\phi(r,t)$ indicates that as long as all the reproduction factors are less than one, the assembly will be subcritical, and the only way to maintain a steady-state condition is to have an external source present. The steady-state value is represented by the second term in the right side of equation (5). If the assembly of fuel and moderator is increased in size, the B_n 's will decrease, and a point will be reached when the lowest B_n , namely, B_1 , will be such that the $(k_{\text{eff}})_1$ is exactly unity. Since B_1 is the lowest eigenvalue, $(k_{\text{eff}})_1$ will be larger than all other possible values of $(k_{\text{eff}})_n$. With the source removed, the second term in equation (5) is zero, the system would be self-sustaining on the lowest eigenvalue, $(k_{\text{eff}})_1 = 1$, and the higher modes would decay to zero, since all other $(k_{\text{eff}})_n$'s are less than one.

The critical equation which represents the condition for a self-sustaining chain reaction is

$$k_{\text{eff}} = \frac{1}{1 + L^2 B_m^2} = 1, \quad (6)$$

as seen from equation (5). The quantity B_m is defined from this equation and is called the material buckling, since it is a function of the multiplying medium only. The geometrical buckling, $B_1 = B_g$, is the lowest eigenvalue that results from solving the wave equation (2). It is evident that if the geometrical buckling of the system equals the material buckling, the system has reached a critical condition.

B. Cylindrical Core Reactor: The critical-state solution of equation (2) must satisfy the boundary conditions of the system. If we consider a bare cylindrical reactor which is critical (no source present), we require physically that $\phi > 0$ within the assembly and equal to 0 at the extrapolated boundaries. Equation (2) in cylindrical coordinates is (see Fig. 18.1)

$$\frac{\partial^2 \Psi}{\partial r^2} + \frac{1}{r} \frac{\partial \Psi}{\partial r} + \frac{\partial^2 \Psi}{\partial z^2} + B_g^2 \Psi = 0. \quad (7)$$

With proper boundary conditions, the solution to equation (7) becomes

$$\Psi(r, z) = A J_0 \left[\frac{2.405 r}{R} \right] \cos \frac{\pi z}{H} \quad (8)$$

and

$$B_g^2 = \left(\frac{2.405}{R} \right)^2 + \left(\frac{\pi}{H} \right)^2,$$

where R and H are the radius and height of the reactor including an extrapolation distance. Since the reactor is critical,

$$B_g^2 = B_m^2 = \left(\frac{2.405}{R} \right)^2 + \left(\frac{\pi}{H} \right)^2. \quad (9)$$

The minimum critical volume for which a cylindrical reactor with a certain B_g^2 can be determined by the Lagrange multiplier method. The minimum value of $\pi R^2 H$ is subject to constraint given by equation (9) and lends

$$V_{\text{min}} = 148.2 / B_g^3.$$

C. Cylindrical Exponential Assembly: In the exponential assembly the neutron production is not sufficient to overcome the leakage loss of neutrons. Hence, the neutron level must be maintained by an external source (see Fig. 18.2). As stated in Section B, we solve equation (2) for its eigenvalues. The boundary conditions again demand that the flux is zero at the extrapolated boundaries of the assembly and that the current flowing into the assembly from the pedestal must be equal to that provided by the source.

Fig. 18.2

Cylindrical Exponential Assembly

The exact form of the solution is strongly dependent on the geometry. It can be shown that for a large subcritical assembly the thermal-neutron balance in a region far from the source, where essentially all the neutrons have resulted from fissions, can be represented by equation (1) with the source term $S(r)$ equal to zero. The wave equation for the flux distribution is given to a good approximation by

$$\nabla^2 \Psi(r) + B_m^2 \Psi(r) = 0, \quad (10)$$

where B_m^2 is the material buckling of the multiplying medium of which the assembly is composed.

In order to solve the wave equation in cylindrical geometry with proper boundary conditions, we first separate variables in the radial and axial directions:

$$\Psi = \theta(r)Z(z).$$

The solution is

$$\Psi(r, z) = \sum_{n=1}^{\infty} A_n J_0(\alpha_n r) \sinh \gamma_n (H - z). \quad (11)$$

Since measurements are made at distances far from the bottom face of the assembly, the flux distribution can be represented with accuracy by the fundamental term ($n = 1$); equation (11) may then be rewritten as

$$\Psi(r, z) = A J_0(\alpha r) \sinh \gamma (H - z)$$

and

$$-\alpha^2 + \gamma^2 + B_m^2 = 0.$$

The radial distribution is

$$\theta(r) = A' J_0(\alpha r),$$

and from the boundary condition that the flux shall be zero at the extrapolated surface of the cylinder,

$$\alpha = 2.405/R,$$

where R is the extrapolated radius. The vertical distribution is

$$Z(z) = C' \sinh \gamma(H - z),$$

where H is the extrapolated height of the experimental cylinder. This can be rewritten as

$$Z(z) = C'' e^{-\gamma Z} [1 - e^{-2\gamma(H-z)}].$$

For measurements made where the term in brackets is approximately one, the distribution in the axial direction can be represented by a single exponential.

Finally,

$$\begin{aligned} B_m^2 &= \alpha^2 - \gamma^2 \\ &= [2.405/R]^2 - \gamma^2. \end{aligned}$$

The critical size of a reactor with the same material buckling as the exponential assembly will be

$$V_m = 148.2/B_g^3 = 148.2/B_m^3.$$

D. Measurements and Calculations of Reactor Parameters: The one-group (all thermal) model chosen to describe neutron behavior in a multiplying system yields the following critical equation:

$$\frac{k}{1 + L^2 B_m^2} = 1.$$

This model is only an approximation, since neutrons which are produced in the system are born as fast neutrons in the fission process. These neutrons then migrate through the medium making collisions with the atoms present in the system. During this slowing-down process some of the neutrons will leak out of the system. The distribution of the neutrons in the system during the slowing-down process will determine the probability of loss by leakage.

A more exact model of the system should therefore include this slowing-down process. The approach which is usually taken for a bare system is to assume that the slowing-down neutron density is everywhere proportional to the thermal-neutron flux. It can be shown that the critical equation then becomes

$$\frac{k\mathcal{L}_f}{1 + L^2B^2} = 1,$$

where $\mathcal{L}_f$ is the fast nonleakage probability, which is a function of the buckling of the assembly. Below is a list of fast nonleakage probabilities based on specific slowing-down models:

- (i) Fermi continuous slowing-down model (for heavy moderating materials):

$$\mathcal{L}_f = e^{-B^2\tau},$$

where τ is the neutron age from fission energies to the thermal energy;

- (ii) two-group model:

$$\mathcal{L}_f = \frac{1}{1 + L_f^2B^2},$$

where L_f^2 , the square of the fast diffusion length, equals $1/6$ of the mean square distance a neutron travels before becoming thermal;

- (iii) multigroups of neutrons:

$$\mathcal{L}_f = \frac{1}{(1 + L_1^2B^2)(1 + L_2^2B^2)(1 + L_3B^2)\dots}$$

where L_i is the fast diffusion length for neutrons in the i th energy group before they are slowed down into the energy group $(i + 1)$.

The two-group model is a fairly good representation of the critical equation for a heavy water system. Hence, the critical equation can be written as

$$\frac{k_\infty}{(1 + L^2B^2)(1 + L_f^2B^2)} = 1.$$

In the exponential experiment the material buckling for a critical reactor of the same composition and structure was measured. Calculations can now be made to determine L , L_f , or k if any two of these parameters are known. The infinite reproduction, k_∞ , is a function of the material of which the reactor is made and equals $\eta\epsilon pf$, as given by the "four-factor formula." Here,

$\eta \equiv$ neutrons produced/neutrons absorbed in fuel;

$\epsilon \equiv$ fast fission factor = $\frac{\text{fast neutrons produced by all fissions}}{\text{fast neutrons produced by thermal fissions}}$;

$p \equiv$ resonance escape probability, the fraction of neutrons escaping parasitic capture while slowing down;

$f \equiv$ thermal utilization, the fraction of thermal neutrons absorbed in fuel.

The calculation of these factors for the reactor involved is by no means simple because the reactor is a heterogeneous system. The evaluation of k is usually based on a combination of theory and experiment. Calculations based on the heterogeneous system in question yield the following approximate values for the four-factor parameters:

$$\eta = 1.328; \quad \epsilon \approx 1.03; \quad p = 0.912; \quad f = 0.970.$$

The diffusion length for the thermal neutrons is defined as

$$L^2 = D/\bar{\Sigma}_a = 140 \text{ cm}^2,$$

where

$\bar{\Sigma}_a$ = average macroscopic absorption cross section;

D = diffusion coefficient.

The fast diffusion length of the system is assumed to be the following:

$$L_f^2 = 130 \text{ cm}^2.$$

The measured value of the material buckling for the D_2O natural uranium exponential assembly is

$$B_m^2 = 7.6 \times 10^{-4} \text{ cm}^{-2}.$$

With these representative values, the following reactor parameters are calculated for a critical reactor with the same composition and lattice structure as the exponential assembly:

1. infinite multiplication factor $k = 1.22$;

2. fast nonleakage probability $\frac{1}{1 + L_f^2 B_m^2} = 0.911;$
3. slow nonleakage probability $\frac{1}{1 + L^2 B_m^2} = 0.905;$
4. minimum critical volume $148.2/B_m^3 = 7.90 \times 10^6 \text{ cm}^3.$

Experimentally, the exponential pile can be used to obtain the fast diffusion length L_f ; the diffusion length L ; and the average macroscopic absorption cross section in the specific lattice of fuel and moderator. The fast diffusion length (age) of fission neutrons is found by comparing material buckling with and without a uniformly disposed poison such as boron. The diffusion length is determined by substituting nonmultiplying fuel elements with the same scattering and absorption properties as the fuel. This procedure makes the assembly a sigma pile, which is normally used for measurements of thermal diffusion length. The average macroscopic absorption cross section, $\bar{\Sigma}_a$, of the assembly can be measured by a simple cadmium ratio measurement of an activation detector. If the response of the detector is known, the cadmium ratio measurement yields the ratio of the thermal-neutron flux to the slowing-down neutron flux per log (e) interval. Using a variation of the basic neutron-balance equation [equation (1)], $\bar{\Sigma}_a$ can be determined.

III. APPARATUS

A. Source: As is clear from the theoretical outline, an exponential assembly requires an external source of neutrons to maintain a steady state. Ideally, the energy distribution of these source neutrons should be equivalent to the neutron spectrum of the lattice. This is approximately realized in practice by the introduction of thermal neutrons into the assembly.

The exponential utilizes a photoneutron source, based on the reaction

In order for this nuclear event to occur, the gamma ray must have a minimum (threshold) energy of 1.66 MeV. A practical source consists of a substance emitting gamma rays with energies greater than threshold, surrounded by an amount of beryllium. Unfortunately, the photoneutron type of reaction is rather common. If one is to avoid producing source neutrons in an unknown manner directly within the multiplying medium, care must be taken in the selection of the gamma emitter. This is particularly true when the moderating material is D_2O , since its photoneutron threshold lies at 2.18 MeV.

A convenient gamma emitter producing copious radiation with the energy above the 1.66-MeV threshold of beryllium but less than the 2.1-MeV threshold of D_2O is Sb^{124} . Nearly 50% of its disintegrations pass through the above range of energies, and the rate of decay ($\tau_{1/2} = 60$ d) is slow enough to provide usefulness over a considerable length of time. Antimony occurs naturally in two isotopic forms: Sb^{121} (58%) and Sb^{123} (42%). If the natural metal is exposed to a slow-neutron flux, neutron capture leads to the formation of Sb^{122} ($\sigma_c \sim 5.7$ b) and Sb^{124} ($\sigma_a \sim 2.5$ b). The former is of short life and emits only low-energy radiation; it is thus of no interest to us. However, very intense sources of Sb^{124} can be made in a flux of slow neutrons such as is available at CP-5. Its 1.7-MeV gamma ray, when incident on beryllium, produces neutrons with an energy of several keV. These relatively low-energy neutrons are moderated to thermal velocities in the carbon pedestal before entrance into the multiplying medium.

The actual source is a right cylinder of antimony, $1\frac{1}{2}$ -in., high and 1-in. in diameter, placed within a $2\frac{3}{4} \times 2\frac{3}{4} \times 3\frac{1}{8}$ -in. block of beryllium. The intensity of the neutron source obtained with a given amount of material is easily calculable. From the particular construction used here, one obtains 10^8 n/sec from 5 C of Sb^{124} . This amount of radiation must be handled with care. The source is entirely removed from the assembly by sliding the carbon source stringer out of the pedestal and back into the lead source cave. A view of this stringer with the source in place is shown in Fig. 18.3.

139-124

Fig. 18.3. Source and Stringer. Antimony is shown removed from the beryllium block.

B. Fuel Assemblies: The basic component is a natural uranium cylinder, 1-in. in diameter and 8-in. long. These are placed within a 0.040-in.-thick aluminum tube so as to form an aluminum-clad natural uranium rod. The weight of each slug of uranium is 1.97 kg. Eight are contained in the fuel rod. The total fuel content is tabulated as follows:

Number of Elements	Fuel Content	
	Weight U/element, kg	Weight Total U/assembly, kg
112	15.76	1765.1

A typical fuel assembly and its components is shown in Fig. 18.4.

139-113

Fig. 18.4. Fuel Rod and Components

C. D₂O Exponential: The D₂O exponential consists of an aluminum tank, 5 ft in diameter and 6 ft tall, containing a natural uranium lattice. This unit rests upon a carbon pedestal. Below and to one side of the exponential tank is placed a rectangular storage container for the heavy water. A pumping system permits the transferring of the D₂O moderator from the storage tank to the exponential. The arrangement is shown in Figs. 18.5 and 18.6.

The fuel elements used are of the type described in Section B above. Each rod is held in a vertical position by aluminum grid plates located at the top and bottom of the tank. These plates determine the geometry of the particular lattice. In this experiment we use a square array measuring 4.9 in. on a side. Figure 18.7 shows the grid plate and fuel lattice before the tank cover is in place.

Any possible outside perturbations are excluded by wrapping the assembly with cadmium. This gives the "black body" boundary condition to be used in the calculations. Since D₂O is expensive, precautions are taken to avoid loss or contamination. The exponential tank is surrounded by an aluminum catch basin to contain any accidental "spill." The entire system is closed to prevent any loss by evaporation or contamination due to H₂O from the atmosphere. Approximately 3,000 kg D₂O are needed in the experiment.

Measurements in the horizontal plane are carried out in one of the three apertures provided for that purpose. These are "through tubes"

Fig. 18.5. Exponential Exp. Layout Bldg. 24

139-135

Fig. 18.6. D₂O Exponential

139-106

Fig. 18.7. D₂O Lattice

which isolate the detector from the D_2O . A selection of similar apertures is available for vertical measurements. In using the detectors, care must be taken not to distort the "through tubes," since the possibility of fracture is always present. By using these tubes it is at no time necessary to expose the D_2O . The relation of the lattice to the detector tubes is shown in Fig. 18.5. The water level is measured in relation to the physical boundary of the tank with a micrometer and a liquid-level sight gauge.

D. Neutron Detectors: The detectors used to measure the spatial neutron distribution in the D_2O assembly are of two types: a current chamber and a proportional counter. In both types the neutron response depends on the reaction

The large thermal-neutron cross section for this event makes the instruments exceedingly sensitive. The response of the boron-coated current chamber is measured as a small ionization current which is proportional to the neutron density at the chamber location. The second type of detector, a BF_3 proportional counter, is a pulse counter. Each event (reaction) occurring in the counter yields a short voltage pulse which is recorded on a conventional scaler. The rate at which these events occur is proportional to the neutron density at the position of the counter.

Both types of detectors are positioned within the assembly by means of remotely controlled drive units. The chamber movement is controlled as to speed and position from a central panel. Electrical stops prohibit overdriving the chamber in either direction. The position of each detector is accurately indicated, in inches, by the control panel dials.

IV. PROCEDURE

The initial part of the experimental procedure will include a visual inspection of the experimental assembly and a short discussion describing the apparatus which will be used in the experiment. Questions regarding the experimental arrangement will be answered by the instructor at this time.

The procedure for measuring the radial and vertical neutron flux distribution follows:

A. Radial Distribution: The measurement is made with the B^{10} current chamber. After turning on the drive units, current amplifier, and chart mechanism, move the detector to one extremity of its travel. With the chart recorder open, simultaneously mark the chart and start the chamber moving through the assembly at a slow speed. You will see the pen record a variation in neutron flux as you move through the exponential. Watch carefully as

your chamber nears the opposite extremity of its travel. Mark the chart when the drive unit stops. You now have a "ready made" plot of the neutron distribution within the exponential. Remove the section of chart for later analysis. The instructor will furnish you with a value for the distance between stops on the drive unit. This will allow you to calibrate the abscissa of your flux traverse.

B. Vertical Distribution: The BF_3 counter is connected to the same type of a drive unit described above and positioned with identical controls. As pointed out previously, this is a pulse counter, so individual measurements must be made at selected points. You may choose your locations for flux measurements in a manner that seems to you to give the best determination of the neutron distribution. The counts taken over a selected time interval at the different counter positions are recorded in a MPC scaler.

V. RESULTS

The results of the experiment will include the following:

1. a linear plot of the radial neutron flux distribution normalized to unity at the center;
2. a semilog plot of the vertical neutron-flux distribution normalized to unity at some arbitrary height;
3. the calculations and resulting material buckling of the exponential assembly.

The experimental neutron-flux distribution normalized as suggested will be fitted to the theoretical distributions given below:

$$\theta(r) = \theta(0)J_0(\alpha r)$$

and

$$Z(z_i) = Z(z_0)e^{-\gamma(z_i-z_0)},$$

where $\theta(0)$ and $Z(z_0)$ equal unity. The constants α and γ can now be easily determined for each experimental point the student chooses with the aid of Bessel and exponential function tables. An average value of α and γ will be used to calculate the material buckling:

$$B_m^2 = \alpha^2 - \gamma^2.$$

4. the minimum critical volume for a reactor of the same composition and lattice arrangement as the exponential assembly is calculated:

$$V_{\min}(\text{cylinder}) = 148.2/B_m^3.$$

VI. ANALYSIS AND CONCLUSIONS

A comparison of the result of experimental measurement of material buckling can be made with the correct value listed in the theory section. Any discrepancies between these two results should be analyzed to determine the possible sources of error. Comments regarding the experiment both from theoretical and experimental standpoints are welcomed by the instructor. This experiment was developed for the purpose of acquainting students with the use of an exponential assembly to determine certain reactor parameters. The theoretical model chosen to describe the experiment should familiarize the students with basic diffusion theory.

VII. BIBLIOGRAPHY

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VIII. APPENDICESA. Discussion Questions

1. Describe an exponential assembly. How does this system differ from a critical reactor?
2. Describe an exponential experiment for determining the material buckling of a certain multiplying medium. What basic theoretical model can be used to describe the system in question? Define the parameters used in this theoretical model.
3. If the exponential experiment yields a material buckling which is negative, what can be said about the multiplying system and why?
4. If the dimensions of an exponential assembly are increased, does the result of measurement of material buckling increase, decrease, or remain constant?
5. What boundary conditions must prevail for the exponential experiment?
6. What is the critical-reactor equation? What conditions must prevail to make a multiplying medium of a given size and geometry critical?
7. Derive the neutron-flux distributions in a cylindrical-base critical reactor where R and H are the radius and height of the reactor including the proper extrapolation distance. Start with the basic diffusion equation in the general theory section with the external source term equal to zero (equation 1).
8. What is the geometrical buckling for the system described in Question 7? What is the minimum critical volume for the reactor as a function of the buckling?
9. The extrapolated height and radius of the exponential assembly used in this experiment are 168.0 cm and 77.9 cm, respectively. The effective reproduction factor of the fundamental mode of the wave equation is

$$k_{\text{eff}} = \frac{k}{(1 + B_1^2 L^2)(1 + B_1^2 L_f^2)}$$

How subcritical is the system in question? (The reactor parameters for this assembly are given in Section II, D.)

10. What type of neutron source is used in this experiment to obtain a steady-state condition in the assembly? What other types of sources could be used?

11. What is a heterogeneous reactor? Why must this type of a system be treated differently from a homogeneous system?
12. What type of neutron detectors are used in this experiment? What basic reaction is utilized in both detector types?

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STUDENT RESULTS

Calculation of α

$$\phi(r) = \phi_0 J_0(\alpha r).$$

Then,

$$\alpha = \frac{J_0^{-1}[\phi_r/\phi_0]}{r}.$$

ϕ_r	ϕ_r/ϕ_0	r , cm	$J_0^{-1}[\phi_r/\phi_0]$	α
1	0.925	6.30	0.553	0.0878
2	0.931	6.30	0.530	0.0841
3	0.825	9.90	0.856	0.0865
4	0.839	9.90	0.819	0.0827
5	0.660	13.50	1.222	0.0905
6	0.700	13.50	1.141	0.0845
7	0.480	18.00	1.556	0.0864
8	0.465	18.00	1.583	0.0879
9	0.298	22.5	1.872	0.0832
10	0.250	22.5	1.955	0.0869

$$\bar{\alpha} = 0.0861 \text{ 1/in.} = 0.0339 \text{ 1/cm}$$

Since

$$\bar{\alpha} = 2.405/R$$

we find

$$R = 27.93 \text{ in.} = 70.94 \text{ cm.}$$

Fig. S18.1
Radial Flux
Distribution

Fig. S18.2. Vertical Flux Distribution

DATAVertical Flux Distribution

Sensitivity of Amp. 0.04

Discriminator 6.27

Position, in.	Counts/2 min	Selected Data Pts.
6	32,744	1
12	25,785	2
18	20,170	3
24	15,907	4
30	12,382	5
36	9,692	6
42	6,999	-
48	4,734	-
54	2,832	-
60	1,003	-
66	173	-
72	130	-

Calculation of γ

$$Z(z_i) = Z(z_0) e^{-\gamma(z_i - z_0)}$$

$$\gamma = \frac{\ln Z(z_i)/Z(z_0)}{Z_0 - Z_i}$$

Data	Ratio $Z(z_i)/Z(z_0)$	ln Ratio	$Z_0 - Z_i$	γ
2/1	0.787	0.2395	6 in.	0.0399
3/1	0.616	0.4845	12 in.	0.0404
4/1	0.486	0.7215	18 in.	0.0401
5/1	0.378	0.9729	24 in.	0.0405
6/1	0.296	1.2174	30 in.	0.0406
3/2	0.782	0.2459	6 in.	0.0410
4/2	0.617	0.9829	12 in.	0.0402
5/2	0.480	0.7339	18 in.	0.0408
6/2	0.376	0.9782	24 in.	0.0408
4/3	0.789	0.2365	6 in.	0.0394
5/3	0.614	0.4878	12 in.	0.0406
6/3	0.481	0.7319	18 in.	0.0407
5/4	0.778	0.2510	6 in.	0.0418
6/4	0.609	0.4959	12 in.	0.0413
6/5	0.783	0.2446	6 in.	0.0408

Average = 0.0405 1/in.

 B_m^2 Calculations

$$B_m^2 = \alpha^2 - \gamma^2$$

$$= (0.0861)^2 - (0.04053)^2$$

$$= 0.007413 - 0.001643 = 0.005770 \text{ 1/in.}^2$$

$$B_m^2 = 0.00577 \text{ 1/in.}^2 = 0.000894 \text{ 1/cm}^2$$

$$B_m = 0.0760 \text{ 1/in.} = 0.0299 \text{ 1/cm}$$

Minimum Critical Volume

$$V_{\min} = \frac{148.2}{B_m^3} = \frac{148.2}{4.39 \times 10^{-4}}$$

$$V_{\min} = 3.36 \times 10^5 \text{ in.}^3 = 5.51 \times 10^6 \text{ cm}^3$$

$$\text{Actual Volume} = \pi r^2 h = \pi (75.5)^2 (164.5) = 2.94 \times 10^6 \text{ cm}^3$$

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THERMAL-NEUTRON DIFFUSION LENGTH IN WATER

by

L. L. Lawyer

ABSTRACT

This experiment is a measurement of thermal-neutron diffusion length in H_2O . This measurement utilizes reactor leakage neutrons as a plane source, indium foils as thermal-neutron detectors, and cadmium absorption coupled with the indium resonance as an epithermal neutron detector. The thermal-neutron diffusion length is evaluated from the parameters of a cadmium-covered indium distribution.

OUTLINE

- I. INTRODUCTION
- II. THEORY
 - A. General
 - B. Infinite Plane Source in Infinite Medium
 - C. Physical Significance of Diffusion Length and Relation to Relaxation Length
 - D. Effect of Finite Dimensions
- III. APPARATUS
- IV. PROCEDURE
- V. RESULTS
- VI. ANALYSIS OF RESULTS AND CONCLUSIONS
- VII. BIBLIOGRAPHY
- VIII. APPENDIX
 - Discussion Questions

I. INTRODUCTION

A neutron life cycle in a thermal reactor moderator or reflector may be divided on an energy basis into two main categories: thermal and epithermal. The distance a neutron travels while its energy is in the thermal portion of this cycle is of prime importance. The thermal-neutron diffusion length is proportional to this distance.

Due, among other things, to the widespread usage of H_2O as a moderator and reflector in current reactors, we have chosen it as the diffusing medium.

Experimental methods of measuring thermal-neutron diffusion length can be separated into two main divisions. The first is the spatial variation of neutron flux from a steady source, and the second is the time variation of neutron flux following a pulse of neutrons. The former method will be employed in this experiment.

II. THEORY

A. General: The general diffusion equation is

$$D\nabla^2\phi - \Sigma_a\phi + S = \frac{\partial n}{\partial t}, \quad (1)$$

where $-D\nabla^2\phi$ is the neutron leakage per unit volume per second, $\Sigma_a\phi$ is the neutron absorption per cm^3 per second, and S is the rate of neutron production.

We concern ourselves with the special case in which the system is in a steady state and the time rate of change of neutron density is zero. In addition, we will restrict ourselves to the region outside the source ($S = 0$). Under these conditions, Eq. (1) becomes

$$D\nabla^2\phi - \Sigma_a\phi = 0$$

or

$$\nabla^2\phi - K^2\phi = 0 \quad (\text{wave equation}),$$

where $K^2 = \Sigma_a/D$ and has units of reciprocal length squared.

B. Infinite Plane Source in Infinite Medium: Assuming that the source is infinite in the x and y planes, then at any given value of z from the source the flux will be independent of x and y , so that ∇^2 will be d^2/dz^2 , and the diffusion equation can be written

$$\frac{d^2\phi}{dz^2} - K^2\phi = 0, \quad (2)$$

with the boundary conditions:

1. the flux must be finite except at $z = 0$;
2. the net current density at the source is one-half the source strength.

The general solution of Eq. (2) can then be written as

$$\phi = Ae^{-Kz} + Ce^{Kz}. \quad (3)$$

By imposing the above boundary conditions, C and A can be evaluated to give $C = 0$ and $A = 1/2KD$. Introducing these results, Eq. (3) becomes

$$\phi = e^{-Kz}/2KD, \quad (4)$$

which describes the steady-state flux distribution in the z direction from an infinite plane source emitting one neutron per cm^2 per second into an infinite medium. We now define the thermal-neutron diffusion length, L , as

$$L = \frac{1}{K} = \sqrt{\frac{D}{\Sigma_a}}. \quad (5)$$

C. Physical Significance of Diffusion Length and Relation to Relaxation Length: To give this quantity physical significance, we consider the mean square distance traveled by a neutron from the point at which it just becomes thermal to the point at which it is absorbed. This is just the second spatial moment of the flux distribution given by Eq. (4).

If ϕ is the flux in neutrons per cm^2 per second at a distance z from the plane source, then the rate at which neutrons will be absorbed in the medium will be equal to $\Sigma_a \phi$ per centimeter cubed per second, where Σ_a is the macroscopic absorption cross section of the medium. In an element of area A , and thickness dz at a distance z from the plane source, the number of neutrons captured per second will be $\Sigma_a \phi A dz$. This is a measure of the probability that a neutron will be absorbed within the element. Therefore, the mean square distance that a neutron travels from its source to where it is absorbed is given by

$$\overline{z^2} = \frac{\int_0^\infty z^2 \Sigma_a \phi A dz}{\int_0^\infty \Sigma_a \phi A dz} \quad (6)$$

Substituting the spatial relationship for the flux as given by Eq. (4), we have

$$\frac{\int_0^\infty z^2 \Sigma_a \frac{e^{-Kz}}{2KD} A dz}{\int_0^\infty \Sigma_a \frac{e^{-Kz}}{2KD} A dz} = \frac{2/K^3}{1/K} = \frac{2}{K^2} \quad (7)$$

Therefore

$$L^2 = \frac{1}{K^2} = \frac{1}{2} \overline{z^2} \quad (8)$$

Also, since $1/L = K$, Eq. (4) may be written in the form

$$\phi = A e^{-z/L}, \quad (9)$$

in which form we see that the diffusion length is equivalent to the "relaxation length," i.e., the distance in which the neutron flux is decreased by a factor of e .

D. Effect of Finite Dimensions: If it were possible for us to duplicate experimentally an infinite plane source in an infinite medium, we could determine the thermal-neutron diffusion length from a set of measurements of the spatial variation of the neutron flux. From Eq. (9), we see that

$\frac{d \ln \phi}{dz} = \frac{-1}{L}$; therefore a log plot of the spatial variation of the neutron flux from an infinite plane source into an infinite medium against z , the distance from the source, should yield a straight line of slope $-1/L$. However, practical measurements must be made with a finite source in a finite medium, and the analysis of the results is therefore not as simple as indicated by our consideration of Eq. (9).

In a manner very similar to that in which we derived Eq. (4), it is shown in Reference 3 that for thermal neutrons diffusing into a long rectangular parallelepiped of moderator, the flux distribution along any line in the z direction that is not too close to a boundary is given by

$$\phi = (\text{const}) x e^{-\gamma z}, \quad (10)$$

where γ is related to the dimensions of the block by

$$\gamma^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \frac{1}{L^2}, \quad (11)$$

with a and b the dimensions, including twice the extrapolation distance, in the plane of the plane thermal-neutron source. In order to determine the effect of finite dimensions on the measurement of L^2 without reproducing the rather lengthy derivation required to obtain Eqs. (10) and (11), let us examine the comparatively simple case of a single finite dimension.

If the neutrons from an infinite plane source diffuse into a slab of infinite extent but of finite thickness, by the same arguments used in the infinite dimension case, we find the general solution given by Eq. (3) to be applicable. To evaluate the arbitrary constants A and C , it is necessary to introduce an additional boundary condition that the flux shall vanish at the extrapolated boundary. If a is the finite dimension of the slab including this extrapolation distance, then

$$\phi_a = A e^{-Ka} + C e^{Ka} = 0; \quad C = -A e^{-2Ka} \quad (12)$$

Equation (3) then becomes

$$\phi = A[e^{-Kz} - e^{-K(2a-z)}]. \quad (13)$$

Using the boundary condition for the neutron current density as before, we evaluate A to be

$$\frac{1}{2KD(1 + e^{-2Ka})};$$

hence,

$$\phi = \frac{e^{-Kz} - e^{-K(2a-z)}}{2KD(1 + e^{-2Ka})}. \quad (14)$$

If, in Eq. (14), we let a be very large, then we see that Eq. (14) becomes identical, as we would expect, with Eq. (4). Of more importance to us at this time, however, is determining an approximate value for a , which is sufficiently large to make Eq. (14) essentially equal to Eq. (4). Let us select the dimension a such that $Ka = 3$; then Eq. (14) is

$$\phi = \frac{e^{-Kz} - e^{[6+3(z/a)]}}{2KD(1 + e^{-6})}.$$

Since we know $z \leq a$, let us choose for comparison a_z of one diffusion length, i.e., $z = L = 1/K$. Then Eq. (14) becomes

$$\phi = \frac{e^{-1} - e^{-5}}{2KD(1.0025)} \frac{0.368 - 0.0067}{2KD(1.0025)},$$

and the correction terms, in parentheses, are seen to be very small for $Ka = 3$. Thus, when the thickness of the medium, including the extrapolation distance, is about $3/K$, or more, the behavior of the flux at $1/K$ from the source is essentially that of an infinite medium. It is generally true that if the thickness is at least three times the diffusion length, it may be treated as an infinite medium at distances greater than one diffusion length from the boundary.

We may explain the effects we have seen as follows: If the thickness is three or more times the diffusion length, most of the neutrons are scattered back before they reach the boundary and the leakage is very small.

Referring to Eqs. (10) and (11), which are the working equations of this experiment, and employing intuition in lieu of mathematical rigor, we note that the only essential change as compared with the infinite-source, infinite-medium case is the replacement of K by γ . Also, we note that the expression for γ^2 given in Eq. (11) contains, in addition to K^2 , two terms which we will term the "leakage terms," which are identical to the geometrical buckling if we consider leakage in only two dimensions (since we measure along the third dimension).

III. APPARATUS

The apparatus for this experiment consists of neutron detectors, a diffusion box, and a large volume of water with a source of thermal neutrons. The general method employed is that of the spatial variation of neutron flux from a steady source.

The neutron detectors are indium foils (see Fig. 19.1). The diffusion box consists of a 60.9 x 60.9 x 60.9-cm aluminum frame covered with 20-mil cadmium (see Fig. 19.2). This box has provision for 24 foils on its

Fig. 19.1
Indium Foils
and Holders

139-2421

Fig. 19.2
Experimental Apparatus

139 2419

V. RESULTS

Using the data obtained by the procedures described in Section IV, make a plot of the spatial variation of the thermal-neutron flux as a function of distance from the front face of the diffusion box. The slope of the plot on semilog paper of the measured flux as a function of distance along the z direction yields the constant γ , from which L may be computed. If applicable, plot the transverse thermal-neutron flux from the appropriate data. Also, it may prove useful to compute and list the absolute flux as determined by a gold-foil irradiation along with the gold-cadmium ratio.

VI. ANALYSIS OF RESULTS AND CONCLUSIONS

This determination of the thermal diffusion length can be expected to agree with the published value⁽²⁾ of 2.85 cm to approximately $\begin{smallmatrix} +0.0\% \\ -8.0\% \end{smallmatrix}$.

VII. BIBLIOGRAPHY

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VIII. APPENDIXDiscussion Questions

The following propositions should be considered prior to arrival at the Argonaut. A discussion of them will be considered a part of this experiment.

1. What is the physical meaning of neutron diffusion length?
2. What facets of reactor design are primarily affected by the value of diffusion length?
3. What background corrections do you expect to make in this experiment and what is their need?
4. Explain the general physical meaning of L^2 and M^2 .
5. In what direction would our value of L^2 be affected by each of the following:
 1. impurities whose only appreciable cross section is that for absorption;
 2. the foil distribution at an angle other than 90° to the plane of the plane thermal-neutron source?
6. A cadmium-covered box, 2 x 2 x 2 ft, is available for measuring L^2 . In order to determine the indium-foil activation due to the fast neutrons present, it is suggested that we cover the foils with cadmium. Discuss the merits of covering the entire box with cadmium as compared to covering each individual foil with cadmium.
7. From your knowledge of the thermal-neutron diffusion length, calculate roughly the effect of the 2-ft dimension on L^2 . Will a box 2 x 2 ft be large enough?

STUDENT RESULTS

Thermal Diffusion Length in H₂O

Position	Counter Number	Total cpm	cpm Minus Background	Normalized to Counter #4	t(min)	Weight (gm)	e- λ t	we- λ t	$\frac{\text{cpm}}{\text{we-}\lambda t}$
Cadmium-covered Foil									
1	3	921	910	927	9	0.1336	0.891	0.119	6,193
2	3	1,030	1,019	1,039	11	0.1373	0.868	0.119	8,731
3	3	767	756	772	13	0.1357	0.846	0.115	6,699
4	3	576	565	577	15	0.1337	0.825	0.110	5,245
5	3	436	425	434	17	0.1353	0.804	0.109	3,982
6	3	311	300	306	19	0.1304	0.784	0.102	3,000
7	4	193	183	183	17	0.1374	0.804	0.110	1,664
8	4	160	150	150	15	0.1383	0.825	0.114	1,316
9	4	423	413	413	12	0.1270	0.857	0.109	3,789
10	4	232	222	222	9	0.1267	0.891	0.113	1,965
11									
Bare Foil									
1	3	23,007	22,996	23,444	6	1,374	0.926	0.1272	184,300
2	3	9,559	9,548	9,734	8	1,362	0.902	0.1228	79,267
3	3	4,163	4,152	4,233	10	1,360	0.880	0.1197	35,363
4	3	1,860	1,849	1,885	12	1,368	0.857	0.1172	16,084
5	3	889	878	895	14	1,408	0.836	0.1177	7,604
6	3	94	83	85	16	1,411	0.814	0.1149	740
7	4	293	283	283	14	1,418	0.836	0.1185	2,388
8	4	200	190	190	12	1,401	0.857	0.1201	1,582
9	4	136	126	126	9	1,291	0.891	0.1150	1,096
10	4	123	113	113	6	1,350	0.926	0.1250	904

Counter Background Standard

#3 10.9 cpm 202,022 cpm } Counter Ratio = 1.02
 #4 9.6 cpm 205,956 cpm }

Position	Bare (cpm/gm) Minus Cd-covered (cpm/gm)
1	184,300 - 6,193 = 178,107
2	79,267 - 8,731 = 70,536
3	35,363 - 6,699 = 28,664
4	16,084 - 5,245 = 10,839
5	7,604 - 3,982 = 3,622
6	740 - 3,000 = -2,260
7	2,388 - 1,664 = 724
8	1,582 - 1,316 = 266
9	1,096 - 3,789 = -2,693
10	904 - 1,965 = -1,061

Fig. S19.1. Diffusion Length in H_2O Fig. S19.2. Thermal Diffusion Length in H_2O

NEUTRON AGE IN WATER

by

L. L. Lawyer

ABSTRACT

This experiment is designed to demonstrate the concepts involved in the slowing-down process of neutrons from fission energy to the energy of the indium resonance (1.46 eV). The neutron age, a parameter describing the behavior of these neutrons in this energy range, is evaluated from a measurement of a spatial distribution of neutrons within a moderator. Thermalized reactor leakage neutrons impinging on a set of Argonaut fuel plates produce a plane source of fission energy neutrons. The moderating medium is water and the detecting devices are cadmium-covered indium foils.

OUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
- VI. PROCEDURE
- V. RESULTS
- VI. ANALYSIS AND CONCLUSIONS
- VII. BIBLIOGRAPHY
- VIII. APPENDIX

Discussion Questions

I. INTRODUCTION

The average free-flight distance a fission neutron travels while slowing down has a direct bearing on the critical size of a reactor. If this average distance is large, then the reactor must be large in order to reduce the fast leakage factor. Therefore, the spatial distribution of neutrons resulting from the diffusion process determines the probability of loss of epithermal neutrons by leakage during the slowing-down process.

The life cycle of a neutron in a thermal reactor, moderator, or reflector can be considered to be divided on an energy scale into thermal and epithermal regions. The Argonaut experiment "Measurement of Thermal Neutron Diffusion Length in H_2O " concerns itself with the fate of the neutron in the thermal portion of such a cycle; this experiment completes the picture by considering the neutron behavior during its epithermal career.

II. THEORY

The spatial distribution of neutrons of various energies resulting from diffusion during the slowing-down process is important because the average (net vector) distance a fission neutron travels while slowing down has a direct bearing on the critical size of a reactor. An approximate approach to the investigation of this distribution is based on the "continuous slowing down model" or "Fermi age model."

Consider a neutron produced at fission energy E_0 in a medium containing only relatively heavy nuclei. It will travel with this energy for a certain time before it collides with a nucleus. After the collision, its energy will be decreased as the square of the mass ratio and will then travel with this constant lower energy until it again has a collision. Due to the lower velocity after each collision, the time between collisions will, on the average, increase. This process will continue until the neutron is thermalized at energy E_{th} . This model is depicted in Fig. 20.1.

Fig. 20.1

Neutron Slowing-down Energy
versus Time

It should be noted that in Fig. 20.1 for a given material the vertical steps in energy are merely ξ , the average logarithmic energy decrement per collision, and are therefore independent of energy, whereas the horizontal steps become increasingly large with decreasing energy.

If the medium consists of nuclei of moderate or high mass number, the spread of energies among the neutrons of the same generation is relatively small; and it is possible to represent the behavior of a large number of individual neutrons by means of an average. As a result, the series of steps may be replaced, without serious error, by a continuous curve. This is the basis of the continuous slowing-down (Fermi-age) model. Here it is assumed that, during slowing down, each neutron velocity is a smooth and continuous function, allowing the use of differential equations to describe a process that is actually discrete. It must be emphasized that this model, even as an approximation, is good only for moderators consisting of elements of high mass number and definitely is not valid for light-element moderators containing hydrogen, i.e., light water. The concepts and mathematical derivations involved in the Fermi-age model, however, will prove themselves useful in the case of a light-element moderator as will be seen later.

The spatial distribution of the slowing density, q , in a nonabsorbing, nonmultiplying medium is represented by the Fermi-age equation:⁽¹⁾

$$\nabla^2 q = \frac{\partial q}{\partial \tau} \quad (1)$$

where the energy-dependent variable $\tau(E)$ is defined as

$$\tau(E) \equiv \int_E^{E_0} \frac{D}{\xi \Sigma_s E} dE. \quad (2)$$

In Eq. (2)

D = diffusion coefficient

ξ = average logarithmic energy decrement per collision

Σ_s = macroscopic cross section for scattering

E = neutron energy

$\tau(E)$ = Fermi age.

At this point it is interesting to note that, if $q(E)$ is a solution of Eq. (1) for the case of no absorption, then the corresponding slowing-down density $q'(E)$ for the case with weak absorption is merely $q(E)$ times the resonance escape probability.

The solution⁽¹⁾ $q(E)$ [or $q'(E)$] of the age equation for the slowing-down density in the case of a plane source of neutrons in a moderating region of infinite extent is

$$q(x, \tau) = \frac{S}{\sqrt{4\pi\tau}} e^{-x^2/4\tau}, \quad (3)$$

where

q = slowing-down density at a particular energy E

S = source strength

τ = the age of the neutrons at the energy E .

To give a more precise physical significance to this "age," we may calculate the second spatial moment of the slowing-down density (see Section II.3 of "Measurement of Thermal Neutron Diffusion Length in H_2O ") denoted by $\bar{x}^2(\tau)$:

$$\bar{x}^2(\tau) = \frac{\int_0^\infty x^2 e^{-x^2/4\tau} dx}{\int_0^\infty e^{-x^2/4\tau} dx} = 2\tau. \quad (4)$$

The "age" τ is, therefore, one-half the mean-square (net vector) distance traveled by a neutron from the time of its emission from the source, i.e., age zero to the time at which its age is τ .

In considering the slowing-down process applied to neutrons in hydrogen, a better approach is through a two-group treatment⁽²⁾ in which the fast group is taken to include all neutrons of energies above a given

energy E . Diffusion theory predicts that the flux ϕ of neutrons of energy E should vary with the distance x from the plane of an infinite plane source in an infinite medium according to the relation

$$\phi_E = \text{constant } e^{-x/L_f}, \quad (5)$$

where L_f is the diffusion length of the fast group. Then, in a manner analogous to that used in the case of thermal neutrons, it can be shown⁽³⁾ that for a finite source in a finite medium

$$\phi_E = \text{constant } e^{-\gamma x}, \quad (6)$$

where γ is given by the relation

$$\gamma^2 = \frac{1}{L_f^2} + \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2; \quad (7)$$

here a and b are the extrapolated dimensions of the moderator in the plane of the source.

If we now calculate the mean-square (net vector) distance that the neutrons of the fast group have diffused in reaching the energy E , we get

$$\bar{x}^2 = \frac{\int_0^\infty x^2 e^{-\gamma x} dx}{\int_0^\infty e^{-\gamma x} dx} = \frac{2}{\gamma^2},$$

or, if we assume leakage to be negligible, i.e., if

$$\gamma^2 = 1/L_f^2,$$

then

$$\bar{x}^2 = \frac{\int_0^\infty x^2 e^{-x/L_f} dx}{\int_0^\infty e^{-x/L_f} dx} = 2L_f^2.$$

We can then, by accepting the definition of "age" for slowing down in hydrogen to be $\bar{x}^2/2$, get the result that the "age" τ at the energy E is equal to the square of the diffusion length of the neutron group having energies between the source energy and the energy E .

Thus, the age of neutrons of specified energy in a given medium can be determined experimentally provided a material is available which has a large activation cross section for neutrons principally of that energy. Indium provides us with such a material, due to its 1.46-eV resonance. The saturated activity A of a cadmium-covered indium foil can be taken to be proportional to the flux ϕ of neutrons having energies in the narrow region of this resonance.

III. APPARATUS

The shielding tank facility, located on the south side of the reactor, will be utilized for this experiment. A plane source of fission neutrons will be generated by a number of Argonaut fuel plates (Fig. 20.2). The detecting devices are indium foils 1.9 cm in diameter, 80 mg-cm² enclosed in 0.0457-cm-thick (18/mil) cadmium cans (Fig. 20.4). These foils will be mounted on a holder provided (Fig. 20.3). A pair of gold foils may be used for monitor foils and to determine the Au-Cd ratio if desired.

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Fig. 20.2. Neutron Converter

Fig. 20.3
Experimental Arrangement

139-2419

Fig. 20.4
Foil and Holders

158-2421

Fig. 20.5. Experimental Geometry

IV. PROCEDURE

Using the apparatus described above, the distribution for neutrons slowing down in water will be determined by activating a group of indium foils distributed perpendicular to the plane of the plane fission source. The saturated activities of these foils will be determined by counting with end-window proportional flow counters.

Since some of the neutrons detected in the above procedure may have originated in the reactor core rather than in the plane source, the magnitude of this activity should be investigated. To do this, we repeat the above procedure with the fuel plates removed from the tank.

The saturated activity of each foil due to neutrons originating in the plane source can then be calculated (see Table 20.1).

Table 20.1

DATA SHEET

Name		Irradiation Period					
Date		Reactor Power					
$(1 - e^{-\lambda\tau})$		Background					
1 Foil Location	2 Activity	3 Activity- Background	4 Time (min)	5 $e^{-\lambda t}$	6 Weight (gm)	7 $1 - e^{-\lambda\tau}$ x 5 x 6	8 Saturated Activity

V. RESULTS

Plot the saturated activity of the background corrected indium foils versus the first power of their distance from the plane fission source and compute τ from the relations indicated in Eqs. (6) and (7).

Next, compute τ from the saturated activity by means of Eq. (4).

Finally, considering Eq. (3), plot the saturated activity of the

background-corrected foils as a function of the square of the distance of their position from the plane source of fission neutrons, using semilog graph paper and compare the resulting curve with that predicted by Fermi-age theory for a nonhydrogenous medium.

VI. ANALYSIS AND CONCLUSIONS

In the analysis of our results we must note particularly that Eq. (3), like the equation of the continuous slowing-down model, is not applicable to a hydrogenous medium such as H_2O and that Eq. (5) is for an infinite medium. With these assumptions in mind, we can then explain the deviations at the end of our plotted curves. Other possible sources of error to be considered are impurities in the water, inaccuracy in foil position, source thickness, and the transverse distribution of the fission neutrons.

VII. BIBLIOGRAPHY

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3. Ibid., pp. 6-54.

VIII. APPENDIXDiscussion Questions

The following propositions should be considered prior to arrival at the Argonaut. A discussion of them will be considered a part of this experiment.

1. What is the physical meaning of neutron age?
2. What facets of reactor design are primarily affected by the value of age?
3. What will be the energy spectrum over which we measure age if we use cadmium-covered indium as the detecting device?
4. B^{10} ionization chambers are in place behind the water tank in our experiment. What effect, if any, will the presence of these chambers have on our results?
5. Express the slowing-down density as a function of neutron age for the following geometries:
 - a. a point source in nonabsorbing media;
 - b. a plane source in nonabsorbing media.
6. Why is the term "age" not strictly proper for hydrogenous media?
7. What background corrections must be made in this experiment, and what is their need?
8. Can you suggest a better foil material than cadmium-covered indium for this experiment?
9. How will you correct the data for leakage?
10. In view of your experience with the Argonaut, do you believe that it is necessary to remove some fuel from the core to compensate for the eight fuel plates used as our plane source of fission neutrons? Give supporting evidence.
11. Explain the general physical meaning of L^2 , M^2 , and τ .
12. Qualitatively, what discrepancies might arise from assuming $q(r, \tau)$ proportional to the saturated activity of the indium foils we irradiate, where $q(r, \tau)$ is the slowing-down density?

13. From what sources, other than nonmoderated neutrons, might we expect to get background neutrons during a background run with detecting foils in the water tank? What is the energy of these neutrons?
14. What factors must be considered in choosing the indium foil thickness and the cadmium-cover thickness to be used for these experiments?
15. How will the results of this experiment be affected by flux leakage on the sides of the tank? Will this be caused by leakage into or out of the tank?
16. How will the σ_a of hydrogen affect our value of τ ?
17. There is a dimension of 4 in. between the plane source of fission neutrons and the edge of the water tank. Do you expect this dimension to be of any importance? Would you increase this distance or decrease it?

STUDENT RESULTS

A set of experimental results are tabulated in Table S20.1 and Table S20.2 and plotted in Figs. S20.1-S20.5. The value of γ from Fig. S20.1 is found to be -0.535 in.^{-1} . Using this value and $a = b = 34 \text{ in.}$ (Fig. S20.2) in Eqs. (6) and (7) we find $\tau = 3.72 \text{ in.}^2 = 24 \text{ cm}^2$.

