WEAKLY RECURRENT POMERONS^{*†}

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ABSTRACT

After reviewing the evidence for weak simultaneous occurrence of one exclusive and one inclusive pomeron, experiments are described that seek two exclusive pomerons, as well as two exclusive and one inclusive. We discuss the theoretical basis for expecting weak but indefinite pomeron recurrence as the energy increases and outline a systematic rapidly-converging fireball expansion that brings order to a confusing picture. The widely-employed two-component model of particle production corresponds to the two leading terms. An experimentally meaningful concept of "bare" pomeron emerges naturally from the fireball expansion, as does the notion of pomeron renormalization and splitting (the schizophrenic pomeron).

I. INTRODUCTION

The status of the pomeron concept continues to be controversial, but substantial evidence now exists both from inclusive and exclusive measurements that the single-pomeron component of reaction amplitudes is at least approximately factorizable.^{1,2} If the pomeron is factorizable, like ordinary Regge poles, one expects that pomerons may occur more than once in a single amplitude. The idea of pomeron recurrence is at first sight baffling from a direct-reaction standpoint (as is pomeron factorizability) but particle physicists have learned to live with concepts that seem natural from one viewpoint and obscure from another. This review will deal with a variety of aspects of pomeron recurrence.

We begin with the experimental evidence that the pomeron recurs in at least one sense and describe other experiments, either already in progress or within sight, that study other forms of recurrence. The vital question of the weakness of pomeron recurrence is then addressed, followed by introduction of the multifireball expansion which helps to clarify a picture that may appear paradoxical. We next discuss the distinction between "bare" and "clothed" pomerons --necessary for theoretical exploitation of the fireball expansion --and explain how different definitions of the bare pomeron may be

* Review delivered at the Fifth International Conference on High Energy Collisions, Stony Brook, Long Island, New York, August 23, 1975.

† This work was supported by the U. S. Atomic Energy Commission.

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useful for different purposes. Particular emphasis is given to the concept of the "schizophrenic pomeron," where the separate identities of P and P' become confused.

II. EXPERIMENTAL EVIDENCE FOR POMERON RECURRENCE

The recently-developed evidence for pomeron recurrence lies in what may be described as large-mass diffractive dissociation, corresponding to Fig. 1. Experiments on $pp \rightarrow p + \text{anything}$ at both

Fig. 1. Large-mass diffractive dissociation of a proton in collision with another proton.

NAL and the ISR, when analyzed to extract the pomeron component^{5,4,5} --which we equate here with the diffractive component--may be represented, after factoring off the pp elastic vertex on the left, by a "pomeron-proton total cross-section"--which corresponds to the right-hand vertex, squared and summed over all particle combinations emerging from that vertex. The result is shown in Fig. 2, taken

Fig. 2. The pomeron-proton total cross section for a pomeron mass squared of -0.15 GeV^2 , as deduced in Ref. 4.

from a paper by Kaidalov, et al.⁴ For large M^2 this analysis is equivalent to a standard triple-Regge analysis, the flat tail of the pomeron-proton total cross section corresponding to a non-vanishing triple-pomeron coupling. That is, if the P - p total

cross section may by the optical theorem formally be related to a forward P-p elastic amplitude, then the high energy limit of that amplitude may be represented by pomeron exchange as in Fig. 3. The

Fig. 3. The pomeron-exchange component of pomeron-proton scattering.

left vertex here is the celebrated coupling parameter $g_{ppp}(t)$, which has been the subject of volumes of theoretical speculation during the past three years. We now know at long last that $g_{ppp}(t)$ does not vanish, at least as measured in pp collisions for $|t| \gtrsim 0.15 \text{ GeV}^2$ where its dependence on t is weak. Whether $g_{ppp}(t)$ vanishes at $t = 0$ is not yet known, but many interesting predictions follow from the quantitative knowledge that now exists about this triple-pomeron vertex.

An alternative way of characterizing the experiment in question is through a rapidity plot, as in Fig. 4. Dominance by the

Fig. 4. A rapidity plot of a reaction that corresponds to large-mass diffractive dissociation.

"exclusive" pomeron $P(t)$ over secondary Regge singularities requires a large gap between the observed proton and the next particle in the rapidity chain. Dominance by the "inclusive" pomeron $P(0)$ requires a large rapidity interval to be subtended by the group of particles that constitute the missing mass.

Figure 4 reminds us of interesting concepts such as the multiplicity of particles produced in pomeron-particle collisions. Since the form of the Pp total cross section (Fig. 2) looks similar to that of an ordinary particle-particle total cross section, we might expect the multiplicity to behave in a similar way,^{6,7,8,9,10} and such indeed appears to be the case when the spanned rapidity intervals correspond.¹¹ One easily thinks of other aspects of pomeron-particle "collisions" that it will be interesting to compare to physical collisions.

