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MASTER

STRUCTURAL RELIABILITY
ANALYSIS TECHNIQUES
AND REFLECTOR EXAMPLE

G. J. DeSalvo

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Introduction

This report discusses the work done by the Structural Analysis group in applying the methods developed by WANL and SNPO for Structural Reliability Analysis.

The report gives a brief explanation of the statistics involved in reliability calculations and a derivation of the basic overlap or load-strength interference method. The material is presented in a way that agrees with the nomenclature and examples of SNPO-C-1, Draft No. 2.

Design type curves have been worked out such that quick estimates can be made of the reliability, assuming theoretical gaussian distributions of an infinite sample size. These are useful for first estimates of the reliability by designers. Methods of calculation are also shown for finite sample size as per the method suggested both by W. Horton and SNPO-C-1.

The reliability theory is applied to the outer web of the A6 reflector to find the probabilities of failure associated with the calculated and experimentally obtained loads. Also, for a given reliability goal, the mean loads and load deviations that can be tolerated for new designs are obtained. Two theories of failure are considered and compared in determining the reflector outer web structural reliability. One is based on the ultimate tensile strength and the other on a fracture toughness approach.

The material presented in this report could serve as a portion of the proposed Reliability Handbook and Documents and as an aid in the study of Reliability by Design and Analysis personnel.

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Distribution Curves

Assume that a large number of material specimens are tested to failure. A plot of the number of observed failures and the corresponding failure stress may appear as shown in Figure 1. If the ordinate scale is altered in such a manner that the area under the frequency curve becomes one, we obtain a probability density curve.

If the distribution curve shown graphically in Figure 2 can be represented mathematically by

$$f(\mu) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{\mu - \bar{\mu}}{\sigma} \right)^2} \quad (1)$$

the distribution may be considered normal, or gaussian. In the above equation $\sigma \equiv$ standard deviation, $\bar{\mu} \equiv$ arithmetic mean and μ extends from $-\infty$ to $+\infty$. When we place the mean of the normal distribution at the origin of the coordinate system and choose the standard deviation as unity, we obtain the so-called "standardized normal distribution," $f_s(\mu)$,

$$f_s(\mu) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \mu^2} \quad (2)$$

We may also find the standardized normal distribution by using the transform

$$X = \frac{\mu - \bar{\mu}}{\sigma} \quad (3)$$

on Equation (1) and keeping the area under the curve equal to unity. The area under the standardized normal curve from $X = -\infty$ to X is given by

$$F_s(X) = \int_{-\infty}^X f_s(\mu) d\mu = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^X e^{-\frac{1}{2} (\mu^2)} d\mu \quad (4)$$

This area cannot be expressed by a known function, but it can be computed and tabulated. It can be shown that this area also represents the probability that some values will be below μ . Table I lists several probabilities associated with their X values.

A graph of the standardized normal curve is shown in Figure 3. In this graph we have indicated the areas included between $X = -1$ and $+1$, $X = -2$ and $+2$, $X = -3$ and $+3$ as equal, respectively, to 68.26%, 95.44%, and 99.74% of the total area which is one.

Basic Reliability

In its simplest form structural reliability should simply be a question of whether the structure will or will not fail. This survival condition can be expressed mathematically as $\mu_m - \mu_L \geq 0$, where μ_L is the load and μ_m is the strength of the structure. However, load and strength are both probabilistic in nature and cannot be represented by a single value.

Consider the case where only the material strength is described statistically. Such a case may be represented by the weight suspended on a rod in tension illustrated in Figure 4. If the weight is less than the strength of the rod, there will be no failure. However, the strength of the supporting rod will vary from batch to batch and should be described statistically. If the weight were exactly equal to the average strength of the rod, $\bar{\mu}_m$, the probability of failure would be 0.5. The situation is shown graphically in Figure 5 with the shaded area representing the failures.

Because the scatter in the material strength is readily obtainable, the average, or mean strength, is not used as the allowable strength. A design minimum strength, S , is defined above which a certain percentage of the material strengths will fall. Figure 6 represents the situation where 99% of the material strengths exceed the design minimum value.

Statistically this says that the design minimum strength is located 2.326 standard deviations below the mean strength, or $S = \bar{\mu}_m - 2.326 \sigma_m$. In this case, 1 percent of the rods used in such a structure would fail and 99 percent would survive. In general

$$S = \bar{\mu}_m - X_m \sigma_m \quad (5)$$

where X_m represents the number of standard deviations between the mean and design strength of the material.

The choice of a material "design minimum" alone does not guarantee a satisfactory structural reliability. Another element, called the factor of safety, is usually introduced to lower the allowable stress to a value defined as the stress intensity limit, or $\overline{SIL}$. The factor of safety, $\overline{FS}$, and the $\overline{SIL}$ are related to the design minimum, S , by

$$\overline{SIL} = \overline{FS} (S) \quad (6)$$

and shown graphically in Figure 7.

Load Distribution and Location

In general, the applied load will also be subjected to variabilities and should be described statistically. These variabilities result in a combined

stress-strength distribution as shown in Figure 8. The shaded area in Figure 8 represents the probability of failure (as will be shown in the next section) and can be seen to be a function of the degree of overlap of the two distributions.

Similar to the design minimum on the strength distribution curve, there can be defined a design limit load, L , below which a certain percentage of the load values must fall. If $\bar{\mu}_L$ is defined as the mean load, the design limit load is defined by

$$L = \bar{\mu}_L + X_L \sigma_L \quad (7)$$

where σ_L is the standard load deviation and X_L represents the number of standard deviations between the limit load and the mean. This load L may be located somewhat below the previously defined $\bar{SIL}$. The ratio of the $\bar{SIL}$ to L is defined by the margin of safety, $\bar{MS}$

$$\bar{MS} = \frac{\bar{SIL}}{L} - 1 \quad (8)$$

These relationships are shown graphically in Figure 9.