Using Eq. (4) and the data shown in Figs. S20.3 and S20.4

$$2\tau = \frac{\int A(x) x^2 dx}{\int A(x) dx} = \frac{180,400}{24,900}$$

$$\tau = 3.62 \text{ in.}^2 = 23.4 \text{ cm}^2.$$

Comparison of age predicted by Eq. (3) and a plot of $A(x)$ versus x^2 is done in Fig. S20.5.

Table S20.1

BACKGROUND COUNTS FOR CADMIUM-COVERED INDIUM FOILS (28 W Saturation Power Irradiation)

Foil No.	Counter		Foil Weight, gm	Delay Time, min		$e\lambda t$		Counts/min		Counts-Background		Corrected Counts	Counts/gm at T=0
	No.	Factor											
1	6	0.956	0.2485	16	36	1.226	1.583	$\frac{1400}{5}$	$\frac{1091}{5}$	265	203	304	1240
2	4	1.008	0.2345	16	48	1.226	1.845	$\frac{1265}{5}$	$\frac{1024}{5}$	238	190	324	1390
3	3	1	0.2436	16	54	1.226	1.992	$\frac{677}{5}$	$\frac{463}{5}$	119	77	150	615
4	5	0.995	0.2412	17	37	1.242	1.604	$\frac{414}{5}$	$\frac{352}{5}$	69	57	88	366
5	6	0.956	0.2487	24	42	1.358	1.709	$\frac{273}{5}$	$\frac{250}{5}$	40	35	54	224
7	4	1.008	0.2355	24	42	1.358	1.709	$\frac{207}{5}$	$\frac{199}{5}$	26	25	39	165
8	3	1.0	0.2363	25		1.845	1.700	$\frac{166}{5}$	$\frac{152}{5}$	17	14	28	119
9	5	0.995	0.2473	25		1.376	1.731	144/5		16	15	24	97
10	6	0.956	0.2482	30		1.466	1.845	121/5		9	10	16	65
11	4	1.008	0.2705	30		1.466	1.58	112/5		7	9	12	44
12	3	1.0	0.2465	31		1.485	1.58	101/5		4	4	6	24
13	5	0.995	0.2495	31		1.485		114/5		9		13	53
14			0.2420										

Background,		Calibration Counts	
Counter	cpm	Counter	Factor
3	16	3	22,784
4	15	4	22,972
5	13	5	22,684
6	15	6	21,876
			1
			1.008
			0.995
			0.956

Table S20.2

DETERMINATION OF τ
COUNTS OF CADMIUM-COVERED INDIUM FOILS WITH PLANE FISSION PLATE IN POSITION
(31.5 W Saturation Power Irradiation)

Foil No.	Counter		Foil Weight, gm	Delay Time, min		$e\lambda t$		Counts/min		Mean Counts/min Counter Background	Corrected Measured, counts/min	cpm/gm	cpm/gm Background (Power Corrected)	Counts/gm (abs)
	No.	Factor		a	b	a	b	a	b					
1	3	1	0.2485	15		1.211		6926/4		1718	2080	8,400	1390	7,010
2	4	1.017	0.2345	15		1.211		9041/4		1247	1535	6,550	1560	4,990
3	3	1	0.2436	38		1.624		2401/5		466	758	3,120	690	2,430
4	4	1.017	0.2412	32		1.504		1782/5		343	524	2,170	410	1,760
5	3	1	0.2487	32	69	1.504	2.412	956/5	1255/10	177/112	268	1,080	250	830
6	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	4	1.017	0.2355	26	69	1.394	2.412	450/5	584/10	77/45	109	464	185	279
8	3	1	0.2363	26	58	1.394	2.096	314/5	554/10	49/41	77	326	134	192
9	4	1.017	0.2473	20	58	1.291	2.096	281/5	412/10	43/28	58	235	109	126
10	3	1	0.2482	20	47	1.291	1.822	207/5	375/10	27/23	39	157	73	84
11	3	1	0.2705	11	47	1.151	1.822	64/2	290/10	18/15	24	89	50	49
12	4	1.017	0.2465	11	38	1.151	1.624	52/2	138/5	13/14	19	77	27	50
13	3	1	0.2495	8		1.11		59/2		16	18	72	59	13
14	4	1.017	0.2420	8		1.11		61/2		17	19	78	60	18
Monitor			0.2487											
A			0.2464									231		
B			0.2408									459		
C			0.2453									636		
D			0.2425									552		
E			0.2450									665		
F			0.2480									674		
G			0.2421									743		
H			0.2555									636		
I			0.2503									713		
J			0.2566									492		
K			0.2572									440		
L			0.2512									227		

Counter Backgrounds 3-14 cpm; 4-13 cpm

Fig. S20.1. Determination of τ

Fig. S20.2. Resonance Activity Distribution at $z = 6$ in.

Fig. S20.3. Plot of $A(x) x^2$ versus x Areas = 180,400 counts, in.²

Fig. S20.4. Plot of $A(x)$ versus x

Fig. S20.5

DIFFUSION PARAMETERS OF MODERATORS
USING PULSED NEUTRON TECHNIQUES

by

W. F. Gilbert and M. Petrovic

ABSTRACT

A moderating medium is subjected to a pulse of fast neutrons. After the slowing down of these neutrons in the medium, the rate of decay of the resulting thermal neutrons is measured by a multichannel time analyzer. The basic diffusion parameters of the moderating medium, D_0 , Σ_a , and L^2 , can be obtained from the decay constants determined for various bucklings.

OUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
- IV. PROCEDURE
- V. RESULTS
- VI. ANALYSIS AND CONCLUSIONS
- VII. BIBLIOGRAPHY
- VIII. APPENDIX

Discussion Questions

I. INTRODUCTION

The pulsed neutron source consists of a pulsed ion source producing deuterium ions which are accelerated to approximately 100-keV energy. The ions bombard either a deuterated copper target or a tritium target and produce neutrons of about 3.5-MeV energy for the D-D reaction, or about 14-MeV for the D-T reaction. The neutron yield from the D-T reaction is about 100 times that from the D-D reaction.

Following each neutron pulse, sufficient time is allowed for the neutrons to become thermalized in the moderator and for the higher modes to decay. The decay of the fundamental mode of the thermal neutrons is followed on a multichannel time analyzer. This procedure is followed for several different bucklings, and analysis of the resulting data yields values of D_0 , Σ_a , and L^2 for the moderating medium.

II. THEORY

The basic equation which describes the diffusion of neutrons at any point in a uniform medium may be written as

$$D_0 \nabla^2 n - \Sigma_a \phi + S = \frac{\partial n}{\partial t}, \quad (1)$$

where n is the neutron density, D_0 the conventional diffusion coefficient as used in Fick's law:

$$J = -D_0 \text{grad } n,$$

Σ_a is the macroscopic absorption coefficient of the media, ϕ is the neutron flux nv , where v is the neutron speed, and S is the contribution from various sources which may be present.

Consider Eq. (1) for the case of thermal neutrons. If we eliminate all sources of thermal neutrons, including those which might become thermal due to slowing down from higher energies, Eq. (1) becomes

$$D_0 \nabla^2 n - \Sigma_a \phi = \frac{\partial n}{\partial t}. \quad (2)$$

If we convert the neutron-density terms to flux terms by the relationship $\phi = nv$, Eq. (2) becomes

$$\frac{D_0}{v} \nabla^2 \phi - \Sigma_a \phi = \frac{1}{v} \frac{\partial \phi}{\partial t}. \quad (3)$$

Upon division by D_0/v we obtain

$$\nabla^2 \phi - \frac{v \Sigma_a}{D_0} \phi = \frac{1}{D_0} \frac{\partial \phi}{\partial t}. \quad (4)$$

Equation (4) may be separated into a space-dependent and a time-dependent term. The space-dependent part is of the form

$$\nabla^2 \phi + B_1^2 \phi = 0, \quad (5)$$

where B_1^2 is geometric buckling for the particular space mode being considered. The allowable space modes are those which satisfy the boundary conditions for the media. In steady-state or criticality computations, only the fundamental mode (the smallest value of B_1^2) need usually be considered. In pulsed-neutron work, a nonequilibrium state is established, and we must consider what happens to all the modes being excited.

If we solve for $\nabla^2 \phi$ in Eq. (5) and substitute in Eq. (4) we have

$$-\left[B_1^2 + \frac{v \Sigma_a}{D_0}\right] \phi_i = \frac{1}{D_0} \frac{\partial \phi_i}{\partial t}, \quad (6)$$

where ϕ_i is now the neutron-flux component corresponding to a specific excited mode. If we multiply by D_0 and divide by ϕ_i , we obtain

$$-[D_0 B_1^2 + v \Sigma_a] = \frac{\frac{\partial \phi_i}{\partial t}}{\phi_i}. \quad (7)$$

Upon separation of the variables, we have

$$\frac{\partial \phi_i}{\phi_i} = -[D_0 B_1^2 + v \Sigma_a] dt. \quad (8)$$

Integration gives

$$\ln \phi_i = - [D_0 B_i^2 + v \Sigma_a] t, \quad (9)$$

or, upon introducing suitable limits,

$$\phi_i = \phi_{0i} e^{-[D_0 B_i^2 + v \Sigma_a] t}, \quad (10)$$

where t is the time for the flux to change from ϕ_{0i} to ϕ_i . This equation is of the form

$$\phi = \phi_0 e^{-\alpha t}, \quad (11)$$

where α is known as the decay constant. Thus

$$\alpha_i = [D_0 B_i^2 + v \Sigma_a]. \quad (12)$$

If a burst of fast neutrons impinges on a moderating medium, the slowing-down time ($\sim 10^{-5}$ sec) is very short compared with the diffusion time ($\sim 10^{-4}$ - 10^{-1} sec). Thus thermal neutrons are produced so rapidly by the slowing-down process that the time effect is much the same as if a burst of thermal neutrons had been received by the medium. The application of such a pulse of thermal neutrons will excite the fundamental and higher modes in the medium, the specific modes and their amplitudes being determined by the geometry of the experiment.

Associated with each mode generated is a value of the geometric buckling, B_i^2 . For higher modes (larger values of i) the value of B_i^2 increases, and the value of the decay constant, α_i , increases. Thus the higher modes will decay more rapidly than the lowest or fundamental mode. If sufficient time is allowed to elapse before the decay constant is measured, only the fundamental mode remains, and the decay constant found experimentally is that for the fundamental mode.

The effect of changes in B_i^2 for the various modes upon the value of α_i is most pronounced if the $D_0 B_i^2$ term is the predominant one in the expression for α_i . Thus, the higher modes will decay most rapidly relative to the fundamental if B_i^2 is made large. We determine then that small volumes of the media being pulsed are most desirable for this experiment, since B^2 increases as the dimensions of the system decrease.

If the decay constant for thermal neutrons in the medium is found after the higher modes are negligible, the value of α found is for the fundamental mode only. The corresponding value of the buckling, B^2 , is the usual geometric buckling used in steady-state measurement.

For a given medium, if the value of α is found for various values of B^2 and plotted linearly versus B^2 , the slope of the curve will determine D_0 , and the α intercept at $B^2 = 0$ will determine $v\Sigma_a$. Thus, Σ_a for the medium can be found at any chosen velocity. D_0 is related to the more usual diffusion coefficient D by

$$D_0 = Dv. \quad (13)$$

The diffusion length L for the medium can be found from the relationship

$$L^2 = D_0/v\Sigma_a = D/\Sigma_a. \quad (14)$$

If we solve for D_0 in Eq. (14) and substitute this value in Eq. (12), we obtain for the fundamental mode

$$\alpha = v\Sigma_a L^2 B^2 + v\Sigma_a \quad (15)$$

or

$$\alpha = v\Sigma_a [1 + L^2 B^2]. \quad (16)$$

The average lifetime of thermal neutrons in an infinite machine ($B^2 = 0$) is

$$\ell_0 = \lambda_a/v = 1/v\Sigma_a. \quad (17)$$

The average lifetime of neutrons in the finite medium is

$$\ell^* = 1/\alpha \quad (18)$$

or

$$\ell^* = 1/[v\Sigma_a(1 + L^2 B^2)]. \quad (19)$$

Substitution for $v\Sigma_a$ from Eq. (17) yields

$$\ell^* = \ell_0/(1 + L^2 B^2). \quad (20)$$

Thus the average lifetime of thermal neutrons in the finite medium is less than in an infinite medium due to neutron leakage.

Diffusion Cooling

Equation (16) indicates that the decay constant α is a linear function of the buckling B^2 . Careful experiments, however, indicate a slightly nonlinear relationship between α and B^2 . This is explained as being due to diffusion cooling, the preferential leakage of neutrons of higher energy from the medium.

This effect is a function of the size of the assembly and therefore a function of the geometric buckling. The smaller the assembly the greater the leakage, resulting in a neutron-energy spectrum in the medium which is shifted in the direction of lower energy. This is equivalent to the effect which would be obtained by cooling the medium; hence the name diffusion cooling for the effect.

The experimental relationship between α and B^2 may be fitted by an equation containing higher power terms of the buckling. Commonly, a fit is made to the equation

$$\alpha = v\Sigma_a [1 + L^2B^2 + CB^4], \quad (21)$$

where C is called the diffusion cooling coefficient.

III. APPARATUS (see Fig. 21.1)

Pulsed-neutron source;
 Multichannel time analyzer (preferably with pulsed-neutron logic and digital readout);
 $B^{10}F_3$ detector, preamplifier, and linear amplifier;
 MPC high-voltage supply, amplifier, and sealer;
 AID linear amplifier.

139-3417

Fig. 21.1. Experimental Apparatus

IV. PROCEDURE

A. Prepare the pulsed-neutron source and multichannel time analyzer for operation as directed by the instructor.

B. Add distilled water to one of the aluminum tanks provided, and determine the dimensions and geometric buckling of the sample by use of the appropriate extrapolation length. (In practice, an extrapolation length would have to be assumed and rough values of the diffusion parameters determined using the resulting geometric bucklings. A better value for the extrapolation length could then be computed and the diffusion parameters again determined. For other than small samples, the iteration method converges quickly, as the extrapolation length has only a small effect on the geometric buckling.)

C. Place the aluminum tank inside the cadmium-lined chamber, and align the center of the sample with the center of the target. Place a small $B^{10}F_3$ counter inside the water tank with the counter at the center of the water sample. Pulse the neutron source until the memory limit of the time analyzer is reached; then print out the accumulated data.

D. Repeat Parts 2 and 3 for several samples of water having different geometric buckling.

E. Plot each set of data on a semilog plot, and determine the decay constant for each geometric buckling.

F. Plot the decay constant versus geometric buckling, and determine the diffusion parameters of water from this plot by means of Eqs. (12) and (14) in Section II.

V. RESULTS

Results to be obtained are:

1. the data plots and the evaluation of the thermal-neutron decay constant for each plot;
2. the plot of decay constant versus buckling, and the evaluations of the diffusion parameters of water from the slope and intercept of this plot.

VI. ANALYSIS AND CONCLUSIONS

The evaluation of the experiment is left as an exercise for the student.

VII. BIBLIOGRAPHY

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2. W. M. Lopez and J. R. Beyster, ibid. 12, 190-202 (1962).

VIII. APPENDIXDiscussion Questions

1. Following the injection of a burst of fast neutrons into a sample of moderating material, what will be the shape of the thermal flux in the moderator as a function of time?
2. During what period of time do we want to measure the flux as a function of time for the purposes of this experiment?
3. What effects are predominant in the data obtained before and following the period selected in Question 2?
4. Are there any limitations on the repetition rate selected for the pulsed-neutron source which are dictated by considerations other than those having to do with the source itself?
5. Which of the common diffusion parameters are determined by the slope of the decay constant versus buckling curve; which from the intercept at $B^2 = 0$; which common parameters can be computed from the two measured parameters?
6. What is the purpose of shielding the moderator sample with cadmium, and why is the counter covered with cadmium with only a window exposed to the sample?
7. The length of the neutron pulse supplied by the sample may vary during the course of the experiment. Will this have any effect on the data obtained?
8. How would the sample sizes selected for this experiment vary with the diffusion length of the moderating material?

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STUDENT RESULTSBuckling Calculations

$$B^2 = \left(\frac{\pi}{a+2t}\right)^2 + \left(\frac{\pi}{b+2t}\right)^2 + \left(\frac{\pi}{c+2t}\right)^2$$

$$t = 0.35 \text{ cm} \quad 2t = 0.70 \text{ cm}$$

Sample 1

$$23.0 \times 23.0 \times 23.0 \text{ cm}$$

$$B^2 = 3\left(\frac{\pi}{23.7}\right)^2 = 3(0.1326)^2 = 3 \times 0.0175 = 0.0525 \text{ cm}^{-2}$$

Sample 2

$$22.9 \times 22.9 \times 11.4 \text{ cm}$$

$$B^2 = 2\left(\frac{\pi}{23.6}\right)^2 + \left(\frac{\pi}{12.1}\right)^2 = 2(0.1331)^2 + (0.2596)^2$$

$$B^2 = 2 \times 0.0177 + 0.0674 = 0.0354 + 0.0674 = 0.1028 \text{ cm}^{-2}$$

Sample 3

$$11.45 \times 11.45 \times 15.3 \text{ cm}$$

$$B^2 = 2\left(\frac{\pi}{12.15}\right)^2 + \left(\frac{\pi}{16.0}\right)^2 = 2(0.2586)^2 + (0.1964)^2$$

$$B^2 = 2 \times 0.0669 + 0.0386 = 0.1338 + 0.0386 = 0.1724 \text{ cm}^{-2}$$

Sample 4

$$11.45 \times 11.45 \times 7.65 \text{ cm}$$

$$B^2 = 2\left(\frac{\pi}{12.15}\right)^2 + \left(\frac{\pi}{8.35}\right)^2 = 2(0.2586)^2 + (0.3762)^2$$

$$B^2 = 2 \times 0.0669 + 0.1415 = 0.1338 + 0.1415 = 0.2753 \text{ cm}^{-2}$$

Decay Data

Sample 1

Ch. No.	Count	Ch. No.	Count
020	00105	010	02514
019	00110	009	03707
018	00188	008	04993
017	00217	007	07031
016	00374	006	09579
015	00441	005	13476
014	00716	004	18045
013	00922	003	24320
012	01316	002	32768
011	01803		

23.0 x 23.0 x 23.0 cm

T = 19°C

Sample 2

Ch. No.	Count	Ch. No.	Count
020	00015	010	01154
019	00024	009	01805
018	00031	008	02643
017	00056	007	04110
016	00096	006	06550
015	00130	005	09827
014	00213	004	14515
013	00327	003	22204
012	00499	002	32768
011	00758		

22.9 x 22.9 x 11.4 cm

T = 19°C

Sample 3

Ch. No.	Count	Ch. No.	Count
020	00001	010	00199
019	00003	009	00329
018	00002	008	00643
017	00007	007	01002
016	00012	006	01755
015	00008	005	03157
014	00026	004	05662
013	00030	003	09540
012	00060	002	16384
011	00110		

2 liters

11.45 x 11.45 x 15.3 cm

T = 19°C

Sample 4

Ch. No.	Count	Ch. No.	Count
020	00000	010	00209
019	00001	009	00433
018	00001	008	00807
017	00003	007	01727
016	00006	006	03553
015	00009	005	07538
014	00015	004	14993
013	00028	003	28650
012	00063	002	32768
011	00094		

1 liter

11.45 x 11.45 x 7.65 cm

T = 19°C

For all data: channel length = 40 + 10 = 50 μ s1/4 in. x 1 in. B¹⁰F₃ counter at center of sample.

Fig. S21.1

Fig. S21.2

Fig. S21.3

Fig. S21.4

Fig. S21.5. Decay Constant
versus Buckling

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NEUTRON TEMPERATURE, AGN-201 REACTOR

by

Jan Spalek

ABSTRACT

The experiment is performed at the AGN-201 reactor. The temperature of neutrons in the reactor core, which is related to their kinetic energy, is measured by two methods. The basis of both methods is the variation of the absorption cross sections of materials with the incident neutron energy and, hence, temperature. One method involves a danger-coefficient technique, the other the measurement of an average cross section. This experiment demonstrates that the temperature of neutrons in the core is higher than the temperature of the moderator atoms.

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- II. THEORY
- III. APPARATUS
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- VIII. APPENDICES
 - A. Discussion Questions
 - B. Derivation of the Number of Neutrons of all Velocities Striking or Crossing a Surface per Unit Area per Unit Time
 - C. Hardening Correction Curve

I. INTRODUCTION

The temperature of neutrons in the core is of importance to reactor design and in the study of the mechanism of moderation. Further, the beam neutron temperature is of great value to the scientist using the neutron beam for research.

Temperatures of beam neutrons were measured at Argonne and Oak Ridge early after World War II. The experimental values showed a marked difference, even those obtained from the same reactor but by different methods. Values of 255°K to 330°K for beam neutrons have been measured; the temperature of neutrons in the core varied from 90°C to 140°C above the moderator temperature. Taking the discrepancies of neutron-temperature measurements into account, the neutrons exhibited about 100°C higher temperature than the moderator atoms. A higher temperature was expected because of the presence of some fast neutrons in the process of slowing down to thermal velocities, and therefore not yet in equilibrium with the moderator atoms. Also, a preferential absorption of the slow neutrons by 1/v material is taking place.

II. THEORY

The average kinetic energy of thermal neutrons is given by the relation

$$\frac{1}{2} m \overline{v^2} = (3/2) kT \quad (1)$$

which is derived in any book on kinetic theory of gases (see Reference 4). T will be referred to as neutron temperature.

A determination of the neutron temperature would therefore involve the measurement of the average kinetic energy of the neutrons, or their "root mean square" velocity ($v_{rms} = \sqrt{\overline{v^2}}$). These quantities, however, are practically not measurable. The root-mean-square velocity is related to the most probable velocity v_0 of the Maxwell distribution of thermal neutrons, the latter velocity being a measurable quantity:

$$v_{rms} = (\overline{v^2})^{1/2} = \left[\frac{\int v^2 dn}{\int dn} \right]^{1/2} = \left[\frac{\frac{4n_T}{\sqrt{\pi}v_0^3} \int_0^\infty v^2 v^2 e^{-v^2/v_0^2} dv}{N} \right]^{1/2} = \left[\frac{3}{2} v_0^2 \right]^{1/2} = \underline{\underline{1.224 v_0}}, \quad (2)$$

where

$$\frac{4n_T}{\sqrt{\pi}v_0^3} v^2 e^{-v^2/v_0^2} dv$$

is the Maxwellian velocity distribution which equals dn , the number of neutrons whose velocity lies between v and $v + dv$.

Therefore,

$$\overline{v^2} = \frac{3}{2} v_0^2. \quad (3)$$

Substitution of (3) into (1) gives $(1/2)m(3/2)v_0^2 = 3/2 kT$, which after simplification is no longer the average kinetic energy of the neutrons. The relation obtained,

$$1/2 m v_0^2 = kT \quad (4)$$

means that the most probable velocity of the Maxwellian distribution corresponds to a kinetic energy kT of the neutrons. Equation (4) is also used to define the "temperature" of neutrons.

The problem of determining T thus becomes one of measuring v_0 . This velocity could be obtained by measuring the relative number of neutrons as a function of velocity with a crystal spectrometer or chopper. The maximum of the distribution thus measured would give v_0 and therefore T from Eq. (4).

The two methods for measuring v_0 used in this experiment are based on the absorption cross section of materials, which are dependent on the neutron velocity and, therefore, on temperature.

The first method measures the cross section of a $1/v$ dependence, at the average velocity $\sigma_{\bar{v}}$ in the reactor core (glory hole). This cross section is related to the cross section at the most probable velocity σ_{v_0} (or σ_{kT}) by the same factor which relates the two velocities (average $\bar{v}$ and most probable v_0) in the Maxwell velocity distribution:

$$\bar{v} = \frac{\int v \, dn}{\int dn} = \frac{\frac{4n_T}{\sqrt{\pi}v_0^3} \int_0^\infty v^3 e^{-v^2/v_0^2} dv}{n_T} = \frac{2}{\sqrt{\pi}} v_0 = 1.128 v_0. \quad (5)$$

Since the absorption cross section of a $1/v$ material is k/v ,

$$\sigma_{\bar{v}} = k/v = k/1.128v_0 = (1/1.128)\sigma_{v_0}. \quad (6)$$

From Eq. (6) the most probable velocity v_0 of the actual Maxwellian distribution in the reactor (at the time $\sigma_{\bar{v}}$ was measured) can be calculated, provided k is known. But k can be determined from a known cross section measured at a given velocity (by crystal spectrometer or chopper).

Such a cross section is very well known for boron:

$$\sigma_{v_0}(\text{B}) = 755 \text{ b at } v_0 = 2200 \text{ m/sec},$$

so that

$$k = 755 \text{ b} \times 2200 \text{ m/sec}. \quad (7)$$

The v_0 of the present distribution in the reactor is thus

$$v_0 = \frac{k}{1.128\sigma_{\bar{v}}} = \frac{755 \text{ b} \times 2200 \text{ m/sec}}{1.128 (\sigma_{\bar{v}} \text{ measured, in b})}. \quad (8)$$

Also, it is known from (4) that the neutron temperature corresponding to 2200-m/sec neutron speed is 293°K, so that

$$(1/2)m(2200 \text{ m/sec})^2 = k(293^\circ\text{K}) \text{ at } v_0 = 2200 \text{ m/sec}$$

and

$$1/2 m v_0^2 = kT \quad \text{at } v_0 \text{ just measured.}$$

Hence,

$$T(\text{in } ^\circ\text{K}) = 293 \left(\frac{v_0}{2200 \text{ m/sec}} \right)^2 \quad (9)$$

is the present neutron temperature in the reactor core. The cross section at the average velocity, σ_v , is measured by the danger coefficient method. This method involves neutron absorption rates and reactivity measurements with the sample material placed inside the reactor core. A detailed description of the method and the procedures to be followed is given in Reference 6.

In the second method the average cross section of a $1/v$ material in the beam issuing out of a reactor (either a beam hole in graphite or a glory hole) is measured by the transmission method. Briefly (see Reference No. 5 for details), the method consists of measuring the attenuation of the neutron beam upon passing through a sample of thickness x ; the ratio of the intensity I_0 of the incident beam to the intensity I of the beam emerging from the sample determines the total cross section:

$$I/I_0 = e^{-N\sigma_T x}$$

or

$$\sigma_T = -\frac{\ln(I/I_0)}{Nx}. \quad (10)$$

Subtracting a known scattering cross section from σ_T gives the absorption cross section desired. If a sample of 50% transmission is used, no correction for the hardening of the beam is necessary, and the σ_T obtained is equal to σ_{v_0} . The basis for this second method is the same as for the first, that is, comparison of a measured velocity-dependent cross section with one at a known velocity to obtain v_0 of the actual neutron velocity distribution in the reactor, and thus the temperature of the neutrons. But the difference in method gives a different type of cross section than the one measured by the first, related by different factors to the cross section at the most probable velocity. A brief derivation of the relating factors necessary in this method is presented below. (For details, see again Reference No. 5, Total Neutron Cross Section.)

The flux distribution of the neutrons in the beam is obtained by multiplying the Maxwellian velocity distribution of the neutrons in the core, (dn) by v . Thus,

$$d(nv) = \frac{4n}{\sqrt{\pi}v_0^3} v v^2 e^{-v^2/v_0^2} dv. \quad (11)$$

However, using a $1/v$ detector ($B^{10}F_3$), the distribution as seen by this detector is again the velocity distribution as in the core (obtained by multiplying the flux distribution by $1/v$ sensitivity of the detector). Now, if a $1/v$ absorber that does not harden the neutron spectrum is placed in the

beam, the observed cross section will be the average cross section $\bar{\sigma}$ for the Maxwell velocity distribution.

$$\bar{\sigma} = \frac{\int \frac{k}{v} n(v) dv}{\int n(v) dv} = \frac{\frac{4n_T k}{\sqrt{\pi} v_0^3} \int_0^{\infty} \frac{v^2}{v} e^{-v^2/v_0^2} dv}{\frac{4n_T}{\sqrt{\pi} v_0^3} \int_0^{\infty} v^2 e^{-v^2/v_0^2} dv} = \frac{2}{\sqrt{\pi}} \frac{k}{v_0} = 1.128 \sigma_{v_0} \quad (12)$$

$$\bar{\sigma} = \frac{2}{\sqrt{\pi}} \frac{k}{v_0 \exp} = 1.128 \sigma_{v_0} \quad (12.1)$$

Equation (12.1) shows that in this case the average cross section $\bar{\sigma}$ is not equal to the cross section at the average velocity $\sigma_{\bar{v}}$ [Eq. (6)]. Also, Eq. (12) was derived under the assumption that the $1/v$ absorber did not harden the neutron spectrum. A thin sample of high transmission (transmission $T = I/I_0$) would approximate the above condition, but then the accuracy of the cross section would be poor due to any small error in the count rates I_0 and I , which differ by a small amount. Therefore, a thicker sample has to be used to increase the accuracy. But the thick sample hardens the neutron beam spectrum that passes through it, and so Eq. (12) no longer holds. A correction for hardening now has to be applied to relate correctly the experimentally obtained cross section $\bar{\sigma}_{\text{exp}}$ with the cross section at the most probable velocity σ_{v_0} . The hardening correction was calculated for samples of different transmission by H. A. Bethe (Reference 2, p. 237) and presented in the form of a curve, which is shown in Appendix C. It is a fortunate circumstance that for a sample of 50% transmission ($\ln T = 0.693$) which gives good accuracy, the hardening correction factor is

unity $\left(\frac{\bar{\sigma}_{\text{exp}}}{\sigma_{v_0}} = 1 \right)$, as can be seen from the graph.

The experimental cross section $\bar{\sigma}_{\text{exp}}$ is then equal to σ_{v_0} directly. It should be pointed out that the relating factor of Eq. (12.1) was calculated for the flux distribution in the beam modified by the $1/v$ detector so that the resultant distribution was again the same as in the core. However, the actual distribution of neutrons in the beam is the flux distribution of Eq. (11) whose $\bar{v}^2$ is higher than v^2 of the velocity distribution of neutrons in the core. Correspondingly, the temperature of the neutrons in the beam is higher than of those in the core. To compute the temperature of the beam neutrons, their $\bar{v}^2$ must first be calculated. Taking a beam of neutrons issuing out of a core, where the neutrons have a Maxwell velocity distribution, only those neutrons that hit the surface of the core will get into the beam. The number of neutrons with speed v striking a surface of a container is $(1/4)v \, dn$ (see Appendix B). Thus the $\bar{v}^2$ of the neutrons in the beam is

$$\overline{v^2} = \frac{\int v^2 \frac{1}{4} v \, dn}{\int \frac{1}{4} v \, dn} = \frac{\frac{4nT}{4\sqrt{\pi}v_0^3} \int_0^\infty v^5 e^{-v^2/v_0^2} dv}{\frac{4nT}{4\sqrt{\pi}v_0^3} \int_0^\infty v^3 e^{-v^2/v_0^2} dv} = 2v_0^2. \quad (13)$$

Summarizing:

$$\text{for the beam: } \overline{v_b^2} = 2v_0^2;$$

$$\text{for the core: } \overline{v_c^2} = 3/2 v_0^2.$$

If the measured velocity v_0 is taken, the temperature of the core is, from $1/2 m v^2 = 3/2 kT$,

$$(1/2)m(3/2)v_0^2 = (3/2)kT_c,$$

while T in the beam is

$$(1/2)m(2v_0^2) = (3/2)kT_b,$$

so that

$$T_{\text{beam}} = (4/3)T_{\text{core}}. \quad (14)$$

If the detector used in the beam method does not have a $1/v$ sensitivity, but is black (like a long $B^{10}F_3$ detector), Eq. (12) no longer holds and an average cross section over the flux distribution is computed:

$$\overline{\sigma} = \frac{\int \frac{k}{v} v n(v) \, dv}{\int v n(v) \, dv} = \frac{\frac{4nTk}{\sqrt{\pi}v_0^3} \int \frac{v^3}{v} e^{-v^2/v_0^2} dv}{\frac{4nT}{\sqrt{\pi}v_0^3} \int v^3 e^{-v^2/v_0^2} dv} = \frac{\sqrt{\pi}}{2} \frac{k}{v_0} = \frac{k}{\overline{v}}$$

or

$$\overline{\sigma} = \frac{\int \frac{k}{v} v \, dn}{\int v \, dn} = k \frac{\int dn}{\int v \, dn} = \frac{k}{\overline{v}}.$$

III. APPARATUS

First Method

AGN-201 Reactor (see Fig. 22.1)
 Boron sample (see Fig. 22.2)
 3 cadmium wires (see Fig. 22.2)
 BF₃ detector (see Fig. 22.3)
 Preamplifier
 Amplifier
 MPC counter

Second Method

Reactor
 Gold sample
 Indium sample
 BF₃ detector
 Preamplifier
 Amplifier
 MPC counter
 Detector and sample mounts

139-3418

Fig. 22.1. AGN-201 Reactor with δ_T Experimental Apparatus

139-3421

Fig. 22.2. Boron and Cadmium Samples with Positioning Rods and Lucite Holders

139-3422

Fig. 22.3. BF₃ Proportional Counter and Sample Holder

IV. PROCEDURE

First Method: The cross section at the average velocity ($\sigma_{\bar{v}}$) is obtained by means of the effect of an absorber on the reactivity of a reactor.

The absorption area (cross section) of a black absorber (like cadmium) will not change with neutron velocity (or temperature). The absorption "area" (cross section) of boron (a $1/v$ absorber), however, will change with neutron temperature. Since an absorber will change the reactivity, we may calibrate the sensitivity (reactivity/area of absorber) of a reactor using a black absorber whose absorption area is constant.

In a given average flux $n\bar{v}$, the number of neutrons hitting a unit surface per second is $1/4 n\bar{v}$ (see Appendix II for derivation). Therefore, the number of neutrons being absorbed per second by a cadmium wire of length ℓ and diameter d ($d \ll \ell$) is equal to the surface of the wire times the number of hitting this surface, or $(\pi \ell d)(1/4)(n\bar{v})$. In other words, the absorption area of cadmium wire for a given average flux $n\bar{v}$ is

$$\pi \ell d/4. \quad (15)$$

This is the "area of absorber" to be used in the sensitivity calibration.

Since the cadmium wire depresses the flux, the sensitivity is measured for three cadmium wires of different lengths, and a plot of reactivity/ cm^2 against the absorption area of the wires is made. The extrapolated value (to "zero" absorption area) gives the sensitivity (ρ/cm^2) for no flux depression. Now, the effect of the boron atoms on reactivity is measured, giving

$$N_{\rho} = \text{number of boron atoms/} \rho.$$

The microscopic cross section of boron at average velocity can then be calculated by dividing the total area per reactivity by the number of boron atoms/reativity:

$$\frac{\frac{\text{Effective area (cm}^2\text{) of black absorber}}{\text{reactivity}}}{\frac{\text{number of B atoms}}{\text{reactivity}}} = \text{area of absorption of a single B atom} = \sigma_{\bar{v}}(\text{B}).$$

Using the relating factors from Section II, calculate T.

Second Method: Mount the sample and detector in line with the glory hole. Measure the transmission of a $1/v$ absorber (of close to 50% transmission to "eliminate" the hardening correction) by taking count rates with sample in and out of beam, and take the corresponding background counts. Calculate $\bar{\sigma}_{\text{exp}}$ using Eq. (10); and thus $\bar{\sigma}_{\text{exp}} = \sigma_{v_0}$ in this case. Then calculate the temperature.

V. RESULTS

First Method

For cadmium wires:

Wire	Length (cm)	Diameter (cm)	Physical Area, cm^2	Absorption Area, cm^2	Sensitivity ($\mu\rho/\text{cm}^2$)
1					
2					
3					
4					
5					

From the plot of sensitivity versus absorption area, sensitivity = $\text{---} \mu\rho/\text{cm}^2$

Calculate "inverse sensitivity" $I = \text{---} \text{cm}^2/\mu\rho$

For boron samples (A and B):

Number of boron atoms = --- (sample A)

= --- (sample B)

$N_\rho = \frac{\text{number of boron atoms}}{\text{reactivity corrected for epicadmium absorption}}, N_\rho(A) = \text{---} \text{atoms}/\mu\rho$

Epicadmium correction $\approx 4\%$

$N_\rho(B) = \text{---} \text{atoms}/\mu\rho$

$N_\rho(\text{Avg.}) = \text{---} \text{atoms}/\mu\rho$

Calculate:

$$\sigma_{\bar{v}}(B) = \frac{I}{N_\rho} = \text{---} \text{cm}^2 = \text{---} \text{b}$$

$$v_0 = \frac{755 \text{ b} \times 2200 \text{ m/sec}}{1.128 [\sigma_{\bar{v}}(B) \text{ b}]} = \text{---} \text{m/sec}$$

$$T_{\text{core}} = 293 \left(\frac{v_0}{2200} \right)^2 \text{ } ^\circ\text{K} = \text{---} \text{ } ^\circ\text{K.}$$

Second Method

$I_0 =$ _____ cpm, I_0 Background = _____ cpm.

Sample	Thickness x (cm)	N (atoms/cm ³)	I (cpm)	I - BG	$\ln(I'/I_0)$	$\bar{\sigma}_{\text{exp}}$

σ_{2200} m/sec (given) = _____, $k = 2200 \times \sigma_{2200}(\text{Au})$ _____

$$\bar{\sigma}_{\text{exp}} = F \sigma_{v_0 \text{ exp}} = \frac{k}{v_0 \text{ exp}}$$

F = Harding correction

$$v_0 \text{ exp} = \frac{k}{\bar{\sigma}_{\text{exp}}} = \text{_____ m/sec}$$

$$T_{\text{core neutrons}} = 293 \left(\frac{v_0 \text{ exp}}{2200} \right)^2 \text{ } ^\circ\text{K} = \text{_____ } ^\circ\text{K}$$

Calculate T using indium cross-section data.

Read on reactor thermometer: T(moderator) = _____ $^\circ\text{C}$ = _____ $^\circ\text{K}$

VI. ANALYSIS AND CONCLUSIONS

This section is to be done by the student. However, the following facts, besides the basic assumptions, should be considered before the analysis:

The neutron beam at the AGN-201 reactor was produced by inserting a polystyrene rod midway into the glory hole.

The B^{10}F_3 detector used may not have been of a $1/v$ sensitivity, depending on its length, pressure, and orientation. If it was positioned with

its axis parallel to the beam, its sensitivity is somewhere between a $1/v$ and that of a black detector, depending on its gas pressure and length.

There may have been impurities in the boron sample; the boron was mounted in polystyrene.

Any other factors which may have affected the results should also be discussed.

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VIII. APPENDICESA. Discussion Questions

1. Explain why the thermal-neutron temperature is expected to be higher than the moderator temperature.
2. Explain what is meant by a black absorber.
3. If the average cross section in the beam ($\bar{\sigma}$) is measured and Eq. (12) used to calculate v_0 , the most probable velocity, to which distribution (that is, the velocity distribution or flux distribution) does this velocity belong?
4. If the detector used in the beam method of measuring were black instead of $1/v$, how would the temperature of the neutrons in the core be calculated?
5. Discuss how the variation of temperature would be detected by use of a $1/v$ and a black absorber, from the viewpoint of cross sections. Do the same from the viewpoint of neutron-absorption rate and reactivity.
6. Discuss the possible deviations of the experimental setup from the theory (from basic assumptions like Maxwell distributions, isotropic flux, equilibrium conditions), and their causes, for the first and second methods.
7. Why does the first method give the cross section at the average velocity ($\bar{\sigma}_v$) and not at some other velocity (say, at the most probable velocity σ_{v_0} , as in the second method)?
8. In what particular case only is the total cross section, as measured by the transmission method by passing the whole spectrum through the sample, equal to the cross section at the most probable velocity?

B. Derivation of the Number of Neutrons of All Velocities Striking or Crossing a Surface per Unit Area per Unit Time

Assumptions:

1. All directions of neutron velocities are equally probable (isotropic flux).
2. Large number of neutrons are in a container ("neutron gas") exerting no forces on one another except during collision.

Assumption (1) can be put into analytic form by assigning to each neutron a vector representing the magnitude and direction of its velocity, and transferring all the vectors to the origin of a sphere of arbitrary radius r . The points of intersection of the velocity vectors (prolonged if necessary) with the surface of the sphere will be uniformly distributed due to assumption (1). The number of these points (which represent the number of neutrons N) per unit area is therefore

$$N/4\pi r^2. \quad (1)$$

The number of points in any element of area dA is then $dN = (N/4\pi r^2) dA$. In spherical coordinates, $dA = r^2 \sin \theta d\theta d\psi$. Thus, the number of points in area dA , which represents the number of neutrons having velocities in a direction between θ and $\theta + d\theta$, and between ψ and $\psi + d\psi$ (as is evident from Fig. 22.4), call it $dN_{\theta\psi}$, is

$$dN_{\theta\psi} = (N/4\pi r^2) r^2 \sin \theta d\theta d\psi = (N/4\pi) \sin \theta d\theta d\psi, \quad (2)$$

or the number of neutrons per unit volume with velocities in directions θ and $\theta + d\theta$, ψ and $\psi + d\psi$ (call it $dn_{\theta\psi}$) is

$$dn_{\theta\psi} = \frac{dN_{\theta\psi}}{V} = \frac{n}{4\pi} \sin \theta d\theta d\psi.$$

But these neutrons have different velocities (not all the velocity vectors terminate on the sphere).

Fig. 22.4
Spherical Coordinate System

Let the number of neutrons per unit volume with speeds between v and $v + dv$ (in Fig. 22.4 it is represented by the number of velocity vectors terminating in a thin shell of radius $r = v$ and thickness dv) be represented by dn_v . Therefore, the number of neutrons per unit volume traveling with speeds between v and $v + dv$ in directions between θ and $\theta + d\theta$, ψ and $\psi + d\psi$, symbolized $dn_{\theta\psi v}$, is

$$dn_{\theta\psi v} = (1/4\pi) dn_v \sin \theta d\theta d\psi. \quad (3)$$

How many of the neutrons with these particular velocities specified above will strike a surface/unit area/time? As many as there are in a volume element of length $v dt$ and area dA , inclined in the directions θ and ψ (see Fig. 22.5). No other neutrons of the above characteristics, given by Eq. (3), will strike the surface dA , as a little reasoning will show. The volume of the element dV is $dV = (dA)(v dt \cos \theta)$; the number of neutrons in this element dV is

$$\begin{aligned} dn_{\theta\psi v} dV &= (1/4\pi) dn_v \sin \theta d\theta d\psi (dA v dt \cos \theta) \\ &= (1/4\pi) v dn_v \sin \theta \cos \theta d\theta d\psi dt dA, \end{aligned} \quad (4)$$

or, the number of neutrons striking a surface per unit area per unit time, in the specified directions and with speed v , is

$$\frac{dn_{\theta\psi v}}{dA dt} = \frac{1}{4\pi} v dn_v \sin \theta \cos \theta d\theta d\psi. \quad (5)$$

Then the total number of neutrons striking a surface from all directions per unit area per unit time with speed v is

$$\frac{dn_v}{dA dt} = \frac{v dn_v}{4\pi} \int_0^{2\pi} \int_0^{\pi/2} \sin \theta \cos \theta d\theta d\psi = (1/4) v dn_v. \quad (6)$$

Fig. 22.5
Volume Element

The total number of neutrons of all velocities colliding with the surface per unit dA/dt is

$$\frac{1}{4} \int v dn_v. \quad (7)$$

But the average speed $\bar{v}$ of neutrons in any distribution dn_v is

$$\bar{v} = \frac{\int v dn_v}{n}; \quad (8)$$

therefore, $\int v dn_v = n\bar{v}$, and the result to be derived, that is, the total number of neutrons of all velocities striking a surface per unit area per unit time, is

$$\frac{1}{4} (n\bar{v}). \quad (9)$$

The last equation was derived irrespective of what the velocity distribution dn_v of the neutrons is.

C. Hardening Correction Curve

Fig. 22.6. Hardening Correction for a Maxwellian Distribution Using a $1/v$ Absorber and a $1/v$ Detector

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STUDENT RESULTS

I. Danger Coefficient Method

Data

MEASUREMENTS OF CADMIUM WIRE REACTIVITY WORTH

Length, cm	Area, cm ²	Area/4, cm ²	T, sec	k _{ex} , μρ	Reactivity Worth, μρ	Worth / $\frac{A}{4}$
5	3.047	0.762	∞	0	1190	1560
4	2.452	0.613	512	190	1000	1640
3	1.851	0.466	236	385	805	1728
2	1.255	0.314	133	630	560	1782
1	0.655	0.164	90.2	885	305	1860
0	-	-	57.2	1190	-	-

Calculations

$$\text{Surface Area} = 2\pi r\ell + 2\pi r^2$$

$$A = (0.598\ell + 0.0568) \text{ cm}^2$$

Reactor period as taken from plot of power rise.

$$T = \frac{t_2 - t_1}{\ln(C_2/C_1)}$$

For the 4-cm absorber

$$T = \frac{300 \text{ sec}}{\ln 1.78} = 512 \text{ sec}$$

From the inhour equation plots, a period of 512 sec corresponds to a k_{ex} of 190 μρ. With no tubing in the reactor k_{ex} is in like manner found to be 1190 μρ; hence, the 4-cm absorber must be with the difference in the two k_{ex} values, 1000 μρ. Division of the worth of the sample by its effective absorption cross-sectional area ($A/4$) yields the reactor sensitivity. An extrapolation of the sensitivities of decreasingly smaller absorbers yields the correct sensitivity with zero flux depression, 1890 μρ/cm².

Fig. S22.1. Power Rises with Cadmium Absorbers at Core Center

Fig. S22.2. Cadmium Tubing Reactivity Worth/cm²

BORON-SAMPLES DATA

Sample	N_T , atoms	T, sec	$k_{ex}, \mu\rho$	Sample Worth, $\mu\rho$	Worth/ N_T
6-5	3.02×10^{20}	2950	35	311	1.03×10^{-18}
3-4	-	266	346	-	-
6-1	1.51×10^{20}	555	177	169	1.12×10^{-18}

Calculations

Boron (sample 6-5)

$$N_T = (\text{sample wt}) \frac{\left(\text{Avogadro No., } \frac{\text{mol.}}{\text{g-atoms}} \right) \left(\frac{\text{Boron}}{\text{atoms}} \right)}{\left(\text{Molecular wt., } \frac{\text{g}}{\text{g-atoms}} \right) \left(\frac{\text{H}_3\text{BO}_3}{\text{mol}} \right)}$$

$$= 0.031 \text{ g } \frac{(6.023 \times 10^{23})}{61.84} (1) = 3.02 \times 10^{20}$$

$$T = \frac{t_2 - t_1}{\ln(C_2/C_1)} = \frac{600 \text{ sec}}{\ln 1.226} = 2950 \text{ sec}$$

Fig. S22.3. Power Rises with Boron Samples

From plot of inhour equation, a T equal to 2950 sec corresponds to a k_{ex} of 35 $\mu\rho$. The Lucite runs yield a period of 266 sec corresponding to a k_{ex} of 346 $\mu\rho$. The worth of Sample 6-5 is therefore 311 $\mu\rho$. The worth of Sample 6-1 is found in the same manner. Assuming that no self-shielding and flux depression is present, the worth per atom minus a 5% correction for resonance absorption yields a value of $1.08 \times 10^{-18} \mu\rho/\text{atom}$.

The cross section of boron based on the black absorber sensitivity is

$$\sigma_{\bar{v}} = \frac{1.08 \times 10^{18} \mu\rho/\text{atom}}{1890 \mu\rho/\text{cm}^2} = 569 \text{ b.}$$

The average velocity is

$$\bar{v} = k/\sigma_{\bar{v}} = (755 \times 10^{-24})(2200)/569,$$

$$\bar{v} = 2920 \text{ m/sec,}$$

and

$$v_0 = 2590 \text{ m/sec.}$$

Since $T(^{\circ}\text{K}) \sim v^2$, the neutron temperature can be determined from the following relationships.

$$\frac{T_{2200}}{T_{v_0}} = \left(\frac{2200}{2590} \right) = 0.720, \quad T_{v_0} = \frac{298}{0.720} = 414^{\circ}\text{K.}$$

II. Total Cross-section Method

Data

(Open beam) $I_0^1 = 2592 \text{ cpm}$ (background, cadmium in beam) 150 cpm.
Thermal neutron beam $I_0 = 2440 \text{ cpm}$.

Sample	Thickness x (cm)	N $\frac{\text{atoms}}{\text{cm}^3}$	I'	BGD	I/I ₀	ln I/I ₀	σ_{exp} (b)
Au	0.1016	0.0591×10^{24}	1508	118	0.570	-0.563	939
In	0.1016	0.0382×10^{24}	1320	99	0.501	-0.692	178

The table is completed by using the basic transmission equation

$$I = I_0 e^{-N\sigma x},$$

from which

$$\sigma_{\text{exp}} = \frac{\ln I/I_0}{-Nx}.$$

For gold,

$$\sigma_{\text{exp}} = \frac{\ln 1390/2440}{-(0.0591 \times 10^{24} \times 0.1016)} = \frac{-0.563}{-0.0060} = 93.9 \text{ b}$$

$$\sigma_{\text{exp}} = \sigma(\text{absorption}) + \sigma(\text{scattering}) = 93.9 \text{ b}$$

$$\sigma_s \text{ for gold equals } 9.3 \text{ b (see BNL-325)}$$

therefore

$$\sigma_a = 84.6 \text{ b}$$

$$\sigma_a = 1.128 \sigma_{v_0} \text{ for } T \rightarrow 1$$

$$\sigma_a = 1.016 \sigma_{v_0} \text{ for } T = 0.57\% \text{ taken from the hardening curve.}$$

Therefore,

$$\sigma_{v_0} = 84.6/1.016 = 83.3$$

$$\sigma_{v_0} = K/v_0 = (98.8)(2200)/v_0$$

$$\sigma_{v_0} = 98.8/83.3(2200) = 2610 \text{ m/sec.}$$

Neutron temperature with gold

$$\left(\frac{v_0}{2200}\right)^2 = \frac{T_n}{298}; \quad T_n = 420^\circ\text{K.}$$

Similar calculations for I_n yields

$$T_n = 298 \left(\frac{2450}{2200}\right)^2 = 369^\circ\text{K.}$$

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REMOVAL CROSS SECTION

by

S. G. Mundim and D. A. Daavettla

ABSTRACT

The neutron removal cross section is a measure of the capability of a material to remove neutrons from the 6-8-MeV energy group to a group of lower energy.

