23,149

PRESSURE VESSEL RELIABILITY
AS A FUNCTION OF ALLOWABLE STRESS

H. G. Arnold

Oak Ridge National Laboratory*
Oak Ridge, Tennessee 37830

Abstract

The probability of failure corresponding to specified levels of allowable design stress was calculated for pressure vessels designed in accordance with the ASME Boiler and Pressure Vessel Code. The analysis was performed for maximum shear stress failure and for cyclic stress failure. The significance of such failure prediction is discussed and a rationale for selecting an allowable stress is presented. Examples are presented that demonstrate the estimation of vessel failure probability as a function of load variation, strength variation, and design safety factor.

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

*Oak Ridge National Laboratory is operated by Union Carbide Corporation, Nuclear Division, under contract with the United States Atomic Energy Commission.

MASTER

PRESSURE VESSEL RELIABILITY
AS A FUNCTION OF ALLOWABLE STRESS

H. G. Arnold

Oak Ridge National Laboratory*
Oak Ridge, Tennessee 37830

Uncertainties in design have been accounted for traditionally by means of a safety margin or "safety factor." The selection of a safety factor is usually based on experience and engineering judgment; some texts even recommend an appropriate safety factor for different areas of design. In recent years, the experience and judgment relied upon for selection of a safety factor have been augmented by analytical methods which permit probabilistic treatment of the known load and strength variables affecting a design problem.¹ The investigation reported herein was undertaken to demonstrate the application of these analytical methods to pressure vessels and to evaluate the allowable stress intensities presently used in pressure vessel design.

By definition, failure occurs in a pressure vessel when an induced stress exceeds a specified allowable level. The physical result may be rupture, crack initiation, yielding, or other manifestations, depending upon the type of induced stress and the manner in which the allowable stress is defined. Although there are many physical forms of vessel failure, there is but one definition. This definition can be expressed in terms of induced strain and allowable strain, induced pressure and

*Oak Ridge National Laboratory is operated by Union Carbide Corporation, Nuclear Division, under contract with the United States Atomic Energy Commission.

allowable pressure, or by many other variations. The definition is expressed herein in terms of an induced stress and an allowable stress to permit evaluation of the Section III ASME Code rules² for pressure design.

The induced stress will generally be the result of an interaction of external loads with a specific geometry of equipment; a calculated stress. The allowable stress will generally result from the interaction of material properties and environmental conditions; a measured material strength. Because of many factors such as the uncertainty of the known loads, the variation in any repeated loads, tolerances in manufactured parts, etc., the induced stress will be capable of having any value in a distribution of values about some average or mean induced stress level. Similarly, variations in material properties and in environment will result in an allowable stress whose actual value is within a range distributed about some mean allowable stress level. The probability that failure will occur is defined as the probability that an induced stress (S_i) will exceed the allowable stress (S_a) for any values of S_i and S_a that exist simultaneously. The analytical process used to calculate this probability is known as probabilistic design.³

The interaction of the induced stress distribution and the allowable stress distribution is illustrated in Fig. 1. Since these are frequency distributions, the probability of failure is a function of the area of overlap of the two curves. When the distributions are defined, probability calculus can be used to calculate the area of overlap of the curves.

Figure 1. Failure Stresses Resulting From Overlap of Two Stress Distributions.

For a probability density function of induced stress, $f(S_i)$, and a probability density function of allowable stress, $g(S_a)$, the probability of failure (Q) is given by the following equation.⁴

$$Q = \int_{-\infty}^{\infty} g(S_a) \int_{-\infty}^{A_n} f(S_i) dS_i dS_a, \quad (1)$$

where A_n is any specified failure value of allowable stress.

Membrane Stress Distributions

In order to use Eq. 1 to calculate failure probability, the distribution functions $f(S_i)$ and $g(S_a)$ must be defined. When data are available, definition of these functions is simply a matter of data analysis. However, in many cases, the density functions of induced stress and allowable stress will not be known during the design stages. A further complication is that Eq. 1 cannot be solved in closed form for many density functions. It is therefore of interest to determine whether pressure vessel data are likely to have typical density functions from which a more specific statement of Eq. 1 can be developed.