It is often convenient to express the variation in a distribution by a dimensionless parameter called the coefficient of variation, C_v . Such a parameter can be defined for the load, strength, and difference distributions, respectively, as

$$C_{vL} = \sigma_L / \bar{\mu}_L \quad (9)$$

$$C_{vM} = \sigma_m / \bar{\mu}_m \quad (10)$$

$$C_{vR} = \sigma_R / \bar{\mu}_m \quad (11)$$

where σ_R is the combined load and strength standard deviation defined in the next section.

Reliability Analysis

The reliability of a component can be determined from the basic concept that a failure probability exists when a given strength value is exceeded by a load. The probability that a load of value μ_L exists is equal to the area of the element $d\mu_L$ or to A_1 on Figure 10, or

$$Pr \left\{ \mu_L - \frac{d\mu_L}{2} \leq \mu_L \leq \mu_L + \frac{d\mu_L}{2} \right\} = f(\mu_L) d\mu_L \quad (12)$$

The probability of strength exceeding μ_L is equal to the shaded area A_2 , or

$$Pr \{ \mu_m > \mu_L \} = \int_{\mu_L}^{\infty} f(\mu_m) d\mu_m = A_2 \quad (13)$$

The probability of success, i.e., the reliability at μ_L , is the product of these two probabilities, or

$$d(Rel.) = f(\mu_L) d\mu_L \int_{\mu_L}^{\infty} f(\mu_m) d\mu_m \quad (14)$$

The component reliability would then be all probabilities of strength being greater than all possible values of load, or

$$Rel. = \int d(Rel.) = \int_{-\infty}^{\infty} f(\mu_L) \left[\int_{\mu_L}^{\infty} f(\mu_m) d\mu_m \right] d\mu_L \quad (15)$$

Similarly, the probability of component failure, or unreliability, is given by

$$Q = \int_{-\infty}^{\infty} f(\mu_L) \left[\int_{-\infty}^{\mu_L} f(\mu_m) d\mu_m \right] d\mu_L \quad (16)$$

$$\text{where} \quad Rel. + Q = 1 \quad (17)$$

The failure distribution function is given by

$$f'(\mu_L) = \frac{dQ}{d\mu_L} = f(\mu_L) \left[\int_{-\infty}^{\mu_L} f(\mu_m) d\mu_m \right] \quad (18)$$

and represents the curve drawn inside the overlapping portion of the load and strength distributions. The unreliability is the area under this curve, which is also given by Equation (16), and hence the shaded area in Figure 8 represents the probability of failure.

Exact analytical solutions to Equation (15) are not known for load and strength distributions other than both gaussian. If $f(\mu_L)$ and $f(\mu_m)$ are gaussian distributions, they may be expressed as

$$f(\mu_L) = \frac{1}{\sigma_L \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{\mu_L - \bar{\mu}_L}{\sigma_L} \right)^2} \quad (19)$$

and

$$f(\mu_m) = \frac{1}{\sigma_m \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{\mu_m - \bar{\mu}_m}{\sigma_m} \right)^2} \quad (20)$$

Reliability is the probability that the strength is in excess of the load or that $\mu_m - \mu_L > 0$. If we designate $R = \mu_m - \mu_L$, then reliability is the probability that $R > 0$. $f(R)$ is defined as the difference distribution of $f(\mu_m)$ and $f(\mu_L)$, and as $f(\mu_m)$ and $f(\mu_L)$ are normally distributed $f(R)$ can be shown to be normally distributed also and given by

$$f(R) = \frac{1}{\sigma_R \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{R - \bar{R}}{\sigma_R} \right)^2} \quad (21)$$

where

$$\bar{R} = \bar{\mu}_m - \bar{\mu}_L \quad (22)$$

and

$$\sigma_R = \sqrt{\sigma_m^2 + \sigma_L^2} \quad (23)$$

The reliability is then given by all probabilities of R being a positive value, hence

$$\begin{aligned} R_{el.} &= Pr \{ R > 0 \} = \int_0^{\infty} f(R) dR \\ &= \frac{1}{\sigma_R \sqrt{2\pi}} \int_0^{\infty} e^{-\frac{1}{2} \left(\frac{R - \bar{R}}{\sigma_R} \right)^2} dR \end{aligned} \quad (24)$$

The function $f(R)$ is shown in Figure 11. If we let $X = \frac{R - \bar{R}}{\sigma_R}$, Equation (24) can be normalized to

$$R_{al.} = \frac{1}{\sqrt{2\pi}} \int_{-x_R}^{\infty} e^{-x^2/2} dx \quad (25)$$

where

$$X_R = \frac{\bar{\mu}_m - \bar{\mu}_L}{\sigma_R} \quad (26)$$

and is of the same form as Equation (4). Therefore the reliability of a strength-load distribution for various values of X_R can be obtained from Table I to give the results in Table II. It can be seen that the reliability associated with two overlapping distributions can be determined by knowing the mean and standard deviation of each distribution.

Working Curves

The following curves were obtained from the relationships presented in the previous section. The load and strength curves are assumed to be normally distributed and based on population statistics.

Figure A shows the probability of failure as a function of the mean load, the mean strength, and the combined coefficient of variation, C_{vR} . The combined coefficient can be calculated from the strength mean and standard deviation and the load standard deviation given the load and strength means and coefficients of variation.

Figure B gives the ratio of the mean strength to the mean load as a function of the coefficients of variation for a given reliability. This particular curve is constructed for a reliability of .99985 or $P = 1.5 \times 10^{-4}$.

Figure C can be interpreted in several ways. Considering only the strength diagram, it gives the ratio of the mean strength to the $\overline{SIL}$ for strength variations up to 30%. Implied are the conditions that 99% ($X_m \approx 2.3$) of the strength values lie above the design minimum and that the $\overline{SIL}$ is 80% this design minimum, i.e., $\overline{FS} = .8$.

