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CORNELL RELATIVISTIC ELECTRON COIL EXPERIMENT (RECE)

(Progress Report)

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I. SUMMARY

During the first half of 1973, the Cornell Relativistic Electron Coil program has made significant progress and achieved very encouraging results in a number of areas.

Measurements on the smaller RECE-Berta facility indicate that strong relativistic electron rings can withstand ("self-heal") considerably larger irregularities in the externally applied magnetic field than had been assumed in the past. At the same time, energy measurements on the trapped electrons showed that the energy losses of these electrons are dominated by classical collisions with the background gas and do not seem seriously enhanced by collective effects. Similarly, a more detailed theoretical analysis of the absolute decay rates of our rings done in this period reaffirms that the decay of our rings can be fully explained by classical collisions without anomalous effects.

In another small pilot facility, a different injection scheme - injection of the electron beams through a magnetic cusp - was successfully tested and led to the generation of field-reversing electron rings. These rings exhibited life times of up to 9 μ sec and general characteristics very similar to those observed on the RECE-Berta rings.

The acceptance tests for the new 5-MeV electron accelerator were successfully completed and the machine in operation. So far, excellent operational and maintenance characteristics were observed, and no problems were encountered.

The new RECE-Christa facility was put into operation and experiments on it started. Some unexpected problems were encountered in the propagation of the new high-energy beams through the beam injector. However, first trapping experiments still led to the generation of almost-field-reversing electron rings. The general characteristics of these rings again are very similar to those observed in the other machines. However, life times increased to up to 125 μsec , i.e. about five times over those found in RECE-Berta. Very encouragingly, these life times agree well with extrapolations of the RECE-Berta results based on a purely collisional scaling.

II. RECE-BERTA EXPERIMENTS

In the last report, the smaller RECE-BERTA facility and the first experimental results obtained from it were described indicating the generation and containment of field-reversing torus shaped electron rings. The initial decay of these rings had been found to scale very closely according to the decay to be expected from the "classical" scattering of the fast electrons in the background gas. Only, after the layer strength had decayed to rather low levels, a more rapid final "dump" set in, exhibiting characteristics similar to the "precessional dump" found in the Livermore Astron experiment.

In the present reporting period, the emphasis of the measurements on RECE-BERTA was directed towards more detailed explorations of the stability of the electron rings against perturbations of the applied magnetic field and towards a determination of the energy and the energy losses of the fast electrons during the lifetime of the layer. First results of these measurements were reported at the annual Spring Meeting of the American Physical Society at Washington in April 1973 and are described in more detail below.

The perturbation measurements indicate that considerably larger asymmetries in the strength of the applied magnetic field along the electron torus seem to be tolerable than hitherto assumed, without creating a significant shortening of the ring life times. The rings are found to move in a way as to "self-heal" the external perturbations. The measurements of the electron energy show that the energy losses in the beam-plasma-neutral-gas system are dominated by simple "classical" energy losses due to collisions with the neutral gas, approximately balancing the energy gain of the electrons resulting from the betatron

type acceleration of these electrons in the decaying self field of the coils. Contributions from non-classical beam plasma interactions like two-stream instabilities etc. fortunately do not seem to add significantly to the energy losses of the fast electrons. This is in good agreement with the mentioned collisional decay of the rings.

In addition to these two series of measurements the analysis of the older ring decay measurements was completed. It appears that not only the scaling of the ring decay is as expected classically but that also the absolute decay times are in quite good agreement with this explanation. More details of these results are described in two publications which have been and are being submitted for publication and copies of which are included in the Appendix.

Furthermore, some more experiments have been performed on the "dump" of the electron torus. The new results support the old findings without particular new clues.

1. Stability of Electron Rings Against Field Errors

In the last report, results of preliminary measurements were reported which gave first indications of a rather good stability of the electron rings in RECE-Berta against perturbations in the magnetic field resulting from a canting of the downstream mirror coil. During this report period, additional experiments were started to obtain a more detailed and quantitative picture of the stability and general behavior of trapped electron rings under the influence of a number of field perturbations. First results of these measurements were reported at the recent Spring Meeting of the American Physical Society at Washington, D. C. Additional experiments will be performed in the coming months.

a. Vertical Perturbation Coil

So far, most data have been obtained for the field configuration shown in Fig. 1. In addition to the normal RECE-Berta arrangement already described in the last report, a vertical perturbation coil was positioned above the minimum of the basic magnetic field configuration, i.e. above the normal equilibrium position of the trapped electron rings. This perturbation coil was energized in series with the basic field coils. The direction of the resulting magnetic field at the position of the trapped electron rings was either parallel ("+") or antiparallel ("-") to the basic mirror field. An example of the magnetic field strength along the vertical radius connecting the tank axis and the center of the perturbation coil is shown in Fig. 2. Due to the variation of the strength this perturbation field over the tank cross section, the rotational symmetry of the basic field was destroyed and a field gradient in vertical direction was introduced. An example of the resulting total field strength dependence (in units of the base field) along the mentioned radius line is shown in Fig. 3 for various strengths and directions of the perturbation

FIGURE 1. Experimental Arrangement for the Measurements with Vertical Perturbation Coil

FIGURE 2. Strength of perturbing field along vertical radius between tank axis and perturbation coil.

FIGURE 3. Vertical distribution of the total external magnetic field strength

coil. For +100 windings, the symmetric distribution of the basic mirror field ("0 Turns") is changed into a monotonic decrease of field strength from the upper to the lower sections of the tank. For opposite direction of the coil, generally the magnetic field still exhibits a maximum within the tank but decreases increasingly faster towards the upper tank wall.

Examples of the results are shown in Fig. 4 for three different cases. The exhibited recordings have been obtained from two probes, one at the tank wall above the tank axis (upper traces), one at the wall below the tank axis (lower traces). Both probes were positioned axially approximately at the position of the trapped rings. As shown in Fig. 4 no really serious changes in the decay of the rings seems to occur between +90 and -20 perturbation turns, though the decay slopes appear to increase somewhat in the +90 case. (The existing difference in total life time is mostly due to differences in initial ring strength.) However, recordings of the upper probe appear to be smaller than the signal of the lower probe in the case +90, indicating a shift of the trapped electron coils towards the lower half of the tank. In the case -20, the situation seems reversed, i.e. the ring appears more in the upper half of the tank. From Fig. 5 which plots the ratio of the two probe signals for various situations, this vertical shift in ring position becomes even clearer. In the cases +70 and +90, the center of the trapped rings is initially positioned below the tank axis and moves even further down during the decay of the rings. From more recent calibration measurements with a dummy coil, it is concluded that the total shift of the ring center amounts to about 2-3 cm. The case -20 appears roughly identical with the case of no perturbation. In both cases, the rings appear to move slightly upward (about 1 cm) during their life time. On the other hand, the rings generally decay very rapidly when a larger negative perturbation is applied (≥ 50).

