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STATISTICAL TREATMENT OF
HOT CHANNEL FACTORS FOR COMPACT REACTORS

AEC Research and Development Report

ATOMICS INTERNATIONAL

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STATISTICAL TREATMENT OF
HOT CHANNEL FACTORS FOR COMPACT REACTORS

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TO
STATISTICAL TREATMENT OF HOT CHANNEL
FACTORS FOR COMPACT REACTORS

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Equation (4), p 22 should read:

$$\Delta T'(Z)_{\text{element}} = \Delta T(Z)_{\text{element}} \cdot I_R$$

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ABSTRACT

This report presents the theoretical development, statistical treatment, and application of hot channel factors in compact SNAP reactors as represented by the SNAP 8 experimental core loading.

Presented are the channel and rod power effects of variation in the concentrations of uranium, hydrogen, and poison in the fuel rods. The statistical distributions of all variables are examined. A random selection of rods and channels is compared to the actual SNAP 8 configuration, with no difference noted. Confidence limits are set on hot rod and hot channel factors. Various other factors and aspects of the problem are discussed.

The results of the study have shown that the hot channel factors may be reduced to less than +2% over the nominal power, as opposed to a hot channel factor of about +10% as previously determined by empirical methods.

I. INTRODUCTION

The application of hot channel factors to SNAP reactor core thermal design allows the designer to predict the upper thermal limit at which the fuel element components will operate. This is particularly important where the core power density is high and thermal limitations are severe. In particular, the maximum fuel and ceramic temperatures must be known accurately, because of the importance of temperature effects on hydrogen loss, radiation damage, and ceramic barrier stability.

The term "hot channel factor", as used herein, denotes that multiple of the nominal power which represents the most extreme power conditions above the nominal case. (The nominal case is derived through theoretical considerations and is the approximate mean power.) Usually this factor is applied to the central core region where the highest power and, normally, the highest temperatures occur.

The statistical analysis developed in this report indicates that the hot channel and hot rod factors may be reduced by an approximate factor of 5 over those previously used.^{1, 2, 3} Also of interest is the failure to discover any significant difference between actual rods and core and "random" rods and core.

The particular fuel assembly analyzed in this paper is the SNAP 8 experimental reactor core loading. This assembly may be considered as being quite representative of most SNAP core loadings.

II. THEORETICAL CONSIDERATIONS AND APPLICATIONS

A study of hot channel factors must take into consideration (1) the variations in uranium loading, hydrogen concentration and poison concentration for the hot rod factors and (2) the accuracy in predicting power distribution, power variation during core life, and variations in the flow distribution for the operational hot channel factors. Some reactor configurations require the consideration of component dimensional tolerances. This item is not usually included in SNAP core studies, since both the nuclear and thermal design calculations normally are based on the most severe tolerances for each case.

The uranium, N_H (hydrogen), and poison loading variations may be determined by statistical analysis of data for the particular production fuel elements of the core under consideration. Although this report deals with one particular fuel loading, the results are general, inasmuch as this loading is representative of present configurations, the fuel is representative of present technology in fuel production control, and the data are treated in a normalized fashion. Hence, the application of the hot channel factors developed in this report to other compact systems is justified.

The extreme variations in N_H , uranium, and poison concentrations, as indicated by the range of the respective sample distributions, may be rather large in relationship to their nominal values: $\pm 4\frac{1}{2}\%$ for uranium and N_H , and $\pm 40\%$ for poison (Figures 1, 2, and 3, and Table I). The resultant extreme power variations (obtained by use of Figures 4 and 5) are, however, somewhat less, especially for poison (Figures 6, 7, and 8, and Table I). When the individual variations are combined in a total fuel element power statistic, their power variation effects are lessened further (Figures 9 and 10 and Table II). That is, the randomness of the constituent effects in production elements tends to allow them to cancel one another, thus forcing the net power variations to group more closely about the nominal value. When the fuel elements are assembled in a core, essentially in a random manner, this "cancelling effect" is again in evidence (Figures 11 and 12). The power of the three-element combination (the lowest basic single channel combination) again more closely approaches the nominal power.

