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**INTERPOLATION IN EXCITATION ENERGY
FOR TWO-DIMENSIONAL TABULAR DATA**

S. T. Perkins

R. J. Doyas

June 1973

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LAWRENCE LIVERMORE LABORATORY
University of California, Livermore, California, 94550

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INTERPOLATION IN EXCITATION ENERGY FOR TWO-DIMENSIONAL TABULAR DATA

Abstract

We present techniques to interpolate two-dimensional tabular arrays that describe either inelastic neutron

scattering through resolved or unresolved levels, or discrete photon production spectra.

Introduction

Recently, we proposed two transformation methods to handle interpolation between tabular arrays of two independent variables.^{1,2} Both methods retain linearly interpolable end points, something simple linear interpolation cannot do, as well as linear interpolation of area. Data containing resolvable line excitations, e.g., inelastic neutron scattering or photon production, cannot be treated by these transforms. Therefore, we suggested² that such line excitations be extracted from the distributions and presented explicitly.

F. G. Perey³ pointed out that an important intermediate category of data exists: emergent neutron spectra from inelastic neutron scattering in which obvious line excitation effects are present, but where the lines are unresolved. An example of this may be seen in the emergent neutron spectra from the $^{60}\text{Ni}(n, n')$ reaction shown in Fig. 10 of Ref. 4. When these spectra are presented in terms of the average excitation energy in ^{60}Ni rather than of the emergent neutron en-

ergy, the unresolvable line excitations at various incident neutron energies are completely aligned. Any valid interpolation method for such data must be derived with the cognizance of this observation.

It has further been noted that the unresolved level excitations are sitting on a background that may be described by an evaporation model.⁵ In principle, this portion of the spectrum can be removed and interpolated with the techniques outlined in Refs. 1 and 2. Additionally, the low-neutron-energy (high-excitation-energy) portion of the spectrum is experimentally poorly resolved,⁴ and an evaluator could use the evaporation model to complete this portion of the spectrum.

An example of emergent neutron spectra through unresolved level excitation is shown in Fig. 1. Two tabular spectra are given at the incident neutron energies, E_1 and E_3 , and we wish to obtain an interpolated spectrum at the incident neutron energy E_2 , which is bounded such that $E'_{\min}(E_2) \leq E' \leq E'_{\max}(E_2)$. Suppose that straight linear interpolation

is used to obtain $\pi(E_2 \rightarrow E')$. This technique falls on several points; the interpolated distribution falls outside the specified bounds, and the interpolated spectrum contains six unresolved levels, rather than, as we show later, the correct number of three. The unit base transformation^{1,2} fares no better, since, although it preserves the intermediate bounds, the interpolated spectrum now contains seven unresolved levels.

It is apparent that an appropriate interpolation scheme must take more of the physics into account — particularly, the alignment of unresolved levels that occurs when they are associated with the excitation energy of the target nucleus. We found a technique that is valid for unresolved inelastic level excitations, with or without the background present; it is equally valid for discrete inelastic level excitations, assuming that they can

Fig. 1. Inelastic neutron spectra as a function of emergent neutron energy. Unresolved levels are numbered by increasing excitation energy. The dashed lines between resonances ② and resonances ③ correspond to constant excitation energy.

be described by numerical delta-function neutron spectra (narrow equal-base triangular distributions).

We also found a method for interpolation of discrete gamma ray production

spectra, assuming that a numerical delta-function representation can be used, since sometimes many levels are present that are not important enough to merit individual representation.

Inelastic Scattering

Assume a tabular array $\pi(E \rightarrow E')$, the probability of a neutron being transferred from incident energy E to emergent energy E' . Let $\pi(E \rightarrow E')$ be known at the points E_j, E'_i . The ground rules imposed on these tabular distributions are as follows:

Rule 1: $\pi(E_j \rightarrow E')$ is piecewise linearly interpolable in E' , i. e., linearly interpolable between $E'_{i-1} \leq E' \leq E'_i$ ($\min \leq i \leq \max$).

Rule 2: The end points of $\pi(E_j \rightarrow E')$ (corresponding to values of E' that bound the distribution) are piecewise linearly interpolable in E , i. e., linearly interpolable between $E_{j-1} \leq E \leq E_j$. Furthermore, the linear relation between end points corresponds to lines of zero probability.

Rule 1 defines the one-dimensional type of interpolation that will be used. Rule 2 is consistent with the physics of the problem, e. g., in the evaporation spectrum, $E'_{\min} = 0$ and $E'_{\max} = E - E_{th}$, where E_{th} is the threshold of the evaporation component. An example of the type of arrays under discussion is shown in Fig. 1.

