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TEMPERATURE DISTRIBUTION AND THERMAL STRESSES
IN
RADIATION-COOLED CIRCULAR CYLINDERS

By

Maurice W. Wildin
William E. Putman, Jr.

August, 1963

Prepared for Sandia Corporation
Under P.O. 51-5134, Task 13
UNM Acct. 290-368

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ABSTRACT

An investigation has been undertaken to study the thermal stresses in a cylindrical NAP fuel rod. The study has consisted of two phases, one analytical and the other experimental.

In the analytical phase, a mathematical model of a fuel rod has been developed in which the rod is considered to be a long solid circular cylinder cooled by emission of thermal radiation. A closed form of solution for the temperature distribution could not be obtained, so numerical results for the temperature and stress distributions were obtained by finite difference methods. The results predict that relatively small stresses are produced when the fuel rod is cooled in a temperature range in which brittle fracture can reasonably be expected.

The experimental phase, which is not yet complete, was undertaken to verify the predicted temperature and stress distributions. Rather than using an actual fuel rod in the experimental program, which would present formidable safety and security problems, a modeling technique is being used. The theory of similitude will be employed to relate the results obtained from the physical model to those which one would expect to obtain from a fuel rod. An embedded strain gage technique is being used to measure internal strains.

TABLE OF CONTENTS

Introduction	1
Analytical Phase of the Investigation	5
Equations for Temperature and Stress	7
Non-dimensionalization of the Equations	10
Solution for the Temperature and Stress Distributions	17
Discussion of Results	24
Experimental Phase of the Investigation	35
Conditions Involving Simulation	36
Environment Simulation	38
Measurements, Model Instrumentation	39
Recording Equipment	42
Experimental Apparatus	43
Bibliography	46
List of Symbols	47
Appendix A; Development of Finite Difference Equations	49
Appendix B; Figures	52

INTRODUCTION

The research to be discussed in this report was performed as a project associated with the aerospace nuclear safety program. The question which motivated this research was whether a cylindrical NAP fuel rod could be caused to fail by suddenly exposing it to the environment encountered by a body as it starts to re-enter the earth's atmosphere. It was thought that if a rod were exposed to such an environment while at a high enough temperature to cause it to cool rapidly, perhaps the thermal stresses thus induced would be large enough to produce failure.

In defining the problem, it was decided at the start that the initial effort would be to determine the set of parameter values which would be most likely to cause the fuel rod to fail. In the event that failure was a strong possibility for this set of values, other sets of values defining less severe conditions would be considered in an attempt to establish a threshold at which failure could be expected to start. In other words, the problem was divorced from the actual conditions to which the rods in a re-entering reactor might be exposed in order to determine whether, and under what conditions, a fuel rod would fail. The problem of devising the means to create these conditions, in case they did not correspond to those which would naturally occur, was considered a separate problem to be attacked later, if it arose.

The decision was also made to neglect the Hastelloy cladding on the fuel rod and the ceramic coating on the inside of the cladding. Due to the thinness of the cladding and the coating, these components of the fuel rod have a relatively small thermal capacity compared with the Uranium-Zirconium hydride fuel-moderator material which constitutes the body of the rod. Furthermore, the Hastelloy cladding has a slightly larger coefficient of linear expansion than the fuel-moderator material up to approximately 1600°F. (1)¹. Hence, it was concluded that the direct effect of the cladding on the stress distribution in the fuel-moderator material, due to surface forces it might exert, as well as the indirect effect, through its influence on the temperature distribution, would be small. The greatest error introduced by neglecting the cladding would probably be due to a difference between the total hemispherical emissivity of the fuel-moderator material and that of the cladding. This would cause an error in the rate of heat transfer from the rod at any instant, thus causing the instantaneous temperature and stress distribution to be in error. The magnitude of the errors thus introduced is difficult to estimate, because it is very difficult to predict what the emissivity of the cladding would be after a period of service in a reactor, but the error could be great. The result of neglecting the cladding and the ceramic layer is that one may reasonably treat the fuel rod as a homogeneous body.

¹Numbers in parentheses refer to corresponding items in the Bibliography.

Application of the above conditions and assumptions leads to a general definition of the problem: First; identify the parameters which are pertinent to the process of induction of thermal stresses in a solid, homogeneous cylinder starting to re-enter the earth's atmosphere. Second; predict the set of parameter values which will be most likely to correspond to failure of a cylinder of Uranium-Zirconium hydride. Third; in case there is a strong possibility of failure, investigate less severe conditions in an attempt to establish a threshold at which failure can be expected to start.

The attack on these problems is being carried out in two phases, one phase being analytical and the other experimental in nature. Since only surface strains could be measured if the actual fuel rod were used in the experimental program, and since the handling of such material presents serious problems, it was decided that the experimental work should be done on a model of the fuel rod. By using the theory of similitude and the embedded strain gage techniques which have been developed in the Mechanical Engineering Department, it was felt that more data could be obtained with less expense and in a simpler manner by modeling the fuel rod than by using an actual fuel rod.

Although the analytical and experimental programs were pursued more or less simultaneously, it was found that the experiment could be better designed if the results of the analysis were employed. This, combined with the relatively

large time required to assemble the experimental apparatus, has caused the experimental program to lag considerably behind the analytical program. At the time this is written, a large number of numerical results have been obtained; however assembly of the experimental equipment is not complete, so no experimental results have been obtained. In the remainder of this report, the analytical attack on the problem will be described, including:

- 1) The equations and boundary conditions required to determine temperature and principal stresses as functions of position and time.
- 2) The non-dimensionalization of the equations and boundary conditions to permit use of the theory of similitude.
- 3) The numerical results obtained through the analytical program and the significance of these results.

The design of the experiment will also be discussed, including the manner in which similitude will be employed and the equipment to be used in establishing the test conditions and obtaining the necessary data.

ANALYTICAL PHASE OF THE INVESTIGATION

The analytical work was first directed towards obtaining answers to the first two phases of the general problem, namely, identifying the important parameters and predicting a set of parameter values which is most likely to lead to failure. The initial step in obtaining answers to both phases was to set up a mathematical model. Once this had been done, the important parameters became obvious; they were the ones appearing in the equations and boundary conditions associated with the model. In trying to establish whether failure is likely, it is necessary to adopt a theory of failure. The selection of such a theory could be a formidable task if one wished to be particular. For the purposes of this investigation, it was felt that an extensive effort in this area could not be justified unless, and until, it had been established that failure was a possibility according to one of the common theories of failure. It is known that the Uranium-Zirconium hydride fuel-moderator material acts in a brittle fashion up to some temperature lying between 1000° and 1500°F , and in a more or less ductile fashion at higher temperatures (2). The maximum principal stress theory of failure might be adopted for the temperature range in which the material is brittle and the maximum shear stress theory for the range in which the material is ductile. However, it makes no difference which of these theories is employed, since in this case they yield identical results. This is because the

maximum principal stress is equal to the maximum difference between the principal stresses. Furthermore, it is noted that re-entry is a "one-shot" process, since, if failure is to occur, it must be produced by a single cooling process. It has been established that the probability of failure of a ductile material due to a single thermal shock is very low, whereas the probability of failure of a brittle material with a single shock is good (3). It was therefore concluded that the investigation of the possibility of producing failure in a NAP fuel rod could be confined to the range of temperatures in which the fuel rod material is brittle. This range extends up to some presently unknown value between 1000° and 1500°F . It followed that the maximum principal stress theory could be employed to predict failure.

With a criterion for failure established, it was possible to make rough estimates of the conditions which were most likely to lead to failure. According to the maximum principal stress theory, failure is most likely when the maximum value of principal stress occurs. Intuitively, it was expected that the maximum value of principal stress would be induced when the rate of cooling from a given initial condition is greatest. This conclusion may not be valid if thermal and mechanical properties are temperature dependent, or if a chemical process involving absorption or rejection of energy is taking place. It was expected that the above intuitive conclusion would be

valid in the present case, since chemical reactions apparently do not control the behavior of the fuel-moderator material until it reaches a temperature of about 1550°F , which lies outside the brittle range of the fuel-moderator material (2).

