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STATISTICAL ERRORS IN SUBCRITICAL
REACTIVITY INFERRED FROM
INVERSE KINETICS ROD-DROP MEASUREMENTS
USING THE THREE-POINT METHOD

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JULY 1973

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ABSTRACT

A study was made to determine the uncertainty in subcritical reactivity as inferred from inverse kinetics rod-drop experiments (using the three-point method) due to the statistical uncertainty inherent in the observed count rate of the neutron sensor. The two methods employed were: a classical propagation of error analysis, and an analysis of simulated repeated rod-drops, with an assumption that the uncertainty in reactivity was due to the detection process itself for both techniques. To test the analysis methods, the reactivity uncertainties for various experimental rod-drop data sets were computed by both methods. There was excellent agreement of the results. The propagation of error analysis may be used on three-point subcriticality measurements to provide an experimenter with an index to the statistical reliability of the inferred reactivity estimate.

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1. INTRODUCTION

A mathematical study was made to determine the uncertainty of a subcritical reactivity estimate inferred from inverse kinetics, rod-drop (IKRD) experiments that could be explained by the statistical uncertainty inherent in the observed count rate from the neutron sensor. Two major objectives of this work were to contrive a way of calculating a priori the precision of an IKRD measurement for specific reactor conditions and to develop a system that would enable an on-line determination of the precision attained in an IKRD measurement without requiring time consuming, multiple rod-drop measurements.

The ability to assess the precision expected in a given rod-drop condition will enable a potential user of IKRD techniques to evaluate the usefulness of this technique. Also, the on-line determination of the precision associated with a given rod-drop measurement will serve as an indicator as to the "goodness" of the results from a statistical analysis.

Two methods were examined: one that may be incorporated in an on-line application, and another, secondary method which was used specifically to verify the results obtained by the first method.

The first method was examined by applying classical propagation of error analysis techniques with an assumption that the uncertainty of the reactivity estimate was due to the detection process itself, i.e., the uncertainty of an observation was not correlated with the uncertainty of any other observation.

The propagation of error analysis was applied to a three-point (3-PT) method of analysis for IKRD subcritical measurements similar to one developed by Carpenter

and Goin¹ (see Appendix for detailed derivation). Bennett² also introduced a 3-PT method which uses "average" fluxes over predetermined intervals of time during the transient associated with the rod drop. He performed an error analysis for the average fluxes required in his 3-PT formulation, but he did not carry through in the error analysis to determine the statistical uncertainty of an inferred reactivity. In his analysis, which included a correlation between the average counts due to fission chains, he demonstrated that for a near critical system in which the detection efficiency is high the correlated component can be important when the uncertainty in the average counts is determined. Conversely, his analysis showed that for highly subcritical systems and low detection efficiency correlation effects are negligible and can be ignored.

In the study reported herein, the correlated components of the uncertainty in the observed counts were neglected because most neutron sensors of current interest (fission chambers in fast systems) have a low efficiency and for the normal shutdown state of interest (nominally 1 to 3 dollars subcritical) the correlated component will be small.

The second, verification method was an error analysis based on repeated rod-drops which were simulated from observed count rate vs time data. With an assumption that there is no correlation between detector count rates, Gaussian statistical theory was applied to numerically create a set of stochastic observed count rates from a single, experimental rod-drop data set. This procedure was repeated N times for the same observed drop, to numerically simulate N repeated rod-drops. From these N sets of data, the initial and final reactivities were estimated N times, from which the most

probable values of the initial and final reactivities and their associated uncertainties were obtained.

Both error analysis techniques were applied to data obtained from experiments performed by ORNL personnel at the Southwest Experimental Fast Oxide Reactor (SEFOR)³ operated by the General Electric Company.

2. THEORETICAL CONSIDERATIONS

The first error analysis method considered was a propagation technique. Uncertainties in the count rates of a single rod-drop were propagated through the 3-PT equations used for calculating the predrop and postdrop reactivity levels. The second error analysis method was based on repeated IKRD experiments. This latter method infers the count rates of repeated rod drops from a single measured drop. The resulting set of reactivities for the repeated drops was determined by 3-PT calculations and tabulated. Then classic uncertainty equations were applied to determine the mean and standard deviation from the set of reactivity values.

2.1 Uncertainty in Reactivity by Propagation of Error Analysis

2.1.1 The Propagation Functions

The only dynamic variables input to the 3-PT formulation of reactivity calibration are the observed detector count rates for each time interval. For this analysis, the count rates are obtained from the output of a fission chamber detector (operated in the pulse mode). The mathematical form of this data is represented as

$$R(i\Delta t) = \frac{1}{\Delta t} \int_{t_i - \Delta t}^{t_i} CR(t) dt, \quad (1)$$

where $R(j\Delta t)$ is the total number of counts observed from $t_j - \Delta t$ to t_j , and Δt is the time interval over which the output of the detector is integrated. The standard deviation, $\delta R(j\Delta t)$, of the observed counts is

$$\delta R(j\Delta t) = [R(j\Delta t)]^{1/2}, \quad (2)$$

for $j \neq 0$. The standard deviation of $R(0)$ may be calculated from as many pre-drop detector counts as desired by classic mean and standard deviation methods.

The reactivity level, ρ , may be expressed in functional form as

$$\rho = \rho(R_0, R_1, R_2, \dots, R_N), \quad (3)$$

where

$R_0 \equiv$ the average detector counts per interval Δt prior to the drop,

$R_1 \equiv$ the total detector counts from $t = 0$ to $t = \Delta t$,

$R_N \equiv$ the total detector counts from $t = (N - 1)\Delta t$ to $t = N\Delta t$.