If we assume that we have a line load at the $\overline{SIL}$, as shown in Figure 12, or a load distributed about the $\overline{SIL}$, as in Figure 13, the $\overline{SIL}$ will also represent the mean load. Implied in this statement, and in all the curves, is the fact that the margin of safety, $\overline{MS}$, is set equal to zero. This yields the maximum probability of failure for the given conditions.

Figure D shows how the probability of failure varies for strength and load coefficients up to 20% wherever the mean load is located at the $\overline{SIL}$.

Special interpretation can be given to the case where $C_{VL} = 0$. Let us assume that this line load represents the limiting load value that can never be exceeded with 100% certainty. The probability of failure calculated will then be a maximum over that obtained for any distributed load with values up to the $\overline{SIL}$.

If the mean load is located at several standard deviations below the $\overline{SIL}$, as illustrated in Figure 14, the probability of failure can be above or below the line load case, depending on the distribution statistics. Figure E shows how the probability of failure varies with different load and strength coefficients for the case where the mean load is located 2.3σ below the $\overline{SIL}$, i.e., 99% of all loads must fall below the $\overline{SIL}$.

Figure F can then be used to locate the mean load relative to the mean strength for this 2.3σ case. The mean load for values of X_L other than 2.3 can also be found from Figure F. In all cases the design load is assumed to be located at the $\overline{SIL}$ for reference.

Distribution Variabilities

The reliability method of analysis requires the knowledge of the failure governing load and strength distributions in the component under the actual application and operation environment. These distributions may be affected by, but are not necessarily limited to, the following factors:

- 1) Stress concentrations
- 2) Type of loading
- 3) Temperature
- 4) Forming or manufacturing processes
- 5) Surface treatment
- 6) Heat treatment
- 7) Type of assembly
- 8) Corrosion and surface environment
- 9) Time

It should be noted that these factors may be considered as strength reducing or load augmenting factors or both.

These factors are usually applied to the mean stress and, unfortunately, little is known of their effect on the stress deviations. In addition, these factors may have distributions of their own and the task of combining these variables to obtain load and strength distributions which are

probabilistically representative of the distributions of these variables is necessary.

Before applying a reliability analysis it is necessary to know the maximum stresses which so combine at the same point in the component as to result in the failure governing stress. The identification of this critical stress point can usually be obtained by inspection of the most likely points and the stress calculated to determine the maximum. It is important that all six stress components be calculated and the proper stress factors applied to the components. Calculation of the principal stresses may or may not be required, depending on the form of failure governing criterion used. It is important to determine the strength which, if exceeded, leads to a probability of failure. For this the strength criterion most applicable to the particular problem at hand should be selected.

Combined Distributions

A method of combining distributions is possible through the application of a complete and consistent algebra of distribution functions. The distributions are assumed to be normal and variables statistically independent. If a complete theory of statistical inference is developed based on the normal distribution alone, then one has in reality a system which may be employed quite generally since it is possible to transform other distributions into the normal form. A meaningful statistical description is possible because the algebra of normal functions is closed, i. e., the sum, difference, product or quotient of two normal functions is again a normal function.

not true!

The operations of taking a sum or difference, forming a product, or obtaining a quotient are necessary when deriving statistical expressions for structural strength and combined total loading. The need for normal-function algebra can be seen in any formula; for example, $S = P/A + Mc/I$. Each of the parameters has a mean, μ , and a standard deviation, σ . A summary of normal-function algebra for several combinations is given below:

1) The sum of two normal functions

$$\text{Mean of sum} = \bar{\mu}_{m+L} = \bar{\mu}_m + \bar{\mu}_L$$

$$\text{Standard deviation of sum} = \sigma_{m+L} = \sqrt{\sigma_m^2 + \sigma_L^2}$$

2) Difference of two normal functions

$$\text{Mean of difference} = \bar{\mu}_{m-L} = \bar{\mu}_m - \bar{\mu}_L$$

$$\text{Standard deviation of difference} = \sigma_{m-L} = \sqrt{\sigma_m^2 + \sigma_L^2}$$

3) Product of two normal functions

$$\text{Mean of product} = \bar{\mu}_{m \cdot L} = \bar{\mu}_m \cdot \bar{\mu}_L$$

$$\text{Standard deviation of product} = \sigma_{m \cdot L} =$$

$$\sqrt{\bar{\mu}_m^2 \sigma_L^2 + \bar{\mu}_L^2 \sigma_m^2 + \sigma_m^2 \sigma_L^2}$$

4) Quotient of two normal functions

$$\text{Mean of quotient} = \bar{\mu}_{m/L} = \bar{\mu}_m / \bar{\mu}_L$$

$$\text{Standard deviation of quotient} = \sigma_{m/L} =$$

$$\frac{1}{\bar{\mu}_L} \sqrt{\frac{|\bar{\mu}_L^2 \sigma_m^2 - \bar{\mu}_m^2 \sigma_L^2|}{\bar{\mu}_m^2 + \bar{\mu}_L^2}}$$

5) Power functions

Consider a function such as $I = bh^3/12$. It is obvious that $h \cdot h \cdot h$ is not a statistically independent function in h . For this reason special treatment must be given to power functions. For square and cubic functions the mean is given by $\bar{\mu}_{h^2} = \bar{\mu}_h^2 + \sigma_h^2$

and $\bar{\mu}_{h^3} = \bar{\mu}_h^3 + 3\bar{\mu}_h\sigma_h^2$, respectively. The standard deviation is $\sigma_{h^2} = h\bar{\mu}_h\sigma_h$. The operations of addition and multiplication are commutative in the algebra of normal functions. The associative law holds for addition. The associative and the distributive laws hold for multiplication. Multiplication of a normal function by a constant is handled by considering constants to be normal functions having mean values and zero standard deviations.

