MAY 15 1964

LADC-6353

MASTER

81929

Convergent Electron Plasma Kinetic Equation

C. W. Hirt

University of California
Los Alamos Scientific Laboratory
Los Alamos, New Mexico

A convergent and consistent kinetic equation is derived for a spatially homogeneous electron plasma using a modified expansion of the B-B-G-K-Y hierarchy equations.

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, in the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

This paper was submitted for publication in the open literature at least 6 months prior to the issuance date of this Microcard. Since the U.S.A.E.C. has no evidence that it has been published, the paper is being distributed in Microcard form as a preprint.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

I. INTRODUCTION

One aim of nonequilibrium statistical mechanics is the development of a kinetic equation of the form

$$\frac{\partial f(x,t)}{\partial t} = A(x;f) \quad (1)$$

which describes the time evolution of the 1-particle distribution function. In Eq. (1) x stands for the six variables $\{x, y\}$ and A depends functionally on $f(x,t)$. With Eq. (1) the initial value of $f(x,t)$ is sufficient to determine its value at any other time. One would also expect such a kinetic equation to give an irreversible approach to equilibrium.

In this paper we develop a kinetic equation for a spatially homogeneous electron plasma. The restriction to a spatially homogeneous plasma considerably simplifies the problem as does the use of the electron plasma model in which the ions are assumed to maintain a constant density n .

The usual starting point for obtaining a kinetic equation is the B-B-G-K-Y hierarchy of equations obtained from successive integrations of the Liouville equation.¹ The hierarchy of equations relate the s -particle distribution functions, $f_s(x_1, \dots, x_s, t)$, where

$$V^{-s} f_s(x_1, \dots, x_s, t) dx_1 \dots dx_s$$

is the probability that electron 1 is within dx_1 of x_1 , etc., and V is the volume of plasma containing N electrons. We will always assume the limit $N, V \rightarrow \infty$, $N/V = n$ finite. In this limit the first two members of the B-B-G-K-Y hierarchy for a spatially homogeneous plasma are

$$\frac{\partial f(x,t)}{\partial t} - \frac{e^2 n}{m} \int dx_2 \frac{\partial \phi_{12}}{\partial r_1} \cdot \frac{\partial}{\partial v_1} f_2(x_1, x_2, t) = 0 \quad (2)$$

$$\left\{ \frac{\partial}{\partial t} + v_1 \cdot \frac{\partial}{\partial r_1} + v_2 \cdot \frac{\partial}{\partial r_2} - \frac{e^2}{m} \frac{\partial \phi_{12}}{\partial r_1} \cdot \frac{\partial}{\partial v_1} - \frac{e^2}{m} \frac{\partial \phi_{21}}{\partial r_2} \cdot \frac{\partial}{\partial v_2} \right\} f_2(x_1, x_2, t) \\ - \frac{e^2 n}{m} \int dx_3 \frac{\partial \phi_{13}}{\partial r_1} \cdot \frac{\partial}{\partial v_1} f_3(x_1, x_2, x_3, t) - \frac{e^2 n}{m} \int dx_3 \frac{\partial \phi_{23}}{\partial r_2} \cdot \frac{\partial}{\partial v_2} f_3(x_1, x_2, x_3, t) = 0 \quad (3)$$

where

$$\phi_{ij} = \phi(|r_i - r_j|) = |r_i - r_j|^{-1} \quad (4)$$

In general, the hierarchy equation for f_s will involve f_{s+1} , so that a solution for f_s implies a solution of the full Liouville equation. The point of introducing the s -particle distribution functions is that f_s , for some s , can be approximated by the lower order distribution functions. One then seeks a solution of the "truncated" hierarchy of equations that will cast Eq. (2) into the form of a kinetic equation, i.e., Eq. (1).

Approximate solutions of the B-B-G-K-Y hierarchy have been obtained by several authors.^{2,3,4} A brief review of these results is given in the next section. A difficulty with these approximate treatments is that they

lead to a kinetic equation containing logarithmically diverging integrals. The purpose of this paper is to derive, consistently, a convergent kinetic equation from a suitably truncated form of the B-B-G-K-Y hierarchy.