Figure 1. Histogram of $\% \Delta C/C_0$ Values for Uranium

Figure 2. Histogram of $\% \Delta C / C_0$ Values for Hydrogen

Figure 3. Histogram of % $\Delta C/C_0$ Values for Poison

TABLE I

SAMPLE STATISTIC VALUES FOR $\% \Delta C/C_o$ AND $\% \Delta P/P_o$

Statistic	$\% \frac{\Delta C}{C_o}$			$\% \frac{\Delta P}{P_o}$		
	U	N_H	Poison	U	N_H	Poison
mean = $\bar{x}$ (est of μ)	0.2721	-0.1073	-0.4517	0.1242	-0.0772	-0.0193
variance = s^2 (est of σ^2)	1.9480	3.7993	207.2949	0.8965	0.2367	0.0195
standard deviation = s	1.3957	1.9492	14.3977	0.9468	0.4865	0.1396
range	-3.85 to 4.25	-4.36 to 4.19	-28.8 to 39.3	-3.0 to 2.5	-1.3 to 1.1	-0.57 to 0.16
99% confidence interval on μ	0.0269	-0.4499	-2.9816	-0.0422	-0.1627	-0.0438
	to 0.5173	to 0.2353	to 2.0782	to 0.2906	to 0.0083	to 0.0052
99% confidence interval on a value	-3.93 to 4.47	-5.95 to 5.74	-43.65 to 42.75	-2.72 to 2.96	-1.54 to 1.38	-0.44 to 0.40
g_1	0.1405	0.3799	0.3077	-0.2212	0.1889	-0.0138
g_2	0.6068	-0.7389	-0.3199	0.6853	-0.6801	0.0198
sg_1	0.1659	0.1659	0.1659	0.1659	0.1659	0.1659
sg_2	0.3303	0.3303	0.3303	0.3303	0.3303	0.3303
t_1	0.8470	2.2901	1.8545	-1.3335	1.1384	-8.2975
t_2	1.8370	-2.2368	-0.9685	2.0746	-2.0589	5.9952
decision on normality	normal	normal	normal	normal	normal	anormal

Figure 4. Concentration-Power Conversion for Uranium and Hydrogen

Figure 5. Concentration — Power Conversion for Poison

Figure 6. Histogram of % Δ Power/ P_o
Values for Uranium

Figure 7. Histogram of $\% \Delta \text{Power} / P_0$ Values for Hydrogen

Figure 8. Histogram of $\% \Delta \text{Power}/P_0$ Values for Poison

Figure 9. Histogram of I_R Values for SNAP 8 Fuel Rods

Figure 10. Histogram of I_R Values for "Random Rods"

TABLE II
SAMPLE STATISTIC VALUES FOR HOT ROD AND
HOT CHANNEL FACTORS

Statistic	Hot Rod Factors		Hot Channel Factors	
	"Random"	SNAP 8	"Random"	SNAP 8
mean = $\bar{x}$ (est of μ)	1.0005	1.0003	1.0003	1.0004
variance = s^2 (est of σ^2)	0.000116	0.000105	0.000034	0.000037
standard deviation = s	0.0108	0.0102	0.0058	0.0061
range	0.966 to 1.031	0.966 to 1.024	0.982 to 1.014	0.980 to 1.015
99% confidence interval on μ	0.9998 to 1.0012	0.9996 to 1.0010	0.9996 to 1.0010	0.9997 to 1.0011
99% confidence interval on a value of I_R or I_C	0.968 to 1.033	0.970 to 1.031	0.983 to 1.018	0.982 to 1.019
g_1	-0.5181	-0.5189	-6.6263	-4.5458
g_2	37.7916	6.9475	1496.3763	863.1462
$^s g_1$	0.1659	0.1659	0.1265	0.1265
$^s g_2$	0.3303	0.3303	0.2523	0.2523
t_1	-3.1229	-3.1279	-52.3851	-35.9374
t_2	114.4073	21.0324	5930.4848	3420.8475
Fisher Test:				
decision on normality	anormal*	anormal*	anormal*	anormal*
χ^2 values critical value	33.41	33.41	23.21	26.22
calculated value	25.4	11.1	14.67	18.73
χ^2 test				
decision on normality	normal	normal	normal	normal
test of $H_0: \mu_R = \mu_S$	accept		accept	
test of $H_0: \sigma_R^2 = \sigma_S^2$	accept		accept	