A method of measuring the removal cross section is described, and a measurement for an iron shielding plate is performed.

OUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
- IV. PROCEDURE
- V. RESULTS
- VI. ANALYSIS AND CONCLUSION
- VII. BIBLIOGRAPHY
- VIII. APPENDIX
 - Discussion Questions

I. INTRODUCTION

The idea of removal cross section was introduced by Albert and Welton to interpret the attenuation of fast neutrons in an exponential way. (See Reference 4.)

The use of the removal cross section involves the idea of a thin shield inside a hydrogenous infinite medium. This means that a fast neutron, once colliding with a nucleus in the shield penetrates into the moderating medium, slows down and is absorbed. This collision with a shield nucleus is the concept of "removing" the neutron from the fast energy group.*

The attenuation of a neutron population after passing through a shield of thickness t can be expressed by

$$I/I_0 = \exp(-\Sigma t), \quad (1)$$

where I and I_0 are the neutron population respectively after and before passing through the shield, and Σ is the macroscopic removal cross section.*

For the particular case of the Argonaut water tank, Eq. (1) has to be corrected, because the distance between source and detector is smaller than the minimum value (120 cm) that allows the use of the simple exponential law. The correction introduced takes into account the source radius, the shielding thickness, the detector-source distance, and the relaxation length of the neutrons where the counter is located.

II. THEORY

If we assume the concept of point kernel to be true for the present case, i.e., the dose received by a detector is a function only of the distance to the emitting source, we have

*This concept is applied to energies of the order of 6 to 8 MeV because neutrons with values smaller or larger may be neglected from an energy-range consideration standpoint.

$$D = G(R)$$

as a result of the emission of a unit point source R cm away from the detector. The dose rate, D , is defined as a measure of the radiation being received at a distance R , and $G(R)$ is called point kernel.

If we have a circular source emitting one particle per sec per unit area, as in Fig. 23.1, the dose rate resultant from the element source $2\pi\rho d\rho$ is

$$G(R) 2\pi\rho d\rho. \quad (2)$$

Integration from zero to the source radius a , gives the total dose rate that comes from the source.

Fig. 23.1
Circular Plane Source

Assuming that the dose rate is related to distance R by the expression

$$G(R) = g(Z) \exp |-(R - Z)/\lambda| \quad (3)$$

where λ is the relaxation length, and the flux at R cm is

$$G(R) = g(R)/4\pi R^2,$$

where $g(R)$ is also a function only of R , we can integrate Eq. (2) to give

$$I_0 = \frac{g(Z) e^{Z/\lambda_0}}{2} \int_Z^{\sqrt{Z^2 + a^2}} \frac{e^{-R/\lambda_0}}{R} dR. \quad (4)$$

If Eq. (4) is transformed into two exponential integrals, we obtain

$$I_0 = \frac{g(Z) e^{Z/\lambda_0}}{2} \left[E_0(Z/\lambda_0) - E_0\left(\sqrt{Z^2 + a^2}/\lambda_0\right) \right], \quad (5)$$

where E_0 is an exponential integral of the argument in parentheses.

Now we notice that for each distance Z the expression in the brackets is constant, as well as the exponential of Z/λ_0 ; we can accordingly rewrite Eq. (5) as

$$I_0 = \frac{1}{2} g(Z) K_0(Z, \lambda_0), \quad (6)$$

where

$$K_0 = e^{Z/\lambda_0} \left[E_0(Z/\lambda_0) - E_0\left(\sqrt{Z^2 + a^2}/\lambda_0\right) \right]. \quad (7)$$

If we put a shielding material between the source and the detector, and reason as above, we find the expression

$$I = \frac{1}{2} g(Z) F(\Sigma), \quad (8)$$

where

$$F(\Sigma) = e^{Z/\lambda} \left\{ E\left[\frac{Z}{\lambda} + \Sigma t\right] - E\left[\left(\frac{Z}{\lambda} + \Sigma t\right) \sqrt{1 + \frac{a^2}{(Z+t)^2}}\right] \right\}. \quad (9)$$

Division of Eq. (6) by Eq. (8), gives

$$I_0/I = K_0/F(\Sigma).$$

Thus, for a given geometric arrangement,

$$F(\Sigma) = \frac{K_0}{I_0/I}. \quad (10)$$

Having once measured I_0 and I , we calculate K_0 by Eq. (7). Then we can find $F(\Sigma)$ by Eq. (10), and finally get Σ from Eq. (9). Curves are given in this experiment report for graphically determining the values of K_0 , $F(\Sigma)$, and based on measurements of I and I_0 , and their respective slopes λ and λ_0 at a given position Z .

For a more detailed derivation, please read Bibliography entry number 1.

III. APPARATUS

The necessary apparatus for this measurement is shown in Figs. 23.2 and 23.3. A detailed list of apparatus follows.

Argonaut reactor.

Water tank.

Fast-neutron source, which is an assembly of 30 fuel plates arranged in 6 groups of 5 plates each. (See Figs. 23.4 and 23.5.).

Shutter mechanism (see Fig. 23.6) consisting of a cadmium sheet with a circular hole and a cadmium plate which can cover this hole. Both are put together and outside the tank front face.

Thermal-neutron filter: a cadmium sheet close to the source.

Two shielding iron plates, each one inch thick, put together to form a 2 in. thick shield.

Detector and traverse mechanism to slide the detector along the tank length (see Figs. 23.7 and 23.8).

Preamplifier and multipurpose counter.

IV. PROCEDURE

It is suggested that the shielding plates be placed in the tank before the measurements because these plates are heavy and difficult to put in the right position close to the neutron source. If this is done, the operation to take them out of the tank for the second run does not introduce sensible errors in the source position.

It is also suggested that the bar which supports the individual iron shield plates be removed and that both plates be bolted together. The plates will then be supported by the bottom of the tank.

The ruler which supplies the readings for the counter position should have a centimeter scale, since the curves for the calculation of removal cross section were plotted in this unit.

The following steps should be performed:

1. Assemble in the tank all the apparatus, including the shielding plates.
2. Close the shutter and drive the tank in.
3. Make the reactor critical and set the channel-1 instrument to an expanded scale in order that the power level can be more accurately reproduced in the second run.

Fig. 23.2. General View of the Assembly

Fig. 23.3. Traversing and Shutter Mechanisms

Fig. 23.4. Neutron Converter

Fig. 23.5. Neutron Converter and Detector

Fig. 23.6. Shutter and Cadmium Mask

Fig. 23.7. Measuring Device

Fig. 23.8. Traversing Mechanism

4. Make a traverse measure along the tank length, being sure to relate each position to the shield face. The measurements should be taken preferably at the following points related to the shield face: 25 - 30 - 35 - 40 - 45 - 50 - cm, because the constant-Z curves from which the calculation will be done were plotted for those values.

5. Without shutting the reactor down, open the shutter, allowing the thermal neutrons to come into the source to produce fission neutrons.

6. Measure the neutron population exactly at the same points as in item 4.

7. Drop the reactor control rods (do not scram the reactor) and remove the shielding plates. Care must be taken not to modify the geometry of the system.

8. Close the shutter and make the reactor again critical at the same power as before; measure the neutron population at the same points as before.

9. Open the shutter and repeat the same measurements.

V. RESULTS

The measurements made in Item IV-4 give the background correction for the neutron population (I_b) with the shielding plate in. The measurements made in Item IV-6 give the gross value (I_g) of the neutron population.

The difference $I_g - I_b = I$ gives the net value of the neutron population behind the shielding plate.

Similarly, the measurements made in Items IV-8 and 9 result, respectively, in values I_{0b} and I_{0g} such that

$$I_{0g} - I_{0b} = I_0,$$

where I_0 is the neutron population as seen directly from the source.

Plot both I and I_0 on a semilog paper versus distance from the shielded face, and shift the I_0 curve to the right by a distance t cm larger than the plotted one, where t is the shield thickness. This shifting is done because the space filled by the shield was filled with water when the former was removed. Accordingly, the distance in water is now increased by t cm.

At each distance, take from the plot the values indicated below:

$$Z(\text{cm}), I_0, \lambda_0(\text{cm}), I, \lambda(\text{cm}), I_0/I,$$

where λ_0 and λ are the relaxation lengths, respectively, measured from the curves I_0 and I at the same point Z . These lengths can be given directly by the e-folding distance of the curve at the considered point.

Enter with the value of Z and λ_0 in Fig. 23.9 to obtain the value of K_0 . Application of Eq. (10) then gives $F(\Sigma)$.

Choosing the curve corresponding to the value of Z being considered (see Figs. 23.10 to 23.15) enter with $F(\Sigma)$ and λ , and find the corresponding value of σ —the microscopic removal cross section.

Repeat the same calculation for every point and find the average microscopic removal cross section for iron.

Fig. 23.9. K_0 versus Z

Fig. 23.10. $F(\Sigma)$ versus λ for $Z = 25$ cm Fig. 23.11. $F(\Sigma)$ versus λ for $Z = 30$ cm

Fig. 23.12. $F(\Sigma)$ versus λ for $Z = 35$ cm Fig. 23.13. $F(\Sigma)$ versus λ for $Z = 40$ cm

Fig. 23.14
 $F(\Sigma)$ versus λ
 for $Z = 45$ cm

Fig. 23.15
 $F(\Sigma)$ versus λ
 for $Z = 50$ cm

VI. ANALYSIS AND CONCLUSION

The method described above gives results in good agreement with the tabulated values of removal cross section.

The conclusion should be considered by the student from the results obtained.

VII. BIBLIOGRAPHY

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VIII. APPENDIXDiscussion Questions

1. Describe the process for removal of fast neutrons.
2. Why are only neutrons with energies from 6 to 8 MeV considered?
3. When I_0 and I curves are plotted, it is observed that there is a portion of I_0 curve which is lower than the I curve. Explain.
4. What is the reason for shifting the I_0 curve to the right?
5. If the removal theory allows a simple exponential treatment for the attenuation of fast neutrons [Eq. (1)], why was it not applied in the measurement described here?
6. The BF_3 counter used is encapsulated in a Lucite tube. Why was this material used, and what other material could be used?
7. What could you suggest to eliminate the gamma counting registered by the BF_3 counter?

STUDENT RESULTS

I. Irradiation Run

The reactor core was a one-slab loading of six fuel boxes in the north quadrant. The removal cross section of iron was measured using the two one-inch-thick, steel shield plates. The shield plates were bolted together and mounted flush against the converter fuel plates. The irradiation power level was 100 W.

II. Raw Measured Data

The raw measured data are the neutron counts per minute versus the distance measured from the shield plate face, and these data are presented in Table S23.1. Here I_0 has already been corrected to the proper length of water path.

Table S23.1

MEASURED DATA

Z, cm	I_0 (gross), cpm	I_0 (background), cpm	I_0 (net), cpm	I (gross), cpm	I_b (background), cpm	I (net), cpm
25.42	60051	12010	48041	23548	4355	19193
30.52	22726	5886	16840	8346	1850	6496
35.52	9832	3205	6627	3433	968	2465
40.62	4307	1783	2524	1532	529	1003
45.72	2542	1360	1182	768	335	433
50.82	1565	1072	493	397	207	190

III. Calculation of the Removal Cross Section

The values of I_0 and I for various values of Z are given in Table S23.2 as determined from Fig. S23.1. The net activities I_0 and I from Table S23.1 are plotted in Fig. S23.1.

Table S23.2

NET I_0 AND I ACTIVITY

Distance z, cm	I_0 , cpm	I, cpm	I_0/I
30	18600	7050	2.64
35	7100	2700	2.63
40	2940	1100	2.67
45	1230	485	2.54
50	545	218	2.50

Fig. S23.1

Net Measured Activity
(cpm) versus Distance
Z from Shield Face

From Fig. S23.1 the slopes (e-folding distance) as a function of Z were determined to be as in Table S23.3.

Table S23.3

SLOPE VERSUS Z

Distance z, cm	λ , cm	λ_0 , cm
30	5.00	5.20
35	5.25	5.50
40	5.80	5.65
45	6.10	6.00
50	6.10	6.20

For each λ_0 the corresponding K_0 was determined from Fig. 23.9. Then $F(\Sigma)$ was calculated by Eq. (10). Finally, σ_r was picked out from Figs. 23.10 through 23.15. The results are given in Table S23.4.

Table S23.4

REMOVAL CROSS SECTIONS

Distance, cm	K_0	$F(\Sigma)$	$\sigma(r)$, b
30	0.0905	0.0343	1.70
35	0.0729	0.0277	1.76
40	0.0600	0.0225	1.83
45	0.0502	0.0198	1.72
50	0.0423	0.0170	1.70

The literature value is 1.98 b.

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ABSORPTION CROSS SECTION MEASUREMENT
IN A GRAPHITE SIGMA PILE

by

K. C. Ruzich

ABSTRACT

A study of thermal-neutron diffusion in a nonmultiplying medium is made. Basic diffusion theory is used to describe the neutron spatial distribution in a rectangular graphite pile with a neutron source placed below the assembly. The measurement of the neutron distribution in the pile is used to obtain the thermal diffusion length of graphite from which the absorption or transport cross sections can be calculated.

OUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
- IV. PROCEDURE
 - A. Irradiation of Indium Foils
 - B. Counting Indium Foils in a Gas Flow Proportional Counter
- V. RESULTS
- VI. ANALYSIS AND CONCLUSION
- VII. BIBLIOGRAPHY
- VIII. APPENDIX
 - Discussion Questions

I. INTRODUCTION

The main purpose of this experiment is to acquaint the student with the basic ideas of neutron diffusion in matter. The experiment consists of measuring the thermal-neutron spatial distribution in a graphite pile and the determination of the diffusion length of graphite. The method to be used was formulated before chain-reacting piles were built. The nonmultiplying pile is called a "sigma pile," because the determination of the diffusion length of the assembly yields either the absorption or transport cross section. The cross sections and diffusion lengths of a certain material are important reactor parameters that must be known in order to estimate the absorption and leakage of neutrons from a chain-reacting pile containing this material.

The sigma pile used in this experiment is a rectangular parallelepiped, approximately $5\frac{1}{2}$ ft square and 6 ft tall. The pile consists of long rectangular graphite blocks, $3\frac{13}{16}$ in. sq, all lying in the same direction. Below the pile is a 2-ft-high graphite pedestal which contains the antimony-beryllium neutron source. Neutron-flux measurements are made in the horizontal and vertical directions, either with a neutron detector or by foil-activation techniques. The neutron distribution is then fitted to the theoretical distribution determined for a pile of this geometry. The fitting of the theoretical distribution with that obtained experimentally yields the parameters necessary for the determination of the thermal-neutron diffusion length.

II. THEORY

The behavior of neutrons as they move through matter is an extremely important problem in reactor physics. A typical neutron path

would be a zigzag pattern of straight-line segments of varying lengths extending from the neutron origin to disappearance by capture (see Fig. 24.1). The probability of capture of the neutron as it moves through the medium is a function of the sum of the straight-line segments of its flight path. In a finite medium, the crow-flight distance of the neutron before capture will be a determining factor in the number of neutrons that leak out of the system.

Fig. 24.1. Typical Neutron Path

The basic theory which is used as a model for our system is taken from the kinetic theory of gases and is called diffusion theory. This theoretical approach to the neutron behavior in the graphite pile is only workable under certain simplifying assumptions about the phenomena occurring. The neutrons in the pile are thought of as being a low-density gas colliding with the atoms in the medium. The theory deals with the average behavior of large numbers of neutrons.

The conservation of neutrons in a homogeneous volume element is expressed as

$$\text{Production} - \text{leakage} - \text{absorption} = \frac{\partial n}{\partial t}, \quad (1)$$

where n equals the number of neutrons per cm^3 . In the steady-state condition, $\partial n / \partial t = 0$, giving

$$S - \mathcal{L} - A = 0, \quad (2)$$

with

- S = neutrons produced in the volume element per sec,
- $\mathcal{L} = DV^2\phi(r)$ neutrons leaking out of the volume element per sec,
- D = diffusion coefficient, which equals $1/3$ of the transport mean path,
- ∇^2 = Laplacian operator,
- $\phi(r)$ = monovelocity neutron flux (nv) at position r ,

$A = \Sigma_a \phi$ neutrons absorbed in the volume element per sec, and
 Σ_a = macroscopic absorption cross section of volume element.

Rewriting Eq. (2), we find

$$S + D\nabla^2\phi(r) - \Sigma_a\phi(r) = 0. \quad (3)$$

This second-order differential equation is called the steady-state diffusion equation. It is applicable only for single-velocity neutrons and at distances greater than three mean free paths from boundaries and from strong sources. In order to solve the equation for a system and determine the neutron spatial distribution, specific boundary conditions must be known from the physical characteristics of the system.

Fig. 24.2. Isotropic Point Source

As an example, imagine a neutron point source of strength S (n/sec) placed in an infinite medium (see Fig. 24.2). The spatial distribution will now be determined from Eq. (3) with ∇^2 in spherical coordinates and S equal to zero everywhere except at $r = 0$:

$$\frac{d^2\phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} - \frac{\Sigma_a}{D} \phi = 0. \quad (4)$$

The ratio D/Σ_a is defined as the square of the diffusion length (L^2) of the medium in question. If we solve Eq. (4), with the condition that the neutron flux is finite in the medium, we obtain

$$\phi(r) = \frac{Ae^{-r/L}}{r}. \quad (5)$$

The constant A is determined from the source condition. Neglecting absorption, the number of neutrons passing through a sphere with the surface area $4\pi r^2$ must equal S ; hence the neutron current J (neutrons/cm²-sec) at position r equals $S/4\pi r^2$, and therefore in the limit as r approaches zero,

$$\lim_{r \rightarrow 0} 4\pi r^2 J = S. \quad (6)$$

According to basic diffusion theory, it can be shown that the neutron current at radius r is

$$J = -D \left[\frac{d\phi}{dr} \right]. \quad (7)$$

Upon substituting Eqs. (5) and (7) in Eq. (6), we find

$$A = S/4\pi D, \quad (8)$$

and Eq. (5) becomes

$$\phi(r) = \frac{Se^{-r/L}}{4\pi Dr}. \quad (9)$$

The number of neutrons being absorbed per cm^3 per sec at position r in the medium is $\Sigma_a \phi$.

We wish to determine the mean square distance neutrons travel from the source before being absorbed. This distance is related to the crow-flight distance we mentioned in Fig. 24.1:

$$\overline{r^2} = \frac{\int r^2 \phi \Sigma_a dV}{\int \phi \Sigma_a dV}. \quad (10)$$

Substituting Eq. (9) into (10) and integrating, with $dV = 4\pi r^2 dr$, from zero to ∞ (Σ_a is a constant), we find

$$\overline{r^2} = 6L^2; \quad L = \sqrt{\overline{r^2}/6}.$$

The physical significance of the diffusion length is now apparent. For a point source of thermal neutrons, the diffusion length is equal to the square root of $1/6$ times the root-mean-square distance travelled by a neutron from point of origin to the point of absorption.

The diffusion length is an important reactor parameter which determines the amount of thermal-neutron leakage out of a finite medium. Of course, in a chain reacting pile the diffusion length is that of a mixture of materials: moderator, fuel, structure, etc. The measurement of L in a nonmultiplying medium is significant in that cross sections can be determined from the measurement. As previously defined, $L^2 = D/\Sigma_a$. The diffusion coefficient equals one-third the transport mean free path, and Σ_a is the inverse of the absorption mean free path. Therefore,

$$L^2 = \lambda_t \lambda_a / 3 = 1/3 \Sigma_t \Sigma_a = 1/3 N^2 \sigma_t \sigma_a,$$

with

- N = number of atoms per cm^3 ,
- σ_t = microscopic transport cross section,
- σ_a = microscopic absorption cross section.

If either σ_t or σ_a are known, the other cross sections can be determined from a measurement of diffusion length.

The experimental arrangement for the measurement of L in a graphite medium is shown in Fig. 24.3. The sides of the parallelepiped are a , b , and c ; the source is located below the pile in a graphite pedestal. Diffusion theory will be used to determine the theoretical neutron spatial distribution in the assembly. In cartesian coordinates Eq. (3) is

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} - \frac{\Sigma_a}{D} \phi = 0 \quad (11)$$

if $S = 0$ in the assembly.

Fig. 24.3. Sigma Pile and Graphite Pedestal

The boundary conditions prevailing are the following:

1. ϕ is finite everywhere.
2. ϕ is equal to zero at the extrapolated boundaries.
(This is a mathematical convenience to establish a simple boundary condition. The extrapolated distance is taken to be equal to $0.71 \lambda t$.)
 - a) At $x = \pm \left(\frac{a'}{2} + 0.71 \lambda t \right)$, $\phi = 0$;
 - b) At $y = \pm \left(\frac{b'}{2} + 0.71 \lambda t \right)$, $\phi = 0$;
 - c) At $z = c' + 0.71 \lambda t$, $\phi = 0$.

3. A source condition at bottom plane of the assembly
($z = 0$).

The assumption is made that the variables x , y , and z are separable, that is

$$\phi(x, y, z) = X(x)Y(y)Z(z). \quad (12)$$

Substitution of this expression for the neutron flux into Eq. (11) and then division by the expression yield

$$\frac{1}{X} \frac{d^2 X}{dx^2} + \frac{1}{Y} \frac{d^2 Y}{dy^2} + \frac{1}{Z} \frac{d^2 Z}{dz^2} - \frac{1}{L^2} = 0. \quad (13)$$

Each of the first three terms in the Eq. (13) can be represented by a constant, since they are independent of each other, and $1/L^2$ is a constant. Eq. (13) may be rewritten as

$$-\alpha^2 - \beta^2 + \gamma^2 - (1/L^2) = 0. \quad (14)$$

Each term in Eq. (13) is here represented by a constant that is a real number, and whose sign was chosen to satisfy a simple boundary condition. Eqs. (13) and (14) produce three second-order differential equations to be solved with the correct boundary conditions. The resulting distributions for the assembly in each variable are,

$$\begin{aligned} X_m &= A_m \cos(m\pi x/a); \\ Y_n &= B_n \cos(n\pi y/b); \\ Z_{mn} &= C_{mn} e^{-\gamma_{mn} z} [1 - e^{-2\gamma(c-z)}], \end{aligned} \quad (15)$$

where a , b , and c are the dimensions of the assembly including the extrapolation distances, and

$$\begin{aligned} m\pi/a &= \alpha; \\ n\pi/b &= \beta. \end{aligned}$$

Here m and n are odd integers: 1, 3, 5, Equation (14) may be rewritten

$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{1}{L}\right)^2 = \gamma_{mn}. \quad (16)$$

Since $(1/L)^2$ is a constant for any set of m 's and n 's, there is a value γ_{mn} satisfying Eq. (16).

A simple solution describing the neutron flux at any position in the assembly is just a product of X , Y , and Z as given in Eq. (15). A complete solution, however, is the summation of all possible products. Hence, summing all possible solutions, Eqs. (15) are substituted into Eq. (12).

$$\phi(x, y, z) = \sum_{mn} \phi_{mn}(x, y, z) = \sum_{mn} A_{mn} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{-\gamma_{mn} z} \left[1 - e^{-2\gamma_{mn}(c-z)} \right], \quad (17)$$

where A_{mn} represents the combined arbitrary constants in Eq. (15). The neutron-flux distribution is represented by a series of harmonic terms plus the fundamental, for which $m = n = 1$. The higher harmonic effects will decay faster as we move in the positive z direction as a result of the factor $e^{-\gamma_{mn} z}$. This is true because γ_{mn} is larger for the higher harmonics, as seen from Eq. (16). If it can be assumed that the fundamental term is the only term contributing to the neutron flux, Eq. (17) can be rewritten as

$$\phi(x, y, z) = A_{11} \cos \frac{\pi x}{a} \cos \frac{\pi y}{b} e^{-\gamma_{11} z} \left[1 - e^{-2\gamma_{11}(c-z)} \right],$$

and, providing $c \gg z$, the bracketed quantity can be assumed to equal one. The neutron flux in the z direction is

$$Z(z) = (\text{const}) e^{-\gamma_{11} z},$$

and the diffusion length is then determined from Eq. (18):

$$1/L^2 = \gamma_{11}^2 = \left(\frac{\pi}{a} \right)^2 + \left(\frac{\pi}{b} \right)^2.$$

III. APPARATUS

The graphite sigma pile is composed of $3 \frac{13}{16}$ in. sq carbon blocks stacked in horizontal layers to form a rectangular parallelepiped measuring $64 \frac{7}{8} \times 65 \frac{5}{8}$ in. in cross section and 72 in. in height. Alternate graphite blocks in the pile have $1 \frac{3}{4}$ -in.-diameter horizontal through holes which are filled with removable graphite plugs. The holes are present to permit loading materials other than graphite into the pile. There are small, rectangular access stringers running horizontally and vertically through the pile. These are used for placements of activation detectors when measuring the flux distribution in the pile. A horizontal and a vertical circular hole are provided for the movement of counters and/or ionization chambers.

The pile stands on a graphite pedestal which contains the neutron source. The pedestal has the same cross-sectional dimensions as the pile and is $24 \frac{1}{2}$ in. high. A cadmium shutter is provided at the interface of the pile and pedestal. The shutter is used to eliminate the thermal neutrons entering the pile from the pedestal. Figures 24.4 and 24.5 give a pictorial representation of the graphite pile.

Fig. 24.4. Schematic of Graphite Sigma Pile

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Fig. 24.5. Graphite Sigma Pile

The neutron source used to provide a steady-state thermal-neutron distribution in the pile is located at the lower center of the graphite pedestal in a removable stringer. The basic nuclear reaction of the source is

gamma rays are captured by beryllium nuclei, and a neutron is emitted. This type of a source is called a photoneutron source because photons (gamma rays) initiate the reaction. In order for this reaction to occur, the gamma ray must have a minimum (threshold) energy of 1.66 MeV. This energy is needed to remove a neutron from the Be^9 nucleus and corresponds to the binding energy of the neutron in question. The gamma rays are produced by radioactive antimony which has a half-life of 60 days. The product gamma ray of interest emitted by Sb^{124} has an energy of 1.7 MeV. The antimony is made radioactive by the reaction

A very intense source of Sb^{124} is made by irradiation of antimony in the thermal-neutron flux of the CP-5 reactor. The actual source (see Fig. 24.6) is a right cylinder of antimony, $1\frac{1}{2}$ in. high and 1 in. in diameter, placed in a small, 3-in. block of beryllium. The strength of the source is approximately 2×10^7 n/sec/Ci of antimony-124. The neutrons produced by the source are of relatively low energy (several keV); hence, the neutrons entering the pile from the source are moderated to thermal velocities in the graphite pedestal.

Measurements of the neutron-flux distribution can be made by foil-activation techniques. Indium has been chosen as the activation detector because of its high activation cross section for thermal neutrons and its convenient half-life of 54 min. The circular foils are $1\frac{1}{8}$ in. in diameter and weigh approximately 0.5 gm. The foils are placed in aluminum planchets and are loaded into the pile after being taped onto foil stringers which extend completely through the pile. The activity of the indium foils will be determined by counting techniques using a gas flow proportional counter (see Fig. 24.7). A conventional linear amplifier and scaling unit (MPC-1) is used. The steel pig shields the small cylindrical chamber from outside irradiation. P_{10} gas (90% argon and 10% methane) is passed through the chamber at a constant, slow rate. The indium foils are placed on a holder and inserted into the counter at different shelf positions which give various counting geometries.

The use of indium foils to determine the relative neutron-flux distribution in the pile requires knowledge of the laws of radioactivity. When indium is placed in a certain neutron flux for a given length of time, some of the indium atoms will capture neutrons, and the following reactions will occur:

139-3416

Fig. 24.6. Antimony-beryllium Photoneutron Source

139-3424

Fig. 24.7. Gas Flow Proportional Counter

Each product of the reaction is radioactive, and its decay scheme is the following:

The ground-state nuclei (In^{116}) decay with a half-life of 13 sec; hence, these nuclei disappear completely in the order of a few minutes after they are produced. The reaction yielding In^{116m} , the meta-stable state of In^{116} , will be utilized in this experiment.

The rate of producing In^{116m} nuclei in an indium foil is

$$R(\text{activations/sec}) = N_T^{115} \sigma_{\text{act}} \phi,$$

where

$$N_T^{115} = \text{total number of atoms of } \text{In}^{115},$$

$$\sigma_{\text{act}} = \text{the microscopic activation cross section per atom,}$$

$$\phi = \text{neutron flux (nv) neutron/cm}^2\text{-sec.}$$

The number of In^{116} atoms decaying per second equals λN , where

$$\lambda = \text{decay constant of } \text{In}^{116m},$$

$$N = \text{number of radioactive } \text{In}^{116m} \text{ atoms.}$$

The rate of change in In^{116m} atoms in the foil equals the rate of production minus the rate of decay:

$$\frac{dN}{dt} = N_T^{115} \sigma_{\text{act}} \phi - \lambda N.$$

If the foils are irradiated in the sigma pile for a time t_1 , the number of radioactive atoms existing in the foil at t_1 is

$$N(t_1) = \frac{N_T^{115} \sigma_{\text{act}} \phi}{\lambda} (1 - e^{-\gamma t_1}).$$

The activity of the foils, $\lambda N(t_1)$, is proportional to the neutron flux in which the foils were irradiated. If λt_1 is large ($t_1 \gg 1/\lambda$), the activity of the foil is said to have reached saturation (the term in the parentheses will then equal one). This is the maximum activity that the foils can obtain at this location.

The problem of determining the neutron-flux distribution in the pile is relatively simple if the activity of the foils can be determined at the time of removal. The foils will proceed to decay after time t_1 , and the activity at any time after the irradiation will be

$$\lambda N(t) = \lambda N(t_1) e^{-\lambda t},$$

where t is the decay time measured since the irradiation ceased. If the foils are placed in a counter and the counting rates are determined, it can be shown that under certain simplifying assumptions

$$\lambda N(t) \sim \dot{C} \text{ and } \lambda N(t) = K \dot{C},$$

where K is the inverse efficiency of the counter. Hence, if the efficiency of the counter is the same for all foils, the counting rates of the foils extrapolated back to removal time will be proportional to the neutron flux, and the spatial distribution can be determined:

$$K \dot{C} e^{\lambda t} = \lambda N(t) e^{\lambda t} = \lambda N(t_1) = N_T^{115} \sigma_{\text{act}} \phi (1 - e^{-\lambda t_1}).$$

An alternative method for measuring the thermal-neutron distribution in the pile requires the use of a $B^{10}F_3$ proportional counter. The counter utilizes the following reaction to detect neutrons:

The alpha and lithium nuclei ionize the BF_3 gas, and a voltage pulse is ultimately produced from each event. The number of events, absorptions, occurring per second is

$$R(\text{absorptions/sec}) = N_T \sigma_a \phi,$$

where

N_T = total number of ${}_5\text{B}^{10}$ atoms,

σ_a = microscopic absorption cross section,

ϕ = neutron flux at detector location

It is easily seen from this expression that the counting rate obtained by the detector is proportional to the neutron flux at the detector location. The B^{10}F_3 detector used in this experiment has an active volume of approximately 12 cm^3 and a sensitivity of one count per second per unit flux. The counter is placed in a fixed position at the end of an aluminum tube which is about 6 ft long. The tube is graduated in equal increments of length. The counter is positioned in the pile by inserting the tube into the vertical or horizontal holes provided. The electronics associated with the detector consist of a preamplifier and a MPC-1 linear amplifier, high-voltage supply, and scaler.

IV. PROCEDURE

A. The first procedure in this experiment consists of two main sections: the irradiation of the indium foils and the determination of the activity of the foils by counting techniques. A step-by-step procedure follows:

1. Mount the indium foils in the vertical and horizontal stringers. Each foil has a number inscribed on it for identification. A long list of foils and the position on the stringer is made on the data sheet (see Fig. 24.8). The foils are placed in their stringer positions and taped firmly in place.
2. The foil stringers are placed in their respective positions in the pile. The antimony-beryllium source is then inserted into the graphite pedestal. The insertion time should be noted on the data sheet.
3. Irradiate the foils in the pile for a minimum of 2 hr (this gives 78% of saturation activity).
4. Remove the neutron source from the graphite pedestal. Note time of removal on the data sheet. Remove stringers from the pile. Check the radioactivity of the stringers and foils with a radiation safety survey instrument to insure no dangerous health conditions are prevailing.
5. Remove the foils from the stringers in preparation for counting. Wait at least 3 min before counting the first foil to make certain that the ground-state nuclei of In^{116} have decayed out. Take a background

Vertical and Horizontal

a = _____ cm

b. = _____ cm

C = _____ cm

DATE _____

BKD. _____ cpm

 τ (Dead Time)
$$\lambda = 0.01281/\text{min.}$$

Start Irradiation: _____

Stop Irradiation: _____

[illegible]

24.17

count with the gas flow proportional counter. Count the foils in the gas flow proportional counter using the foil holders available for this purpose. Two counters are used: one for the vertical foils and the other for the horizontal foils. Count each foil on the same shelf, if possible, so that no correction has to be made for different counter geometries. Collect all the data necessary to determine the correct counting rates at a certain time. The data sheets should be filled in completely.

6. The background should be measured again after all the foils have been counted. Complete the data sheet, making the necessary calculations to obtain the true counting rate per gram of each foil.

B. The alternative procedure with a $B^{10}F_3$ counter includes the following steps:

1. With the help of the instructor, set up the experimental apparatus (counting system).
2. Move the neutron source into the graphite pedestal.
3. Place the aluminum tube containing the neutron detector into the horizontal hole. Measure the counting rates at different positions in the pile so as to determine the spatial distribution in the horizontal direction. All the data should be listed on the data sheet provided.
4. Measure the vertical neutron spatial distribution using the vertical hole provided.

V. RESULTS

The results of the experiment should include the following:

1. A linear horizontal plot of the thermal-neutron flux normalized to one at the center of the assembly. The resulting curve should be compared with the expected theoretical distribution:

$$X(x) = A \cos (\pi/a)x,$$

where a equals the pile dimension in the horizontal direction plus two terms $0.71 \lambda_{tn}$. The transport mean free path for graphite is 2.70 cm. A plot of the theoretical distribution should be superimposed over the experimental distribution.

DATA SHEET

BF₃ COUNTER TRAVERSE

DATE _____

Tube Position	Reference Position	Total Counts	Time	Counting Rate	Tube Position	Reference Position	Total Counts	Time	Counting Rate

2. A semilog vertical plot of the thermal-flux distribution is made. The data will yield a straight-line plot everywhere except near the boundaries of the pile. The theoretical distribution is (see Section II).

$$Z(z_i) = Z(z_0) e^{-\gamma(Z_i - Z_0)}.$$

The parameter γ is determined from the end points of the straight-line portion of the plot:

$$\gamma = \frac{\ln[Z(z_0)/Z(z_1)]}{Z_1 - Z_0}.$$

3. The diffusion length is calculated from the derived expression

$$\frac{1}{L^2} = \gamma^2 - \left(\frac{\pi}{a}\right)^2 - \left(\frac{\pi}{b}\right)^2,$$

where a and b are the horizontal dimensions of the pile plus the extrapolated distances.

4. From the calculated value of L^2 , determine the absorption cross section for the pile graphite:

$$L^2 = D/\Sigma_a = \lambda_{tr}/3N\sigma_a; \quad \sigma_a = \lambda_{tr}/3NL^2.$$

VI. ANALYSIS AND CONCLUSIONS

This section is left to the student as an exercise. A comparison of the experimental results of σ_a and L^2 should be made with the most current values listed in the literature. Any conclusion can be considered to be completed.

VII. BIBLIOGRAPHY

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VIII. APPENDIXDiscussion Questions

The following questions should be considered prior to arrival at the pile. If possible, all the questions will be discussed and answered before the beginning of the experimental work.

1. Define the following pile parameters: Σ_a , Σ_{tr} , D , λ_{tr} , and L^2 .
2. What basic theory is used to describe neutron behavior in the sigma pile? Where did the theory originate, and what conditions must prevail in order to make the theory applicable?
3. Describe a sigma pile experiment to determine the diffusion length and the absorption cross section of the assembly.
4. How is the neutron spatial distribution measured in this experiment?
5. Derive the expression relating detector activity to the neutron flux. What is meant by saturated activity?
6. Why was indium chosen as the activation detector to be used in this experiment? What type of nuclear particles are given off when In^{116m} decays? Which of these particles are counted in the gas flow proportional counter?
7. What is the sensitivity of a BF_3 counter? Must it be known before the neutron-flux distribution can be determined?
8. What other methods can be used to measure the absorption cross section of graphite?
9. What precautions should be taken in this experiment to ensure that no one is exposed to radiation?

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STUDENT RESULTSDATAHorizontal Traverse

Position, in.	Total Counts	Time (min)	cpm	Normalized
65	11,762	7	1,680	0.105
61	13,154	3	4,385	0.274
57	27,592	4	6,898	0.431
63	19,177	2	9,588	0.599
49	23,318	2	11,659	0.729
45	40,879	3	13,626	0.852
41	30,183	2	15,091	0.943
37	31,570	2	15,785	0.987
33	31,994	2	15,997	1.000
29	31,030	2	15,515	0.970
25	28,763	2	14,381	0.899
21	25,533	2	12,766	0.798
17	21,193	2	10,591	0.662
13	24,620	3	8,206	0.513
9	11,213	2	5,606	0.350
5	11,626	4	2,906	0.1816
1	6,295	20	314	0.0196

Vertical Traverse

Position, in.	Total Counts	Time (min)	cpm
67	95,393	1	95,393
63	65,951	1	65,951
59	45,515	1	45,515
55	31,569	1	31,569
51	22,085	1	22,085
47	15,651	1	15,651
43	11,050	1	11,050
39	23,682	3	7,894
35	39,340	7	5,620
31	15,862	4	3,965
27	17,070	6	2,845
23	24,020	12	2,002
19	11,361	8	1,420
15	10,479	11	952
11	10,433	17	614
7	7,195	21	343

Calculations

Diffusion Length:

$$\frac{1}{L^2} = \gamma^2 - \left(\frac{\pi}{a}\right)^2 - \left(\frac{\pi}{b}\right)^2$$

where a and b equal the physical dimensions plus the extrapolated distance.

$$\begin{aligned} \frac{1}{L^2} &= \gamma^2 - \left(\frac{\pi}{66.4}\right)^2 - \left(\frac{\pi}{67.1}\right)^2 \\ &= \gamma^2 - 0.00443. \end{aligned}$$

From the measured value of γ ,

$$1/L^2 = (0.0867)^2 - 0.00443 = 0.00309 \text{ (in in.}^{-2}\text{)}.$$

$$L^2 = 323.6 \text{ in.}^2 = 2088 \text{ cm}^2$$

$$L = 18.0 \text{ in.} = 45.7 \text{ cm.}$$

Absorption cross section

$$\sigma_a = \lambda_{tr}/3NL^2.$$

Now

$$\lambda_{tr} = 2.70 \text{ cm}$$

$$N = \rho(6.023 \times 10^{23})/12.01,$$

with

$$\rho = 1.6 \text{ g/cm}^3.$$

Therefore

$$N = 0.0803 \times 10^{24} \text{ atoms/cm}^3$$

Accordingly,

$$\begin{aligned} \sigma_a &= 2.70 \text{ cm}/3(0.0803 \times 10^{24})(2.09 \times 10^3) \\ &= 5.36 \times 10^{-27} \text{ cm}^2 = 0.00536 \text{ b.} \end{aligned}$$

Fig. S24.1. σ -pile

Fig. S24.2. Horizontal Flux Traverse

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TOTAL NEUTRON CROSS SECTION

by

Jan Spalek

ABSTRACT

The theory and technique of the measurement of the total cross section by the transmission method is presented. The cross sections of various materials are measured. The measurement of the cross section of different thicknesses of a material illustrates the hardening of the neutron spectrum with increasing penetration into the material. Finally, the effect of the sensitivity of the type of detector used on the cross section obtained is illustrated.

OUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
- IV. PROCEDURE
- V. RESULTS
- VI. ANALYSIS AND CONCLUSIONS
- VII. BIBLIOGRAPHY
- VIII. APPENDICES
 - A. Discussion Questions
 - B. Theory of Total Neutron Cross Sections

I. INTRODUCTION

The experiment is performed at the Argonaut with a beam of neutrons which emerge from a one-inch square hole in the external thermal column. The hole, which is made in a graphite stringer, penetrates the thermal column to within a few inches of the reactor core. The beam is collimated in a Masonite-cadmium collimator and passed through a $B^{10}F_3$ proportional counter. Measurements of the intensity of the neutron beam are made in terms of counting rates by the detector.

Total cross-section determinations of sample materials are made by measuring the transmission of the beam through the material. The transmission is defined as the ratio of the intensities of the beam with and without the sample material inserted in the path of the neutrons. After corrections for background (epi-cadmium neutrons) are made on the counting rates, the total cross section for thermal neutrons can be calculated from the basic attenuation equation. This expression states that the fractional loss of the beam intensity per unit path length is constant.

The experiment demonstrates the simplicity of the method since total cross sections are obtained by simple direct measurements. The early average values for the cross sections for many elements were measured by the same method. Today, the time-of-flight and crystal-spectrometer cross-section measurements use the transmission method as a basis with mono-energetic neutrons. Values of the total cross section obtained in this experiment are useful, although they are only average values weighted against the neutron-energy distribution in the beam. Under certain simplifying experimental conditions, the average cross section measured can be related to a specific cross section at the most probable velocity of the neutron density and distribution.

II. THEORY

The derivation of the equation involving the cross section to be measured is presented below. For a detailed discussion, see Appendix II and/or reference books.

A beam containing I_0 neutrons/cm² is attenuated in passing through a material of thickness x . The decrease ($-dI$) in neutrons/cm²/sec due to passage through thickness dx of material is proportional to the number of neutrons/cm²/sec incident on the thickness dx , and to the number (N) of nuclei/cm³ in the thin layer dx . The proportionality constant is the total cross section (σ_T) to be measured. The equation is

$$-\frac{dI}{dx} = IN\sigma_T.$$

Integration and substitution of the condition that $I = I_0$ when $x = 0$ gives

$$I = I_0 e^{-N\sigma_T x}.$$

The transmission T of a material is defined as the ratio of neutron flux at point x to the initial flux:

$$T = \frac{I}{I_0} = \frac{\text{relative flux at point } x}{\text{relative initial flux}} = \frac{\text{count rate with sample in beam}}{\text{count rate without sample in beam}},$$

so that $T = e^{-N\sigma_T x}$. Taking the natural logarithm of this equation, we obtain for the cross section

$$\sigma_T = -\frac{\ln T}{Nx}.$$

The T and x are measured by this experiment, and N is calculated from the knowledge of the density of the material whose transmission was measured, its atomic weight A , and Avogadro's number, N_0 :

$$N = \rho N_0 / A.$$

For the interpretation of the cross section thus obtained (which is actually an average) and the correction for hardening of the beam, see Appendix II.

III. APPARATUS (see Fig. 25.1)

Table with detector mount, collimator, and sample holder.
Samples of Au, Ab, In, Cd, and Fe.

139-2416

Fig. 25.1. Experiment Apparatus at Argonaut Thermal Column

Beam catcher and shutter.

Neutron detector (1-in. diameter, 12 cm Hg pressure of $B^{10}F_3$ gas, operating voltage ~ 1100 V).

Preamplifier, amplifier, scaler (multipurpose counter), high and voltage supply. Optional: oscilloscope.

IV. PROCEDURE

Close the beam shutter and position the apparatus in line with the expected neutron-beam path. Place a beam catcher behind the $B^{10}F_3$ detector.

Open the beam shutter and align the apparatus so that neutrons will pass through the collimator and sample into the detector.

Discriminate against low-noise and γ -ray pulses with the pulse-height selector on the multipurpose counter.

Take count rates without sample in beam, with sample in beam, and epi-cadmium background count rates with and without sample in beam. (Epi-cadmium background is obtained with a thin (0.010-in.) cadmium sheet in the beam.)

Do the above for as many of the different materials provided, and for several thicknesses of each (to show the hardening effect), as desired or as time permits.

A simple data sheet is given below for gold.

Material	Thickness, cm	I' for Open Beam	I _{BCd} with Cadmium in Beam	Thermal-neutron Intensity, $I = I' - I_{BCd}$	Transmission $T = I/I_0$
None	-				
Au 1	0.0254				
Au 2	0.0508				
Au 3	0.1016				
None	-				
Au 4	0.1524				
Au 5	0.2032				
Au 6	0.2540				
Au 7	0.3048				
None	-				

V. RESULTS

For each sample whose transmission was measured, calculate the total experimental microscopic cross section by means of the basic transmission equation:

$$T = e^{-\sigma T N x}$$

The total cross section will consist of two components, scattering and absorption. In the case of gold and indium, the beam will be attenuated primarily by neutron absorption; in the case of aluminum and iron, scattering will furnish the main contribution. Listed below are the absorption and scattering cross sections of the materials in question (taken from BNL-325):

Material

$$\text{Au} \quad \bar{\sigma}_s = 9.3 \times 10^{-24} \text{ cm}^2$$

$$\text{In} \quad \bar{\sigma}_s = 2.2 \times 10^{-24} \text{ cm}^2$$

$$\text{Al} \quad 1.128 \times \sigma_a (2200 \text{ m/s}) = 0.26 \times 10^{-24} \text{ cm}^2$$

$$\text{Fe} \quad 1.128 \times \sigma_a (2200 \text{ m/s}) = 2.85 \times 10^{-24} \text{ cm}^2$$

These values are to be subtracted from the experimental total cross sections to obtain the absorption cross sections of gold and indium, and the average scattering cross sections of aluminum and iron. The experimental absorption cross sections will be corrected for hardening by use of Fig. 25.2 and connected to the cross section at the most probable velocity of the neutron-density distribution. The experimental measurements for scattering cross section will yield an average value over the neutron-density distribution.

Fig. 15.2

Hardening Correction for a
Maxwellian Distribution
Using a $1/v$ -absorber and a
 $1/v$ -detector.

VI. ANALYSIS AND CONCLUSIONS

To be done by the student. (Also see Appendix II.)

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1. S. Glasstone and M. C. Edlund, Nuclear Reactor Theory, D. Van Nostrand Co., Inc., New York (1957).
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VIII. APPENDICES

A. Discussion Questions

The following propositions should be considered prior to arrival at the Argonaut. If possible, the questions will be discussed before the beginning of experimental work.

1. State the essentials of the transmission method. (What determines the cross section?)
2. Should the distance between the sample and the detector be small or large? Justify your answer.
3. What are "efficiency" and "sensitivity" of a detector? Distinguish between them.
4. Do we need to know the efficiency of the detector used in the transmission experiment in order to calculate the cross section?
5. Do we need to know the energy sensitivity of the detector in order to interpret the results?
6. What is "hardening" of the beam? To what is it due?
7. Why is the beam collimated? How much (to what size) should the beam be collimated compared with sample diameter?
8. Does it make any difference what materials are used to support the sample and counter?
9. What does a detector see? (To what does a detector respond?)
10. Why do we subtract the count rate due to epithermal neutrons from the measured count rates? What kind of cross section would be obtained by subtracting the room background only (with epi-cadmium neutrons blocked off)?
11. How would we eliminate the fast neutrons for a room-background count?
12. Show how to calculate the thickness of cadmium that would eliminate the thermal neutrons.
13. Why do we not use a very thick sample of cadmium for the epi-cadmium background counts?

14. Is the average cross section and the cross section for the average velocity one and the same thing? If not, try to write down the relationship between them.
15. In this experiment a number of samples of different transmission values for each material are used. Is more than one sample of each material essential for determining its cross section?

B. Theory of Total Neutron Cross Sections

Introduction

Since knowledge of the various cross sections is very important in the study of the materials used in a reactor, and practically all of neutron research deals with the measurement of cross sections, a basic detailed discussion of the subject is presented.

This discussion can be found in many books dealing with nuclear physics. (see references). The following section is an extract from the "many books" intended to save time for those who wish to review the subject.

Nuclear Cross Sections

To describe quantitatively the interactions of bombarding particles with atomic nuclei the concept of cross section was introduced. In this sense the term "cross section" is a measure of the probability of occurrence of a given process under certain conditions. To illustrate the concept of nuclear cross section, it can be visualized as the cross-sectional (or target) area presented by a nucleus to an incident neutron. If we further visualize the nucleus as a sphere of radius r cm, and think of the neutrons as point projectiles, then the target area or cross section σ of each nucleus is $\sigma = \pi r^2 \text{ cm}^2$. This simple picture illustrates only one type of cross section, the geometrical, and considers only collisions of neutrons with target nuclei. But it serves to derive a formula for cross section that can be extended to any kind of interaction of neutrons with atomic nuclei. Today, there are experimental values for many kinds of cross sections (absorption, scattering, activation, etc.) for different materials, and several types of each (microscopic, macroscopic, atomic, nuclear, differential).

In the early studies of X rays it was observed that the intensity I of a parallel beam of X rays was reduced by passing through a material. The decrease in intensity, $-dI$, was proportional to the thickness dx of the material and to the intensity of the beam:

$$-dI = \mu I dx, \tag{1}$$

where μ is the factor of proportionality called the absorption coefficient, or the total absorption coefficient of the material.