VERTICAL PERTURBATION COIL SHOTS 10.

OFF AXIS PROBES ABOVE & BELOW ELECTRON RING

PARALLEL
+ 90 turns
(+90 P)

NO COIL

ANTIPARALLEL
-20 turns
(-20 P)

Fig. 4. Sample recordings for vertical perturbation coil.

FIGURE 5. Signal ratios for upper and lower probes with vertical perturbation coil.

Longtime confinement in this case is obtained only when the perturbation coil is moved further away from the tank. A similar behavior though more pronounced is indicated from Fig. 6 for the case of added toroidal field B_0 .

The described results can be understood roughly as follows. When a vertical magnetic field gradient is introduced, the rings move into the regions of small field until the wall stabilizes them against further displacement. From a rough analysis of the data (in agreement with theoretical arguments) it appears that a new equilibrium position is established when the sum to the externally applied fields and the fields resulting from the image currents in the wall is roughly equal on both sides of the ring. Due to the decrease of the strength of the image currents the ring position then shifts during the ring decay further towards the wall. At the same time, the corresponding reduction in effectively available radial space leads to an increase in the decay rate. However, these increases in the decay rate become significant only for assymetries of the order of 1% of the base field, which is considerably larger than it was assumed possible so far.

The mentioned difference in ring reaction to large positive and large negative perturbations can be explained from Fig. 3. For positive perturbations, the average field gradient remains approximately constant when the rings move downward. For comparable negative perturbations, on the other hand, the resulting field gradients generally are somewhat larger, and probably more important, they increase when the rings move towards the upper wall. Thus, the rings are moved even further and the decay rate is seriously increased. This effect is basically a result of the nonlinearity of the perturbation fields along the vertical radius, and therefore, reduced when the coil is moved away from the tank.

FIGURE 6. Signal ratios with vertical perturbation field and toroidal field B_θ .

More detailed measurements on the physics of these ring displacements are in progress.

b. Horizontal Perturbation Coil

Similarly, first investigations were performed using the horizontal orientation of the perturbing coil indicated in Fig. 7. In this case, the vertical variation of the perturbing field strength (see Fig. 8) is similar as in Fig. 3 though its actual values are larger for equal number of turns. However, the perturbing field is directed perpendicular to the basic mirror field. Some resulting recordings of the same probes used in the earlier measurements are shown in Fig. 9. Again significant reductions of the life time seem to occur only when the perturbing field strength becomes about 1/2% of the base field. The rather abrupt change between the cases of 20 turns and 30 turns may indicate the onset of some instability. Additional measurements indicate that the equilibrium positions of the electron rings may somewhat canted for perturbations smaller than this limit. More detailed measurements on these points are planned.

FIGURE 7. Position of horizontal perturbation coil.

FIGURE 8.. Strength of perturbing field from horizontal coil along vertical tank radius.

HORIZONTAL PERTURBATION COIL SHOTS

17.

OFF AXIS PROBES ABOVE & BELOW

ELECTRON RING

NO COIL

20 TURNS

30 TURNS

Fig. 9. Sample recordings of magnetic probes at upper and lower tank wall with horizontal perturbation coil.

2. Energy Measurements of the Trapped Electrons

The experiments described in this section are aimed at measuring the energy of the relativistic electrons trapped in RECE-Berta. Measurements of this type are important both with respect to a comparison of the absolute values of the decay rates during the collisional decay of the rings with theoretical predictions and with respect to possible non-classical interactions and energy losses of the fast electrons in the surrounding background plasma which have been predicted by some theorists.

So far, the claim of a collisional decay of the electron rings mainly rested on the pressure dependence of the decay rates, and more important, on their dependence on the atomic number Z of the background gas. These dependencies are rather independent of the actual energy of the trapped electrons, as long as it is in the 100 keV range or above. However, for a more direct comparison of the absolute decay rates with theoretical predictions, the energy of the electrons has to be known. In our earlier rough comparisons, it had been assumed that the energy of the trapped electrons equalled the maximum of the measured diode voltage. However, it had not actually been shown that these most-energetic electrons were the ones that were trapped.

On the other hand, measurements of the time dependence of the electron energies can give indications as to the energy losses of the fast electrons in the background plasma.

First results of these measurements which are described below were reported in the recent Spring Meeting of the American Physical Society at Washington (copy of the abstract is included in the Appendix). They

indicate that the energy of the trapped electrons generally agrees to within 10-20% with the measured maximum of the diode voltage, and that no serious non-classical energy losses of the electrons seem to exist.

a. Experimental Arrangements:

The X-ray energy was to be measured by X-ray absorption techniques. For this purpose the experimental arrangement shown in Fig. 10 was constructed. A gold-coated tungsten wire, .0007" in diameter, was stretched across the RECE-Berta vacuum tank, radius at an axial position close to the normal center of the electron rings. The bremsstrahlungs X-rays generated by the impinging fast electrons were observed in forward direction by three separate NaI-scintillator photomultiplier combinations. For background suppression, these detectors were heavily shielded with 5-10 cm of lead. Collimator channels in the front face of this shield permitted the detectors to "see" a 4"-long span of the wire around the half-radius point of the tank. For a further reduction of the X-ray signals resulting from electrons impinging upon the tank wall, an additional $\frac{1}{4}$ " lead shield covered the field windings leaving open only a $\frac{1}{4}$ " wide slit at the wire position. After calibration measurements, the apertures of the three detectors were covered with various absorbers, and thus, the average absorption of the X-rays determined for each shot.

b. Experimental Procedures:

First, with the wire in place and with all three detector apertures covered with identical absorbers (either .15" of Sn or 2x.08" of lead),

Fig. 10. Experimental arrangement for energy measurements.