Invoking the standard formula for error propagation through an equation of N variables, the uncertainty in ρ may be expressed as

$$(\delta\rho)^2 = \sum_{i=0}^N \left(\frac{\partial\rho}{\partial R_i} \right)^2 (\delta R_i)^2 + \text{Correlated Terms}. \quad (4)$$

The uncertainties, δR_i , are assumed to arise from the detection process, thereby implying that δR_ℓ is not correlated to δR_i for $\ell \neq i$. Hence, all correlated terms of Eq. (4) become zero.

Before the specific functional form of Eq. (4) is developed, the equations used for the inference of ρ by the 3-PT method shall be reviewed in the next section.

2.1.2 Three-Point Method of Reactivity Inference

Explicit information about the subcritical reactivity, ρ , is contained in the point kinetics equations, i.e.,

$$\frac{dn(t)}{dt} = \frac{\rho(t) - \beta}{\Lambda} n(t) + \sum_{i=1}^D \lambda_i C_i(t) + S, \quad (5)$$

and

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\Lambda} n(t) - \lambda_i C_i(t), \quad (6)$$

for $i = 1, 2, \dots, D$. The variables in Eqs. (5) and (6) are common and defined in the Appendix.

The neutron population, $n(t)$, may be found from the output of a pulse-mode neutron detector by

$$CR(t) = \frac{W(t)n(t)}{\Lambda}, \quad (7)$$

where $W(t)$ is the detection efficiency and $CR(t)$ is the observed detector count rate.

Combining this expression with Eqs. (5) and (6) yields

$$\frac{\Lambda}{W(t)} \frac{d[CR(t)]}{dt} = \frac{\rho(t) - \beta}{W(t)} CR(t) + \sum_{i=1}^D \lambda_i C_i(t) + S \quad (8)$$

and

$$\frac{dC_i(t)}{dt} = -\lambda_i C_i(t) + \frac{\beta_i}{W(t)} CR(t). \quad (9)$$

Assume that the reactor is initially subcritical at ρ_i . If a rod is dropped at time $t = 0$ and seats at time $t = t_s$, and if the detection efficiency is postulated

to remain constant, Eqs. (8) and (9) may be combined and integrated over time greater than t_s for constant final reactivity, ρ_f [see Eq. (68)]. This equation will have two unknowns, ρ_f and a source term. Hence, by choosing two distinct time intervals in the required range of the drop transient, a system of two equations in two unknowns is generated:

$$\rho_f \int_{t_0}^{t_1} CR(t) dt + S'(t_1 - t_0) = \sum_{i=1}^D [C'_i(t_1) - C'_i(t_0)] , \quad (10)$$

and

$$\rho_f \int_{t_1}^{t_2} CR(t) dt + S'(t_2 - t_1) = \sum_{i=1}^D [C'_i(t_2) - C'_i(t_1)] , \quad (11)$$

where $C'_i(t) = WC_i(t)$, $S' = WS$, and $t_0 \geq t_s$.

In order to utilize an optimum amount of data, the time points t_0 , t_1 , and t_2 should be chosen on the following premises: (a) t_0 should be very close to, yet still greater than, t_s ; (b) t_1 should be chosen after t_s while the count rate transient is still significantly decaying, as this portion is most sensitive to ρ_f , yet should also be sufficiently larger than t_0 to reduce statistical errors (~ 25 sec after t_s); and (c) t_2 should be near the end of the transient when the reactor is approaching its final steady-state condition (~ 100 – 300 sec after t_s).

By making the following notations,

$$\Delta t_1 = t_1 - t_0 , \quad (12)$$

$$\Delta t_2 = t_2 - t_1 , \quad (13)$$

$$B_1 = \int_{t_0}^{t_1} CR(t) dt , \quad (14)$$

$$B_2 = \int_{t_1}^{t_2} CR(t) dt , \quad (15)$$

$$A_1 = \sum_{i=1}^D [C'_i(t_1) - C'_i(t_0)] , \quad (16)$$

and

$$A_2 = \sum_{i=1}^D [C'_i(t_2) - C'_i(t_1)] , \quad (17)$$

the solution of Eqs. (10) and (11) can be written as

$$\rho_f = \frac{A_1 \Delta t_2 - A_2 \Delta t_1}{B_1 \Delta t_2 - B_2 \Delta t_1} \quad (18)$$

and

$$S' = \frac{B_1 A_2 - B_2 A_1}{B_1 \Delta t_2 - B_2 \Delta t_1} . \quad (19)$$

These are the expressions used in the error propagation equation, Eq. (4) in the development of the algorithm for δp_f . This development is presented in the next section.