For X rays μ is analogous to the macroscopic absorption cross section Σ_a for neutrons. The only difference is that μ represents the fraction of energy removed from the beam of X rays per cm length, whereas Σ_a represents the fraction of neutrons absorbed per unit length.

The equation for the attenuation of a beam of neutrons is identical in form to that for X rays [Eq. (1)]:

$$- dI/I = \Sigma_a dx. \quad (2)$$

In this form, since the left-hand side represents the fraction of neutrons absorbed in the path dx , $\Sigma_a dx$ is equal to this fraction, but is preferably referred to as the probability that neutrons will be absorbed in path dx . To describe what happens microscopically, X-ray experimenters introduced the atomic absorption coefficient μ_a defined in terms of the total absorption coefficient μ as

$$\mu_a = \mu_A / \rho N_0, \quad (3)$$

where A is the atomic weight, ρ the density of the element, and N_0 Avogadro's number. The dimensions of μ_a are those of an area/atom.

As an example, the atomic absorption coefficient for copper is about $50 \times 10^{-21} \text{ cm}^2/\text{atom}$. If we picture the atom as a sphere of radius $2 \times 10^{-8} \text{ cm}$, then we can say that this atom has a geometrical cross section of about $12 \times 10^{-16} \text{ cm}^2$. From this example it is seen that the atomic absorption coefficient, also called the absorption cross section, is very much smaller than the geometrical cross section, in this case.

The concept of nuclear cross section can be extended to other types of nuclear processes. Since we are primarily interested in neutrons, we define the nuclear cross section σ for a specific reaction as the average number of individual processes occurring per incident neutron in the beam per nucleus:

$$\sigma \equiv C / N_A I \text{ cm}^2 \text{ per nucleus}, \quad (4)$$

where C is the number of individual nuclear processes, N_A is the number of atoms per cm^2 of material, and I the number of neutrons in the incident beam per cm^2 . The above definition was defined by assuming a uniform beam of I neutrons impinging perpendicularly on a target one atomic layer thick. Rearrangement of Eq. (4) gives

$$N_A \sigma = C / I. \quad (5)$$

The right-hand side represents the fraction of the incident neutrons which interact with the target nuclei. Therefore $N\sigma$ can be thought of as the fraction of the surface which is capable of undergoing the given reaction. Thus, in 1 cm^2 of surface, $N\sigma \text{ cm}^2$ is effective. Since 1 cm^2 contains N nuclei, the quantity $\sigma \text{ cm}^2$ is the effective area per single nucleus for the reaction. This interpretation of σ leads to the use of the term cross section, but it is generally not related to the geometrical cross section.

Nuclear cross sections are usually in the range from 10^{-22} to 10^{-26} cm^2 per nucleus. A useful unit is the barn; by definition,

$$1 \text{ barn(b)} = 10^{-24} \text{ cm}^2 \text{ per nucleus.}$$

Derivation of Transmission Equation

Let a beam containing I_0 neutrons per cm^2 strike a material with N nuclei per cm^3 from the left (see Fig. 25.3).

Fig. 25.3

Transmission Experiment

The number of nuclei present in a thin layer dx of the material is $N dx$ nuclei per cm^2 . This is equivalent to the quantity NA in the defining equation (4); therefore, $Ndx \sigma$ is the fraction of the neutrons falling on this layer which react. Hence, $N\sigma dx$ is then set equal to $-dI/I$, where $-dI$ is the decrease in neutrons per cm^2 due to passage through thickness dx of the material:

$$-dI/I = N\sigma dx. \quad (6)$$

Integration yields

$$\ln I = -N\sigma x + \ln I_0, \quad (6.1)$$

where I_0 is the intensity of the beam before entering the material (that is, at $x = 0$). Therefore,

$$I_x = I_0 e^{-N\sigma x}, \quad (6.2)$$

where I_x is the number of neutrons penetrating the material of thickness x , I_0 is the number of neutrons falling on the material (initial flux), and N is the number of nuclei per cm^3 in the target material. Equation (6.2) is used in the transmission experiment.

The transmission T of a material is defined as the ratio of flux at point x to the initial flux:

$$T = I_x/I_0 = e^{-N\sigma x}. \quad (7)$$

In different notation, since the intensity of the beam, I , is the flux density nv (usually called flux), n the number of neutrons in beam per cm^3 , and v the velocity of neutrons in cm/sec , Eq. (7) can be written as

$$T = nv/(nv)_0 = e^{-N\sigma x}. \quad (7.1)$$

Upon taking natural logarithms of both sides, we find $\ln T = -N\sigma x$, so that

$$\sigma = -\frac{\ln T}{Nx}, \quad (7.2)$$

where

$$\begin{aligned} T = \frac{I_x}{I_0} &= \frac{nv}{(nv)_0} = \frac{\text{relative flux at point } x}{\text{relative initial flux}} \\ &= \frac{\text{count rate with sample in beam}}{\text{count rate without sample in beam}}. \end{aligned}$$

If the thickness x of the material is small, the value of transmission T may be close to unity. This would greatly reduce the accuracy of the cross section obtained. It is preferable to use a sample thick enough to attenuate the beam more so that T is much less than 1. But with increasing sample thickness the Maxwell velocity distribution is increasingly distorted due to greater (preferential) absorption of slow neutrons by $1/v$ capture. This change in the Maxwellian velocity distribution is called hardening of the beam (see Fig. 25.4).

Fig. 25.4

Hardening of the Maxwell Speed Distribution (v^2) for a Transmission of 50%.

The cross section given by Eq. (7.2) is called microscopic, to distinguish it from $N\sigma$, called the macroscopic cross section and symbolized Σ . Therefore $\Sigma = N\sigma \text{ cm}^{-1}$, where N is the number of nuclei per cm^3 . Consequently Σ is the total cross section of the nuclei in 1 cm^3 of material. (Its dimensions are cm^{-1} .)

Given ρ , the density of the target material, and its atomic weight A , ρ/A is the number of g atoms per cm^3 . The number of atomic nuclei N per cm^3 is then

$$N = (\rho/A) N_0$$

and

$$\Sigma = (\rho N_0/A) \sigma \quad (8)$$

is the resultant relationship between macroscopic and microscopic cross section.

The transmission method is the most direct procedure for the measurement of cross sections. It gives the total cross section with good accuracy, for it involves only the ratio of counting rates with and without the sample in beam. The efficiency of the detector is not needed. But the sensitivity (with respect to neutron velocity) of a detector does enter into the interpretation of measured cross section. The total cross section σ_T determined experimentally is the sum of the absorption (σ_a) and scattering (σ_s) cross sections, for it makes no difference to the detector how the neutrons are removed from the beam in the attenuation process.

Since the cross section depends on the neutron velocity or energy, the experiment gives σ_T of a material for the particular energy or energies of neutrons used. When the beam intensity is low, it is not practical to use a velocity selector (crystal spectrometer or a chopper) to select a particular energy. So the whole energy spectrum of the neutrons coming out of a reactor is passed through the sample material, and the total cross section obtained (σ_T) should be called experimental ($\sigma_T \text{ exp}$), for then it is an average over the above spectrum. The variation of $\sigma_T \text{ exp}$ is mostly due to that of the absorption cross section σ_a . The value of σ_a varies not only for different materials, but also usually varies greatly with the neutron energy. There are two types of variation of the absorption cross section σ_a .

In the first one, σ_a varies according to the " $1/v$ law":

$$\sigma_a = (\sigma_a)_0 v_0/v, \quad (9)$$

where σ_a is the absorption cross section at velocity v and $(\sigma_a)_0$ the absorption cross section at velocity $v_0 = 2200 \text{ m/sec}$.

The second type is called "resonance" absorption. This is made evident by a sharp rise in absorption at one or more particular energies.

Finally, the two components of the total cross section can be still subdivided further. Thus, there are two types of scattering, elastic and inelastic:

$$\sigma_s = \sigma_e + \sigma_i,$$

and two types of absorption, capture and fission:

$$\sigma_a = \sigma_c + \sigma_f.$$

When a neutron is captured by a nucleus, the compound nucleus may be radioactive, and the measurement of this activity gives the activation cross section (see activation cross section experiment). The fission cross section is the measure of the probability that a compound nucleus will fission into two almost equal parts.

Thermal-neutron Distributions

The fission neutrons from a fuel assembly surrounded by a moderator are slowed down by a number of elastic scattering collisions with the moderator atoms. The final average kinetic energy of the neutrons is approximately the same as that of the atoms (or molecules) of the moderator. Since the number of moderator atoms having speeds within a certain range is described by the Maxwell distribution function of molecular speeds, this same function may be used to describe the neutrons that are in thermal equilibrium with the moderator atoms (that is, thermal neutrons):

$$dn = \frac{4nT}{\sqrt{\pi} v_0^3} v^2 e^{-v^2/v_0^2} dv, \quad (10)$$

where:

dn = number of neutrons per unit volume in the speed range dv at the speed v ;

nT = total number of neutrons per unit volume;

v_0 = most probable speed, that is, the speed at which $n(v)$ is a maximum.

A graph of Eq. (10) is shown in Fig. 25.5.

Fig. 25.5
Maxwellian Distribution

The ordinate [the ratio dn/dv , or the coefficient of dv in (10)] is the Maxwell distribution of molecular (neutron) speeds; the number of neutrons per unit volume with speeds between v and $v + dv$ is represented on the graph by the area of the narrow vertical strip of width dv .

The average kinetic energy of the thermal neutrons is

$$\frac{1}{2} m \overline{v^2} = \frac{3}{2} kT, \quad (11)$$

where k is the Boltzmann constant ($= 1.380 \times 10^{-16}$ erg/°K), m the neutron mass, and T the absolute temperature in °K. The speed corresponding to this energy is the root mean square speed:

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{3kT/m}. \quad (11.1)$$

The most probable speed v_0 in the Maxwellian distribution (10) corresponds to the energy kT of the neutrons:

$$\frac{1}{2} m v_0^2 = kT. \quad (12)$$

Therefore the value of the most probable speed (shown in Fig. 25.5) is

$$v_0 = \sqrt{2kT/m}. \quad (12.1)$$

Equation (10) gives the density of neutrons with speed v as a function of v . But the experimental measurements are done in terms of flux. Since flux is nv , the flux distribution can be obtained from Eq. (10) by multiplying by v , giving

$$d(nv) = \frac{4nT}{\sqrt{\pi} v_0^3} v^3 e^{-v^2/v_0^2} dv. \quad (13)$$

This equation describes the flux distribution within the reactor core. Its graph in Fig. 25.6 shows that the most probable flux is at a speed greater

than the most probable speed v_0 . To find the value of the speed at which the flux is maximum, differentiate Eq. (13) and set the result equal to zero, giving

$$v = \sqrt{\frac{3}{2}} v_0 = 1.22 v_0. \quad (14)$$

The actual (measured) speed distribution of thermal neutrons in a reactor core was obtained by Ringo at Argonne, and it was seen to follow the Maxwellian fairly closely.

Fig. 25.6
Comparison of Flux and
Density Distributions

Experimental Cross Section

The flux distribution in a beam hole reaching through the reflector to the reactor core will also be given by Eq. (13). But the flux distribution as seen by a detector of a particular sensitivity placed in the beam will be given by Eq. (13), the flux distribution in the beam, multiplied by the sensitivity (with respect to neutron speed) of the detector.

If a detector has $1/v$ sensitivity (such as $B^{10}F_3$ detector), the measured distribution of count rate against neutron velocity will be

$$d(nv) = \frac{4nT}{\sqrt{\pi} v_0} \frac{v^3}{v} e^{-v^2/v_0^2} dv,$$

which is again just the Maxwell speed distribution (10). If in conjunction with a $1/v$ detector, a $1/v$ absorber of cross section $k/\bar{v}$ is used in the transmission measurement, then the observed cross section for the whole beam will be the average cross section $\bar{\sigma}$ for the Maxwell distribution:

$$\bar{\sigma} = \frac{\int \frac{k}{v} r(v) dv}{\int r(v) dv} = \frac{\frac{4nk}{\sqrt{\pi} v_0^3} \int_0^\infty \frac{v^2}{v} e^{-v^2/v_0^2} dv}{\frac{4nT}{\sqrt{\pi}} v_0^3 \int_0^\infty v^2 e^{-v^2/v_0^2} dv} = \frac{2}{\sqrt{\pi}} \frac{k}{v_0} = 1.128 \sigma_{v_0}. \quad (15)$$

When the detector is black, meaning that all neutrons that strike the detector react, the reaction rate (counting rate) will be proportional to the neutron flux $d(nv)$. The observed cross section must be weighted against the flux distribution.

$$\begin{aligned}\bar{\sigma} &= \frac{\int \frac{k}{v} vn(v)dv}{\int vn(v)dv} = \frac{\frac{4nTk}{\sqrt{\pi} v_0^3} \int \frac{v^3}{v} e^{-v^2/v_0^2} dv}{\frac{4nT}{\sqrt{\pi} v_0^3} \int v^3 e^{-v^2/v_0^2} dv} \\ &= \frac{\sqrt{\pi}}{2} \frac{k}{v_0} = \frac{k}{\bar{v}}\end{aligned}\quad (15.1)$$

The experimental microscopic cross section for a $1/v$ absorber and black detector is the cross section for a neutron having the average velocity of the density distribution. [As seen below in Eq. (16)]

Table 25.1

$$f(n) = \int_0^{\infty} x^n e^{-ax^2} dx$$

n	f(n)	n	f(n)
0	$\frac{1}{2} \sqrt{\frac{\pi}{a}}$	1	$\frac{1}{2a}$
2	$\frac{1}{4} \sqrt{\frac{\pi}{a^3}}$	3	$\frac{1}{2a^2}$
4	$\frac{3}{8} \sqrt{\frac{\pi}{a^5}}$	5	$\frac{1}{a^3}$
6	$\frac{15}{16} \sqrt{\frac{\pi}{a^7}}$	7	$\frac{1}{a^4}$

If n is even, $\int_{-\infty}^{\infty} x^n e^{-ax^2} dx = 2f(n)$

If n is odd, $\int_{-\infty}^{\infty} x^n e^{-ax^2} dx = 0.$

In order not to confuse the several average cross sections, a clear idea of the three particular speeds of the Maxwell speed distribution must be reached. Also, the validity of many formulas dealing with thermal neutrons depend on the correct interpretation of the speed v appearing in them as a particular speed in the Maxwellian distribution.

Two of the speeds, the root mean square v_{rms} and the most probable speed v_0 appeared before in terms of energy [see Eqs. (11) and (12)]. For clarification, the above two speeds, and the arithmetic average speed $\bar{v}$, will now be derived.

Using Table I we found for the average speed

$$\bar{v} = \frac{\int v \, dn}{n} = \frac{\int_0^{\infty} v \frac{4n}{\sqrt{\pi} v_0^3} v^2 e^{-v^2/v_0^2} dv}{n} = \frac{2}{\sqrt{\pi}} v_0 = 1.128 v_0, \quad (16)$$

where dn is the number of neutrons per unit volume with speed v from the Maxwellian speed distribution (10).

The root mean square speed v_{rms} , as in the average kinetic energy of neutrons [see Eq. (11)], is

$$v_{rms} = \sqrt{\overline{v^2}} = \left[\frac{\int v^2 dn}{n} \right]^{1/2} = \left[\frac{4n}{n v_0^3 \sqrt{\pi}} \int_0^{\infty} v^4 e^{-v^2/v_0^2} dv \right]^{1/2} = \sqrt{\frac{3}{2}} v_0. \quad (17)$$

The most probable speed v_0 is the maximum of the Maxwell distribution. Setting the derivative with respect to v of Eq. (10) equal to zero, we obtain

$$\frac{d}{dv} \left(\frac{dn}{dv} \right) = \frac{4n}{\sqrt{\pi} v_0^3} \left[2v e^{-v^2/v_0^2} - \frac{2v^3}{v_0^2} e^{-v^2/v_0^2} \right] = 0. \quad (18)$$

Thus,

$$v_{max} = v_0.$$

The relative magnitudes of these speeds are shown on the speed distribution function (dn/dv) graph in Fig. 25.7.

Fig. 25.7

Important Velocities in a
Maxwellian Distribution

Now it is seen that the observed average cross section from Eq. (15) is 1.128 times larger than the cross section at the most probable speed $\sigma v_0 = k/v_0$. Also, the observed average cross section $\bar{\sigma}$ is not the same as the cross section for the average velocity $\sigma \bar{v}$, for

$$\sigma \bar{v} = \frac{k}{\bar{v}} = \frac{k}{1.128 v_0} = \frac{\sigma v_0}{1.128}. \quad (19)$$

A use of the average cross section as given by Eq. (15) is to determine experimentally the neutron temperature. Obtaining $\bar{\sigma}$ experimentally determines v_0 from (15) and therefore the temperature T from Eq. (12), provided $\sigma = k/v$ is known.

Various correction factors; interpretation of the experimental cross section; accuracy.

The total experimental cross section for a sample material is obtained (from the two count rates) by use of the equation

$$\sigma_{T\text{exp}} = \frac{\ln(1/T)}{Nx}$$

If a very thin sample of $1/v$ absorber is used (transmission close to unity), and a detector of $1/v$ sensitivity, then the $\sigma_{T\text{exp}}$ obtained is the average $\bar{\sigma}$ for the Maxwell spectrum. If we wish to compare this value with the listed value of σ_{v_0} for the material, which is tabulated at the most probable speed v_0 , the relating factor is

$$\bar{\sigma}_{T\text{exp}} = 1.128 \sigma_{v_0}.$$

In the case of high transmission, however, the accuracy is very poor, as can be seen from the following:

$$T = e^{-Nx\sigma}$$

so that

$$\ln T = -Nx\sigma$$

Differentiation yields

$$dT = -Nx e^{-Nx\sigma} d\sigma$$

and

$$\frac{dT}{T} = -Nx d\sigma = \frac{\ln T}{\sigma} d\sigma$$

The fractional error in the transmission measurement is related to the accuracy of the calculated cross section by

$$\frac{d\sigma}{\sigma} = \frac{1}{\ln T} \frac{dT}{T}$$

For transmissions close to unity $1/\ln T$ is a large number and the error in the cross-section determination is large. If the transmission is $0.95 \pm 1\%$,

$$\frac{d\sigma}{\sigma} \approx \frac{1}{\ln 0.95} (1\%) = 19.5\%.$$

If the transmission is $0.50 \pm 1\%$,

$$\frac{d\sigma}{\sigma} = \frac{1}{\ln 0.50} (1\%) = 1.45\%.$$

It is obviously desirable from the standpoint of accuracy to use thick samples in the measurement of total cross sections.

With sample thickness increased, however, hardening (distortion) of the Maxwellian distribution occurs. The σ_{Texp} is then no longer the average cross section $\bar{\sigma}$. Because the Maxwellian distribution changes shape as it penetrates the absorber, σ_{Texp} is therefore an average over and average distribution seen by the sample's atoms.

The average cross section $\bar{\sigma}$ cannot be measured directly as a function of absorber thickness. Nevertheless, H. A. Bethe obtained a curve for this function by numerical integration for a $1/v$ material and a $1/v$ detector; this is shown in Fig. 25.2. This curve gives the ratio of the experimental absorption cross section ($\sigma_a exp$) to σ_a at v_0 , plotted against transmission. It can be used to correct for hardening if the sample used has $1/v$ absorption cross-section large compared with the scattering. From this it is also seen that for transmission close to 50% $\sigma_a exp / \sigma_a v_0 = 1$, and therefore the measured σ_{exp} is equal to the σ_{v_0} which is given in the tables.

Since values obtained for 50% transmission also have very good accuracy, such a procedure is most commonly used in cross-section measurements.

The above applies to the absorption cross section only and only for a $1/v$ material. Nevertheless, if the $1/v$ material has an absorption cross section much larger than the scattering cross section, Fig. 25.4 can be used to correct for hardening.

However, if a material is predominantly a scatterer, the total cross section may be used to obtain the average thermal scattering cross section by subtracting the average absorption $\sigma_{\text{abs}} = 1.128 \sigma_{v_0}$, if known from other measurement (e.g., from a pile oscillator), for there is almost no hardening in a predominant scatterer. But the average scattering cross section, which is fairly constant in the thermal region, has no simple relationship to cross section (σ_s) at particular velocity, because σ_s , unlike σ_a , varies irregularly with velocity in the thermal region.

The preceding theory is a special case of a general quantum-mechanical treatment of neutron cross sections. With high neutron-flux sources (reactors), and the development of velocity selectors (choppers, spectrometers) which permit measurements at chosen velocities, the cross-section values obtained are more accurate, but the methods much more complex. The above theory shows how the first cross-section measurements were made. The presentation of this simple method is further justified by the fact that many cross sections have been measured as averages in the same way, and for some materials this simple method is still very useful.

STUDENT RESULTS

MEASUREMENT OF σ_T WITH A $1/\nu$ DETECTOR

Material	Thickness, cm	Open Beam I' , cpm	Cd in Beam I_{BGD} , cpm	Thermal-neutron Intensity $I = I' - I_{BGD}$, cpm		Transmission $T = I/I_0$
None	-	15943	1450	14493		-
Au 1	0.0254	13760	1347	12413		0.856
Au 2	0.0508	11736	1250	10486		0.724
Au 3	0.1016	8970	1082	7888		0.544
None	-	16013	1445	14568		-
Au 4	0.1524	6923	921	6002		0.413
Au 5	0.2032	5510	897	4613		0.318
Au 6	0.2540	4401	801	3600		0.248
Au 7	0.3048	3484	720	2764		0.191
None	-	15873	1430	14443		-
	$-\ln T$	$Nx(10^{24})$, atoms/cm ²	σ_T , b	$\sigma_a(\text{exp})$, b	F	$\sigma_a(v_0)$, b
Au 1	0.1555	0.00150	103.6	94.3	1.085	86.9
Au 2	0.3230	0.00300	107.7	98.4	1.05	93.7
Au 3	0.6088	0.00600	101.5	92.2	1.01	91.3
Au 4	0.8870	0.00900	98.6	89.3	0.98	91.1
Au 5	1.146	0.0120	95.4	86.1	0.95	90.6
Au 6	1.401	0.0150	93.6	84.3	0.92	91.6
Au 7	1.656	0.0180	91.9	82.6	0.90	91.8

$$\sigma_a(v_0) = 91.0 \text{ b}$$

Sample Calculation: for Sample Au 1

$$T = \frac{I' - I_{BGD}}{I_0' - I_{0BGD}} = \frac{13760 - 1347}{15945 - 1442} = \frac{12413}{14503} = 0.856.$$

Since

$$T = e^{-N\sigma_T x},$$

we find

$$\sigma_T = \frac{-\ln T}{Nx} = \frac{0.1555}{0.0591 \times 10^{24} \times 0.0254} = 103.6 \text{ b}$$

$$\sigma_a(\text{exp}) = \sigma_T - \sigma_S = 103.6 - 9.3 = 94.3 \text{ b.}$$

From the hardening correction curve,

$$\sigma_a(\text{exp}) = F \sigma_a(v_0).$$

With $T = 0.856$ and $F = 1.085$, we find

$$\sigma_a(v_0) = 94.3/1.085 = 86.9 \text{ b.}$$

The average value for all right samples is obtained as

$$\overline{\sigma_a(v_0)} = \frac{\sum_{n=1}^8 \sigma_a(v_0)}{8} = 91.0 \text{ b.}$$

Fig. S25.1. Transmission versus Thickness

NEUTRON ABSORPTION CROSS-SECTION
MEASUREMENT USING PULSED NEUTRON TECHNIQUES

by

W. F. Gilbert

ABSTRACT

A moderating medium such as water is pulsed with bursts of fast neutrons. The decay of the resulting thermal flux is observed with a multichannel time analyzer. A known amount of neutron-absorbing material is added to the moderating medium, and a new decay rate of the thermal flux is observed. If the scattering properties of the moderating medium have not been appreciably disturbed by the addition of the material, the absorption cross section of the added material is obtained from the two observed decay constants.

OUTLINE

- I. INTRODUCTION
 - II. THEORY
 - III. APPARATUS
 - IV. PROCEDURE
 - V. RESULTS
 - VI. ANALYSIS
 - VII. BIBLIOGRAPHY
 - VIII. APPENDIX
- Discussion Questions

I. INTRODUCTION

Neutron-absorption cross sections have largely been measured by transmission methods associated with time-of-flight or spectrometer techniques and by pile oscillator methods. The first method involves the subtraction of the scattering cross section, which is often large compared with the absorption cross section; the latter involves comparison with a standard. A third method, capable of high accuracy for a Maxwellian spectrum of neutrons and involving the scattering cross section only as a minor correction, is presented in this experiment.

In this method, the time dependence of the thermal-neutron flux in a source-free pure water system is compared with that of a dilute solution having the same geometric buckling. In the limiting case of very dilute solutions, the difference between the two decay rates will be due to the added absorption of the solute. In the case of more concentrated solutions, corrections must be made for the change in diffusion coefficient and macroscopic absorption cross section of the water due to displacement by the solute.

II. THEORY

The decay of monoenergetic thermal neutrons in a sourceless non-multiplying moderating medium is expressed by the equation

$$\phi_1 = \phi_0 e^{-[D_0 B^2 + v \Sigma_{am}]t} = \phi_0 e^{-\alpha_1 t} \quad (1)$$

where D_0 is the moderator diffusion coefficient ($= Dv$), B^2 the geometric buckling of the sampling, Σ_{am} is the macroscopic absorption of the moderator, and α_1 is known as the decay constant for thermal neutrons in the moderating medium. From Eq. (1) we see that

$$\alpha_1 = D_0 B^2 + v \Sigma_{am} \quad (2)$$

The actual decay consists of the sum of many terms having the form of Eq. (1) due to the various geometric modes which may be present. If sufficient time is allowed to elapse before the decay is studied, only the fundamental mode will be present, and the result is a single exponential having the form of Eq. (1).

If an absorber is added to the moderating medium and the geometry maintained the same, the neutron-flux decay is expressed by the equation

$$\phi_2 = \phi_0 e^{-[D_0 B^2 + v \Sigma_{am} + v \Sigma_{ax}]t} = \phi_0 e^{-\alpha_2 t}, \quad (3)$$

where the subscript x refers to the added absorber. The new decay constant is

$$\alpha_2 = D_0 B^2 + v \Sigma_{am} + v \Sigma_{ax}. \quad (4)$$

If we divide Eq. (3) by Eq. (1) we obtain

$$\phi_2/\phi_1 = e^{-v \Sigma_{ax} t} = e^{-\alpha' t}, \quad (5)$$

where α' is an equivalent decay constant due only to the added absorber:

$$\alpha' = v \Sigma_{ax}. \quad (6)$$

Thus we eliminate the properties of the moderating medium and have an equation relating only to the added absorber. Equation (6) may also be obtained by subtracting Eq. (2) from Eq. (4):

$$\alpha_2 - \alpha_1 = \alpha' = v \Sigma_{ax}. \quad (7)$$

We may obtain α' experimentally either by finding the decay constant for thermal neutrons with and without added absorber and subtracting, or by dividing the two decay curves, point by point, and finding the slope of the resulting decay curve.

Since the decay actually observed is that of a Maxwellian velocity distribution rather than that of a monoenergetic group of neutrons, the $v \Sigma_a$ terms in Eqs. (1-7) are actually averaged over the Maxwellian velocity distribution. Thus,

$$v \Sigma_a = N \overline{v \sigma_a} = N \frac{\int \sigma_a(v) v n(v) dv}{\int v n(v) dv}. \quad (8)$$

If σ_a has a $1/v$ -dependence, as many materials do, then $\overline{v \sigma_a}$ is independent of the neutron spectrum, and the cross section may be obtained from the equation

$$\overline{v\sigma_a} = v\sigma_a(v). \quad (9)$$

If σ_a does not have a $1/v$ -dependence, then $\overline{v\sigma_a}$ may be written in terms of an effective cross section:

$$\overline{v\sigma_a} = v_0 \hat{\sigma}_a, \quad (10)$$

where $v_0 = 2200$ m/sec (2.2×10^5 cm/sec) and $\hat{\sigma}_a$, the effective cross section, is related to the 2200-meter/sec cross section by the equation

$$\hat{\sigma}_a = g\sigma_{a2200}. \quad (11)$$

For elements with $1/v$ -cross sections, $g = 1$. For many other elements, g has been computed by Wescott.⁽²⁾ For many elements, g is very nearly one.

The previous derivations have been based on the assumption that when the absorber is added to the moderator, the moderator remains unchanged. The added absorber often displaces moderator nuclei, such as usually occurs when a material is dissolved in another. Then both D_0 and Σ_{am} are affected, and the observed change in decay constant is due to these changes as well as the added absorber. These effects must be corrected for if a correct absorption cross section is to be obtained.

If the change in the number of moderator nuclei per unit volume is small, then the geometric buckling can be assumed to be unchanged. This is because the change in macroscopic scattering cross section of the sample affects the geometric buckling only through its effect upon the extrapolation length, and this effect is usually small.

Referring to Eq. (4), we find the change in decay constant due only to a change in D_0 to be

$$\Delta\alpha = B^2\Delta D_0. \quad (12)$$

Since D_0 is inversely proportional to the macroscopic scattering cross section, we may write

$$D_{01}N_{m1} = D_{02}N_{m2}, \quad (13)$$

where the subscripts 1 and 2 refer to conditions before and after the addition of the absorber, respectively. We may express the change in D_0 as

$$\Delta D_0 = D_{02} - D_{01} = D_{01} \frac{N_{m1}}{N_{m2}} - D_{01} \quad (14)$$

or

$$\Delta D_0 = D_{01} \left(\frac{N_{m1}}{N_{m2}} - 1 \right) = D_{01} \left(\frac{V_2}{V_1} - 1 \right), \quad (15)$$

where V_1 and V_2 are the moderator volumes before and after adding the absorber, respectively. Substitution of Eq. (15) into Eq. (12) gives the diffusion-coefficient correction to the decay constant as

$$\Delta \alpha = B^2 D_{01} \left(\frac{V_2}{V_1} - 1 \right) \quad (16)$$

The change in decay constant due only to a change in Σ_{am} is found from Eq. (4) as

$$\Delta \alpha = v \Delta \Sigma_{am} \quad (17)$$

But

$$\begin{aligned} \Delta \Sigma_{am} &= \Sigma_{am2} - \Sigma_{am1} = \Sigma_{am1} \frac{N_2}{N_1} - \Sigma_{am1} = \Sigma_{am1} \left(\frac{N_2}{N_1} - 1 \right) \\ &= \Sigma_{am1} \left(\frac{V_1}{V_2} - 1 \right) \end{aligned} \quad (18)$$

Thus

$$\Delta \alpha = v \Sigma_{am1} \left(\frac{V_1}{V_2} - 1 \right) \quad (19)$$

Equations (16) and (19) represent corrections to the observed decay constant due to the change of the moderator volume when the absorber is added. For absorbers of large cross section, such as boron, the small amount added may make these corrections negligible. For absorbers of small cross section, the corrections may be very appreciable.

The added absorber is usually in the form of a compound soluble in the moderator. For such a compound, the observed additional cross section is the sum of the cross sections for the various elements making up the compound. A particular cross section may be obtained by forming a compound with elements known to have negligible absorption cross sections. If this is not possible, a compound may be formed with an element whose absorption cross section has been accurately determined and the unknown cross section determined by a difference technique.

III. APPARATUS

Pulsed neutron source
Multichannel time analyzer
Aluminum containers
 $B^{10}F_3$ detector, preamplifier, and linear amplifier
MPC high-voltage supply, amplifier, and scaler
Distilled water
Absorber solutions

Fig. 26.1
Experimental
Apparatus

139-3417

IV. PROCEDURE

A. Prepare the pulsed neutron source and multichannel time analyzer for operation as directed by the instructor.

B. Place a known amount of distilled water in an aluminum tank, and determine the dimensions and geometric buckling of the sample by use of the appropriate extrapolation length. The extrapolation length may be determined from a previous experiment or from the published diffusion parameters for water.

C. Pulse the water sample to the memory limit of the multi-channel time analyzer. Print out the data, and plot the decay curve on semilog paper. Determine the decay constant for the water sample.

D. To a known amount of distilled water, add the absorbing material selected and dissolve it completely in the water. Again measure the volume of solution obtained. If the increase is appreciable, corrections to the observed cross section must be made as outlined in Section II.

E. Place the same volume of solution as was used in Part B in the same aluminum can. Pulse the sample to the memory limit of the time analyzer. Print out the data, and plot the decay curve on semilog paper. Determine the decay constant for the solution sample.

F. Subtract the decay constants obtained in Parts C and E.

G. Divide the decay curves obtained in Parts C and E, point by point, to obtain a new decay curve due to the added absorber. Determine the new decay constant, and compare with that obtained in Part F.

H. If the increase in volume upon addition of the solute was noticeable in Part D, compute the corrections in the decay constant due to the change in diffusion coefficient and absorption cross section of the water. Correct the decay constant found in Parts F and G for these changes.

I. By use of the corrected decay constant due to the added absorber, compute the exponential microscopic absorption cross section for the added material. If more than one absorption cross section is appreciable in the compound used, subtract the known absorption cross section from the total to obtain the unknown cross section. If the cross section is not $1/v$, use the non- $1/v$ correction factor to obtain the microscopic absorption cross section at 2200 m/sec.

V. RESULTS

The results to be obtained will consist of the neutron-decay curves for the water sample and the sample with added absorber. The ratio curve obtained by dividing the curves point by point should also be plotted. From the decay constants found from these curves, the absorption cross section of the added absorber is computed.

VI. ANALYSIS AND CONCLUSIONS

The student should give considerable attention to possible sources of error in the experiment. In particular, a comparison of the errors involved in the two methods used for determining the net decay constant should be made.

VII. BIBLIOGRAPHY

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2. C. H. Westcott, CRRP-787.

VIII. APPENDICES

Discussion Questions

1. Why are equal volumes of water and solution used in the two runs? Under what conditions would we lose the advantage this usually produces?
2. Discuss the possible advantages of each of the two methods used to determine the net change in decay constant.
3. Must the two samples be irradiated to the same monitor count? Why?
4. The scattering effect of the solute in the solution has been neglected in the corrections considered. Why?
5. If the solute displaces an appreciable amount of water, what factors are changed, and what effect do they have on the observed decay constant?
6. Why is the change in extrapolation length due to the addition of solute neglected? Under what conditions might it become important?
7. Must the temperature of the solutions be considered in this experiment? Why?
8. Would this experiment produce more accurate results for absorbers of large cross section or small cross section?

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STUDENT RESULTSABSORBER DATA

Sample 11.5 x 11.5 x 15.1 cm 2 liters

H₃BO₃ (mol wt = 61.84)

2 g H₃BO₃/liter of solution

$$N = \frac{2 \times 6.02(10^{23})}{61.84 \times 1000} = 1.947(10^{19}) \text{ B atoms/cm}^3$$

Negligible increase in volume

AlCl₃ · 6H₂O (Formula wt = 241.44)

50 g AlCl₃ · 6 H₂O/liter at solution

$$N' = \frac{50 \times 6.02(10^{23})}{241.44 \times 1000} = 1.247(10^{20}) \text{ mol/cm}^3$$

$$N = 3 \times 1.247(10^{20}) = 3.741(10^{20}) \text{ Cl atoms/cm}^3$$

Mol wt H₂O = 18.02 g/mol

$$6 \text{ H}_2\text{O} = 6 \times 18.02 = 108.12 \text{ g H}_2\text{O/mol AlCl}_3 \cdot 6 \text{ H}_2\text{O}$$

$$\text{H}_2\text{O in 50 g AlCl}_3 \cdot 6 \text{ H}_2\text{O} = \frac{108.12}{241.44} \times 50 = 22.4 \text{ g}$$

Increase in volume = 27 cm³/liter

Increase in volume due to AlCl₃ = 27 - 22 = 5 cm³/liter

<u>Ch. No.</u>	<u>Count</u>	<u>Ch. No.</u>	<u>Count</u>	<u>Ch. No.</u>	<u>Count</u>	<u>Ch. No.</u>	<u>Count</u>
015	00047	008	01439	015	00014	008	01522
014	00065	007	02421	014	00020	007	03155
013	00116	006	04023	013	00047	006	06188
012	00168	005	07111	012	00089	005	12563
011	00254	004	12295	011	00183	004	25408
010	00461	003	21871	010	00343	003	47320
009	00792	002	35012	009	00732	002	02932

H₂O

H₃BO₃
2 g/liter

Ch. No.	Count	Ch. No.	Count
015	00026	008	00787
014	00026	007	01604
013	00054	006	03163
012	00072	005	06409
011	00132	004	13168
010	00204	003	25680
009	00421	002	29531

50 g/liter

Channel time = 50 μs /channel
for all data

Fig. S26.1

Fig. S26.2

Fig. S26.3

CROSS SECTION CALCULATIONS

Boron

$$\alpha(\text{H}_3\text{BO}_3) = 14.22(10^3) \text{ sec}^{-1}$$

$$\alpha(\text{H}_2\text{O}) = 11.00(10^3) \text{ sec}^{-1}$$

$$\alpha' = \alpha(\text{H}_3\text{BO}_3) - \alpha(\text{H}_2\text{O}) = (14.22 - 11.00)10^3 = 3.22(10^3) \text{ sec}^{-1}$$

$$\alpha' = v \Sigma_{\text{ax}}$$

$$\Sigma_{\text{ax}} = \frac{\alpha'}{v} = \frac{3.22(10^3) \text{ sec}^{-1}}{2.2(10^5) \text{ cm/sec}} = 1.464(10^{-2}) \text{ cm}^{-1}$$

$$\sigma_{\text{ax}} = \frac{\Sigma_{\text{ax}}}{N} = \frac{1.464(10^{-2})}{1.947(10^{19})} = 7.52(10^{-22}) \text{ cm}^2$$

$$\sigma_{\text{ax}}(\text{Boron}) = 752 \text{ b}$$

Chlorine

$$\alpha(\text{AlCl}_3 \cdot 6 \text{H}_2\text{O}) = 13.90(10^3) \text{ sec}^{-1}$$

$$\alpha(\text{H}_2\text{O}) = 11.0(10^3) \text{ sec}^{-1}$$

$$\alpha' = (13.90 - 11.0)10^3 = 2.90(10^3) \text{ sec}^{-1}$$

Correction for displaced moderator

$$v = 2.2(10^5) \text{ cm/sec} \quad B^2 = 0.175 \text{ cm}^{-2}$$

$$D_{01} = vD = 2.2(10^5) 0.164 = 3.61(10^4) \text{ cm}^2/\text{sec}$$

$$\Sigma_{a1} = 0.022 \text{ cm}^{-1}$$

$$\frac{V_1}{V_2} = \frac{1000}{1005}$$

$$\Delta\alpha(\text{due to change in } D) = B^2 D_{01} \left(\frac{V_2}{V_1} - 1 \right)$$

$$\begin{aligned} \Delta\alpha &= 0.175 \times 3.61(10^4) \left(\frac{1005}{1000} - 1 \right) = 0.175 \times 3.61(10^4) \times \frac{5}{1000} \\ &= 30 \text{ sec}^{-1} \end{aligned}$$

$$\Delta\alpha(\text{due to change in } \Sigma_a) = v \Sigma_{a1} \left(\frac{V_1}{V_2} - 1 \right)$$

$$\begin{aligned} \Delta\alpha &= 2.2(10^5) 2.2(10^{-2}) \left(\frac{1000}{1005} - 1 \right) = 2.2(10^5) 2.2(10^{-2}) \left(-\frac{5}{1000} \right) \\ &= -22 \text{ sec}^{-1} \end{aligned}$$

$$\alpha''(\text{corrected}) = \alpha' + 22 - 30 = 2.90(10^3) + 22 - 30 = 2.89(10^3)$$

$$\alpha'' = v \Sigma_{ax}$$

$$2.89(10^3) = 2.2(10^5) \Sigma_{ax}$$

$$\Sigma_{ax} = 1.314(10^{-2}) \text{ cm}^{-1}$$

$$\sigma_{ax} = \frac{\Sigma_{ax}}{N} = \frac{1.314(10^{-2})}{3.741(10^{20})} = 3.51(10^{-23}) \text{ cm}^2$$

$$\sigma_{ax}(\text{Cl}) = 35.1 \text{ b}$$

ACTIVATION CROSS SECTION

by

W. F. Gilbert

ABSTRACT

A method of computing activation cross sections by determination of the absolute disintegration rate of a foil irradiated in a known thermal flux is described. The method can also be used to determine an unknown thermal flux in which a foil has been irradiated, provided that an activation cross section for the foil material is assumed. The method used could be applied to many materials which become activated by thermal neutrons and whose resulting beta activity has a sufficiently long half-life so that counting of the sample is not a problem. The techniques involved in absolute beta counting are emphasized.

OUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
- IV. PROCEDURE
- V. RESULTS
- VI. ANALYSIS AND CONCLUSIONS
- VII. BIBLIOGRAPHY
- VIII. APPENDICES
 - A. Discussion Questions
 - B. Beta-counting Considerations
 - C. Thermal-neutron Self-shielding Effects in Thin Foils

I. INTRODUCTION

The experiment described here yields by measurement two fundamental reactor parameters: absorption cross section and thermal-neutron flux, plus the important experience of doing absolute counting.

The absolute number of disintegrations per unit time in an irradiated foil is a function of flux, time, foil weight, characteristic decay constant, and activation cross section. If the first four are known and also the absolute disintegration rate, the activation cross section can be determined. Similarly, if all are known except the thermal flux, it can be determined.

II. THEORY

If a material which can be activated by a thermal neutron flux is irradiated in such a flux, the net rate of increase of radioactive nuclei is given by

$$\frac{dN'}{dt} = \phi_{th} N \sigma_{act} - \lambda N', \quad (1)$$

where

ϕ_{th} = the thermal-neutron flux, n/cm^2 -sec;

N = number of nuclei in the sample subject to activation by thermal neutrons;

σ_{act} = effective cross section of each nucleus for the activation reaction, cm^2 ;

N' = the number of radioactive nuclei present in the sample at any time t ;

λ = decay constant of the radioactive nuclei, sec^{-1} ;

t = time, sec.

The term $\phi_{\text{th}} N \sigma_{\text{act}}$ is the rate of formation of radioactive nuclei in the sample, and $\lambda N'$ is the rate of decay of these nuclei.

If the sample is irradiated to saturation, the rate of decay equals the rate of formation:

$$\frac{dN'_s}{dt} = 0 = \phi_{\text{th}} N \sigma_{\text{act}} - \lambda N'_s$$

or

$$\lambda N'_s = \phi_{\text{th}} N \sigma_{\text{act}}, \quad (2)$$

where N'_s is the number of radioactive nuclei in the sample at saturation. If the sample is then removed from the flux, the initial activity is given by

$$\frac{dN'_s}{dt} = - \lambda N'_s$$

or, from Eq. (2),

$$\frac{dN'_s}{dt} = \phi_{\text{th}} N \sigma_{\text{act}} \text{ (disintegrations/sec)}. \quad (3)$$

For a given sample N is easily determined and, if the saturated disintegration rate is determined, the activation cross section may be found if the thermal flux in which the sample was irradiated is known. Conversely, the thermal flux may be computed if an activation cross section is assumed.

Most neutron-activated nuclei decay with beta emission and, since one beta particle is emitted for each disintegration, an absolute beta-emission rate determined for the sample is equal to the disintegration rate.

If the sample used has a $1/v$ -absorption cross section, as is true of most materials, the reaction rate is proportional to neutron density in the sample. The neutrons may be assumed to have any convenient single velocity, but the cross section must correspond to the velocity chosen. Then ϕ_{th} is

$$\phi_{\text{th}} = N_0 v_0, \quad (4)$$

where N_0 is the neutron density (n/cm^3) and v_0 the arbitrary velocity of the neutrons usually chosen as the most probable velocity of a Maxwellian thermal-neutron distribution at room temperature ($2.2 \times 10^5 \text{ cm/sec}$). We may compute N as

$$N = N_a W/A, \quad (5)$$

where

N_a = Avogadro's number;

A = atomic weight of sample;

W = weight of sample, g.

Equations (1), (2) and (3) are correct only if each atom of the sample is exposed to the same thermal flux. If the sample is in the usual form of a foil which has appreciable thickness, the thermal flux is decreased inside the foil due to the absorption of thermal neutrons as they traverse the foil. This self-shielding effect will be compensated for in computing the equivalent activity the foil would have had if the thermal flux were not depleted by the self-absorption of the foil.

When a foil is irradiated at a location where there is an appreciable epithermal flux present, the activity observed may have to be corrected to determine the portion of the induced activity due only to the thermal flux. The activity of the foil due to thermal neutrons and corrected for the self-shielding of the foil for thermal neutrons can be expressed as

$$A_{th} = AR/\alpha, \quad (6)$$

where

A_{th} = equivalent thermal-neutron-induced activity of the foil;

A = observed total activity of the same foil at the end of irradiation;

R = fraction of total foil activity due to thermal neutrons;

α = thermal neutron self-shielding correction factor (relative activation per unit mass).

Now R is related to the usual cadmium ratio (CR) as follows:

$$A'_{th} = A - FA_{Cd}, \quad (7)$$

where

A_{th} = that part of the A due to thermal neutrons;

A_{Cd} = observed activity of the same foil when irradiated under the same conditions but cadmium-covered;

F = correction factor to compensate for resonance neutrons absorbed by the cadmium cover.

Rearrangement yields

$$A - A_{th} = A_{Cd}F,$$

upon division by A we obtain

$$1 - \frac{A_{th}}{A} = \frac{A_{Cd}F}{A}.$$

But $A/A_{Cd} = (CR)$, by definition, and $A_{th}/A = R$. Therefore,

$$1 - R = F/(CR)$$

or

$$R = \frac{(CR) - F}{(CR)}. \quad (8)$$

The correction for the activity due to epithermal neutrons can be made by using either R in Eq. (6) or by direct use of Eq. (7).

The thermal-neutron self-shielding factor of the foil, α , is computed by diffusion theory. This factor is based upon an isotropic flux; it is therefore implied that the foil was irradiated in a medium having good scattering properties for thermal neutrons. The calculation of the self-shielding factor is considered in Appendix C.

III. APPARATUS (see Fig. 27.1)

Reactor

Standard pile

2π and end-window proportional counters

Suitable foils with cadmium and aluminum (see Figs. 27.2 and 27.3) covers.

or

$$\Delta D_0 = D_{01} \left(\frac{N_{m1}}{N_{m2}} - 1 \right) = D_{01} \left(\frac{V_2}{V_1} - 1 \right), \quad (15)$$

where V_1 and V_2 are the moderator volumes before and after adding the absorber, respectively. Substitution of Eq. (15) into Eq. (12) gives the diffusion-coefficient correction to the decay constant as

$$\Delta \alpha = B^2 D_{01} \left(\frac{V_2}{V_1} - 1 \right). \quad (16)$$

The change in decay constant due only to a change in Σ_{am} is found from Eq. (4) as

$$\Delta \alpha = v \Delta \Sigma_{am}. \quad (17)$$

But

$$\begin{aligned} \Delta \Sigma_{am} &= \Sigma_{am2} - \Sigma_{am1} = \Sigma_{am1} \frac{N_2}{N_1} - \Sigma_{am1} = \Sigma_{am1} \left(\frac{N_2}{N_1} - 1 \right) \\ &= \Sigma_{am1} \left(\frac{V_1}{V_2} - 1 \right). \end{aligned} \quad (18)$$

Thus

$$\Delta \alpha = v \Sigma_{am1} \left(\frac{V_1}{V_2} - 1 \right). \quad (19)$$

Equations (16) and (19) represent corrections to the observed decay constant due to the change of the moderator volume when the absorber is added. For absorbers of large cross section, such as boron, the small amount added may make these corrections negligible. For absorbers of small cross section, the corrections may be very appreciable.

The added absorber is usually in the form of a compound soluble in the moderator. For such a compound, the observed additional cross section is the sum of the cross sections for the various elements making up the compound. A particular cross section may be obtained by forming a compound with elements known to have negligible absorption cross sections. If this is not possible, a compound may be formed with an element whose absorption cross section has been accurately determined and the unknown cross section determined by a difference technique.

III. APPARATUS

Pulsed neutron source
Multichannel time analyzer
Aluminum containers
 $B^{10}F_3$ detector, preamplifier, and linear amplifier
MPC high-voltage supply, amplifier, and scaler
Distilled water
Absorber solutions

Fig. 26.1
Experimental
Apparatus

139-3417

IV. PROCEDURE

A. Prepare the pulsed neutron source and multichannel time analyzer for operation as directed by the instructor.

B. Place a known amount of distilled water in an aluminum tank, and determine the dimensions and geometric buckling of the sample by use of the appropriate extrapolation length. The extrapolation length may be determined from a previous experiment or from the published diffusion parameters for water.

C. Pulse the water sample to the memory limit of the multi-channel time analyzer. Print out the data, and plot the decay curve on semilog paper. Determine the decay constant for the water sample.

D. To a known amount of distilled water, add the absorbing material selected and dissolve it completely in the water. Again measure the volume of solution obtained. If the increase is appreciable, corrections to the observed cross section must be made as outlined in Section II.

E. Place the same volume of solution as was used in Part B in the same aluminum can. Pulse the sample to the memory limit of the time analyzer. Print out the data, and plot the decay curve on semilog paper. Determine the decay constant for the solution sample.

F. Subtract the decay constants obtained in Parts C and E.

G. Divide the decay curves obtained in Parts C and E, point by point, to obtain a new decay curve due to the added absorber. Determine the new decay constant, and compare with that obtained in Part F.

H. If the increase in volume upon addition of the solute was noticeable in Part D, compute the corrections in the decay constant due to the change in diffusion coefficient and absorption cross section of the water. Correct the decay constant found in Parts F and G for these changes.