the output signals of the three detectors were equalized by adjusting the respective multiplier voltages. Thereafter, the three detector apertures were covered with different lead absorbers (1, 2, or 3 times 2.50 g/cm^2 of lead) in addition to the original Sn absorber, and thus, the time dependence of the absorption determined. To check what percentage of the recordings was due to electrons impinging on the tungsten wire, some shots were taken with the wire removed. Examples of the recordings in both cases are given in Fig. 11. The recordings with the wire in place show the increasing absorption of the X-rays in the lead. The noise apparent on the X-ray detector recordings is due mostly to statistical fluctuation of the low X-ray intensity. The electrical noise is comparatively small as evident from the recordings with the wire out. The absence of significant readings in the latter recordings indicate also that most of the X-rays recorded with the wire in position are generated at the wire, i.e. by electrons which are still trapped. For a determination of the absorption ratios at a given time, the recordings of the photomultiplier detectors were smoothened over 2-3 μsec and the remaining electric noise contributions subtracted.

c. Evaluations and Results:

To deduce the energy of the trapped electrons from the measured absorption ratios, it was first assumed that the energy spectrum of these electrons is approximately monoenergetic. Then, for a few energies, the thin-target bremsstrahlung spectrum of fast electrons was multiplied with transmission ratios for the various absorbers shown in Fig. 12. (The same figure shows also as example the bremsstrahlung spectrum used

TYPICAL X-RAY DATA

WIRE OUT

PM 1

PM 2

PM 3

ELECTRON RING
B FIELD PROBES

Fig. 11. Sample traces from
energy measurements.

WIRE IN

X-RAY TRANSMISSION FROM TUNGSTEN WIRE TO NaI CRYSTALS WITH THE ADDITION OF ABSORBERS.

$$\text{Sn} = 1.91 \text{ g}\cdot\text{cm}^{-2}$$

$$\text{Pb} = 2.50 \text{ g}\cdot\text{cm}^{-2}$$

for an electron energy of 500 keV.) The resulting transmitted spectra then were multiplied with the energy dependent detection efficiency of the NaI-scintillator crystals, and the overall detector response was calculated by integration. The resulting signal ratio curves for various absorber combinations are shown in Fig. 13.

Using these curves, the experimentally observed absorption ratios were converted to corresponding electron energies. Examples of the resulting time dependence of the electron energies as obtained from measurements with one and two lead absorbers are shown in Figs. 14 and 15 for a few shots. The fluctuations of these curves in time reflect the influence of the remainders of the mentioned statistical intensity fluctuations in the X-ray signals.

All of these curves indicate initial electron energies roughly between 400-450 keV. This result agrees quite reasonably with the measured peaks in the diode voltage of 450-500 kV, i.e. the electrons seem to be trapped mostly during the peak of the accelerating voltage, and they do not appear to lose a significant amount of their energy in the process.

Furthermore, the measured electron energies do not appear to change significantly in time. This fact agrees with the observed constancy-in-time of the average ring radii mentioned in our last report. It can be understood as an approximate balancing of the collisional energy losses of the fast electrons in the background hydrogen gas and the betatron-type energy gains of the electrons due to the decay of the negative magnetic self-field flux within the electron coil. At 400 mTorr hydrogen at which most of these measurements were performed, the collisional energy loss amounts to about 5 keV/ μ sec. On the other hand,

Fig. 13

DETECTOR SIGNAL RATIOS AS A
FUNCTION OF ELECTRON ENERGY AND
ABSORBER.

Fig. 14. Time dependence of electron energies in ring.

Fig. 15

the measured decay rates of the magnetic field changes on the tank axis infer betatron-type electron energy gains of 6-8 keV/ μ sec for infinitely long and infinitely thin electron layers of the measured radii. Energy gains for rings of finite length and significant thickness are estimated somewhat smaller. Thus, non-classical energy losses of the fast electrons certainly are limited to a few keV/ μ sec, i.e. to less than the classical losses.

The limit for the energy losses due to collective beam-plasma interactions is satisfying. The collision rate for the slow plasma electrons is quite high under present conditions. However, from known ionization cross sections for fast electrons in hydrogen and from known momentum-transfer cross-sections for slow electrons, it is estimated that the product $\omega_p \tau$ of the plasma frequency ω_p of the slow electrons and their collision time τ reaches up to 50-200. Since, at the same time, the ratio of fast electron density n_b to plasma density n_p is around 10^{-2} - 10^{-3} , the collision frequency $1/\tau$ for the slow electrons is still quite comparable or even smaller than the fastest predicted growth rates for collective two-stream interactions, and the mentioned experimental result may indicate that these interactions may not be quite as strong as sometimes assumed. The same question now has to be investigated in RECE-Christa where lower collision rates for the slow electrons hopefully are achievable.

So far, a monoenergetic electron spectrum had been assumed in all evaluations. Additional information of the actual width of the spectrum in principle can be obtained from other simultaneously measured absorption ratios, e.g. from the ratio of the signals for one and three lead

absorbers are similar. Some first experiments in this direction were tried, but fluctuations in this case become quite large. From preliminary data, it appears possible that some spread in the spectrum may be present, but these data still are very uncertain.

Overall, the mentioned experiments and results only reflect the first stage of the energy measurements. More detailed investigations including a higher collection efficiency and thus smaller statistical fluctuations are in preparation and should be finished over the coming 3-4 months.

3. Analysis of Collisional Decay of Electron Rings

In our last report, we described investigations of the dependence of the negative slope of the magnetic probe signals on the background hydrogen pressure and on the pressure of high-Z admixtures. The observed linear dependences of these decay rates with the pressures and with the high-energy multiple scattering cross section of the various gases was interpreted as an indication that the ring decay is dominated by purely classical collisions of the fast electrons in the background gas. During this report period we now tried to check if also the absolute values of these decay rates agreed with this interpretation. For this purpose, first, a theoretical analysis of the diffusional decay of geometrically idealized electron rings was performed, and then, after some adaptation of these predictions to the existing ring geometry, they were compared with the experimental findings. At present this analysis is not fully completed; but, the present results of this analysis which are contained in a completed theoretical paper and a first draft of our experimental paper (see Appendix) show quite reasonable agreement, thus again supporting the conclusion of a mostly classical decay of our electron rings.

Details of the mentioned new theoretical treatment of the collisional diffusion of an idealized self-compressed ring of relativistic electrons is described in a forthcoming publication, "Diffusion of Strong Stable E-Layers" by Dr. S. Humphries. The paper has been submitted for publication in the Physics of Fluids and a copy of it is enclosed in the Appendix. Based on earlier work of Dr. Humphries on the diffusional decay of the weak electron rings, the paper treats the decay of strong

electron rings for the cases, 1) when a strong toroidal magnetic field $B_\theta \gg B_z$ is applied, and ii) when no such field exists and the confining field B_z can be considered constant over the dimensions of the rings. In addition, the paper considers the corresponding self-consistent fast-electron density distributions in the rings and the relation between the energy losses of the electrons by collisions and related diffusional decay of these rings.