A distribution function of yield strength computed from 16 values obtained⁵ from A533 Grade B steel is shown in Fig. 2. The data points are plotted on a normal probability scale and very closely approximate a straight line, thereby fulfilling the criterion of a normally distributed yield strength. Similar results were obtained for ultimate strength. If this steel plate were to be used in a pressure vessel, the assumption of a normally distributed allowable stress for this vessel in a light-water-cooled reactor system would be reasonable. However, more data would be required to generalize the normal distribution to cover all steel.

Figure 2. Distribution Function of Yield Strength Plotted on Normal Probability Scale.

The induced stress distribution function can be approximated by a normal distribution if there is a very high incidence of pressure fluctuation on either side of the design pressure. For example, 10^6 cycles of ± 100 psi, 200 cycles of ± 120 psi, and 5 cycles of ± 500 psi can be treated as a normally distributed induced stress.

For the specific case in which the induced stress and the allowable stress are both normally distributed variables, Eq. 1 becomes⁴

$$Q = \frac{1}{(2\pi)^{1/2}} \int_m^{\infty} e^{-u^2/2} du, \quad (2)$$

where Q is the probability of failure,

$$m = \frac{-\bar{S}_i + \bar{S}_a}{(\sigma_{S_i}^2 + \sigma_{S_a}^2)^{1/2}}, \quad (3)$$

and

$$u = \frac{S - \bar{S}_i + \bar{S}_a}{(\sigma_{S_i}^2 + \sigma_{S_a}^2)^{1/2}} = \frac{S - \bar{S}}{\sigma_s}, \quad (4)$$

where

S = the stress level for which the probability of failure is calculated,

$\bar{S}_i$ = the mean value of induced stress,

$\bar{S}_a$ = the mean value of allowable stress,

σ_{S_i} = the standard deviation of induced stress, and

σ_{S_a} = the standard deviation of allowable stress.

The solution of Eq. 2 for several values of m calculated by using Eq. 3 is given in Table 1. The probability of failure resulting from normally distributed stresses is therefore given by Eq. 3 and the data

Table 1. Values of
Normal Integral

m	Q
5.0	2.87×10^{-7}
4.5	3.40×10^{-6}
4.0	3.12×10^{-5}
3.5	2.33×10^{-4}
3.0	1.35×10^{-3}
2.8	2.56×10^{-3}
2.6	4.66×10^{-3}
2.4	8.20×10^{-3}
2.2	0.0139
2.0	0.0227
1.8	0.0359
1.6	0.0548
1.4	0.0807
1.2	0.1151
1.0	0.1586
0.5	0.3085
0.0	0.50

in Table 1. By using the coefficient of variation V_s (the ratio of the standard deviation to the mean),¹ where

$$V_s = \sigma_s / \bar{S}, \quad (5)$$

and the ratio of allowable to induced stress

$$M = \bar{S}_a / \bar{S}_i, \quad (6)$$

Eq. 3 yields the value of the coupling parameter

$$n = \frac{M - 1}{\left(V_{S_a}^2 M^2 + V_{S_i}^2 \right)^{1/2}} \quad (7)$$

for use in Table 1.

When either the induced stress or the allowable stress is not normally distributed, Eq. 1 must be solved for the particular distribution function involved. When both are normally distributed, fairly simple calculations are all that is required.

Calculation of Induced Stress

If internal pressure is a distributed random variable, the stress resulting from internal pressure will be a distributed random variable. If the geometry of the vessel (radius and thickness, for example) is also a distributed random variable, the stress will have a probability distribution function resulting from the interaction of the probability density functions of pressure and geometry. The calculation of this probability density function results in the induced stress distribution function used in Eqs. 1, 3, and 5.

As long as normal probability distribution functions are involved, the calculation is straightforward. The methods of maximum likelihood estimation are used to derive the algebraic operations with two normal distribution functions x and y . This derivation, wherein $\bar{x}$ and $\bar{y}$ are means of the respective distributions and σ_x and σ_y are the respective standard deviations, results in the following equations.⁶

$$\text{If} \quad x + y = w, \quad (8)$$

$$\bar{x} + \bar{y} = \bar{w} \quad (9)$$

$$\text{and} \quad \sigma_x^2 + \sigma_y^2 = \sigma_w^2. \quad (10)$$

$$\text{If} \quad xy = u, \quad (11)$$

$$\bar{x}\bar{y} = \bar{u} \quad (12)$$

and
$$\sigma_x^2 \sigma_y^2 + \bar{x}^2 \sigma_y^2 + \bar{y}^2 \sigma_x^2 = \sigma_w^2 . \quad (13)$$