To the end of selecting the set of conditions which produces the maximum rate of cooling, it was assumed that heat input to a fuel rod by convection, due to re-entry heating, and by radiation reflected and emitted from the sun, the moon, and the earth and other planets would be negligible. Under this assumption, the net rate of heat transfer from a rod is equal to the rate of emission of thermal radiation by the rod. Since the rod has a length-to-diameter ratio of ten, it was assumed that heat transfer from the ends could be neglected. With the additional assumptions that the material is homogeneous and isotropic and that the temperature is initially uniform throughout the rod, the temperature distribution at any later instant is a function only of radial position. (The assumption of homogeneity appears to be substantiated by Woolsey (4). The assumption of isotropy cannot be substantiated, since the manner of construction of the rod is not known. An initially uniform temperature is as probable as any other distribution.)

Equations for Temperature and Stress

Application of one more assumption, that the fuel-moderator material has a high absorption coefficient, so that radiation is emitted only from the surface of the rod, allowed the heat transfer portion of the problem to be formulated in

terms of the following differential equation, initial condition, and boundary conditions:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(rK \frac{\partial T}{\partial r} \right) = \rho c \frac{\partial T}{\partial t} \quad (1a)$$

$$T(\bar{r}, 0) = T_i$$

$$\frac{\partial T}{\partial r} (0, t) = 0, \quad t \geq 0 \quad (1b)$$

$$-K \frac{\partial T}{\partial r} (\bar{a}, t) = \sigma \epsilon T^4 (\bar{a}, t), \quad t > 0$$

The differential equation for the temperature distribution was formulated in such a manner that variable thermal conductivity and specific heat could be taken into account. (A check on the variations of density with temperature revealed that such variations would be quite small, even with large changes in temperature; so it was decided to treat density as a constant. Emissivity was also treated as a constant.) The first condition is the initial condition, while the second condition specifies the radial symmetry of the temperature distribution about the axis of the rod. The last condition is obtained by equating the heat flux at the surface of the rod by conduction to that by radiation. The boundary and initial conditions follow directly from the assumptions stated previously. Before discussing the means used to obtain numerical results for the temperature distribution, we shall discuss the equations for the principal stresses and strains and the non-dimensionalization

of these equations along with the equation and attendant conditions for the temperature distribution.

Since no step changes in temperature occur when a body is cooled by emission of thermal radiation, the quasistatic theory of thermoelasticity may be employed. That is, inertia terms in the equations of equilibrium may be neglected, as McKinley and Ju pointed out in an earlier report from the Mechanical Engineering Department (5). If one makes the assumption that plane sections normal to the axis of the cylinder remain plane and if one applies the boundary conditions of no radial nor axial external constraints, the solutions for the principal components of the stress and strain tensors are known. Heap (11) presents the results for these components, treating Young's modulus and Poisson's ratio as constants and the coefficient of linear expansion as a function of temperature. For a solid cylinder which is initially unstressed and at the temperature T_i , the principal stresses are given by

$$\begin{aligned}\sigma_z &= \frac{E}{1-\nu} \left[\frac{2}{a^2} \int_0^{\bar{a}} \left(\int_{T_i}^T \alpha dT \right) \bar{r} d\bar{r} - \int_{T_i}^T \alpha dT \right] \\ \sigma_r &= \frac{E}{1-\nu} \left[\frac{1}{a^2} \int_0^{\bar{a}} \left(\int_{T_i}^T \alpha dT \right) \bar{r} d\bar{r} - \frac{1}{\bar{r}^2} \int_0^{\bar{r}} \left(\int_{T_i}^T \alpha dT \right) \bar{r} d\bar{r} \right] \quad (2) \\ \sigma_\theta &= \frac{E}{1-\nu} \left[\frac{1}{a^2} \int_0^{\bar{a}} \left(\int_{T_i}^T \alpha dT \right) \bar{r} d\bar{r} + \frac{1}{\bar{r}^2} \int_0^{\bar{r}} \left(\int_{T_i}^T \alpha dT \right) \bar{r} d\bar{r} - \int_{T_i}^T \alpha dT \right]\end{aligned}$$

It may be noted that if the functional dependence of α on T is known, equations (2) enable one to compute the distribution of principal stresses corresponding to any temperature distribution. Thus, to determine the state of stress at any instant one may solve equations (1a) and (1b) for the temperature distribution at that instant and then substitute this distribution into equation (2). The calculation of the temperature distribution as a function of time was therefore the key analytical problem.

Non-dimensionalization of the Equations

To obtain solutions for the stress and temperature distributions which are more widely applicable, as well as to facilitate modeling the thermal stresses, the equations for the stress and temperature distributions were non-dimensionalized. Denoting reference values by the subscript R, the following dimensionless quantities were defined.

$$\varphi = T/T_R, \quad r = \bar{r}/\bar{r}_R, \quad \lambda^K = K/K_R, \quad \lambda^C = C/C_R, \quad (3)$$

$$\lambda^\alpha = \alpha/\alpha_R, \quad \tau = \frac{K_R t}{\rho C_R \bar{r}_R^2}, \quad N_R = \frac{\bar{r}_R^3 \epsilon \sigma T_R}{K_R}, \quad \Sigma_i = \frac{(1-\nu) \sigma_i}{E \alpha_R T_R}$$

Substitution of these dimensionless quantities into equations (1a) and (1b) for the temperature distribution yields

$$\frac{1}{r} \frac{\partial}{\partial r} (r \lambda^K(\varphi) \frac{\partial \varphi}{\partial r}) = \lambda^C(\varphi) \frac{\partial \varphi}{\partial \tau} \quad (4a)$$

$$\varphi(r, 0) = T_i/T_R = \varphi_i$$

$$\frac{\partial \varphi}{\partial r}(0, \tau) = 0, \quad \tau \geq 0$$

$$\frac{\partial \varphi}{\partial r}(\bar{a}/\bar{r}_R, \tau) = - \frac{N_R}{\lambda^K(\varphi)} \varphi^4(\bar{a}/\bar{r}_R, \tau), \quad \tau > 0 \quad (4b)$$

Substitution from (3) into the expressions (2) for the principal stresses yields:

$$\begin{aligned}\Sigma_z &= \frac{2}{\bar{a}^2/\bar{r}_R^2} \int_0^{\bar{a}/\bar{r}_R} \left(\int_{\varphi_i}^{\varphi} \lambda^\alpha d\varphi \right) r dr - \int_{\varphi_i}^{\varphi} \lambda^\alpha d\varphi \\ \Sigma_r &= \frac{1}{\bar{a}^2/\bar{r}_R^2} \int_0^{\bar{a}/\bar{r}_R} \left(\int_{\varphi_i}^{\varphi} \lambda^\alpha d\varphi \right) r dr - \frac{1}{r^2} \int_0^r \left(\int_{\varphi_i}^{\varphi} \lambda^\alpha d\varphi \right) r dr \\ \Sigma_\theta &= \frac{1}{\bar{a}^2/\bar{r}_R^2} \int_0^{\bar{a}/\bar{r}_R} \left(\int_{\varphi_i}^{\varphi} \lambda^\alpha d\varphi \right) r dr + \frac{1}{r^2} \int_0^r \left(\int_{\varphi_i}^{\varphi} \lambda^\alpha d\varphi \right) r dr - \int_{\varphi_i}^{\varphi} \lambda^\alpha d\varphi\end{aligned}\quad (5)$$

Similarity is said to exist between the temperature and stress distributions in the model and prototype if equations (4a), (4b), and (5) have identical forms for the model and prototype.

Turning attention first to the differential equation and the initial and boundary conditions for the temperature distribution, it is observed that several conditions must be satisfied in order for similarity to exist. The initial condition requires that the initial dimensionless temperature distributions must be the same for the model and prototype. Using the subscript m to denote the model and the subscript p to denote the prototype, this condition requires that

$$\left\{ \varphi(r, 0) \right\}_m = \left\{ \varphi(r, 0) \right\}_p$$

or

$$\left\{ \frac{T_i}{T_R} \right\}_m = \left\{ \frac{T_i}{T_R} \right\}_p$$

If the initial temperatures in the model and prototype are selected as the reference temperatures for the model and prototype respectively, this condition will be satisfied without restricting the choice of initial temperatures for either the model or prototype. Hence, it was decided to let

$$\left\{ T_R \right\}_m = \left\{ T_i \right\}_m$$

and

$$\left\{ T_R \right\}_p = \left\{ T_i \right\}_p$$

or, in short, to let

$$T_R = T_i$$

The boundary condition,

$$\frac{\partial \phi}{\partial r} (0, \tau) = 0$$

is satisfied by axisymmetric temperature distributions in both the model and prototype. In addition to a uniform initial temperature this requires that the rate of heat transfer be uniform over the surface and that the material be homogeneous and isotropic. Hence, the emissivity and reflectivity of the surface of the model must be uniform and the surface temperature must be uniform. It is observed that the satisfaction of this boundary condition does not aid in selecting reference values; rather, it gives some insight into the physical conditions which must be observed in constructing the model and establishing the test conditions.