The data are now transformed (usually by a processing code) to excitation energy, in which the levels are aligned. The average excitation energy (neglecting recoil), W , is defined as

$$W = E - E'. \quad (1)$$

The transformed distribution, $\pi(E \rightarrow W)$, is then equal to

$$\pi(E \rightarrow W) = \pi(E \rightarrow E - E'). \quad (2)$$

This transformation, performed on the Fig. 1 data, is shown in Fig. 2. The task now is to determine the interpolated distribution at incident energy E_2 . Linear interpolation may be used on these transformed data because of the level alignment that has occurred. Care, however, must be taken to renormalize the interpolated area to account for the changing end points.

The first step is to determine the bounds $W_{\min}$ and $W_{\max}$ on the distribution at E_2 . From Rule 2,

$$W_{\min}(E_2) = W_{\min}(E_1) + \{W_{\min}(E_3) - W_{\min}(E_1)\} \left(\frac{E_2 - E_1}{E_3 - E_1} \right), \quad (3)$$

with a similar result for $W_{\max}(E_2)$.

The E, W plane is now to be divided into five regions between E_1 and E_3 as follows:

Fig. 2. Inelastic neutron spectra as a function of average excitation energy. Unresolved levels are numbered by increasing excitation energy. The dashed lines are lines of constant excitation energy, separating the five regions identified by numbers in squares.

Region	W bounds: $W^-(i) \leq W \leq W^+(i)$	
1	$\{\min [W_{\min}(E_1), W_{\min}(E_3)]\}$	4 $\{\min [W_{\max}(E_1), W_{\max}(E_3)]\}$
	$\leq W \leq W_{\min}(E_2)$	$\leq W \leq W_{\max}(E_2)$
2	$W_{\min}(E_2) \leq W \leq \{\max [W_{\min}(E_1),$	5 $W_{\max}(E_2) \leq W \leq \{\max [W_{\max}(E_1),$
	$W_{\min}(E_3)]\}$	$W_{\max}(E_3)]\}$
3	$\{\max [W_{\min}(E_1), W_{\min}(E_3)]\}$	
	$\leq W \leq \{\min [W_{\max}(E_1),$	
	$W_{\max}(E_3)]\}$	

Within any specific region, the partial areas over the initial distribution, $A_1(E)$, are equal to

$$A_1(E) = \int_{W^-(i)}^{W^+(i)} \pi(E \rightarrow W) dW. \quad (4)$$

Additionally, areas summed over several consecutive regions are defined, for example, as

$$A_2^4(E) = \sum_{i=2}^4 A_i(E). \quad (5)$$

The total area,

$$A(E) \equiv A_1^5(E), \quad (6)$$

is considered to be a known linearly interpolable function, e.g., equal to unity for our probability distributions.

Specifications within the various regions are now considered separately:

Region 1: This region lies below the lower bound of the interpolated distribution and hence there is no contribution. This is identified by the cross-hatched area in Fig. 2.

Region 2: In this region, one must be cognizant of Rule 2. For a specific excitation energy, linear interpolation is used between the distribution at E_1 and a value of zero at the interaction with $W_{\min}(E)$. For a more comprehensive visual representation (although not explicitly required for the calculation), the area in this region is extrapolated onto the plane at E_3 , where it is of course negative (shown by the hatched area in Fig. 2). The value of E for which the probability is zero is denoted by $E_{\text{in}}(W)$. The following two cases require investigation:

$W_{\min}(E_1) < W_{\min}(E_3)$: In this case, the projected area is onto the plane at E_3 (as in Fig. 2). The value of $E_{\text{in}}(W)$ is given by

$$E_{\text{in}}(W) = E_2 + [E_3 - E_2]$$

$$\times \left[\frac{W - W_{\min}(E_2)}{W_{\min}(E_3) - W_{\min}(E_2)} \right], \quad (7)$$

with the corresponding extrapolated distribution point by

$$\pi(E_3 \rightarrow W) = \pi(E_1 \rightarrow W) \left[\frac{E_{\text{in}}(W) - E_3}{E_{\text{in}}(W) - E_1} \right]. \quad (8)$$

$W_{\min}(E_1) > W_{\min}(E_3)$: The projected area here is onto the plane at E_1 . The value of $E_{\text{in}}(W)$ is given by

$$E_{\text{in}}(W) = E_2 - [E_2 - E_1]$$

$$\times \left[\frac{W - W_{\min}(E_2)}{W_{\min}(E_1) - W_{\min}(E_2)} \right], \quad (9)$$

with the corresponding extrapolated distribution point by

$$\pi(E_1 \rightarrow W) = \pi(E_3 \rightarrow W) \times \left[\frac{E_{\text{in}}(W) - E_1}{E_{\text{in}}(W) - E_3} \right]. \quad (10)$$

Once the distributions are specified at E_1 and E_3 , the interpolated distribution is obtained by linear interpolation,

$$\pi(E_2 \rightarrow W) = \pi(E_1 \rightarrow W) + [\pi(E_3 \rightarrow W) - \pi(E_1 \rightarrow W)] \left(\frac{E_2 - E_1}{E_3 - E_1} \right). \quad (11)$$

Region 3: Distributions in this region are directly linearly interpolable, as in Eq. (11).