III. WEAKNESS OF POMERON RECURRENCE

In order to assess the magnitude of pomeron recurrence let us assume that high-energy total cross sections are at least roughly factorizable:

$$\sigma_{AB}^{\text{tot}} \approx g_A g_B. \quad (3.1)$$

Then

$$g_p^2 \approx 40 \text{ mb} \approx 100 \text{ GeV}^{-2}$$

or

$$g_p \approx 10 \text{ GeV}^{-1}. \quad (3.2)$$

Now we have seen that ($g_p = g_{ppp}$)

$$g_p g_p \approx 1 \text{ mb},$$

so

$$g_p \approx \frac{1}{40} g_p \approx 0.25 \text{ GeV}^{-1}. \quad (3.3)$$

Knowledge of this number allows a large number of deductions.

The ratio to the elastic cross section of the cross section for large-mass single diffractive dissociation (S.D.D.), per unit $\log$ of missing mass squared, can easily be deduced to be approximately¹²

$$\frac{1}{\sigma_{AA}^{\text{el}}} \frac{d\sigma_{AA}^{\text{S.D.D.}}}{d \ln M^2} \approx \frac{2}{M^2} \frac{g_p}{g_A}, \quad (3.4)$$

the factor 2 corresponding to an approximate absence of t dependence in g_p so that the width of the S.D.D. distribution is

twice that of elastic scattering (we are neglecting the slope of the pomeron trajectory).^{3,4} Since $(M^2)_{\max}$ is proportional to s , if we are to keep the left gap in Fig. 4 larger than some minimum, the right-hand side of (3.4) measures the growth rate per unit $\ln s$ of the integrated diffractive-dissociation cross section:

$$\frac{d\sigma_{AA}^{S.D.D.}}{d \ln s} \approx 4 \frac{g_P}{g_A} \sigma_{AA}^{el} \quad (3.5)$$

We have here added another factor 2 so as to count dissociation of either incident particle. For the special case where particle A is a proton, we thus have

$$\begin{aligned} \frac{d\sigma_{pp}^{S.D.D.}}{d \ln s} &\approx \frac{4}{40} (\sigma_{el}^{pp} \approx 7 \text{ mb}) \\ &\approx 0.7 \text{ mb.} \end{aligned} \quad (3.6)$$

This rate of increase is too small to account single-handedly for the observed ISR rise in the pp total cross section, but the correspondence in order of magnitude is remarkable. Many theorists have conjectured a connection.^{10,13,14,15,16,17}

Let us consider next a two-proton inclusive measurement, with big rapidity gaps adjacent to each proton, as shown in Fig. 5. Such

Fig. 5. A rapidity configuration controlled by the pomeron-pomeron cross section.

a measurement can be interpreted in terms of the pomeron-pomeron "total cross section," as shown in Fig. 6. We might also describe

Fig. 6. The contraction of Fig. 5.

this process as "diffractive dissociation" of the pomeron $P(t_1)$ in collision with particle 2 (or alternatively, as diffractive dissociation of $P(t_2)$ in collision with particle 1). Such a designation suggests a simple estimate--based on the assumption that the probability of diffractive dissociation of the pomeron in a pomeron-particle collision is roughly the same as that of one particle in a particle-particle collision. This latter probability has been measured for protons as roughly 10%. Thus the integrated cross section for events of type of Fig. 6 might be guessed as 10% of the total large-mass (single) diffraction-dissociation cross section.

Assiduous efforts are being made in 200 GeV π^-p and pp bubble-chamber experiments^{10,19} to identify the special (exclusive) reaction shown in Fig. 7, which corresponds to pomeron dissociation

Fig. 7. An important special case of Fig. 6 currently under study in NAL bubble-chamber experiments.

into a $\pi^+\pi^-$ pair. Although the accessible rapidity intervals at NAL are marginal for unambiguous isolation of any pomeron-pomeron component, preliminary results are compatible with the above rough expectation. Higher statistics may be able to produce a definitive measurement.

The two-proton inclusive measurement at ISR energies would be much easier to analyze. Let us consider how the analysis might proceed via the triangle plot advocated in Ref. (19). First define

$$y_1 = \ln \frac{1}{1 - x_1} \quad \text{and} \quad y_2 = \ln \frac{1}{1 - x_2},$$

the two rapidity gaps adjacent to the observed protons, and enter the observed events on the plot shown in Fig. 8. Because

Fig. 8. The triangle plot employed in the analysis of Ref. (19). The circle in the center of the diagram designates the region where two exclusive and one inclusive pomeron should dominate.