If the second boundary condition

$$\frac{\partial \varphi}{\partial \bar{r}} (\bar{a}/\bar{r}_R, \tau) = \frac{-N_R}{\lambda^K(\varphi)} \varphi^4(\bar{a}/\bar{r}_R, \tau)$$

is to have the same form for the model and prototype, it must be evaluated at the same value of the dimensionless radial coordinate. So we must have

$$\left\{ \bar{a}/\bar{r}_R \right\}_m = \left\{ \bar{a}/\bar{r}_R \right\}_p$$

This condition may be satisfied without restricting the outside radius of the model or prototype by letting

$$\begin{aligned} \left\{ \bar{r}_R \right\}_m &= \left\{ \bar{a} \right\}_m \\ \left\{ \bar{r}_R \right\}_p &= \left\{ \bar{a} \right\}_p \end{aligned}$$

or, in short, by letting

$$\bar{r}_R = \bar{a} \tag{7}$$

The form of the second boundary condition then becomes

$$\frac{\partial \varphi}{\partial \bar{r}}(1, \tau) = \frac{-N_R}{\lambda^K(\varphi)} \varphi^4(1, \tau)$$

This boundary condition will have the same form for model and prototype if

$$\left\{ N_R \right\}_m = \left\{ N_R \right\}_p.$$

Thus N_R is identified as an important independent dimensionless parameter in the phenomenon of cooling of a long cylinder by emission of thermal radiation. Of course, one ensures equality of N_R for the model and prototype by selecting for the model values of the parameters comprising N_R which give the value

of N_R that one wishes to duplicate.

The condition that $\lambda^K(\varphi)$ have exactly the same form for the model and prototype must also be satisfied if the differential equation (4a) is to have the same form for the model and prototype. The same is true of $\lambda^C(\varphi)$. This type of condition cannot be arbitrarily satisfied as was the case with those already discussed. It is obvious that the functional dependence of λ^K or λ^C on φ must be of the same type, i.e., linear, quadratic, exponential, etc., for the model and prototype. It is known from the data presented in references (2) and (6) that both the specific heat and thermal conductivity of the prototype can be approximated very closely by linear functions up to the maximum temperature of interest (2). (Since the property values of the prototype material were measured in either a hydrogen or argon atmosphere, there is considerable doubt concerning the validity of using these values to predict what will occur in a re-entry environment. Their use is justified on the basis that they are the only available values.) So the relation between thermal conductivity and temperature, for example, may be written

$$K = K_a + b(T - T_a)$$

where K_a is the thermal conductivity corresponding to some arbitrary temperature, T_a . Substituting for K and T in terms of λ^K and φ gives

$$\lambda^K(\varphi) = \left(\frac{K_a}{K_R} - \frac{bT_a}{K_R} \right) + \frac{bT_i}{K_R} \varphi \quad (9)$$

To preserve identity of form for model and prototype, λ^K must be a linear function of ϕ for the model and we must have

$$\left\{ \frac{bT_i}{K_R} \right\}_m = \left\{ \frac{bT_i}{K_R} \right\}_p$$

$$\left\{ \frac{K_a}{K_R} - \frac{bT_a}{K_R} \right\}_m = \left\{ \frac{K_a}{K_R} - \frac{bT_a}{K_R} \right\}_p .$$

Although these equalities cannot be satisfied simply by thoughtful choices for K_a , K_R , and T_a , it appears that the task of satisfying them would be considerably simplified by letting

$$K_R = K_i$$

$$K_a = K_i$$

and

$$T_a = T_i \tag{10}$$

for both the model and prototype. These choices insure that the initial values of λ^K will be equal for the model and prototype. Substituting from (10) into equation (9) and defining

$$F = \frac{bT_i}{K_i} \tag{11}$$

results in a simple form for the relation between dimensionless thermal conductivity and temperature

$$\lambda^K(\phi) = (1-F) + F\phi , \tag{12}$$

where F must be the same for model and prototype to insure similarity. Similar reasoning leads to the same type of relation between dimensionless specific heat and temperature

$$\lambda^C(\phi) = (1-H) + H\phi \tag{13}$$

where

$$H = e T_i \frac{1}{C_i} \quad (14)$$

and e is the slope of the curve relating specific heat to temperature. It is to be noted that the requirements that F and H be equal for the model and prototype are, in essence, two conditions which restrict the choice of the initial temperature. This is the case, since the slopes b and e are fixed for a given material in a given temperature range and since C_i and K_i depend on T_i .

The final form of the differential equation and conditions which the temperature distribution must satisfy is

$$\frac{1}{r} \frac{\partial}{\partial r} (r \lambda^K(\varphi) \frac{\partial \varphi}{\partial r}) = \lambda^C(\varphi) \frac{\partial \varphi}{\partial \tau} \quad (15a)$$

$$\varphi(r, 0) = 1$$

$$\frac{\partial \varphi}{\partial r}(0, \tau) = 0, \quad \tau \geq 0 \quad (15b)$$

$$\frac{\partial \varphi}{\partial r}(1, \tau) = \frac{-N_R}{\lambda^K(\varphi)} \varphi^4(1, \tau), \quad \tau > 0$$

where

$$\lambda^K(\varphi) = (1-F) = F\varphi \quad (12)$$

and

$$\lambda^C(\varphi) = (1-H) + H\varphi \quad (13)$$

in the temperature range of interest.

Application of the reference values selected above to the stress equations (5) gives

$$\Sigma_z = 2 \int_0^1 \left(\int_1^\varphi \lambda^\alpha(\varphi) d\varphi \right) r dr - \int_1^\varphi \lambda^\alpha(\varphi) d\varphi$$

$$\begin{aligned}\Sigma_r &= \int_0^1 \left(\int_1^\varphi \lambda^\alpha(\varphi) d\varphi \right) r dr - \frac{1}{r^2} \int_0^r \left(\int_1^\varphi \lambda^\alpha(\varphi) d\varphi \right) r dr \\ \Sigma_\theta &= \int_0^1 \left(\int_1^\varphi \lambda^\alpha(\varphi) d\varphi \right) r dr + \frac{1}{r^2} \int_0^r \left(\int_1^\varphi \lambda^\alpha(\varphi) d\varphi \right) r dr \\ &\quad - \int_1^\varphi \lambda^\alpha(\varphi) d\varphi .\end{aligned}\tag{16}$$

If $\lambda^\alpha(\varphi)$ has the same form for model and prototype, the stress equations are identical for model and prototype. The initial value of λ was chosen as the reference value; i.e.,

$$\alpha_R = \alpha_i ,$$

so that

(17)

$$\lambda^\alpha(\varphi) = \frac{\alpha}{\alpha_i}$$

It was not necessary to choose a form for $\lambda^\alpha(\varphi)$, since the data for the prototype (6) indicated that α was a very weak function of temperature. Hence $\lambda^{\alpha=1}$ was used in all the stress calculations.

Solution for the Temperature and Stress Distributions

The calculation of $\varphi(r, \tau)$ was the main problem, since the dimensionless stress components can be calculated from equations (16) as soon as $\varphi(r, \tau)$ is known. To calculate $\varphi(r, \tau)$ one must solve equation (15a) subject to the conditions (15b). This is not a simple task, due to the non-linear boundary condition at $r=1$. Additional difficulty is encountered if λ^K and λ^C are not treated as constants, i.e., if F and H are not zero.