2.1.3 The Algorithm to Calculate Reactivity Uncertainty

The form of the observed data to be analyzed is given by Eq. (1). Therefore, Eqs. (14)–(17) must be adjusted to be compatible with this structure of the data. This is accomplished by defining L , M , and N such that

$$t_0 = L(\Delta t) , \quad (20)$$

$$t_1 = M(\Delta t) , \quad (21)$$

and

$$t_2 = N(\Delta t) . \quad (22)$$

We also let the following be true:

$$t_i = i(\Delta t) . \quad (23)$$

Then, Eqs. (14)–(17) may be written as

$$B_1 = \sum_{i=L+1}^M R_i , \quad (24)$$

$$B_2 = \sum_{i=M+1}^N R_i, \quad (25)$$

$$A_1 = \sum_{i=1}^D [C_i'(M\Delta t) - C_i'(L\Delta t)], \quad (26)$$

and

$$A_2 = \sum_{i=1}^D [C_i'(N\Delta t) - C_i'(M\Delta t)]. \quad (27)$$

The $C_i'(j\Delta t)$'s are functions of the count rate or observed counts for each interval [see Eq. (9)]. Hence, A_1 , A_2 , B_1 , and B_2 are all functions of R_i , and the chain rule may be applied to Eq. (18) to derive

$$\frac{\partial \rho_f}{\partial R_i} = \frac{\partial \rho_f}{\partial A_1} \frac{\partial A_1}{\partial R_i} + \frac{\partial \rho_f}{\partial A_2} \frac{\partial A_2}{\partial R_i} + \frac{\partial \rho_f}{\partial B_1} \frac{\partial B_1}{\partial R_i} + \frac{\partial \rho_f}{\partial B_2} \frac{\partial B_2}{\partial R_i}. \quad (28)$$

The partials of ρ_f with respect to A_1 , A_2 , B_1 , and B_2 were evaluated and listed in Table 1.

The partials $\partial B_1 / \partial R_i$ and $\partial B_2 / \partial R_i$ are determined from Eqs. (24) and (25) as

$$\frac{\partial B_1}{\partial R_i} = \begin{cases} 0, & i \leq L \\ 1, & L < i \leq M \\ 0, & i > M \end{cases} \quad (29)$$

$$\frac{\partial B_2}{\partial R_i} = \begin{cases} 0, & i \leq M \\ 1, & M < i \leq N \\ 0, & i > N \end{cases} \quad (30)$$

The evaluation of the partials $\partial A_1 / \partial R_i$ and $\partial A_2 / \partial R_i$ requires a solution for $C_i(j\Delta t)$ to be in terms of the integral count R_i , that is, a function of the form $C_i(j\Delta t) = C_i(R_1, R_2, \dots, R_i)$. This may be accomplished by the following recursive relationship derived from Eq. (9) for constant detector integration interval, Δt (for details, see Appendix).

$$C_i'(t_i + \Delta t) = C_i'(t_i) e^{-\lambda_i \Delta t} + \frac{\beta_i}{\lambda_i} \frac{R_i}{\Delta t} \left(1 - e^{-\lambda_i \Delta t} \right) \quad (31)$$

where

$$C_i(0) = \frac{\beta_i}{\lambda_i} \frac{R_0}{\Delta t}, \quad (32)$$

and R_0 is the average, predrop, steady-state integral count for time interval Δt .

For convenience, we write the following equations:

$$\gamma_{1i} = e^{-\lambda_i \Delta t} \quad (33)$$

and

$$\gamma_{2i} = 1 - \gamma_{1i}. \quad (34)$$

Table 1. Partial derivatives for use in Eq. (28)

$$K = \frac{\partial p_f}{\partial A_1} = \frac{\Delta t_2}{B_1 \Delta t_2 - B_2 \Delta t_1}$$

$$T = \frac{\partial p_f}{\partial A_2} = - \frac{\Delta t_1}{B_1 \Delta t_2 - B_2 \Delta t_1}$$

$$U = \frac{\partial p_f}{\partial B_1} = - \frac{A_1 \Delta t_2 - A_2 \Delta t_1}{(B_1 \Delta t_2 - B_2 \Delta t_1)^2} \Delta t_2$$

$$V = \frac{\partial p_f}{\partial B_2} = \frac{A_1 \Delta t_2 - A_2 \Delta t_1}{(B_1 \Delta t_2 - B_2 \Delta t_1)^2} \Delta t_1$$

After a time Δt , the following relationships can be expressed:

$$C_i'(\Delta t) = C_i(0) \gamma_{1i} + \frac{\beta_i}{\lambda_i} \frac{R_1}{\Delta t} \gamma_{2i} ,$$

or

$$C_i'(\Delta t) = \frac{\beta_i}{\lambda_i} \frac{R_0}{\Delta t} \gamma_{1i} + \frac{\beta_i}{\lambda_i} \frac{R_1}{\Delta t} \gamma_{2i} .$$

Let $C_i'(\Delta t) = \frac{\beta_i}{\lambda_i} F_{1i}$, where

$$F_{1i} \equiv \frac{R_0}{\Delta t} \gamma_{1i} + \frac{R_1}{\Delta t} \gamma_{2i} . \quad (35)$$

Expanding further, it can be shown that for the N^{th} time step

$$C_i'(N\Delta t) = \frac{\beta_i}{\lambda_i} F_{Ni} , \quad (36)$$

where

$$F_{Ni} \equiv F_{(N-1)i} \gamma_{1i} + \frac{R_N}{\Delta t} \gamma_{2i} . \quad (37)$$

$C_i'(N\Delta t)$ represents the i^{th} delay-group precursor population times detector efficiency $N\Delta t$ seconds after the drop point. $R_N/\Delta t$ is the average count rate over the N^{th} time interval. This recursive technique is described in the Appendix.