$y_1 + y_2 \lesssim Y \approx \ln s$, all events will lie within the indicated (large) triangular region, while the events of possible relevance to pomeron-pomeron collision interpretation fall within the smaller triangle where both y_1 and y_2 are $\gtrsim 2$. The heavily-populated corner regions correspond to single low-mass diffraction and to detect the pomeron-pomeron effect it is necessary to get beyond the tails of these peaks and establish a flat intervening distribution, that may depend on $y_1 + y_2$ near the diagonal boundary, but not on $y_1 - y_2$. Another important feature is that this distribution, relative to the diagonal boundary, must not depend on the size of the triangle, i.e. on the total energy. If such a distribution can be established it may be interpreted by factorization of Fig. 6 in terms of

$$\sigma_{PP}^{\text{tot}}(M^2) \quad (3.7)$$

where the logarithm of the missing mass M^2 is proportional to $Y - y_1 - y_2$. (An average over t_1 and t_2 has here been implied, but the analysis in principle may keep these variables fixed.) We may expect to find a form for $\sigma_{PP}^{\text{tot}}(M^2)$ similar to that of Fig. 2, but now the height of the flat tail will be

$$\sigma_{PP}^{\text{tot}}(M^2) \xrightarrow{M^2 \text{ large}} \mathcal{E}_P^2 \approx 25 \text{ } \mu\text{b.}$$

The region within the triangle plot where this limit may be relevant is indicated by the circle, where y_1 , y_2 and $Y - y_1 - y_2$ are all simultaneously large.²⁰

The preceding effect may be described as 3-pomeron in character, two of the pomerons being exclusive and one inclusive. A different

3-pomeron experiment is indicated by the rapidity plot of Fig. 9,

Fig. 9. The rapidity configuration corresponding to double diffractive dissociation into two large masses.

where the large gap comes in the middle. This configuration, which may be described as double-diffractive dissociation leading to two large masses, is more difficult to observe because of the necessity for excluding neutral particles from the central gap. To the extent that $\alpha_p(t) \approx 1$, both of our 3-pomeron examples give contributions to the total cross section of order $g_p^2 \ln^2 s$. It is easy to see that whenever the total number of pomerons in sequence is N , built from a combination of inclusive and exclusive, the relative contribution to the total cross section is of order

$$(g_p \ln s)^{N-1}. \quad (3.8)$$

Except at enormously high energies, the smallness of g_p makes pomeron recurrence improbable.

The weakness of pomeron recurrence is equivalent to a small probability for large rapidity gaps. In general, the distribution of rapidity gaps has the structure shown in Fig. 10, the flat tail

Fig. 10. A typical rapidity-gap distribution.

corresponding to the (exclusive) pomeron. Outside the fragmentation regions, this distribution is supposed to have a universal form, and it is shown in Ref. (12) that the height of the tail is given by $\epsilon_p(\Delta)$, where

$$\epsilon_p = \frac{1}{16\pi} \int g_p^2(t) dt, \quad (3.9)$$

and $\langle \Delta \rangle$ is the mean gap size. We do not yet know the behavior of $g_p(t)$ for large $|t|$, but it would be surprising if the t -width were much greater than 1 GeV^2 . On that basis one estimates²¹

$$\epsilon_p \lesssim 0.002. \quad (3.10)$$

Why is this universal dimensionless parameter so small? No clean answer has yet been given, but theoretical connections have been suggested with a number of other measured small hadronic parameters. Calculations by Sorensen²² and Shankar²³ show that multiperipheral phase space, with its emphasis on small t -values along the chain (which corresponds to the small average transverse momenta of produced particles and which may be related to the small pion mass) automatically leads to triple-Regge couplings that are smaller for higher trajectories than for lower. The ABFS model, in particular, may be used to compute g_{ppp} , $g_{ppp'}$, etc. in terms of measured elastic and total cross sections, and the results are in qualitative agreement with experiment. This circumstance indicates that to the extent ϵ_p is small, particle-particle high-energy cross sections ought to be regarded as small, a puzzling statement at first sight, in view of the apparent near-saturation of unitarity bounds. In fact, however, these bounds are approached only for small angular momentum; equivalently, one may say that the hadronic "size" is much smaller than permitted by the small rest-mass of the pion. Now why is long-range pion exchange weak? Partly because of the Adler condition and partly because of small pionic coupling constants (e.g. $f_{\pi N}^2 = 0.08$) or, if you prefer, because of small resonance partial widths for pion decay. Why are pion-channel partial widths small? At least partly, because of a high overall multiplicity of channels related to SU_3 and SU_6 symmetry patterns. By such a chain of reasoning it may be possible to infer a connection between the smallness of ϵ_p and the reciprocal of the number of different low-mass mesons.