If
$$x/y = v , \quad (14)$$

$$\bar{x}/\bar{y} = \bar{v} \quad (15)$$

and
$$\frac{1}{\bar{y}^2} \left(\frac{\bar{x}^2 \sigma_y^2 + \bar{y}^2 \sigma_x^2}{\bar{y}^2 + \sigma_y^2} \right) = \sigma_v^2 . \quad (16)$$

To calculate the mean and standard deviation of primary membrane stress resulting from normally distributed internal pressure and normally distributed vessel dimensions, Eq. 8 through 16 are used in the traditional equations of stress analysis. By the maximum shear stress theory, failure results for principal stresses $S_1 > S_2 > S_3$ if

$$\frac{S_1 - S_3}{2} \geq \frac{S_y}{2} , \quad (17)$$

where S_y = the yield strength of the material as measured in a uniaxial tensile specimen.² For regions in a cylinder remote from discontinuities,

$$S_h = S_1 = (pr)/t , \quad (18)$$

where

S_h = hoop stress,

p = internal pressure,

r = inside radius of the cylinder,

t = thickness of cylinder wall, and

$$S_3 = -p . \quad (19)$$

For the normal probability distribution function of the variables S_h and p , Eq. 9 is used to obtain the mean induced stress

$$S_1 = \frac{1}{2} (\bar{S}_h + \bar{p}) \quad (20)$$

and Eq. 10 is used to obtain the standard deviation

$$\sigma_{S_i} = \frac{1}{2} \left(\sigma_{S_h}^2 + \sigma_p^2 \right)^{1/2} . \quad (21)$$

Application of Eqs. 8 through 16 to the hoop stress is simplified if the ratio of cylinder radius to wall thickness is calculated first.

Let

$$y = (ar)/t , \quad (22)$$

where a = an invariant multiplier that is a function of geometry ($a = 1$ for a cylinder and $1/2$ for a sphere). From Eqs. 15 and 16,

$$\bar{y} = (a\bar{r})/\bar{t} \quad (23)$$

and

$$\sigma_y = \left[\frac{1}{\bar{t}^2} \left(\frac{\bar{r}^2 \sigma_t^2 + \bar{t}^2 \sigma_r^2}{\bar{t}^2 + \sigma_t^2} \right) \right]^{1/2} (a) . \quad (24)$$

Then for

$$S_h = py , \quad (25)$$

Eqs. 12 and 13 yield

$$\bar{S}_h = \bar{p}\bar{y} \quad (26)$$

and

$$\sigma_{S_h} = (\sigma_p^2 + \sigma_y^2 + \bar{p}^2 \sigma_y^2 + \bar{y}^2 \sigma_p^2)^{1/2} . \quad (27)$$

Equations 26 and 27 can be substituted into Eqs. 20 and 21 to define the normal probability distribution function of induced stress.

Example Analysis

A numerical example is presented here to illustrate the manner in which the analysis methods are used to compute failure probability. A cylinder with specified design conditions typical of those for a pressure vessel in a pressurized-water nuclear reactor is considered in this

example, and the radius (r) and shell thickness (t) of this vessel are treated as invariants because their coefficients of variation are small when compared with those for pressure and yield strength. The specified design conditions for this vessel are as follows.

$\bar{p}$ = mean or average internal pressure = 2250 psi,

σ_p = standard deviation of internal pressure = 86 psi,

$\bar{S}_y$ = mean yield strength of vessel material = 57,500 psi,

σ_{S_y} = standard deviation of the yield strength = 3068 psi,

$\bar{S}_u$ = mean ultimate tensile strength of vessel material = 83,000 psi,

σ_{S_u} = standard deviation of ultimate tensile strength = 4650 psi

S_m = allowable stress intensity as specified in Section III of the

ASME Boiler and Pressure Vessel Code² = 26,700 psi,

r = radius of cylinder = 91 in.