* See Appendix B

Figure 11. Histogram of I_C Values for SNAP 8 Core

Figure 12. Histogram of I_C Values for "Random Core"

It should be noted that, while the hydrogen variation (N_H) makes an important contribution to the hot channel and rod factors, the importance of this variable decreases during reactor operation. This is because of the variations in hydrogen loss in the production elements and the effect of hydrogen redistribution due to the axial temperature profile imposed on the element during operation. Both of these effects tend to lower the maximum power in each rod. For example, in the case of SNAP 8, the redistribution diminished the maximum power to about 3% below the nominal maximum.⁵

A. DEVELOPMENT OF HOT ROD EFFECTS

To determine temperature effect of the hot rod at the maximum temperature point in the ceramic fuel centerlines or other element areas of interest (Figure 13), the axial position of the hot spot must be established. This must be accomplished by means of an iterative technique, since the axial position of maximum temperature varies with both core radial position and power. Curves such as those shown in Figure 15 are generated for this purpose. The core radial and axial power distributions must be known, so that the power at the particular core radial position and axial elevation, $P(R, Z)$, may be used in the subsequent calculations. In SNAP cores, the radial power distribution follows a chopped cosine function with a peak/average value of about 1.3 and a peak/maximum value of about 1.7. The axial power distribution has the same general functional shape with a peak/average value of about 1.4 and a peak/minimum value of about 3.5.

The axial temperatures may be calculated by use of the equations given in Reference 1 and are given here in a condensed form. Slab geometry is assumed for all fuel element components except the fuel slug and coolant gap. The effect of the (tri-cusp channel) fuel geometry is included in the temperature drop calculations.¹ Thus,

$$T_{\max} = T_{\text{inlet}} + \Delta T(Z)_{\text{coolant}} + \Delta T(Z)_{\text{element}} \quad , \quad \dots (1)$$

where $T_{\max}$ is the maximum temperature at the hottest axial point Z in the particular element component (ceramic, etc.) examined, and its constituents on the right hand side are defined as follows (see Figure 13):

Figure 13. Nominal Rod and Channel

Figure 14. Hot Rod and Channel

$\Delta T(Z)_{\text{element}}$ is the sum of the radial temperature rises through the coolant film, cladding, ceramic, hydrogen gap and fuel slug (or to whatever radial point is being examined).

$\Delta T(Z)_{\text{coolant}}$ is simply the axial coolant temperature rise from the inlet to the elevation Z^* (see Figure 14). Thus, in the nominal case under discussion, at constant reactor radius R

$$\Delta T(Z)_{\text{coolant}} = 1/2 \frac{\int_0^Z P(R, Z) dZ}{M(R) C_m L} \quad , \quad \dots (2)$$

where

$P(R, Z)$ = the power, Btu/hr, generated at station Z along core radius R

$M(R)$ = the core coolant flow rate at R , lb_m/hr, assumed constant in R at all Z ,

C_m = the heat capacity of the coolant, Btu/lb-°F, and

L = core length, ft.

To assess the effect of the hot rod factor I_R on T_{max} , rises in $\Delta T(Z)_{\text{coolant}}$ and $\Delta T(Z)_{\text{element}}$ must be considered. For the former, the nominal case may be modified to include (1) the flow variations from the nominal, $M'(R)^\dagger$ and (2) the hot channel contribution from the rod in question:[§]

$$\Delta T'(Z)_{\text{coolant}} = 1/2 \frac{\int_0^Z P(R, Z) dZ}{M'(R) C_m L} \cdot \frac{I_R}{3} \quad , \quad \dots (3)$$

where I_R is the hot rod factor and represents the contribution to the channel from that rod in the three-rod channel.

*It is assumed that an analysis has been made to determine the height Z at which T_{max} occurs.

† $M'(R)$ is figured to be $M(R) \pm 10\%$, which represents the usual accuracy in shaping flow⁽¹⁾

§ The rod in question will ordinarily be what we shall call the "statistically hottest rod," which will be our term for the upper 99% confidence limit on a sample value from the population of rod power values as taken from the frequency distribution of the sample values.