In both cases, in particular in case ii) which is closest to the experimental situation, a circular small cross section of the ring is assumed with a minor radius $a \ll R$ (R = major radius of the ring). Furthermore, the assumption of a constant current density within the ring leads to decoupling of the motions of the fast electrons around the center line of the ring in z -direction (axially) and r -direction (in the direction of the major radius). The extension of the rings in z - and r -direction is to be limited by material objects absorbing all fast electrons impinging on it. These objects are assumed to be positioned at distances δz and δr from the circular center line of the ring, with $\delta z \approx \delta r \ll R$. The energy of the electrons is assumed constant.

Under these assumptions, the distribution functions in z - and r -direction can be separated into various "modes" which decay independently of each other. The "lowest-order mode" is the one that decays slowest and, therefore, will generally dominate most of the observed decay. Neglecting all high-order modes, i.e. assuming that they already have decayed away when the observations are made, the relative distribution functions will be constant in time, and the total number $N_T(t)$ of

fast electrons contained in the ring will decrease linearly in time according to

$$N_T(t) = N_T(0) - \frac{N_T(0) (2.405)^2 r_g^2 D}{\xi(0) \delta z} t \quad (1)$$

where $N_T(0)$ is the number of electrons at time $t = 0$; $r_g = R$ is the Larmor radius of the fast electrons in the applied external magnetic field B_0 ; the "strength parameter" $\xi(0) = 4I/acB_0$ (I = total ring current, c = velocity of light). The diffusion coefficient D is given by

$$D = \frac{16\pi c n_i}{\gamma^2 \beta^3} \sum_i U_i Z_i (Z_i + 1) r_e^2 e^{\ln \frac{b_{\max,i}}{b_{\min,i}}} \quad (2)$$

where n_i = density of i^{th} neutral gas species, Z_i = its atomic number, $r_e = 2.8 \times 10^{-13}$ cm = natural electron radius, $\beta = v/c$, $\gamma = (1 - \beta^2)^{-1/2}$, $b_{\max,i}$ = shielding distance of the neutral atom species, $b_{\min,i}$ = minimum impact parameter important for scattering from the same species.

The linear decay of the ring strength required by Eq. (1) has been observed experimentally, as described in our last report. Also, the dependence on the pressure and composition of the background gas as it was checked and described in the last report is contained in the summation term of Eq. (2) exactly as it was observed. This dependence is independent of the exact theoretical treatment and of the shape, etc. of the rings. However, for a further comparison of the absolute values of the decay rates, the exact geometry of the rings and of the limiting boundaries are important and have to be taken into account. Unfortunately, as mentioned before, the theoretical treatment could be developed

only for an idealized case which does not agree directly with the experimentally present parameters.

Therefore, the theoretical results have to be adapted to the experiment. Some details of this adaptation are to be described in a forthcoming paper reporting on our experimental results, "Observation of Stable Equilibrium and Classical Diffusion of Field Reversing Relativistic Electron Coils", by D. A. Phelps et al., a draft copy of which is included in the Appendix. In brief, the most serious difference between the given experimental parameters and the theoretical assumptions is that the length and -- presumably -- the thickness of the experimentally observed rings are quite comparable with or even larger than the observed radii, and not, as assumed, small against them. Similarly important is the fact that the rings do not have a definite material limitation in axial direction, but can and do expand axially during their decay.

For a modification of the final formulae (1) and (2), the first discrepancy mainly concerns the effective initial strength parameter $\xi(0)$ which contains the assumed minor radius a . Correspondingly, it appears necessary to adapt this parameter to the conditions present in the experiment. Unfortunately, the theoretical development does not give a clear indication how that adaptation has to be done, and a few recipes can be considered the results of which may disagree roughly by factors 1-1.5. The most reasonable way seemed to us to retain the physical meaning of this parameter in our adaptation. In its original definition for $a \ll R$, $\xi(0) = 4I(0)/acB_0$, this parameter was equal to the initial (in time) difference of the magnetic field at the outer and

the inner perimeter of the ring divided by the vacuum field strength B_0 , i.e. $\xi(0) = (B_{out} - B_{in})/B_0$, or proportional to the maximum current density per cm axial length. In this respect the original definition of $\xi(0)$ is equal to the definition of the same parameter used at Livermore. The value of this parameter can then be directly calculated for a given ring configuration and strength from the machine code mentioned in the last report.

Similarly, for a given ring geometry, the magnetic field changes $\delta B_A(t)$ measured on the tank axis are proportional to the number of electrons $N_T(t)$ in the ring. Thus Eq. (1) can be directly rewritten in the form

$$\delta B_A(t) = \delta B_A(0) - \delta B_A(0) \frac{2.405^2 r_D}{\xi(0) \delta z} t \quad (3)$$

from which the experimentally determined slopes $d\delta B_A/dt$ can be calculated:

$$d\delta B_A/dt = (\delta B_A(0)/\xi(0)) (2.405^2 r_D/\delta z) \quad (4)$$

In that formula (4), the factor $(\delta B_A(0)/\xi(0))$ is only geometry-, but not strength-dependent. Therefore, the experimentally determined slopes should be roughly independent of the initial ring strength, as it was actually observed, and this factor can be calculated from the experimentally observed geometry. The results of this calculation performed for various assumptions on the radial density distribution of the rings agree quite closely.

Another adaptation has to be performed concerning δz . As mentioned before, no actual axial material limiter existed close to the rings in the experiment. However, also the decoupling of the axial and radial motion of the electrons as it was assumed in the theory does not appear sufficiently satisfied under the experimental conditions, and most of the electrons seem to get lost on the wall instead of (as Humphries concludes) axially. Correspondingly, it appears reasonable to take δz equal to the difference between wall radius and measured ring radius, i.e. $\delta z \approx 10\text{cm}$. Similarly, δz could be taken equal to one-half of the measured full axial length of the rings, i.e. about 10-15 cm.

The corresponding numerical analysis is not yet fully complete. But, preliminary results indicate that the magnetic field decay rates calculated under these assumptions ($\delta z = 12\text{ cm}$) appear smaller than the experimentally determined rates by a factor of about 1.5. A discrepancy of this size does not appear significant, in view of the mentioned uncertainties in the adaptation of Eq. (1) to the actual experimental conditions and the uncertainties in the determination of the relevant experimental parameters. Furthermore, the axial length of the experimental rings is increasing during the decay, as mentioned before, and this leads to a more rapid decrease of the measured magnetic probe signals. From a preliminary analysis, it seems that this effect can roughly account for an enhancement factor of about 1.5 of the decay rates. Thus, the agreement between the experimentally measured decay rates and the theoretical predictions appears quite reasonable and encouraging, and certainly within an overall uncertainty of roughly a factor of 2.