The rules of Section III of the ASME Code² require that the minimum shell thickness ($t_{\min}$) for this cylinder be

$$t_{\min} = \frac{\bar{p}r}{S_m - \bar{p}} = \frac{2250(91)}{26,700 - 2250} = 8.37 \text{ in.} \quad (28)$$

The corresponding mean value of the hoop stress (from Eq. 26)

$$\bar{S}_h = \frac{\bar{p}r}{t} = \frac{2250(91)}{8.37} = 24,462 \text{ psi} , \quad (29)$$

and the standard deviation of hoop stress (from Eq. 22)

$$\sigma_{S_h} = \frac{\sigma_p r}{t} = \frac{86(91)}{8.37} = 935 \text{ psi} . \quad (30)$$

The mean value of induced stress (from Eq. 20)

$$\bar{S}_i = \frac{\bar{S}_h + \bar{p}}{2} = \frac{24,462 + 2250}{2} = 13,356 \text{ psi} , \quad (31)$$

and the standard deviation of induced stress (from Eq. 21)

$$\sigma_{S_i} = \frac{1}{2} \left(\sigma_{S_h}^2 + \sigma_p^2 \right)^{1/2} = \frac{940}{2} = 470 \text{ psi} . \quad (32)$$

The values of $\bar{S}_y/2$ and $\sigma_{S_y}/2$ and the results determined from Eqs. 31 and 32 are used in Eq. 3 to determine the value of m to be used in Table 1.

$$m = \frac{-13,356 + 28,750}{\left[(470)^2 + \left(\frac{3068}{2} \right)^2 \right]^{1/2}} = 9.59 . \quad (33)$$

From Table 1, the probability of failure corresponding to a value of $m = 9.59$ is essentially zero.

The probability of failure corresponding to several values of shell thickness for the vessel and specified design conditions was calculated in the same manner, and the resulting data are given in Table 2.

Table 2. Maximum Shear Stress
Failure Probability as a Function of
Shell Thickness for a Cylinder

t (in.)	m	Q
4.00	0.3	0.3
4.25	2.0	2.3×10^{-3}
4.50	2.8	2.5×10^{-3}
4.75	3.5	2.3×10^{-4}
5.0	4.8	2.0×10^{-7}
8.37	9.6	~ 0

Examination of the data in Table 2 reveals that the probability of failure is very low for shell thicknesses over 5 in. From the given data, it can be concluded that a minimum shell thickness of 8.37 in. provides more than adequate assurance against membrane failure defined by the maximum shear stress theory of failure. If a minimum thickness of

8.37 in. were not mandatory, the value of 5 in. would probably be adequate for the conditions specified in this example because of the significant improvement in reliability realized from the 1/4-in. increase in thickness from 4.75 in. to 5 in.

ASME Allowable Stress Intensities

The value of allowable stress intensity given in Section III of the ASME Code² is usually equal to the lower fraction (N) of the minimum specified yield strength (S_y) or a smaller fraction of the minimum specified ultimate tensile strength (S_u).⁷ Examination of the allowable stress intensities in Section III² reveals that the values for ferritic materials are based on one-third of the minimum specified ultimate tensile strength of the material, while the values for austenitic materials are based on two-thirds of the minimum specified yield strength of the material. The ratio of ultimate strength to tensile strength for ferritic materials is such that one-third of the ultimate strength (S_u) is equal to $0.55S_y$. Thus, the values of allowable stress intensity given in Section III are based on the equivalent of $0.55S_y$ and $0.667S_y$, depending upon the type of material used.

Data collected from three sources^{5,8,9} indicate that the average value of the actual yield strength of either ferritic or austenitic material will be 1.35 times greater than the minimum specified yield strength of the material. Therefore, the stress distribution probability density function of yield strength has a mean value

$$\bar{S}_y = 1.35S_{y \text{ min}} \quad (34)$$

If it is assumed that $\bar{S}_y$ is the allowable stress and S_m is the induced stress, the mean-allowable-stress-to-mean-induced-stress ratio (M) becomes (from Eq. 6)

