THE STRUCTURE OF ANOMALOUS LEPTON HADRON

INTERACTIONS AND $\pi^0 \rightarrow e^- e^+$

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ABSTRACT

The contribution of anomalous lepton-quark (hadron) interaction $\sim \bar{l} \gamma_\mu q X_\mu$ to $\pi^0 \rightarrow e^- e^+$ is very strongly dependent on the form of O . Unless experiments reveal an unexpectedly large branching ratio ($> 7 \cdot 10^{-6}$) for the process, O must have (V - A) or (S - P) structure.

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* This research was supported in part by the U. S. Atomic Energy Commission.

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One way to explain¹ the very large observed cross section for $e^-e^+ \rightarrow \text{hadrons}$,² is to introduce a new form of interaction between leptons and quarks, that is,

$$\mathcal{L}_{an} = -ig_{an} \bar{l} O_{\mu} q X_{\mu} + h.c. \quad (1)$$

mediated by massive X particles, where O is some unknown form of (4×4) matrix.

In a process with $s \ll m_x^2$ one may think of Eq. (1) leading to a four Fermion point interaction characterized by a dimensional coupling, G_{an} given by

$$\frac{G_{an}}{\sqrt{2}} = \frac{g_{an}^2}{m_x^2} \quad (2)$$

where m_x is the mass of the X particle. Then for a high energy semi-leptonic process, with $m_l^2, m_q^2 \ll s \ll m_x^2$, which conventionally proceeds via a one photon exchange electromagnetic interaction, the cross section, from dimensional consideration alone, assumes the form,

$$\sigma \sim \frac{\alpha^2}{s} \left[1 + c_1 \sqrt{2} \frac{G_{an} s}{4\pi\alpha} + c_2 \frac{G_{an}^2 s^2}{8\pi^2 \alpha^2} \right] \quad (3)$$

where c_1 and c_2 are some unimportant constants. This expression for the cross section is obtained quite independently of what O is in Eq. (1).³ To explain the observed cross section for the process $e^-e^+ \rightarrow \text{hadrons}$ one can show that the anomalous 4 point coupling must be such that

$$\frac{G_{an}}{\alpha} = f \quad (4)$$

with $0.1 \lesssim f \lesssim 0.5$ depending upon the underlying quark model for hadrons.

To discover the structure for O , which may be crucial for an experimental recognition of the signatures of such interactions in high energy phenomena, let us examine the decay $\pi^0 \rightarrow e^- e^+$. Now there are two masses in the problem m_e^2 and m_π^2 . The contribution due to the anomalous mechanism can go as $G_{an}^2 m_\pi^5$ or $G_{an}^2 m_\pi^3 m_e^2$ depending upon the structure of O so that the branching ratio

$$R_\pi = \frac{\Gamma(\pi^0 \rightarrow e^- e^+)}{\Gamma(\pi^0 \rightarrow 2\gamma)} \quad , \quad (5)$$

that results would be widely different; that is, different by a factor of

$$\frac{m_\pi^2}{m_e^2} \sim (8 \times 10^4) \quad .$$

We can see this as follows. Quite generally, for the four Fermion interaction of Fig. (1b), deduced from Fig. (1a), we are concerned with an interaction Hamiltonian of the form:

$$H_{int} = \frac{G_{an}}{\sqrt{2}} \int \left[\bar{\ell}(x) \circ_\mu^i q(x) \right] \left[\bar{q}(x) \circ_\mu^i \ell(x) \right] d^4x \quad (6)$$

where i takes the value 1 to 5, correspondingly O^1 to O^5 denote the usual S , V , P , A , and T form. On Fierz transformation this becomes,

$$H_{int} = \frac{G_{an}}{\sqrt{2}} \sum_{j=1,2,\dots,5} a^{ij} \int \left[\bar{\ell}(x) F_\mu^j \ell(x) \right] \left[\bar{q}(x) F_\mu^j q(x) \right] d^4x \quad (7)$$

where F^1 to F^5 are again the usual S , V , P , A , and T Dirac matrices and a^{ij}

are the corresponding numerical coefficients which are given by the (5×5) matrix:

$$a^{ij} = \frac{1}{4} \begin{pmatrix} 1 & 4 & 6 & -4 & 1 \\ 1 & -2 & 0 & -2 & -1 \\ 1 & 0 & -2 & 0 & 1 \\ 1 & 2 & 0 & 2 & -1 \\ 1 & -4 & 6 & 4 & 1 \end{pmatrix} \quad (8)$$

Using the Fierz transformed form of the anomalous interaction (Eq. (7)) we can easily calculate the resulting width for the decay mode $\pi^0 \rightarrow e^- e^+$. We find⁴:

$$\begin{aligned} F^S, F^P, \text{ or } F^T \text{ gives } \Gamma(\pi^0 \rightarrow e^- e^+) &\approx \frac{G^2 m_\pi^5}{4\pi} \left(1 - 4 \frac{m_e^2}{m_\pi^2}\right)^{\frac{1}{2}} \\ F^V \text{ gives } \Gamma(\pi^0 \rightarrow e^- e^+) &= 0 \\ F^A \text{ gives } \Gamma(\pi^0 \rightarrow e^- e^+) &= \frac{G^2 m_\pi^3 m_e^2}{4\pi} \left(1 - 4 \frac{m_e^2}{m_\pi^2}\right)^{\frac{1}{2}} \end{aligned} \quad (9)$$

Now we use Eqs. (8) and (9) to construct the branching ratios for various possible structures for O . The results are shown in Table I. The possible contributions to the branching ratio from the anomalous mechanism fall into 3 categories:

- (a). any simple form S , V , T , A , or P or $(V+A)$ for O each would give $R_\pi \sim 10^{-3}$;
- (b). a very cumbersome form $\frac{1}{4}[4S - 2V + 2A - 4P]$ for O which on Fierz transformation yields pure V would give $R_\pi = 0$; and,

(c). the form $\frac{1}{4}[-4S - 2V + 2A + 4P]$ for O which on Fierz transformation yields pure A , or the simple form $(S - P)$ or $(V - A)$ for O yield $R_{\pi} \sim 10^{-8}$.