Now, the values $\partial A_1/\partial R_i$ and $\partial A_2/\partial R_i$ may be directly computed. Substitution from Eq. (36) into Eq. (26) and differentiation gives

$$\frac{\partial A_1}{\partial R_i} = \sum_{i=1}^D \left[\alpha_i \left(\frac{\partial F_{Mi}}{\partial R_i} - \frac{\partial F_{Li}}{\partial R_i} \right) \right] , \quad (38)$$

where

$$\alpha_i = \frac{\beta_i}{\lambda_i} . \quad (39)$$

By starting with Eq. (35) and applying Eq. (37), F_{Ni} may be expanded as follows:

$$F_{1i} = \frac{R_0}{\Delta t} \gamma_{1i} + \frac{R_1}{\Delta t} \gamma_{2i} ,$$

and

$$F_{2i} = F_{1i} \gamma_{1i} + \frac{R_2}{\Delta t} \gamma_{2i} ;$$

or

$$F_{2i} = \frac{R_0}{\Delta t} \gamma_{1i}^2 + \frac{R_1}{\Delta t} \gamma_{1i} \gamma_{2i} + \frac{R_2}{\Delta t} \gamma_{2i} ,$$

and

$$F_{3i} = F_{2i} \gamma_{1i} + \frac{R_3}{\Delta t} \gamma_{2i} ;$$

or

$$F_{3i} = \frac{R_0}{\Delta t} \gamma_{1i}^3 + \frac{R_1}{\Delta t} \gamma_{1i}^2 \gamma_{2i} + \frac{R_2}{\Delta t} \gamma_{1i} \gamma_{2i}^2 + \frac{R_3}{\Delta t} \gamma_{2i}^3 .$$

In general,

$$F_{Ni} = \frac{R_0}{\Delta t} \gamma_{1i}^N + \frac{\gamma_{2i}}{\Delta t} \left(R_1 \gamma_{1i}^{N-1} + R_2 \gamma_{1i}^{N-2} + \dots + R_i \gamma_{1i}^{N-i} + \dots + R_{N-1} \gamma_{1i} + R_N \right), \quad (40)$$

for arbitrary N ,

By using Eqs. (36), (38), and (40), and calculating the partials, the following expressions are obtained:

$$\sum_{i=1}^D W_{ij} = \frac{\partial A_1}{\partial R_i} = \sum_{i=1}^D \left[\frac{\alpha_i \gamma_{2i}}{\Delta t} \left(\gamma_{1i}^{M-i} - \gamma_{1i}^{L-i} \right) \right] \quad (41)$$

for $0 < j \leq L$,

$$\sum_{i=1}^D Y_{ij} = \frac{\partial A_1}{\partial R_i} = \sum_{i=1}^D \left(\frac{\alpha_i \gamma_{2i}}{\Delta t} \gamma_{1i}^{M-i} \right) \quad (42)$$

for $L < j \leq M$, and

$$\frac{\partial A_1}{\partial R_i} = 0 \quad (43)$$

for $M < j$. Also,

$$\sum_{i=1}^D W_{i0} = \frac{\partial A_1}{\partial R_0} = \sum_{i=1}^D \left[\frac{\alpha_i}{\Delta t} \left(\gamma_{1i}^M - \gamma_{1i}^L \right) \right] \quad (44)$$

By similar means, the following can also be shown,

$$\sum_{i=1}^D X_{ij} = \frac{\partial A_2}{\partial R_i} = \sum_{i=1}^D \left[\frac{\alpha_i \gamma_{2i}}{\Delta t} \left(\gamma_{1i}^{N-i} - \gamma_{1i}^{M-i} \right) \right] \quad (45)$$

for $0 < j \leq M$,

$$\sum_{i=1}^D Z_{ij} = \frac{\partial A_2}{\partial R_i} = \sum_{i=1}^D \left[\frac{\alpha_i \gamma_{2i}}{\Delta t} \gamma_{1i}^{N-i} \right] \quad (46)$$

for $M < j \leq N$,

$$\frac{\partial A_2}{\partial R_i} = 0 \quad (47)$$

for $N < j$, and

$$\sum_{i=1}^D X_{i0} = \frac{\partial A_2}{\partial R_0} = \sum_{i=1}^D \left[\frac{\alpha_i}{\Delta t} (\gamma_{1i}^N - \gamma_{1i}^M) \right] \quad (48)$$

All partial derivatives used in Eq. (28) are now known for all values of R_i .

Substitution of Eqs. (29), (30), and (41)–(48) and equations in Table 1 into Eq. (28)

and then inclusion of Eq. (28) in Eq. (4) yields for a final result

$$\begin{aligned} (\delta p_f)^2 = & \sum_{j=0}^L \left[K \left(\sum_{i=1}^D w_{ij} \right) + T \left(\sum_{i=1}^D x_{ij} \right) \right]^2 (\delta R_i)^2 \\ & + \sum_{j=L+1}^M \left[K \left(\sum_{i=1}^D \gamma_{ij} \right) + T \left(\sum_{i=1}^D x_{ij} \right) + U \right]^2 (\delta R_i)^2 \end{aligned}$$

$$+ \sum_{i=M+1}^N \left[T \left(\sum_{j=1}^D Z_{ij} \right) + V \right]^2 (\delta R_i)^2. \quad (49)$$

The defined variables K , T , U , and V are denoted in Table 1, and the defined variables W_{ij} , X_{ij} , Y_{ij} , and Z_{ij} are denoted in Eqs. (41)-(48).