Figure 15. Typical Core Temperature Distribution

The nominal case of $\Delta T(Z)_{\text{element}}$ is modified quite simply:

$$\Delta T(Z)_{\text{element}} = \Delta T(Z)_{\text{element}} \cdot I_R \quad \dots (4)$$

Hence the modification of Equation 1 for consideration of the hot rod factor:

$$T'_{\text{max}} = T_{\text{inlet}} + \Delta T'(Z)_{\text{coolant}} + \Delta T'(Z)_{\text{element}} \quad \dots (5)$$

B. DEVELOPMENT OF HOT CHANNEL EFFECTS

While the hot rod effects are an important consideration in the analysis of a specific or general fuel rod, the hot channel effects are the more meaningful from the standpoint of overall core design. The method of hot channel application is quite similar to the hot rod application method described in Section A, except that in the channel application the three rod tri-cusp channel combination is considered from the standpoint of the "statistically hottest channel" as a contributor, third note, p 20. The hottest of the three rods is examined for its effect on the channel and the channel's effect on it.

Again examining the axial and radial temperature rises separately, the following modifications of the coolant and element temperature changes are made:

$$\Delta T''(Z)_{\text{coolant}} = 1/2 \frac{\int_0^Z P(R, Z) dZ}{M'(R) C_m L} \cdot I_C \quad \dots (6)$$

where I_C is the hot channel factor, and

$$\Delta T''(Z)_{\text{element}} = \Delta T(Z)_{\text{element}} \cdot I_{R^*} \quad \dots (7)$$

where I_{R^*} is the hot rod factor for the hottest rod in the hot channel. Hence the resultant modification of Equation 1 (Figure 14):

$$T''_{\text{max}} = T_{\text{inlet}} + \Delta T''(Z)_{\text{coolant}} + \Delta T''(Z)_{\text{element}} \quad \dots (8)$$

Again, curves similar to those of Figure 15 are generated.

III. STATISTICAL TREATMENT

A. INTRODUCTION

The original data were taken from fuel element data compiled by J. E. Caldwell.⁶ These data consisted in part of fuel element serial numbers, fuel rod serial numbers, grams uranium per rod, fuel rod average N_H , and milligrams Sm_2O_3 per inch. The data were recorded by birdcage, with 12 rods in each of 17 birdcages and 11 rods in one birdcage, making a total of 215 rods.

The data were complete with one exception, that being a Sm_2O_3 reading in birdcage VI. This reading was supplied by the laboratory and included in the analysis. The stated accuracy for Sm_2O_3 determination was $\pm 10\%$. No statement of accuracy was determined for uranium or N_H .

B. HOT ROD FACTORS ANALYSIS

The three measurements, uranium, N_H , and Sm_2O_3 from the original data will be denoted by x_i , $i = 1, 2, 3$, respectively. Individual measurements will be denoted by x_{ij} , $j = 1, 2, \dots, 215$.

The data from the two groups ("random" and actual rods) were treated as follows. Associated with each x_i was a nominal value, as explained earlier, which will be called here x_{io} . For each x_{ij} , the statistic

$$\left(\% \frac{\Delta C}{C_o} \right)_{ij} = \frac{x_{ij} - x_{io}}{x_{io}} \cdot 100$$

was first computed. This "percent change in concentration" was then transformed to a "percent change in power" by use of the graphs in Figures 4 and 5, and denoted by $(\% \Delta P/P_o)_{ij} = p_{ij}$. The sample frequency distribution of the p_{ij} was then investigated in the hope that normality would be observed (see Appendix A and Table I), thus enabling the use of some sort of normal theory in combining the p_{ij} to get a "total percent change in power from nominal" statistic.

No correlation was observed between any of the three variables. The correlation coefficients (r) are given below. A correlation coefficient of about ± 0.176 or greater would be necessary for correlation to be considered significant.

	Uranium and N_H	Uranium and Poison	N_H and Poison
r for $\% \frac{\Delta C}{C_0}$	-0.107	0.007	-0.029
r for $\% \frac{\Delta P}{P_0}$	-0.109	-0.036	0.060

Although normality could safely be assumed for the frequency distributions of N_H and uranium, such was definitely not the case for Sm_2O_3 , the sample frequency distribution of the latter being markedly skewed to the left. This skewness shows up in all of the subsequent derivative distributions (see Figures 8 to 12).