It may be recalled that the pure electromagnetic process has been estimated to yield $R_{\pi} \sim 10^{-8}$,^{5,6} which is of the same order of magnitude as the contribution from $(V - A)$ or $(S - P)$ form of anomalous interaction. Such a small branching ratio is, of course, hard to detect experimentally, but if O in Eq. (1) has the simple form S , V , T , A , P , or $(V + A)$, then R_{π} would be greatly enhanced. Experimental information on this decay mode would, therefore, be very useful in furthering our understanding of anomalous lepton hadron interactions in particular, and, $e^+e^- \pi^0$ vertex in general. Since this decay mode, proceeding via lowest order electromagnetic interaction, takes place only to order α^4 , whereas the anomalous interactions allow the process to occur to order G_{an}^2 , it presents a unique opportunity to study such interactions where they are rather unimpeded by the much stronger electromagnetic interactions.

If we assume that such anomalous interactions obey (μ, e) universality, which is a very reasonable assumption because the NAL experiments on $pp \rightarrow \ell^+ \ell^- + x$ have not revealed any significant differences between electrons and muons, then we may use the experimental results on $\pi^0 \rightarrow \mu^+ \mu^-$ to put constraints on the structure for O . Experimentally, it is found that^{7,8}

$$(a). \quad R_{\pi} = \frac{\Gamma(\pi^0 \rightarrow \mu^+ \mu^-)}{\Gamma(\pi^0 \rightarrow 2\gamma)} = (5.9 \pm 2.2) \times 10^{-5} \quad (10)$$

$$(b). \quad \Gamma(\eta^0 \rightarrow 2\gamma) = (1.00 \pm 0.22) \text{ keV} \quad (11)$$

yielding,

$$\Gamma(\eta^0 \rightarrow \mu^+ \mu^-) = (6 \pm 3) \times 10^{-11} \text{ GeV} \quad (12)$$

Now we construct the 4th column of Table I for $\eta^0 \rightarrow \mu^+ \mu^-$ and see that if the anomalous interaction Eq. (1) obeys electron-muon universality, then the experimental result Eq. (12) constrains O in Eq. (1) to be $(V - A)$ or $(S - P)$.⁹

This structure for anomalous lepton hadron interaction coupled with the fact that $G_{an} \gg G_F$, where G_F is the conventional four point weak coupling, immediately tells us that we should see significant lepton charge asymmetry in $pp \rightarrow \ell^+ \ell^- x$ and in $\ell^+ N$ versus $\ell^- N$ scattering, and also a significant amount of parity violation in muonic atoms. Quantitative estimates of the expected violations and other consequences will be presented in a separate publication. Experiments to test such asymmetries would be very illuminating.

REFERENCES

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- 3). We are assuming here that the interference term between electromagnetic and X interactions is non-vanishing.
- 4). We are neglecting the contribution from the conventional weak neutral current since $G_{\text{weak}} \ll G_{\text{anomalous}}$.
- 5). S. M. Berman and D. A. Geffen, *Nuovo Cimento* 18, 1192 (1960).
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- 9). Since the electromagnetic contribution is much, much smaller than that of the anomalous coupling with $O = S, V, T, A, P$ or $(V+A)$, this conclusion is not expected to change by the interference between the two processes.

Figure Captions.

Figure 1. Fig. 1a shows Feynman diagram for the process $\pi^0 \rightarrow e^- e^+$ through the anomalous mechanism. Fig. 1b shows the same process as a four fermion point interaction.

TABLE I : The widths and branching ratios of π^0 and η^0 with different forms of anomalous interactions.

Form for $\mathcal{O}$	a). $\Gamma(\pi^0 \rightarrow e^- e^+)$	b). $R_\pi = \frac{\Gamma(\pi^0 \rightarrow e^- e^+)}{\Gamma(\pi^0 \rightarrow 2\gamma)}$	b). $\Gamma(\eta^0 \rightarrow \mu^+ \mu^-)$
$S, V, T, A, P, \text{ or } (V+A)$	$\sim G_{\text{an}}^2 \frac{m_\pi^5}{4\pi} K$	$\sim 2.3 \times 10^{-3}$	$\sim 6.0 \times 10^{-9}$
$\frac{1}{4} [4S - 2V + 2A - 4P]$	0	0	0
$(V-A) \text{ or } 2(S-P)$	$\sim \frac{G_{\text{an}}^2 m_\pi^3 m_e^2}{4\pi} K$	$\sim 3.1 \times 10^{-8}$	$\sim 2.2 \times 10^{-10}$

$$\text{a). } K = \left(1 - \frac{4m_e^2}{m_\pi^2} \right)^{\frac{1}{2}} \quad \approx 1 \text{ for } \pi^0 \rightarrow e^- e^+$$

$$= \left(1 - \frac{4m_\mu^2}{m_\eta^2} \right)^{\frac{1}{2}} \quad \text{for } \eta^0 \rightarrow \mu^+ \mu^-$$

b). Numerical values are computed using $f = 0.3$.

Fig. 1a

Fig. 1b