I. By use of the corrected decay constant due to the added absorber, compute the exponential microscopic absorption cross section for the added material. If more than one absorption cross section is appreciable in the compound used, subtract the known absorption cross section from the total to obtain the unknown cross section. If the cross section is not $1/v$, use the non- $1/v$ correction factor to obtain the microscopic absorption cross section at 2200 m/sec.

V. RESULTS

The results to be obtained will consist of the neutron-decay curves for the water sample and the sample with added absorber. The ratio curve obtained by dividing the curves point by point should also be plotted. From the decay constants found from these curves, the absorption cross section of the added absorber is computed.

VI. ANALYSIS AND CONCLUSIONS

The student should give considerable attention to possible sources of error in the experiment. In particular, a comparison of the errors involved in the two methods used for determining the net decay constant should be made.

VII. BIBLIOGRAPHY

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VIII. APPENDICES

Discussion Questions

1. Why are equal volumes of water and solution used in the two runs? Under what conditions would we lose the advantage this usually produces?
2. Discuss the possible advantages of each of the two methods used to determine the net change in decay constant.
3. Must the two samples be irradiated to the same monitor count? Why?
4. The scattering effect of the solute in the solution has been neglected in the corrections considered. Why?
5. If the solute displaces an appreciable amount of water, what factors are changed, and what effect do they have on the observed decay constant?
6. Why is the change in extrapolation length due to the addition of solute neglected? Under what conditions might it become important?
7. Must the temperature of the solutions be considered in this experiment? Why?
8. Would this experiment produce more accurate results for absorbers of large cross section or small cross section?

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STUDENT RESULTSABSORBER DATA

Sample 11.5 x 11.5 x 15.1 cm 2 liters

H₃BO₃ (mol wt = 61.84)2 g H₃BO₃/liter of solution

$$N = \frac{2 \times 6.02(10^{23})}{61.84 \times 1000} = 1.947(10^{19}) \text{ B atoms/cm}^3$$

Negligible increase in volume

AlCl₃ · 6H₂O (Formula wt = 241.44)50 g AlCl₃ · 6 H₂O/liter at solution

$$N' = \frac{50 \times 6.02(10^{23})}{241.44 \times 1000} = 1.247(10^{20}) \text{ mol/cm}^3$$

$$N = 3 \times 1.247(10^{20}) = 3.741(10^{20}) \text{ Cl atoms/cm}^3$$

Mol wt H₂O = 18.02 g/mol

$$6 \text{ H}_2\text{O} = 6 \times 18.02 = 108.12 \text{ g H}_2\text{O/mol AlCl}_3 \cdot 6 \text{ H}_2\text{O}$$

$$\text{H}_2\text{O in 50 g AlCl}_3 \cdot 6 \text{ H}_2\text{O} = \frac{108.12}{241.44} \times 50 = 22.4 \text{ g}$$

Increase in volume = 27 cm³/literIncrease in volume due to AlCl₃ = 27 - 22 = 5 cm³/liter

<u>Ch. No.</u>	<u>Count</u>	<u>Ch. No.</u>	<u>Count</u>	<u>Ch. No.</u>	<u>Count</u>	<u>Ch. No.</u>	<u>Count</u>
015	00047	008	01439	015	00014	008	01522
014	00065	007	02421	014	00020	007	03155
013	00116	006	04023	013	00047	006	06188
012	00168	005	07111	012	00089	005	12563
011	00254	004	12295	011	00183	004	25408
010	00461	003	21871	010	00343	003	47320
009	00792	002	35012	009	00732	002	02932

H₂OH₃BO₃
2 g/liter

Ch. No.	Count	Ch. No.	Count
015	00026	008	00787
014	00026	007	01604
013	00054	006	03163
012	00072	005	06409
011	00132	004	13168
010	00204	003	25680
009	00421	002	29531

50 g/liter

Channel time = 50 μs /channel
for all data

Fig. S26.1

Fig. S26.2

Fig. S26.3

CROSS SECTION CALCULATIONS

Boron

$$\alpha(\text{H}_3\text{BO}_3) = 14.22(10^3) \text{ sec}^{-1}$$

$$\alpha(\text{H}_2\text{O}) = 11.00(10^3) \text{ sec}^{-1}$$

$$\alpha' = \alpha(\text{H}_3\text{BO}_3) - \alpha(\text{H}_2\text{O}) = (14.22 - 11.00)10^3 = 3.22(10^3) \text{ sec}^{-1}$$

$$\alpha' = v \Sigma_{\text{ax}}$$

$$\Sigma_{\text{ax}} = \frac{\alpha'}{v} = \frac{3.22(10^3) \text{ sec}^{-1}}{2.2(10^5) \text{ cm/sec}} = 1.464(10^{-2}) \text{ cm}^{-1}$$

$$\sigma_{\text{ax}} = \frac{\Sigma_{\text{ax}}}{N} = \frac{1.464(10^{-2})}{1.947(10^{19})} = 7.52(10^{-22}) \text{ cm}^2$$

$$\sigma_{\text{ax}}(\text{Boron}) = 752 \text{ b}$$

Chlorine

$$\alpha(\text{AlCl}_3 \cdot 6 \text{H}_2\text{O}) = 13.90(10^3) \text{ sec}^{-1}$$

$$\alpha(\text{H}_2\text{O}) = 11.0(10^3) \text{ sec}^{-1}$$

$$\alpha' = (13.90 - 11.0)10^3 = 2.90(10^3) \text{ sec}^{-1}$$

Correction for displaced moderator

$$v = 2.2(10^5) \text{ cm/sec} \quad B^2 = 0.175 \text{ cm}^{-2}$$

$$D_{01} = vD = 2.2(10^5) 0.164 = 3.61(10^4) \text{ cm}^2/\text{sec}$$

$$\Sigma_{a1} = 0.022 \text{ cm}^{-1}$$

$$\frac{V_1}{V_2} = \frac{1000}{1005}$$

$$\Delta\alpha(\text{due to change in } D) = B^2 D_{01} \left(\frac{V_2}{V_1} - 1 \right)$$

$$\begin{aligned} \Delta\alpha &= 0.175 \times 3.61(10^4) \left(\frac{1005}{1000} - 1 \right) = 0.175 \times 3.61(10^4) \times \frac{5}{1000} \\ &= 30 \text{ sec}^{-1} \end{aligned}$$

$$\Delta\alpha(\text{due to change in } \Sigma_a) = v \Sigma_{a1} \left(\frac{V_1}{V_2} - 1 \right)$$

$$\begin{aligned} \Delta\alpha &= 2.2(10^5) 2.2(10^{-2}) \left(\frac{1000}{1005} - 1 \right) = 2.2(10^5) 2.2(10^{-2}) \left(-\frac{5}{1000} \right) \\ &= -22 \text{ sec}^{-1} \end{aligned}$$

$$\alpha''(\text{corrected}) = \alpha' + 22 - 30 = 2.90(10^3) + 22 - 30 = 2.89(10^3)$$

$$\alpha'' = v \Sigma_{ax}$$

$$2.89(10^3) = 2.2(10^5) \Sigma_{ax}$$

$$\Sigma_{ax} = 1.314(10^{-2}) \text{ cm}^{-1}$$

$$\sigma_{ax} = \frac{\Sigma_{ax}}{N} = \frac{1.314(10^{-2})}{3.741(10^{20})} = 3.51(10^{-23}) \text{ cm}^2$$

$$\sigma_{ax}(\text{Cl}) = 35.1 \text{ b}$$

ACTIVATION CROSS SECTION

by

W. F. Gilbert

ABSTRACT

A method of computing activation cross sections by determination of the absolute disintegration rate of a foil irradiated in a known thermal flux is described. The method can also be used to determine an unknown thermal flux in which a foil has been irradiated, provided that an activation cross section for the foil material is assumed. The method used could be applied to many materials which become activated by thermal neutrons and whose resulting beta activity has a sufficiently long half-life so that counting of the sample is not a problem. The techniques involved in absolute beta counting are emphasized.

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 - A. Discussion Questions
 - B. Beta-counting Considerations
 - C. Thermal-neutron Self-shielding Effects in Thin Foils

I. INTRODUCTION

The experiment described here yields by measurement two fundamental reactor parameters: absorption cross section and thermal-neutron flux, plus the important experience of doing absolute counting.

The absolute number of disintegrations per unit time in an irradiated foil is a function of flux, time, foil weight, characteristic decay constant, and activation cross section. If the first four are known and also the absolute disintegration rate, the activation cross section can be determined. Similarly, if all are known except the thermal flux, it can be determined.

II. THEORY

If a material which can be activated by a thermal neutron flux is irradiated in such a flux, the net rate of increase of radioactive nuclei is given by

$$\frac{dN'}{dt} = \phi_{th} N \sigma_{act} - \lambda N', \quad (1)$$

where

ϕ_{th} = the thermal-neutron flux, n/cm^2 -sec;

N = number of nuclei in the sample subject to activation by thermal neutrons;

σ_{act} = effective cross section of each nucleus for the activation reaction, cm^2 ;

N' = the number of radioactive nuclei present in the sample at any time t ;

λ = decay constant of the radioactive nuclei, sec^{-1} ;

t = time, sec.

The term $\phi_{\text{th}} N \sigma_{\text{act}}$ is the rate of formation of radioactive nuclei in the sample, and $\lambda N'$ is the rate of decay of these nuclei.

If the sample is irradiated to saturation, the rate of decay equals the rate of formation:

$$\frac{dN'_s}{dt} = 0 = \phi_{\text{th}} N \sigma_{\text{act}} - \lambda N'_s$$

or

$$\lambda N'_s = \phi_{\text{th}} N \sigma_{\text{act}}, \quad (2)$$

where N'_s is the number of radioactive nuclei in the sample at saturation. If the sample is then removed from the flux, the initial activity is given by

$$\frac{dN'_s}{dt} = -\lambda N'_s$$

or, from Eq. (2),

$$\frac{dN'_s}{dt} = \phi_{\text{th}} N \sigma_{\text{act}} \text{ (disintegrations/sec)}. \quad (3)$$

For a given sample N is easily determined and, if the saturated disintegration rate is determined, the activation cross section may be found if the thermal flux in which the sample was irradiated is known. Conversely, the thermal flux may be computed if an activation cross section is assumed.

Most neutron-activated nuclei decay with beta emission and, since one beta particle is emitted for each disintegration, an absolute beta-emission rate determined for the sample is equal to the disintegration rate.

If the sample used has a $1/v$ -absorption cross section, as is true of most materials, the reaction rate is proportional to neutron density in the sample. The neutrons may be assumed to have any convenient single velocity, but the cross section must correspond to the velocity chosen. Then ϕ_{th} is

$$\phi_{\text{th}} = N_0 v_0, \quad (4)$$

where N_0 is the neutron density (n/cm^3) and v_0 the arbitrary velocity of the neutrons usually chosen as the most probable velocity of a Maxwellian thermal-neutron distribution at room temperature ($2.2 \times 10^5 \text{ cm/sec}$). We may compute N as

$$N = N_a W/A, \quad (5)$$

where

N_a = Avogadro's number;

A = atomic weight of sample;

W = weight of sample, g.

Equations (1), (2) and (3) are correct only if each atom of the sample is exposed to the same thermal flux. If the sample is in the usual form of a foil which has appreciable thickness, the thermal flux is decreased inside the foil due to the absorption of thermal neutrons as they traverse the foil. This self-shielding effect will be compensated for in computing the equivalent activity the foil would have had if the thermal flux were not depleted by the self-absorption of the foil.

When a foil is irradiated at a location where there is an appreciable epithermal flux present, the activity observed may have to be corrected to determine the portion of the induced activity due only to the thermal flux. The activity of the foil due to thermal neutrons and corrected for the self-shielding of the foil for thermal neutrons can be expressed as

$$A_{th} = AR/\alpha, \quad (6)$$

where

A_{th} = equivalent thermal-neutron-induced activity of the foil;

A = observed total activity of the same foil at the end of irradiation;

R = fraction of total foil activity due to thermal neutrons;

α = thermal neutron self-shielding correction factor (relative activation per unit mass).

Now R is related to the usual cadmium ratio (CR) as follows:

$$A'_{th} = A - FA_{Cd}, \quad (7)$$

where

A_{th}^1 = that part of the A due to thermal neutrons;

A_{Cd} = observed activity of the same foil when irradiated under the same conditions but cadmium-covered;

F = correction factor to compensate for resonance neutrons absorbed by the cadmium cover.

Rearrangement yields

$$A - A_{th}^1 = A_{Cd}F,$$

upon division by A we obtain

$$1 - \frac{A_{th}^1}{A} = \frac{A_{Cd}F}{A}.$$

But $A/A_{Cd} = (CR)$, by definition, and $A_{th}^1/A = R$. Therefore,

$$1 - R = F/(CR)$$

or

$$R = \frac{(CR) - F}{(CR)}. \quad (8)$$

The correction for the activity due to epithermal neutrons can be made by using either R in Eq. (6) or by direct use of Eq. (7).

The thermal-neutron self-shielding factor of the foil, α , is computed by diffusion theory. This factor is based upon an isotropic flux; it is therefore implied that the foil was irradiated in a medium having good scattering properties for thermal neutrons. The calculation of the self-shielding factor is considered in Appendix C.

III. APPARATUS (see Fig. 27.1)

Reactor

Standard pile

2π and end-window proportional counters

Suitable foils with cadmium and aluminum (see Figs. 27.2 and 27.3) covers.

139-2426

Fig. 27.1. Components Used in Counting of Indium Foils

139-2424

Fig. 27.2. 2π Counter

139-3424

Fig. 27.3. Gas Flow Proportional Counter

IV. PROCEDURE

An aluminum-covered indium foil is irradiated in the reactor and is counted in an end-window counter to determine its initial activity. A thick aluminum backing for the foil is used while counting, and the observed count rate is corrected to saturation.

A count rate versus absorber curve for the foil is found by using aluminum absorbers to about 2 g/cm^2 . The gamma portion of the curve is extrapolated back to zero absorber to determine the gamma correction factor f_γ . The gamma contribution is subtracted from the curve and the resulting beta count rate versus absorber curve is plotted for absorber thicknesses to 50 mg/cm^2 . This curve is extrapolated to determine the counter window correction factor, f_w .

The foil is counted in a known geometry counter and in the end-window counter, with a thick aluminum backing in each case. By use of gamma and counter window corrections, the geometry factor f_g for the end-window counter is found. The foil is counted in the end-window counters with and without aluminum backing to determine the backscattering correction factor f_{bs} .

An aluminum-covered indium foil is irradiated in the standard pile and counted with aluminum backing in the end-window counter. The observed count rate is corrected to saturation.

Cadmium-covered indium foils are irradiated in the reactor and the standard pile, and counted with aluminum backing in the end-window counter. The count rates are corrected to saturation, and the cadmium ratio determined for the reactor and the standard pile. The saturated count rate of both aluminum-covered foils is corrected for epithermal activity, and the thermal flux in the reactor is computed from knowledge of the thermal flux in the standard pile.

The equivalent saturated thermal-neutron-induced activity of the aluminum-covered foil irradiated in the reactor is computed, and the absolute thermal-neutron-induced disintegration rate of the saturated foil is determined. The activation cross section of indium is then computed.

V. RESULTS

Results obtained in the experiment are primarily the values obtained for the reactor thermal flux and the activation cross section of indium. The correction factors which must be found as part of the experiment are also of considerable interest, and the data leading to them should be carefully presented.

VI. ANALYSIS AND CONCLUSIONS

The value obtained for the activation cross section of indium should be compared with the accepted values obtainable from various sources, and possible sources of error in the experiment may be investigated.

VII. BIBLIOGRAPHY

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VIII. APPENDICES

A. Discussion Questions

The following propositions should be considered prior to arrival at the Argonaut. If possible, the questions will be discussed before the beginning of experimental work.

1. When a foil has been irradiated and is then counted, what factors must be considered if the saturated count rate of the foil is to be found?
2. Explain the physical concepts involved in microscopic absorption, scattering, and activation cross sections.
3. What relationship necessarily exists between absorption and activation cross sections?
4. Are cross sections in any way related to the radius of the nucleus?
5. Do (n, γ) reactions in the foil contribute to the observed count rate?
6. A foil which emits both beta particles and gamma rays is counted in a counter which is sensitive to both. What shape would you expect the curve of observed count rate versus absorber thickness to have?
7. How can we determine the count rate due only to beta particles from the count rate versus absorber curve?
8. How can we determine the counter window correction factor for beta particles from the count rate versus absorber curve?
9. The beta-particle emission from a foil of appreciable thickness is not isotropic, but is found to be predominant in the direction perpendicular to the plane of the foil. What is the reason for this observed effect?
10. What effect does the preferential forward scattering of beta particles from a foil have on the apparent geometry factor of the counter when the foil is counted in an end-window counter?
11. Would the gamma correction factor for indium be a function of shelf position when an end-window counter is used?
12. What effect would we expect the foil thickness to have on the back-scattering correction factor?
13. Why must the saturated count rate of a cadmium-covered indium foil be corrected before it is subtracted from the bare-foil, saturated count rate to determine the activity due only to thermal neutrons?

14. Why is the relative activity per unit mass of a foil less when irradiated in an isotropic flux as compared with a collimated flux?
15. The self-absorption of beta particles by a foil is found to increase rapidly with foil thickness for thin foils and then to increase more slowly for thicker foils. What property of beta particles produces this effect?
16. In this experiment the flux depression in the medium surrounding the foil has been neglected. What conditions in the experiment cause this effect to be very small? Under what condition might the effect have to be considered?

B. Beta-counting Considerations

Since each disintegration in the foil produces one beta particle, the corrected observed activity is related to the absolute disintegration rate by the many factors associated with the counting of the foil. This may be expressed as

$$A_{th} = \frac{dN'}{dt} f_g f_w f_e f_\gamma f_{bs} f_s, \quad (9)$$

where

A_{th} = equivalent thermal-neutron-induced activity of the foil;

$\frac{dN'}{dt}$ = absolute disintegration rate of the foil at the end of irradiation due to thermal neutrons;

f_g = counter-geometry correction factor;

f_w = counter-window-absorption correction factor;

f_e = counter efficiency for β particles;

f_γ = gamma-background correction factor;

f_{bs} = correction factor for beta-particle backscattering from the foil-support material;

f_s = foil beta-particle self-absorption and self-scatter correction factor.

The counter geometry correction factor, f_g , cannot in general be computed if an end-window counter tube is used, because beta-emitting foils exhibit a very strong preferential forward-scattering effect which causes many more beta particles to enter the counter window than would be expected on the basis of the solid angle subtended by the window. The equivalent geometry factor can be obtained experimentally by counting a similar foil in the counter and also in another counter having known geometry, such as a 2π counter. An additional correction for different gamma backgrounds may have to be made in the two counting rates since the gamma emission of the foil does not exhibit the strong forward-scattering effect of the beta particles. Different counter-window correction factors may also have to be considered.

The gamma background correction factor, f_γ , can be found by counting the foil with the use of absorbers of various thicknesses. After a certain critical thickness of absorber has been reached, all the beta particles are removed; the remaining count rate is due to the gamma emission of the foil. The result will be a straight line if plotted on semilog graph paper for absorbers above the critical thickness. This straight line can be extrapolated back to zero absorber to determine the gamma contribution in the original count rate.

The counter-window correction factor, f_w , is found by subtracting the gamma contribution from the total count rate versus absorber curve and then plotting the first part of the resulting beta-activity versus absorber curve on semilog paper. The result is a straight line which can be extrapolated back to give the correction for the counter tube window and the counter air window.

The counter efficiency for beta particles, f_e , can be considered as unity.

The correction factor for beta particle backscattering from the foil support material, f_{bs} , can be found by counting the foil on the supporting material and on a film support of negligible thickness.

The foil beta particle self-absorption and self-scatter correction factor, f_s , is the only correction factor which is strongly dependent on the foil thickness (see Fig. 27.4). The absorption increases very rapidly with increasing foil thickness for thin foils because of the absorption of low-energy beta particles, and then increases more slowly for greater foil thicknesses. This correction factor can be found by using foils of various thicknesses and observing their relative unity as the foil thickness approaches zero gives the relative count rate to be expected at any foil thickness due to the foil self-absorption and self-scattering. The relative count rates are usually determined in 2π or 4π counting systems to eliminate extraneous effects, such as preferential forward scattering.

Fig. 27.4. Beta Particle Self-absorption and Self-scattering Correction Factor for Indium Foils

The disintegration rate of the foil at the end of irradiation, dN'/dt , is obtained from Eq. (9) and can be converted to dN'_s/dt , the saturated disintegration rate, by the relationship

$$\frac{dN'}{dt} = \frac{dN'_s}{dt} (1 - e^{-\lambda t_1}),$$

where λ is the decay constant and t_1 the irradiation time of the foil.

The thermal flux in which the foil was irradiated can be found by comparing the saturated activity of similar foils exposed in the unknown flux and in a known flux in a standard pile. In both cases, the cadmium ratio must be found and each activity corrected to determine the thermal component. If similar foils are used and they are counted under identical conditions, all other correction factors pertaining to the counting of foils are identical for both and therefore do not need to be considered.

C. Thermal-neutron Self-shielding Effects in Thin Foils

Introduction

Materials which become active when exposed to neutrons are commonly used to detect and measure neutron flux. In order to obtain useful counting rates in moderate amounts of the irradiated materials the cross section for activation must be reasonably large and the total absorption cross section will be as large or larger.

It is usually assumed that the material to be activated is irradiated in a uniform neutron flux, either collimated or isotropic as the case may be. This condition can only be approached in practice by using a sample which captures a negligible fraction of the neutrons which enter the sample. Because of the relatively large absorption cross section, the sample must be extremely thin to meet this condition. Such samples are difficult to prepare and very likely to be damaged in handling.

For convenience in handling, samples to be irradiated are commonly prepared as foils which are physically rather thin but which do not satisfy the requirement that they absorb a negligible number of neutrons. In such foils the neutron flux in the interior region of the foil is decreased by the absorption of neutrons in the outer regions of the foil. Since the activation produced at any point in the foil is proportional to the neutron flux at that point, the total activation produced in such a foil is obviously less than that which would have been produced had the flux been constant throughout the foil.

It is therefore necessary to determine correction factors by which the observed activity of a foil can be converted to the activity which would have been obtained in the absence of self-shielding in the foil. Such corrections are usually necessary only when the absolute disintegration rate in the foil is to be determined from the neutron flux in which the foil was irradiated and the activation cross section.

Correction for the self-shielding in a foil may or may not be necessary depending on the nature of the foil and the precision of the measurement to be made. For an indium foil, 5 mil thick, exposed in an isotropic flux, the error introduced by neglecting the self-shielding is about 16 percent. For the same foil in a collimated flux the error is only about 5 percent.

I. Collimated Neutron Flux

Consider a collimated thermal-neutron flux ϕ_0 incident at right angles upon the surface of a foil of thickness t , as shown in Fig. 27.5. For such a collimated flux the neutron current J_0 into the surface of the

foil is equal to the flux ϕ_0 . If the foil has an absorption cross section which is not constant with respect to neutron energy (such as the $1/v$ cross section of many materials), neutrons of certain energies are absorbed preferentially. As a result, the neutron flux observed anywhere in the foil or that emerging from the foil will have a different energy distribution than that entering the foil, and the number of neutrons captured at any point is difficult to compute.

Fig. 27.5. Collimated Flux Incident on a Foil

If the foil is sufficiently thin so that relatively few neutrons are captured in the foil, the shift in the energy distribution of the flux throughout the foil is small and may be neglected for many calculations. The assumption of a sufficiently thin foil is made in deriving the following expressions.

The number of neutrons removed from the beam per cm^2 per second in an incremental foil thickness dx at x is $\phi_x \Sigma_T dx$; therefore,

$$d\phi_x = - \phi_x \Sigma_T dx$$

or

$$\frac{d\phi_x}{\phi_x} = \Sigma_T dx. \quad (1)$$

Integration gives

$$\ln \phi_x = -\Sigma_T x + \ln A$$

or

$$\phi_x = A e^{-\Sigma_T x}, \quad (2)$$

when $x = 0$, $\phi_x = \phi_0$; therefore $A = \phi_0$ and

$$\phi_x = \phi_0 e^{-\Sigma_T x}. \quad (3)$$

At $x = t$;

$$\phi_t = \phi_0 e^{-\Sigma_T t}. \quad (4)$$

The transmission T_{coll} of the foil for this collimated flux is defined as the ratio of the transmitted flux to the incident flux:

$$T_{\text{coll}} = \phi_t / \phi_0 = e^{-\Sigma_T t}. \quad (5)$$

The number of reactions, R , which occur for each cm^2 of foil surface per second is equal to the reduction in flux in passing through the foil:

$$R = \phi_0 - \phi_t = \phi_0(1 - T_{\text{coll}}). \quad (6)$$

The number of reactions, R' , which would occur for each cm^2 of foil surface per second if the flux were ϕ_0 throughout the foil is

$$R' = \phi_0 \Sigma_T t = \phi_0(t/\lambda) = \phi_0 x', \quad (7)$$

where λ is the mean free path of a neutron in the foil material and x' is the foil thickness in mean free paths.

The ratio of actual reactions in the foil to those which would have occurred if the flux had remained constant at ϕ_0 throughout the foil is known as the self-shielding factor of the foil, α :

$$\alpha_{\text{coll}} = \frac{R}{R'} = \frac{1 - T_{\text{coll}}}{x'}. \quad (8)$$

Since any activity induced in the foil is proportional to the number of neutron reactions in the foil, α is also the ratio of the activity induced in the foil to that which would have been induced by a constant flux ϕ_0 throughout the foil.

The average mean free path of thermal neutrons in the foil having a $1/v$ cross section is the mean free path of neutrons having the average velocity of the Maxwellian thermal velocity distribution. For such a distribution

$$\bar{v} = 1.128 v_0, \quad (9)$$

where v_0 is the most probable neutron velocity.

For a $1/v$ absorber,

$$\bar{\sigma} = \sigma_0 / 1.128, \quad (10)$$

where σ_0 is the microscopic absorption cross section at the velocity v_0 . (T_{coll} and α_{coll} are plotted as a function of foil thickness in mean free paths in Figs. 27.7 and 27.8.)

II. Isotropic Neutron Flux

Consider a foil of thickness given as x' mean free paths in an isotropic scattering medium and assume that the foil has a negligible effect on the flux in the medium. Also consider an imaginary hemisphere of radius r (much smaller than the foil) as shown in Fig. 27.6.

Fig. 27.6. Isotropic Flux Incident on a Foil

Since the scattering is isotropic, the number of neutrons, dN' , scattered into the hemisphere through any area dA is proportional to dA :

$$dN' = K dA = K 2\pi r^2 \cos \theta d\theta. \quad (11)$$

Of this number, those which will pass through the incremental surface of the foil ds at O is proportional to the solid angle subtended by ds at the point of entrance through the surface of the sphere:

$$d\Omega = \frac{ds \sin \theta}{r^2}. \quad (12)$$

Since the neutrons scattered into the hemisphere have a uniform distribution over 2π steradians, the probability of scattering through ds is

$$\frac{d\Omega}{2\pi} = \frac{ds \sin \theta}{2\pi r^2}. \quad (13)$$

Thus the number of neutrons scattered inward through dA and passing through ds is

$$\begin{aligned} dN &= dN' \frac{d\Omega}{2\pi} = \frac{K ds 2\pi r^2 \cos \theta}{2\pi r^2} \sin \theta d\theta \\ &= K ds \cos \theta \sin \theta d\theta. \end{aligned} \quad (14)$$

The total number of neutrons passing through ds is obtained by integrating over the surface of the hemisphere:

$$\begin{aligned} N &= \int dN = K ds \int_0^{\pi/2} \cos \theta \sin \theta d\theta \\ &= \frac{K ds}{2} \left[\sin^2 \theta \right]_0^{\pi/2} = \frac{K ds}{2}. \end{aligned} \quad (15)$$

The neutron current J_0 through the surface is

$$J_0 = \frac{N}{ds} = \frac{K ds}{2ds} = K/2. \quad (16)$$

From diffusion theory* this current in one direction through a plane surface in an isotropic scattering medium is $\phi/4$ in the absence of any flux gradient.

*S. Glasstone and M. C. Edlund, The Elements of Nuclear Reactor Theory, D. Van Nostrand Co., Inc., New York (1952), p. 94.

Thus, $K/2 = \phi/4$ or

$$K = \phi/2. \quad (17)$$

The restriction originally placed upon r is seen to be justified as the results are independent of r .

If we now consider the same neutrons which were scattered through the surface of the foil and find J_x , the number which pass through the foil and out the opposite side, we can define the transmission factor T_{iso} for the foil in an isotropic flux:

$$T_{iso} = J_x/J_0. \quad (18)$$

Of the number dN which passed through ds at an angle θ , the number which emerge from the other side of the foil is

$$dN_x = dN e^{-x' \sec \theta}. \quad (19)$$

Substituting for dN from Eq. (14) and for K from Eq. (17),

$$dN_x = \frac{1}{2} \phi ds e^{-x' \sec \theta} \cos \theta \sin \theta d\theta. \quad (20)$$

Then

$$N_x = \int dN_x = \frac{\phi ds}{2} \int_0^{\pi/2} e^{-x' \sec \theta} \cos \theta \sin \theta d\theta \quad (21)$$

and

$$J_x = \frac{N_x}{ds} = \frac{\phi}{2} \int_0^{\pi/2} e^{-x' \sec \theta} \cos \theta \sin \theta d\theta. \quad (22)$$

Integration by parts yields

$$J_x = \frac{\phi}{2} e^{-x'} - \frac{\phi x'}{4} \int_0^{\pi/2} e^{-x' \sec \theta} \sin \theta d\theta. \quad (23)$$

A second integration by parts yields

$$J_x = \frac{\phi}{2} e^{-x'} - \frac{\phi x'}{2} e^{-x'} + \frac{x'^2}{2} \int_0^{\pi/2} e^{-x' \sec \theta} \frac{\sin \theta}{\cos \theta} d\theta. \quad (24)$$

Upon multiplying and dividing the remaining integral by $x' \cos \theta$, we have

$$J_x = \frac{\phi}{2} e^{-x'}(1 - x') + \frac{\phi x'^2}{2} \int_0^\pi e^{-x' \sec \theta} \frac{x' \sin \theta \cos \theta}{x' \cos^2 \theta} d\theta, \quad (25)$$

which can be reduced to

$$J_x = \frac{\phi}{2} e^{-x'}(1 - x') + \frac{\phi x'^2}{2} \int_{x'}^\infty \frac{e^{-t}}{t} dt \quad (26)$$

where $t = x' \sec \theta$. Thus,

$$J_x = \frac{\phi}{2} e^{-x'}(1 - x') + \frac{\phi x'^2}{2} E_1(x'), \quad (27)$$

where $E_1(x')$ is the exponential integral $\int_{x'}^\infty \frac{e^{-t}}{t} dt$.

Equation (27) can be rewritten as

$$J_x = \frac{\phi}{2} \left[e^{-x'}(1 - x') + x'^2 E_1(x') \right]. \quad (28)$$

The transmission is then

$$T_{iso} = \frac{J_x}{J_0} = \frac{\phi/2 \left[e^{-x'}(1 - x') + x'^2 E_1(x') \right]}{\phi/2} \quad (29)$$

or

$$T_{iso} = e^{-x'}(1 - x') + x'^2 E_1(x'). \quad (30)$$

For a foil exposed in an isotropic flux, the number of reactions in the foil per cm^2 of surface and per second due to the flux on one side is

$$J_0 - J_x = \frac{\phi}{4} - \frac{\phi}{4} T_{iso} = \frac{\phi}{4} (1 - T_{iso}). \quad (31)$$

When consideration of the neutrons entering the foil from both sides is taken, the number of reactions, R , is twice the above or

$$R = \frac{1}{2} \phi (1 - T_{iso}). \quad (32)$$

If the foil were exposed in such a way that no self-shielding were possible, the number of reactions per cm^2 of surface per second would be

$$\phi \Sigma_T = \phi t / \lambda = \phi x', \quad (33)$$

where t is the thickness of the foil (in cm), λ is the mean free path of neutrons in the foil (in cm), and x' is the thickness of the foil expressed in mean free paths. The ratio of the actual reactions in the foil to the number of reactions in the absence of self-shielding is the thermal neutron self-shielding factor:

$$\alpha_{\text{iso}} = \frac{1}{2} \phi \frac{(1 - T_{\text{iso}})}{\phi x'} = \frac{1 - T_{\text{iso}}}{2x'}. \quad (34)$$

As in the case of the collimated flux, the average mean free path for a $1/v$ absorber in a Maxwellian thermal-neutron flux is the mean free path for neutrons having the average velocity of the distribution.

T_{iso} and α_{iso} are plotted as a function of foil thickness in mean free paths in Figs. 27.7 and 27.8.

Fig. 27.7. Thin Foil Transmission Factor

Fig. 27.8. Relative Foil Activation

STUDENT RESULTSDATA AND CALCULATIONSIndium Foil Data

$$A = 114.76 \text{ g/mole} \quad 95.8\% \text{ In}^{116}$$

$$W = 134.0 \text{ mg} \quad \text{Area} = \frac{\pi - D^2}{4} = 1.885 \text{ cm}^2$$

$$\text{Diameter} = 1.55 \text{ cm} \quad W/\text{Area} = \frac{134.0}{1.885} = 71.1 \text{ mg/cm}^2$$

$$N = \frac{W \text{ Na}}{A} = \frac{134.0(10^{-3}) \times 6.02(10^{23})}{114.76} = 7.03(10^{20}) \text{ atoms/foil}$$

$$N_{115} = N \times 0.958 = 7.03(10^{20})0.958 = 6.73(10^{20}) \frac{\text{atoms In}^{115}}{\text{foil}}$$

$$\Sigma_T(2200\text{m/sec}) = 7.34 \text{ cm}^{-1} \quad (\text{ANL-5800, p. 524})$$

$$\bar{\Sigma}_T = \frac{7.34}{1.128} = 6.51 \text{ cm}^{-1}$$

$$\bar{\lambda} = \frac{1}{\bar{\Sigma}_T} = \frac{1}{6.51} = 0.1537 \text{ cm.}$$

$$x(\text{foil thickness}) = \frac{W/\text{Area}}{\xi} = \frac{71.1(10^{-3})}{7.28} = 9.77(10^{-3}) \text{ cm.}$$

$$x'(\text{foil thickness in MFP}) = \frac{x}{\bar{\lambda}} = \frac{9.77(10^{-3})}{0.1537} = 0.0636 \text{ MFP}$$

$$\alpha = 0.878 \quad (\text{from Fig. 27.4})$$

$$f_s = 0.415 \quad (\text{from graph})$$

$$f_e = 1.00 \quad \text{for both counters}$$

GAMMA AND WINDOW CORRECTION FACTORS

Decay Time min	Al Absorber mg/cm ²					Beta Only c/m
		c/m	c/m-BG	e ^{-λt₁}	c/m-BG e ^{-λt₁}	
9	0	78.9 × 10 ³	78.9 × 10 ³	0.891	88.5 × 10 ³	85.1 × 10 ³
11	2.32	74.9 × 10 ³	74.9 × 10 ³	0.868	86.3 × 10 ³	82.9 × 10 ³
13	5.56	69.7 × 10 ³	69.7 × 10 ³	0.846	82.4 × 10 ³	79.0 × 10 ³
15	12.79	62.5 × 10 ³	62.5 × 10 ³	0.825	75.7 × 10 ³	72.3 × 10 ³
17	27.49	50.4 × 10 ³	50.4 × 10 ³	0.804	62.7 × 10 ³	59.3 × 10 ³
19	35.77	43.9 × 10 ³	43.9 × 10 ³	0.784	56.0 × 10 ³	52.6 × 10 ³
21	49.04	35.2 × 10 ³	35.2 × 10 ³	0.764	46.1 × 10 ³	42.7 × 10 ³
23	55.65	31.7 × 10 ³	31.7 × 10 ³	0.744	42.6 × 10 ³	
25	134	8.92 × 10 ³	8.91 × 10 ³	0.725	12.3 × 10 ³	
27	210	4.04 × 10 ³	4.03 × 10 ³	0.707	5.70 × 10 ³	
29	348	2.45 × 10 ³	2.44 × 10 ³	0.689	3.54 × 10 ³	
31	444	2.29 × 10 ³	2.28 × 10 ³	0.672	3.39 × 10 ³	
33	544	2.06 × 10 ³	2.05 × 10 ³	0.655	3.13 × 10 ³	
35	861	1.92 × 10 ³	1.91 × 10 ³	0.638	2.99 × 10 ³	
37	1730	1.65 × 10 ³	1.64 × 10 ³	0.622	2.64 × 10 ³	
39	2100	1.47 × 10 ³	1.46 × 10 ³	0.606	2.41 × 10 ³	

BG = 12 c/m

Window = 0.5 mg/cm² plastic + 2.2 cm Air @ 1.18 mg/cm²
 = 0.5 + 2.6 = 3.1 mg/cm²

Fig. S27.1. Beta Absorption Curve

Standard Pile Foils

$$\lambda = 1.2836(10^{-2}) \text{ min}^{-1} \quad t_{1/2} = 54.0 \text{ min}$$

$$\text{Irradiated in hole \#18} \quad \phi_{th} = 6611 \text{ n/cm}^2 - \text{sec}$$

$$t_0 = \text{irradiation time} \quad t_1 = \text{decay time to start of count}$$

$$\Delta t = \text{counting time}$$

Bare foil

$$t_0 = 100 \text{ min} \quad t_1 = 11 \text{ min} \quad \Delta t = 20 \text{ min}$$

$$C = 38,439 \text{ counts/20 min} \quad BG = 233 \text{ counts/20 min}$$

$$C' = C - BG = 38,206 \text{ counts/20 min}$$

$$A_s = \frac{C' \lambda}{(1 - e^{-\lambda t_0}) e^{-\lambda t_1} (1 - e^{-\lambda \Delta t})}$$

$$= \frac{38,206 \times 1.2836(10^{-2})}{(1 - e^{-100\lambda}) e^{-11\lambda} (1 - e^{-20\lambda})} = \frac{38,206 \times 1.2836(10^{-2})}{0.723 \times 0.868 \times 0.2264}$$

$$A_s (\text{bare}) = 3455 \text{ counts/min}$$

Cadmium covered foil

$$t_0 = 157 \text{ min} \quad t_1 = 9 \text{ min} \quad \Delta t = 20 \text{ min}$$

$$C = 8704 \text{ counts/20 min} \quad BG = 250 \text{ counts/20 min}$$

$$C' = C - BG = 8454 \text{ counts/20 min}$$

$$A_s = \frac{C' \lambda}{(1 - e^{-\lambda t_0}) e^{-\lambda t_1} (1 - e^{-\lambda \Delta t})} = \frac{8454 \times 1.2836(10^{-2})}{0.867 \times 0.891 \times 0.2264}$$

$$A_s (\text{Cd covered}) = 621 \text{ counts/min}$$

$$A_s' = A_s (\text{bare}) - F A_s (\text{Cd covered})$$

$$= 3455 - 1.065 \times 621 = 3455 - 662 = 2783 \text{ counts/min}$$

$$F = 1.065 \text{ for } 71.1 \text{ mg/cm}^2 \text{ foils in 20 mil Cd covers}$$

(ANL-5800, p. 490)

Reactor FoilsBare foil

$$t_0 = 30 \text{ min} \quad t_1 = 9 \text{ min} \quad C = 78,900 \text{ counts/min} \quad \text{BG} = 8 \text{ c/min}$$

$$A_s = \frac{C}{(1 - e^{-\lambda t_0}) e^{-\lambda t_1}} = \frac{78,900}{(1 - e^{-30\lambda}) e^{-9\lambda}} = \frac{78,900}{0.3196 \times 0.891}$$

$$= 276,000 \text{ c/min}$$

Cadmium covered foil

$$t_0 = 30 \text{ min} \quad t_1 = 6 \text{ min} \quad C = 18,897 \text{ counts/min}$$

$$A_s = \frac{C}{(1 - e^{-\lambda t_0}) e^{-\lambda t_1}} = \frac{18,890}{(1 - e^{-30\lambda}) e^{-6\lambda}} = \frac{18,890}{0.3196 \times 0.926}$$

$$= 63,900 \text{ c/min}$$

$$A'_s = A_s (\text{bare}) - F A_s (\text{Cd covered}) = 276,000 - 1.065 \times 63,900$$

$$= 276,000 - 68,000 = 208,000 \text{ counts/min}$$

$$\frac{\phi_{\text{th}}(\text{reactor})}{\phi_{\text{th}}(\text{std. pile})} = \frac{A'_s(\text{reactor})}{A'_s(\text{std. pile})}$$

$$\phi_{\text{th}}(\text{reactor}) = 6611 \times \frac{208,000}{2783} = 4.94(10^5) \text{ n/cm}^2 \cdot \text{sec}$$

Back scattering coefficient

Time, min	Backing Material	c/min	$e^{-\lambda t_1}$	$\frac{\text{c/min}}{e^{-\lambda t}}$
0	Mylar 2.5 mg/cm ²	47,939	1.00	47,940
2	Thick Aluminum	48,162	0.975	49,400

$$f_{\text{bs}} = \frac{49,400}{47,940} = 1.031$$

Geometry factor

Time, min	Counter	c/min	$e^{-\lambda t_1}$	$\frac{c/\text{min}}{e^{-\lambda t}}$
0	End Window	9,989	1.00	9,989
3	2π	30,871	0.962	31,300

$$\frac{A(2\pi)}{A(\text{EW})} = \frac{f_g(2\pi) f_w(2\pi) f_{bs}(2\pi) f_\gamma(2\pi)}{f_g(\text{EW}) f_w(\text{EW}) f_{bs}(\text{EW}) f_\gamma(\text{EW})}$$

$$f_\gamma(2\pi) \approx f_\gamma(\text{EW}) \quad f_{bs}(2\pi) = 1.060 \quad (\text{from references})$$

$$f_g(2\pi) = 0.50 \quad f_w(2\pi) = 1.00 \quad (\text{no window})$$

$$\frac{31,300}{9,989} = \frac{0.50 \times 1.00 \times 1.060}{f_g(\text{EW}) \times 0.964 \times 1.031}$$

$$f_g(\text{EW}) = \frac{9,989 \times 0.050 \times 1.060}{31,300 \times 0.964 \times 1.031} = 0.170$$

Activation cross section calculation

$$A_{th} = \frac{A'_s(\text{reactor})}{\alpha} = \frac{dN'}{dt} f_g f_w f_e f_\gamma f_{bs} f_s$$

$$\frac{208,000}{0.878} = \frac{dN'}{dt} 0.170 \times 0.964 \times 1.00 \times 1.040 \times 1.031 \times 0.415$$

$$\frac{dN'}{dt} = 3,250,000 \text{ disintegrations/min}$$

$$= \frac{3,250,000}{60} \text{ d/sec} = 54,200 \text{ d/sec}$$

$$\frac{dN'}{dt} = \phi_{th} N \sigma_{act}$$

$$54,200 = 4.94(10^5) 6.73(10^{20}) \sigma_{act}$$

$$\sigma_{act} = 163(10^{-24}) \text{ cm}^2 = 163 \text{ barns}$$

ABSOLUTE NEUTRON FLUX SPECTRUM
MEASUREMENTS, ARGONAUT REACTOR

by

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ABSTRACT

In this experiment, measurements are made of the fast, resonance, and thermal-neutron fluxes at the same location in the Argonaut core by means of induced-activation detectors. Thoroughly discussed are the selection and preparation of the foil materials (detectors), the calibration of the counting apparatus, the determination of the induced activity from the foil-counting data, and, finally, the calculation of the particular neutron flux from the activity data.

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I. INTRODUCTION

Neutron-induced transmutations occur in certain stable elements when they are placed in a reactor, and the activity of the radioactive products can be measured with an appropriate counting system. If the response (cross-section variation with energy) of the detector is known, the neutron flux can be determined.

The neutrons in a reactor are spread over a large energy range - from fission energies down to the thermal region. Many detectors have responses which are well suited for measuring the neutron flux in certain parts of this energy range. In this experiment measurements will be made of the neutron flux at a point in the core of the Argonaut by means of induced-activation detectors with desired responses throughout the neutron-energy spectrum. The results which will be obtained will be strongly dependent on the theory involved in the experiment, since the detector response and the shape of the neutron energy spectrum are assumed to be known.

II. THEORY

A detailed writeup of the theory involved in this experiment can be found in Appendix B.

III. APPARATUS

- A. Argonaut Reactor
- B. Induced-activation Detectors (see Fig. 28.1)
 - 1. Gold-foil discs
 - 2. Indium-foil discs
 - 3. Manganese-foil discs
 - 4. Standard gold foils
 - 5. Sodium carbonate pellets
 - 6. Phosphorus pellets
 - 7. Sulfur pellets
- C. Aluminum and Cadmium Foil Covers
- D. Counting Systems
 - 1. GM end-window counter
 - 2. NaI scintillation counter
- E. Detector Holders for Reactor Irradiation (see Fig. 28.1)

IV. PROCEDURE

A. Selection and Preparation of the Activation Detectors to be Used for the Flux Measurements

1. Thermal Flux

Gold and indium will be used to measure the thermal flux in the Argonaut reactor. The thermal activation cross sections for these materials are well known. In the thermal region of the neutron-flux spectrum, both materials can be assumed to be $1/v$ detectors. In order to ensure that no self-shielding or flux depression occurs during the irradiation, the detector material is made extremely thin. A known mass of each detector material is irradiated to a desired activity. The material is then counted on a Geiger-Mueller counter, using a beta absorber and only counting gammas.

The material is placed in an evacuated bell jar (see Fig. 28.2) and evaporated onto polystyrene discs, 15/16 in. in diameter. The discs are removed from the bell jar and counted with the same Geiger-Mueller system. The counting rate at this time is extrapolated back to the initial counting period. The ratio of the counting rates determines the ratio of the mass of the material on the polystyrene disc to the original mass.

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Fig. 28.1. Detectors with Holders and Spacers

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Fig. 28.2. Vacuum System and Bell Jar

This method requires that the efficiency of the Geiger-Mueller system remains the same for both the disc and the original material. Gamma counting with the Geiger-Mueller system is advantageous in this case since the efficiency of the system will depend primarily on the geometry factor. Self-absorption and forward scattering can be considered the same (negligible) in both cases. Most of the error in the mass determination will come directly from the Geiger-Mueller system itself. Since the initial counting rate is fairly high, it becomes necessary to consider the dead time of the counting system. However, this determination should not make the error involved in the mass measurement greater than about 10 percent.

2. Resonance Flux

Gold and indium discs of the same type as used for measurements of the thermal flux are used to measure the slowing-down density at specific energies. In the region of interest, these detector cross sections are fairly well known in terms of resonance activation integrals (based on a $1/E$ neutron-flux spectrum being present). However, since each material exhibits a main resonance peak which makes up most of the activation integral, an indication of the neutron flux at these resonance energies can be made. The importance of having thin detectors is most

prominent in this region. The peak cross section at these resonance energies rises to 30,000 b; hence, self-shielding inside a thick detector will be an extremely large factor.

Cadmium covers of 20-mil thickness are chosen to separate the thermal activation from the resonance activation. The cadmium difference method will be used with manganese and sodium detectors, along with indium and gold detectors, to determine whether or not the neutron spectrum has a $1/E$ dependence. The manganese detectors are made up as foil discs in the same way as the gold and indium. It will be necessary only to determine the relative weights of these detectors, and this can be done by irradiating two discs in the same flux and counting the activation produced. The sodium detectors are pressed pellets of Na_2CO_3 , one inch in diameter and approximately $1/8$ in. thick. They are sprayed with Krylon to prevent flaking. Each pellet is weighed to determine the number of sodium atoms in the pellet.

3. Fast Flux

The fast flux is measured with a sulfur detector, based on the reaction $\text{S}^{32}(\text{n},\text{p})\text{P}^{32}$. The sulfur detectors are made up in pellet form, one inch in diameter and $1/4$ in. thick. Sulfur is easily pressed into pellets without the use of a binder. The pellets are sprayed with Krylon for protection. Precipitated sulfur, USP grade, is used.

A phosphorus pellet, made of red phosphorus and the same size as the sulfur pellet, is used for the calibration of the end-window Geiger-Mueller counter. The amorphous red phosphorus powder is mixed with a small amount of binder (~ 4 w/o polyvinyl alcohol powder) before being pressed. The pressed pellet is also sprayed with Krylon.

The thickness of the pellets is purposely made larger than the range of the P^{32} beta particle ($\beta = 1.70$ MeV; range, ~ 800 mg/cm²). This ensures that the active weight of the pellets that the counter "sees" is the same.

A standard gold foil (49.0 mg/cm², one square cm in area) is used to measure the thermal-neutron flux in which the phosphorus pellet is irradiated.

B. Calibration of the Counting Systems for Determining the Induced Activity of the Detectors

1. Geiger-Mueller End-window Counter

A Geiger-Mueller end-window counter (see Fig. 28.3) is used for counting the P^{32} beta particles. There are five shelf positions at varying distances from the tube. The shelf positions are calibrated

relative to adjacent shelves and then to a particular shelf which is absolutely calibrated. To make this absolute calibration, a phosphorus pellet is irradiated in a well-thermalized flux along with a standard gold foil. The gold foil is counted with a calibrated Geiger-Mueller counter which has been calibrated previously by use of a standard pile of known flux. From this measurement, the thermal flux in which the phosphorus was irradiated can be determined. The phosphorus pellet is then counted with the end-window counter. As previously stated, the range of the beta particle determines the active weight which is counted. The counting rate is given by

$$C_p K = \phi_{th} \sigma_p N_p V_p [1 - e^{-\lambda_{32} T_p}],$$

where

C_p = corrected counts per sec at end of irradiation;

λ_{32} = P^{32} decay constant (min^{-1});

T_p = irradiation time (min);

N_p = number of P^{31} atoms per cm^3 ;

σ_p = P^{31} activation cross section, cm^2 ;

ϕ_{th} = thermal flux, neutrons/ cm^2/sec ;

V_p = active volume of the pellet;

K = inverse of the counter efficiency.