In the face of this fact, it was decided to use the somewhat standard procedure of using the product of the "power", $1 + p_{ij}$, as a "total" for the j^{th} rod. This is denoted by

$$p_{T_j} = \prod_i (1 + p_{ij}) \quad .$$

Since the data is normalized, it is evident that p_{T_j} also represents the hot rod factor for the j^{th} rod. Thus,

$$p_{T_j} = I_{R_j} \quad .$$

Two groups of data were analyzed in the hot rod analysis. The first was the data from the original 215 rods. The second was data from 215 "random rods" which was obtained as follows. Two hundred and fifteen numbered chips corresponding to the original rods were placed in a box and drawn without replacement in groups of three. In each triplet, the uranium reading for the first rod, the N_H reading for the second, and the poison (Sm_2O_3) reading from the third were combined to give a rod total (as explained above) for a random rod. Two hundred and fifteen such triplets were drawn.

The sample frequency distribution of I_R was then investigated for both groups of data. Although some deviation from normality was indicated, it was decided that normality should be assumed for the purposes of this analysis (see Appendix B). Under this assumption, the statistics shown in Table II were calculated. They are the sample mean, $\bar{I}_R$ (also called $\bar{x}$), the sample variance,

$s_{I_R}^2$, and the sample standard deviation, s_{I_R} . Utilizing these statistics, it may be stated with 99% confidence that the true value of the mean of the population from which the sample of I_R was drawn lies between the confidence limits given. With the same confidence, it may be stated that no particular value of I_R will be observed beyond the limits shown. In the confidence interval, I_R represents the actual value of any member of the population from which the sample was drawn.

It was decided to test the two hypotheses necessary to ascertain whether or not the "random rods" and the actual rods came from the same population (see Appendix B). The results indicated that they did.

C. HOT CHANNEL FACTORS ANALYSIS

The total hot rod factors may be combined to get a hot channel factor by the following method. Suppose that the three rods, A, B, and C form a channel. Combine the total hot rod factors of the three rods by the simple formula

$$I_C = \frac{I_{RA} + I_{RB} + I_{RC}}{3} ,$$

to obtain the hot channel factor for that particular channel.

There are 372 tri-cusp channels in the SNAP 8 core. Two separate sets of data were analyzed, one a set of 372 I_C values representing the SNAP 8 core as it is actually constructed, the other a set of data from a "random core". This latter set of data was obtained by drawing at random three fuel elements, calculating I_C , and repeating this operation 372 times to obtain that number of data points. The sampling technique was to place 215 numbered chips, which represented the fuel elements, in a box, and drawing three at a time, without replacement until the box was exhausted, then starting over, until 372 triplets representing channels were obtained. No attempt was made to construct a hypothetical core per se.

The two sets of data were analyzed first to ascertain whether normality could be assumed. The results were as indicated by Table II as interpreted in view of Appendix B. Then each set of data was used to calculate the other statistics in Table II, as for the hot rod factors. A fuller explanation of these

statistics and their interpretation is given in Reference 8. As can be seen, the results were fairly similar. Again, as for the rods, it was decided to test two hypotheses necessary to determine any significant difference between the "random core" sample and the actual core sample. The hypothesis $H_0: \mu_S = \mu_R$ was tested (i. e., did the two samples come from populations with the same mean?) The conclusion was to accept H_0 . The hypothesis $H_0: \sigma_S^2 = \sigma_R^2$ was tested similarly, with the same results. Both tests were at 99% confidence. Since the mean and variance completely describe a normal population, it may be assumed that both samples came from the same population, i. e., that there was no essential difference between the actual core and a "random core" insofar as hot channel factors are concerned, and, again, inasmuch as the discussion in Appendix B applies.

NOMENCLATURE

C = Concentration
 C_o = Nominal concentration
 C_p = Heat capacity, $\text{Btu/lb}_m - ^\circ\text{F}$
 I_C = Hot channel factor
 I_R^* = Hot rod factor in the hot channel
 I_R = Hot rod factor
 L = Fueled length of core, ft
 M = Coolant flow rate, lb_m/hr
 P = Power, Btu/hr
 P_o = Nominal power
 r = Fuel element radial coordinate
 R = Core radial coordinate
 T = Temperature, $^\circ\text{F}$
 ΔT = Temperature difference, $^\circ\text{F}$
 Z = Core axial coordinate or distance, ft

Superscripts

' = Hot rod condition
" = Hot channel and/or hot rod in hot channel condition

APPENDIX A

TESTS OF NORMALITY IN THE DISTRIBUTION OF A SAMPLE

There are two methods available for a test of normality of a sample distribution. Both were applied to the data upon which this paper is based, with varying results.