The uncertainty in initial reactivity, $\rho_i(t < 0)$, is dependent on the uncertainty in the source term S' of Eq. (19). The quantity $\delta S'$ is calculated in the same fashion as $\delta \rho_f$ was. Therefore, in light of Eq. (4),

$$(\delta S')^2 = \sum_{i=0}^N \left(\frac{\partial S'}{\partial R_i} \right)^2 (\delta R_i)^2, \quad (50)$$

where, as in Eq. (4), correlated terms are neglected. Applying the chain rule to Eq. (19) yields

$$\frac{\partial S'}{\partial R_i} = \frac{\partial S'}{\partial A_1} \frac{\partial A_1}{\partial R_i} + \frac{\partial S'}{\partial A_2} \frac{\partial A_2}{\partial R_i} + \frac{\partial S'}{\partial B_1} \frac{\partial B_1}{\partial R_i} + \frac{\partial S'}{\partial B_2} \frac{\partial B_2}{\partial R_i}. \quad (51)$$

The partials of S' with respect to A_1 , A_2 , B_1 , and B_2 are presented in Table 2. The other partials are exactly the same as those calculated in the computation of $(\delta \rho_f)^2$. Hence, use of the equations in Table 2 and Eqs. (50), (51), and (41)-(48) leads to

Table 2. Partial derivatives for use in Eq. (51)

$$P = \frac{\partial S'}{\partial A_1} = - \frac{B_2}{B_1 \Delta t_2 - B_2 \Delta t_1}$$

$$Q = \frac{\partial S'}{\partial A_2} = \frac{B_1}{B_1 \Delta t_2 - B_2 \Delta t_1}$$

$$G = \frac{\partial S'}{\partial B_1} = - \frac{B_1 A_2 - B_2 A_1}{(B_1 \Delta t_2 - B_2 \Delta t_1)^2} \Delta t_2 + \frac{A_2}{B_1 \Delta t_2 - B_2 \Delta t_1}$$

$$H = \frac{\partial S'}{\partial B_2} = \frac{B_1 A_2 - B_2 A_1}{(B_1 \Delta t_2 - B_2 \Delta t_1)^2} \Delta t_1 - \frac{A_1}{B_1 \Delta t_2 - B_2 \Delta t_1}$$

$$\begin{aligned}
 (\delta S')^2 = & \sum_{i=0}^L \left[P \left(\sum_{j=1}^D w_{ij} \right) + Q \left(\sum_{j=1}^D x_{ij} \right) \right]^2 (\delta R_i)^2 \\
 & + \sum_{i=L+1}^M \left[P \left(\sum_{j=1}^D y_{ij} \right) + Q \left(\sum_{j=1}^D x_{ij} \right) + G \right]^2 (\delta R_i)^2 \\
 & + \sum_{i=M+1}^N \left[Q \left(\sum_{j=1}^D z_{ij} \right) + H \right]^2 (\delta R_i)^2,
 \end{aligned} \tag{52}$$

where G , H , P , and Q are defined in Table 2.

The initial reactivity, ρ_i , may be calculated by solving Eqs. (8) and (9) at steady state, or

$$\rho_i = - \frac{S'}{R_0/\Delta t} \quad (53)$$

The uncertainty in ρ_i is given by

$$(\delta\rho_i)^2 = \sum_{j=0}^N \left(\frac{\partial\rho_i}{\partial R_j} \right)^2 (\delta R_j)^2 \quad (54)$$

Using

$$E = \frac{1}{R_0/\Delta t} \quad (55)$$

then the partials $\partial\rho_i/\partial R_j$ may be written as

$$\frac{\partial\rho_i}{\partial R_0} = \frac{\partial\rho_i}{\partial S'} \frac{\partial S'}{\partial R_0} + \frac{\partial\rho_i}{\partial E} \frac{\partial E}{\partial R_0} \quad (56)$$

for $j = 0$, and

$$\frac{\partial\rho_i}{\partial R_j} = \frac{\partial\rho_i}{\partial S'} \frac{\partial S'}{\partial R_j} \quad (57)$$

for $j \neq 0$. Therefore, directly differentiating Eq. (53) and using this result in Eq.

(54) leads to

$$\begin{aligned}
(\delta p_i)^2 = & \left(\frac{\Delta t}{R_0} \right)^2 \left(\frac{\partial S}{\partial R_0} \right)^2 (\delta R_0)^2 + \frac{S'^2 (\Delta t)^2}{R_0^4} - \frac{2S' (\Delta t)^2}{R_0^3} \frac{\partial S}{\partial R_0} (\delta R_0)^2 \\
& + \left(\frac{\Delta t}{R_0} \right)^2 \sum_{i=1}^N \left(\frac{\partial S}{\partial R_i} \right)^2 (\delta R_i)^2 . \quad (58)
\end{aligned}$$

By substituting $(\delta S')^2$ from Eq. (50), the final expression is

$$(\delta p_i)^2 = \frac{S'^2 (\Delta t)^2 (\delta R_0)^2}{R_0^4} - \frac{2S' (\Delta t)^2}{R_0^3} \frac{\partial S}{\partial R_0} (\delta R_0)^2 + \left(\frac{\Delta t}{R_0} \right)^2 (\delta S')^2 , \quad (59)$$

where all values are known or previously calculated.

Equations (49) and (59) may be applied to any rod-drop transient where the count rates are in a form of integral counts per constant time interval. For real cases, conventions for determining the rod-drop point must be established so that the limits of integration of Eqs. (14) and (15) may be chosen to incorporate the largest amount of data possible for constant ρ . Errors in the predrop count rate, (δR_0) , can be reduced significantly by increasing the amount of data used in computing R_0 , hence making the third term in Eq. (59) the dominant term.