Fig. 28.3
End-window
GM Counter

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The efficiency can be expressed as follows:

$$\frac{K}{(V\rho)_p} = \frac{\phi_{th}\sigma_p N_0 S_p [1 - e^{-\lambda_{32} T_p}]}{C_p A_p},$$

where

N_0 = Avogadro's number;

A_p = atomic weight of phosphorus;

ρ_p = density of phosphorus pellet;

$(V\rho)_p$ = active weight of P;

S_p = content of P^{32} in the pellet, w/o 100.

This is an accurate expression for the efficiency of both the phosphorus and sulfur pellets if V_p does not differ appreciably from V_s , the active volume of the sulfur pellet. If these volumes are quite different, the overall geometry factors of each pellet would differ also. Fortunately, this is not the case, for the densities of the two types of pellets are almost equivalent.

The activation for the sulfur pellet can be expressed as

$$KC_s = \phi_f \sigma_0 N_s V_s [1 - e^{-\lambda_{32} T_s}],$$

where the factors involved are the same as in the previous case, but with the proper subscripts, and

ϕ_f = the integrated flux above 2.9 MeV, and

σ_0 = microscopic cross section for $S^{32}(n, p)P^{32}$.

Rewriting,

$$\phi_f = \frac{KC_s A_s}{\sigma_0 \rho_s V_s N_0 k_{32} [1 - e^{-\lambda_{32} T_p}]}$$

with

k_{32} = isotopic content of S^{32} .

Since the half-life of P^{32} is 14.3 days, $\lambda_{32} T$ is small for a one-hour irradiation, so that we may set

$$1 - e^{-\lambda_{32}T} = \lambda_{32}T.$$

Substituting $K/(V\rho)_p$ for $K/(V\rho)_s$ and cancelling terms,

$$\phi_f = C_s \sigma_p A_s T_p \phi_{th} S_p / C_p \sigma_0 A_p T_s k_{32}.$$

To eliminate an S^{35} beta produced by thermal activation in the sulfur pellet, a thin, 40-mg/cm² absorber should be kept over the window when beta counting. This absorber must also be present for the counter calibration with the phosphorus pellets.

2. Scintillation Counter

A 2 x 2-in. NaI scintillation counter is used to count gamma rays. The efficiency of this system for counting gamma rays is much higher than that of a Geiger-Mueller counter. The counting system is calibrated for the gold and indium detectors. The method used to calibrate the counter consists of the following:

- a) Indium and gold foils are made of the same size as the foil discs, but much thicker (Au, ~55 mg/cm²; In, ~93 mg/cm²).
- b) These foils are irradiated in the standard pile where the thermal flux and the cadmium ratios are known.
- c) The activity of each foil is calculated, making the proper corrections for self-shielding and resonance activation.
- d) The foils are counted on the five shelves of the scintillation counter.
- e) The corrected counting rates are extrapolated back to the removal time. The efficiency of each shelf position is determined by the ratio of the activity to the counting rate at the time of removal.

The efficiency of the counter measured in this manner will be somewhat in error since the calibration foils are much thicker than the foil discs. This error should be small compared with the error involved in the standard pile flux (10 percent and possibly greater). The self-absorption of gamma rays in the thick foils will only be about one percent. This is the primary reason for counting gamma rays instead of beta particles. In order to calibrate a counter, a thick foil had to be used to obtain enough activation in the standard pile. If the beta particles were counted, the self-absorption loss would be as high as 20 percent for the gold foil used in the calibration.

The manganese foil discs and the sodium pellets do not require an absolute measurement of activity, since only their cadmium ratios are necessary for the experiment. The manganese will be counted with an end-window Geiger-Mueller counter, and the sodium with the NaI scintillation counter.

C. Irradiation of the Selected Detectors in the Core of the Argonaut Reactor

Ideally, all of the detectors should be irradiated at a certain position in the reactor. This would require numerous irradiation runs at a certain power level. To eliminate some of these runs, the detectors, in pairs, are irradiated at different locations along a vertical line of the core. As seen in Fig. 28.4, the irradiation position in the core is located in the center graphite wedge between fuel boxes Nos. 15 and 16.

Fig. 28.4

Aluminum Water-tight
Detector Tube with
Graphite Spacers

A long aluminum tube containing the activation detectors is inserted vertically into a hole which has been made in the wedge. The activation detectors are placed in the tube in the vertical arrangement shown in Fig. 28.4. Graphite cylinders of various heights are used as spacers to separate the different detector locations. The aluminum tube is water-tight, and the open top lies approximately 2 in. above the water reflector when inserted in the wedge.

Two separate runs are made, the first with the detectors covered with aluminum (bare), and the second with cadmium covers. An indium foil disc will be irradiated with the sulfur pellet in both irradiations. These detectors will monitor the flux to ensure that the power level is the same for both irradiations. The indium will also serve to compare the bare activations at positions (A), (B), and (C). From previous irradiations it has been found that the cadmium ratio is constant throughout the region where the detectors are placed. This indicates that the thermal-to-resonance flux ratio remains constant. The thermal flux at positions (A) and (C) should be approximately the same, since these locations are symmetrical about the horizontal centerline near the center of the core.

The irradiation time for each run will be one-half hour at 100 W ($\sim 1 \times 10^9$ thermal flux) for a one-slab loading. The reactor will be brought to the desired power level on a period less than 30 sec. This will eliminate any correction needed for the activation of detectors during startup.

During the first irradiation, a phosphorus pellet and standard gold foil will be placed in the J-10 Stringer, inside the external thermal column. This is the pellet that will be used to calibrate the end-window counter for P^{32} betas. The counter has also been standardized for the standard gold foil; hence, it can be used to determine the thermal flux in which the phosphorus is irradiated.

D. Determination of Induced Activity by Counting on the Calibrated Counters

After the detectors are removed from the reactor, they will be taken to the staff counting room in Building 24 to be counted.

NaI Scintillation Counter (see Fig. 28.5)

- (1) Gold foil discs
- (2) Indium foil discs
- (3) Sodium carbonate pellets

Fig. 28.5
Single-channel
Analyzer

The calibration of this system for gold and indium is listed in Table 28.1. A 444-mg/cm² aluminum absorber is placed on Shelf 1.

Geiger-Mueller End-window Counter

- (1) Sulfur pellets
- (2) Phosphorus pellets
- (3) Standard gold foil
- (4) Manganese foil discs

The standardization of the end-window Geiger-Mueller counter with standard gold foil (~50 mg/cm²) is given in Table 28.2. The listed values are for the standard foil holder in three shelf positions; no absorber was used.

Table 28.1

Gold Calibration		Indium Calibration	
H. V. 1050 Window Level 52.0 Integrate Coarse Gain #4 Amplifier Gain 1500		H. V. 1050 Window Level 56 Integrate Coarse Gain #2 Amplifier Gain 1500	
Shelf	dpm/cpm	Shelf	dpm/cpm
1	-	1	-
2	28.7	2	152
3	49.6	3	296
4	76.6	4	478
5	109.0	5	773

Table 28.2

AuS ⁺ d Calibration High Voltage 700 V Discriminator Setting 1.75 Resolving Time 6.8×10^{-6} min Foil Holder #2	
Shelf	dpm/cpm
1	6.43
2	11.7
3	21.1

E. Calculation of the Neutron Flux

In Table 28.3 are listed the constants of each detector material needed for the flux calculations. The thermal activation cross

sections given are those for a neutron velocity of 2200 m/sec. The activation reaction and the most prominent decay particles of the product are listed. The resonance activation integrals (taken from Reference 3) have a lower cutoff energy of 0.5 eV. Alpha, the ratio of the resonance component of the resonance activation to the $1/v$ component, is listed for those detectors used in the cadmium difference method.

Table 28.3

DETECTOR CONSTANT

Detector	Reaction	Thermal Cross Section, b	Activation Resonance Integral, b	Half Life	Decay Constant, min ⁻¹	α
⁷⁹ Au ¹⁹⁷	$n + \text{Au}^{197} \rightarrow \text{Au}^{198}$ β MeV = 0.960(98.6%) γ MeV = 0.412(100%)	98.8	1530	2.7 day	1.78×10^{-4}	35.4
⁴⁹ In ¹¹⁵	$n + \text{In}^{115} \rightarrow \text{In}^{116m}$ β MeV = 1.00(51%) 0.87(28%) 0.60(21%) γ MeV = 1.09(54%) 1.27(75%) 0.41, 1.49, 2.09 (25% each)	155	2554	54.2 min	1.28×10^{-2}	37.0
Mn	$n + \text{Mn}^{55} \rightarrow \text{Mn}^{56}$ β MeV = 2.81(50%) 1.04(30%) 0.65(20%) γ MeV = 0.845(100%) 1.81(30%) 2.13(20%)	13.2	13.8	2.58 hr	4.48×10^{-3}	1.47
Na	$n + \text{Na}^{23} \rightarrow \text{Na}^{24}$ β MeV = 1.34(100%) γ MeV = 1.38(100%) 2.76(100%)	0.53	0.29	15.0 hr	7.70×10^{-4}	0.316
P	$n + \text{P}^{31} \rightarrow \text{P}^{32}$ β MeV = 1.70(100%)	0.23		14.3 day	3.37×10^{-5}	
S	$\text{S}^{32} + n \rightarrow \text{P}^{32} + p^+$	Reaction Cross Section 0.30	Effective Cutoff Energy 2.9 MeV	14.3 day	3.37×10^{-5}	

The counting rates obtained in Section D for each detector should first be extrapolated back to the time at which the detector was removed from the reactor. This counting rate divided by the detector weight will be denoted by c_0 .

1. Thermal Flux

From the activities of irradiated identical bare and cadmium-covered detectors, the thermal activation can be determined:

$$(c_0)_{th} = (c_0)_{bare} - x(c_0)_{Cd},$$

where x is the correction factor dealing with that portion of the resonance activation lost when the cadmium-covered detector is irradiated. For all the detectors except indium, x equals one; for indium, it is slightly greater than one (1.035). The thermal activities are determined for the gold and indium detectors, since the counter efficiency is known for these materials:

$$A_{th} = Kc_0W_b = \sigma_p n_0 v_p N_T [1 - e^{-\lambda T}],$$

where

v_p = most probable neutron velocity;

W_b = weight;

n_0 = neutron density;

N_T = total number of detector atoms.

The thermal flux, that is the total neutron density times the velocity of 2200 m/sec, is determined from this fundamental relationship. This quantity should be transformed to the more correct representation of the thermal-neutron flux as a monoenergetic neutron (neutron density $\times$ the average velocity). A temperature correction, assuming the neutron temperature is that of the moderator, should also be made.

2. Resonance Flux

The cadmium difference method for the determination of the $1/E$ flux dependence requires the measurement of the cadmium ratio of the gold, indium, manganese, and sodium detectors. The cadmium ratio is expressed as

$$R_{Cd} - 1 = \frac{(c_0)_{th}}{(c_0)_{Cd}}.$$

When the value of $(R_{Cd} - 1)$ for each detector is substituted into the equation

$$\frac{\phi_r}{\phi_{th}} = \frac{2.37}{(RCd-1)(1+\alpha)},$$

the ratio of ϕ_r/ϕ_{th} should be approximately the same. If this is found to be true, the differential flux spectrum within the energy response region of the detectors will be given by

$$\phi(E) = \phi_r/E.$$

The calculation of the flux per $\ln e$ interval (ϕ_r) can be made with each of the detector materials since the thermal flux is known.

Separate calculations to measure the slowing-down density with the gold and indium detectors will also be made. The saturated activity of the detector can be expressed as

$$A_\infty = \frac{Kc_0 W_{Cd}}{1 - e^{-\lambda T}} = \frac{q(E_n)}{\xi \Sigma_s} N_T \int_{0.5}^{\infty} \sigma \frac{dE}{E},$$

where

W_{Cd} = weight of cadmium-covered detector;

K = inverse counter efficiency;

E_n = resonance peak energy.

The slowing-down density at the detector resonance energy will be determined from this expression. The slowing-down power in the Argonaut reactor at the resonance energy of the detectors can easily be determined:

$$(\Sigma_s \xi)_{core} = (\Sigma_s \xi)_{H_2O} + (\Sigma_s \xi)_{graphite}.$$

The molecular and atomic densities of H_2O and graphite are taken from Reference 4, p. 9, and the scattering cross sections from BNL-325:

$$\Sigma_s \xi = 0.01945(43.7)(0.92) + 0.016944(4.8)(0.158) = 0.795 \text{ cm}^{-1}.$$

3. Fast Flux

The sulfur threshold detector is used to measure the integrated fast flux above 2.9 MeV. In Part B, the equation used to determine the fast flux is derived in conjunction with a phosphorus-pellet calibration of the Geiger-Mueller counter:

$$\phi_F = \frac{C_s \sigma_p A_s T_p S_p}{C_p \sigma_s A_p T_s k_{32}} \phi_{th}$$

The thermal flux ϕ_{th} in which the phosphorus pellet was irradiated is measured with a standard gold foil. The equation used for this calculation is

$$A = K c_0 W_{STD} = \sigma_p \phi_{th} N_T (1 - e^{-\lambda T}),$$

where

K = inverse efficiency of the standard end-window counter;

W_{STD} = weight of standard foil;

σ_p = activation cross section for gold;

N_T = total number of gold atoms.

V. RESULTS

The results of the experiment, calculated from the previously derived equations, consist of the following:

- A. Thermal-neutron flux (neutron density $\times 2200 \text{ m/c}$).
- B. Resonance neutron flux per $\ln e$ energy interval.
- C. Ratio of ϕ_r/ϕ_{th} for Au, In, Mn, and Na.
- D. Integrated fast neutron flux above 2.9 MeV.

A table should be made up containing the result obtained with each detector in its respective neutron-flux region.

VI. ANALYSIS AND CONCLUSIONS

Since a number of detectors were used to measure both the thermal and the resonance flux, a comparison of their results can be made. Measurement of the resonance flux is dependent upon a $1/E$ neutron spectrum being present in the reactor core. The cadmium difference method was used to determine if this dependency exists. What explanation can be given if it is found that a $1/E$ spectrum is not present?

Many errors can arise in the procedure used for this experiment. The measured weight of the foil disc detectors could involve a large error. The technique used to calibrate the counters for measuring the detector activity could be refined to give more accurate results. Careful analysis of the results would, of course, be a lengthy task in itself; it is suggested that each student try to examine the more prominent errors that could have arisen and why there are any deviations from the expected theoretical results.

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VIII. APPENDICES

A. Discussion Questions

The following is a list of topics and questions which will be discussed prior to the performance of the experiment. Students are requested to go over the available material and be prepared to join in the discussion.

1. Consider the relationship between neutron flux, activation rate, and detector activity.
 - a. What is the fundamental equation describing the activation rate of a detector?
 - b. After a detector has been activated in a reactor for time t , what is the simple relationship describing its activity?
 - c. What is the saturated activity?
2. Consider the determination of detector activity by counting methods.
 - a. If the activity of a detector is measured during an interval between t_2 and t_3 , what will the expression for the saturated activity be? Assume that the counting system has an efficiency of one. What assumptions can be made about this equation?
 - b. What methods can be used to calibrate a counting system to be used for determining the detector's activity?
3. Measurement of the thermal flux is performed by using a $1/v$ detector and a cadmium-ratio measurement to eliminate any activation occurring above the thermal region.
 - a. What is meant by a $1/v$ detector? Why is it advantageous to use such a detector to measure the thermal flux? What is actually measured by the detector?
 - b. What is meant by hardening of the flux spectrum in the detector in question? How can this be eliminated?
 - c. What is the energy distribution of neutrons in a weakly absorbing medium where the neutrons are in thermal equilibrium with the moderating atoms?
 - d. What is the most probable velocity of the neutrons in the Maxwell-Boltzmann distributions? The average velocity?

- e. How can the thermal flux be represented as a monoenergetic flux?
 - f. How does the temperature of the moderator compare with that of the neutrons? If the neutron temperature is assumed equal to the moderator temperature, what correction must be made to the monoenergetic flux representation?
 - g. Since the $1/v$ detector is placed in the core of the reactor, neutrons of higher energies than the thermal neutrons are also present. How can these neutrons be eliminated? What factors are involved in this method?
 - h. From a cadmium-ratio measurement it is possible to determine the activation in the detector due to neutrons below some effective cut-off energy. Under what circumstances does this represent only the thermal activation?
4. Measurement of the neutron flux in the slowing-down region necessitates the use of resonance activation detectors. The thermal activation is eliminated by a cadmium-ratio measurement.
- a. What type of an energy distribution will the neutron flux per unit energy exhibit in this region? What conditions must prevail?
 - b. What is the slowing-down density? Is it constant in this region? If absorption is present what factor is involved?
 - c. Where does the $1/E$ neutron spectrum join the thermal-neutron distribution?
 - d. What is the activation resonance integral? What two components make it up? By what methods can it be determined? Will the integral be dependent upon the cadmium-cover thickness which is needed to eliminate thermal activation?
 - e. Explain self-shielding and flux depression in detector foils. How will a resonance detector be effected by these factors?
 - f. What is the cadmium difference method for determining a $1/E$ neutron flux dependence?
5. Measurement of the integrated fast neutron flux with threshold detectors.
- a. How are the fast neutrons produced in a reactor? What type of an energy spectrum will fast neutrons have in a U^{235} reactor?
 - b. What is the energy dependence of a threshold reaction initiated by neutrons? Are these reactions exothermic or endothermic? List three threshold reactions which can be initiated by fast neutrons.

- c. Determine the threshold energy by mass considerations for the reaction $S^{32}(n,p)P^{32}$.
- d. At the threshold energy what is the kinetic energy of the products? If the energy of the neutrons is greater than E_T , what happens to the excess energy?
- e. When one of the products is an alpha particle or proton, the electrostatic potential barrier of a compound nucleus must be considered. What effect does this have on the reaction cross section? How does the energy of the neutron above E_T affect the penetrability?
- f. If the cross section for a threshold reaction is known, explain how the integrated fast flux above some effective E can be measured. What assumptions must be made?

B. Measurements of Neutron Flux by Induced Activation

Detector Activity

The rate at which a detector will be activated when placed in a neutron flux ϕ can be expressed as

$$dr = N(x,y,z)\sigma_{\text{act}}(E)\phi(E,x,y,z) dE dV,$$

where

$N(x,y,z)$ = target atoms/unit volume at (x,y,z) ;

$\sigma_{\text{act}}(E)$ = microscopic activation cross section;

$\phi(E,x,y,z)$ = neutron flux/unit energy at (x,y,z) .

If the neutron flux is assumed to be constant within the detector and N_T is equal to the total number of detector atoms, the total activation rate is

$$r = N_T \int_0^{\infty} \sigma(E)\phi(E) dE \quad (1)$$

or

$$R = \frac{r}{N_T} = \int_0^{\infty} \sigma(E)v n(E) dE,$$

where

R = activation rate/target atom;

$n(E)$ = neutron density/unit energy;

v = neutron velocity.

The activation rate will remain constant with time if the neutron flux remains constant and the number of target atoms destroyed by activation is very small compared with the total number of target atoms.

The activity of the detector at any time during its irradiation is determined from the following:

$$\text{Rate of change of radioactive atoms} = \text{rate of production} - \text{rate of decay}$$

or

$$\frac{dN}{dt} = r - \lambda N,$$

where

N = number of radioactive atoms at time t , and

λ = decay constant for N .

Solution of this differential equation with $N = 0$ at time $t = 0$, gives

$$N = \frac{r}{\lambda} (1 - e^{-\lambda t}).$$

Therefore, the activity of the detector at time t is

$$N\lambda = r(1 - e^{-\lambda t}). \quad (2)$$

At saturation, where $dN/dt = 0$, we obtain $\lambda t \approx \infty$. Then

$$N_{\infty}\lambda = r = N_T \int_0^{\infty} \sigma(E) \phi(E) dE.$$

The number of radioactive atoms and the activity of these atoms can easily be determined at the time the detector ceases to be irradiated. Following the irradiation, the number of radioactive atoms and the activity of these atoms will decrease according to the well-known decay law

$$N = N_0 e^{-\lambda t},$$

or

$$N\lambda = N_0 \lambda e^{-\lambda t},$$

where

N_0 = number of radioactive atoms at $t = 0$ (end of irradiation);

t = time after the irradiation,

λ = decay constant.

If a measurement of this activity is taken between t_2 and t_3 , the total number of counts will be

$$kC_T = \int_{t_2}^{t_3} N\lambda dt = \int_{t_2}^{t_3} N_0\lambda e^{-\lambda t} dt$$

where

k = inverse efficiency of the counter.

Then

$$kC_T = N_0(e^{-\lambda t_2} - e^{-\lambda t_3})$$

or

$$kC_T = N_0 e^{-\lambda t_2} [1 - e^{-\lambda(t_3 - t_2)}]$$

From this measurement, N_0 can be determined:

$$\lambda N_0 = \frac{kC_T \lambda}{e^{-\lambda t_2}(1 - e^{-\lambda t_c})}, \quad (3)$$

where $t_3 - t_2 = t_c$.

If $t_c \ll t_{1/2}$, then $\lambda t_c \ll 0.693$ and

$$1 - e^{-\lambda t_c} \cong \lambda t_c.$$

Accordingly,

$$\lambda N_0 = kC_T/t_c e^{-\lambda t_2}$$

or, for a more exact relationship,

$$\lambda N_0 = kC_T/t_c e^{-\lambda t'_2}$$

with $t'_2 = t_2 + \frac{1}{2}t_c$. Relating this activity measurement to the neutron flux by Eqs. (1) and (2):

$$\frac{N_0 \lambda}{1 - e^{-\lambda T}} = r = N_T \int_0^\infty \sigma(E) \phi(E) dE \quad (4)$$

with T the irradiation time.

Neutron-flux Measurements

The neutrons in a thermal nuclear reactor are distributed over a large energy range. Neutrons are continuously being created by the fission process (for U^{235}) at energies ranging from 0.05 to 15 MeV. These neutrons are quickly slowed down by elastic collisions with a moderating material. The slowing-down process continues until the neutrons are in thermal equilibrium with the molecules of the medium. This is the thermal-neutron region, in which the net energy exchange between the medium and the neutrons is zero.

The neutron spectrum can be divided in three regions (see Fig. 28.6). In each region certain detector materials have a desired activation response which can lead to a measurement of the flux.

Fig. 28.6
Neutron Flux
Spectrum

Thermal Region

In the thermal region of the neutron-energy spectrum, the kinetic energies of the neutrons will be distributed statistically according to the Maxwell-Boltzmann distribution law. This distribution, derived from the kinetic theory of gases, is applicable to the neutron gas only if the medium in which the neutrons are present is weakly absorbing. The velocity distribution is

$$dn = n(v) dv = \frac{4\pi n_0}{(2\pi kT/m)^{3/2}} v^2 \exp\left(-\frac{1/2mv^2}{kT}\right) dv, \quad (5)$$

where

n_0 = total neutron density;

k = Boltzmann constant;

T = neutron temperature (absolute).

With $E = 1/2 mv^2$, the energy distribution can be written as

$$dn = n(E)dE = \frac{2\pi n_0}{(\pi kT)^{3/2}} E^{1/2} \exp\left(-\frac{E}{kT}\right) dE. \quad (5a)$$

The most probable velocity, that is, where $n(v)$ is a maximum, is found by differentiating the neutron velocity distribution and setting the result equal to zero:

$$\begin{aligned} v_p &= (2kT/m)^{1/2}; & E_p &= kT \\ &= (1.648 \times 10^8 T)^{1/2}. \end{aligned} \quad (6)$$

At room temperature (20°C), v_p 2200 m/sec, corresponding to an energy of 0.25 eV. The average velocity of the neutrons is obtained from the expression

$$\begin{aligned} \bar{v} &= \frac{\int_0^\infty n(v)v \, dv}{\int_0^\infty n(v) \, dv} \\ &= \frac{2}{\sqrt{\pi}} v_p. \end{aligned} \quad (7)$$

The type of detector used in the thermal region normally exhibits an activation cross section which varies as $1/v$. The activation rate of a $1/v$ detector is independent of the velocity of the neutrons:

$$\begin{aligned} r &= N_T \int_0^\infty \sigma(E)\phi(E) \, dE \\ &= N_T \int_0^\infty \sigma(E)n(E)v \, dE \\ &= kN_T \int_0^\infty n(E) \, dE. \end{aligned}$$

with $\sigma = k/v$, we obtain

$$r = kN_T n_0. \quad (8)$$

It is obvious that the activation rate is a measurement of neutron density, not neutron flux. Hence, a change in neutron velocity (that is, of temperature) would not be detected because the change in flux would be cancelled by the change in cross section. To determine the neutron flux from the measured activation rate it is customary to use the cross section at the most probable velocity, σ_p , so that

$$\frac{r}{N_T \sigma_P} = v_P n_0. \quad (9)$$

This is why the thermal cross sections tables are those for 2200 m/sec. For materials that do not follow the $1/v$ law in the thermal region, a correction can be calculated to determine its equivalent $1/v$ cross section. A ratio of the actual activation to that of a $1/v$ material of the same value for the 2200-m/sec cross section can be calculated from the known cross-section curve of the material:

$$F = \frac{\int n(v) v \sigma \, dv}{\int n(v) v (k/v) \, dv} = \frac{1}{n_0} \int \frac{n(v) \sigma}{k/v} \, dv,$$

The quantity F will vary slowly with the temperature of the neutron distribution.

A more strict representation of the neutron flux would be the use of the average velocity times the neutron density (number of neutrons crossing unit area per sec). This is determined by dividing the reaction rate by the cross section at the average velocity of the Maxwellian neutron distribution:

$$r/N_T \sigma_0 = \bar{v} n_0. \quad (10)$$

The flux is now the neutron density multiplied by the average velocity of the Maxwellian neutron distribution. This is still not an absolute determination of the neutron flux since the temperature of the neutron was not taken in account. The assumption was made that the Maxwellian neutron distribution is that corresponding to a temperature of 20°C. If it is assumed that the neutron temperature is the same as that of the moderator, $T_N = T_M$, the average velocity of the Maxwellian distribution at this temperature can easily be evaluated:

$$\begin{aligned} \bar{v} &= \frac{2}{(\pi)^{1/2}} v_P = \frac{2}{(\pi)^{1/2}} \left(\frac{2kT_N}{m} \right)^{1/2} \\ &= \left(\frac{8kT_N}{\pi m} \right)^{1/2} \end{aligned}$$

Accordingly,

$$\frac{\bar{v}_{293^\circ}}{\bar{v}_{T_M}} = \left(\frac{293}{T_N} \right)^{1/2} \quad (11)$$

It must be noted that the energy distribution of neutron is not exactly Maxwellian insofar as it corresponds to a neutron temperature which is slightly higher than that of the moderator. This shift in the Maxwellian peak is caused by the preferential absorption of neutrons of lower energy (spectrum hardening) and by the steady addition of neutrons at higher energies which have been slowed down from their fission energies.

Slowing-down Region

Fast neutrons are slowed down from their fission energies by elastic collisions with the atoms of the moderator. If it is assumed that Q fast neutrons are being created per cm^3 per sec in a large volume of moderator, the number of neutrons per cm^3 slowing down past E per sec can be expressed as

$$\begin{aligned} q(E) &= \frac{N(E)\Delta E}{\Delta t} \\ &= N(E)(\Delta E)v\Sigma_s \end{aligned}$$

where $N(E)$ is the neutron density per unit energy at E and Δt the time it takes the neutrons to pass through ΔE . Here,

$$\begin{aligned} \Delta t &= \frac{\lambda_s}{v} \\ &= \frac{\text{scattering mean free path}}{\text{velocity corresponding to } E} \end{aligned}$$

and

$$\lambda_s = 1/\Sigma_s.$$

If ΔE is small,

$$\frac{\Delta E}{E} = \Delta \ln E = \xi$$

where ξ is the average logarithmic energy decrement per collision. Then

$$q(E) = N(E)v\xi E\Sigma_s$$

In the absence of absorption, $q(E) = Q$. Rearrangement gives

$$N(E)v\Sigma_s dE = (Q/E\xi) dE,$$

that is,

number of scattering collision = influx gain,

and

$$\phi(E) dE = \frac{Q dE}{\sum_s \xi_s E} \quad (12)$$

Hence, neglecting neutron absorption and leakage, the neutron flux in the slowing-down region is proportional to $1/E$. The energy distribution will actually be a sum over many $1/E$ distributions, each one extending up to a particular energy in the fission spectrum.

The activation rate of a material is (see above)

$$r = N_T \int_0^{\infty} \sigma(E) \phi(E) dE$$

which we can now write as

$$r = \frac{QN_T}{\sum_s \xi_s} \int_{E_1}^{E_2} \sigma(E) \frac{dE}{E}, \quad (13)$$

where E_1 to E_2 represents the slowing-down region. The assumption is made that the scattering cross section remains constant with energy. The upper limit of the resonance activation integral, E_2 , can be considered equal to infinity since all the activation in the resonance detectors will take place at the lower energies of the $1/E$ spectrum. The constant in front of the activation integral is the neutron flux in any ($\ln e$) energy interval (1 to 2.72 eV, 10 to 27.2 eV, etc.).

A detector placed in the core of a reactor is subject to activation from the whole neutron spectrum. To measure the ratio of the thermal response of a detector to its resonance response, a cadmium ratio must be made. The cadmium ratio is defined as

$$R_{Cd} = \frac{\text{thermal response} + \text{resonance response}}{\text{resonance response}}$$

If the detector sensitivity to the neutron flux is known, the response of the detector will be a measure of the neutron flux:

$$R_{Cd} = \frac{\int_0^{\infty} \phi(E) \sigma(E) dE}{\int_{E_1}^{\infty} \phi(E) \sigma(E) dE} = \frac{\text{Bare-detector activation}}{\text{Cadmium-covered activation}}$$

$$R_{Cd} = \frac{\int_0^{E_1} \phi(E) \sigma(E) dE + \int_{E_1}^{\infty} \phi(E) \sigma(E) dE}{\int_{E_1}^{\infty} \phi(E) \sigma(E) dE}$$

or

$$R_{Cd}^{-1} = \frac{\int_0^{E_M} n(E) v \sigma dE + \int_{E_M}^{E_1} \phi(E) \sigma dE}{\frac{Q}{\Sigma_s \xi} \int_{E_1}^{\infty} \frac{\sigma dE}{E}} \quad (14)$$

The first term in the numerator is the activation caused by thermal neutrons in the Maxwellian distribution. The second term is the activation caused by the $1/E$ spectrum from E_M , the energy at which the $1/E$ spectrum joins the Maxwellian distribution, to some cadmium cutoff energy E_1 . Actually, the true distribution of the flux near E_M will be quite difficult to describe since it is a transition region. The thermal cutoff energy (E_M) is normally defined by the condition that at E_M the slowing-down density equals the absorption under the Maxwell distribution at lower energies:

$$\int_0^{E_M} \phi(E) \Sigma(E) dE = q = \phi(E_M) \xi \Sigma_s E_M.$$

This can be written as

$$\bar{\Sigma}_a \int_0^{E_M} \phi(E) dE = \xi \Sigma_s E_M \phi(E_M)$$

with

$$\phi(E) = vn(E);$$

$$\phi(E_M) = vn(E_M)$$

and $n(E)$ the Maxwellian distribution. The flux per unit energy, $\phi(E)$, in the $1/E$ spectrum and the Maxwellian distribution are, of course, equal at E_M . By substituting the Maxwell neutron distribution for $\phi(E)$ and integrating, it can be shown that

$$\frac{\Sigma_a}{\Sigma_s \xi} = \left[\frac{E_M}{kT_N} \right]^2 e^{-E_M/kT_N},$$

where T_N is the neutron temperature. Use of the constants for the Argonaut reactor similar to those found in Reference 4, page 11, for a one-slab loading:

$$\Sigma_a \cong 0.0722 \text{ cm}^{-1};$$

$$\xi \Sigma_s = 1.06 \text{ cm}^{-1};$$

$$kT_N = 0.0256 \text{ eV}, \quad (T_N = 25^\circ\text{C})$$

gives a value of 0.205 eV for E_M .

If the crude assumption is made that the neutron flux has a $1/E$ distribution from E_M to E_1 , a calculation can be made to determine whether or not activation in this region is appreciable for a $1/v$ detector.

$$\begin{aligned} N_T \int_{E_M}^{E_1} \phi(E) \sigma(E) dE &= \frac{N_T Q}{\Sigma_s \xi} K \int_{E_M}^{E_1} \frac{dE}{E^{3/2}} \\ &= N_T \frac{Q}{\Sigma_s \xi} 2K \left[\frac{1}{\sqrt{E_M}} - \frac{1}{\sqrt{E_1}} \right]. \end{aligned}$$

With $E_M = 0.205 \text{ eV}$, $E_1 = 0.500 \text{ eV}$, and $K = \sigma_{th} \sqrt{E_{th}} = \sigma_{th} \sqrt{0.025}$,

$$\text{Activation rate from } E_M \text{ to } E_1 = \frac{Q}{\Sigma_s \xi} N_T \sigma_{th}(0.25).$$

The ratio of thermal activation to the calculated activation is

$$\frac{vn_0 \sigma_{th}}{\frac{Q}{\Sigma_s \xi} (0.25 \sigma_{th})} = \frac{\phi_{th} \sigma_{th}}{\phi_r (0.25 \sigma_{th})}.$$

Since ϕ_{th} is always much larger than ϕ_r [neutron flux per $(\ln e)$ interval], it can be assumed in most cases that the resonance activation in this region is negligible compared with the thermal activation.

The lower limit of the resonance activation integral will be an effective cutoff energy depending on the thickness of the cadmium cover and if the detector is placed in an isotropic or beam neutron flux.

A cadmium thickness of 20 mils corresponds to an 0.5-eV effective cutoff energy in an isotropic flux. Some neutrons above 0.5 eV will be absorbed and others below 0.5 eV will be transmitted through the cadmium. The neutron flux that would pass through the cadmium cover would have a shape similar to that in Fig. 28.7.

Fig. 28.7

Cadmium Cutoff Curve

Even if the shape of $\phi(E)$ is not known inside the foil cover, it is still possible to determine (providing the cadmium cover eliminates all thermal activation) the effective cutoff energy. By using a thin, $1/v$, cadmium-covered detector, the activation rate below the effective cutoff energy can be made equal to the activation rate lost above the effective cutoff energy (E_1):

$$\int_0^{E_1} \phi(E) \sigma(E) dE = \int_{E_1}^{\infty} \phi^*(E) \sigma(E) dE - \int_{E_1}^{\infty} \phi(E) \sigma(E) dE$$

where

$\phi(E)$ = flux/unit energy inside cadmium cover;

$\phi^*(E)$ = flux/unit energy outside the cover which is a $1/E$ spectrum;

$\sigma(E) \sim 1/v = K/\sqrt{E}$.

Then,

$$\begin{aligned} \text{measured activation rate} &= \frac{QN_T}{\Sigma_s \xi} \int_{E_1}^{\infty} \sigma \frac{dE}{E} \\ \text{(in the core of a reactor)} &= \frac{QN_T}{\Sigma_s \xi} K \int_{E_1}^{\infty} \frac{dE}{E^{3/2}} \\ &= \frac{QN_T}{\Sigma_s \xi} K 2 \left(\frac{1}{\sqrt{E_1}} \right) \end{aligned}$$

If the neutron flux per ($\ln e$) interval ($Q/\Sigma_s \xi$) and the thermal cross section of the detector are known, E_1 can be evaluated from a single irradiation in the core of the reactor. The value of the effective cadmium cutoff energy for a certain thickness of cadmium should hold for any detector whose $\sigma(E)$ exhibits a $1/v$ dependence in the cutoff energy region. If a detector has a resonance activation peak near this cutoff energy, the error in a cadmium-ratio measurement would be quite large. An approximate

correction for the loss of resonance activation (extra gain in thermal activation) in the cadmium cover could be determined by irradiating the detector with different thicknesses of cadmium. A plot of activation versus cadmium-cover thickness followed by a linear extrapolation back to zero cadmium thickness yields the approximate correction. The ratio of A_0/A_{20} is the correction factor for a 20-mil cadmium cover. This factor when multiplied by the resonance activation gives the approximate total resonance response. This quantity subtracted from the bare-detector activation gives the true thermal response.

In considering the resonance response of the detector, the activation resonance integral can be divided in two separate parts: the $1/v$ component and the resonance component. Thus,

$$\int_{E_1}^{\infty} \sigma(E) \frac{dE}{E} = \int_{E_1}^{\infty} \sigma_{1/v} \frac{dE}{E} + \int_{E_1}^{\infty} \sigma_r \frac{dE}{E}$$

The $1/v$ component is easily related to the thermal cross section:

$$\frac{\sigma_{th}}{\int_{E_1}^{\infty} \sigma_{1/v} \frac{dE}{E}} = \frac{K 1/(E_{th})^{1/2}}{K \int_{0.5}^{\infty} \frac{dE}{E^{3/2}}} = \frac{1/\sqrt{0.025}}{2 \left[\frac{1}{\sqrt{0.5}} \right]} = \frac{1}{0.448} = 2.23$$

$$\int_{E_1}^{\infty} \sigma_{1/v} \frac{dE}{E} = 0.45 \sigma_{th}$$

As previously calculated, the resonance activation lost for a $1/v$ detector covered with 20-mil cadmium is

$$\int_{E_M}^{E_1} \sigma(E) \frac{dE}{E} = 0.25 \sigma_{th}$$

Therefore, when considering a $1/v$ detector which has no resonance activation peaks, approximately one-third of the resonance activation is lost in the cadmium cover. This loss need not be considered when making a cadmium-ratio measurement since the thermal activation is so much greater than the resonance activation.

The resonance component of the activation integral is due to the resonance peaks of the detector. The type of resonance detector can be of two varieties: (1) one with many resonance peaks which give an accumulative effect to the total resonance integral; (2) one with a single resonance

peak attributing to the resonance integral. Only the latter type shall be considered here. The resonance integral for this type can be calculated from the Breit-Wigner formula.

In most cases, the resonance integrals have been found by direct measurement using a material with a well-known $\sigma(E)$ as a standard detector. The values of the resonance activation integrals will be taken from Reference 3, where gold was taken as the standard. The major resonance at 5 eV was measured with a crystal spectrometer, and the small contribution from the next six resonance were taken from neutron time-of-flight spectrometer studies. The cadmium ratio of gold was compared with other resonance detectors, and from this comparison the relative ratio of the resonance integral was determined:

$$\frac{[R_{Cd-1}]_{Au}}{[R_{Cd-1}]_x} = \frac{\phi_{th} \sigma_{thAu} / \phi_r \int_{Au} \sigma \frac{dE}{E}}{\phi_{th} \sigma_{thx} / \phi_r \int_x \sigma \frac{dE}{E}}$$

or

$$\int_x \sigma \frac{dE}{E} = \frac{[R_{Cd-1}]_{Au}}{[R_{Cd-1}]_x} \frac{\sigma_{thx}}{\sigma_{thAu}} \int_{Au} \sigma \frac{dE}{E}.$$

The cadmium cutoff is chosen so that the $1/v$ component of the resonance integral is equal to $0.44 \sigma_{th}$ or $E_1 \approx 0.5$ eV in an isotropic flux.

The standard detector, gold, has a resonance activation integral of 1558 b, of which the resonance component attributes 1513 and the $1/v$ component 45 b. The thermal cross section for gold was taken as 98 b for the measurements.

If it is desired to obtain the slowing-down density at a certain energy, resonance detectors of the type just discussed could be used. The saturated activity of the detector (gold or indium) would be expressed as

$$\begin{aligned} A_\infty &= N_T \int_{E_1}^{\infty} \sigma(E) \phi(E) dE \\ &= \frac{qN_T}{\xi \Sigma_s} \int_{E_1}^{\infty} \sigma(E) \frac{dE}{E}. \end{aligned}$$

Although the activation is noted as an integral effect, almost all the activation will take place in a small energy region around the resonance peak of the detector.

It must be remembered that there are many assumptions made in a measurement of this kind: (1) The medium in which the detector is placed has negligible absorption in the slowing-down region. This will depend on the resonance escape probability of the system. In the Argonaut reactor, p is between 0.95 and 0.97, values obtained both by direct measurement and calculations. The loss of neutrons by absorption while slowing down is attributed mostly to capture in U^{238} . Since the Argonaut is a heterogeneous system, the slowing-down neutron distribution in the fuel elements shall be depleted at the absorption resonance peaks of U^{238} (its first resonance peak lies at 6.7 eV and contribution over half the total resonance absorption integral). As the neutrons move away from the fuel plates into the H_2O moderator, the depleted portion of the neutron slowing-down spectrum becomes less apparent since neutrons of higher energies are slowed down into this region. Hence, inside the moderator the resonance absorption effect should be smeared out with a net loss of neutrons below the energy at which the U^{238} absorption resonance peaks are located. Since this loss is small in the Argonaut (<5%) it can be neglected. (2) Because our system is of finite size, leakage occurs as the neutrons are being slowed down. Neglect of this leakage could lead to a large error. In a light-water-moderated reactor, the fast neutron leakage is the predominant leakage factor. Thermal leakage is small since the diffusion length (L) is very small. The so-called age (τ) in a light water system is small also, but it definitely makes up most of the migration length of the neutrons. The fast leakage, therefore, is expected to affect strongly the slowing-down density in the assumed $1/E$ spectrum. Since the first flight of the fission neutrons makes up a large portion of the age (fission to thermal), most of the leakage occurs for neutrons of high energy. This is true because the mean free path of fast neutrons in H_2O is long compared with that for lower-energy neutrons. This indicates that the assumption that the scattering cross section for the system is constant with energy is incorrect. The scattering cross section for H actually varies quite sharply at high energies, increasing down to 10,000 eV. The cross section then levels off until the thermal region is reached; it then rises sharply due to the chemical binding effect of the water molecules.

Each assumption that has been made to describe the slowing-down region does lead to an error which would be difficult to determine in most cases.

Returning again to the resonance-type detectors, the activation equation should be stated as

$$A_{\infty} = N_T \int_{E_1}^{\infty} \sigma(E) \frac{q(E)}{\xi \Sigma_s(E)} \frac{dE}{E}$$

where $q(E)$ and Σ_s are functions of energy. To determine $q(E)$ at a certain energy it would be necessary to have a detector which would only absorb neutrons over a small energy interval. This would allow the assumption that $q(E)$ and $\Sigma_s(E)$ are constant over this small energy region. The resonance detectors (gold and indium) to a first approximation do possess this quality. The contributions other than the main resonance peak can normally be assumed small. If these assumptions hold, the measurement of ϕ_r will determine the flux at the resonance energy. The value of ϕ_r/E_r will be the flux per unit energy at E_r . If the flux spectrum does exhibit a $1/E$ dependence over a large portion of the resonance region, the differential flux can be stated as ϕ_r/E .

The cadmium difference method can be used to determine whether or not a $1/E$ neutron slowing-down spectrum is present. This method requires the use of a number of resonance detectors which have different responses over the resonance region. The cadmium ratio of a resonance detector has been defined as

$$R_{Cd} - 1 = \frac{\int_0^{E_M} \phi(E) \sigma dE + \int_{E_M}^{E_1} \phi(E) \sigma dE}{\frac{Q}{\Sigma_s \xi} \int_{E_1}^{\infty} \sigma \frac{dE}{E}}$$

The resonance activation can be divided as

$$\begin{aligned} \phi_r \int_{E_1}^{\infty} \sigma \frac{dE}{E} &= \phi_r \left[\int_{E_1}^{\infty} \sigma_{1/v} \frac{dE}{E} + \int_{E_1}^{\infty} \sigma_r \frac{dE}{E} \right] \\ &= \phi_r (1 + \alpha) \int_{E_1}^{\infty} \sigma_{1/v} \frac{dE}{E} \end{aligned}$$

where

$$\alpha = \frac{\int \sigma_r \frac{dE}{E}}{\int \sigma_{1/v} \frac{dE}{E}}$$

For a 0.20-mil cadmium cover, $E_1 = 0.5$ eV, so that

$$\int_{0.5}^{\infty} \sigma_{1/v} \frac{dE}{E} = 0.45 \sigma_{th}$$

and, as previously calculated,

$$\int_{E_M}^{E_1} \phi(E) \sigma dE \approx 0.25 \sigma_{th} \phi_r.$$

Hence,

$$R_{Cd} - 1 = \frac{\phi_{th} \sigma_{th} + 0.25 \phi_r \sigma_{th}}{0.45 \sigma_{th} \phi_r (1 + \alpha)}.$$

Assuming $\phi_{th} \gg \phi_r$, we find

$$\frac{\phi_r}{\phi_{th}} = \frac{2.22}{(R_{Cd} - 1)(1 + \alpha)}. \quad (15)$$

Cadmium ratio measurements are made with a number of resonance detectors at a certain position in the reactor core. If α is known for each detector and the slowing-down spectrum is of the $1/E$ type, ϕ_r/ϕ_{th} determined for each detector should be approximately the same.

If the thermal flux is known at this position, ϕ_r can be determined, and the differential flux over the range of energy determined by the detectors can be given as

$$\phi(E) = \phi_r/E.$$

Fast Region

The fast neutrons are produced in the fission process. These neutrons have not been moderated by collisions. The energy range of the fission neutrons varies from over 10 MeV down to the thermal region. The fission neutrons are divided into two categories: prompt and delayed neutrons. The prompt neutrons represent over 99 percent of the total fission neutrons. The energy distribution of the delayed and prompt neutrons differs considerably. The delayed neutrons are produced at much lower energies than the average energy of the prompt neutrons. The effects of the delayed neutrons will not be considered, since measurement of the fast-neutron flux will be made above 2.9 MeV. The fast-neutron spectrum (for prompt neutrons) for U^{235} fission is well represented by the equation

$$\phi(E) \sim e^{-E/0.965} \sinh \sqrt{2.29 E}. \quad (16)$$

The region of interest will be above 2.9 MeV, which is above the average energy of this fast neutron energy spectrum. The higher-energy neutrons which are slightly moderated but still remain in the region of interest will not distort the flux spectrum. This is true because the number of neutrons

per unit energy in the fission spectrum increases rapidly with decreasing energy above the average neutron energy.

The method which will be used to measure the integrated fast flux is called the "Trice" method. The reaction rate upon irradiation in an unknown flux $\phi(E)$ is

$$r = \int_0^{\infty} N_T \sigma(E) \phi(E) dE.$$

If an element is considered that exhibits a $\sigma(E)$ which remains equal to zero until some threshold energy is reached and then rises to some saturated value, we may write

$$r = N_T \int_{E_T}^{\infty} \sigma(E) \phi(E) dE.$$

The reaction that takes place is endothermic since the incident neutron must reach a certain energy before the reaction can take place. The threshold reactions that can be initiated by fast neutrons are (n,p) , (n,α) , (n,n') , and $(n,2n)$. These reactions can be detected by activation methods only if the product nucleus is radioactive. If the Q value is found for a certain threshold reaction by mass considerations, the threshold energy of the reaction will be given by

$$E_T = Q(A+1)/A,$$

where A is the atomic weight of the target nucleus. The threshold energy is greater than Q since a fraction of the energy of the incident neutron becomes translational energy of the compound nucleus. If the incident neutron possesses energy above E_T , the additional energy will appear as kinetic energy of the products (inversely proportional to their masses). As the energy of the incident neutron is increased above E_T , the cross section will increase to a saturated value for the (n,p) and (n,α) reactions, because the compound nucleus possesses an electrostatic potential barrier which the alpha particle or proton must penetrate. The ability of the particle to penetrate the barrier will be dependent on the velocity, electrical charge of the particle, and the charge of the final product nucleus.

The threshold detector which will be used to measure the fast neutron flux in the Argonaut utilizes the $S^{32}(n,p)P^{32}$ reaction. The cross section used for this reaction was idealized to a step function and assumed constant at some saturated value after an effective threshold energy is reached.

The effective threshold energy was determined from the following calculational technique:

$$r = N_T \int_0^{\infty} \sigma(E) \phi(E) dE = N_T \sigma_0 \int_{E_{\text{eff}}}^{\infty} \phi(E) dE,$$

where σ_0 is the constant saturated cross section for the reaction. Rearrangement yields

$$\int_{E_E}^{\infty} \phi(E) dE = r/N_T \sigma_0.$$

The activation rate will be determined by numerical integration. The variation of reaction cross section with energy is known, and the neutron flux per unit energy, $\phi(E)$, is assumed to correspond to the previously stated U^{235} fission spectrum. The value of $\phi(E)$ versus energy is tabulated in Reference 5, where

$$\int_0^{14} \phi(E) dE = 1.$$

For the $S^{32}(n,p)P^{32}$ reaction,

$$N_T \int_0^{\infty} \sigma(E) \phi(E) dE = r = N_T (0.0672); \quad \sigma_0 = 300 \text{ mb};$$

$$\frac{r}{N_T \sigma_0} = \frac{0.0672}{0.300} = \int_{E_{\text{eff}}}^{\infty} \phi(E) dE = 0.224$$

from the $\phi(E)$ tabulation for $E_{\text{eff}} = 2.9 \text{ MeV}$.