II. REVIEW

Several authors^{2,3,4} have expanded the B-B-G-K-Y hierarchy in powers of a small parameter, $g = 4\pi R_L/R_D = 1/nR_D^3$, where $R_L = e^2/\theta$ is the Landau length, $R_D = (4\pi e^2 n/\theta)^{-1/2}$ is the Debye length, and $\theta = k_B T$ is the kinetic temperature. In many interesting plasmas g is a very small number. For example, in a typical hot plasma $n = 10^{14} \text{ cm}^{-3}$, $T = 10^5 \text{ }^\circ\text{K}$, and $g = 3.01 \times 10^{-5}$.

To expand the hierarchy in powers of g one introduces dimensionless variables using the Debye length, R_D , as a unit of length and the inverse of the plasma frequency, $\omega_p^{-1} = (4\pi e^2 n/m)^{-1/2}$, as a unit of time. In terms of dimensionless variables Eq. (2) is unchanged, but in Eq. (3) the terms containing the direct interaction,

$$\frac{e^2}{m} \frac{\partial \phi_{12}}{\partial \mathbf{r}_1} \cdot \frac{\partial \mathbf{f}_2}{\partial \mathbf{v}_1}, \quad i = 1, 2 \quad (5)$$

have a coefficient of g . The appearance of g in the dimensionless form of Eq. (3) makes a power series solution of the hierarchy attractive.

The expansion in powers of g automatically truncates the hierarchy. To zero order in g ,

$$f_2(x_1, x_2, t) = f(x_1, t) f(x_2, t)$$

and Eq. (2) becomes the Landau-Vlasov (self consistent field) equation. The Vlasov kinetic equation gives an irreversible approach to spatial uniformity (Landau damping). It does not, however, give an approach to equilibrium in velocity space, since the equation is satisfied by any spatially uniform, time independent, function of the velocity.

To obtain the kinetic equation to first order in g it is necessary to make two further assumptions. First, a suitable boundary condition must be chosen for f_2 . Second, the time dependence of f_2 must be assumed to occur only through a functional dependence on $f(x,t)$. This latter assumption is often referred to as the "adiabatic hypothesis." The spatially homogeneous kinetic equation has been obtained under these assumptions by Balescu,² Guernsey,³ and Lenard⁴ (the B-G-L equation),

$$\frac{\partial f(\underline{v}_1, t)}{\partial t} = \frac{2e^4 n}{m^2} \frac{\partial}{\partial \underline{v}_1} \cdot \int d\underline{v}_2 \int d\underline{k} \frac{k k \delta[k \cdot (\underline{v}_2 - \underline{v}_1)]}{k^4 |e(k, k, \underline{v}_2)|^2} \left\{ f(\underline{v}_2, t) \frac{\partial f(\underline{v}_1, t)}{\partial \underline{v}_1} - f(\underline{v}_1, t) \frac{\partial f(\underline{v}_2, t)}{\partial \underline{v}_2} \right\} \quad (6)$$

where

$$e(k, k, \underline{v}_2) = 1 + \frac{\omega_p^2}{k^2} \int d\underline{v} \frac{1}{k \cdot (\underline{v}_2 - \underline{v}) - i\nu} k \cdot \frac{\partial f(\underline{v}, t)}{\partial \underline{v}} \quad (7)$$

and the limit $\nu \rightarrow 0^+$ is assumed. Eq. (6) has a logarithmic divergence at large k . This divergence indicates that the expansion of the B-B-G-K-Y hierarchy in powers of g is not correct. The origin of the divergence lies in the neglect (to first order in g) of the direct interaction terms (5), which are not small for all values of $|\underline{r}_1 - \underline{r}_2|$. This is analogous

to the divergence that occurs in the plasma test particle problem⁶ when one attempts a linearization of the Vlasov equation.