It thus seems unlikely that ϵ_p will indefinitely be regarded as an independently small parameter of mysterious origin. General S-matrix principles will almost certainly be found to connect this parameter with already-familiar hadronic attributes. At the same time the value of ϵ_p (or g_p) constitutes a convenient basis for describing a wide variety of experimental phenomena.

IV. THE ORIGIN OF POMERON RECURRENCE

If the pomeron is a factorizable Regge pole, then its recurrence is natural and unavoidable. Models that emphasize crossed-reaction unitarity lead in this way to factorizable pomerons, but models that focus on direct-reaction unitarity do not identify the pomeron as a Regge pole and find its recurrence to be unnatural. It is therefore interesting that there have been identified at least two sources of pomeron recurrence which do not insist on a Regge-pole status for the pomeron.

The first might be called the multiple-Deck effect. If we consider for example the reaction $pp \rightarrow pp\pi^+\pi^-$, we might employ the pion exchange model of Fig. 11, where the cross section is propor-

Fig. 11. Diagram depicting the pion-exchange pole in the amplitude for the reaction $pp \rightarrow pp\pi^+\pi^-$.

tional to the product of two elastic πp cross sections,

$$\sigma_{\pi^- p}^{el}(s') \sigma_{\pi^+ p}^{el}(s''), \quad (4.1)$$

evaluated at the appropriate subenergies. Now the form of the elastic $\pi^\pm p$ cross section is as shown in Fig. 12. The ordinary

Fig. 12. The general form of the πp elastic cross section.

Deck effect arises from the region where one of the two subenergies is near a low-energy resonance while the other subenergy lies in the flat region above the prominent resonances. But if the total energy is sufficiently large, there will be a part of phase space for which both s' and s'' lie in the flat region. In terms of rapidity this region is as shown in Fig. 13, where the respective πp elastic

Fig. 13. Final-state rapidities when both s' and s'' are large in Fig. 11.

cross sections have a dependence on the corresponding large rapidity gaps that is equivalent to assuming pomeron dominance of these gaps. We are thus led by the pion-exchange model to the behavior that was designated in Fig. 7 as a 2 (exclusive) pomeron effect.

Shankar²⁴ has shown that all physical effects associated with recurrent pomerons will necessarily be present in models based on factorizable pion poles. Any pion-pole residue is a product of physical pion amplitudes, each of which is known to have high energy behavior of at least the single-pomeron type. One may choose not to associate the term "pomeron" with this high-energy behavior, but the physical content of the predicted recurrence turns out to be equivalent.

Although pion-pole models (like all models) have defects, they have been consistently successful in giving qualitative predictions about diffractive phenomena. The Deck description of the A_1 region is well known, but it may be less well known that the order of magnitude and t -dependence of large-mass diffractive dissociation was correctly predicted on the basis of measured pion elastic and total cross sections.²² That is to say, the quantity g_p and other triple-Regge couplings were successfully estimated through π -exchange in advance of measurement. We draw the readers' attention in this connection to the pion-exchange model's unequivocal prediction that $g_p(t)$ does not vanish at $t = 0$.

An apparently independent basis for pomeron recurrence may be found in the notion of short-range order in rapidity. As one proceeds along the rapidity chain, memory seems to become lost after 4 or 5 gaps. Since there is only a tiny probability for a large gap to occur, it is likely at any given point along the chain that the

last preceding large gap has been "forgotten." The consequent notion that the probability for a large gap is independent of position along the rapidity chain is evidently equivalent to the indefinite recurrability of exclusive pomerons.

Readers may find disagreeable the thought that some models--possessing unquestionable physical relevance--give no hint that the pomeron may recur, while a variety of other considerations do suggest recurrence. Such a situation, however, has ample precedent in hadron physics and has served to stimulate the bootstrap conjecture. Were any single model the source of all understanding of strong interactions, the ingredients of that model could legitimately be regarded as "fundamental." The absence of any completely adequate model supports the bootstrap idea that fundamental ingredients do not exist.

V. SYSTEMATIC FIREBALL EXPANSION

The concept of pomeron recurrence can be exploited more systematically if we decompose the total cross section according to the number of large rapidity gaps that appear in the distribution of produced particles.²⁵ Such a decomposition is portrayed in Fig. 14,

Fig. 14. The fireball expansion for the imaginary part of the forward elastic amplitude.

where the circles with internal crosses denote produced-particle clusters containing no large gaps. It has become customary to refer to such clusters as "fireballs," although the term fireball has in the past sometimes been used with other meanings. The wiggly lines denote the large gaps that separate fireballs.