Figure 28.8 shows that this idealized method assumes that no neutron below the effective energy contributes to the reaction and that all the neutrons above E_{eff} are assumed to have the same cross section, σ_0 . The reaction rate is held constant with area A being made equivalent to area B. If a U^{235} fission spectrum is definitely present and the detector cross section for the reaction has the proper shape, the accuracy of the measurement will depend on the determinations of the detector activity. The fast flux above 2.9 MeV for the sulfur detector will be expressed as

$$\phi_f = \int_{2.9}^{\infty} \phi(E) dE = \frac{r}{N_T \sigma_0}. \quad (17)$$

Fig. 28.8

Fission Spectrum and Cross Section
for $S^{32}(n,p)P^{32}$ Reaction

STUDENT RESULTSIRRADIATION DATA

Position	Run 1 (Al covered)	Run 2 (Cd covered)
A	Au ²⁵ 935 μ g In ³¹ 922 μ g	Au ¹⁵ 859 μ g In ³⁵ 965 μ g
B	S "1" - In ³⁸ 452 μ g	S "2" - In ³⁹ 448 μ g (not Cd covered)
C	Mn ⁴⁵ 1.00 unit wt. In ³⁷ 819 μ g	Mn ⁴⁴ 1.65 unit wt. In ³³ 800 μ g
D	Na "A" 2.68 g	Na "B" 2.81 g
J-10	P "1" - Au std. 50.1 mg	

Start irradiation 11:35
Stop irradiation 12:05

Start irradiation 13:30
Stop irradiation 14:00

Reactor period 30 sec
Reactor power 100 W

Reactor period 30 sec
Reactor power 100 W

COUNTING DATA

In ³⁷ Avg	cpm	Time	NaI counter; Hv 1050 V Window level 56.0% Integrate Gain 2 444 mg/cm ² abs on shelf 1 In foil on shelf 2 Bgd ≈ 0 $\lambda A_0 = (3.36 \times 10^4)(1.52 \times 10^2 \text{ dpm/cpm})e^{\lambda \times 82 \text{ min}}(1/60)$ $= 2.44 \times 10^5 \text{ dps}$
	34,291	13:24	
	34,462	13:26	
	33,111	13:28	
	32,625	13:30	
	33,622		
In ³⁸ Avg	cpm	Time	NaI counter; (Conditions same as for In ³⁷) Bgd ≈ 0 $\lambda A_0 = (1.74 \times 10^4)(1.52 \times 10^2)e^{\lambda \times 91 \text{ min}}(1/60)$ $= 1.41 \times 10^4 \text{ dps}$
	17,739	13:35	
	17,437	13:36	
	17,331	13:37	
	17,009	13:38	
	17,379		
Na "A" Avg (-bgd)	cpm	Time	NaI counter; (Conditions same as for In ³⁷) Pellet on shelf 4 Bgd = 72 cpm
	127,198	13:40	
	127,966	13:41	
	126,506	13:42	
	127,697	13:43	
	127,270		
cpm at end of irradiation = $1.27 \times 10^5 \times 2^{\lambda \times 97} = 1.38 \times 10^5 \text{ cpm}$			

In ³¹	cpm	Time	NaI counter; (Conditions same as for In ³⁷) Bgd $\approx$ 0 $\lambda A_0 = (2.59 \times 10^4)(1.52 \times 10^2 \text{ dpm/cpm})e^{\lambda \times 105} (1/60)$ $= 2.51 \times 10^5 \text{ dps}$
	26,111	13:48	
	26,058	13:49	
	25,860	13:50	
	25,711	13:51	
Avg	25,935		
Au ²⁵	cpm	Time	NaI counter HV 1050 V Window level 53% Integrate Gain 4 444 mg/cm ² abs. on shelf 1 Au foil on shelf 2 Bgd 141 cpm $\lambda A_0 = (4.60 \times 10^3)(2.87 \times 10^1 \text{ dpm/cpm})e^{\lambda \times 111 \text{ min}} (1/60)$ $= 2.25 \times 10^3 \text{ dps}$
	4,724	13:54	
	4,790	13:55	
	4,792	13:56	
	4,671	13:57	
Avg (-bgd)	4,603		
Mn ⁴⁵	cpm	Time	GM counter; HV 700 V, shelf 2 Resolving time = $6.8 \times 10^{-6} \text{ min}$ Bgd $\approx$ 40 cpm
	11,120	13:36	
	10,802	13:37	
	10,902	13:38	
	10,850	13:39	
Avg	10,920		
Dead time correction = $\frac{1}{1 - (10,920)(6.8 \times 10^{-6})} = 1.08$ Avg (-bgd) = $(10,950 \times 1.08 - 40) = 11,780 \text{ cpm}$ cpm at end of irradiation = $11,780 \times e^{\lambda \times 93 \text{ min}} = 1.785 \times 10^4 \text{ cpm}$			
Austd	cpm	Time	GM counter; HV 700 V, shelf 3 Resolving time = $6.8 \times 10^{-6} \text{ min}$ Bgd = 40 cpm
	15,620	13:41	
	15,581	13:42	
	15,642	13:43	
Avg	15,620		
Dead time correction = $\frac{1}{1 - (1.562 \times 10^4)(6.8 \times 10^{-6})} = 1.12$ Avg (-bgd) = $(15,620 \times 1.12 - 40) = 17,470 \text{ cpm}$ $\lambda A_0 = (17,470)(21.1 \text{ dpm/cpm})e^{\lambda \times 97 \text{ min}} (1/60)$ $= 6.24 \times 10^3 \text{ dps}$			
P "1"	Counts	Time	GM counter; HV 700 V 40 mg/cm ² abs. on shelf 1 Pellet on shelf 2 Bgd $\approx$ 40 cpm
	3,911	13:45-	
		13:48	
	3,955	13:49-	
		13:52	
Avg (-bgd)	1,271 cpm		
S "1"	Counts	Time	GM counter; (Conditions same as for P "1") Bgd $\approx$ 40 cpm
	7,522	13:55-	
		13:58	
	7,490	13:59-	
		14:02	
Avg (-bgd)	2,462 cpm		

Na "B"	cpm	Time	NaI counter; (Condition identical to Na "A") Bgd = 63 cpm
	6,083	14:47	
	5,540	14:48	
	5,829	14:49	
	5,685	14:50	
Avg (-bgd) 5,721			
cpm at end of irradiation = $(5.721 \times 10^3)e^{\lambda \times 47 \text{ min}} = 5.91 \times 10^3 \text{ cpm}$			
In ³³	cpm	Time	NaI counter; (Conditions identical to In ³⁷) Bgd ≈ 0 $\lambda A_0 = (2.05 \times 10^4)(1.52 \times 10^2)e^{\lambda \times 60 \text{ min}}(1/60)$ $= 1.12 \times 10^5 \text{ dps}$
	20,899	14:59	
	20,928	15:00	
	20,098	15:01	
	20,234	15:02	
Avg 20,540			
In ³⁹	cpm	Time	NaI counter; (Conditions identical to In ³⁷) Bgd ≈ 0 $\lambda A_0 = (2.43 \times 10^4)(1.52 \times 10^2)e^{\lambda \times 65 \text{ min}}(1/60)$ $= 1.42 \times 10^5 \text{ dps}$
	24,968	15:05	
	24,308	15:06	
	23,699	15:07	
	23,721	15:08	
Avg 24,274			
In ³⁵	cpm	Time	NaI counter; (Conditions identical to In ³⁷) Bgd ≈ 0 $\lambda A_0 = (1.906 \times 10^4)(1.52 \times 10^2)e^{\lambda \times 72 \text{ min}}(1/60)$ $= 1.21 \times 10^5 \text{ dps}$
	19,242	15:12	
	19,111	15:13	
	19,001	15:14	
	18,922	15:15	
Avg 19,060			
Au ¹⁵	cpm	Time	NaI counter; (Conditions identical to Au ²⁵) Bgd = 124 cpm $\lambda A_0 = (2.09 \times 10^3)(28.7 \text{ dpm/cpm})e^{\lambda \times 78 \text{ min}}(1/60)$ $= 1.015 \times 10^3 \text{ dps}$
	2,254	15:18	
	2,178	15:19	
	2,229	15:20	
	2,199	15:21	
Avg (-bgd) 2,091			
Mn ⁴⁴	cpm	Time	GM counter; (Conditions identical to Mn ⁴⁵) Bgd 40 cpm
	2,310	14:58	
	2,351	14:59	
	2,149	15:00	
	2,241	15:01	
Avg (-bgd) 2,223			
cpm at the end of irradiation = $(2.22 \times 10^3)e^{\lambda \times 60 \text{ min}} = 2.90 \times 10^3$			
S "2"	Counts	Time	GM counter; (Conditions identical to S "1")
	7,511	15:03-	
		15:06	
	7,549	15:07-	
		15:10	
Avg (-bgd) 2,510 cpm			

DATA TABLE

Foil or Pellet	Weight	λA_0 or cpm at end of irradi.	$\frac{\lambda A_0}{\text{wt.}}$	Thermal Neutron Activity	Resonance Neutron Activity	R_{cd}^{-1}	$1 + \alpha$
Au ²⁵	935 μg	2.25×10^3 dps	2.41 dps/ μg	1.22 dps/ μg		1.03	36.4
Au ¹⁵	859 μg	1.02×10^3 dps	1.19 dps/ μg		1.19 dps/g		
In ³¹	922 μg	2.51×10^5 dps	272 dps/ μg	143 dps/ μg		1.14	38.0
In ³⁵	965 μg	1.21×10^5 dps	125 dps/ μg		125 dps/g		
In ³⁷	819 μg	2.44×10^5 dps	298 dps/ μg	153 dps/ μg		1.09	38.0
In ³³	800 μg	1.12×10^5 dps	140 dps/ μg		140 dps/g		
Mn ⁴⁵	1.00 Relative Weight	1.79×10^4 cpm	17.9 Normalized			9.20	2.47
Mn ⁴⁴	1.65 Relative Weight	2.90×10^3 cpm	1.76 Normalized				
Na "A"	2.68 g	1.38×10^5 cpm	5.52×10^4 cpm/g			23.3	1.32
Na "B"	2.81 g	5.91×10^3 cpm	2.10×10^3 cpm/g				
In ³⁸	452 μg	1.41×10^4 dps	312 dps/ μg	161 dps/ μg			
In ³⁹	448 μg	1.42×10^4 dps	317 dps/ μg	Total (nvt) Comparison Run 1 Run 2 = 1			
S "1"	-	2.46×10^3 cpm	2.46×10^3 cpm				
S "2"	-	2.51×10^3 cpm	2.51×10^3 cpm				
P "1"	-	1.27×10^3 cpm					
Au (std)	50.1 mg	6.24×10^3 dps					

CalculationsThermal Flux

$$\lambda A_0 = Kc_0 W = \sigma_p n_0 v_p N_T (1 - e^{-\lambda T})$$

Au²⁵

$$\phi_{th} = n_0 v_p = \frac{\lambda A_0}{\sigma_p N_T (1 - e^{-\lambda T})} = \frac{\lambda A_0}{\sigma_p N_T \lambda T}$$

$$\phi_{th} = \frac{(1.22)(935)}{(98.8 \times 10^{-24}) \left[(935 \times 10^{-6}) \frac{(6.023 \times 10^{23})}{197} \right] (1.782 \times 10^{-4})(30.5)}$$

$$\phi_{th} = \frac{1.22}{1.645 \times 10^{-9}} = 7.40 \times 10^8$$

In³¹

$$\phi_{th} = \frac{(143)(922)}{(155 \times 10^{-24})(922 \times 10^{-6}) \frac{(6.023 \times 10^{23})}{115} (1 - e^{-1.28 \times 10^{-2} \times 30.3})}$$

$$= \frac{143}{2.63 \times 10^{-7}} = 5.45 \times 10^8$$

In³⁷

$$\phi_{th} = \frac{153(819)}{(155 \times 10^{-24})(819 \times 10^{-6}) \frac{(6.023 \times 10^{23})}{115} (1 - e^{-1.28 \times 10^{-2} \times 30.5})}$$

$$\phi_{th} = 5.82 \times 10^8$$

In³⁸

$$\phi_{th} = \frac{161(452)}{(155 \times 10^{-24})(452 \times 10^{-6}) \frac{(6.023 \times 10^{23})}{115} (1 - e^{-1.28 \times 10^{-2} \times 30.5})}$$

$$= 6.13 \times 10^8$$

Resonance FluxAu¹⁵

$$\phi_r = \frac{\lambda A_0}{N_T \left(\int \sigma \frac{dE}{E} \right) [1 - e^{-\lambda T}]}$$

$$\phi_r = \frac{(1.19)(859)}{(859 \times 10^{-6}) \frac{(6.023 \times 10^{23})}{197} (1530 \times 10^{-24}) [(1.782 \times 10^{-4})(305)]}$$

$$\phi_r = \frac{1.19}{2.54 \times 10^{-8}} = 4.68 \times 10^7$$

In³⁵

$$\phi_r = \frac{125(965)}{(965 \times 10^{-6}) \frac{(6.023 \times 10^{23})}{115} (2554 \times 10^{-24}) (1 - e^{-\lambda T})}$$

$$= \frac{125}{4.34 \times 10^{-6}} = 2.84 \times 10^7$$

In³³

$$\phi_r = \frac{140}{4.34 \times 10^{-6}} = 3.24 \times 10^7$$

$$\frac{\phi_r}{\phi_{th}} = \frac{2.37^*}{(R_{Cd} - 1)(1 + \alpha)}$$

Au Position A

$$\frac{\phi_r}{\phi_{th}} = \frac{2.37}{(2.03 - 1)(1 + 35.4)} = 0.063$$

In Position A

$$\frac{\phi_r}{\phi_{th}} = \frac{2.37}{(2.14 - 1)(1 + 37)} = 0.054$$

In Position C

$$\frac{\phi_r}{\phi_{th}} = \frac{2.37}{(2.09 - 1)(1 + 37)} = 0.059$$

Mn Position D

$$\frac{\phi_r}{\phi_{th}} = \frac{2.37}{(10.2 - 1)(1 + 1.47)} = 0.104$$

Na Position E

$$\frac{\phi_r}{\phi_{th}} = \frac{2.37}{(24.3 - 1)(1 + 0.32)} = 0.077$$

Fast Flux CalculationsAu^{STD.}

$$\begin{aligned} \phi_{th} &= \frac{\lambda A_0}{\sigma_p N_T (1 - e^{-\lambda T})} \\ &= \frac{6.24 \times 10^3}{(98.8 \times 10^{-24}) \cdot 0.501 \times \frac{6.023}{197} \times 10^{23} (1.782 \times 10^{-4} \times 30.5)} \\ &= \frac{6.24 \times 10^3}{8.24 \times 10^{-5}} = 7.57 \times 10^7 \text{ n/cm}^2\text{sec} \end{aligned}$$

*Obtained for Cd cutoff $E_1 = 0.55 \text{ eV}$.

S-P

$$\phi_F = \frac{C_s \sigma_p A_s T_p S_p}{C_p \sigma_0 A_p T_{sk_{32}}} \phi_{th}$$

$$= \frac{(2.48 \times 10^3)(0.23 \times 10^{-24})(32)(T_p)(.96)}{(1.27 \times 10^3)(0.30 \times 10^{-24})(31)(T_s)(.96)} (7.57 \times 10^2)$$

$$\phi_F = 1.17 \times 10^8$$

$$\phi_F = \frac{\int_{2.9}^{\infty} \phi(E) dE}{\int_0^{\infty} \phi(E) dE} = 0.22$$

$$\phi_{\text{Fission}} = 5.31 \times 10^8 \text{ n/cm}^2 \text{ sec}$$

Results

Position	Detector	ϕ_{th}	ϕ_r	ϕ_r/ϕ_{th}
A	Au	7.40×10^8	4.68×10^7	0.063
A	In	5.45×10^8	2.84×10^7	0.054
B	In	6.13×10^8	-	-
C	In	5.82×10^8	3.24×10^7	0.057
D	Mn	-	-	0.104
E	Na	-	-	0.077
B	S	$\phi_f = \int_{2.9}^{\infty} \phi(E) dE = 1.17 \times 10^8$ $\phi_{\text{Fission}} = \int_0^{\infty} \phi(E) dE = 5.31 \times 10^8$		

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NEUTRON DIFFRACTION

by

Jan Spalek

ABSTRACT

In the first part of this experiment the diffraction of neutrons by a single crystal is demonstrated and neutron wave properties experimentally verified. To do this, a crystal spectrometer is aligned in a beam of neutrons and a rocking curve is measured. The resolution of the spectrometer is calculated. The energy spectrum of neutrons coming out of the reactor beam hole is then measured and compared with the calculated theoretical distribution. The approximately Maxwellian shape of the beam spectrum is thus shown. The use of the experimental spectrum plot as a direct method for obtaining the effective neutron temperature is demonstrated.

OUTLINE

- I. INTRODUCTION
- II. THEORY
 - A. Neutron Waves
 - B. Diffraction
 - C. Grating Space in Crystals (d)
 - D. Derivation of the Bragg Law
 - E. Resolution
 - F. Calculation of the Diffracted Beam Intensity
 - G. Calculation of the Theoretical Maxwell Distribution Curves for Comparison with Experimental Graphs
- III. APPARATUS
- IV. PROCEDURE
- V. RESULTS
- VI. ANALYSIS AND CONCLUSIONS
- VII. BIBLIOGRAPHY
- VIII. APPENDICES
 - A. Discussion Questions
 - B. Design and Construction of a Single-crystal Neutron Spectrometer for the Argonaut

I. INTRODUCTION

The first experiments that demonstrated neutron diffraction were done a few years before the advent of atomic reactors. In 1936, a group of scientists used a radium-beryllium source to provide neutrons which they deflected by single crystals mounted at such an angle as to reflect the neutrons into a detector. If the crystals were rotated from the critical angle, a sharp decrease in the neutrons arriving at the detector was noted. However, because the radium-beryllium source is weak and, additionally, emits neutrons of many energies, the neutrons detected by the counter were not sufficient to provide any quantitative information about the spectrum. Neutron spectrometers were therefore not developed until atomic reactors provided neutron beams of sufficient intensities. The first neutron spectrometer was constructed at the Argonne National Laboratory in 1945. The measured spectrum of thermal-column radiation and cross sections at different energies using the spectrometer were reported by W. H. Zinn and W. J. Sturm in 1947 Physical Review (References 4 and 5).

The modern spectrometers are now used not only as monochromators to measure nuclear cross sections as a function of energy by the transmission method, but the diffraction method (the measurement of diffracted beams as a function of the diffraction angle) has a wide application in problems of nuclear physics: the measurement of nuclear scattering amplitudes, magnetic scatterings, studies of light atoms among heavy elements, etc., as well as the investigation of the structure of materials and in crystallography.

II. THEORY

A. Neutron Waves. The motion of neutrons is determined by wave mechanics. A beam of neutrons can be thought of as neutron radiation (neutron rays or waves) with wave properties. In our experiment, the wave properties of neutrons are verified by demonstrating that neutrons are diffracted by a single crystal. From wave theory, the wavelength of neutrons of velocity v is given by

$$\lambda = h/mv \quad (1)$$

where $h = 6.624 \times 10^{-27}$ erg-sec is the Planck constant and $m = 1.675 \times 10^{-24}$ g the neutron mass.

B. Diffraction. In general, diffraction is a scattering process involving two essentials: waves (rays) capable of interference and a set of regularly arranged scattering centers (the atoms of a crystal). These centers (atoms and nuclei) scatter the incident waves in all directions, but only in some particular directions will the scattered waves be in phase so as to reinforce each other and form a diffracted beam.

In a crystal, the atoms are arranged periodically in parallel planes, so that the waves scattered by the crystal atoms are out of phase (destructive interference) in most directions of scattering. In a few directions reinforcement (waves in phase) takes place, and a diffracted beam is formed. That is why, in spite of scattering by a single atom being isotropic, diffracted beams are formed only in a few directions.

There is a fundamental distinction between X-ray and neutron diffraction: X rays are scattered by the electrons of an atom, while neutrons (except in a few cases of magnetic materials) are scattered by the nuclei of the atoms. For this reason, the X-ray scattering amplitude is proportional to the atomic number of the element, whereas the neutron scattering amplitude shows a relatively small variation with atomic number. The scattering of neutrons by nuclei is a very complicated process and cannot be covered in the brief theory necessary for this experiment. The student is referred to Neutron Diffraction by Bacon (Reference 1 in Bibliography). It is only mentioned here that the scattering of neutrons by single nuclei is the same in all directions (isotropic). Another distinction to be remembered is that between diffraction and reflection. At first sight, these two appear to be the same: angle of incidence is equal to the angle of reflection for both. However, they differ fundamentally in the following:

1. Diffraction (scattering) by the crystal is produced by all the atoms in the incident beam path, whereas reflection of visible light is done on the surface of the reflecting medium.

2. The diffraction takes place only at the angles of incidence which satisfy the Bragg law (to be derived later); reflection of light can take place at any angle of incidence.
3. Almost all light rays are reflected by a mirror, whereas the diffracted beam intensity is only a few percent ($\approx 2\%$) of the incident beam.

Nevertheless, in common speech the term reflection (from crystals) is used although diffraction is actually meant.

C. Grating Space (d) in Crystals. To use the spectrometer, it is necessary to know the distance between the atomic planes (the grating space) of the crystal. This space varies from substance to substance and for a given crystal depends on the directions in which it is cut. This is evident

Fig. 29.1. Grating Space in Crystals

from Fig. 29.1, which shows only a two-dimensional picture of the atomic planes for clarity. From the figure it is also seen that the number of scattering centers varies in different directions. The rule for a given crystal is: the greater the value of d between planes, the greater the number of atoms in these planes. In order to calculate d for a given crystal from X-ray crystallographic data (see Handbook of Physics and Chemistry), the notation is illustrated as follows:

Figure 29.2 shows a unit cell; the three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ are called the crystallographic axes of the cell (lattice). The directions of lines in a unit cell (lattice) are described by indices such as are shown in Fig. 29.3. The Miller indices (h , k , l) describe the orientation of planes in a lattice. Some planes identified by Miller indices are shown in Fig. 29.4. The distance d between planes depends on these indices (h , k , l) d .

Fig. 29.2. Unit Cell

Fig. 29.3. Direction Lines

Fig. 29.4. Miller Indices

In a cubic crystal, the distance $d_{(1,0,0)}$ for $hkl = (1,0,0)$ is equal to $|\vec{a}|$ and may be found in tables. For the cubic crystal, $a = b = c$. In our case, the crystal (LiF) was cut parallel to the $(1,1,1)$ plane, and the formula for d is

$$d_{(h,k,l)} = \frac{a}{\sqrt{h^2 + k^2 + l^2}} = \frac{a}{\sqrt{1 + 1 + 1}} = \frac{a}{\sqrt{3}}, \quad (2)$$

where a is the lattice constant for LiF.

D. Derivation of the Bragg Law. Consider a beam of neutron waves incident at an angle θ on the atomic planes of a crystal. The planes are a distance d apart. Take only wave 1

Fig. 29.5. Bragg Reflection

and 2 of the beam, whose wavelength is λ , incident on nuclei M and N, respectively (see Fig. 29.5). These nuclei will scatter the waves 1 and 2 in all directions. However, the scattered waves will reinforce each other when they are in phases, that is, when the difference in their paths is equal to their wavelength λ (or an integral multiple of λ). The difference in their paths is clearly from Fig. 29.5 equal to

$$ON + NP = n\lambda,$$

where n is an integer. From the figure, $ON = d \sin \theta$ and $NP = d \sin \theta$; therefore,

$$n\lambda = 2d \sin \theta, \quad (3)$$

which is the Bragg law. The diffractions with $n > 1$ will not be considered here, because their intensity decreases rapidly with decreasing wavelength in the regions of the spectrum where the spectrometer is used, and the reflectivity of the crystal planes decreases also rapidly.

E. Resolution. The Bragg law was derived under ideal conditions (the use of a monochromatic, perfectly parallel beam and of a perfect crystal). In practice, the diffracted beam has a wavelength spread that depends on the divergence of the incident beam (this beam is not monochromatic) and the mosaic spread of the crystal. (The mosaic spread is taken up in Section F below.)

Consider, first, a large perfect crystal and a parallel (well-collimated) incident beam of neutron waves of many wavelengths (such as from a beam hole). At a given Bragg angle θ , the diffraction will occur over a very small angular range near the Bragg angle, and not only exactly at the Bragg angle (see Reference 1). This range is less than 1 sec of arc for neutrons (see Reference 1). This means that only neutrons of wavelength λ (determined by the Bragg angle) or very close to λ will be diffracted and results in a diffracted beam of very low intensity. The incident beam must therefore be made large in cross section (by use of a wide collimator) to increase the intensity. This, however, limits the resolution of the spectrometer in the following way.

Figure 29.6 shows a collimator of width w and length L . From the point P , which is on the edge of the reactor core, radiate neutrons of all wavelengths and directions. Consider the two limiting directions D_1 and D_2 . For a given position (AB) of the crystal, of all the neutrons incident on the crystal at an angle θ_1 (direction D_1) only those of wavelength λ_1 determined by θ_1 (from the Bragg law) will be diffracted in the direction D_1 . For

Fig. 29.6. Experimental Arrangement

the same position of the crystal, of all the neutrons incident along D_2 those of wavelength λ_2 will be diffracted along D_2' . The angle of incidence θ_2 is different from θ_1 by $\delta\theta$. The wavelength λ_2 is different from λ_1 by $\delta\lambda$, i.e., $\lambda_2 = \lambda_1 - \delta\lambda$. The diffracted beam (from the incident beam originating at the point P only) will thus consist of neutron waves of wavelength range $\delta\lambda$. To determine $\delta\lambda$, differentiate the Bragg law:

$$\delta\lambda = 2d \cos \theta \delta\theta. \quad (4)$$

But $\theta_1 - \theta_2 = \delta\theta$ is known, and it can be seen from Fig. 29.6 that $\delta\theta = \alpha$, the collimator angle. Also from the figure, $\alpha = w/L$; therefore,

$$\delta\lambda = 2\alpha d \cos \theta = 2(w/L) d \cos \theta, \quad (5)$$

and the wavelength spread of a part of the total diffracted beam is thus determined by the collimator angle.

The total diffracted beam is composed of neutrons originating at all points (not just P) at the collimator end inside the reactor. To determine the wavelength range $\delta\lambda$ of the total diffracted beam, imagine point P to move across w , the width of the collimator. The total spread of the angle of incidence of neutrons on the crystal is then seen to be equal to twice the collimator angle ($= 2\alpha$), and therefore the wavelength range of the total diffracted beam is

$$\delta\lambda = 4\alpha d \cos \theta. \quad (6)$$

This width $\delta\lambda$ limits the resolution of the spectrometer.

In spectroscopy, the resolving power of an instrument is defined as the power to distinguish between two components of radiation which have wavelength very close together:

$$\text{resolution or resolving power} = \lambda/\delta\lambda, \quad (7)$$

where $\delta\lambda$ is the difference between the two wavelengths (given by formula 6), and λ is their mean value. The resolving power of X-ray instruments with ranges for about 500 to 30,000, whereas it is about 20 for neutron spectrometers. In this experiment, we are interested mainly in the wavelength range $\delta\lambda$ that the spectrometer selects out of the polychromatic beam.

If the crystal were perfect, only neutrons of wavelength λ_1 with directions deviating from D_1 (see Fig. 29.6) by a maximum of 1 sec of arc will be Bragg reflected. (For theoretical calculations of the above mentioned and following results see Reference 1 and Reference 2, Ch. 3-4). But all real single crystals are mosaic, that is, possess structural

imperfections which break up the arrangement of atoms in regular planes (see Fig. 29.7). The length of these blocks is about 5000 Å, and the maximum angle of disorientation between the blocks (called mosaic spread) varies from a few minutes up to one degree, depending on the crystal. From the figure, it is seen how mosaic spread increases reflectivity, for now neutrons of wavelength λ_1 can be diffracted over an angular range about D_1 corresponding to the mosaic spread. This increase is from 10 to 100 times (see Reference 1). This is a second fact which limits the resolution of the spectrometer.

Fig. 29.7
Mosaic Spread Effects

F. Derivation of the Observed Spectrum. The distribution of thermal neutron in a reactor, of Maxwellian, may be expressed as

$$\delta n_1 = K_1 v^2 e^{-v^2/v_0^2} \delta v, \quad (8)$$

where δn_1 is the number of neutrons in a velocity interval δv about a velocity v ; v_0 is the most probable neutron velocity; and K_1 is a constant.

In the scattering of a neutron beam from the reactor, δn (the number of neutrons scattered into the beam at a given velocity) is proportional to that velocity, so that

$$\delta n = v \delta n_1 = K_1 v^3 e^{-v^2/v_0^2} \delta v. \quad (9)$$

Equation (9) represents the neutron velocity distribution in the beam extracted from the reactor.

The response of the spectrometer and detector system to this beam spectrum can be divided into several distinct parts.

a. Detector Response. The detector used in this experiment is assumed to be black. Such a detector will have a response independent of the neutron energy or velocity and thus responds to the neutron density. If another detector, such as one having a k/v response, were used in the experiment, the observed spectrum would be affected accordingly.

b. Crystal Response. The reflectivity of a crystal, which determines the number of neutrons reflected (diffracted) at any velocity, varies as the inverse of E or v^2 . The diffracted spectrum is then

$$\delta n' = \delta n/v^2 = K_1 v e^{-v^2/v_0^2} \delta v. \quad (10)$$

c. Resolution Effects. The angular resolution of the spectrometer is essentially constant, and is determined largely by the collimation of the beam and the mosaic spread of the crystal. The resolution in terms of velocity varies with the angle as is shown below.

The deBroglie wavelength is given by

$$\lambda = h/mv. \quad (1)$$

Differentiation gives

$$\delta \lambda = - (h/mv^2) \delta v. \quad (11)$$

By the Bragg law,

$$\lambda = 2d \sin \theta. \quad (4)$$

Differentiation gives

$$\delta \lambda = 2d \cos \theta \delta \theta. \quad (12)$$

Upon equation of Eqs. (11) and (12) we obtain

$$-(h/mv^2) \delta v = 2d \cos \theta \delta \theta. \quad (13)$$

Solution for δv gives

$$\delta v = -(2dm/h) v^2 \cos \theta \delta \theta \quad (14)$$

or

$$\delta v = K_2 v^2 \cos \theta \delta \theta, \quad (15)$$

where

$$K_2 = -2dm/h.$$

Substitution of Eq. (15) into Eq. (10) yields

$$\delta n'' = K_3 v^3 e^{-v^2/v_0^2} \cos \theta \delta \theta \quad (16)$$

or

$$\frac{\delta n''}{\delta \theta} = K_3 v^3 e^{-v^2/v_0^2} \cos \theta,$$

where $\delta n''$ is the response of a black detector to the diffracted neutrons of velocity v and having an angular resolution $\delta \theta$. Since the experimental response is the count rate of the detector and $\delta \theta$ is constant, Eq. (16) can be rewritten as

$$C = K' v^3 e^{-v^2/v_0^2} \cos \theta, \quad (17)$$

where C is the observed count rate at angle θ . Dividing Eq. (17) by $\cos \theta$, we obtain

$$C' = C / \cos \theta = K' v^3 e^{-v^2/v_0^2}. \quad (18)$$

This distribution is the same as that obtained by multiplying the neutron distribution in the reactor by v . This then is the same distribution as the thermal-neutron flux spectrum in the reactor.

G. Calculation of the theoretical Maxwell distribution curves for comparison with experimental graphs. The experimental curve (see Student Results) shows the variation of counting rate (number of neutrons counted per minute) in the diffracted beam for different angles of the crystal. In other words, this curve shows the distribution of wavelengths (or velocities or energies) in the diffracted beam. The shape of this curve will not be exactly the same as the shape of the curve giving the distribution of wavelengths in the incident beam from the reactor, because the diffracted spectrum curve shape depends on the reflectivity of the crystal, which varies with wavelength (see Section F, as well as References 1 and 9). However, the experimental curve is sufficient to demonstrate that the (reactor) neutron wavelength distribution is approximately Maxwellian.

The Maxwellian distributions used in the following calculations has been discussed in detail in ANL-6410 (Reference 6). For conversion from " λ " notation to " v " notation, see Appendix B.

To convert from crystal glancing angle to wavelength, velocity, or energy, we have:

from Eq. (2) for LiF: $d_{(111)} = 4.01 \text{ \AA} / \sqrt{3} = 2.32 \times 10^{-8} \text{ cm.}$

From Eq. (3): $\lambda = 4.64 \times 10^{-8} \sin \theta.$ (19)

From Eq. (1): $v = 3.95 \times 10^{-3} / \lambda = 8.51 \times 10^4 / \sin \theta.$ (20)

The kinetic energy (in eV) is given by

$$KE = \frac{1}{2} mv^2 = \frac{1}{2} \times \frac{1.675 \times 10^{-24}}{1.602 \times 10^{-12}} \left(\frac{8.51 \times 10^4}{\sin \theta} \right)^2 = \frac{3.78 \times 10^{-3}}{\sin^2 \theta}. \quad (21)$$

(Note: The above and following calculations are within 12-in. slide-rule accuracy.)

The flux distribution (beam radiation as seen by a black detector) is given by

$$\frac{d(nv)}{dv} = \frac{4n}{\sqrt{\pi} v_0^3} v^3 e^{-v^2/v_0^2}, \quad (22)$$

where $d(nv)$ = number of neutrons/cm² leaving the beam hole per second with velocities between v and $v + dv$ in the range dv , n the total number of neutrons per unit volume, and v_0 = most probable speed of the Maxwell velocity distribution. By differentiation of Eq. (22) and setting the derivative equal to zero, the most probable flux is at the speed

$$v_{mp} = \sqrt{\frac{3}{2}} v_0 = 1.22 v_0. \quad (23)$$

Therefore, the corresponding most probable speed of the velocity distribution is

$$v_0 = v_{mp} / 1.22. \quad (24)$$

To calculate some points of the theoretical flux distribution (22) in order to compare them with the experimental flux points, we note from the graph given in Student Results the value of v_{mp} and the count rate at that velocity. Rewriting Eq. (22) by lumping the constants in K , we obtain

$$\frac{d(nv)}{dv} = F = K v^3 e^{-v^2/v_0^2}. \quad (25)$$

Substitution of the values from the peak of the experimental curve into Eq. (25) determines the value of K .

III. APPARATUS (see Fig. 29.8)

Spectrometer (see Fig. 29.9); four LiF crystals; BF_3 detector of 40-cm Hg pressure, Multi-purpose Counter with high-voltage supply and amplifier; preamplifier.

IV. PROCEDURE

- A. Align the spectrometer with instructor's help.
- B. Measure the rocking curves at $\theta = 12^\circ, 16^\circ$, and 23° .
- C. Reposition the crystal table to the peak of the rocking curve, and then sweep the spectrum from 2θ (the arm angle) of 18° to 56° , taking count rates at each angle setting. Also take corresponding background counts at each angle by offsetting the crystal about $+2$ and -2 from the mean position (from the peak of the rocking curve). This offsetting for background count is determined by the half-width of the rocking curve, which is $\sim 2^\circ$. Plot the points on graph paper during the experiment, in order to see if some "bad" points need retaking. Plot also the "net" flux points (background subtracted).
- D. As soon as the net flux-plot data are sufficient to determine the peak of the flux spectrum (the most probable flux), fit the constant K of Eq. (25) to this peak value, and calculate about 10 points of the theoretical flux over the range of the experimental spectrum. See Section II G and Student Results to make this calculation. Make sure you substitute the proper v_{mp} and v_0 (into formula 25) determined by the peak of your spectrum.
- E. Plot the theoretical spectrum curve over the experimental and compare.
- F. In the above, the velocity at each angle had to be computed. Replot the experimental and theoretical flux spectrum against an increasing velocity scale, since no additional calculations are necessary. For the experimental replot, first draw a "best" line through the net flux plot and then take points from it.
- G. If time permits, calculate the "temperature" of the beam neutrons.

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Fig. 29.8. Experimental Apparatus

139-2705

Fig. 29.9. Crystal Spectrometer

VI. ANALYSIS AND CONCLUSIONS

To facilitate the analysis and before too many conclusions are drawn, it is pointed out here that only the basic theories and major factors involved in diffraction technique suitable for an introductory experiment were treated. Keeping the above and the purpose of this experiment as described in the abstract in mind, the following facts (besides others) should be considered in the student analysis and conclusions of this experiment:

Four lithium fluoride crystals were used, which were cut with faces parallel within $\pm 2^\circ$ of the (111) plane.

There is a neutron-flux variation due to manual controls of the reactor.

A wide collimator was used.

High background (detector relatively close to the crystal) exists.

Low count rates are due to the low power of the reactor.

A small detector, not completely black ($B^{10}F_3$, 40 cm Hg pressure, 14 in. long, axis parallel to the beam), was used.

VII. BIBLIOGRAPHY

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VIII. APPENDICESA. Discussion Questions

1. What is diffraction? What are the two essentials for diffraction?
2. Distinguish diffraction from reflection. State the three fundamental differences.
3. What is the difference between X-ray scattering and neutron-wave scattering?
4. Distinguish between scattering by a single atom and scattering by the atoms in a crystal.
5. How would you obtain neutrons of 0.3 eV ($\pm$ the $\delta\lambda$ of the instrument) from the spectrometer? (Indicate the necessary calculations.)
6. What determines the wavelength spread $\delta\lambda$ of the diffracted beam?
7. How would you obtain the maximum resolution out of this spectrometer?
8. Why should the large opening of the collimator be partitioned by vertical neutron-absorbing strips?
9. What information can be obtained from the graph about the beam spectrum? Interpret the shift of the experimental spectrum with respect to the theoretical.
10. How would you calculate the neutron-beam temperature from a given experimental flux plot?
11. Name the fields of application of the spectrometer.
12. Discuss the neutron-scattering phase.

B. Design and Construction of a Single-crystal Neutron Spectrometer for the Argonaut

This Appendix points out the major design criteria used in the construction of the present spectrometer, and suggests many improvements possible for this or a new spectrometer. The spectrometer shown in Fig. 29.9 and described below was constructed as a prototype and a demonstration unit. With it, the diffraction of neutrons at low reactor power (about 200 W) was demonstrated, and the spectrum of the neutron radiation was measured in a student experiment at the International Institute.

Design and Construction Criteria

1. Very low flux level ($\sim 800 \text{ n/cm}^2\text{-sec}$) at the beam hole
2. Student use
3. Simplicity and economy
4. Available space
5. Use of materials at hand
6. Room for improvement and/or improvisation
7. Speedy construction
8. No massive shielding necessary

Some Design Considerations in More Detail

By far the major factor which influenced the design of the spectrometer was the very low neutron flux at the end of the Argonaut beam hole. At the maximum power level ($\sim 200 \text{ W}$) that could be used at this training reactor compatibly with other experiments, the neutron flux at the beam hole was only $\sim 1000 \text{ n/cm}^2\text{-sec}$, compared with $10^5\text{-}10^7 \text{ n/cm}^2\text{-sec}$ commonly used for diffraction measurements.

The low-flux criterion jointly with the "student use" consideration (for relative ease in alignment) demanded that the spectrometer be placed as close as possible to the beam hole. This, in turn, limited the available space because of the particular construction of the Argonaut near the beam hole, and so a shorter than desirable detector arm resulted.

Another major design and construction factor was "simplicity and economy." It was desired to produce a prototype spectrometer for minimum cost, since it was not known that the instrument will be able to measure the Argonaut reactor spectrum at the very low neutron flux previously mentioned. Only rocking curves were obtained in a preliminary measurement with crude apparatus. Therefore, the instrument was made to be as simple and economical as possible, and yet to have considerable accuracy so that the spectrum

could be measured. Finally, an improved version of the spectrometer was planned to be constructed for student use at a reactor of higher power, the Juggernaut (200 kW).

The present simple instrument used at the Argonaut at a power level of ~ 100 W has surpassed expectations. An experiment on neutron diffraction was written for this spectrometer, and graphs of the Argonaut neutron spectrum measured by the students the first time the diffraction experiment was offered are attached. Also attached are additional graphs of the Argonaut neutron spectrum measured by this instrument showing the effect of different experimental arrangements on the spectrum and background.

General Description of the Spectrometer

Refer to Figs. 29.10-29.12. The angular motion of the spectrometer mechanism is initiated (manually) by the detector arm, which is coupled to the crystal-supporting table through gears in a 1 to 2 ratio. The pivot end of the detector arm is coupled to the base plate by 2 thrust ball bearings, the assembly being held together by a hollow bolt. The swinging end of the detector arm is supported and rolls on a ball-bearing wheel, and can be locked at different angular positions to the base plate by a bolt with a wing nut.

Fig. 29.10. Spectrometer Details

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Fig. 29.11. Crystal Spectrometer at the Reactor

139-2632

Fig. 29.12. Gearbox and Crystal Mounting Details

The detector arm and its pivot bearings are the main weight- and thrust-supporting parts of the spectrometer, the detector shield being the heaviest object that the spectrometer has to support. The 1-to-2-ratio gear box and the crystal-supporting table are, mechanically considered, the appendages to the base plate and detector-arm assembly. The "fast" shaft of the gear box is fastened in the hollow bolt of the pivot end of the detector arm by means of a screw in the arm. This shaft is normally locked to the arm with this screw. The gear box is bolted to the base plate near the pivot point. The slow shaft of the gear box supports the table with the crystal. The table is locked to this shaft through an improvised screw-and-nut mechanism that permits a small angular motion of the table with respect to the shaft. By turning this screw, the face of the crystal on the table may be positioned at exactly one-half the detector arm angle. This is determined by the peak of the crystal rocking curve. Ceasing the turning of the screw, the table is left "automatically" locked to the detector arm through the gear box, leaving the instrument ready to "sweep" the spectrum. The detector arm angle is read off the circumference of the base plate. The base-plate semicircle was calculated to be machined to such a radius that would make 1-mm division of a standard tape (glued to the circumference of the base plate) equivalent to $1/5$ of a degree ($= 12$ min) of arc. This eliminated the graduating of the base plate by a machine. The spectrometer is therefore capable of reading detector angles of 12 min, and half of this angle is the crystal (glancing) angle.

List of Suggested Improvements

As mentioned, many improvements are possible for this spectrometer.

The present spectrometer has worked satisfactorily in the student experiment of measuring the neutron spectrum at the Argonaut reactor. Several of the measured spectra are shown in Student Results. The variations exhibited by the spectra, the rocking curves, and background are due to the changes in collimation, reactor flux level, the number of diffracting crystals used, and the distance between the crystal and the detector.

The following improvements should be done on this one or a new spectrometer if the design criteria may be changed. Those numbered 1, 2, and 3 could be done now on this spectrometer. However, it is not guaranteed that #1 and #2 will improve the spectra shown in the graphs. The narrower collimator may reduce the beam strength too much, and the wider detector may also raise the background by counting too many of the scattered neutrons from the crystal. In fact, it should be kept in mind that the following suggested improvements were not tested experimentally, and some of them may prove not to be improvements at all due to other factors; but some of the purely mechanical improvements will be obviously advantageous.

1. Better collimator (see Fig. 29.13).

Fig. 29.13. Better Collimator

2. Larger diameter detector (vertically).
3. Separate vertical adjustment of the detector arm.
4. Longer detector arm (will necessarily mean a larger radius base plate).
5. Reproducible mounting of the detector shield.
6. Gear and pinion crystal table adjusting mechanism (angularly).
7. Longer vertical adjustment of the crystal table.
8. Lock for the table to the base plate (for a detector rocking curve).
9. Fast, 2 position crystal background offset. (From the peak of the rocking curve position.)
10. One large single crystal.

The Only Items That Were Purchased Outside the Laboratory

Four LiF single crystals, cut along the 111 plane ($\pm 2^\circ$), purchased at The Harshaw Chemical Company, 1945 East 97th Street, Cleveland 6, Ohio, Tel. RAndolph 1-8300.

Four gears, 48 pitch, 1/4 in. bore, clamp hub: 72 teeth, anti-backlash, #SR 414253-72 (Reeves Catalog No.), pitch diameter 1.5000; 96 teeth, anti-backlash, #SR 414253-96, pitch diameter 2.000; 72 teeth, #SR 54252-72, pitch diameter 1.500; 96 teeth, #SR 54252-96, pitch diameter 2.000, purchased at Reeves Instrument Corporation, Roosevelt Field, Garden City, New York, Tel. PIONEER 6-8100.

One $B^{10}F_3$ detector, 40 cm Hg press. 1950 oper. volt, 1 in. diameter, 14 in. long. Purchased at Nancy Wood Counter Laboratory, 1525 East 53rd Street, Chicago 15, Illinois, Tel. FAirfax 4-1114.

Estimated Cost of the Spectrometer

The cost of the metal for the construction of the spectrometer, except the base plate, is negligible. The major part of the cost is taken up by the purchased items listed on the preceding page, and the necessary counting equipment to operate the BF_3 detector.

Excluding the associated counting equipment, and the cost of the labor involved in the construction of the spectrometer, the approximate cost is:

4 LiF crystals	about \$160
1 BF_3 detector	" \$100
4 gears	" \$ 60
2 thrust bearings	" \$ 10
Aluminum base plate 1/2 in. x 2 ft x 2 ft.	" \$ 15
	<u>\$345</u>

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STUDENT RESULTSDETERMINATION OF COUNTER ZERO POSITION

Position in Degrees	Counts/0.4 min
-1.5	18,681
-1.0	21,017
-0.5	23,905
0	24,140
+0.5	24,129
+1.0	22,889
+1.5	21,196
+2.0	18,548

Reactor power = 10 W.

Counter zero position = $+0.2^\circ$ Crystal Rocking Curve

Table Vernier Position in Degrees	Counts/min
0	191
1	194
2	348
3	795
4	1,084
5	1,163
6	1,136
7	905
8	618
9	362
11	108
13	87
15	85
5.25	1,160

Reactor power = 200 W.

Crystal peak position = 5.2°

Fig. S29.1

Fig. S29.2

REACTOR NEUTRON SPECTRUM

Crystal Angle θ degrees	Neutron Velocity v m/sec	C" Counts/2 min	Background Counts/2 min BG	C Counts/2 min C"-BG	$\cos \theta$	$\frac{C'}{\cos \theta}$
6	8,154	1,350	1,060	290	0.995	292
8	6,124	928	502	426	0.990	430
10	4,908	1,496	353	1,143	0.985	1,160
12	4,099	2,137	283	1,854	0.978	1,898
13	3,789	2,582	390	2,192	0.974	2,250
14	3,523	2,542	264	2,278	0.970	2,350
15	3,293	2,654	381	2,273	0.966	2,350
16	3,092	2,579	237	2,342	0.961	2,440
17	2,915	2,512	282	2,230	0.956	2,330
18	2,758	2,250	179	2,089	0.951	2,180
19	2,618	2,266	188	2,078	0.946	2,200
20	2,493	2,101	162	1,939	0.940	2,060
21	2,378	1,942	162	1,780	0.934	1,905
22	2,275	1,706	155	1,551	0.927	1,674
23	2,181	1,713	172	1,541	0.921	1,672
24	2,095	1,426	187	1,239	0.914	1,355
26	1,944	1,202	160	1,042	0.899	1,159
28	1,815	1,050	148	902	0.883	1,020
30	1,705	913	159	754	0.866	871

Calculation of K

$$C = Kv^3 e^{-v^2/v_0^2}$$

at peak of flux spectrum

$$v = 3250 \text{ m/sec}$$

$$v_0 = \frac{v_{mp}}{1.225} = 2660 \text{ m/sec}$$

$$C = 2400$$

$$2400 = K (3250)^3 e^{-(3250/2660)^2}$$

$$2400 = K 34.3 (10^9) e^{-1.50} = K 34.3 (10^9) 0.223$$

$$K = \frac{2400}{34.3(10^9) 0.223} = 3.14(10^{-7})$$

CALCULATION OF THEORETICAL FLUX SPECTRUM

$$v_0 = 2660 \text{ m/sec}$$

$$v_0^2 = 7.076 (10^6) \text{ m}^2/\text{sec}$$

$$K = 3.14 (10^{-7})$$

θ	v	v^2 $\times 10^{-4}$	v^3 $\times 10^{-9}$	v^2/v_0^2	e^{-v^2/v_0^2}	$v^3 e^{-v^2/v_0^2}$ $\times 15^{-7}$	$Kv^3 e^{-v^2/v_0^2}$
6	8,154	66.5	542	9.40	0.000083	4.5	15
8	6,124	37.5	230	5.30	0.0050	115	361
10	4,908	24.1	118.2	3.41	0.0331	390	1,225
12	4,099	16.80	68.9	2.37	0.094	648	2,035
13	3,789	14.36	54.4	2.03	0.132	718	2,255
14	3,523	12.41	43.7	1.754	0.174	760	2,386
15	3,293	10.84	35.7	1.532	0.216	771	2,421
16	3,092	9.56	29.6	1.351	0.259	767	2,408
17	2,915	8.50	24.8	1.201	0.300	744	2,336
18	2,758	7.61	21.0	1.075	0.342	718	2,255
19	2,618	6.85	17.94	0.968	0.380	682	2,141
20	2,492	6.21	15.48	0.878	0.416	644	2,022
21	2,378	5.65	13.45	0.798	0.450	605	1,900
22	2,275	5.18	11.77	0.732	0.481	566	1,777
23	2,181	4.76	10.37	0.673	0.510	529	1,661
24	2,095	4.39	9.19	0.620	0.538	494	1,551
26	1,944	3.78	7.35	0.534	0.586	431	1,353
28	1,815	3.29	5.98	0.465	0.628	376	1,181
30	1,705	2.91	4.96	0.411	0.663	329	1,033

Fig. S29.3

Temperature of the beam from experimental flux plot:

$$v_{mp} = 3250 \text{ m/sec}, \therefore v_0 = 2660 \text{ m/sec.}$$

Generally, the "temperature" of neutrons is related to their average kinetic energy by the formula

$$\frac{1}{2} m \overline{v^2} = \frac{3}{2} kT$$

"Temperature" of beam neutrons (flux) = T_b ,

$$\frac{1}{2} m \overline{v_{\text{flux}}^2} = \frac{3}{2} k T_b$$

$$\overline{v_{\text{flux}}^2} = \frac{\int v^2 d(nv)}{\int d(nv)} = 2 v_0^2 \quad (\text{see Reference 6}),$$

therefore

$$T_{\text{beam}} = \frac{\frac{1}{2} m (2 v_0^2)}{\frac{3}{2} k} = 525^\circ\text{K} = 252^\circ\text{C.}$$

Also, for the reactor core, where the neutrons have a Maxwell velocity distribution

$$\frac{d(n)}{dv} = \frac{4n}{\sqrt{\pi} v_0^3} v^2 e^{-v^2/v_0^2}$$

The root-mean-square velocity (squared) of the neutrons in the core is

$$v_c^2 = \frac{\int v^2 d(n)}{\int d(n)} = \frac{3}{2} v_0^2.$$

Therefore the temperature of the neutrons in the core

$$T_c = \frac{\frac{1}{2} m \left(\frac{3}{2} v_0^2 \right)}{\frac{3}{2} k} = 429^\circ\text{K} = \underline{156^\circ\text{C}}.$$

NEUTRON TIME-OF-FLIGHT SPECTROMETRY

by

Jan Spalek

ABSTRACT

The experiment discusses the time-of-flight technique for the selection of neutrons of definite energy ranges from a neutron spectrum. The Fermi slow neutron chopper is described and is used to measure the reactor beam spectrum, as well as the total neutron cross section of several materials as a function of velocity.

OUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
- IV. PROCEDURE
 - A. Procedure Using the Original Chopper Electronics
 - B. Procedure Using the TMC 256 Channel Analyzer
- V. RESULTS
- VI. BIBLIOGRAPHY
- VII. APPENDICES
 - A. Discussion Questions
 - B. Detailed Theory
 - 1. The opening time of the first gate.
 - 2. Number of neutrons in a given velocity range.
 - 3. The photocell angle.
 - 4. Calculation of the velocity range of neutrons in a channel; resolution.
 - 5. Burst shape and transmission of the shutter.
 - 6. The "cutoff function."
 - 7. Chopper applications: Spectrum and cross section measurement; cold neutrons, scattering cross section of filter materials ("crystal effects"); neutron resonances.

I. INTRODUCTION

The basis of the time-of-flight technique is the production of poly-energetic neutron bursts of relatively short duration, letting the neutrons in each burst spread out over the flight path according to their velocities, and counting the neutrons at successive time intervals in corresponding channels, electronically selected. Each channel then represents the number of neutrons within a certain velocity range. The resolution in $\mu\text{sec}/\text{m}$ and the counting rate per channel are the essential characteristics that describe the quality of a time-of-flight velocity selector.

If the neutron bursts are produced by a rotating shutter of a neutron chopper (Fig. 30.1), the velocity ranges in each channel depend on three variables:

- 1) shutter angular velocity (rps);
- 2) flight path length;
- 3) the time limits within which the detector and its associated equipment define the arrival of a neutron.

If any of these quantities are varied, neutron spectra or cross sections at different velocities can be measured.

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Fig. 30.1. Fermi Chopper on the Argonaut Reactor

II. THEORY

The essential theory with the equations necessary for the performance of this experiment are given below. For detailed discussion of the theory, refer to Appendix B and the references listed in the Bibliography.

Figure 30.2 shows a diagram of the Fermi chopper. The shutter is a hollow steel cylinder, inside of which is a "sandwich" of alternate cadmium and aluminum sheets. The aluminum sheets, due to their small neutron cross section, are the "slits" that allow the neutrons to pass through when the slits are within $1\frac{1}{2}^\circ$ of the horizontal position (parallel to the neutron beam). Due to the angular speed (ω) of the shutter, the slits are "open" for the time it takes the shutter to rotate 3° .

Fig. 30.2. Shutter in the Open Position

The neutron bursts, produced twice per revolution, are of time duration T_n :

$$T_n = S/\omega R,$$

where S is the slit width (aluminum thickness) and $2R$ is the slit length. This equation is a good approximation for the burst duration of neutrons of velocities above $4\omega R^2/S$. Below this velocity, the neutron-burst durations get smaller and shrink to zero for neutrons of "cutoff velocity." The shutter will cut off any thermal neutrons whose velocity is below the minimum velocity which would enable them to traverse the slit during the time the slit is open. This velocity is equal to $\omega R^2/S$.

The burst duration T_n and the channel time width (the "gate time" τ) are the main factors determining the velocity spread (Δv) of the neutrons in a channel. The proper choice of three independent variables

(ω, L, B) of the Fermi chopper, as shown in Fig. 30.1, will determine a certain neutron velocity range for the first channel.

The gate time τ is determined from the angular speed of the photocell-exciter light ray, which is equal to 2ω , and the angular spacing between the photocells. The first photocell signal opens the first gate; the second photocell signal closes the first gate and opens the second, etc.

As a first approximation, in which the burst duration is ignored, the fastest neutron recorded in channel #1 would have a velocity

$$v_f' = L/t_0, \quad (6')^*$$

and the slowest velocity,

$$v_s' = \frac{L}{t_0 + \tau}. \quad (6')$$

The opening time of the first channel is

$$t_0 = \frac{\beta}{2 \times \text{rps} \times 360^\circ}. \quad (3)$$

Considering the burst duration T_n jointly with the gate time τ , these velocities may be expressed as

$$v_f = \frac{L}{t_0 - \frac{1}{2}T_n}; \quad v_s = \frac{L}{t_0 + \tau + \frac{1}{2}T_n}, \quad (6)$$

and the extreme velocity range of the neutrons in channel #1 is $v_f - v_s$.

Equations (6) were derived for the case in which some of the neutrons are considered to be present at the ends of the burst; however, the probability for neutrons to be there is zero. To calculate a more realistic or effective velocity range in a channel, a "resolution function" was introduced. This takes into account the probabilities of recording a certain velocity neutron by the time-of-flight technique. The resolution R is defined as the full width of the resolution function curve at half-maximum divided by the flight path.

The effective Δv in a channel is

$$\Delta v = \frac{R}{(t_{F1}/L)^2} = \bar{v}^2 R,$$

*All equation numbers refer to the same equation derived in Appendix B.

where t_{F1} is the time of flight of the mean-velocity neutron in a channel.
For channel 1,

$$t_{F1} = t_0 + \frac{1}{2}\tau_1; \quad \bar{v}_1 = L / \left(t_0 + \frac{1}{2}\tau_1 \right).$$

For channel 2,

$$t_{F1} = t_0 + \tau_1 + \frac{1}{2}\tau_2; \quad \bar{v}_2 = L / \left(t_0 + \tau_1 + \frac{1}{2}\tau_2 \right).$$

The resolution for this instrument at 60 rps and a 2-m flight path is

$$\text{Res.} = 48.25 \times 10^{-6} \text{ sec/m.}$$

III. APPARATUS

There are two major components of the apparatus: the mechanical chopper, which should be considered jointly with the neutron detector for optimum performance, and the electronics necessary for the selection of neutrons of different velocities.

1. The mechanical chopper with the detector is shown in Fig. 30.3.
2. The electronic apparatus is shown in Figs. 30.4 and 30.5 and diagrammatically in Fig. 30.6.

Additional necessary electronic equipment are:

Preamplifier (for the BF_3 detector) Model 219A, Atomic Instrument Co.

AID Amplifier, Model 218, Baird Atomic

Multipurpose counter, ANL Model MPC-1

Factor of 2 attenuators.

Test equipment:

Exponential pulse generator

Tektronix oscilloscope Type 541

139-2856

Fig. 30.3. Fermi Slow-neutron Chopper

139-2852

Fig. 30.4. Electronics Associated with the Fermi Chopper

139-2855

Fig. 30.5. Rear View of Chopper Electronics

Fig. 30.6. Fermi Chopper Electronics

IV. PROCEDURE

A. Procedure Using the Original Chopper Electronics

1. Electronic apparatus connections. Connect according to Fig. 30.6.

Checkout procedure:

Run the chopper shutter at 60 rps - indicated by reading of "60" on the rate meter and 2 spots on the scope. Measure the photocell output at the input of the gate circuit chassis using a Tektronix oscilloscope. The photocell pulses should be within -30 to -40 V peaks (check-point No. 1 in Fig. 30.7). Set the output of the exponential pulse generator to -30 V, and feed it into the pulse-distributor chassis. Set the pulse-distributor switch to "gate check" and measure the pulse at the counter pulse input of the gate circuit. It should read -6 V (see the corresponding pulse shapes in Fig. 30.6 at check-point No. 2).

Fig. 30.7. Checkout Diagram

Next measure the output of the gate circuit at each channel. The output pulse should be -18 V. Then check the scalers and registers by running them for one minute with the switch on the pulse distributor in position "A." All the scalers should count and have comparable number of counts. The variations in the counts registered in each channel are governed by the gate times.

2. Set the chopper shutter and the BF_3 detector in line with the beam hole and connect the BF_3 detector through the preamplifier and the AID amplifier to the pulse-distributor terminal input No. 7. The detector should be positioned at exactly 2 m from the center of the shutter. Then bring the reactor up to power of about 200 W.

3. Open the beam hole and rotate the chopper shutter by hand to get the maximum count rate out of the detector. The shutter slits are then aligned parallel to the beam. This open position of the shutter can be verified by the reflected light ray which should be now horizontal (parallel with neutron beam). Now realign the BF_3 detector for the maximum count rate, using the monitoring MPC counter.

As a final check, run the chopper at 60 rps and count for about 10 min with the pulse-distributor switch on B. G. count, and repeat with the switch on "run." The registered counts in channel 6 on B. G. run should be approximately equal to the counts in the MPC monitor; the counts in channels 1 to 5 for both runs should be about equal.

4. The measurement of the length of time each gate is open is done with the Tektronix oscilloscope by taking the signal from the cathode of the 6J6 tube, pin No. 7 on the gate chassis.

5. Set the chopper speed at 60 rps, and observe its stability on the scope on the tachometer chassis. There should be 2 spots on the scope at 60 rps, and they should be stationary if the speed is stable. To get neutrons of the desired velocity in the first gate, calculate the necessary photocell angle setting for this rotational speed of the shutter and the flight path as given by the detector distance from the shutter. All the formulas are given in Section II and Appendix B. As a guide, the necessary calculations are performed in Section V.

6. The run. With the photocell angle setting a portion of the thermal spectrum is measured in the 6 channels. Take 10-min readings of: open beam, background (by inserting cadmium in the beam), and the transmission of the provided sample. Repeat these measurements after resetting the photocell angle, so that another portion of the spectrum can be measured.

B. Procedure Using the TMC 256-channel Analyzer: A second or alternative procedure is used to increase the accuracy of the experiment in the limited time available for its performance. In this procedure the 256-channel analyzer (see Fig. 30.8) is substituted for the six photocell

139-2853

Fig. 30.8. TMC 256-channel Analyzer

channels used in procedure A. The TMC is used in conjunction with a pulse neutron time base logic. The number of analyzer channels used is sixty four. Each channel has an $80\text{-}\mu\text{sec}$ data-collecting time and there is a $10\text{-}\mu\text{sec}$ read and write time interval between successive channels. The first photocell pulse of the chopper is used to trigger the analyzer at zero time when the shutter plates are parallel to the neutron beam. This is easily accomplished by setting the first photocell slit at a position 0° from the path of the neutron beam. A block diagram of the alternative experimental arrangement is shown in Fig. 30.9.

Fig. 30.9. Block Diagram of Experimental Arrangement

The analyzer trigger sequence is shown in Fig. 30.10. The base time is taken from a 100-kc oscillator.

Fig. 30.10. Trigger Sequence

It is advisable to run through most of procedure A as carefully as possible in order to get acquainted with the chopper and its associated electronics. Once this has been accomplished, the use of the multichannel analyzer as a data-gathering device is easily understood. A step-by-step procedure for this section is listed below:

1. Run through procedure A as carefully as time permits.
2. Connect TMC multichannel analyzer as shown on the block diagram (Fig. 30.9).
3. Set photocell No. 1 so that the reflected light strikes its slit when the aluminum shutter plates are exactly parallel to the neutron beam.
4. Make a trial run with a constant neutron source placed near the detector. Each channel should have the same number of counts after the trial run is completed.
5. With the channel length set at 80 μ sec, and the delay time and background rates at two, make the following runs at 60 rps for approximately one-half to one hour each:
 - a. open beam with no absorber present;
 - b. filter beam with no absorber present. The filter used is 10-mil cadmium, which eliminates thermal neutron and determines the epicadmium background;
 - c. open beam with absorber (either Al, Au, or Be);
 - d. filtered beam with absorber.

V. RESULTS

The calculations required to obtain the results of this experiment are somewhat difficult and repetitious. The results and calculations for both procedures listed above are for the most part identical.

The initial values of the Fermi chopper parameters (ω , β , and L) will be given by the instructor. The normal values of these parameters which will be used are:

$$\omega = 2\pi f = 377 \text{ radians/sec (} f = 60 \text{ rps);}$$

$$L = 2\text{m;}$$

$$\beta = \frac{L \times 2 \times F \times 360^\circ}{v} = \frac{8.64 \times 10^4}{v},$$

where v is the desired average velocity (m/sec) in the first channel. The channel lengths are determined by observing the gating pulse in procedure A

or by the channel-length setting in procedure B. The average velocities and velocity ranges in each channel i are calculated from the equation

$$\bar{v}_i = L/(t_{F1})_i.$$

The flight time for the channel i is the time elapsed from the shutter opening to the middle of the channel time. The velocity range is determined from the expression

$$\Delta v = 10^{-6} v^2 R,$$

where R is the resolutions of the chopper in units of $\mu\text{sec}/\text{m}$:

$$\text{Res.} = 48.3 \mu\text{sec}/\text{m} \text{ for } \tau = 100 \mu\text{sec}$$

$$\text{Res.} = 40.5 \mu\text{sec}/\text{m} \text{ for } \tau = 80 \mu\text{sec}$$

$$\text{Res.} = 28.5 \mu\text{sec}/\text{m} \text{ for } \tau = 40 \mu\text{sec}.$$

The burst time of the shutter at 60 rps was used in all three resolution determinations listed above. The velocity range which is of interest lies above $v = 4R/T_n$, 900 m/s since neutrons above this velocity all have the same burst time

$$T_n = S/\omega R = 94.5 \mu\text{sec},$$

with

$$S = 0.0794 \text{ cm (shutter-plate thickness);}$$

$$\omega = 377 \text{ radians/sec;}$$

$$R = 2.23 \text{ cm (average slit radius).}$$

After the average velocity and velocity range of each channel is calculated, the following results of the experiment will be determined:

1. A plot of the neutron-density distribution will be made and compared with the expected Maxwellian distribution. The distribution as seen from the corrected counts in each channel versus velocity will be of the form

$$C_i \sim v^4 e^{-v^2/v_0^2},$$

where v_0 is the most probable velocity of the neutron-density distribution,

$(dn/dv \sim v^2 e^{-v^2/v_0^2})$. The v^4 distribution from the resulting data arises because the neutron-density measurements in each channel are made over equal time increments and $dv \sim v^2 dt$.

2. The total cross section of a selected material (Au, Al, or Be) will be made at different velocities corresponding to their respective channels. The basic transmission method for determining the total cross section states that the fractional loss of neutrons per unit path length of material is constant. This yields the well-known transmission equation

$$I = I_0 e^{-Nx\sigma_T},$$

where, in our case,

I_0 = net counting rate with no sample;

I = net counting rate with sample;

N = number of atoms per cm^3 ;

x = sample thickness;

σ_T = total microscopic cross section per atom.

Rearrangement gives

$$\sigma_T = \frac{\ln(I/I_0)}{Nx}.$$

The calculation of σ_T for each channel will be the cross section for neutrons possessing the average velocity of that channel. A plot of σ_T versus the energy corresponding to the average velocity in each channel will be made and compared with the values listed in BNL-325. In order to determine the kinetic energy of a neutron from its velocity, the following relationship is given:

$$v = 1.38 \times 10^4 \sqrt{E},$$

where v has units of m/sec and E is in electron Volts.

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VII. APPENDICES.A. Discussion Questions

1. Explain the time-of-flight technique. (How are the neutrons separated into energy ranges?)
2. Name the three things that can be varied to measure the whole thermal-neutron spectrum with this chopper. Is it sufficient to vary any single one of them to sweep the whole spectrum?
3. Does the BF_3 detector count all the thermal neutrons that go through it? If yes, are all of these counts registered?
4. Fixing the angular speed of the shutter and the flight path, what factor would determine the fastest neutron that you wish to count in the first channel?
5. Write the equation for the "cutoff" velocity of the shutter, given its angular speed, radius, and the slit thickness.
6. How is the chopper resolution defined?
7. What measurement can be done to check the chopper resolution experimentally?
8. Given the slit width $S = 0.079$ cm, its length $2R = 4.46$ cm, and the angular speed = 60 rps, what is the burst time T_n of the shutter for neutrons moving at high velocities?
9. In what portion of the transmission curve should the chopper be operated? How can this be accomplished?

B. Detailed Theory

The Fermi slow-neutron chopper with its electronics, which is used in this experiment, utilizes the time-of-flight technique for selection of neutrons of small energy ranges out of a beam of neutrons of all energies. The rotating shutter of the chopper, shown in cross section in Fig. 30.11.

Fig. 30.11. Shutter

is used only to produce bursts of neutrons of certain duration. The neutrons in a burst (there are two bursts per revolution) are then allowed to fly a certain distance (the flight path), in which they spread out according to their velocities. At the end of the flight path a BF_3 detector, "on" all the time, is made to record periodically all the neutrons that pass through it. The output neutron pulses of the BF_3 are fed through amplifiers into timed gate circuits where they are stopped and rejected if the gates are closed, and registered in any one of several electronic recording channels if the gates let them through. A double-faced mirror, mounted on the end of the shutter, reflects a light beam from a lamp mounted above it. As the mirror rotates, the reflected light beam will excite the photocells successively. When the light beam reaches the first photocell, the pulse produced is used in an electronic tachometer, and also the same pulse opens the first gate.

The first gate stays open until the pulse from the second photocell closes it. This second pulse, which also opens the second gate, comes at a later time τ , the "gate width," given by the angular speed of the light beam and the distance between photocell slits. The pulses from the following photocells always close the preceding gate and open the succeeding gate.

1) The opening time of the first gate is set (by the photocell angle) to correspond to the time of flight of a neutron of a certain velocity that we wish to count in the first channel. The gate may be closed in the shortest possible time for best resolution or after a longer time for more counts in a channel, but at the loss of resolution. The counts in each channel, then, represent:

2) the number of neutrons in a velocity range, the velocity range being determined by the following factors:

1. the length of the electronic time channel (the gate time τ),
2. the duration T_n and the shape of the neutron burst;

3. neutron path length in the detector (L);
4. the delay time in the ion-collection action of the detector (the detector "jitter");
5. the neutron time of flight (t_f).

In this experiment, the fourth factor will be neglected because the detector delay time is much less than the burst and gate times. Also, the neutron path in the detector can be neglected because it is small compared with the neutron burst and to the gate time. With the Fermi chopper, the opening of the first gate is done by means of a light ray reflected off a double-faced mirror mounted on the rotating shutter (see Fig. 30.12). The mirror is inclined 45° to the shutter slits, so that when the shutter is open, the reflected ray is horizontal. Now, the time of flight of the neutron that we want to detect must be equal to the time spent by the exciter light ray in traversing the angle between its initial horizontal position and the first photocell. Therefore,

- 3) the photocell angle β is determined by:

the shutter speed, which is one-half of the angular speed of the light ray;
the flight path;
the velocity of the desired neutron.

Therefore, given the angular speed of the shutter, ω , the flight path L , and the desired neutron velocity, v_n , the relating formulas are:

$$\text{Time of flight } t_f = L/v_n. \quad (1)$$

Set the opening time of first gate at

$$t_0 = t_f, \quad (2)$$

with

$$t_0 = \frac{\beta}{2 \times (\text{rps}) \times 360}, \quad (3)$$

Fig. 30.12. Experimental Arrangement

where β is the angle setting in degrees of the first photocell, and rps refers to the number of revolutions made by the shutter per second.

From Eqs. (1) and (2),

$$\text{photocell angle } \beta = (L/v_n) \times 2 \times (\text{rps}) \times 360. \quad (4)$$

4) Calculation of the velocity range of neutrons in a channel; resolution. This section is discussed in detail in order to understand the complexity of the problem and the necessity for simplifications.

It is very important to refer the times of flight of different neutrons to the same (arbitrary) zero time in order to calculate correctly the velocity range in a channel. A convenient zero time for the Fermi chopper is when the photocell exciter light beam is horizontal and the shutter is, by construction, fully open. Equation (3) gives, therefore, the opening time of the first channel (t_0) directly referred to zero time. The closing time of the channel is t_0 plus the gate length τ . This gate length (or channel width) can be determined in two ways: from the time it takes the exciter light beam to travel between photocell slits, or by measuring the gate length by means of an oscilloscope (described in Section IV). The slight time difference between the two measurements is negligible in this case (that is, for the slow-neutron chopper). The closing time t_c is therefore,

$$t_c = t_0 + \tau. \quad (5)$$

If the neutrons were released by the shutter at the same time, the velocity range in a channel would be given by the difference between the fastest (v_f) and the slowest (v_s) neutron recorded:

$$v_f' = \frac{L}{t_0}; \quad v_s' = \frac{L}{t_0 + \tau} \quad (6')$$

(see Fig. 30.13). It is emphasized that Eq. (6') does not take the burst duration (T_n) into account. Also, as was mentioned in part 2, the detector length is neglected. In the above and following derivations, it is very important not to mix the positions and flight times of neutrons.

Fig. 30.13. Time and Distance Variables

Consider now the burst shape. First, if it were rectangular (could be produced by the shutter shown in Fig. 30.13), the burst duration T_n would be determined by the time between the opening and the closing of the shutter, and would be equal to the full width of the burst shape curve (see Fig. 30.14). From Fig. 30.14 and by correct analysis of the physical limits imposed by the shutter mechanism, the new extreme neutron velocities in a channel can be calculated.

Fig. 30.14. Burst Shape

Because of the constant interplay of position space and time "space" in the graphical representations in the figures, and the corresponding physical situations, and the necessary simplifying approximations, a detailed analysis follows. Refer to Fig. 30.14. A beam of neutrons of "all" velocities is assumed to be hitting the "knife edge" shutter, and at each position q_1, q_2, q_3 , all velocities are present. The shutter opens instantly, releasing at the same time and from the same place, neutrons of all velocities. The shutter stays open for a time T_n (the burst duration), during which the faster neutrons pull out in front of the slower ones. At the instant the shutter closes, the physical situation of the burst is shown in Fig. 30.14. Only the fastest neutron (v_1) is in the front of the burst ($v_1 > v_2 > v_3$), while all velocity neutrons exist at the end of the burst. To each neutron velocity (v_n) corresponds a burst shape curve which represents the number of neutrons per unit time at a given time in the burst.

The bursts for neutrons of different velocities produced by a rotating shutter of the Fermi chopper have a great variation of time duration at a given angular speed of the shutter (see Section 5 on p. 30.29). The difference between burst duration for neutrons of velocity $v = 4\omega R^2/S$ and "infinite" velocity is only $2 \mu\text{sec}$ out of $100\text{-}\mu\text{sec}$ bursts, $100 \mu\text{sec}$ being the burst duration of the "infinite" velocity neutrons at 60 rps of the shutter [see Eqs. (7)]. Considering only the neutrons of velocities $v_n > 4\omega R^2/S$, this will be still in the region of interest. Putting this restriction on the "knife" shutter, it can be assumed, therefore, that all the burst shape curves

(of the neutrons in the burst) have equal base (equal time duration). This approximation will introduce an error of 2% maximum and will greatly simplify the calculations. The resultant burst shape is shown in Fig. 30.14, where the zero time is referred to the center of the burst, which is midway between opening and closing of the shutter. Now it can be seen that, due to the burst duration, there will be faster (v_f'') and slower (v_s'') neutrons than v_f' and v_s' which were released at the same time.

Figure 30.15 shows the corresponding physical situation. Figure 30.15a shows the positions. Figure 30.15b shows the time intervals.

Fig. 30.15. Time and Distance Variables

From Fig. 30.15 it may be seen that the slowest possible neutron (n_2) will be the one at the front of the burst which enters the detector just at the closing time of the gate. The corresponding time of flight for neutron n_2 , over the flight path L , will be obtained as follows: At zero time, neutron n_2 has already flown for a time $\frac{1}{2} T_n$ over the flight path L . It still has to fly over the remaining portion of L for the length of time from zero to the closing of the gate at time t_c . Therefore, the total time of flight of the slowest neutron, n_2 , is

$$t_s'' = \frac{1}{2} T_n + t_c = \frac{1}{2} T_n + t_0 + \tau \quad (6''a)$$

By similar reasoning, the fastest possible neutron (n_3) is the one at the end of the burst which gets counted in the detector at the opening time of the channel (t_0). The total time of flight for n_3 is then

$$t_f'' = t_0 - \frac{1}{2} T_n. \quad (6''b)$$

The extreme velocity range in a channel is then $v_f'' - v_s''$:

$$v_f'' = \frac{L}{t_f''} = \frac{L}{t_0 - \frac{1}{2} T_n}; \quad (6''c)$$

$$v_s'' = \frac{L}{t_0 + \tau + \frac{1}{2} T_n}. \quad (6''d)$$

Now, in the following considerations, with only a few additional approximations, the above Eqs. (6'') will be shown to define the extreme velocity in a channel of the Fermi chopper which uses a rotating shutter.

Fig. 30.16. Shutter Positions and Resulting Burst Shape

The flight path L is measured from the center of the shutter slit. The position of the fully open slit corresponds to zero.

time. At zero time the position of the center of the burst is known to be at the center of the slit, as can be verified by the analysis of the intensities of the neutrons that get through the shutter, or by considering the symmetries of the burst and the shutter and the uniformity of motions.

The burst produced by the rotating rectangular slit is roughly triangular in shape (see Section 5 on p. 30.29). No error is introduced in the calculation of the extreme velocity range in a channel if the burst shape is considered exactly triangular (see dotted triangle in Fig. 30.15b) and Eqs. (6''), in which we use the actual neutron flight path for neutrons in a triangular shape burst, the extreme velocity range in a channel for this chopper is found to be

$$\Delta v_{\text{ext}} = v_f - v_s = \frac{L}{t_0 - \frac{1}{2}T_n} - \frac{L}{t_0 + \tau + \frac{1}{2}T_n}. \quad (6)$$

All the above Eqs. (6, 6', and 6'') hold strictly only with the following restrictions:

$$t_0 > 2T_n; \quad T_n \approx \tau.$$

In Eq. (6), T_n is the only unknown quantity left. To determine it, consider the neutron burst produced by a rotating rectangular slit of a slow chopper. Its duration varies with the neutron velocity and, of course, with the rotational speed ω of the shutter. For neutrons of high velocity,

$$v > 4\omega R^2/S = 900 \text{ m/sec for } 60 \text{ rps (where } 2R = 4.4 \text{ cm is the slit length and } S = 0.079 \text{ cm slit width),}$$

the duration of the burst is given by the physical opening and closing of the slit:

$$T_n = S/\omega R. \quad (7a)$$

But for neutron speeds of $v < 4\omega R^2/S$,

$$T_n = 4\sqrt{S/\omega v} - 4R/v. \quad (7b)$$

Equation (7b) applies only for neutrons of $v < 900 \text{ m/sec}$ (see Reference 1). The burst shape also depends on neutron velocities and angular speed of the shutter, and its form is given by Eqs. (10a to d) of the next section.

The burst duration being thus given by Eqs. (7), the extreme velocity range in a channel can now be calculated by application of Eq. (6), into which the corresponding opening time and channel width must be substituted.

The burst shapes show that for any neutron velocity there are practically no neutrons either at the start or on the end of the burst. In other words, the probability of a neutron being at either end of the burst is zero. Since the (extreme) velocity range in a channel was calculated by taking the neutrons at the extreme ends of a burst, the probability that such a velocity range actually exists in a channel during measurements is zero. To calculate a more realistic or effective velocity range in a channel, a "resolution function" which takes into account the probabilities (or uncertainties) of recording a given velocity neutron by the time-of-flight technique was introduced.

As mentioned in Part 2 of this Appendix, the velocity spread of the neutrons in a channel depends on the length of the neutron burst, the gate time width, the neutron path in the detector, the detector "jitter" (which is the time of charge collection in the detector), and the neutron time of flight in the detector. The first four factors, which are timing uncertainties in the neutron flight time (see References 1, 4, and 5), combine to give a "resolution function," whose full width at half-maximum (Δt), divided by the flight path (L), is usually defined as the resolution

$$(R = \Delta t/L),$$

(or the resolving power) of the instrument. The manner in which these uncertainties in time of flight measurement combine to give a resolution function is shown in Fig. 30.17. This "combination" is a mathematical

Fig. 17

Time Uncertainties

$$dE = mv dv$$

$$= mv \left(-v \frac{dt}{t} \right) = -mL^2 \frac{dt}{t^3}$$

$$= -2E dt/t.$$

convolution. For this experiment, the major factors contributing to the resolution function are the burst duration T_n and the gate time τ only. The convolution of the two curves representing these factors is carried out graphically in Fig. 30.18.

The velocity and kinetic energy of a certain neutron are expressed in the following relationships, using appropriate units,

$$v = L/t; \quad E = \frac{1}{2}mv^2. \quad (8)$$

Upon differentiating both expressions we obtain

$$dv = -L \frac{dt}{t^2}; \quad dv = -v \frac{dt}{t};$$

An uncertainty or spread in time (Δt) will therefore yield the same fractional uncertainty in velocity and twice the fractional uncertainty in energy.

$$\frac{\Delta v}{v} = \left| \frac{\Delta t}{t} \right|; \quad \frac{\Delta E}{E} = 2 \left| \frac{\Delta t}{t} \right|.$$

Rearrangement of these expressions gives

$$\Delta v = \left| v^2 \frac{\Delta t}{L} \right|; \quad \Delta E = \left| m v^3 \frac{\Delta t}{L} \right|.$$

Fig. 30.18. Graphical Convolution of T_n and τ

By definition, $\Delta t/L$ is the resolution of the chopper, denoted by R_1 and the velocity or energy spread in a specific channel is

$$\Delta v = \bar{v}^2 R; \quad \Delta E = \frac{2.83}{\sqrt{m}} \bar{E}^{3/2} R,$$

with

$$\bar{v} = L/t_{F1}; \quad \bar{E} = \frac{1}{2} m \bar{v}^2; \quad t_{F1} = t_0 + \frac{1}{2} \tau.$$

As an example, if the chopper has a resolution of $50 \mu\text{sec}/\text{m}$, the velocity spread for a channel with an average velocity of 1000 m/sec will be 50 m/sec (5%). This velocity corresponds to 0.014 eV and would have an energy spread of 0.0014 eV (10%). A channel whose average velocity is 6000 m/sec will have a velocity range of 1800 m/sec (30%). This channel's average E would be 0.188 eV and the energy spread 0.113 eV (60%).

The resolution, as defined above, is currently used in most time-of-flight measurements and corresponding literature. Since, by its use, the effective velocity (or energy) range in a channel can be calculated, it will be adopted in this experiment. An illustration of the calculation of the velocity range in a channel using resolution function for this experiment follows:

First, the resolution function curve has to be obtained by convoluting the two main contributing curves of this setup: the burst duration, and the gate time length. The resolution function curve depends on the shape of these two. If the burst curve is rectangular and the gate time curve is also rectangular, the convoluted resolution curve is a triangle (Fig. 30.19a).

Fig. 30.19. Resolution Functions.

In this experiment, the burst-shape curve can be approximated by a triangle. The resultant resolution function is shown in Fig. 30.19b. Notice that the time of flight t_f is measured from center of the burst to midpoint of the channel (gate time), while the chopper photocell angle gives the opening time of the gate (t_0). The time of flight t_{F1} corresponds, therefore, to the mean velocity $\bar{v}$ in a channel and is given by

$$t_{F1} = t_0 + \frac{1}{2} \tau. \quad (9a)$$

Looking again at Fig. 30.18b, if the full width at the base of the resolution function curve is taken, $\Delta t_b = T_n + \tau$. If Δt_b is divided by the flight path L , we obtain resolution R_{ex} , which substituted into Eq. (9), would then give a velocity range Δv_{ex} which would be very close to that given by Eqs. (6) - the extreme velocity range. In the above case, however, there would be

no need to go through the convoluting of the curves to get the resolution function from which to calculate the extreme velocity range. Equations (6) would be much simpler to use. That was the purpose in their [Eq. (6)] derivation: given the physical situation at the chopper (and no "definition of resolution"), calculate the extreme velocity range in a channel. However, it is the effective velocity range that is of more interest; for its calculation, the standard resolution is simpler to use. Again, from Fig. 30.19b, the resolution R (for effective velocity range) is therefore the full width Δt at half-maximum, divided by the flight path L . This R substituted into Eq. (9) gives the effective velocity range in a channel.

Note that there are only six channels with this chopper, which are capable of covering only a portion of the thermal spectrum at a given shutter velocity, flight path, and (first) photocell angle β . If any of these quantities are varied the whole neutron spectrum can be measured. But the experimentally measured spectrum differs from the spectrum of neutrons coming out of the reactor beam hole because the revolving shutter of the chopper does not transmit neutrons of different velocities equally. To determine the "true" relative spectrum, then, the shutter transmission, which depends on the form of the burst, and the sensitivity of the detector must be taken into account. The transmission of the shutter $[f(x)]$ is discussed and calculated next. However, to calculate the absolute neutron spectrum incident on the shutter, the fraction F , which gives the ratio of the recorded (measured) number of neutrons in a velocity range Δv to the number of neutrons in the same range Δv incident on the shutter, must be applied to the measured spectrum.

5) Burst shape and transmission of the shutter. For a perfectly parallel neutron beam and a revolving shutter of rectangular (parallel) slits, the shape of the neutron burst $J(v, t)$ is given by

$$J(v, t) = 1 + \frac{2\omega R}{S} t; \quad \frac{S}{2\omega R} \leq t \leq -\frac{2R}{v}; \quad (10a)$$

$$= 1 - \frac{\omega v}{4S} \left(t - \frac{2R}{v}\right)^2; \quad -\frac{2R}{v} \leq t \leq 0; \quad (10b)$$

$$= 1 - \frac{\omega v}{4S} \left(t + \frac{2R}{v}\right)^2; \quad 0 \leq t \leq \frac{2R}{v}; \quad (10c)$$

$$= 1 - \frac{2R\omega}{S} t; \quad \frac{2R}{v} \leq t \leq \frac{S}{2\omega R}, \quad (10d)$$

where S is the slit width, $2R$ the slit length, ω the angular speed of shutter, and v the neutron velocity.

This expression is given in Reference 1 and may be derived by a detailed analysis of the passage of perfectly collimated, different-velocity neutrons through the rotating rectangular slit.

For neutrons of infinite velocity passing through a rectangular slit of a rotating shutter, the burst shape is a triangle (see Fig. 30.20).

Fig. 30.20. Burst Shape for a Rectangular Slit

This shape can be deduced from a graphical analysis of the uniformly rotating slit and steady flow of neutrons (see Fig. 30.21).

Fig. 30.21. Burst Shape Analysis

Transmission of the rotating shutter:

In Fig. 30.19 each curve represents a burst shape for neutrons of a certain velocity only. For neutrons of different velocity, another burst-shape curve has to be drawn. An elementary area $J(v, t)dt$

represents the relative number of neutrons transmitted at any time between the opening and closing of the shutter. The whole area under the burst-shape curve for neutrons of velocity v represents the total number of neutrons of this velocity transmitted. The ratio of the area under this curve to the area for a burst of neutrons of infinite velocity is (see Reference 1) a measure of the transmission of the rotor for neutrons of velocity v . From Eqs. (10) the following expression for the transmission function can be derived for the burst shape and the above definition of transmission:

$$f(x) = 1 - \frac{8}{3} x^4; \quad 0 \leq x \leq 0.5; \quad (11a)$$

$$f(x) = \frac{16}{3} x (1-x)^2 \left(1 + \frac{x}{2}\right); \quad 0.5 \leq x \leq 1, \quad (11b)$$

where $x = R\sqrt{\omega}/Sv$ and $f(x)$ is the transmission function (see Fig. 30.22). This transmission function, $f(x)$, is used to correct the measured spectrum to obtain the relative neutron spectrum incident on the shutter.

Fig. 30.22. Transmission Function versus Velocity

6) The "cutoff function." For a given angular velocity of the slit, neutrons of velocity

$$v_n < \frac{\omega R^2}{S} \quad (12)$$

will not get through the slit at all. They are "cut off" by the shutter. This can be seen from the comparison of the times spent by the neutron in traversing the slit of length $2R$, and the time between the opening and the closing of the slit. The neutron takes the time

$$t_n = \frac{2R}{v} \quad (13)$$

to traverse the slit, and the slit stays open for

$$t_s = \frac{2S}{R\omega}. \quad (14)$$

If, therefore, $t_n > t_s$, or

$$\frac{2R}{v_n} > \frac{2S}{R\omega}, \quad (15)$$

the cutoff velocity v_n is directly from Eq. (15):

$$v_n < \frac{\omega R^2}{S},$$

which proves Eq. (12) above.

It may happen that the fast neutrons of a burst will catch up with the slow neutrons of the preceding burst before these reach the detector. This is called "overlapping." However, with the cutoff property of the rotating shutter for a given angular velocity of the slit, neutrons of velocity v_n given by Eq. (12) will not get through. This cutoff neutron velocity is also related to the flight path, which should be less than $\pi R^2/S$, that is,

$$L < \frac{\pi R^2}{S} \quad (16)$$

(see Fig. 30.23 for definition of symbols). The comparison between the

Fig. 30.23
Chopper Variables

time of flight (t_{F1}) of the slow neutrons and the time between bursts, called the duty cycle of the shutter (t_d), will prove the relation (16). To avoid overlap,

$$t_{F1} \leq t_d; \quad t_{F1} = L/v_n; \quad t_d = \pi R/\omega R = \pi/\omega;$$

substituting these, we obtain

$$L/v_n \leq \pi/\omega.$$

But

$$v_n = \omega R^2/S,$$

therefore,

$$\frac{L}{\omega R^2/S} \leq \frac{\pi}{\omega},$$

or

$$L \leq \pi R^2/S;$$

which is Eq. (16).

If the relative transmission of the rotor [see Eq. (11)] is plotted against the ratio of time of flight to the time between bursts, instead of against x as in Fig. 30.22, the curve is called the chopper cutoff function. See Fig. 30.24 and compare with Fig. 30.22. Sometimes the cutoff function

Fig. 30.24

Transmission Function

$f(x)$ is plotted against the neutron time of flight t_{F1} only, since the time between bursts (the cycle, of duty time) is constant for a given ω . The geometry of the slit determines the shape of the cutoff function curve, while the angular speed of the slit determines the neutron cutoff velocity (see Fig. 30.24 and Reference 5). It is desirable to operate the chopper in the region where the transmission of the shutter is unity. In Fig. 30.24 this region is the straight portion of the two curves, and it is seen that the cigar-shaped slit is better. Also from the figure, the cutoff velocity is determined from the points where $f(x)$ is zero.

7) Chopper applications.

Spectrum measurement: The slow-neutron spectrum can be measured by this chopper as indicated on the end of Section 4. After the proper corrections in Section 5 are applied, in order to compare this experimental spectrum with the theoretical Maxwell distribution, the following must be considered: If it is assumed that the neutrons are in complete thermal equilibrium in the reactor core, the number with velocities between v and $v + dv$ will be given by

$$n(v) dv = cv^2 \exp(-v^2/v_0^2) dv, \quad (17)$$

where v_0 is the most probable velocity of the Maxwell speed (or density) distribution. (The thermal neutron distributions are discussed in detail in the experiments of Reference 7; see also References 6 and 8.)

The neutrons emerging from a reactor beam hole, however, are described by a flux distribution

$$n(v) v dv = c_1 v^3 \exp(-v^2/v_0^2) dv.$$

Since the neutrons are detected by a BF_3 detector which utilizes the $B(n,\alpha)$ reaction proportional to $1/v$, the distribution of the detected neutrons should be again the velocity distribution $n(v)$ of Eq. (17). When some parameter ϕ other than v is used to express the form of the velocity distribution, then

$$F(\phi) d\phi = n(v) dv = C[(dv/d\phi) n(v)] d\phi.$$

In the chopper measurement, equal intervals of time of flight, t , of the neutrons over the path length L are used to give

$$F(t) dt = C[n(v) dv/dt] dt = Cn(v) v^2 dt,$$

because

$$t = C/v; \quad dt = (C_1/v^2) dv; \quad dv/dt = C_2 v^2.$$

In another form,

$$F(t) dt = K v^4 \exp(-v^2/v_0^2) dt = K t^{-4} \exp(-v^2/v_0^2) dt. \quad (18)$$

Therefore, the measured distribution should be compared with the v^4 distribution of Eq. (18) if plotted on velocity scale, or to the t^{-4} distribution if the time scale is used.

The chopper is also used extensively for the measurement of the total cross sections at different neutron energies. In this application the time-of-flight spectrometer is superior to the crystal spectrometer, because it can measure the cross sections of a sample at several different energies (equal to the number of channels) at the same time.

In the case of a fast neutron chopper, an additional advantage of a chopper over the crystal monochromator is the constancy of the counting rate with energy for the former in the region of dE/E spectrum (see Reference 8). Also, neutron resonances are measured mainly with the fast chopper (see Reference 4).

From the slow-neutron chopper, measurements of the transmission of the cold neutrons (4- to 20-Å range) through crystals, cross sections can be obtained that give information on the nuclear forces and lattice vibrations in crystals. To get enough of the cold neutrons, whose intensity is low, slow-neutron filters are used to reduce the intensity of the "hot" neutrons.

The above subjects are very interesting, but are lengthy and complex, and mainly of importance to the nuclear physicist. They cannot be discussed in this introductory experiment; the interested student is therefore referred to Reference 8.

However, since by the use of slow-neutron filters, due to their "crystal cutoff" property, the resolution of the chopper and the velocity of neutrons in a channel can be verified experimentally, a brief discussion on filters follows. (The student should, nevertheless, consult References 8 and 9.)

Slow-neutron filter materials as, for example, Be, BeO, Fe, C, and Bi have the property of scattering all the neutrons of wavelength below a certain maximum value λ_m , but letting pass through "all" the neutrons of $\lambda > \lambda_m$. Out of an incident beam of thermal neutrons, then, a BeO filter will let pass through without scattering only the cold neutrons of $\lambda > 4$ Å. This property of a filter can be explained by referring to the Bragg equation of crystal diffraction:

$$n\lambda = 2d \sin \phi.$$

The filter material contains many crystal grains oriented at random (a polycrystal). When a neutron of wavelength λ encounters a crystal grain which is at such an angle that the Bragg equation is satisfied, this neutron will be diffracted out of the beam path. Since there are many grains of all orientations (ϕ varies from 0 to 90°) and many planes of different spacings d , all the neutrons of wavelength up to

$$\lambda_m = 2d_m$$

will be scattered out of the beam. This wavelength, equal to twice the largest lattice spacing d_m , is called the "cutoff wavelength." The remaining neutrons of wavelengths $\lambda > \lambda_m$, which cannot satisfy the Bragg law, will therefore pass through the filter undiffracted. However, some of these cold neutrons can still be removed from the beam by thermal inelastic scattering in the filter. The inelastic scattering is strongly temperature dependent, so that the transmission of the cold neutrons can be increased by cooling the filter.

The scattering cross sections as functions of velocity of some filter materials are shown in Fig. 30.25. These cross sections were calculated from the "free atom" cross sections and the crystal structure of materials in Reference 8.

Fig. 30.25. Total Cross Section versus Wavelength

Although the distribution of the filtered neutrons is not monoenergetic, the exceedingly sharp cutoff gives an identifiable wavelength of extreme accuracy (that of the interplanar crystal spacing). Since the cross sections for neutrons of wavelength near filter cutoffs can be easily measured with the slow chopper, they can be used for an experimental check on the calculated resolution and velocity in the following way:

The position of the break in the cross section is the experimental check on the calculation of neutron velocity, and the sharpness of the break is the measure of the resolution width of the instrument. As an example, take the transmission of an iron powder near cutoff wavelength measured by the chopper. For an ideal case, a very sharp break in the cross section should occur at a wavelength of 4.053 Å, which is the value of $2d$ for the (110) planes in iron. But, due to the resolution of the chopper, this break appears less sharp. The reciprocal of the measured transmission of iron powder is plotted on logarithmic scale (see Fig. 30.26) so that linear distances from the base line are proportional to the cross section. The cross section is expected to rise towards long wavelength on both sides of the break. The transmission curve will thus show a well defined maximum and minimum. The difference in wavelength between them is the practical resolution width of the chopper (see Reference 9).

Some of the theory and equations in the preceding sections apply strictly for idealized conditions. Example: perfect collimation,

no "edge effects" (no leakage of neutron through edges of the shutter), etc. Nevertheless, the equations are used in the procedure and serve as good approximations.

Fig. 30.26

Reciprocal Transmission versus
Neutron Wavelength

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STUDENT RESULTS

Determination of $\bar{v}$ and Δv in Each Channel

Channel No.	t_{F1} (μsec)	$\bar{v}$ (m/sec)	$(\bar{v})^2$ $\times 10^6$ (m^2/sec^2)	$\bar{v}^2 R = \Delta v$ (m/sec)
2	370	5405	29.2	1183
3	460	4348	18.9	766
4	550	3636	13.2	535
5	640	3125	9.77	395
6	730	2740	7.51	304
7	820	2439	5.95	241
8	910	2198	4.83	196
9	1000	2000	4.00	162
10	1090	1835	3.37	136
11	1180	1695	2.87	116
12	1270	1575	2.48	100
13	1360	1471	2.16	87.5
14	1450	1379	1.90	77.0
15	1540	1299	1.69	68.3
16	1630	1227	1.51	61.0
17	1720	1163	1.35	54.7
18	1810	1105	1.22	49.4
19	1900	1053	1.11	44.9
20	1990	1005	1.01	40.9
21	2080	962	0.924	37.4

$$\bar{v} = \frac{2 \text{ meters}}{t_{F1}}$$

$$\Delta v = \bar{v}^2 R$$

$$R = 40.5 \times 10^{-6} \text{ s/m}$$

Velocity transmission limits

$$v_1 = \frac{4R}{T_n} = \frac{4 \times 2.3}{94.4 \times 10^{-6}} = 975 \text{ m/s} \quad v_{\text{cutoff}} = \frac{R}{T_n} = 244 \text{ m/s}$$

Fig. S30.1. Total Cross Section for Au^{197}

TOTAL CROSS SECTION OF Au¹⁹⁸

Channel No.	Counts (Open Beam)	Counts (Au 40 mil)	$T = I/I_0$	$\ln T$	σ_T (b)	E (eV)
2	4,955	3,501	0.704	0.352	58.7	0.153
3	9,887	6,821	0.690	0.372	62.0	0.0992
4	12,327	8,523	0.688	0.376	62.6	0.0694
5	12,357	7,911	0.640	0.448	74.6	0.0508
6	9,731	6,122	0.630	0.464	77.3	0.0394
7	8,813	4,817	0.545	0.616	103	0.0316
8	6,594	3,471	0.529	0.640	106	0.0254
9	5,438	2,558	0.472	0.754	126	0.0210
10	4,075	1,870	0.460	0.780	130	0.0177
11	3,100	1,331	0.431	0.846	141	0.0151
12	2,642	941	0.356	1.04	172	0.0130
13	2,214	934	0.422	0.871	145	0.0114
14	1,756	531	0.303	1.20	200	0.0100
15	1,339	597	0.446	0.811	135	0.00886
16	1,035	385	0.372	0.990	165	0.00781
17	874	485	0.556	0.588	98	0.00710
18	497	-	-	-	-	0.00690
19	509	-	-	-	-	0.00083
20	420	-	-	-	-	0.00530

Transmission equation

$$T = \frac{I}{I_0} = e^{-\sigma_T N t}$$

where,

 σ_T = the total microscopic cross sectionsN = number of Au atoms per cm³ (0.0591×10^{24})

t = Au thickness (0.040 in.).

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Channel No.	Counts/ $\bar{v}^2$ $\times 10^{-3}$	Normalized Counts/ v^2
2	0.170	0.117
3	0.523	0.361
4	0.934	0.644
5	1.265	0.872
6	1.296	0.893
7	1.481	1.020
8	1.365	0.941
9	1.360	0.938
10	1.209	0.834
11	1.080	0.745
12	1.065	0.734
13	1.025	0.707
14	0.924	0.637
15	0.792	0.546
16	0.642	0.443
17	0.647	0.446
18	0.407	0.281
19	0.458	0.316
20	0.415	0.286

Theoretical Distribution

$$n(v) = Av^2 e^{-v^2/v_0^2}$$

At

$$v = v_0 = 2450 \text{ m/s}$$

$$n(v) \text{ equals } 1$$

$$A = \frac{1}{(2450)^2} = 1.667 \times 10^{-7}$$

$$n(v) = 1.667 \times 10^{-7} v^2 e^{-v^2/v_0^2}$$

v	n(v)
0	0
500	0.109
1,000	0.383
1,500	0.700
2,000	0.930
2,500	0.999
3,000	0.910
3,500	0.720
4,000	0.504
4,500	0.314
5,000	0.176
5,500	0.089
6,000	0.041
6,500	0.017
7,000	0.006

Fig. S30.2. Neutron Velocity Distribution