To investigate the possibility of an exact solution for some special case, while retaining the important features of the problem, it was temporarily assumed that thermal conductivity and specific heat were constant, so that

$$\lambda^K = 1$$

and

$$\lambda^C = 1$$

The non-linear boundary condition remains, however, and prevents the use of product solutions superposition. The use of Duhamel's Theorem in conjunction with the use of the Laplace Transform to determine the variation of surface temperature with time was investigated. Abarbanel (7) has employed a technique making use of these tools to determine the temperature distribution in a semi-infinite solid and a solid sphere when radiation is emitted from the surfaces of these types of bodies. However, he was able to obtain results for only very short or very long times. Abarbanel gave no results for cylindrical coordinates. An investigation of the technique for cylindrical coordinates led to the conclusion that the amount of labor involved would be difficult to justify by the results obtained, especially since there was no assurance that results could be obtained for sufficiently large values of time to permit the calculation of the maximum stress.

Since no promising method could be found which would yield a closed form of solution, and since it was considered desirable to investigate the effects of variable properties,

the decision was made to try finite difference techniques. After a search of the literature, and discussions with Mr. Bob Thompson of the numerical analysis group at Sandia Corporation, it was decided to employ finite difference techniques similar to those described by Richtmyer (8). The development of the difference equations is presented in Appendix A. Denoting increments of the position and time variables by Δr and $\Delta \tau$, respectively; defining the parameters j and n by

$$r = j\Delta r$$

$$\tau = n\Delta \tau ;$$

and using subscript j and superscript n to denote evaluation of a quantity at the corresponding values of r and τ , the set of difference equations is:

$$-A_j \varphi_{j+1}^{n+1} + B_j \varphi_j^{n+1} + C_j \varphi_{j-1}^{n+1} = D_j \quad (18a)$$

where, for

$$1 \leq j < J: \quad (18b)$$

$$A_j = \theta (\lambda^K)_j^n \quad (18b)$$

$$B_j = 2\theta (\lambda^K)_j^n + \frac{(\Delta r)^2}{\Delta \tau} (\lambda^C)_j^n \quad (18c)$$

$$C_j = \theta (\lambda^K)_j^n \quad (18d)$$

$$D_j = \left\{ (1-\theta) (\lambda^K)_j^n + (\lambda^K)_{j+1}^n - \left(1 - \frac{1}{j}\right) (\lambda^K)_j^n \right\} \varphi_{j+1}^n + \left\{ (1-\theta) (\lambda^K)_j^n \right\} \varphi_{j-1}^n - \left\{ \left[2(1-\theta) - \left(1 - \frac{1}{j}\right) \right] (\lambda^K)_j^n + (\lambda^K)_j^n - \frac{(\Delta r)^2}{\Delta \tau} (\lambda^C)_j^n \right\} \varphi_j^n, \quad (18e)$$

for

$$j=0: A_j=1, B_j=-1, C_j=D_j=0, \quad (18f)$$

and for

$$j=J+1, \\ A_j=0, B_j=1, C_j=-1, D_j = \frac{-N_R \Delta r (\phi_{J+1}^n)^4}{(\lambda^K)_J^n} \quad (18g)$$

It may be noted that the latter difference equation is linear, since ϕ_{J+1}^n and $(\lambda^K)_J^n$ may both be obtained from the calculations for the preceding time step. Other differencing schemes could have been used which would be closer approximations to the actual boundary condition; however, these schemes would have produced a non-linear equation instead of (18g). The advantage of using a linear equation was felt to outweigh the possible need for finer time steps in order to obtain more accurate results.

From the initial condition, one obtains $\phi_j^0=1$ for $0 \leq j \leq J+1$. Hence equations (18a) comprise a set of $J+2$ equations in $J+2$ unknowns:

$$\phi_0^{n+1}, \phi_1^{n+1}, \dots, \phi_{J+1}^{n+1}.$$

This set of $J+2$ equations is solved for successively larger integer values of n , i.e., for successive increments of time. If $\theta=0$, the equations may be solved explicitly for ϕ_j^{n+1} , $0 \leq j \leq J+1$. If $\theta \neq 0$, the equations for the ϕ_j^{n+1} are implicit, and must be solved by iteration or by use of an algorithm such as that described by Richtmyer (8) (Section 5, Chapter VI).

The finite difference equations were programmed for the CDC 1604 digital computer by Mrs. Ann Shiver, who also performed a number of check calculations. Since the results for the dimensionless temperature distribution were not obtained in closed form, it was necessary to evaluate the integrals in equations (16) numerically. Mrs. Shiver also prepared these equations for the computer.

To initiate a computation, one must specify the values of a number of parameters:

$N_R, F, H, J, \theta, S \left[-\frac{(\Delta r)^2}{\Delta \tau} \right]$, the time interval at which the stress distributions are to be calculated, and the time interval at which results are printed out by the computer. The first three parameters, N_R, F , and H , are associated with either the model or the prototype, so their values are determined by the physical situation that one wishes to study. If the thermal properties are treated as constants, $F=H=0$, and the physical situation may be considered to be specified by the value of N_R . The parameters J, θ , and S are associated with the finite difference equations. Their values determine the stability and/or the accuracy of the results obtained from these equations.

Since

$$\Delta r = \frac{1}{J+1}$$

the value of J determines the size of Δr . The value of θ determines whether the set of difference equations is implicit or explicit and also enters into the stability criterion.

Although no stability criterion has been derived for the type of finite difference equations which arose in this problem, a heuristic criterion presented by Richtmyer (8) was employed. (A check of the applicability of this criterion to the present set of equations was carried out by Mr. Thompson. The numerical results exhibited stability where the criterion was satisfied and instability when it was violated by a small margin. Hence it was concluded that the criterion was applicable to the present set of equations.) The form of the criterion for this set of equations is

$$\frac{(\lambda^K)_j^n}{s} < \frac{1}{2-4\theta}$$

or

$$\frac{\Delta\tau}{(\Delta r)^2} (\lambda^K)_j^n < \frac{1}{2-4\theta} \quad (19)$$

if $0 \leq \theta < 1/2$. There is no restriction if $1/2 \leq \theta \leq 1$. (Since the slope of the thermal conductivity-versus-temperature curve is positive and since the temperature at every point decreases with time, $(\lambda^K)_j^n \leq 1$ for all n . Hence the simpler, but more severe criterion

$$\frac{\Delta\tau}{(\Delta r)^2} < \frac{1}{2-4\theta}$$

may be employed.) In all the calculations performed to date, aside from test or check calculations, $\theta=0.75$ was employed, so the equations were stable for all values of $\Delta\tau$ and Δr .

To obtain results of satisfactory accuracy, the values of both Δr and $\Delta \tau$ must be limited. To obtain a numerical estimate of the accuracy of the temperatures obtained from the finite difference equations, a check calculation was included in the computer program. The check calculation was based theoretically on the fact that the heat passing through the surface of the cylinder from the initial instant to any later time is equal to the decrease in internal energy of the cylinder in the same time interval. The equations expressing this energy balance have the dimensionless form

$$\frac{Q'(\tau)}{2\pi a^2 \rho C_i T_i} = N_R \int_0^\tau \phi^4(1, \tau) d\tau = \int_0^1 [1 - \lambda^C(\phi) \phi(r, \tau)] r dr \quad (20)$$

where $Q'(\tau)$ denotes the heat transferred from the cylinder per unit length, from the start of the cooling process to any later time, τ . The integrals in these equations were evaluated numerically and printed out with the results for the stress and temperature distributions to permit comparison. Although some error is incurred in the numerical evaluation of the integrals in equations (20), this method of checking the results was felt to be desirable, since it provides more than just a check on the computer program. It is, however, no more than a check on the consistency of the results. By using this check it was decided that results of satisfactory accuracy could be obtained with $\Delta r \leq 0.05$ and $\Delta \tau \leq 0.001$. Smaller values of these increments were required as N_R increased, $\Delta r = .02$ and $\Delta \tau = .0001$ being employed for $N_R = 10$.

Numerical results were obtained for both the stress and temperature distributions from the initial instant up to the value of τ at which the maximum value of stress³ occurred, for values of N_R ranging from 0.0129 through 10.0. Additional calculations of the temperature distribution, but not the stress distributions, were carried out for $0 \leq \tau \leq 1.0$, for values of N_R ranging from 0.0129 to 5.0. These results appear to be of use, judging from articles appearing in the literature, (9, 10); but they could not be located in the literature. Most of the calculations were made with constant thermal property values ($\lambda^K = \lambda^C = 1$)⁴, but calculations were carried out with various combinations of F and H to help determine the effects of variable properties. The reason why variable properties were not considered more extensively is because it was found that variation of thermal conductivity and specific heat with temperature did not have much effect on the maximum value of stress for a given value of N_R , as will be shown further on.