The first of the two methods, and the one most heavily relied upon here, is the moment-comparison method of Fisher,⁷ with utilization of "Students" t-distribution to test the hypothesis of normality as described by Snedecor.⁸ This method compares functions of the first four moments of the normal distribution to the corresponding sample estimates of these moments. The statistics involved are given below. All summations are from $i = 1$ to n , where n is the sample size and x_i is an individual value from the sample.

Sums of powers of deviations from the mean:

$$S_2 = \sum x_i^2 - \frac{1}{n} (\sum x_i)^2$$

$$S_3 = \sum x_i^3 - \frac{3}{n} (\sum x_i) (\sum x_i^2) + \frac{2}{n} (\sum x_i)^3$$

$$S_4 = \sum x_i^4 - \frac{4}{n} (\sum x_i^3) (\sum x_i) + \frac{6}{n} (\sum x_i^2) (\sum x_i)^2 - \frac{3}{n} (\sum x_i)^4$$

k - statistics:

$$k_1 = \bar{x} = \frac{1}{n} \sum x_i$$

$$k_2 = s^2 = \frac{1}{n-1} S_2$$

$$k_3 = \frac{n}{(n-1)(n-2)} S_3$$

$$k_4 = \frac{n}{(n-1)(n-2)(n-3)} \left[(n+1) S_4 - 3 \left(\frac{n-1}{n} \right) S_2^2 \right]$$

g - statistics (functions of the third and fourth moments):

$$g_1 = K_3/K_2^{3/2}$$

$$g_2 = K_4/K_2^2$$

Standard deviation of the g - statistics:

$$s_{g_1} = \sqrt{6n(n-1)/(n-2)(n+1)(n+3)}$$

$$s_{g_2} = \sqrt{24n(n-1)^2/(n-3)(n-2)(n+3)(n+5)}$$

t - statistics:

$$t_1 = g_1/s_{g_1}$$

$$t_2 = g_2/s_{g_2}$$

The above statistics are applied to each sample of size n in the following way. Compute all of the given statistics and observe first the value of g_1 . This statistic gives a measure of skewness, one way in which a frequency distribution may deviate from normality. If $g_1 > 0$, the distribution is skewed to the right; if $g_1 = 0$, there is no skewness and the distribution is symmetric about the mean; if $g_1 < 0$, the distribution is skewed to the left.

Next evaluate g_2 , a measure of kurtosis or "peakedness", which is another possible deviation from normality. If $g_2 = 0$, there is no deviation from normality insofar as kurtosis is concerned. If $g_2 < 0$, this negative kurtosis indicates that the curve is "flat-topped", with an excess of moderate deviations. If $g_2 > 0$, this positive kurtosis indicates a very peaked curved, with too few moderate deviations.

From the above it may be seen that if both g_1 and g_2 are zero, the frequency distribution of the population from which the sample came may be assumed to be the normal distribution, with mean μ estimated by $\bar{x}$ and variance σ^2 estimated by s^2 . Now the question arises as to how close to zero the two values may be to assume "for all practical purposes" that they are indeed zero. That is, how large must g_1 and g_2 be before they are significantly different

from zero? To answer this question, use the respective t-statistics, which have "Students" t-distribution, in the test of the null hypothesis $H_0: g_1 = 0$ and $H_0: g_2 = 0$ against their respective alternative hypothesis, $H_A: g_1 \neq 0$ and $H_A: g_2 \neq 0$. Take the case of g_1 :

The null hypothesis is

$$H_0: g_1 = 0 \quad ,$$

which is tested against the alternative

$$H_A: g_1 \neq 0 \quad .$$

To do this, compare the value of t_1 with the tabulated value of "students" t-distribution for an infinite number of degrees of freedom, and at the pre-determined desired confidence level. For a 99% confidence in rejection of H_0 , which was the confidence used in this paper, the tabulated value is $t_{0.01} = 2.576$. If

$$|t_1| > t_{0.01} \quad ,$$

the null hypothesis H_0 may be rejected with 99% confidence of doing so correctly. Otherwise, if

$$|t_1| \leq t_{0.01} \quad ,$$

the null hypothesis is accepted, that is, the statement is made that the sample came from a population which is symmetric about the mean.