Recently there have appeared two papers concerned with the elimination of this divergence. The first of these, due to Willis,⁷ describes a method of obtaining $f_2(x_1, x_2, t)$ by employing an expansion about equilibrium. Willis considers the deviation from equilibrium to be an additive correction to the equilibrium state,

$$f_2 = f_{2 \text{ equil.}} + f_2'$$

and proceeds to calculate f_2' (to first order in the plasma parameter g) for the spatially homogeneous electron plasma. The analysis presented by Willis contains a number of errors; the most significant is the assertion that the Fourier transform of his term [3d] is zero for the spatially homogeneous case. When the appropriate corrections are made one discovers that this term gives the dominant contribution to f_2' . The kinetic equation determined with the corrected f_2' does not converge.

The second paper is due to Baldwin.⁵ Baldwin proposes to solve Eq. (3) by a modified expansion in the plasma parameter. The essence of his method is to add and subtract a shielded potential ϕ_{12}^λ , λ denoting the shielding distance, to the pure Coulomb potential ϕ_{12} ,

$$\psi_{12} = \phi_{12}^\lambda + (\phi_{12} - \phi_{12}^\lambda)$$

The first term in ψ_{12} is assumed to be of zero order in g while the second

term is assumed to be first order. The requirement that ϕ_{12}^λ reduce to ϕ_{12} for small $|r_{m1} - r_{m2}|$ means that $(\phi_{12} - \phi_{12}^\lambda)$ vanishes for small $|r_{m1} - r_{m2}|$, and can therefore be treated as first order in g for all values of $|r_{m1} - r_{m2}|$. The remaining potentials are cutoff below a small separation distance b_0 , e.g.,

$$\phi_{11} = \begin{cases} |r_{m1} - r_{m1}|^{-1}, & |r_{m1} - r_{m1}| > b_0 \\ b_0^{-1}, & |r_{m1} - r_{m1}| \leq b_0 \end{cases} \quad (i \neq 1, 2)$$

This cutoff is necessary to eliminate divergent integrals that arise in the course of the derivation, but cancel in the final result. Baldwin uses ϕ_{12} in place of ϕ_{12}^λ and develops a solution for f_2 in powers of g . He obtains a convergent kinetic equation consisting of two terms. The first term is a Boltzmann collision integral for the shielded potential ϕ_{12}^λ , and the second term is a B-G-L like term, Eq. (6), in which $|\epsilon|^{-2}$ is replaced by

$$|\epsilon|^{-2} - [\tilde{\phi}^\lambda(k)/\tilde{\phi}(k)]^2$$

the latter going to zero as $k \rightarrow \infty$. The notation $\tilde{\phi}(k)$ indicates the Fourier transformation of $\phi(r)$.

The method used by Baldwin is analogous to the Vlasov method of combining the binary collision and linear Vlasov treatments of the plasma test particle problem.⁶ In the test particle problem Vlasov used a binary collision treatment for collisions with impact parameter less than some distance L , ($R_L \ll R_D$). For collisions with impact parameter greater than L , Vlasov used a solution of the linearized Vlasov equation in which the

test particle is treated as a small perturbation. In the same spirit, Baldwin separates ψ_{12} into two parts, ϕ_{12}^λ of zero order in g and $(\phi_{12} - \phi_{12}^\lambda)$ of order g . The zero order term in ψ_{12} leads to a Boltzmann collision integral which corresponds to binary interactions. The first order term in ψ_{12} leads to the B-G-L like term which corresponds to a perturbation solution of the hierarchy.

This separation of ψ_{12} and the awkward treatment necessary for the potentials ϕ_{11} does not give a consistent derivation of a kinetic equation.