Discussion of Results

In the preliminary analysis it was reasoned that increasing the rate of cooling would increase the maximum stress and thus improve the probability of failure. That the hypothesis in this line of reasoning is correct can be shown by means of the results presented in Figures 1 and 2. (See Appendix B).

³The term "maximum" is employed herein to denote the greatest value of stress calculated for a given set of input parameters. Hence, it refers to the maximum with respect to both position and time.

⁴It will be recalled that the coefficient of linear expansion was treated as a constant, as pointed out previously. Hence $\lambda^\alpha = 1$ was used in all the calculated results.

Figure 1 shows that with constant thermal properties the maximum dimensionless principal stress, which is a tensile stress, increases with increasing values of N_R . For a cylinder of a given size, surface condition, and material, larger values of N_R correspond to higher initial temperatures. Since the rate of heat transfer from the surface of the cylinder is proportional to N_R , as shown by the boundary condition (15b), one would expect the rate of energy loss to increase as N_R increases. Figure 2 confirms that this is the case. It shows that the dimensionless heat transfer from the cylinder in a given interval of time becomes greater as N_R increases. The actual heat transferred per unit length of the cylinder up to any time τ , $Q'(\tau)$, is directly proportional to the dimensionless heat transfer and to the initial temperature. Therefore larger values of N_R are associated with greater amounts of heat transfer from the cylinder in a given time, hence with larger rates of heat transfer.

The maximum principal stress is directly proportional to the maximum dimensionless principal stress and to the initial temperature. Since the maximum dimensionless principal stress increases as N_R increases, as shown in Figure 1, it follows that higher values of maximum principal stress are produced by higher rates of cooling.

Figure 3 illustrates an earlier statement, that the maximum principal stress is affected relatively little by variation of the thermal properties with temperature. The maxima of these

curves are the maximum dimensionless principal stresses defined previously. It can be seen from this figure that the maximum dimensionless principal stress increases as the variation of properties with temperature becomes stronger. However, the increase is only about 2%, even with the strong variation to which the upper curve corresponds. In addition, the value of N_R for which these results were obtained is very large compared to the values of N_R which are appropriate to a fuel rod. (Unity, as compared to approximately 0.04.) Since the effect of variable properties must be less with decreasing rates of cooling; i.e., with smaller values of N_R , one may conclude that the effect of variable properties on the maximum principal stress is quite small for the conditions of primary interest.

Since higher rates of cooling produce higher values of the maximum principal stress and correspond to higher initial temperatures, it might be concluded that the conditions which are most likely to produce failure include the maximum initial temperature which a material can withstand. However, as pointed out previously, radiation quenching is a "one-shot" process; and there is a very low probability of failure due to a single such process unless the material is in a brittle condition. It was also noted earlier that the behavior of the Uranium-Zirconium hydride alloy changes from brittle to ductile somewhere between 1000° and 1500°F. The maximum principal stress produced by an initial temperature of 1500°F will be somewhat greater than the maximum stress that can be induced

when the fuel-moderator material is in a brittle condition. Hence, results were calculated for an initial temperature of 1500°F. To obtain the maximum possible rate of heat transfer for this temperature, it was assumed that the body is black, i.e., $\epsilon=1$.

The other parameter values pertinent to a cylindrical NAP fuel rod are:

$$\bar{a} = 0.625 \text{ in}$$

$$\alpha = 4.56 \times 10^{-6} \text{ in/in}$$

$$E = 11.0 \times 10^6 \text{ lb/in}^2$$

$$\nu = 0.3$$

$$\frac{K}{\rho C} = 0.277 \frac{\text{FT}^2}{\text{HR}}$$

This set of parameter values gives $N_R=0.0395$.

The more important results obtained for this case with

$$\lambda^K = \lambda^C=1,$$

are presented in Figures 4, 5, 6, and 7. Some of the results for the principal stresses have been converted into dimensional form, using the parameter values listed above, to give an idea of the order of magnitude of the stresses. Figure 4 shows that the axial stress at the outer surface increases quite rapidly, peaks at about $\tau=0.2$, and then decreases rather slowly. The axial stress at this surface is particularly significant because, as Figure 5 shows, this stress is instantaneously at least as great as the highest tensile stress. Thus, the highest point on the curve in Figure 4 gives the maximum

principal stress for this case. It is observed that the value of this stress is 1260 lb/sq. in. The real time at which this maximum occurs is about 7.4 seconds.

Figure 5 presents the principal stress distributions at approximately the instant when the maximum principal stress occurs. This figure shows that the axial and circumferential components are equal at the outer surface, where the radial stress vanishes. This is necessarily the case, since it was assumed that no radial external constraints are applied and since the sum of the radial and circumferential components equal the axial component. Only one curve is needed to represent the stress distributions in the rod, because of the assumption of a very long cylinder. It should be recognized, however, that the stresses within about one diameter of the ends of a rod will depart significantly from the distributions in Figure 5.

From Figure 6 it can be seen that the dimensionless axial stress at the center of the cylinder, the largest compressive stress, lags the dimensionless axial stress at the surface, the largest tensile stress. Since the actual stresses are in the same proportion as the dimensionless stresses, this statement also holds for the actual stresses. Examination of the expression for the axial component of stress would lead one to expect this, since the temperature at the center of the cylinder decreases less rapidly than the surface temperature. It may also be noted from Figure 6 that the peak compressive stress is lower than the peak tensile stress.

Figure 7 is a plot of the ratios of the surface and center temperatures to the initial temperature, as a function of time. The high values of these ratios through the time at which the maximum stress occurs (≈ 0.2) indicates that the temperature drop is rather slow. In fact, the surface temperature has dropped only about 49° when the maximum stress occurs. This rather small change in temperature validates the assumption of constant thermal properties for this case.

On the basis of the reasoning presented earlier in this report, the maximum tensile stress which can be induced by radiation quenching of a cylindrical NAP fuel rod in a brittle condition is no larger than 1260 pounds per square inch. In order to decide whether failure of such a fuel rod is likely, it is necessary to compare this result with strength data for the Uranium-Zirconium hydride alloy of which these fuel rods are composed. The strength data are rather sparse. A value of 8340 lb/sq. in. is reported in reference 2 for ultimate tensile strength at room temperature; but valid data was obtained from only one test, due to difficulties encountered in handling and machining the material - difficulties which were attributed to its brittleness⁵. Maximum stress values obtained from disk and ring specimens at room temperature are also reported in reference 2. The maximum stress in the disk specimen was only about 2500 lb/sq. in., but it apparently failed in shear rather than in tension. The maximum stress in the ring at room temperature was about 6,000 lb/sq. in.

⁵Note that the results for ultimate tensile strength of the hydrided and unhydrided materials presented in Table I of reference 2 have apparently been interchanged.

and at 1000°F it was about 6,300 lb/sq. in. The strength of the fuel rod material used in the NAP Developmental Reactor which had a uranium content of 9.85 percent by weight and a hydrogen content of 1.8 percent is discussed in reference 4. The author of that report remarks about the great difficulty involved in trying to obtain data from tensile tests due to the brittleness of the material at room temperature, as did the authors of reference 2. The investigators whose results are presented in reference 4 abandoned the effort to obtain meaningful tensile test data and substituted the transverse bend test. More consistent results were reported from this test, and a value of 24,000 lb/sq. in. was settled upon for the ultimate tensile strength. This value is almost three times the value obtained from the single tensile test reported in reference 2.

Regardless of which of these values are employed, however, use of the maximum principal stress theory of failure leads to the conclusion that the probability of failure of a fuel rod due to radiation quenching is very low, since the maximum principal stress is much lower than any of the strength values. The validity of this criterion may be challenged, however. Perhaps the most powerful reason for questioning its validity is the remark in Section VA of reference (2) that there were numerous fine cracks throughout the material from which those investigators were attempting to prepare specimens. Presumably,

the same fine cracks would exist in the material in an actual fuel rod; and, if so, they could produce large stress concentrations. If the temperature in a fuel rod were to pass through several cycles in the course of operation of a NAP reactor, these cracks might be propagated. Even if they were not propagated, the stress concentrations which they produce would enhance the probability of failure during a radiation quench.