Combining Reliabilities

The reliability of a product composed of several elements may be computed from the logical sum of the component reliabilities. The units described below are those whose elements are assumed to be independent and whose operation can be described in discrete terms as either a "success" or a "failure" over a specified time interval. The reliability of an element of the system is denoted by r and the unreliability, or the probability of failure, given by $(1-r)$, is denoted by q .

Consider the overall reliability of the following basic systems:

1) Series system

The reliability of a system of n elements in series as shown in Figure 14.1 is given by

$$R = r_1 r_2 r_3 \cdots r_n = \prod_{i=1}^n r_i$$

If all of the elements have a high reliability (say $r_i > .95$), a useful approximation is

$$R = 1 - (g_1 + g_2 + g_3 + \cdots + g_n) = 1 - \sum_{i=1}^n g_i$$

2) Parallel systems

The reliability of a unit with n parallel elements as shown in Figure 14.2 is given by

$$R = 1 - (g_1 g_2 g_3 \cdots g_n) = 1 - \prod_{i=1}^n g_i$$

3) Series - Parallel configurations

The reliability of a series of n basic parallel units as shown in Figure 14.3 is given by

$$R = \prod_{j=1}^n (1 - g_{1j} g_{2j} g_{3j} \cdots g_{mj})$$

In general, for series-parallel configurations where the elements are in parallel and the units are in series, the first step is to evaluate the reliability of each unit. The product of the unit reliabilities is equal to the system reliability.

4) Parallel-Series configurations

The reliability of a system of n -series elements in parallel as shown in Figure 14.4 is given by

$$R = 1 - \prod_{i=1}^n (1 - r_{i1} r_{i2} r_{i3} \cdots r_{in})$$

In general, for parallel-series configurations where the elements are in series and the paths are in parallel, the first step is to obtain the path reliability by calculating the product of the element reliabilities. One minus the product of the path unreliabilities is equal to the system reliability.

A comparison of the last two systems indicates that series-parallel configurations result in higher reliability than equivalent parallel-series configurations. It might also be noted that for element reliabilities greater than .90, there is usually little practical advantage to having more than two parallel elements.

Other combinations are necessary for systems exhibiting partial redundancy. In this case at least k out of m parallel paths, where $1 < k < m$, must be satisfactory for system success. Similarly, additional considerations are necessary where conditional probabilities are involved. Such would occur wherever an individual element or path failure had a definite effect on the operation of the surviving paths.

Sample Statistics

The quantity of statistical load and strength data available is usually quite limited. Consequently, the statistical parameters (i.e., the mean and standard deviation) must be derived from limited data which can practically be obtained consistent with present cost and time allotments.

The material presented so far has been entirely in terms of the population mean, μ , and standard deviation, σ . In practice, these quantities are unknown, and it is necessary to estimate their values from the sample mean and standard deviation, $\bar{X}$ and s , and the sample size N . In addition, different samples taken from the same population may have different means and standard

variations. It can be shown that for random samples of size N the mean, $\bar{X}$, is normally distributed having $\bar{\mu}$ for its mean and $\sigma/\sqrt{N}$ for its standard variation.

Confidence Limits

Confidence limits for the mean, which are of the form

$$\bar{X} \pm \frac{\bar{x} \sigma}{\sqrt{N}} \quad (27)$$

are computed in such a way that in a fraction γ of the samples from which they are computed, they will include the mean of the distribution, $\bar{\mu}$. If, in addition, σ is unknown, then the confidence limits can be obtained from

$$\bar{X} \pm \frac{\bar{t} s}{\sqrt{N}} \quad (28)$$

where $\bar{t}$ represents a value based on the student's distribution. The standard deviation may also vary from sample to sample and can also be given confidence limits based on the chi-square distribution.

Tolerance Limits

The tolerance limits considered here are symmetrical about the mean, and they are based on the assumption that the underlying statistical universe may be described by a normal distribution. When the true mean and standard deviation, $\bar{\mu}$ and σ , of this normal distribution are known, tolerance limits are formed by adding to and subtracting from the mean a certain multiple K of the standard deviations; the appropriate multiple is easily obtained from tables of the normal distribution and depends upon what proportion of the universe is to be included within the limits.

Instead of specifying a two-sided interval that will contain a percentage of the product, it is frequently convenient to specify a one-sided limit such that a fixed percentage of the product will have a quality greater than (or less than) this limit. For example, 95% of the population is included in the interval $\bar{\mu} - 1.645 \sigma$ to ∞ .

In most practical situations, the true values of $\bar{\mu}$ and σ are not known and tolerance limits must be estimated from the mean $\bar{X}$ and the standard deviation s of a sample of N observations. While the limits $\bar{\mu} - 1.645 \sigma$ to ∞ will include 95% of the population, clearly the same assertion cannot be made of the limits $\bar{X} - 1.645 s$. The proportion included between the limits $\bar{X} - Ks$ and infinity depends, for example, on whether the sample mean is unusually large, reasonably close to the population mean, or unusually small. The sample standard deviation is also subject to chance fluctuations that affect the proportion included between the limits. In fact, if a value of K is fixed, no two samples from a given universe will lead to the same tolerance limits, since both $\bar{X}$ and s will vary from sample to sample.

It is clear then that the proportion of the universe included between the limits $\bar{X} \pm Ks$ is a random variable, and that it is impossible to determine K so that the limits will always contain a specified proportion P of the population. It is possible, however, to determine K in such a way that in a large series of samples from normal populations a certain proportion γ of the intervals $\bar{X} \pm Ks$ will include P or more of the population. γ is called "the confidence coefficient," since it is a measure of the confidence with which we may assert

that a given tolerance range includes at least P of the population.