III. METHOD OF SOLUTION

To obtain a consistent, but still approximate, solution of Eq. (3) the 3-particle distribution function is written as

$$\begin{aligned} f_3(x_1, x_2, x_3, t) = & f(v_{m1}, t) f_2(x_2, x_3, t) + f(v_{m2}, t) f_2(x_1, x_3, t) \\ & + f(v_{m3}, t) f_2(x_1, x_2, t) - 2f(v_{m1}, t) f(v_{m2}, t) f(v_{m3}, t) \\ & + T(x_1, x_2, x_3, t) \end{aligned} \quad (8)$$

The B-B-G-K-Y hierarchy is truncated by neglecting the 3-particle correlation function, $T(x_1, x_2, x_3, t)$. This is an uncontrolled approximation, but an apparently reasonable one. With this approximation Eq. (3) reduces to a linear integral equation for f_2 ,

$$\begin{aligned} & \left\{ \frac{\partial}{\partial t} + v_1 \cdot \frac{\partial}{\partial r_1} + v_2 \cdot \frac{\partial}{\partial r_2} - \frac{e^2}{m} \frac{\partial \phi_{12}}{\partial r_1} \cdot \frac{\partial}{\partial v_1} - \frac{e^2}{m} \frac{\partial \phi_{21}}{\partial r_2} \cdot \frac{\partial}{\partial v_2} \right\} f_2(x_1, x_2, t) \\ & - \frac{e^2 n}{m} \frac{\partial f(v_1, t)}{\partial v_1} \cdot \int dx_3 \frac{\partial \phi_{13}}{\partial r_1} f_2(x_2, x_3, t) - \frac{e^2 n}{m} \frac{\partial f(v_2, t)}{\partial v_2} \cdot \int dx_3 \frac{\partial \phi_{23}}{\partial r_2} f_2(x_1, x_3, t) \\ & - \frac{\partial}{\partial t} \{ f(v_1, t) f(v_2, t) \} = 0 \end{aligned} \quad (9)$$

where use has been made of Eq. (2). A modified correlation function is introduced according to the definition

$$f_2(x_1, x_2, t) = C_2(|r_1 - r_2|) \{ f(v_1, t) f(v_2, t) + h_2(x_1, x_2, t) \}. \quad (10)$$

In equilibrium, the modified correlation function $h_2(x_1, x_2, t)$ is zero, $f(v, t)$ is the Maxwell-Boltzmann velocity distribution,

$$f^{(0)}(v) = \left(\frac{m}{2\pi\theta} \right)^{3/2} \exp(-mv^2/2\theta) \quad (11)$$

and $C_2(|r_1 - r_2|)$ is the equilibrium correlation function.

In Section IV the equilibrium correlation function is determined. We will see that a good approximation to $C_2(r)$ is $\exp\{-R_L \psi(r)\}$ where $\psi(r)$ is the Debye-Huckel potential.

In Section V the modified correlation function h_2 is calculated, to first order in the plasma parameter, with the "adiabatic hypothesis," in which h_2 depends on time only through a functional dependence on $f(v, t)$.

In Section VI the kinetic equation is obtained and in Section VII the results are discussed.

IV. EQUILIBRIUM CORRELATION FUNCTION

$C_2(|\mathbf{r}_1 - \mathbf{r}_2|)$ is determined from Eq. (9), in which f_2 is given by Eq. (10) (with $h_2 = 0$), and $f(\mathbf{v}, t)$ is given by Eq. (11). If we write

$$C_2(r) = \exp[-R_L \psi(r)]$$

where $R_L = e^2/\theta$ then $\psi(r)$ must satisfy

$$\frac{d\psi(r)}{dr} - \frac{d\phi}{dr} - \frac{1}{R_L R_D^2} \frac{d\phi}{dr} \exp\{R_L \psi(r)\} \int_0^r dr' r'^2 \{\exp[-R_L \psi(r')] - 1\} = 0 \quad (12)$$

An approximate solution for $\psi(r)$ is obtained by expanding the last term in powers of $R_L \psi$. To lowest order in the expansion, the solution that vanishes at infinity is the Debye-Huckel potential,

$$\psi(r) = \frac{1}{r} \exp(-r/R_D) \quad (13)$$

where $R_D = (4\pi e^2 n/\theta)^{-1/2}$ is the Debye length. Thus $C_2(r)$ is given approximately by

$$C_2(r) = \exp\left\{-\frac{R_L}{r} e^{-r/R_D}\right\} \quad (14)$$

An exact solution for $\psi(r)$ can only be obtained numerically. One would expect an exact solution to replace the shielding factor $\exp(-r/R_D)$ with a more complicated function of r . For our purposes, however, Eq. (14) will be sufficient.