Leaving the discussion of this particular case, the question arises as to what can be done to increase the maximum stress besides increasing the initial temperature. Since the maximum stress increases as N_R increases, we might first consider what else can be done to increase the value of N_R . From the definition:

$$N_R = \frac{\bar{a}\sigma\epsilon T_i^3}{K_1}$$

it is clear that increasing the emissivity will help. However, the emissivity can be no larger than unity, so the increase that can be gained by this method has an absolute limit.

Making the outside radius larger will also increase N_R . While there is no absolute limit on the value of $\bar{a}$ there surely is a practical limit, determined by the reactor design criteria.

Larger values of N_R may also be obtained by making the thermal conductivity smaller. This might be accomplished by using a material of a different composition, or by including voids in the material and thus reducing the effective conductivity.

All of these parameters do not appear in the expressions for dimensionless stress, so their only effect on the maximum stress is through N_R .

A number of curves have been included in an attempt to give a broader and clearer picture of the phenomenon of radiation cooling of a long circular cylinder.

Figure 8 is a plot of the dimensionless time at which the maximum stress occurs, versus the parameter N_R . It is observed that the elapsed time at which the maximum stress occurs decreases quite rapidly with increases in N_R when that parameter is small, but that the rate of decrease becomes quite small at larger values of N_R . Since larger values of N_R correspond to larger cooling rates, and since real time is directly proportional to dimensionless time for a cylinder of given size and material, it follows that increasing the rate of cooling has little effect on the elapsed time to reach maximum stress when the rate of cooling is high. Because both branches of the "hyperbola" appear in this figure it is reasoned that the range of conditions covered by the calculated results is rather broad.

Figures 9 and 10 present the distributions of the principal stresses at the instant of maximum stress, divided by the maximum stress, and show the dependence of these distributions on N_R . The data presented in these figures, along with the values for the maximum dimensionless stress in Figure 1, permit one to calculate the distributions of the principal stresses at the time when the maximum stress occurs for various values

of N_R . From Figures 9 and 10 it may be concluded that the gradient of the dimensionless axial and tangential stresses near the outer surface becomes sharper as N_R , i.e., the rate of cooling, increases. Furthermore, as the rate of cooling increases, the dimensionless stresses at the center of the cylinder lag further behind the surface stresses, as indicated by the smaller negative values of the dimensionless stresses at the center for larger values of N_R . These results follow logically from the expressions for the dimensionless principal stresses and from the results for the dimensionless temperatures presented in Figures 11 through 16.

These figures give the ratio of temperature to initial temperature as a function of time and N_R at various radial positions. By comparing the results in Figures 11 through 15 with those in Figure 16, for example, it is seen that the interior temperatures lag behind the surface temperature in a much more pronounced fashion for large values of N_R than for small values of N_R . This is what one would expect, since diffusion becomes the controlling process when the surface cooling rate becomes large, corresponding to low "surface resistance" to heat transfer. Figure 16 shows that the surface temperature decreases very rapidly at higher rates of cooling, whereas it decreases quite slowly for small cooling rates. Hence, the assumption of a constant emissive power at the surface of a cylinder should yield a very close approximation

to the maximum value of stress if the value of N_R is less than roughly 0.05. Of course, the validity of this assumption also depends on the magnitude of T_i .

Figure 17 shown the effects of variable thermal properties on the temperatures at the surface and the center, for a value of N_R corresponding to a relatively strong rate of cooling. The solid curves show the temperature variation with time when properties are constant, while the broken curves give the same sort of results with variable properties. Even with this strong rate of cooling, the temperatures at small values of time are affected very little by variation of properties. At later times the effect becomes appreciable, and appears to continue to grow. Figure 17 shows that the type of thermal property variation considered in this case, which involves curves relating thermal conductivity and specific heat to temperature with positive slopes, results in a lower dimensionless temperature at a given position and time. Figure 3 shows that this type of property variation also produces higher stresses at a given location and time. Thus, even though results were not calculated for the stresses at larger values of t , it can be concluded that the effect of the type of property variation considered here is to increase the dimensionless principal stresses above those induced with constant properties, to a greater extent as time passes.

EXPERIMENTAL PHASE OF THE INVESTIGATION

The experimental phase was undertaken to verify the temperature and stress distributions predicted from the analytical solution. This involves the design, construction, and operation of an apparatus which will reproduce the conditions upon which the analytical solution is based. These conditions include the boundary and initial conditions and other assumptions used in the analysis.

Model

The cylindrical element discussed in the analytical phase is undesirable to use in the test. The reason is the radiation and security problems involved. Also, temperature and strain can be measured only on the surface because, due to the nature of the element, difficulty is incurred when embedding measuring devices (strain gages and thermocouples).

In order to overcome the objections to the actual element, a model will be used which consists of epoxy and aluminum powder. Use of this material will make possible the embedding of thermocouples and strain gages for internal temperature and strain measurements. This particular technique of embedding gages in epoxy-aluminum models has been used successfully in the Mechanical Engineering Department at The University of New Mexico.

Similitude will be used to relate model and prototype (actual cylindrical element) as discussed in the analytical portion of the report. Relating the nondimensional parameters

previously described, the diameter of a suitable model and the initial temperature required for the test are determined. A three-inch diameter will be used for the test cylinder, and the temperature required will be considerably lower than experienced in the prototype.

Conditions Involving Simulation

Referring to the boundary conditions, initial conditions, and other assumptions used in the analytical solution, these conditions must be satisfied in the experimental phase.

Radiative heat transfer must be the primary means of transfer of heat from the surface of the element. This means heat transfer from the surface by convection and conduction must be made negligible. This will be accomplished effectively in the experiment by maintaining a chamber pressure of about 10^{-5} millimeters of mercury.

Initially, the model must be at a uniform temperature in order to reproduce the condition on which the analysis is based. This means that the model will be heated throughout to a uniform temperature and then held at this temperature until time to conduct the test.

The surroundings to which the cylindrical element radiates must be at 0°K . from the standpoint of radiant heat transfer. In the radiant heat transfer part of the solution, the difference of the fourth power of the prototype and surrounding temperatures appears. In order to solve this problem analytically, the temperature of the surroundings must be 0°K .. Therefore,

the surroundings for the model must be close to 0°K. to simulate the conditions used in the analytical solution.

The surroundings to which the cylindrical model radiates must also have the characteristics of a black body and consequently an emissivity of one. This means all the radiant energy that is emitted from the element must be absorbed by the surroundings.

The temperature in the cylindrical prototype is assumed to be constant in the longitudinal and circumferential directions and varies only in the radial direction. Hence, in the model, the temperature must vary only in the radial direction and remain constant in the other two directions as stated.

The stresses at the ends of the cylindrical prototype are different from those at the center cross section. The reason for this is the end effects, or conditions which exist near the ends of a long cylinder. The conditions present at the ends of the cylinder should diminish at some distance from the ends, thus leaving a short length on either side of the center cross section where the stress distribution will remain essentially constant.

This model was designed on the basis of a 5 to 1 ratio between the length and diameter. Since the diameter is 3 inches, the length will be 15 inches. The reason for the 5 to 1 ratio is to insure that the end effects are negligible at the center.

Environment Simulation

There are several things to consider in order to produce the desired environment for this experiment.

To simulate an environment where radiative heat transfer dominates and convective and conductive effects are negligible a good vacuum apparatus is used. The apparatus should be capable of producing vacuums in the range of 10^{-5} to 10^{-6} millimeters of mercury which is sufficient to eliminate convective and conductive heat transfer effects.

To simulate the surroundings to which the prototype will be exposed, the model will be placed concentrically inside a cylindrical heat sink (Figure 18).

The heat sink should satisfy the requirements for the surroundings contingent on the assumptions discussed below. Liquid nitrogen will be used as the cooling fluid in the heat sink. This should maintain the inside cylindrical surface of the heat sink at 85°K . The difference between the actual temperature of liquid nitrogen (77°K .) and the expected surface temperature of 85°K . is due to heat transfer losses through the copper wall of the heat sink by conduction and also due to radiant energy absorbed from the apparatus inside the heat sink. The fourth power of 85°K . is small compared to the fourth power of the range of temperatures experienced in the model during the experiment. Therefore, this temperature of 85°K . will simulate the 0°K . used in the analytical solution.

The heat sink has been made longer than the model to help reduce the end effects.