Intuitively, it seems reasonable that the values of K for use with sample means and standard deviations will be somewhat larger than the values used if the true mean and standard deviation are known. Table III presents such factors K for various combinations of confidence coefficient γ , proportion P , and sample size N . For example: if $\bar{\mu}$ and σ are known, the items above $\bar{\mu} - 2.326 \sigma$ include 99% of the population; however, if $\bar{\mu}$ and σ are estimated from a sample of 10 items then $\bar{X} - 3.981s$ is required to include 99% of the total population with 95% confidence.

Tolerance limits should not be confused with confidence limits for the mean or other parameters of the distribution. While both are of the form $\bar{X} \pm Ks$, confidence limits for the mean are computed in such a way that in a fraction of the samples from which they are computed they will include the mean of the distribution. Tolerance limits, in contrast, are computed in such a way that in a fraction γ of the samples from which they are computed they will include at least a fraction P of the items in the distribution. Thus, as the sample size increases, the value of K for a confidence interval diminishes toward zero; whereas, the value of K for a tolerance interval diminishes toward the normal deviate including a proportion P of the population.

Allowable Stress Calculation

For raw data the sample mean may be computed from

$$\bar{X} = \sum_{i=1}^N \left(\frac{x_i}{N} \right) \quad (29)$$

and the sample standard deviation from

$$s = \sqrt{\frac{\sum_{i=1}^N (x_i - \bar{x})^2}{N-1}} \quad (30)$$

The design minimum strength, S , based on a probability P at a confidence level γ for N samples is given by

$$S = \bar{x}_m - k_m s_m \quad (31)$$

The design limit load, L , based on a probability P at a confidence level γ for N samples is given by

$$L = \bar{x}_L + k_L s_L \quad (32)$$

The stress intensity limit, $\overline{SIL}$, is related to S by the equation

$$\overline{SIL} = \overline{FS} (S) \quad (33)$$

and is related to the design limit load, L , by the margin of safety, $\overline{MS}$, as follows

$$\overline{MS} = \left(\overline{SIL} / L \right) - 1 \quad (34)$$

$$S/L = (1 + \overline{MS}) / \overline{FS} \quad (35)$$

Sample coefficients of variation can also be defined similar to the population coefficients as

$$c_{vm}' = s_m / \bar{x}_m \quad (36)$$

$$\text{and } c_{vL}' = s_L / \bar{x}_L \quad (37)$$

Reliability Calculations

The reliability calculated from sample statistics must be associated with a confidence level. The following method relates the sample reliability to the population reliability at a given confidence level.

Assume that an equation involving the sample statistics, instead of the population statistics, can be devised in a manner similar to that used in developing Equation (26) to give

$$X_R' = \frac{\bar{X}_m - \bar{X}_L}{\hat{S}} \quad (38)$$

$$\text{where } \hat{S} \equiv \sqrt{s_m^2 + s_L^2} \quad (39)$$

The value of X_R' depends upon the equivalent number of combined samples, N , the confidence level and the combined population reliability. The value of X_R' will always be greater than the value of X_R with the difference approaching zero as N approaches infinity. This relationship can be expressed mathematically by

$$X_R' = \frac{X_R + \sqrt{X_R^2 - ab}}{a} \quad (40)$$

$$\text{where } a = 1 - \frac{X_Y^2}{2(N'-1)} \quad (41)$$

$$\text{and } b = X_R^2 - \frac{X_Y^2}{N'} \quad (42)$$

The value of X_Y represents the number of standard deviations associated with the confidence limits. Figure G ^{σ_H} shows this relationship between X_R' , N' , and X_R (expressed in terms of reliability) for a 90% confidence limit, i.e., $X_Y = 1.2816$.

The value of N' , the equivalent combined number of samples, is given by

$$N' = \frac{(\hat{S}^2)^2}{\frac{(S_L^2)^2}{N_L} + \frac{(S_m^2)^2}{N_m}} \quad (43)$$

A sample coefficient of variation of reliability can also be defined by

$$C_{vR}' = \frac{\hat{S}}{\bar{X}_m} \quad (44)$$

It should be noted that the previously plotted working graphs can be used for approximating sample reliabilities if sample statistics are substituted for population statistics, i. e., $\bar{\mu} \leftarrow \bar{X}$. Instead of directly substituting s for σ we can increase the reliability confidence level by modifying the sample variation by K/X . Recall that K is the tolerance factor, depending upon the number of samples and the confidence level, and X is the number of standard deviations associated with the reliability. For example, consider a material whose standard deviation is found from 15 samples to be 1000. The modified standard deviation based on a 99% reliability and a 95% confidence level is given by

$$\sigma_m = \frac{K_m s_m}{X_m} = \frac{(3.52)(1000)}{2.33} = 1510$$

Similarly the modified coefficient of variation is given by 15.1% instead of 10%.

Numerical Example

The following statistics have been developed for a material from sample tests:

$$\begin{aligned} \bar{X}_m &= 99,100 \text{ PSI} \\ S_m &= 6,710 \text{ PSI} \\ N_m &= 15 \end{aligned}$$

The load distribution (from limit load calculations) reveals that the standard deviation is one-tenth of the mean load. Treating this data as population statistics, we get

$$\begin{aligned}\bar{X}_L &= \bar{\mu}_L \\ S_L &= \sigma_L = .1 \bar{\mu}_L \\ N_L &= \infty\end{aligned}$$

Find $\bar{\mu}_L$ and the probability of failure for a factor of safety of .8 and a margin of safety of zero. It is also given that both the design strength and the limit load should be evaluated for a 99% probability at a 95% confidence limit.