It is useful further to distinguish between low-mass and high-mass fireballs as in Fig. 15.²⁶ Inserting the notation of Fig. 15 into Fig. 14 yields Fig. 16, where the numbers beneath the separate terms indicate estimates of the partial cross sections in millibarns at typical NAL energies.^{15,27,28} Note that the second term--

$$\begin{array}{c}
 \bigcirc \otimes = \text{[vertical bar]} + \text{[rectangle with X]} \\
 m \lesssim 2 \text{ GeV} \quad m \gtrsim 2 \text{ GeV}
 \end{array}$$

Fig. 15. Distinction between high-mass and low-mass fireballs.

$$\begin{array}{c}
 \bigcirc \otimes = \text{[diagonal lines]} + \text{[wavy lines]} + \text{[diagonal lines, ln s]} + \text{[diagonal lines, ln s]} \\
 40 \qquad 26 \qquad 12 \qquad 1 \qquad 1 \\
 + \text{[diagonal lines, ln s]} + \text{[diagonal lines, ln^2 s]} + \text{[diagonal lines, ln^2 s]} + \dots \\
 .2(?) \qquad 0 \qquad 0
 \end{array}$$

Fig. 16. The expansion into low and high-mass fireballs. Terms 3 through 7 were separately discussed in Section III, using different language.

two low-mass fireballs--contains elastic scattering. The numerical estimates of course depend on the definition of "large" gap, which is usually taken to be anything greater than 2-3 units of rapidity. NAL energies provide a total rapidity interval insufficient to go beyond the first five terms of the expansion, but at the highest ISR energies the sixth and seventh terms should enter the picture.

Roughly speaking a pair of wiggly lines in Fig. 16 corresponds to the exclusive pomeron of our earlier discussion while a box corresponds to the inclusive pomeron. The reader should be able to

identify terms 3 through 7 with the discussion above in Sec. III. To the extent of such a correspondence, the relative energy dependence of the various terms is as indicated in Fig. 16, so the rate of convergence of the expansion necessarily deteriorates as the total energy increases. The expansion will nevertheless continue to be useful at all conceivably accessible energies.

All terms of the fireball expansion beyond the first might be described as diffractive dissociation, so although the latter constitutes a minority of the total cross section at presently accessible energies, the idea of (exclusive) pomeron recurrence implies that diffractive dissociation must ultimately dominate. As the rapidity chain becomes longer and longer, in other words, it will eventually become overwhelmingly probable that at least one large gap should appear.

The chain expansion of Fig. 16 acquires dynamical content through the association of its links with factors that are presumed to repeat. At least two different attitudes are possible: One may focus on the leading terms of a Regge asymptotic expansion in order to represent the links, or one may exploit the link weakness (characterized by parameters such as ϵ_p) to develop a perturbative calculational method. The latter approach will be deferred until Sec. VI.

An important distinction must be made between inclusive and exclusive links. An exclusive link (pair of wiggly lines) should in principle be representable by the leading physical Regge poles and branch points, but for an inclusive link (fireball) there can be no such representation--even for the large-mass component--because the first term (single-fireball) in the series of Fig. 16 necessarily grows less rapidly than does the total cross section. One may show, in fact, that if $\alpha_p(0)$ characterizes the asymptotic power behavior of σ_{tot} , then a lower power α_0 , sometimes called the "bare" pomeron, characterizes the asymptotic behavior of the single-fireball component of the total cross section.^{27,29}

The difference between $\alpha_p(0)$ and α_0 depends on the precise definition of a single fireball, i.e. on the minimum rapidity gap Δ that is considered "large". If Δ is chosen to be smaller than $\epsilon_p^{-1/2} \approx 20$ but so large that pomeron dominance of each gap is an accurate approximation, then the order of magnitude of $\alpha_p(0) - \alpha_0$ can be shown to be $\sqrt{\epsilon_p} \approx 0.04$.¹⁵ In other words if $\alpha_p(0) \approx 1$, the asymptotic energy dependence of the single-fireball component, so defined, is roughly

$$\begin{array}{c} \diagup \diagdown \\ \square \\ \diagdown \diagup \end{array} \propto s^{-\sqrt{\epsilon_p}}, \quad (5.1)$$

The multifireball (diffractive) components also carry the bare

pomeron power α_0 , but their additional logarithmic factors cause them to add up to a total cross section characterized by $\alpha_p(0)$.³⁰

If one chooses to represent the gaps in the fireball expansion by a finite sum of physical Regge singularities and the large-mass fireballs by a corresponding sum of "bare" singularities, then the infinite series may be evaluated by standard multiperipheral methods to yield the Regge expansion of the total cross section.^{15,27,28,32} One finds not only a leading factorizable pole--to be identified with the physical pomeron at $t = 0$ --but also a series of secondary poles³³ and branch points. One evidently may pose in principle a bootstrap requirement of self-consistency between input and output physical singularities, but the problem is complicated and depends in an obscure fashion on the handling of the "bare" singularities. No important progress can yet be reported on this front.