To insure an emissivity of one on the inside surface of the heat sink, the copper used in construction is bent into a corrugated pattern (Figure 18) and then painted with cat-a-lac-flat black, a commercial paint.

To bring the cylindrical model to the desired temperature and then hold it until ready to test, a collapsible heater has been built and tested (Figure 19). This heater will surround the model during heating and until the temperatures in the model are uniform throughout. Also, between the heater and the heat sink are two collapsible radiation shields used to separate the heat sink from the heater and model (Figure 19). By remote manipulation the heater and shields will be pulled up thus exposing the model directly to the heat sink.

Measurements, Model Instrumentation

In the design of the experiment a list is needed of the measurements required. There are several measurements to be made during the course of the experiment. Basically, however, the two types of measurements are:

- 1) temperature, using thermocouples; and
- 2) strain, using strain gages.

The strain gages and thermocouples will be mounted both on the surface and inside of the model in order to obtain the desired data (Figure 20).

For a comparison of theoretical and experimental results a series of temperatures is required along the radial axis from the longitudinal axis to the outside surface of the model. By substituting these temperatures into the stress equations developed in the analytical phase, curves of stress versus radial position can be plotted. These curves can then be compared with the theoretical curves calculated in the analytical phase.

Strain gages will be used to determine the stresses at various points in the model. These values of stress can be used as a comparison with the experimental results obtained from the thermocouples.

To mount the gages and thermocouples internally, the model will be cut radially at the center cross section halfway between the ends of the cylinder. The smooth surface of the cut will be used for the mounting surface for these measuring devices.

On the surface of the cut, thermocouples will be mounted at four different points along the radial axis (Figure 20). One thermocouple will be mounted at the center of the radial axis, two at intermediate points and one near the surface. When the two halves are sealed together the thermocouples will be imbedded in the cylindrical model.

As a check on the stress values determined from the thermocouples a 90° strain gage rosette will be mounted near the surface; and another rosette will be mounted on the same radius with one of the intermediate points. According to the equations determined in the analytical phase the sum of the radial and circumferential stress equals the longitudinal stress; so with two strain gages, the three principal stresses can be determined at the intermediate point. Near the surface the rosette will be mounted in the longitudinal and circumferential directions. The radial stress will be zero near the surface or very close to zero.

To verify that the temperature remains uniform in the longitudinal direction in the model throughout the test several thermocouples will be mounted at strategic points in the model. Two will be mounted near the surface on either side of the center cross section (Figure 20).

To prove the assumption that the strain remains constant for several inches on either side of the center of the cylindrical model, a strain gage will be mounted near each end and one-half way between the end and the center of the element (Figure 20). With these gages, it can be determined where the end effects are no longer present in the model.

A thermocouple will be mounted on the inside surface of the heat sink to determine the actual temperature the model sees when exposed to the cold surroundings. It is important

to know what this temperature is because if it gets too close to the temperature of the model, the assumption that it simulates 0°K . will no longer be valid.

Recording Equipment

To record the data, equipment is required which will simultaneously record temperatures and strains from the various positions in the model (Figure 21). This is needed since this experiment involves transient thermal stresses. The temperatures and strains in the model will vary rapidly enough that all the readings from the strain gages and thermocouples must be recorded simultaneously.

For this data recording, an oscillograph (C.E.C. recorder) with 18 channels will be used.

To record the signal from a strain gage the lead wires from the gage will be fed into a wheatstone bridge circuit board (Figure 21) which contains 10 bridge circuits. Each bridge circuit will connect to one strain gage. Power for this circuit board is supplied by a high-current, low-voltage power supply.

From this board one leg of the bridge circuit for each gage will be connected to the oscillograph. This completes the circuit for recording the signal from a strain gage.

The leads from a thermocouple will be connected directly to the recorder. An ice bath will be used as a reference

junction in the circuit (Figure 21). Due to the extremely small voltage generated in the thermocouple, very sensitive galvanometers are required in the recorder.

Both the strain gages and thermocouples will be calibrated prior to the test.

Experimental Apparatus

The following equipment is being acquired for the experiment:

1. A vacuum apparatus which includes a mechanical pump, an 18-inch diameter by 30-inch bell jar, a liquid freon cooling system, a diffusion pump, a liquid nitrogen cold trap, a thermocouple vacuum gage, and a hot filament ionization vacuum gage.
2. A 110-liter liquid nitrogen bottle which will be rented to supply liquid nitrogen to the heat sink.
3. Thermocouple wire and strain gages for instrumenting the cylindrical model.
4. Two mechanical manipulators which will be mounted in the base plate. Along with the manipulators, various gears and pulleys are needed.
5. Two electrical lead throughs mounted in the base plate for strain gage leads and heater connections. Each of these lead throughs has 8 pins for electrical connections.
6. Two pass throughs mounted in the base plate for bringing the liquid nitrogen into and out of the vacuum system.

7. An oscillograph with 18 channels for simultaneous recording.
8. A strain gage bridge circuit board which contains 10 bridge circuits.
9. A power supply to be used with the strain gage bridge circuit board. This is a high current, low voltage power supply.
10. Two variacs for controlling the heater input.
11. A McCleod gage to calibrate the hot filament ionization gage.

The following equipment is being constructed for the experiment:

1. A hoist for the bell jar.
 2. An implosion screen to put over the bell jar.
 3. A heat sink which will be cooled with liquid nitrogen.
 4. A model cylinder made of a mixture of epocast and aluminum powder (120 mesh).
 5. Inside the heat sink will be two collapsible radiation shields made of wire wrapped with a foil coating.
- These will be movable and will collapse into about half their stretched length.

6. Inside the two radiation shields will be a collapsible resistance heater surrounding the model. This heater will consist of nichrome wire wrapped with fiberglass cloth and aluminum foil for support.
7. Outside the heat sink will be two radiation shields made of mylar-coated chrome film (Figure 19) to prevent radiation from the outside of the bell jar from reaching the heat sink.
8. A tubular lead out through the base plate for the thermocouple wires.

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LIST OF SYMBOLS

Symbol	Quantity	Dimensions*
$\bar{a}$	Outside radius	L
b	Slope of curve relating thermal conductivity to temperature	$Ft^{-1}\theta^{-2}$
C	Specific heat of solid	$L^2T^{-2}\theta^{-1}$
e	Slope of curve relating specific heat to temperature	$L^2T^{-2}\theta^{-2}$
E	Young's Modulus	FL^{-2}
F	Slope of curve relating dimensionless thermal conductivity to dimensionless temperature. Defined by equation (11).	1
H	Slope of curve relating dimensionless heat capacity to dimensionless temperature. Defined by equation (14).	1
K	Thermal conductivity	$FT^{-1}\theta^{-1}$
N_R	Dimensionless parameter related to surface heat flux, defined as $\bar{a}\epsilon\sigma T_i^3/K_i$	1
$\bar{r}$	Radial position	L
r	Dimensionless radial coordinate	1
t	Time	T
T	Absolute temperature	θ

* L,T,F, θ denote dimensions of length, time, force, and temperature, respectively.

Symbol	Quantity	Dimensions
α	Coefficient of linear thermal expansion	θ^{-1}
ϵ	Total hemispherical emissivity	1
λ	Used with superscript K, C, or α to indicate a dimensionless property	1
ν	Poisson's ratio	1
ϕ	Ratio of temperature to initial temperature, T/T_i	1
ρ	Mass density	$L^{-4}FT^2$
σ	Stefan-Boltzmann Constant	$L^{-1}FT^{-1}\theta^{-4}$
$\Sigma_r, \Sigma_z, \Sigma_\theta$	Dimensionless principal stress components	1
$\sigma_r, \sigma_z, \sigma_\theta$	Principal stress components	FL^{-2}
τ	Dimensionless time, $\frac{K_i t}{\rho c_i \bar{a}^2}$	1
θ	A parameter whose value determines whether the finite-difference equations are implicit or explicit	1

Superscripts:

n	Indicates the time at which a variable is evaluated
$\left. \begin{matrix} k \\ c \\ \alpha \end{matrix} \right\}$	Used with λ to indicate dimensionless thermal conductivity, specific heat, and coefficient of linear thermal expansion, respectively.

Subscripts:

i	Devotes initial value of a quantity
j	Indicates the radial position at which a variable is evaluated.
r	Denotes radial component
z	Denotes axial component
θ	Denotes circumferential component
m	Denotes a quantity pertaining to the model
p	Denotes a quantity pertaining to the prototype.