V. MODIFIED CORRELATION FUNCTION

The equation for h_2 is obtained from Eq. (9) and Eq. (10) together with the normalization $\int dv f(v, t) = 1$. Because of its length and complexity, and because we are only interested in determining h_2 to lowest order in g , we have not included the full h_2 equation. To obtain the equation and the subsequent expansion is straightforward. The expansion procedure consists of introducing dimensionless variables using R_D and ω_p^{-1} as units of length and time. The 1-particle distribution function $f(v, t)$ is assumed to be of zero order in g while h_2 is assumed to be first order in g . Furthermore, the equilibrium correlation function can be expanded

$$\exp\{-R_L \psi(r)\} = 1 - R_L \psi(r) + O(g^2)$$

With these assumptions, and the "adiabatic hypothesis" which requires $\partial h_2 / \partial t \propto O(g^2)$, the equation for h_2 , to first order in g , is

$$\begin{aligned} & (v_1 - v_2) \cdot \frac{\partial}{\partial r} h_2(r, v_1, v_2) - \frac{e^2}{m} \frac{\partial \psi(r)}{\partial r} \left(\frac{\partial}{\partial v_1} - \frac{\partial}{\partial v_2} \right) f(v_1, t) f(v_2, t) \\ & - \frac{e^2 n}{m} \frac{\partial f(v_1, t)}{\partial v_1} \cdot \int dx_3 \frac{\partial \phi(|r - r_3|)}{\partial r} h_2(-r_3, v_2, v_3) \\ & + \frac{e^2 n}{m} \frac{\partial f(v_2, t)}{\partial v_2} \cdot \int dx_3 \frac{\partial \phi(|r - r_3|)}{\partial r} h_2(r_3, v_1, v_3) \\ & - R_L f(v_1, t) f(v_2, t) (v_1 - v_2) \cdot \frac{\partial}{\partial r} \psi(r) = 0 \end{aligned} \quad (15)$$

where $\underline{r} = \underline{r}_1 - \underline{r}_2$. The time argument of h_2 has been omitted, since h_2 now depends on time functionally through $f(\underline{v}, t)$.

A solution of Eq. (15) is obtained by introducing the Fourier transformation

$$\tilde{h}_2(\underline{k}, \underline{v}_1, \underline{v}_2) = \int d\underline{r} \exp(i\underline{k} \cdot \underline{r}) h_2(\underline{r}, \underline{v}_1, \underline{v}_2) \quad (16)$$

The Fourier transformation of Eq. (15) is

$$\begin{aligned} & (\underline{v}_1 - \underline{v}_2) \cdot \underline{k} \tilde{h}_2(\underline{k}, \underline{v}_1, \underline{v}_2) - \frac{e^2 n}{m} \tilde{\psi}(\underline{k}) \underline{k} \cdot \left(\frac{\partial}{\partial \underline{v}_1} - \frac{\partial}{\partial \underline{v}_2} \right) f(\underline{v}_1, t) f(\underline{v}_2, t) \\ & - \frac{e^2 n}{m} \underline{k} \cdot \frac{\partial f(\underline{v}_1, t)}{\partial \underline{v}_1} \tilde{\phi}(\underline{k}) \int d\underline{v}_3 \tilde{h}_2^*(\underline{k}, \underline{v}_2, \underline{v}_3) \\ & + \frac{e^2 n}{m} \underline{k} \cdot \frac{\partial f(\underline{v}_2, t)}{\partial \underline{v}_2} \tilde{\phi}(\underline{k}) \int d\underline{v}_3 \tilde{h}_2(\underline{k}, \underline{v}_1, \underline{v}_3) \\ & - R_L f(\underline{v}_1, t) f(\underline{v}_2, t) \tilde{\psi}(\underline{k}) \underline{k} \cdot (\underline{v}_1 - \underline{v}_2) = 0 \end{aligned} \quad (17)$$