III. GENERATION OF ELECTRON RINGS BY CUSP INJECTION

At the end of the last report period, a small program to test other injection schemes for the production of strong relativistic electron coils was started. As first example, a small experiment in which the electrons were to be injected into a magnetic mirror through a magnetic cusp field was assembled. The experiments done on this machine during this report period were very successful showing the generation and actual trapping of fully field-reversing electron rings with $\delta B/B_0 \lesssim 125\%$ on axis and a total life-time of up to 10 μsec . The electron coils thus produced were very similar in behavior to the ones produced in RECE-Berta. Details of this experiment are described in a paper, "Generation of Field-Reversing Electron Rings by Neutralized-Cusp Injection into a Magnetic Mirror Trap", by R. E. Kribel et al. which has already been accepted for publication by Plasma Physics. A copy of this paper is included in the Appendix. Thus, a brief description in the following paragraphs will suffice.

The experimental arrangement is described in Fig. 16. The electron beam (nominally 450 keV and 10-20 kA) generated by the water Blumlein normally used in our RECE-Berta experiments is injected through a magnetic cusp consisting of two homogenous magnetic field regions opposing each other. It is well known that electrons entering the cusp with a velocity directed parallel to the first set of field lines will be deflected by going through the actual cusp region (in which the field lines are directed radially) to convert part or all of their parallel velocity component into velocity perpendicular to the homogeneous

Fig. 16 Experimental arrangement for cusp-injection experiments

magnetic field in the second section of the cusp. A small magnetic mirror then is expected to trap all or part of the electrons which have lost most of their parallel velocity.

The cross sectional drawing of Fig. 16 shows all essential parts. The accelerator diode (cathode-anode) is extended into the homogeneous field region. The magnetic cusp is made of two opposing field coils (on both sides of the iron ring), and the magnetic mirror is produced by additional moveable mirror coils. As in RECE-Berta, a conducting wall is provided and the entire cusp and trap region is filled with several hundred μ Torr of hydrogen. Magnetic field changes generated by trapped electron coils are recorded by magnetic probes positioned at the mirror center on the axis and at the wall.

Various magnetic field configurations were used (see Fig. 17). The best results so far have been obtained using configuration II of that figure. After some experimenting using this configuration and hydrogen pressures of about .9 Torr, magnetic probe recordings of the type shown in Fig. 18 (295 gauss/div., 1 μ sec/div.) were obtained. As mentioned before, these recordings are quite similar to the ones obtained from RECE-Berta. In particular, they show again the relatively slow linear decay preceding a faster final "dump" when the ring strength has decayed to rather small values. Again, no gross instabilities are apparent for strong rings. Extrapolating the linear decay curve, the initial field changes generated by the trapped electron ring at the axis are measured to be about 125% of the externally applied field, i.e. again clear field reversal has been achieved. Additional

Fig. 17. Field configurations used for successful trapping experiments

Fig. 18. Magnetic probe recording in cusp-injection
experiment showing field reversal

measurements with high-Z gas admixtures showed, as in RECE-Berta, that thin rings actually resulted from trapped fast electrons. This was the first time that this aim had been reached in a machine other than RECE-Berta.

Further experiments on this apparatus are planned if time permits to test possible advantages or disadvantages of this injection scheme with respect to trapping efficiency and reproducibility. In particular, it is intended to use this machine in the coming month for the first pilot experiments to test beam injection and trapping in a highly ionized plasma.

IV. COMPLETION OF NEW 5-MEV ACCELERATOR

1. Completion of Tests and Acceptance of Accelerator

As described in our last report, the basic tests of the Marx generator part of the new accelerator had been essentially completed with success, but voltage tests on the Blumlein section had led to an electric break within one of the polyethelyne support rolls of the middle section. This section was scheduled for replacement by the end of January.

Therefore, at the beginning of this report period, first the testing of the Marx generator section was completed, with the damaged Blumlein replaced by a dummy load. These tests leading to successful full voltage runs were performed without significant problems or delays, and the test results were as expected.

At the end of January, the Blumlein testing was resumed. However an additional breakdown of the replaced Blumlein support leg was encountered at a voltage level of around 3 MeV. Detailed analysis of the broken legs showed in both cases a bulk breakdown within the material itself. Further testing revealed some manufacturing faults of the subcontractor like voids and even small metal parts in the material. New parts were manufactured under very close supervision by Maxwell personnel, and the new material has given no problems since.

After this second replacement of the Blumlein supports, the basic tests of the coupled Marx-Blumlein section, i.e. with the diode replaced by a CuSO_4 dummy resistor proceeded with only minor problems to the end of February.

At this time, the official tests agreed to earlier (see last report) were started. The first series of 90 shots at full voltage into a CuSO_4 dummy load of 35Ω instead of the diode were concluded without hitch. The RMS scatter of the recorded diode voltage maximum was about 3.3%, i.e. clearly less than the required 4%.

In the ring-through tests, the sequence of shots was reversed from the original arrangement, i.e. first, with a Blumlein switch setting corresponding to an output voltage of 5 MeV, the charging voltage was reduced in steps from the maximum 80 Kv. This was to be done until the switch break occurred after the maximum of the Marx voltage. This condition resulted when the charging voltage was reduced to about 40 kV, corresponding to a normal diode voltage of 2.5 MV. Thus, automatically also the first part of this section of tests was successfully concluded. No problems were encountered in these shots. The fact that the Blumlein switch was able to break even when only one half of the necessary normal charging voltage was applied is very encouraging. It means that the accelerator will be able to stand without damage relatively large operational deviations, including possible misadjustments or machine caused prefires.

The two short-circuit shots at full voltage also were completed without problems. The short-circuit consisted of a copper braid connecting the inner and outer Blumlein cylinders. Thus, the effective inductance of the short was identical to that present in the case of diode shorting.