One old observation is nevertheless worth reiterating. The strength of the two-pomeron cut in σ_{tot} (i.e. in the infinite fireball sum) can be shown to be much weaker than might be supposed from the size of the second term (two low-mass fireballs) in Fig. 16,²⁷ a term which taken alone corresponds to a purely branch-point effect. Most of this two-fireball contribution, in other words, is used to renormalize the bare pomeron and shows up in the properties of the physical pomeron. One practical lesson is to expect the total cross section to exhibit simpler asymptotic properties than any of its components. In particular, when using the Mueller-Regge-pole theory of inclusive cross sections, normalization by the total cross section is theoretically more defensible than by the inelastic cross section. Another lesson is to beware of statements about Regge-cut discontinuities based on dynamical models that fail to consider renormalization.

Ambiguities of renormalization also are emphasized by a variation on the multifireball expansion that assumes a bare pomeron at $\alpha_0 = 1$ and restores the Froissart limit by superimposing direct-reaction eikonalization on the standard multiperipheral summation.^{16,17}

VI. THE SCHIZOPHRENIC POMERON

An alternative, more systematic, way to exploit the multifireball expansion may be based on the weakness of pomeron couplings or, equivalently, on the relative improbability of large rapidity gaps. The perturbation approach to pomeron recurrence has been discussed for a number of years⁶ but without the quantitative knowledge now available concerning the magnitude of quantities like g_p . Use of the perturbative approach leads to the concept of the schizophrenic pomeron,³⁷ which recent work by Dash³⁸ has revived as a promising theoretical tool.

The perturbative method starts by ignoring large rapidity gaps--which means ignoring all but the single-fireball term. Simple

multiperipheral models, such as that of ABEFST,^{39,40} suggest that for the single fireball the leading Regge singularity with vacuum quantum numbers will be an isolated simple pole somewhere between $J = 0.5$ and $J = 1.0$. If the maximum gap Δ allowed within the fireball is chosen to be $\lesssim 2$, secondary vacuum singularities of the single fireball lie well below this "bare" pomeron, which in turn is below the bare pomeron that arises when Δ is chosen sufficiently large ($\gtrsim 4$?) as to ensure physical-pomeron dominance of gaps larger than Δ . Roughly speaking one has the option of starting with a well-isolated bare pomeron by choosing a small value of Δ or of prescribing a larger Δ and accepting a more complicated spectrum of leading bare singularities. After inclusion of the multifireball components the physical difference resulting from different choices of Δ must of course disappear.⁴¹

It was pointed out several years ago³⁷ that for moderate values of the total energy the single leading bare pole (achieved by choosing Δ small but large enough to encompass the major inter-particle resonances) averages the effect of the full physical Regge spectrum--which may be complicated. In particular the physical effects in total and elastic cross sections associated with the combination of a P trajectory near $J = 1$ and a P' trajectory near $J = \frac{1}{2}$ may be well approximated by a single bare trajectory lying at an intermediate position and with a residue roughly equal to the sum of P and P' residues. Dash has recently confirmed this phenomenon with respect to large-mass diffractive dissociation. He has shown that except at the highest ISR energies the complicated fits involving combinations of P and P' trajectories may be replaced by a simple fit involving a single bare pomeron for both inclusive and exclusive links located at $\alpha_0 = 0.85$. Evidently the value of g_0 , the triple bare-pomeron coupling, must be larger than that of g_P .

The phenomenological similarity of P and P' couplings has been noted previously.⁴³ What remains to confirm the idea that these two trajectories share a common origin in a perturbative treatment of the fireball expansion is to show that summing the fireball series can produce the required splitting between P and P' . Earlier work⁴² had showed that a first-order effect of the two-pomeron branch point was indeed to split the bare pomeron into two poles, but this previous discussion was based on the two physical-pomeron branch point and associated the magnitude of the splitting with the magnitude of ϵ_P . The experimental result that

$\sqrt{\epsilon_P} \lesssim 0.04$ seemed incompatible with the observed P - P' separation of ≈ 0.5 in J .^{37,15}