Before we obtain a solution of Eq. (17) it is expedient to see what form of h_2 is required for the kinetic equation. The spatially homogeneous kinetic equation is obtained by inserting Eq. (10) in Eq. (2),

$$\begin{aligned} \frac{\partial f(\underline{v}_1, t)}{\partial t} &= \frac{e^2 n}{m} \frac{\partial}{\partial \underline{v}_1} \cdot \int d\underline{r}_2 \frac{\partial \phi(|\underline{r}_1 - \underline{r}_2|)}{\partial \underline{r}_1} \exp\{-R_L \psi(|\underline{r}_1 - \underline{r}_2|)\} \cdot \\ & \int d\underline{v}_2 h_2(\underline{r}_1 - \underline{r}_2, \underline{v}_1, \underline{v}_2) \end{aligned} \quad (18)$$

Using the Fourier representations for the functions in the integrand,

$$\frac{\partial f(v_1, t)}{\partial t} = - \frac{e^2 n}{(2\pi)^3 m} \frac{\partial}{\partial v_1} \cdot \int dk Q(k) \int dv_2 \tilde{h}_2(k, v_1, v_2) \quad (19)$$

where

$$Q(k) = \int dr \frac{\partial \phi(r)}{\partial r} \exp\{-R_L \psi(r) - ik \cdot r\} \quad (20)$$

Since Q is an odd function of k only the imaginary part of h_2 will contribute to Eq. (19)

$$\frac{\partial f(v_1, t)}{\partial t} = - \frac{e^2 n i}{m(2\pi)^3} \frac{\partial}{\partial v_1} \cdot \int dk Q(k) \operatorname{Im} \int dv_2 \tilde{h}_2(k, v_1, v_2) \quad (21)$$

Therefore, we require only a knowledge of $\operatorname{Im} \int dv_2 \tilde{h}_2(k, v_1, v_2)$ to obtain the kinetic equation.

A solution of Eq. (17) for $\operatorname{Im} n \int dv_2 \tilde{h}_2(k, v_1, v_2)$ can be obtained in a manner exactly analogous to that used by Lenard for a similar equation.* The only tricky point is the prescription given to the pole which arises when Eq. (17) is divided by $k \cdot (v_1 - v_2)$. We replace $k \cdot (v_1 - v_2)$ by $k \cdot (v_1 - v_2) - ikv$ where the limit $v \rightarrow 0^+$ is implied. This prescription corresponds to the boundary condition $h_2 \rightarrow 0$ when

$$|r_1 - r_2| \rightarrow \infty; (r_1 - r_2) \cdot (v_1 - v_2) < 0$$

Note that the time reversible character of our equations is destroyed with this boundary condition.

The solution with this boundary condition is

*See Eq. (3) and following of reference (4).

$$\begin{aligned} \text{Im } n \int d\mathbf{v}_2 \tilde{h}_2(\mathbf{k}, \mathbf{v}_1, \mathbf{v}_2) = \\ - \frac{\omega_p^2}{4} \int d\mathbf{v}_2 \frac{\tilde{\psi}(\mathbf{k}) [1 + (kR_D)^{-2}]}{|e(\mathbf{k}, \mathbf{k} \cdot \mathbf{v}_2)|^2} \mathbf{k} \cdot \delta[\mathbf{k} \cdot (\mathbf{v}_1 - \mathbf{v}_2)] \left\{ f(\mathbf{v}_1, t) \frac{\partial f(\mathbf{v}_2, t)}{\partial \mathbf{v}_2} - f(\mathbf{v}_2, t) \frac{\partial f(\mathbf{v}_1, t)}{\partial \mathbf{v}_1} \right\} \end{aligned} \quad (22)$$

where $e(\mathbf{k}, \mathbf{k} \cdot \mathbf{v}_2)$ is given by Eq. (7).