From Table III we get $K_m = 3.52$. Also $K_L = X_L = 2.326 = X_m$

The design minimum strength, S , is given from Equation (31), as

$$S = \bar{X}_m - K_m s_m = 99,100 - 3.52 (6,710) = 75,500 \text{ psi}$$

The stress intensity limit, $\overline{SIL}$, from Equation (33) is

$$\overline{SIL} = \overline{FS}(S) = .8 (75,500) = 60,400 \text{ psi}$$

Since the margin of safety, $\overline{MS}$, is zero, Equation (34) gives

$$\overline{MS} = \overline{SIL}/L - 1 = 0, \therefore L = \overline{SIL}$$

where L is the design limit load. The mean load $\bar{X}_L$ can be found from Equation (32)

as

$$L = \overline{SIL} = \bar{X}_L + K_L s_L$$

or

$$\bar{X}_L (1 + 2.326(.10)) = 60,400 \text{ psi}$$

$$\bar{X}_L = \bar{\mu}_L = 60,400 / 1.2326 = 49,000 \text{ psi}$$

The reliability calculation requires finding X'_R and N' . From Equation

(39)

$$\hat{s} = \sqrt{s_m^2 + s_L^2} = \sqrt{(6,710)^2 + (4,900)^2} = 8,300 \text{ psi}$$

From Equation (38), $X'_R = \frac{\bar{X}_m - \bar{X}_L}{\hat{s}}$, or

$$X'_R = \frac{99,100 - 49,000}{8300} = 6.03$$

From Equation (43),

$$N' = \frac{(\hat{s}^2)^2}{\frac{(s_L^2)^2}{N_L} + \frac{(s_m^2)^2}{N_m}} = \frac{(8300)^4 15}{(6710)^4} = 35.3$$

From Figure G, $G_{\sigma H} R = .97$ @ 90% confidence.

If, as a first approximation, we had corrected the sample data to that for a population we could have then used Figure E, with $C_{VL} = 10\%$ and $C_{VM} = \frac{K_m s_m}{\bar{X}_m \bar{X}_m} = \frac{(3.52)(6710)}{(2.33)(99,100)} = 10.1\%$, to obtain $P = 4 \times 10^{-6}$ or $R = .956$. The mean load can then be obtained from Figure F as 49,000 psi.

Reflector Reliability Examples

Several examples have been calculated to answer the following questions:

- 1) What are the probabilities of failure associated with the A6 calculated and experimentally obtained loads?
- 2) For a given reliability goal, what mean loads and load variations can be tolerated?

Two theories of failure are considered in determining the structural reliability. One is based on the ultimate tensile strength and the other on a fracture toughness approach. The most likely point of failure is chosen to be at the inside surface of the control drum hole at the outer web, as shown in Figure 15.

The stress, σ_1 , at this point is given in Table V for various conditions. In order to determine a lower limit on reliability, the stresses given in Table V are assumed to be limiting values at the conditions given or that all loads occur at this value with a 100% probability. This can be represented statistically by a line load of mean value $\bar{X}_L$ and $s_L = 0$, Figure 16. If loads less than X_L do occur, yet with no probability of the occurrence of loads exceeding X_L , the reliability will be higher than the limiting case.

Based on a fracture mechanics approach, the outer web element can be represented by either of the notched specimens shown in Figure 17. The crack depth, a , is taken to be 1/10 of the thickness, h . Figure 17 (b) represents a simplified element where the maximum stress, σ_1 , is assumed to act uniformly across the element, replacing the $\bar{\sigma}$, M system of Figure 17 (a). Since the two elements give similar results, with the stress intensity factor for the latter case being slightly higher, the simplified model, Figure 17 (b), is used. Then

$$K_1 = \sigma_1 \sqrt{\pi a} f\left(\frac{2a}{h}\right) = .3 \sigma_1$$

Table V also gives the stress intensity factors for $h = .205"$ and $a = 1/10h$ associated with the corresponding maximum stress.

Calculation of reliabilities were made using the method previously described to relate the sample reliability to a population reliability at a 90% confidence level.

Table VI gives the probabilities of failure (based on 10,000) for the loads given in Table V and the properties given in Table IV.

As a reference it is convenient to know the stress intensity limit ($\overline{SIL}$), determined by certain specifications, for the material being considered. The specifications locate the design minimum, S , at a 99% probability with a 95% confidence limit and the $\overline{SIL}$ at 80% of S . Figure 18 shows these quantities for irradiated beryllium @ 230°R.

For a given reliability goal the mean load can be calculated. Also, for non-line load distributions, the percentage of loads that must fall below the $\overline{SIL}$ can be found. Table VII shows these values for irradiated beryllium at 230°R that yield a reliability of .99985 at 90% confidence with the load variations of 0 and 25%.

Also of interest is what reliability is obtained if 99% of the loads are required to fall below the $\overline{SIL}$. This states that the mean load is to be placed 2.3σ below the $\overline{SIL}$. For irradiated beryllium @ 230°R, the values shown in Table VIII can be obtained.

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(11)

Fig. 1 Frequency distribution curve

Fig. 2 Probability density curve

Fig. 3 Standardized normal distribution

(F.2)

Fig. 4 Rod subjected to constant tension

Fig. 5 Applied load at mean strength

Fig. 6 Load-stress, the system having a reliability of .99

(F3)

Fig. 7 Allowable stress determination

Fig. 8 Load and strength distribution with probability of failure given by shaded area.

Fig. 9 Load-strength distributions and relationships

(F 4)

Fig. 10 Areas for determining reliability by interference

Fig. 11 Distribution of difference function of strength and load.