$$M = \bar{S}_y / S_m = \bar{S}_a / \bar{S}_i = 2.02 \quad (35)$$

when the allowable-stress-intensity-to-minimum-specified-yield-strength ratio (N) = 2/3. If it is assumed that both the coefficient of variation of allowable stress (V_{S_a}) and the coefficient of variation of induced stress (V_{S_i}) have a value of 0.05⁵ from Eq. 7 the value of the coupling parameter m = 9.03. Values of M = 2.02 and m = 9.03 correspond to a failure probability (Q) of essentially zero, as indicated by the data in Table 3. Similar calculations for 0.55 $\bar{S}_y$ (corresponding to one-third of ultimate tensile strength) result in approximately the same failure probability. It can therefore be concluded that the probability of failure by yielding is nil when values of induced stress are allowed to reach the ASME Code² allowable stress intensities. As indicated in Table 3, it is only when allowable stress intensity values are allowed to approach 7/8 of the minimum specified yield strength that the chances of failure become significant.

Table 3. Failure Probability (Q) for an Allowable Stress Intensity Equal to a Fraction (N) of Minimum Specified Yield Strength

N	M	m	Q
1	1.35	4.16	1×10^{-5}
7/8	1.53	5.82	1×10^{-6}
3/4	1.80	7.80	~0
2/3	2.02	9.03	~0
5/8	2.16	9.74	~0
1/2	2.7	11.9	~0
1/3	4.05	14.7	~0
3/8	3.60	13.9	~0
1/4	5.4	16.05	~0

The conclusion that failure probability based on ASME Code² allowable stress intensity values is very low is valid as long as uncertainty is not introduced by the analytical model. As indicated by the values in Table 4, increasing values of V_{S_1} (corresponding to increasing values of standard deviation in the calculated induced stress that might be the result of analytical uncertainties) result in significant failure probability only when V_{S_1} exceeds 0.1 or when the standard deviation exceeds about 10% of the mean value of S_1 . Since it is possible to have this variation in yield strength alone for some materials, it is entirely likely that additional variables are present that make the present allowable stress intensity values the highest one would choose for most applications. The analytical techniques presented here can be of help in assessing the adequacy of existing allowable stress values when dealing with such design uncertainties.

Table 4. Failure Probability (Q) as a Function of the Coefficient of Variation of Induced Stress (V_{S_1})

V_{S_1}	M	Q
0.05	9.02	~0
0.075	6.03	10^{-6}
0.10	4.53	3.4×10^{-6}
0.15	3.01	1.35×10^{-3}
0.2	2.26	1×10^{-2}
0.3	1.51	7×10^{-2}
0.5	0.91	0.15
1.0	0.45	0.3

In the preceding numerical example, it was shown that a vessel design (thickness, for example) can be judged by probabilistic analysis methods. In this discussion of ASME Code² allowable stress intensities, it was shown that the allowable stress itself can be judged by these methods. Finally, application of these methods provides a comparative technique for dealing with design uncertainty. The most important aspect of this type of analysis is probably the evaluation of such questions as what happens if the design is changed so much? It can be clearly seen that there are cases when a lot is gained for very little change, while very little is gained in other cases.

Fatigue Analysis

The probability of failure resulting from the repeated application of cyclic stresses consistent with the definition of failure used herein is the probability that the number of stress amplitudes of a specified level exceeds the allowable number of stress amplitudes for that stress level. The probability of failure so defined is a function of the number of stress cycles as well as the amplitude of the stress. The distribution function must therefore include the effects of both the stress level and the number of stresses. One method of including both would be to develop the probability that a number of stresses exceed an allowable number for a specified stress level. Another method would be to develop the probability that an induced stress exceeds an allowable stress for a given number of stress cycles. This latter method

was selected for analysis of ASME Code² allowable stresses to be consistent with the preceding development of a steady-state failure probability in terms of an induced stress exceeding an allowable steady-state stress.

In the preceding development of failure probability, the induced and allowable stresses were assigned a distribution invariant with time (or with the number of stresses). In the cyclic analysis, an induced stress distribution and an allowable stress distribution must be specified for each number of cycles of interest in the evaluation. These stress distributions may not be the same for each cyclic life. For example, a stress may be log normally distributed for one life and exponentially distributed for another.¹⁰ This increases the complexity of the statement of failure probability.

Cyclic Distribution Functions

The distribution function for induced cyclic stresses is generally not known. Induced stresses are usually specified in terms of a number of cycles of a stress of an exact value, and significant variations from that exact stress value are treated as a number of stresses of a different value.² The distribution of induced stress for any specific cyclic life can therefore usually be treated as invariant unless extreme accuracy is required in the estimate of failure probability.

