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GENERATOR TIME IN A SUBCRITICAL FAST REACTOR

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VERIFICATION OF A NEW METHOD FOR CALCULATING AND MEASURING
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ABSTRACT

We have verified a new method for calculating and measuring the generation time (Λ) in a subcritical fast reactor using a recently developed transfer and scattering kinetics scheme (TASK). The method is an extrapolation of a four-node fluctuations theory in which the generation time in a thermal reactor was shown to be equal to the slope of the tangent of the phase angle from the power spectra between a high-energy-sensitive and low-energy-sensitive detector. The TASK method directly computes the prompt die-away time constant and the Λ based on the tangent method. (Present experimental techniques do not provide a direct measure of Λ for subcritical states.)

To substantiate our TASK calculations, we compared Λ with the experimental value at critical and computed Λ using the standard lambda (λ) mode and omega (ω) mode definitions for critical and for three subcritical states of the Fast Test Reactor.

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I. INTRODUCTION

The determination of the shutdown state of a system is desirable for safety and economic reasons (we adapt the commonly used parameter "reactivity," ρ , as a measure of the shutdown). Unfortunately, reactivity is not an observable quantity and must be inferred from experimentally observable quantities and calculated coefficients. Reactivity is usually expressed as a ratio of averages which are not amenable to direct measurement. Furthermore, since ρ does not appear in the equations which describe the behavior of neutrons in the reactor, its definition is completely arbitrary. However, ρ is expected to be zero at critical and to globally represent the neutron balance of the system.

The definition of ρ involves an arbitrary weighting function W . Once W is selected, ρ is defined and it becomes possible to develop the point kinetics equations and perhaps, depending on the selection of W , to develop an expression which relates ρ to an observable quantity (an inhour equation). For such developments the definitions of the neutron generation time, Λ , and the effective delayed neutron fraction $\bar{\beta}_1$ are established according to the selected W . $\bar{\beta}_1$ and Λ are generally measurable only as ratios and their experimental separation is difficult.

We have verified a new method for both calculating and measuring the generation time Λ in a fast reactor independent of $\bar{\beta}_1$ using a recently developed transfer and scattering kinetics (TASK) scheme. The method is an extrapolation of a four-node fluctuation theory which has been demonstrated successfully for thermal systems.

II. REVIEW OF METHODS FOR CALCULATING GENERATION TIME

The selection of the weighting function used in developing the point kinetics equations from the Boltzmann equation defines reactivity, neutron generation time, and the effective delayed neutron fraction. This approach is discussed in the excellent papers of Preskitt et al.¹ and Henry.² It has become customary, based on the work of Henry, to use ϕ_λ and ϕ_λ^* as modal functions in defining Λ -- we refer to this as the λ -mode definition. Preskitt relates ρ_λ (commonly used by reactor designers) to the prompt die-away time constant ω_p using ϕ_λ^* and ϕ_ω as the modal functions. We refer to this as the ω -mode definition of Λ . For a purely analytical approach to developing the point kinetics equations, it does not matter whether one uses the λ - or ω -mode definition of Λ . It is only when we attempt to relate ρ to an observable quantity that the definition of Λ is fixed.

We develop the ω -mode definition of Λ by following the work of Preskitt.¹ We select the λ -mode definition of reactivity

$$\rho_\lambda = 1 - \frac{1}{\lambda_0} \quad (1)$$

where λ_0 is the largest eigenvalue of the λ -mode equation which is generally solved by standard reactor design codes.

$$[L + \frac{1}{\lambda_0} f_\lambda P] \phi_\lambda = 0 \quad (2)$$

where

P and L are production and loss operators,

ϕ is the flux vector, and

$$f_\lambda \equiv (1 - \beta_T) f_p + f_d \beta \quad (3)$$

We choose to relate this definition of ρ_λ to an observable quantity. To do this we write the standard balance equations

$$(L + (1 - \beta_T)f_p P)\phi + f_d \bar{\lambda} C + s = v^{-1} \frac{\partial \phi}{\partial t} \quad (4)$$

and
$$\beta P \phi - \bar{\lambda} C = \frac{\partial C}{\partial t} \quad (5)$$

where C is the precursor vector and $\bar{\lambda}$ the decay constant.