Fig. 12 Line load at SIL

(FS)

Fig. 13 Distributed load at $\bar{s}$

Fig. 14 mean load located below $\bar{s}$

Fig. 14.1 Series system of n -elements

Fig. 14.2 System of m -elements in parallel

Fig. 14.3 Series of multiple elements in parallel

Fig. 14.4 Parallel-series configuration

Fig. 15 Location of most probable point failure

(F7)

Fig. 16 Line load distribution

Fig. 17 Fracture mechanics models

Fig. 18 stress intensity limits for irradiated Beryllium
 @ 250°K based on a) ultimate tensile strength, b) fracture toughness

FIG. A

OVERALL PROBABILITY
(INVOLVING ONLY C_{VA} , $C_{V\sigma}$, $\bar{\mu}_m$, $\bar{\mu}_\sigma$)

MODEL

DATE

FOR $R = .99985$ OR $P = 1.5 \times 10^{-4}$

FIG. B

SIGNATURE _____

DATE _____

CURVE NO. _____

Fig. C

$X_L = 0$
 $X_M = 2.3$
 $\overline{FS} = .80$
 $\overline{MS} = 0$

MODEL

DATE

$$X_L = 2.3$$

$$X_M = 2.3$$

$$\overline{FS} = .80$$

$$\overline{MS} = 0$$

MODEL

DATE

FIG. E

FIG. F

WITH 90% CONFIDENCE

Table I - Probabilities ($F_s(x)$) associated with location X on standardized normal distribution

X	-3	-2	-1	0	1	2	3
$F_s(x)$	.0013	.0228	.1587	.5000	.8413	.9772	.9987

Table II - Reliability of a strength-load distribution for various values of X_R

X_R	-3	-2	-1	0	1	2	3
Rel.	.0013	.0228	.1587	.5000	.8413	.9772	.9987

Table III - see page T2

Table IV - see page T3

Table V - Maximum stress, σ_1 , at outer web of reflector and corresponding stress intensity, K_1 .

	PRE-TEST		POST-TEST	
	DATA BOOK VALUE	DATA BOOK VALUE WITH END EFFECTS	START OF LIFE	END OF LIFE
T	230 °R	230 °R	193 °R	192 °R
σ_1	14,000 psi	32,000 psi	41,000 psi	15,000 psi
K_1	4,200 psi-in	9,600 psi-in	13,200 psi-in	19,500 psi-in

TABLE III

TOLERANCE FACTORS FOR NORMAL DISTRIBUTIONS

Factors K such that the probability is γ that at least a proportion $1 - \beta$ of the distribution will be less than $\bar{x} + Ks$ (or greater than $\bar{x} - Ks$), where $\bar{x}$ and s are estimates of the mean and the standard deviation* computed from a sample of size n

n	$\gamma = 0.75$					$\gamma = 0.90$					$\gamma = 0.95$					$\gamma = 0.99$				
	0.25	0.10	0.05	0.01	0.001	0.25	0.10	0.05	0.01	0.001	0.25	0.10	0.05	0.01	0.001	0.25	0.10	0.05	0.01	0.001
3	1.464	2.501	3.152	4.396	5.805	2.002	4.258	5.310	7.340	9.651	3.801	6.158	7.655	10.852	13.857					
4	1.256	2.131	2.680	3.726	4.910	1.972	3.157	3.957	5.437	7.128	2.619	4.103	5.115	7.012	9.215					
5	1.152	1.961	2.463	3.421	4.507	1.698	2.742	3.400	4.666	6.112	2.149	3.407	4.202	5.741	7.501					
6	1.087	1.860	2.336	3.213	4.273	1.540	2.491	3.091	4.242	5.556	1.895	3.006	3.707	5.062	6.612	2.819	4.408	5.409	7.331	9.510
7	1.013	1.791	2.250	3.126	4.118	1.433	2.333	2.891	3.972	5.201	1.732	2.755	3.399	4.641	6.061	2.490	3.856	4.739	6.411	8.313
8	1.010	1.740	2.100	3.012	4.008	1.360	2.219	2.755	3.783	4.955	1.617	2.582	3.188	4.353	5.686	2.252	3.496	4.287	5.811	7.566
9	0.984	1.702	2.141	2.977	3.924	1.302	2.133	2.649	3.611	4.772	1.532	2.451	3.031	4.143	5.414	2.085	3.242	3.971	5.389	7.014
10	0.964	1.671	2.103	2.927	3.858	1.257	2.065	2.568	3.532	4.629	1.465	2.355	2.911	3.981	5.203	1.954	3.018	3.739	5.075	6.603
11	0.947	1.616	2.073	2.885	3.804	1.219	2.012	2.503	3.441	4.515	1.411	2.275	2.815	3.852	5.036	1.854	2.897	3.557	4.828	6.284
12	0.933	1.624	2.018	2.851	3.760	1.188	1.966	2.448	3.371	4.420	1.366	2.210	2.736	3.747	4.900	1.771	2.773	3.410	4.633	6.032
13	0.919	1.606	2.026	2.822	3.722	1.162	1.928	2.403	3.310	4.311	1.329	2.155	2.670	3.659	4.787	1.702	2.677	3.290	4.472	5.826
14	0.909	1.591	2.007	2.796	3.690	1.139	1.895	2.363	3.257	4.274	1.296	2.108	2.614	3.585	4.690	1.645	2.592	3.189	4.336	5.651
15	0.899	1.577	1.991	2.776	3.661	1.119	1.866	2.329	3.212	4.215	1.268	2.068	2.566	3.520	4.607	1.596	2.521	3.102	4.224	5.507
16	0.891	1.566	1.977	2.756	3.637	1.101	1.842	2.299	3.172	4.161	1.242	2.032	2.523	3.463	4.534	1.553	2.458	3.028	4.124	5.374
17	0.883	1.551	1.961	2.739	3.615	1.083	1.820	2.272	3.136	4.118	1.220	2.001	2.486	3.415	4.471	1.514	2.405	2.962	4.038	5.268
18	0.876	1.511	1.951	2.723	3.595	1.071	1.800	2.249	3.106	4.078	1.200	1.974	2.453	3.370	4.415	1.481	2.357	2.906	3.961	5.167
19	0.870	1.536	1.912	2.710	3.577	1.058	1.781	2.228	3.078	4.011	1.183	1.949	2.423	3.331	4.364	1.450	2.315	2.855	3.893	5.078
20	0.865	1.528	1.913	2.697	3.561	1.046	1.765	2.208	3.052	4.009	1.167	1.926	2.396	3.295	4.319	1.424	2.275	2.807	3.832	5.003
21	0.859	1.520	1.923	2.686	3.545	1.035	1.750	2.190	3.028	3.979	1.152	1.905	2.371	3.262	4.276	1.397	2.241	2.768	3.776	4.932
22	0.851	1.514	1.916	2.675	3.532	1.025	1.736	2.174	3.007	3.952	1.138	1.887	2.350	3.233	4.238	1.376	2.208	2.729	3.727	4.866
23	0.849	1.508	1.907	2.665	3.520	1.016	1.724	2.159	2.987	3.927	1.126	1.869	2.329	3.206	4.204	1.355	2.179	2.693	3.680	4.806
24	0.845	1.502	1.901	2.656	3.509	1.007	1.712	2.145	2.969	3.904	1.114	1.853	2.309	3.181	4.171	1.336	2.154	2.663	3.638	4.755
25	0.842	1.496	1.895	2.647	3.497	0.999	1.702	2.132	2.952	3.882	1.103	1.838	2.292	3.158	4.143	1.319	2.129	2.632	3.601	4.706
30	0.825	1.475	1.869	2.613	3.454	0.966	1.657	2.080	2.884	3.794	1.059	1.778	2.220	3.064	4.022	1.249	2.029	2.516	3.446	4.508
35	0.812	1.458	1.849	2.588	3.421	0.942	1.623	2.041	2.833	3.730	1.025	1.732	2.166	2.994	3.934	1.195	1.957	2.431	3.334	4.364
40	0.803	1.445	1.834	2.568	3.395	0.923	1.598	2.010	2.793	3.679	0.999	1.697	2.126	2.941	3.866	1.154	1.902	2.365	3.250	4.255
45	0.795	1.435	1.821	2.552	3.375	0.908	1.577	1.986	2.762	3.638	0.978	1.669	2.092	2.897	3.811	1.122	1.857	2.313	3.181	4.168
50	0.788	1.426	1.811	2.538	3.358	0.894	1.560	1.965	2.735	3.604	0.961	1.646	2.065	2.863	3.766	1.096	1.821	2.296	3.124	4.096
∞	0.67	1.25	1.645	2.23	3.005															