2.2 Uncertainty in Inferred Reactivity by Simulated Repeated Rod-Drops

The simulated repeated drops are formed from the measured values of the $R(j\Delta t)$'s [see Eq. (1)] of a single drop. All the observed $R(j\Delta t)$'s are statistically randomized by a Gaussian random number generator, using these $R(j\Delta t)$'s as the most probable count

and $\delta R(j\Delta t)$ from Eq. (2) as the standard deviation. Normal 3 PT calculations are applied to the generated count rates, $R(j\Delta t)/\Delta t$, to determine a reactivity level ρ_1 .

Repetition of the procedure N times (i.e., simulating N identical rod drops) produces N values of ρ . Assuming no correlation between Δt 's, the standard deviation of ρ can be determined by

$$(\delta\rho)^2 = \sum_{\ell=1}^N \frac{(\rho_{\ell} - \bar{\rho})^2}{N-1}, \quad (60)$$

where $\bar{\rho} = \sum_{\ell=1}^N \frac{\rho_{\ell}}{N}$, and ρ_{ℓ} is the reactivity value from the ℓ^{th} generated data set.

Equation (60) holds for both initial and final reactivity values.

3. RESULTS

The variance Eqs. (49), (59), and (60) were applied to data from IKRD experiments³ performed by ORNL personnel at SEFOR. The error propagation functions, Eqs. (49) and (59), were solved by a computer code that incorporated the error analysis into the 3-PT calculations. The 3-PT initial and final reactivities and their respective fractional deviations, $\delta\rho/\rho$, are shown, respectively, in columns 2, 3, 6, and 7 of Table 3. For the more subcritical ranges with inherently increased statistical scatter due to the lower detection rate, the deviations are larger. The rod-drop flux transients for cases 1 and 4 of Table 3 (which had fractional deviations for ρ_f of

Table 3. Results of rod-drop error analyses

Case No.	Error Propagation		Repeated Rod Drops		Error Propagation		Repeated Rod Drops	
	ρ_i (-\$)	ρ_f (-\$)	ρ_i (-\$)	ρ_f (-\$)	$\delta\rho_i/\rho_i$	$\delta\rho_f/\rho_f$	$\delta\rho_i/\rho_i$	$\delta\rho_f/\rho_f$
1 ^a	0.06	1.14	0.06	1.14	0.004	0.002	0.000	0.001
2	0.18	1.12	0.18	1.12	0.002	0.001	0.000	0.001
3 ^a	2.37	3.55	2.47	3.71	0.079	0.077	0.078	0.080
4	3.24	3.94	3.23	3.94	0.047	0.046	0.043	0.043
5	1.11	2.37	1.11	2.36	0.005	0.005	0.006	0.006
6	1.18	2.44	1.18	2.45	0.006	0.005	0.006	0.006
7	1.18	2.43	1.18	2.43	0.006	0.005	0.006	0.006
8	1.11	2.36	1.11	2.37	0.006	0.005	0.004	0.004

^aThe detector used to measure these rod drops was farther from the core than for the other rod drops.

0.002 and 0.046 respectively) are shown in Figs. 1 and 2 to give a qualitative indication of the count rate scatter and the respective fractional variance in reactivity.

Equation (60) was applied to the 3-PT reactivities of eight simulated rod drops created from an experimental drop, for example Figs. 1 and 2. The points in these figures were taken to be the most probable count rate values in the simulation process. The mean and standard deviation of the eight reactivities are shown in columns 4, 5, 8, and 9 of Table 3.

Fig. 1. Detector counts vs time for rod-drop case 1, where $\rho_i = -0.06$ dollar and $\rho_f = -0.14$ dollars.

Fig. 2. Detector counts vs time for rod-drop case 4, where $\rho_i = -3.24$ dollars and $\rho_f = -3.94$ dollars.

The agreement between the reactivities and fractional variances of the two methods are in excellent agreement.

The statistical variance of the error propagation and the total observed variance in the reactivity measurement were compared using the data from SEFOR for different rod drops under identical reactor conditions. Cases 5, 6, 7, and 8 are four such rod drops. The statistical fractional variances of these four cases averaged 0.006, whereas the total observed fractional variance was 0.017. This larger observed fractional variance may be attributed to systematics (e.g., difficulties in achieving identical drop conditions).

4. CONCLUSIONS AND RECOMMENDATIONS

The purpose of this study was to determine the uncertainty in subcritical reactivity by a propagation of error technique. This primary method was verified by a second method using simulated repeated rod-drops. These two methods agreed well and gave reasonable results for the experimental data analyzed. This agreement was deemed suitable for verification of the results and for the procedural development of each method. The primary method of uncertainty determination provides a statistical precision value for a 3-PT IKRD reactivity calibration under stated reactor conditions. The off-line computational method employed in this study may be easily incorporated to on-line computations to yield a precision to an IKRD measurement in an efficient and time saving manner.

Further work should be considered in finding the correlation factors for this 3-PT formulation in view of possible higher detection efficiencies in the future and for very low subcritical levels.

Investigations into reactivity variance contributions due to inaccuracies in the other measured variables of the point kinetics equations (e.g., λ_i and β_i) should also be made.