The fourth group of tests concerned the voltage insulating capability of the diode envelope at full voltage. For this purpose, the

diode including its envelope and an experimental cathode and anode assembly was installed. The generated electron beam was to be stopped in a carbon block. In these experiments first some irregularities occurred: i) X-ray dosimeters monitoring the integrated hard X-ray intensity in forward direction indicated only a very small X-ray output (less than 1/50 of what was expected from theoretical calculations, by Maxwell, and from interpolations of results from other machines, from Sandia Corp.); ii) the recorded voltage and current traces showed irregular spikes. The exact origin and final significance of these findings was not directly clear. At that point, further testing was halted by Maxwell. After a two-week delay during which Maxwell performed some computer investigations of the earlier results, the tests were resumed on March 31, with the result that the diode shorting indicated in the earlier test could be traced clearly to a simple "gap closure", i.e., a shorting of the anode-cathode gap during the electron emission, which could be avoided by proper changes in the cathode geometry. The diode envelope very clearly was able to withstand the applied voltage of over 5 MV. In the same shots, diode currents of over 147 kA were measured. A carbon calorimeter which intersected most of the electron beam indicated total beam energies of over 25 kJoule which agrees well with what was expected. X-ray dosimeters showed a maximum integrated dose of 15 radm^2 per shot in forward direction when the beam was intersected by a carbon block. This value was still somewhat smaller than what we had expected, but could be shown to be realistic.

Due to the full and successful completion of all tests agreed upon, the final acceptance of the new accelerator was signed on April 1.

2. X-ray Dose Measurements in Personnel Areas

After the installation of the vacuum diode when electron beams were produced for the first time, numerous X-ray dose measurements were performed using various types of dosimeters. Except in the direct vicinity of the diode (see above), i.e. in the so-called "Pit" or target area where the RECE-Christa machine stands and which is surrounded by concrete walls, no X-ray doses serious for the human body were observed. A number of operational and apparative safety precautions (see below) are to prevent that people can be in that target area during a machine firing. All other areas where personnel may be found regularly have been monitored very carefully since the initial beam operation. At no time and no such position have the measured dose rates exceeded more than 1/20 of the legally acceptable level. Therefore, X-ray dose rates in personnel areas are expected to be easily within safe limits even under heavy machine usage conditions.

3. Added Safety Features and Peripherals

Since the acceptance of the accelerator, emphasis was given to the completion of the safety system in particular to prevent machine firing with personnel present in the target area. For this purpose, the built-in interlock system was expanded to include a positive "go-ahead button" at the entrance to the target area which has to be pushed in order to make machine firing possible. Thus, the machine operator

is forced to check the target area for remaining personnel, before closing the interlocked entrance gate. For additional safety, four more "panic buttons" which when pushed prevent and stop any machine operation are situated easily accessible in the dangerous target area. A loud horn signal is sounded during the full charging and firing cycle of the machine. In addition, a series of green ("safe"), yellow ("ready to operate") and red ("dangerous, machine charging") signal lamps between operating area and target area indicate the present status of the interlock system. In addition, of course, every person has his personal dosimeter which is read at least once per month.

An oil fill switch was introduced in the Marx tank to prevent oil overflow during the filling of the tank. A removable roof on the Marx tank is to prevent any accidental connection (crane cables, etc.) of the Marx inside with the outside. It also shields the Marx tank from dust, etc.

An insurance approved sprinkler system around the accelerator is scheduled for installation in August.

The recent completion of the oil tank housing outside the laboratory building eliminates all access of outsiders to the oil storage and pumping area.

4. Operation of the Accelerator since Acceptance

Since acceptance of the accelerator from Maxwell, Inc., approximately 300 shots were fired at various voltage levels, mostly around 3-4 MV,

but including some shots at full voltage. So far no misoperations or breakdowns occurred, except a very few prefires which mainly resulted from insufficient purging of the Marx gaps.

V. START OF RECE-CHRISTA EXPERIMENTS

After a short familiarization phase, experiments with the new accelerator started beginning in April. Initially beam propagation studies had to be performed. As described below, the task to generate and propagate these high-energy beams reproducibly into our tank proved more difficult than we had expected, and the investigations are still going on. In spite of these difficulties, first injection experiments into the RECE-Christa tank were very successfully performed. After some experimenting, trapping of sizable electron rings was achieved. These rings exhibited a behavior very similar to that known from the RECE-Berta and from the cusp-injection experiments, i.e. after a short transition phase of a few μsec , a rather smooth and almost linear decay was observed leading to a rather rapid dump after the ring strength had decayed to small values. So far, axial magnetic field changes close to the reversal point and total lifetimes of up to 125 μsec , i.e. clearly above the 100 μsec mark, have been achieved. These lifetimes agree quite reasonably with the values expected from a scaling of the RECE-Berta lifetimes.

1. Generation and Propagation of High-Energy Beams

After familiarization with the accelerator, experiments on the generation and propagation of these high-energy electron beams were started. The first group of measurements concentrated on the generation of sufficiently strong electron beams with a diameter of around 2-3 inches. This target diameter was chosen since a diameter of this size

appeared optimal as diameter for the injector tube into the confinement tank.

For this purpose, a segmented graphite calorimeter was placed behind the anode foil. Its inner section had a diameter of 2.5"; the outer part consisted of a graphite ring with 2.5" ID and 6" OD. The investigated cathodes included 4" diameter hemisphere cathodes made of graphite, aluminum or stainless steel, and similar hemisphere cathodes with 1-2" diameter extensions at the tip. The diode voltages used were mostly between 3-4 MV with excursions down to 2 MV and up to 5 MV.

From the results, it appears that a good concentration of the electron beam into the 2" diameter can be achieved with the large hemisphere cathodes only at small cathode-anode spacings at which the diode tends to exhibit gap closure already before the end of the voltage pulse. Better results were obtained from the cathodes with 1-2" extensions at the tip. In this case, generally total electron energies of about 4-5kJ could be concentrated into the 2.5" center using diode voltages of about 3.5 MV.

In these shots, a serious cathode erosion problem became apparent. The graphite and aluminum cathodes exhibited serious cratering and deformations of the surface after a few shots. The stainless steel cathodes seemed melted and glazed at the surface after the first few shots; after a few more, it also became very irregular and pitted. Since then a number of other cathode materials were investigated, including molybdenum, tungsten, tantalum, heavy metal, elkenite, and tungsten sprayed cathodes. Of these materials, so far the best results

i.e. longest lifetimes, were achieved with tungsten sprayed cathodes. In this case, at least a few shots could be made without serious apparent damages of the surfaces. It is hoped that it will not be necessary to exchange these cathodes after every shot, but to be able to run a number of shots without braking the vacuum system. This would seriously enhance the speed of our experiments.

After these investigations, beam propagation experiments were started. For this purpose, the inner part of the calorimeter was removed, and drift tubes of 2" and 4" diameter were coupled to the diode. For beam guidance, these tubes were lined on the inside with a copper mesh which permitted photographic observation of the resulting beams. Both tubes were 6 ft. long approximately equal to the length of the transfer tube into vacuum tank. At the end of these tubes a calorimeter measured the total transmitted energy. An X-ray pinhole camera was positioned perpendicular to the tubes.