* s is the square-root of the unbiased estimate of the variance (cf. Sec. 8.7).

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Table IV material Properties of Beryllium - S-200

UNIRRADIATED :

Temp.	<u>U.T.S.</u> 193°R (-267°F)	220°R (-230°F)
$\bar{X}_m$	44,860 psi	46,000 psi
Δ_m	4,240 psi	4,280 psi
N_m	56	59

F.T.

$\bar{X}_m$	16,320 psi - $\sqrt{in}$	16,700 psi - $\sqrt{in}$
Δ_m	1,050 psi - $\sqrt{in}$	1,160 psi - $\sqrt{in}$
N_m	8	7

IRRADIATED :U.T.S.

$\bar{X}_m$	41,500 psi	43,000 psi
Δ_m	3,810 psi	3,930 psi
N_m	56	59

F.T.

$\bar{X}_m$	9,300 psi - $\sqrt{in}$	9,300 psi - $\sqrt{in}$
Δ_m	710 psi - $\sqrt{in}$	710 psi - $\sqrt{in}$
N_m	6	6

Table VI - AB Reflector outer web stresses and probability of failure for $C_{UL} = 0$.

	PRE-TEST		POST-TEST	
	PREDICTED BEFORE RUN	INCLUDING INDUCTION EFFECTS	START OF LIFE	END OF LIFE
σ_i	14,000 psi	32,000 psi	49,000 psi	65,000 psi
K_I	9,200 psi-in	9,600 psi-in	13,200 psi-in	19,500 psi-in
Temp.	230 °R	230 °R	193 °R	192 °R
$P_{UTS} / 10^4$	$5 \times 10^{-7} @ 90\%$	$6.5 @ 90\%$	4800 @ 90%	9997.9796 @ 90%
R_{UTS}	(.9105)	(.9935)	(.52)	(4×10^{-8})
$P_{FT} / 10^4$	.12 @ 90%	6200 @ 90%	300 @ 90%	9997.9796 @ 90%
R_{FT}	(.9488)	(.38)	(.97)	(10^{-13})

* Based on unirradiated properties

NOTE: R_{FT} for $P / 10^4$ goal = 1.5; K_I based on crack depth equal to .1 of web thickness.

Table VII Design stress for outer web reflector using irradiated Beryllium data at 200 °R to give a probability of failure of $1.5 / 10,000$

	σ_i		X_L	$\overline{SIL}$
	$C_{UL} = 0$	$C_{UL} = 25\%$	$C_{UL} = 25\%$	
Based on UTS	27,300 psi @ 90%	20,000 psi	1.12 (87% < 5)	25,600 psi
Based on F.T.	18,800 psi @ 90% (5,640 psi-in)	14,000 psi @ 90% (4,200 psi-in)	.352 (64% < 5)	15,200 (4,570 psi-in)

(TS1)

Table VIII - Probability of failure and mean load for Beryllium reflector outer web using irradiated properties at 230°R. The 2.3 σ loads are matched at the $\bar{S}_{TL}$ for a C_{TL} value of 25%.

	$\bar{U}_L$	$P / 10,000 + (R)$
Based on UTS	16,250 psi	.04 @ 90% (.95)
Based on F.T.	9,650 psi (2,890 psi ...)	.001 @ 90% (.97)