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5. APPENDIX: THREE-POINT METHOD OF DETERMINING SUBCRITICAL REACTIVITIES

The kinetic behavior of a subcritical reactor may be represented by the point kinetics model for a reactivity-independent neutron generation time, i.e.,

$$\frac{dn(t)}{dt} = \frac{\rho(t) - \beta}{\Lambda} n(t) + \sum_{i=1}^D \lambda_i C_i(t) + S ; \quad (61)$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\Lambda} n(t) - \lambda_i C_i(t) , \quad (62)$$

where

$n(t)$ = neutron population,

$C_i(t)$ = i^{th} delayed neutron precursor population,

λ_i = decay constant of the i^{th} delayed group,

β_i = effective delayed neutron fraction of the i^{th} group,

$$\beta = \sum_{i=1}^D \beta_i ,$$

Λ = neutron generation time,

$\rho(t)$ = reactivity,

S = source neutrons produced per unit time (assumed constant),

D = number of neutron delay groups.

The neutron population, $n(t)$, is related to the output of a fission chamber detector (operated in the pulse mode) by

$$CR(t) = \frac{W(t)n(t)}{\Lambda} , \quad (63)$$

where $CR(t)$ is the detector count rate and $W(t)$ is the detector efficiency.

The mathematical form of the detector output is

$$R(i\Delta t) = \int_{t_i}^{t_i + \Delta t} CR(t) dt, \quad (64)$$

where $R(i\Delta t)$ is the total count of the detector from t_i to $t_i + \Delta t$, and Δt is the detector integration interval. $CR(t)$ in Eq. (64) is an undefined function, whereas $R(i\Delta t)$ is known definitely; therefore, the mean detector count rate for the i^{th} time interval is found from the output of the detector as

$$CR(i\Delta t) = \frac{R(i\Delta t)}{\Delta t}. \quad (65)$$

Substitution of Eq. (63) into Eqs. (61) and (62) yields

$$\frac{\Lambda}{W(t)} \frac{dCR(t)}{dt} = \frac{[\rho(t) - \beta]}{W(t)} CR(t) + \sum_{i=1}^D \lambda_i C_{i-1}(t) + S \quad (66)$$

and

$$\frac{dC_i(t)}{dt} = -\lambda_i C_i(t) + \frac{\beta_i}{W(t)} CR(t). \quad (67)$$

Let the rod-drop occur at $t = 0$. The above equations may then be integrated over some finite time span during the transient following the drop. If $W(t)$ does not change during the drop and the lower integration limit (t_m) is large enough to permit the rod to seat, thereby making $\rho(t)$ a constant after this time, Eqs. (66) and (67) become

$$[CR(t_f) - CR(t_m)] \frac{\Lambda}{W} = \frac{\rho - \beta}{W} \int_{t_m}^{t_f} CR(t) dt + \sum_{i=1}^D \lambda_i \int_{t_m}^{t_f} C_i(t) dt + \int_{t_m}^{t_f} S dt, \quad (68)$$

and

$$C_i(t_f) - C_i(t_m) = -\lambda_i \int_{t_m}^{t_f} C_i(t) dt + \frac{\beta_i}{W} \int_{t_m}^{t_f} CR(t) dt. \quad (69)$$

Combining Eqs. (68) and (69) gives

$$\begin{aligned} [CR(t_f) - CR(t_m)] \frac{\Lambda}{W} &= \frac{\rho}{W} \int_{t_m}^{t_f} CR(t) dt - \frac{\beta}{W} \int_{t_m}^{t_f} CR(t) dt \\ &+ \sum_{i=1}^D \frac{\beta_i}{W} \int_{t_m}^{t_f} CR(t) dt - \sum_{i=1}^D [C_i(t_f) - C_i(t_m)] + \int_{t_m}^{t_f} S dt. \end{aligned} \quad (70)$$

Solving the above equation for ρ and rearranging terms leads to

$$\rho \int_{t_m}^{t_f} CR(t) dt = \Lambda [CR(t_f) - CR(t_m)] + \sum_{i=1}^D [C_i'(t_f) - C_i'(t_m)] - S'(t_f - t_m), \quad (71)$$

where $C_i'(t) = WC_i(t)$ and $S' = WS$.

Values for $C_i'(t)$ are based on a recursive solution of Eq. (67)

$$\frac{dC_i(t)}{dt} + \lambda_i C_i(t) = \frac{\beta_i}{W} CR(t). \quad (67)$$

By multiplication through by the integration factor and integration over the detector integration interval Δt , Eq. (67) becomes

$$\int_{t_i}^{t_i + \Delta t} d \left[C_i(t) e^{\lambda_i t} \right] = \frac{\beta_i}{W} \int_{t_i}^{t_i + \Delta t} CR(t') e^{\lambda_i t'} dt' . \quad (72)$$

The function $CR(t)$, as shown by Eq. (65), is constant over the interval Δt and can be taken outside of the integral and set equal to a constant, $CR(t_i)$, the average count rate from t_i to $t_i + \Delta t$. Carrying out the integration of Eq. (72), we obtain

$$C_i(t_i + \Delta t) = C_i(t_i) e^{-\lambda_i \Delta t} + \frac{\beta_i}{W \lambda_i} CR(t_i) \left[1 - e^{-\lambda_i \Delta t} \right] \quad (73)$$

and

$$C_i(0) = \frac{\beta_i}{\lambda_i} \frac{CR_0}{W} , \quad (74)$$

where CR_0 is the steady-state count rate prior to the drop ($t < 0$). Let

CR_1 = average count rate from $t = 0$ to $t = \Delta t$;

CR_2 = average count rate from $t = \Delta t$ to $t = 2\Delta t$;

$\vdots$

CR_n = average count rate from $t = (N - 1)\Delta t$ to $t = N\Delta t$.