Region 4: This is entirely similar in context to Region 2. Again, there are two cases requiring investigation:

$W_{\max}(E_1) < W_{\max}(E_3)$: In this case, the projected area is onto the plane at E_1 (as in Fig. 3). The value of $E_{in}(W)$ is

$$E_{in}(W) = E_2 - [E_2 - E_1]$$

$$\times \left[\frac{W_{\max}(E_2) - W}{W_{\max}(E_2) - W_{\max}(E_1)} \right], \quad (12)$$

with the corresponding extrapolated distribution point given by Eq. (10).

$W_{\max}(E_1) > W_{\max}(E_3)$: Here, the projected area is onto the plane at E_3 . The value of $E_{in}(W)$ is

$$E_{in}(W) = E_2 + [E_3 - E_2] \times \left[\frac{W_{\max}(E_2) - W}{W_{\max}(E_2) - W_{\max}(E_3)} \right], \quad (13)$$

with the corresponding extrapolated distribution point given by Eq. (8). The interpolated distribution at E_2 is again given by Eq. (11).

Region 5: This region is exactly similar in context to Region 1.

Fig. 3. Gamma-ray production spectra as a function of photon energy. The discrete photons are numbered according to increasing energy.

The interpolated distribution at E_2 , as shown in Fig. 2, has been completely specified. However, as is obvious with the neglect of Regions 1 and 5 and the inclusion of negative areas in Regions 2 and 4, normalization has not been preserved. This is easily rectified by re-normalization. Defining the normalized distribution at E_2 by $\pi_n(E_2 \rightarrow W)$, one obtains

$$\pi_n(E_2 \rightarrow W) = \pi(E_2 \rightarrow W) \frac{A_n(E_2)}{A(E_2)}, \quad (14)$$

where $A(E_2)$ is given by Eq. (6) and $A_n(E_2)$, obtained by linear interpolation of initial areas, is equal to

$$A_n(E_2) = A(E_1) + [A(E_3) - A(E_1)] \times \left(\frac{E_2 - E_1}{E_3 - E_1} \right). \quad (15)$$

The normalized distribution may then be transformed back to the emergent neutron energy by Eqs. (1) and (2).

Gamma-Ray Production

The tabular array is $\pi(E \rightarrow E_\gamma)$, the probability of a neutron of energy E creating a photon of energy E_γ , and is tabulated at the points E_j and E_{j+1} . The first ground rule of the proceeding section will be used (exchanging E_γ for E') along with the following additional rules:

Rule 3: The probability for each discrete photon is described by a numerical delta function with linear interpolation performed in E , i. e., between $E_{j-1} \leq E \leq E_j$.

Rule 4: No extrapolation in incident energy will be permitted. This requires that the complete probability distribution be specified at the bounds in E (threshold energy and upper energy at which the photon no longer appears) for any specific photon.

An example of this type of array is shown in Fig. 3. The discrete photons, ① and ⑤, at E_3 are taken to designate the threshold of these specific photons, since they do not appear in the distribution at E_1 . Similarly, for the discrete photon

③, E_1 is the upper neutron energy at which this photon appears, since it does not appear in the E_3 spectra. The combination of Rules 3 and 4 results in interpolation being performed only when the photon energies appear in the distributions at both E_1 and E_3 .

The partial area over a discrete photon, A_1 , is equal to

$$A_1(E) = \int_{E_{\gamma i-1}}^{E_{\gamma i+1}} \pi(E \rightarrow E_\gamma) dE_\gamma. \quad (16)$$

The total area at a given E , $A(E)$, is

$$A(E) = \sum_i A_1(E) \quad (17)$$

and is a known linearly interpolable function, e. g., equal to unity for our probability distributions. Since the photon energies are already aligned, direct linear interpolation, Eq. (11) with E_γ exchanged for W , may be used to yield

the intermediate distribution of E_2 as shown in Fig. 3. This interpolated distribution is no longer normalized, but may be renormalized, analogous to Eq. (14), by

$$\pi_n(E_2 \rightarrow E_\gamma) = \pi(E_2 \rightarrow E_\gamma) \frac{A_n(E_2)}{A(E_2)}, \quad (18)$$

where $A_n(E_2)$ is given by Eq. (15). The photon spectra at incident neutron energy E_2 is then completely specified.

Conclusions

We have discussed two supplemental interpolation techniques for two-dimensional tabular arrays. One of these allows interpolation for inelastic scattering in which the levels may be either resolved or unresolved; linearly interpolable end points are maintained. The other technique describes interpolation for discrete photon-production spectra. While neither technique preserves area,

renormalization is easily performed on the intermediate distribution, which is linearly interpolable in final energy.

These present techniques, along with the interpolation methods previously described for continuum type spectra,^{1,2} should allow a valid description of most neutron- and photon-production spectra encountered in neutronics and photonics analysis.

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