VI. KINETIC EQUATION

The kinetic equation obtained from Eq. (21) and Eq. (22) is

$$\begin{aligned} \frac{\partial f(\mathbf{v}_1, t)}{\partial t} = - \frac{e^4 n_i}{8\pi m^2} \frac{\partial}{\partial \mathbf{v}_1} \cdot \int d\mathbf{v}_2 \int d\mathbf{k} \tilde{\psi}(\mathbf{k}) \left[1 + \frac{1}{k^2 R_D^2} \right] \frac{Q(\mathbf{k}) \mathbf{k} \delta[\mathbf{k} \cdot (\mathbf{v}_1 - \mathbf{v}_2)]}{|e(\mathbf{k}, \mathbf{k} \cdot \mathbf{v}_2)|^2} \\ \cdot \left(\frac{\partial}{\partial \mathbf{v}_1} - \frac{\partial}{\partial \mathbf{v}_2} \right) f(\mathbf{v}_1, t) f(\mathbf{v}_2, t) \end{aligned} \quad (23)$$

If the Debye-Huckel potential is used for $\psi(r)$, Eq. (13), then

$$\tilde{\psi}(\mathbf{k}) = 4\pi R_D^2 / (1 + k^2 R_D^2) \quad (24)$$

It was shown in Appendix B of reference (6) that to the first order in

$$g = 4\pi R_L / R_D$$

$$Q(\mathbf{k}) = - (2\pi)^2 i \text{hei}_2[(4R_L k)^{\frac{1}{2}}] \mathbf{k} / k^2 \quad (25)$$

where ⁸

$$\text{her}_v(x) \pm i \text{hei}_v(x) = H_v^{(1)}[x \exp(\pm i 3\pi/4)] \quad (26)$$

One should note the following properties:

$$|\text{hei}_2(x)| \leq \frac{1}{\pi}; \quad \text{hei}_2(x) \sim -\left(\frac{2}{\pi x}\right)^{\frac{1}{2}} \cos[(2)^{\frac{1}{2}}x + 9\pi/8] \exp[-(2)^{\frac{1}{2}}x] \quad (27)$$

Using Eqs. (24) and (25) in Eq. (23) results in

$$\begin{aligned} \frac{\partial f(v_1, t)}{\partial t} = & - \frac{2\pi e^4 n}{m^2} \frac{\partial}{\partial v_1} \cdot \int dv_2 \int dk \text{hei}_2[(4R_L k)^{\frac{1}{2}}] \frac{kk}{k^4} \\ & \cdot \frac{\delta[k \cdot (v_1 - v_2)]}{|e(k, k \cdot v_2)|^2} \left\{ f(v_2, t) \frac{\partial f(v_1, t)}{\partial v_1} - f(v_1, t) \frac{\partial f(v_2, t)}{\partial v_2} \right\} \end{aligned} \quad (28)$$

In view of expressions (27), it is easy to see that this kinetic equation involves no diverging integrals.

It is interesting to note that if the equilibrium correlation function, $\exp\{-R_L \phi(r)\}$, is replaced by unity in Eq. (20) then

$$Q_m = 4\pi i k / k^2$$

and the kinetic equation obtained is identical to the B-G-L equation, Eq. (6). It is important to retain the equilibrium correlation function, however, since this guarantees the convergence of the kinetic equation.

VII. DISCUSSION

Equation (28) is the spatially homogeneous electron plasma kinetic equation, in which the modified pair correlation function $h_2(x_1, x_2, t)$ and $Q(k)$ have been calculated to the first order in the plasma parameter $g = 4\pi R_L/R_D$. We have also used the "adiabatic hypothesis," where the time dependence of h_2 appears through a functional dependence on $f(v, t)$.