The fatigue life (allowable stress) of materials subjected to cyclic stress is a different situation in which there are significant variations with respect to both cyclic life and stress level.⁷ To

avoid the complexity involved in the determination of several specific functions for the allowable cyclic stress distribution, the versatility of the Weibull distribution function will be used to generalize the statement of the distribution of allowable stress for all numbers of cycles and stress levels.¹¹

The two-parameter Weibull distribution is given in Eq. 36. The probability that a stress will exceed a specified value in a Weibull

$$Q = 1 - e^{-(S_1/\theta)^b} \quad (36)$$

distribution is obtained by using induced stress (S_1) as the independent variable. The parameters θ and b in Eq. 36 are determined by analysis of the allowable stress distribution for cyclic failure. Equation 36 therefore states the probability that an invariant induced stress will exceed an allowable stress with a Weibull distribution that is characterized by different values of the parameters θ and b for each cyclic life.

Cyclic Rupture

Data on the number of stress amplitude cycles that resulted in the propagation of a crack through the shell (hereinafter referred to as cyclic rupture) of pressure vessels were obtained by the Pressure Vessel Research Committee and reported by The American Society of Mechanical Engineers.⁷ These data are illustrated in Fig. 3. To convert these data from a life distribution to a stress distribution, a logarithmic S-N curve was assumed,¹⁰ a least-squares line was fitted through the data points, and lines parallel to the least-squares line were drawn

Figure 3. Family of S-N Curves Constructed for Pressure Vessels That Failed by Rupture as a Result of Cyclic Internal Pressure.

through each data point to construct a family of logarithmic S-N curves.¹¹ The number of data points on an S-N curve was then assumed to represent the number of failures resulting from the stress amplitude (S_A) corresponding to the intersection of the S-N curve with a specified cyclic life. The failure distribution function for each cyclic life was then determined by assuming a Weibull distribution.

The Weibull parameters obtained by this analysis are shown in Fig. 4 as a function of cyclic life. These parameters result in the Weibull distribution of allowable stress for a specific cyclic life when substituted into Eq. 37. The probability that a specified number of cycles

$$Q = 1 - e^{-(S_A/\theta)^b} \quad (37)$$

of induced stress amplitude will cause rupture is calculated by substituting the induced stress amplitude and the Weibull parameters for that number of cycles into Eq. 37.

Several values of rupture probability were calculated as a function of stress amplitude and cyclic life by using Eq. 37 and the Weibull parameters shown in Fig. 4. These values of failure probability are illustrated in Fig. 5 as S-N curves of a constant failure probability. The S-N curve for low-carbon alloy steel given in Section III of the ASME Boiler and Pressure Vessel Code² is shown as a dashed line in Fig. 5, and it corresponds to a probability of failure by cyclic rupture greater than one chance in 100. Such a result should be expected since, as stated in the ASME Boiler and Pressure Vessel Code, the S-N curves do not necessarily result in a factor of safety for cyclic life inasmuch as they were only corrected to compensate for the difference between test data and operating conditions.⁷

Figure 4. Cyclic Rupture Weibull Parameters for Pressure Vessels as a Function of Number of Cycles.

Figure 5. Family of S-N Curves Constructed for the Probabability of Rupture in Pressure Vessels Subjected to Cyclic Internal Pressure.

It can be concluded that rupture resulting from cyclic loads is a credible event (greater than one chance in 100) if the design stress amplitudes are permitted to reach the values allowable by the S-N curve in Section III of the ASME Boiler and Pressure Vessel Code.² A factor of safety for cyclic life would substantially decrease this failure probability. For example, Fig. 5 shows that designing for the ASME Code allowable stress amplitude at 10^6 cycles would result in a failure probability of 10^{-3} at 10^5 cycles, which would represent a considerable improvement in the failure probability.

Attention should also be directed to the typical practice of assuming that 10^6 cycles is equivalent to infinite life.⁷ The cyclic life for the rupture data from which the failure probability was determined did not exceed 10^5 cycles. However, at 10^5 cycles, there was no indication that stress amplitudes would asymptotically approach some value at 10^6 cycles. Therefore, care should be exercised in extrapolating the cyclic failure probability data in Fig. 5 to lives greater than 10^6 cycles.