Dash's work, however, emphasizes that it is sensible in a perturbative approach to use a bare exclusive pomeron to approximate the interfireball gaps as well as a bare inclusive pomeron for the individual fireballs. The relevant parameter for the splitting

mechanism is then g_0 , which Dash finds to be several times as large as g_P . Using Dash's estimates of $\alpha_0 \approx 0.85$ and $g_0 \approx 2 \text{ GeV}^{-1}$, the splitting mechanism based on a 2 bare-pomeron branch point leads to $\alpha_P \approx 1$ and $\alpha_P \approx 0.55$ ($t = 0$) with roughly equal residues. The schizophrenic pomeron concept thus seems to be back in business; if it survives, the relationship of the pomeron to other Regge singularities will have lost much of its mystery.⁴⁴

We comment that in higher approximations there will be further singularity splitting, so that it may not be profitable to speak of the "exact" Regge spectrum. The perturbative approach should nevertheless allow a controlled and systematic theoretical description of all experimental phenomena at accessible energies, in the same spirit as one approaches electromagnetic phenomena through the perturbative rules of quantum electrodynamics.

VII. RELATIONSHIP TO THE TWO-COMPONENT MODEL

It may be useful in conclusion to identify a connection between the multifireball expansion and the so-called two-component model.⁴⁵ Roughly speaking, the latter corresponds to including the first two terms of Fig. 6, namely, the single fireball and the two low-mass fireballs. The single fireball is usually represented in the two-component model by a P trajectory near $J = 1$ plus perhaps a P' near $J = \frac{3}{2}$, rather than by a bare pomeron at an intermediate location. Also the gap between the two low-mass fireballs is represented by a pomeron with intercept near $J = 1$. Fireballs are thus being implicitly defined here by a fairly large maximum internal gap ($\Delta \gtrsim 5$?).

Since the single-fireball component is describable in an inclusive sense by simple Mueller-Regge-pole theory, it exhibits the corresponding short-range order in rapidity. The two low-mass fireball component, in contrast, exhibits the long range correlations characteristic of an exclusive pomeron link. Taken together, these first two components have been able to correlate an encouraging number of features of the observed multiplicity distributions.⁴⁵ Attention is now turning to the next term in the expansion, corresponding to one low-mass and one high-mass fireball.^{46,10} An example of the effect of the third term is its filling in of an experimentally unwanted dip at intermediate multiplicities that tends to appear in the two-component approximation. We have noted that the third and higher terms will become progressively more important as the energy increases.

The success of the two-component model gives support to the multifireball expansion. At the same time the latter provides a systematic guide to future improvement of the theory of multiplicity distributions.

REFERENCES

1. D. Leith, SLAC-PUB-1263, June (1963).
2. M. Jacob, Proceedings of the XVI International Conference on High Energy Physics, Vol. 3, p. 373 (1972).
3. F. Sannes, et al., Phys. Rev. Letters 30, 766 (1973).
4. A. B. Kaidalov, et al., Physics Letters 4B, 493 (1973).
5. A. Capella, H. Hogasen and V. Rittenberg, SLAC-PUB-1776 (1973).
6. B. Webber, Nuovo Cim. Lett. 3, 424 (1972).
7. D. M. Tow, SLAC Preprint 1110 (1972).
8. R. N. Cahn, SLAC Preprint 1121 (1972).
9. C. F. Chan, Phys. Rev. D8, 179 (1973).
10. W. R. Frazer and D. R. Snider, Physics Letters B45, 136 (1973).
11. C. Quigg, Proceedings of the Berkeley Conference of the APS Division of Particles and Fields, August (1973).
12. H. D. I. Abarbanel, et al., Phys. Rev. Lett. 26, 937 (1971).
13. G. F. Chew, Physics Letters 44B, 169 (1973).
14. A. Capella, et al., Phys. Rev. Letters 31, 497 (1973).
15. W. Frazer, D. R. Snider and Chung I. Tan, UCSD preprint 10-FL0-127 (1973).
16. J. Finkelstein, Columbia University preprint CO-2271-17 (1973).
17. D. Azati, L. Caneschi and M. Ciafaloni, CERN preprint TH-1676 (1973).
18. M. Derrick, et al., ANL preprint HEP-7339 (1973).
19. D. M. Chew, et al., LBL preprint 2106 (1973).
20. When $Y - y_1 - y_2$ is large the two particle inclusive distribution is predicted by Mueller-Regge theory to be related to the product of single-particle inclusive distributions:

$$\frac{d^2\sigma_{AB}}{dy_2 dy_1} \underset{Y-y_1-y_2 \text{ large}}{\approx} \frac{1}{\sigma_{AB}^{\text{tot}}} \frac{d\sigma_{AB}}{dy_1} \frac{d\sigma_{AB}}{dy_2}.$$