The validity of Eq. (28) depends on the approximate solution for $h_2(x_1, x_2, t)$. In obtaining Eq. (15) for h_2 it was assumed that quantities like $R_L\psi(r)$ are small, but this is certainly not correct for all values of r . An order of magnitude estimate for h_2 , for small r , is easily obtained from Eq. (15). When r is small the integral terms in Eq. (15) are unimportant and h_2 is governed primarily by the equation

$$\begin{aligned} \frac{\partial}{\partial r} \cdot \left\{ (v_{m1} - v_{m2}) h_2 - \frac{e^2}{m} \psi(r) \left(\frac{\partial}{\partial v_{m1}} - \frac{\partial}{\partial v_{m2}} \right) f(v_{m1}, t) f(v_{m2}, t) \right. \\ \left. - R_L \psi(r) (v_{m1} - v_{m2}) f(v_{m1}, t) f(v_{m2}, t) \right\} = 0 \end{aligned} \quad (29)$$

From Eq. (29) we see that the approximate solution for h_2 , which breaks down when $h_2 \approx f(v_{m1}, t) f(v_{m2}, t)$, fails for those values of r where

$$R_L \psi(r) \gtrsim 1$$

However, the 2-particle distribution function Eq. (10) contains the factor $\exp[-R_L \psi(r)]$. Therefore, the approximate solution for h_2 fails at the same time that the distribution function is rapidly going to zero.

This case is marginal, and the kinetic equation (28) must be, at least, qualitatively correct. Certainly the lack of divergent integrals in Eq. (28) is a distinct improvement over the B-G-L equation.

The previous discussion assumed the plasma to be arbitrarily far from the equilibrium state. However, if the plasma is near equilibrium, say

$$f(v, t) = f^{(0)}(v) + \lambda h(v, t)$$

where $f^{(0)}(v)$ is the Maxwell-Boltzmann equilibrium distribution, Eq. (11), and λ is a measure of the deviation from equilibrium, then to zero order in λ the last two terms in Eq. (29) cancel, and one finds that

$$h_2 \approx f^{(0)}(v) f^{(0)}(v_2) \text{ for those values of } r \text{ where}$$

$$R_L \psi(r) \geq \lambda^{-1} \quad (30)$$

Thus, the approximate solution for h_2 is valid except for those values of r satisfying (30), and in this region the distribution function f_2 is reduced by an exponential factor $\approx \exp(-1/\lambda)$. We conclude that the kinetic equation becomes exact as the equilibrium state is approached.

It may be remarked that the kinetic equation, Eq. (23) or Eq. (28), exhibits an irreversible approach to a Maxwell-Boltzmann velocity distribution. The proof of the corresponding statement for the B-G-L equation has been given by Guernsey³ and Lenard.⁴ It is a trivial matter to extend these proofs to Eq. (23).

VIII. ACKNOWLEDGEMENT

The author wishes to thank Dr. G. W. Ford for his interest and many helpful suggestions.

The author also wishes to acknowledge financial assistance from the Office of Naval Research under Navy Theoretical Physics, Contract No. Nonr 1224 (15) and, in part, the National Science Foundation, Contract No. G-P 1536.

Publication of this paper was performed under the auspices of the United States Atomic Energy Commission.

REFERENCES

1. N. N. Bogoliubov, Problems of Dynamic Theory in Statistical Physics, AEC-tr-3852 (1960).
2. R. Balescu, Phys. of Fluids 5, 52 (1962).
3. R. L. Guernsey, Thesis, The University of Michigan (1960).
4. A. Lenard, Ann. of Phys. 10, 390 (1960).
5. D. E. Baldwin, Phys. of Fluids 5, 1523 (1962).
6. C. W. Hirt, (note ^{see} to Editor . . .)
7. C. R. Willis, Phys. of Fluids 5, 219 (1962).
8. C. S. Whitehead, Quar. Jour. of Math. 42, 316 (1911).