So far, a rather wide variety of diode parameters and hydrogen gas pressures in the drift tube (.1-10 Torr) were investigated. Fig. 19 gives examples of the optical recordings. The beam mostly exhibits spiralling or break-up, etc., and only in relatively few cases a good propagation of the beams was observed. Unfortunately, so far no parameter range has been found in which reproducibly a good beam propagation was obtained. In all cases there seemed to be a rather statistical switching between the various beam behaviors. Similarly, the calorimeter data show a large scatter and pure reproducibility, and even the correlation with the optical observations is not well established. The X-ray pinhole pictures generally exhibit a structure

Fig. 19. Examples of optical recordings of propagation of a 3.5 KeV-100kA beam in gases

similar to the optical pictures and a reasonable correlation prevails. At present, the origin of this pure reproducibility is not yet clear, and extensive further studies will have to be done.

2. First Trapping Experiments

In spite of our dissatisfaction with the results of our propagation studies, first preliminary beam injection experiments into the RECE-Christa confinement tank were started the end of May. As to be expected from the poor reproducibility of the beam propagation, also the injection studies showed large fluctuations. However, it was possible to separate out some experimental parameters which appear to be reasonably favorable for beam trapping, and the trapping of sizable rings was achieved in a few cases.

The experimental RECE-Christa tank was made operable together with the completion of the first experiments with the new accelerator. Its design had been described already in the last report. In the first preliminary trapping experiments, the tank was used without axial conductor and without wall resistors. The injection angle was approximately 10 deg. For charge neutralization, hydrogen pressures of .1 - 2 Torr were maintained by the mechanical pump. The base pressure of other gases varies from 5 - 15 mTorr. The magnetic field was maintained for a few seconds, so that all transients had disappeared. For diagnostics, 4 magnetic probes were placed at various positions at the wall. The absence of the axial conductor which was to increase the probability for finding suitable trapping conditions, did not allow

2

a direct determination of the axial magnetic field changes. But, these field changes could be roughly inferred from the field changes measured at the wall, using the ratios of these changes as they had been observed in good agreement both in RECE-Berta and in the cusp experiment ($\delta B_{\text{axis}}/\delta B_{\text{wall}} = 4 - 4.5$). An open-shutter camera observed the tank from its side.

As in the early RECE-Berta experiments, a wide variety of magnetic field configurations and pressures was investigated. In most cases, at best, short-time trapping ($< 20 \mu\text{sec}$) of weak electron layers was observed. The most successful magnetic field configuration (similar to the one mostly used presently in RECE-Berta) is shown in Fig. 20. In this case, several times, the trapping of electron rings with life times of the order of $60 - 80 \mu\text{sec}$ was observed. The characteristics of the probe recordings were quite similar to those observed in RECE-Berta and in the cusp. After a short initial transition period the recordings indicated a smooth decay of the rings until the ring strength had decreased to comparatively small values whereupon a faster "dump" set in. In two cases, the life time of the rings exceeded the $100 \mu\text{sec}$ mark. The best recordings observed so far are exhibited in Fig. 21 (two different time scales for the same probe recording), where a ring life time of $125 \mu\text{sec}$ is seen. The initial field change measured by the wall probe amounted to about 17 % of the externally applied vacuum field. From that value, an axial field change of at least 70 - 80 % can be inferred, and due to the slight displacement of the observed electron ring (see photograph) center from the magnetic probe position, the actual field change may even have been slightly higher. The comparable smallness of the recordings of the other probes, and also the photograph, again indicate an axial extension of the ring roughly equal to its diameter.

Fig. 20. Magnetic field configuration used in successful trapping experiments (see Fig. 21)

RECE-CHRISTA EXPERIMENT

SHOT N° 602
 $V_{\text{BANK}} = 52$ volts
 $P_{\text{TANK}} = 770$ microns
 $P_{\text{DRIFT TUBE}} = 2.5$ torr

VERTICAL: 30G/Div

SWEEPS:

UPPER: $5\mu\text{S}/\text{Div}$

LOWER: $20\mu\text{S}/\text{Div}$

SIDE VIEW OF
LAYER

Fig. 21. Recordings for largest trapped ring in RECE-Christa

The observed life time of the ring is quite comparable to the time expected from a scaling of the RECE-Berta times. In the latter case, typically life times of 16 - 20 μ sec are observed for field reversing rings using a hydrogen pressure of around 400 mTorr. If this time still is dominated in RECE-Christa by the scattering of the fast electrons, then it should scale (according to Equ. (2) from page 32) proportionally to $\gamma^2 \beta^3$. (The remaining factor is roughly identical in both cases.) The maximum voltage measured for the case of Fig. 21 was 3.6 MeV. Assuming that the energy of the trapped electrons corresponded to this voltage, a life time ratio of approximately 20 would have to be expected for the two cases, if identical gas pressures are used. However, the scattering of the fast electrons in the gas composition of Fig. 21 should correspond roughly to that in hydrogen of 1100 - 1200 mTorr pressure (770 mTorr hydrogen filling, and about 10 mTorr of unknown gases, most likely air or similar). Thus, the expected life time ratio shrinks to about 7, with an observed ratio of 5 - 6. The latter difference could easily be made up by differences in the assumed parameters and in geometry. Furthermore, the energy losses of the fast electrons in the in the hydrogen gas ^{are different} in the RECE-Berta experiments, this loss corresponds to about 15 - 20 % of the electron energy during the life time of the rings; in RECE-Christa this percentage increases to over 30 %.

Further experiments of this type will be performed after additional studies on beam generation and propagation.