The same procedure applies to testing $g_2 = 0$, utilizing t_2 , with an accepted hypothesis implying the sample comes from a population with negligible deviation from normality insofar as kurtosis is concerned.

If both of the hypothesis are accepted, normality may be safely assumed. If one of them is rejected, consideration must be given to how the sample distribution seems to deviate from normality, and how this will affect the particular problem at hand (see Appendix B).

The second test of normality applied to the data is the well-known chi-square goodness-of-fit test. This test compares the actual number of observations in the several intervals of the histogram of a sample distribution with the corresponding areas under a theoretical normal distribution with mean $\mu = \bar{x}$ variance $\sigma^2 = s^2$. In this test, an χ^2 statistic which is a function of the observed and expected frequencies in each interval of the histogram is calculated and compared to a tabulated value of the χ^2 distribution called the critical value. If the calculated χ^2 is less than the tabulated χ^2 the deviations of observed frequencies from expected frequencies are not great enough to reject an assumption of normality. The χ^2 test was applied to those distributions of Table II which failed the Fisher test. In every case, the χ^2 test did not reject the hypothesis of normality which the Fisher test had rejected (see Table II). Hence the dilemma of having two valid tests of a hypothesis giving contradictory results.

To arrive at a working compromise between the two contradictory tests (contradictory in this case) something must be known about the nature of the tests; the Fisher test is very sensitive. Small deviations are given large notice, especially when the sample size is large and the variance small, as was the case. It should also be pointed out here that a t value a great deal larger than $t_{0.01}$ does not necessarily mean a terrible fit. It simply means that since t was larger than $t_{0.01}$, H_0 must be rejected. This does not tell how far away from a true normal the distribution may be.

The χ^2 test, on the other hand, is a lenient test when the sample size is large. It will allow some deviation from normality in such cases.

The decision was made to assume normality. A discussion of some of the reasons is given in Appendix B.

APPENDIX B

ON THE ASSUMPTION OF NORMALITY IN THE CASE OF ANORMAL DISTRIBUTIONS

As will be seen from Table II, the Fisher test for normality resulted in a rejection of the hypothesis of normality for most of the distributions. The χ^2 test, however, accepted normality.

Examining the results of the Fisher test more closely reveals that in all cases where skewness was found to deviate significantly from symmetry, the statistic g_1 was negative. This indicates that the distribution was skewed to the left, i. e., it had an excess of moderate deviations on the right of the mean, and a shortened "tail" on the right. The region of the distribution which is of primary interest herein is the right tail, which corresponds to the higher values of the variable under consideration. Therefore, if normality were to be assumed, there would actually be a slight unknown safety margin in setting upper confidence limits, because the hypothetical normal distribution which is used to set confidence limits probably extends a little farther up the scale than the actual population distribution on the right-hand side. Hence an assumption of normality would not be too distasteful, especially in view of the favorable χ^2 test results. (See Figure 16 for an illustration of these considerations.)

The effect of an erroneous assumption of normality on the tests of the hypothesis of equal means and variances is another matter. Insofar as the distributions are normal, the tests and results thereof are valid, and this is about the best that can be done under the circumstances.

The possibility that other less well known probability density functions than the normal might fit the data better was considered. It appeared upon cursory examination that a beta distribution (Pearson's Type I distribution) might apply. However, for the SNAP 8 core sample of hot channel factors, when the theoretical moments for the beta distribution were set equal to the sample moments and the parameters α and β (or p and q) of the beta distribution were solved for, values of $\alpha = 2.89$ and $\beta = 2.37$ (or $p = 3.89$ and $q = 3.37$) were obtained. The nearest tabulated distribution⁹ available without tedious three-way interpolation was that for $\alpha = 3$ and $\beta = 2.5$. This theoretical distribution, when compared with the sample distribution via the χ^2 test, resulted in a rejection of the hypothesis that the beta distribution was the correct distribution.

Figure 16. Illustration of Normality Consideration

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