APPENDIX A

DEVELOPMENT OF FINITE DIFFERENCE EQUATIONS

It is desired to develop a set of finite difference equations corresponding to the differential equation

$$\frac{1}{r} \frac{\partial}{\partial r} \left(\lambda^K r \frac{\partial \phi}{\partial r} \right) = \lambda^C \frac{\partial \phi}{\partial \tau} \quad (\text{A-1})$$

and boundary conditions

$$\frac{\partial \phi}{\partial r} (0, \tau) = 0 \quad (\text{A-2})$$

$$\frac{\partial \phi}{\partial r} (1, \tau) = - \frac{N_R}{\lambda^K} \phi^4(1, \tau) \quad (\text{A-3})$$

To this end, let

$$r = j \cdot \Delta r$$

$$\tau = n \cdot \Delta \tau$$

so that

$$\phi(r, \tau) = \phi(j \cdot \Delta r, n \cdot \Delta \tau)$$

and denote the numerical approximation to

$$\phi(j \cdot \Delta r, n \cdot \Delta \tau) \text{ by } \phi_j^n.$$

Approximate the right-hand side of equation (A-1) by

$$(\lambda^C)_j^n \frac{\phi_j^{n+1} - \phi_j^n}{\Delta \tau}.$$

To simplify the resulting finite difference approximation for the left-hand side, it is first differentiated to give

$$\frac{1}{r} \frac{\partial}{\partial r} \left(\lambda^K r \frac{\partial \phi}{\partial r} \right) = \lambda^K \frac{\partial^2 \phi}{\partial r^2} + \left(\frac{\partial \lambda^K}{\partial r} + \frac{\lambda^K}{r} \right) \frac{\partial \phi}{\partial r} \quad (\text{A-4})$$

The first term on the right side of equation A-4 is approximated by

$$\theta(\lambda^K)_j^n \frac{(\varphi_{j+1}^{n+1} - 2\varphi_j^{n+1} + \varphi_{j-1}^{n+1})}{(\Delta r)^2} + (1-\theta)\lambda^K \frac{(\varphi_{j+1}^n - 2\varphi_j^n + \varphi_{j-1}^n)}{(\Delta r)^2}$$

where θ is a parameter ranging from zero through one. The second term on the right side of equation A-4 is approximated by

$$\left[\frac{(\lambda^K)_{j+1}^n - (\lambda^K)_j^n}{\Delta r} + \frac{(\lambda^K)_j^n}{j \cdot \Delta r} \right] \left[\frac{\varphi_{j+1}^n - \varphi_j^n}{\Delta r} \right]$$

This differencing scheme may be used since stability of the finite difference equations is dependent mainly on the highest order term (8).

Substitution of the finite difference approximations for the terms in equation (A-1) leads to the set of equations

$$-A_j \varphi_{j+1}^{n+1} + B_j \varphi_j^{n+1} - C_j \varphi_{j-1}^{n+1} = D_j \quad (\text{A-5})$$

for

$$1 \leq j \leq J,$$

where

$$A_j = C_j = \theta(\lambda^K)_j^n \quad (\text{A-6})$$

$$B_j = 2\theta(\lambda^K)_j^n + \frac{(\Delta r)^2}{\Delta \tau} (\lambda)_j^n \quad (\text{A-7})$$

$$D_j = \left[(1-\theta)(\lambda^K)_j^n + (\lambda^K)_{j+1}^n - (1-\frac{1}{j})(\lambda^K)_j^n \right] \varphi_{j+1}^n \quad (\text{A-8})$$

$$+ (1-\theta)(\lambda^K)_j^n \varphi_{j-1}^n - \left\{ \left[2(1-\theta) - (1-\frac{1}{j}) \right] (\lambda^K)_j^n - \frac{(\Delta r)^2}{\Delta \tau} (\lambda^C)_j^n \right\} \varphi_j^n$$

and where J is defined by

$$(J + 1)\Delta r = 1 \quad (\text{A-9})$$

The difference approximation employed for the derivative in the boundary condition

$$\frac{\partial \varphi(0, \tau)}{\partial r} = 0$$

is
$$\frac{\varphi_1^{n+1} - \varphi_0^{n+1}}{\Delta r} = 0$$

which yields

$$\varphi_0^{n+1} = \varphi_1^{n+1} \quad (\text{A-10})$$

The difference approximation employed for the left side of the boundary condition

$$\frac{\partial \varphi}{\partial r}(1, \tau) = - \frac{N_R}{\lambda^K} \varphi^4(1, \tau)$$

was
$$\frac{\varphi_{J+1}^{n+1} - \varphi_J^{n+1}}{\Delta r}$$

and for the right side

$$\frac{-N_R}{(\lambda^K)_j^n} (\varphi_{J+1}^n)^4$$

Substitution of the approximations leads to

$$\varphi_J^{n+1} = \varphi_{J+1}^{n+1} + \frac{N_R \Delta r}{(\lambda^K)_j^n} (\varphi_{J+1}^n)^4 \quad (\text{A-11})$$

The set of equations (A-5), (A-10), and (A-11) constitute a set of J+2 equations for the J+2 unknowns $\varphi_0^{n+1}, \varphi_1^{n+1}, \dots, \varphi_{J+1}^{n+1}$.

APPENDIX B
FIGURES

Figure 1
VARIATION OF MAXIMUM PRINCIPAL STRESS WITH THE PARAMETER N_R

FIGURE 2
HEAT TRANSFERRED FROM CYLINDER AS A FUNCTION OF
TIME, FOR VARIOUS VALUES OF N_R

FIGURE 3

EFFECT OF VARIABLE THERMAL PROPERTIES ON AXIAL STRESS AT
OUTER SURFACE

FIGURE 4
VARIATION OF MAXIMUM STRESS WITH TIME

FIGURE 5

NORMAL STRESS DISTRIBUTIONS AT APPROXIMATE INSTANT OF MAXIMUM STRESS

FIGURE 6
VARIATION OF AXIAL STRESS WITH TIME AT AXIS AND OUTER SURFACE

FIGURE 7
 VARIATION OF DIMENSIONLESS SURFACE AND CENTER TEMPERATURE
 WITH TIME.

FIGURE 8
VARIATION OF TIME AT WHICH MAXIMUM
STRESS OCCURS WITH N_R .

FIGURE 9

RATIO OF AXIAL STRESS AT INSTANT OF
MAXIMUM STRESS, TO MAXIMUM STRESS, AS
A FUNCTION OF RADIAL POSITION AND N_R .

FIGURE 10
RATIOS OF PRINCIPAL STRESSES AT INSTANT
OF MAXIMUM STRESS, TO THE MAXIMUM STRESS,
AS FUNCTIONS OF RADIAL POSITION AND N_R .

FIGURE 11
VARIATION OF DIMENSIONLESS CENTER
TEMPERATURE WITH DIMENSIONLESS TIME

FIGURE 12
VARIATION WITH DIMENSIONLESS TIME OF
THE DIMENSIONLESS TEMPERATURE AT
TWO-TENTHS OF THE OUTER RADIUS

FIGURE 13
VARIATION WITH DIMENSIONLESS TIME OF
DIMENSIONLESS TEMPERATURE AT FOUR-
TENTHS OF THE OUTER RADIUS.

FIGURE 14

VARIATION WITH DIMENSIONLESS TIME OF
THE DIMENSIONLESS TEMPERATURE AT SIX-
TENTHS OF THE OUTER RADIUS.

FIGURE 15

VARIATION WITH DIMENSIONLESS TIME OF
THE DIMENSIONLESS TEMPERATURE AT EIGHT-
TENTHS OF THE OUTER RADIUS.

FIGURE 16
VARIATION OF DIMENSIONLESS SURFACE
TEMPERATURE WITH DIMENSIONLESS TIME.

FIGURE 17
EFFECT OF VARIABLE SPECIFIC HEAT
AND THERMAL CONDUCTIVITY ON TEMPERATURE.

FIGURE 18
HEAT SINK

FIGURE 19
EXPLODED VIEW OF SPACE SIMULATOR COMPONENTS AND CYLINDRICAL MODEL.

FIGURE 20
INSTRUMENTATION OF MODEL

FIGURE 21
EXTERNAL ELECTRONIC EQUIPMENT