The cyclic failure data shown in Fig. 3 are a combination of data from two research installations on two kinds of material. All of the data points were considered together in the foregoing data analysis, and a very close fit to the assumed Weibull distribution was not achieved. However, the distribution achieved is accurate enough to permit order-of-magnitude comparisons. Thus, the data as analyzed result in a probability that failure by rupture will occur irrespective of the material and data source.

Conclusions

Application of probabilistic analysis methods to pressure vessel design was demonstrated by assuming normal distribution functions for induced stress and allowable stress and by using representative data for a typical pressure vessel. It was concluded that the minimum wall thickness required by the rules of Section III of the ASME Boiler and Pressure Vessel Code² will result in a very low probability of failure ($< 10^{-6}$) by the maximum shear stress theory as long as the standard deviation of induced stress or allowable stress does not exceed 10% of the mean value.

The allowable stress intensities of Section III,² based on either two-thirds of the minimum specified yield strength or one-third of the minimum specified ultimate tensile strength of the material, also result in an acceptably low probability of failure when it is assumed that the induced stress is permitted to reach the allowable stress intensity.

Cyclic failure probability was analyzed by assuming a Weibull distribution of allowable stresses. The S-N curve for low-alloy carbon steel presented in Section III² corresponds to a failure probability of about 10^{-2} , or one chance in 100, based on vessel rupture data compiled by the Pressure Vessel Research Committee.⁷ It is therefore recommended that a factor of safety be added to the data obtained from that S-N curve when it is used in vessel design.

The methods of analysis demonstrated herein provide an additional and important tool for designers of pressure vessels. These methods

allow the designer to analytically assess the uncertainty in a design and judge the adequacy of the safety factor used in the design on a basis other than prior experience. They also allow the designer to determine whether there is a value of some design parameter, such as vessel wall thickness, above which very little increase in reliability is gained for a corresponding increase in the value of that parameter. As well as these benefits to the component designer, application of probabilistic analysis methods in component design would also result in benefits to the system designer by providing him with an estimate of the component's failure rate to be used in the system reliability analysis.

References

1. D. Kececioglu and E. B. Haugen, "A Unified Look at Design Safety Factors, Safety Margins, and Measures of Reliability," pp. 520-530 in 1968 Annals of Assurance Sciences, Seventh Reliability and Maintainability Conference, The American Society of Mechanical Engineers, New York, 1968.
2. "ASME Boiler and Pressure Vessel Code, Section III, Nuclear Power Plant Components," The American Society of Mechanical Engineers, New York, 1971.
3. E. B. Haugen, Probabilistic Approaches to Design, John Wiley and Sons, Inc., New York, 1968.
4. W. H. Von Alven, Ed., Reliability Engineering, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1964.
5. D. A. Canonico and R. G. Berggren, "Tensile and Impact Properties of Thick-Section Plate and Weldments," USAEC Report ORNL-TM-3211, Oak Ridge National Laboratory, Oak Ridge, Tennessee, January 1971.
6. C. Mischke, "A Method of Relating Factor of Safety and Reliability," pp. 537-542 in Journal of Engineering for Industry, August 1970.

7. "Criteria of the ASME Boiler and Pressure Vessel Code for Design by Analysis in Sections III and VIII, Division 2," The American Society of Mechanical Engineers, New York, 1969.
8. R. D. Wylie and W. J. McGonnagle, Quality Control in Fabrication of Nuclear Pressure Vessels, Rowman and Littlefield, Inc., New York, 1964.
9. J. N. Robinson, Inspection Engineering Department, Oak Ridge National Laboratory, personal communication relative to unpublished statistical survey performed on properties of as-received type-304 stainless steel to H. G. Arnold, Oak Ridge National Laboratory, January 1972.
10. E. R. Forrester and V. H. Thevenow, "Designing for Expected Fatigue Life," pp. 511-519 in 1968 Annals of Assurance Sciences, Seventh Reliability and Maintainability Conference, The American Society of Mechanical Engineers, New York, 1968.
11. W. Weibull, Chapter II in Fatigue Testing and the Analysis of Results, Pergamon Press, New York, 1961.