Assume that we have a stable period with an observable decay constant ω_p .

$$\phi = \phi_p e^{\omega_p t} \quad (6)$$

$$C = C_p e^{\omega_p t} \quad (7)$$

Substitution of Eqs. 6 and 7 into Eqs. 4 and 5 and subsequent elimination of C yields

$$[(L - \frac{\omega_p}{v} - f_1 P) \phi_\omega = 0 \quad (8)$$

where

$$f_1 = f_\lambda - \frac{\omega_p f_d \beta}{[\omega_p I + \bar{\lambda}]} \quad (9)$$

To relate ρ_λ and ω_p , we use the adjoint λ -mode equations (adjoint of Eq. 2).

$$[L^* + \frac{1}{\lambda_0} (f_\lambda P)^*] \phi_\lambda^* = 0 \quad (10)$$

Multiplication of Eq. 10 by ϕ_ω and subsequent integration over all phase space yields

$$\int_P \phi_\omega L^* \phi_\lambda^* = - \frac{1}{\lambda_0} \int_P \phi_\omega (f_\lambda P)^* \phi_\lambda^* \quad (11)$$

By "adjointness" we can write Eq. 11 as

$$\int_P \phi_\lambda^* L \phi_\omega = - \frac{1}{\lambda_0} \int_P \phi_\lambda^* f_\lambda P \phi_\omega \quad (12)$$

Multiplication of Eq. 8 by ϕ_λ^* and integration over all phase space, p , yields

$$\int_P \phi_\lambda^* L \phi_\omega = \int_P \phi_\lambda \frac{\omega_P}{v} \phi_\omega - \int_P \phi_\lambda (f_1 P) \phi_\omega \quad (13)$$

Substitution of Eq. 12 into Eq. 13 gives

$$-\frac{1}{\lambda_0} \int_P \phi_\lambda^* f_\lambda P \phi_\omega = \int_P \phi_\lambda^* \frac{\omega_P}{v} \phi_\omega - \int_P \phi_\lambda^* f_1 P \phi_\omega \quad (14)$$

From Eq. 1

$$-\frac{1}{\lambda_0} = (\rho_\lambda - 1) \quad (15)$$

Thus

$$\rho_\lambda - 1 = \omega_P \frac{\int_P \phi_\lambda^* \frac{1}{v} \phi_\omega}{\int_P \phi_\lambda^* f_\lambda P \phi_\omega} - \frac{\int_P \phi_\lambda^* f_1 P \phi_\omega}{\int_P \phi_\lambda^* f_\lambda P \phi_\omega} \quad (16)$$

To obtain the inhour equations, we define the ω -mode generation time as

$$\Lambda_\omega = \frac{\int_P \phi_\lambda^* \frac{1}{v} \phi_\omega}{\int_P \phi_\lambda^* f_\lambda P \phi_\omega} \quad (17)$$

and substitute Eq. 3 into Eq. 16 to obtain

$$\rho_\lambda = \omega_P \Lambda_\omega + \sum_i \frac{\bar{\beta}_i \omega_P}{\omega_P + \lambda_i} \quad (18)$$

where the effective delay fraction $\bar{\beta}_i$ is defined as

$$\bar{\beta}_i = \beta_i \frac{\int_P \phi_\lambda^* f_d^i P \phi_\omega}{\int_P \phi_\lambda^* f_\lambda P \phi_\omega} \quad (19)$$

If no attempt were made to relate ρ_λ^* to the observable ω_P , one could follow a similar procedure using ϕ_λ as the weighting function and derive the λ -mode definition of Λ , i.e.,

$$\Lambda_{\lambda} = \frac{\int_P \phi_{\lambda}^* \frac{1}{v} \phi_{\lambda}}{\int_P \phi_{\lambda}^* f_{\lambda} P \phi_{\lambda}} \quad (20)$$

III. REVIEW OF CORRELATION METHODS FOR DETERMINING GENERATION TIME

A two-energy group reactor noise correlation development by Nieto et al.³ and by Ackermann⁴ suggested that neutron lifetimes* could be directly related to tangent of the angle of the cross power spectral density (CPSD) between the two energy groups. That is

$$\tan \psi_{12}(\omega) = \frac{\text{Im } \psi_{12}(\omega)}{\text{Re } \psi_{12}(\omega)} = -\omega \ell \quad (21)$$

where

ψ_{12} is the CPSD between groups 1 and 2,
 ω is the angular frequency,
 and
 ℓ is the lifetime.