The single-particle distributions become flat in the central region, with a magnitude given by (3.4), so the predicted density in the center of the triangle plot for pp collisions is

$$\begin{aligned} \frac{d^2\sigma_{pp}}{dy_1 dy_2} &\approx \frac{(\sigma_{pp}^{el})}{\sigma_{pp}^{\text{tot}}} 4 \left(\frac{g_p}{g_p} \right)^2 \sigma_{pp}^{el} \\ &\approx \frac{1}{6} \frac{4}{(40)^2} 7 \text{ mb} \approx 3 \text{ } \mu\text{b}. \end{aligned}$$

21. Note that although the distribution of Fig. 10 is not normalizable with a flat tail, the area under the tail at any finite energy is of order $\langle \Delta \rangle \epsilon_p^{-1} n$.
22. C. Sorensen, Phys. Rev. D6, 2554 (1972).
23. R. Shankar, private communication, Lawrence Berkeley Laboratory (1973).
24. R. Shankar, LBL preprint 2068 (1973).
25. G. F. Chew, Phys. Rev. D7, 934 (1973).
26. G. F. Chew, Phys. Rev. D7, 3525 (1973).
27. M. Bishari, G. F. Chew and J. Koplik, LBL-2129 (1973).
28. A. Capella and M-S Chan, Stanford University preprint (1973), to be published in the Physical Review.
29. J. Koplik, Phys. Rev. D7, 558 (1973).
30. We see here a difficulty with the conjecture that large-mass diffraction is responsible for the ISR-observed increase in the total pp cross section. Roughly speaking, the multifireball model predicts an increase in large-mass diffraction just sufficient to compensate the decrease of the single-fireball cross section. If threshold effects are included, there may be a temporary interval in which the total cross section rises, but estimates of the threshold effect in pomeron recurrence show it to be inadequate to account for the ISR increase in σ_{pp}^{tot} , 27, 31.
31. M. Bishari and J. Koplik, LBL-2130 (1973).
32. K. A. Ter-Martirosyan, Phys. Letters 44B, 179 (1973).
33. Some of these secondary singularities are complex poles on the physical J-sheet that produce a damped oscillation in the asymptotic behavior of the total cross section.^{34,35,31} This oscillation corresponds to the threshold effect alluded to in Ref. 30.
34. G. F. Chew and D. M. Snider, Physics Letters 31B, 75 (1970).
35. G. F. Chew, Physics Letters 44B, 169 (1973).
36. A complete list of references may be found in Ref. 15.
37. G. F. Chew and D. R. Snider, Phys. Rev. D1, 3453 (1970) and D3, 420 (1971).
38. J. Dash, University of Washington preprint RLO-1388-660, July (1973); also ANL/HEP-7333.
39. L. Bertocchi, S. Fubini and M. Tonin, Nuovo Cim. 25, 626 (1962); D. Amati, A. Stanghellini and S. Fubini, *ibid* 26, 6 (1962).
40. G. F. Chew, T. Rogers and D. R. Snider, Phys. Rev. D2, 765 (1970).
41. The effect here can be understood clearly and explicitly in the ABFST model,^{39,40} where the kernel of the J-projected equation is given by an integral over the energy of the $\pi\pi$ elastic cross section that runs from the $\pi\pi$ threshold to infinity.⁴² Dividing this integral into two portions--one covering the strong low energy resonances and the other covering the weak high energy tail--achieves the equivalent of our fireball

decomposition according to gap size when the solution of the equation is expanded in powers of the weak tail-contribution to the kernel. If the dividing line between the two subenergy regions is shifted, the definition of a single fireball is shifted, but the solution of the complete equation evidently remains unchanged.

42. H. D. I. Abarbanel, et al., *Annals of Physics* 73, 156 (1972).
43. R. Carlitz, M. B. Green, and A. Zee, *Phys. Rev. Letters* 26, 1515 (1971).
44. As pointed out in Ref. 37), the splitting mechanism automatically leads to a pomeron trajectory, with positive curvature and smaller than normal slope at negative t , that for positive t passes through the physical f_0 . The P' trajectory is roughly linear with normal slope for negative t but disappears onto an unphysical sheet for positive t .
45. K. Fialkowski, *Phys. Lett.* 41B, 379 (1972) and 43B, 61 (1973);
H. Harari and E. Rabinovici, *Phys. Lett.* 43B, 43 (1973);
L. Van Hove, *Phys. Lett.* 43B, 65 (1973);
C. Quigg and J. D. Jackson, NAL-THY-93 preprint (1973);
W. Frazer, R. Peccei, S. Pinsky and C.-I. Tan, *Phys. Rev.* D7, 2647 (1973).
46. R. G. Roberts and D. P. Roy, Rutherford preprint RL-73-065, T64, July (1963).