Also let

$$\gamma_{1i} = e^{-\lambda_i \Delta t} \quad (75)$$

and

$$\gamma_{2i} = 1 - \gamma_{1i} . \quad (76)$$

Equation (73) after a time Δt gives

$$C_i(\Delta t) = C_i(0)\gamma_{1i} + \frac{\beta_i}{W} \frac{CR_1}{\lambda_i} \gamma_{2i} ,$$

or

$$C_i(\Delta t) = \frac{\beta_i}{\lambda_i} \frac{CR_0}{W} \gamma_{1i} + \frac{\beta_i}{\lambda_i} \frac{CR_1}{W} \gamma_{2i} ,$$

or

$$C_i(\Delta t) = \frac{\beta_i}{\lambda_i W} [CR_0 \gamma_{1i} + CR_1 \gamma_{2i}] .$$

Let

$$F_{1i} \equiv CR_0 \gamma_{1i} + CR_1 \gamma_{2i} . \quad (77)$$

Proceeding to the next time interval,

$$C_i(2\Delta t) = \frac{\beta_i}{\lambda_i W} F_{1i} \gamma_{1i} + \frac{\beta_i}{\lambda_i W} CR_1 \gamma_{2i} ,$$

or

$$C_i(2\Delta t) = \frac{\beta_i}{\lambda_i W} F_{2i} ,$$

where

$$F_{2i} = F_{1i} \gamma_{1i} + CR_{2i} \gamma_{2i} .$$

After N time intervals,

$$C_i(N\Delta t) = \frac{\beta_i}{\lambda_i W} F_{Ni} , \quad (78)$$

or

$$C'_i(N\Delta t) = WC_i(N\Delta t) = \frac{\beta_i}{\lambda_i} F_{Ni} , \quad (79)$$

where

$$F_{Ni} = F_{(N-1)i} \gamma_{1i} + CR_{Ni} \gamma_{2i} . \quad (80)$$

Thus, by knowing the count rate history, $C_i(t)$ can be calculated in a recursive manner for any time $N\Delta t$.

Returning to Eq. (71), one may observe that the first term on the right-hand side is the only term involving exact points. The significance of this term is small compared to the other terms. The value of Λ is about 10^{-6} , and $R(t_m)$ and $R(t_p)$ are about

10^6 . Hence, the term $\Lambda[CR(t_f) - CR(t_m)]$ is about 10^0 , compared with the $C_i(t)$'s ($\sim 10^6$) and the minimum value of $\sim 10^6$ for the integral count rate. For these reasons, this term is considered negligible and dropped; hence, Eq. (71) becomes

$$\rho \int_{t_m}^{t_f} CR(t) dt = \sum_{i=1}^D [C_i'(t_f) - C_i'(t_m)] - S'(t_f - t_m) . \quad (81)$$

The unknowns ρ and S' are constants after a rod drop. We now apply Eq. (81) individually to two separate time intervals following a drop from t_0 to t_1 and t_1 to t_2 , such that t_0 is a time shortly after the rod has seated, t_1 is a time chosen in the significant portion of the transient where the delay group populations are still significant [as this is particularly sensitive to ρ (~ 25 sec)], and t_2 is a time near the final steady-state portion of the transient. Then we can write two separate equations in two unknowns:

$$\rho \int_{t_0}^{t_1} CR(t) dt + S'(t_1 - t_0) = \sum_{i=1}^D [C_i'(t_1) - C_i'(t_0)] \quad (82)$$

and

$$\rho \int_{t_1}^{t_2} CR(t) dt + S'(t_2 - t_1) = \sum_{i=1}^D [C_i'(t_2) - C_i'(t_1)] . \quad (83)$$

Next we define several terms, as follows:

$$\Delta t_1 = t_1 - t_0 , \quad (84)$$

$$\Delta t_2 = t_2 - t_1 , \quad (85)$$

$$B_1 = \int_{t_0}^{t_1} CR(t) dt , \quad (86)$$

$$B_2 = \int_{t_1}^{t_2} CR(t) dt , \quad (87)$$

$$A_1 = \sum_{i=1}^D [C_i'(t_1) - C_i'(t_0)] , \quad (88)$$

and

$$A_2 = \sum_{i=1}^D [C_i'(t_2) - C_i'(t_1)] . \quad (89)$$

Then, Eqs. (82) and (83) may be cast in matrix form as

$$\begin{pmatrix} B_1 & \Delta t_1 \\ B_2 & \Delta t_2 \end{pmatrix} \begin{pmatrix} p \\ S' \end{pmatrix} = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} , \quad (90)$$

with solutions

$$\rho = \frac{A_1 \Delta t_2 - A_2 \Delta t_1}{B_1 \Delta t_2 - B_2 \Delta t_1} \quad (91)$$

and

$$S' = \frac{B_1 A_2 - B_2 A_1}{B_1 \Delta t_2 - B_2 \Delta t_1} \quad (92)$$

A noteworthy advantage to the preceding method of calculation is that use is made of the entire count-rate history of the rod drop. Cases where statistical fluctuations are relatively high may then be analyzed with some assurance that the variances will be averaged together, and individual points will not be heavily weighted with significance.

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