Ackermann⁵ investigated this effect by performing experiments in a small H₂O Pool Critical Assembly (PCA). His results were obtained by using two low-energy sensitive detectors - one in the core and one outside the core. The assumption was that neutrons counted by the detector outside the core were the high-energy neutrons leaking from the core and thus the CPSD between these two simulated ψ_{12} in Eq. 21. He measured both the real and imaginary components of ψ_{12} from which $\tan \psi_{12}$ was constructed. His results indicated unexpected space energy effects. He concluded that a high-energy-sensitive detector must be located in the core along with the low-energy-sensitive detector in order to determine lifetime adequately.

Based on Ackermann's results, Penland⁶ developed a four-nodal point neutron fluctuation theory which allowed two energy groups and two space points - one for the core and one representing the reflector. Penland's equations were too complex to allow a simple closed-form solution as per Eq. 21 and required computer solutions. He was, however, able to relate ψ_{12}^C in the core and ψ_{12}^R in the reflector to the core and reflector lifetimes respectively. Penland followed his analytical work with an investigation in the PCA in which he placed a low-energy-sensitive and a high-energy sensitive detector in the core and an identical pair of detectors in the reflector. He was able to relate the $\tan \psi_{12}^C$ and $\tan \psi_{12}^R$ to the core and reflector lifetimes.

Alexander⁷ extended these nodal developments to three energy groups and continuous space using diffusion theory. His thesis was that by employing a space-dependent noise equivalent source simply distributed according to the fission source in space and energy, one could compute directly the dynamic fluxes (Fourier transformed fluxes) for the system. Then one could form directly the auto power spectral density (APSD) and the CPSD between any two detectors. To demonstrate his thesis he modeled the PCA and the detectors used by Penland and computed Penland's results. His results were indeed impressive. He was able to compute lifetimes quite accurately. His encouraging results provided impetus for extending this type of an approach to fast reactors using transport theory.

*Nieto et al. used the nodal model based on the lifetime formulation. Lifetime (inverse destruction probability per unit time) is related to generation time (inverse production probability per unit time) through the system multiplication factor. In particular, they are numerically equal at delayed critical.

IV. GENERATION TIME FOR FAST SYSTEMS

We have extrapolated the two-detector correlation method for determining generation time to critical and subcritical fast reactors in continuous space and energy using transport theory. We used a recently developed transfer and scattering kinetics scheme (TASK)⁸ for computing the dynamic fluxes and developed ancillary programming to compute the APSD and CPSD for arbitrary detector types (i.e., ²³⁵U fission counters, ²³⁸U fission counters, ³He counters, proton-recoil, etc.) located anywhere in the system. The prompt die-away time constant ω_p and the Λ based on the tangent method were obtained from the APSD and CPSD.

To perform numerical experiments we chose a simple one-dimensional representation of the homogeneous mockup of the Fast Test Reactor.⁹ The model is defined in Fig. 1. It should be reemphasized that one can never really measure a global quantity like Λ , thus we are not computing either the ω - or λ -mode definition of Λ with TASK, but rather the physical lag time between high-energy-sensitive and low-energy-sensitive detectors which in the presence of no contamination should be Λ .

To substantiate our TASK⁸ calculations, we compared Λ with the experimental value at critical and computed Λ using the lambda (λ) mode and omega (ω) mode definitions for critical and for three-subcritical states of the Fast Test Reactor. The λ - and ω -mode calculations were implemented using the one-dimensional transport code ANISN.¹⁰

The same neutronics model and multigroup constants were used in all calculations. The model was a one-dimensional, 13-region representation of the FTR, using an S_4 quadrature and a buckling correction for leakage. The multigroup constants were in five energy groups, with a P_3 scattering expansion. The low-energy-sensitive ²³⁵U fission counter used in the experiment was simulated in TASK for computing the prompt die-away time constants. A high-energy-sensitive detector with a ²⁴⁰Pu fission response was used in conjunction with the ²³⁵U detector response to compute Λ directly.

V. RESULTS

The calculated values of Λ are summarized in Table 1 and may be compared with the measured value at critical. The λ -mode and ω -mode Λ values are computed according to Eqs. 17 and 20 using static fluxes generated by ANISN. The Λ values labeled "TASK, ρ_λ , ω_p method" were generated by using the inhour equations, Eq. 18, ρ_λ and the ω_p computed directly by TASK and β of 0.00297.

The procedure for obtaining the $\tan\theta$ values is depicted in Figs. 2 and 3. Figure 2 shows typical calculated values of the real and imaginary parts of CPSD_{12} , where 1 and 2 refer to the ²³⁵U and ²⁴⁰Pu fission counters respectively. Figure 3 contains plots of $\tan\theta$ for the -7 dollar case shown in Fig. 2 and for the -30 dollar case.

Conclusions from the numerical experiments are:

1. The TASK $\tan\theta$ method yields values of Λ which agree with both the critical experimental value and values computed by standard methods.
2. Values of Λ generated using the TASK ρ_λ , ω_p method are consistent with the ω -mode values except for the -28.15 dollar case in which modal contamination is extreme.

O X	CORE	CORE	CORE	CORE ZONE 2	REFLECTOR + CONTROL MATERIAL	REFLECTOR	SHIELD
	ZONE 1	ZONE 1 + LOOPS	ZONE 1				

O - ^{235}U Fission Counter

X - ^{240}Pu Fission Response

Fig. 1. Calculational Model

Table 1. Generation Time for the FTR-3, ZPR9

Reactivity (ρ_λ) (-%)	Neutron Generation Time (μ sec)				Experimental	
	λ -Mode	ω -Mode	TASK ρ_λ, ω_p Method	TASK Tan θ Method		
0.0	0.761	0.761	0.767	0.758	0.750	0.018
1.17	0.735	0.741	0.744	0.743	-	
6.89	0.668	0.697	0.708	0.700	-	
28.15	0.496	0.571 ^a	0.515 ^a	0.510	-	

^aA single ω_p is difficult to ascertain for this shutdown.

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3. In principle, an experiment can be performed in a fast system using a high-energy-sensitive and a low-energy-sensitive detector to determine Λ . However, there are certain practical limitations to making such a measurement. Several of these are discussed in the appendix.

APPENDIX

COMMENTS ON THE SUBCRITICAL MEASUREMENT OF Λ IN FAST SYSTEMS

No measurements of Λ have been made in fast systems using the $\tan^{-1}$ method. However, this method appears to be attractive for core characterization during start-up testing of fast systems to separate generation time from the effective delayed-neutron fraction. The method is probably not usable in high-gamma environments owing to restrictions on the high-energy detector. For this study we used the fission of ^{240}Pu as the high-energy sensor; however, such a detector may not be practical owing to availability of pure materials and assembly problems. We also did a few calculations using a simulated ^{238}U fission counter and the calculated results were satisfactory. Again, this detector is unsatisfactory for practical use because of its small fission cross sections and hence low detection efficiency (neutrons detected per fission in the core). To make practical noise measurements in fast systems, a detector efficiency of 10^{-6} or greater is desirable.

For core characterization measurements in a cold clean assembly, the combination of commercially available ^{235}U fission counters and proton-recoil scintillators appear to be more than adequate for making such measurements of generation time.

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LIST OF FIGURES

Fig. 1. Calculational Model

Fig. 2. CPSD between a ^{235}U and a ^{240}Pu detector in the FTR-3 for $\rho_\lambda = -6.89\%$

Fig. 3. $\tan \theta$ for two ρ_λ values.