VI. PUBLICATIONS AND PRESENTATIONS AT SCIENTIFIC MEETINGS

Presentations at Scientific Meetings

1. "Energy Changes of Relativistic Electrons Trapped in RECE-Berta", A.C. Smith, C.E. Swannack, D.A. Phelps, and H.H. Fleischmann, paper presented at the Spring Meeting of the American Physical Society, Washington, D.C., April 1973
2. "Stability of High-Current Electron Rings Trapped in RECE-Berta Against External Field Perturbations", R.A. Meger, D.A. Phelps, and H.H. Fleischmann, paper presented at the Spring Meeting of the American Physical Society, Washington, D.C., April 1973

Papers to be Published in Scientific Journals

1. "Generation of Field-Reversing Electron Rings by Neutralized-CUSP Injection into a Magnetic Mirror Trap", R.E. Kribel, K. Shinsky, D.A. Phelps, and H.H. Fleischmann, accepted for publication in PLASMA PHYSICS
2. "Use of Strong Relativistic Electron Rings for the Confinement of Thermonuclear Plasmas", H.H. Fleischmann, IEEE Trans. NS-20, June 1973
3. "Diffusion Theory of Strong Stable E Layers", S. Humphries, Jr., submitted for publication in Physics of Fluids
4. "Observation of Stable Equilibrium and Classical Diffusion of Field-Reversing Relativistic Electron Coils", D.A. Phelps, A.C. Smith, D.M. Woodall, and H.H. Fleischmann, to be submitted for publication in Physical Review.

VII. APPENDIX

for the Washington, DC Meeting of the

American Physical Society

23-26 April 1973

Date

Physical Review
Analytic Subject Index
Number 35

Bulletin Subject Heading
in which Paper should be placed
Plasma

Energy Changes of the Relativistic Electrons
Trapped in RECE-BERTA.* A.C. SMITH, JR., C.E. SWANNACK,
D.A. PHELPS, and H.H. FLEISCHMANN, Cornell U.--The energy
of relativistic electrons in the trapped electron rings of
RECE-BERTA¹ were determined by X-ray absorption spectroscopy.
For this purpose, a .0007" tungsten wire was positioned at the
axial fringes of the trapped rings. The time dependence of the
absorption spectrum of the resulting X-rays was determined at each
shot from the simultaneous recordings of three balanced NaI(Tl) detectors,
each with different absorber thicknesses. A calibration of the
absorption curves was obtained using a low current electron beam.
Preliminary results indicate energy losses which are mainly
determined by collisions with the neutral gas.

¹J. J. Bzura, J. T. Fessenden, H. H. Fleischmann, D. A. Phelps,
A. C. Smith, Jr., and D. M. Woodall, Phys. Rev. Lett. 29, 256 (1972).

*Work supported by the Empire State Atomic Development Associates, Inc., and by the Office of Naval Research.

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Physical Review
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Plasma

Stability of High-Current Electron Rings Trapped in RECE-BERTA Against External Field Perturbations.* R.A. MEGER, D.A. PHELPS, and H.H. FLEISCHMANN, Cornell U.--The influence of various perturbations of the external field on the geometry, decay and final dump of the electron rings trapped in the RECE-BERTA experiment¹ were investigated with and without the presence of poloidal magnetic fields. The perturbations included canting of plane of the downstream mirror, azimuthal assymetries of the poloidal magnetic field by dipole, quadrupole and octupole field components and small perturbations by localized magnetic dipoles. Preliminary results indicate a high degree of "self-healing" of the electron coils against rather strong perturbations.

¹J. J. Bzura, T. J. Fessenden, H. H. Fleischmann, D. A. Phelps, A. C. Smith, Jr., and D. M. Woodall, Phys. Rev. Lett. 29, 256 (1972).

*Work supported by the Empire State Atomic Development Associates, Inc., and by the Office of Naval Research.

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GENERATION OF FIELD-REVERSING ELECTRON RINGS BY
NEUTRALIZED-CUSP INJECTION INTO A MAGNETIC MIRROR TRAP⁺

By

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⁺This work was supported by the Empire State Atomic Development Association, the Office of Naval Research and in part by the United States Atomic Energy Commission.

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ABSTRACT

Reversal of the magnetic field on the axis of a magnetic mirror trap has been attained by injection of a high-current relativistic electron beam through a space-charge neutralized non-adiabatic cusp. A 10-20 kA -350 kV pulsed beam of electrons produced circulating electron rings with diamagnetic fields on the mirror axis up to 185% of the vacuum field and lifetimes up to 9.5 μ sec. The resulting electron rings exhibit characteristics similar to those generated by perpendicular injection.

I. Introduction

As proposed by Christofilos in 1958, the Astron experiment at Livermore was designed to trap cylindrical layers of enough relativistic electrons to reverse the magnetic field on the axis of a magnetic mirror by tangential injection and "stacking" of a series of 5 MeV-600 A pulses of electrons. To date, the largest layers in this machine resulted from single pulse injection and produced approximately 35% field suppression on axis. In another approach, using single-pulse tangential injection of a high-current electron beam, Andrews et. al. (1971) and Bzura et. al. (1972) reported the generation of field reversing electron rings lasting up to 20 μ s.

Friedman (1970), based on earlier work by Sinel'nikov and Akshanov (1963), injected high-current beams along the mirror axis. He used a non-adiabatic cusp to convert the parallel energy of the electrons into perpendicular energy and thereby transform the beam into a circulating electron ring. This injection technique could be important in that it admits a broader spread in axial velocities than the conventional tangential injection and may therefore lead to E-layers of longer axial length. However, the intensity of the circulating electron layer attained in Friedman's experiment was less than 35% of that necessary for axial field reversal. Due to the use of a foilless diode, the background pressure was 2×10^{-4} Torr and consequently there was no space charge neutralization. It is surmised that the limited transmission of electrons through the cusp was caused by the build up of a space charge in the cusp region as the electrons lost axial energy.

In this experiment space charge effects were virtually eliminated

by introducing a background of hydrogen gas to permit charge neutralization as the electron beam transits the cusp. As a result, a 10-20 kA pulsed beam of 350 keV and 60 nsec has produced circulating electron layers with diamagnetic fields on the mirror axis up to 185% of the vacuum field and lifetimes up to 9.5 μ sec. These electron rings exhibit characteristics very similar to those generated by perpendicular injection.

II. Experimental Arrangement

The magnetic cusp-mirror trap geometry is shown in Figure 1. An annular electron beam was produced in a diode consisting of a annularly shaped cathode (7 cm diameter 1.2 cm thickness) and a .025 mm thick Ti foil anode. This foil separated the high vacuum diode region from the cusp and trap regions in which the background pressure was varied from 0.2 - 2.0 Torr. A base magnetic field was produced by a solenoidal coil wound on the outside of the vacuum chamber. Various movable mirror coils were then added to produce the desired mirror field. The vacuum chamber was a 15 cm diameter lucite tube lined with a metal tube of 13 cm I. D.

The magnetic cusp was formed by two oppositely wound solenoid sections and was accentuated with a soft iron ring which concentrated the radial magnetic flux thus minimizing the axial extent of the cusp at the beam radius. A magnetic mirror trap was located just downstream of the cusp region. Several field configurations were used with varying well depths and mirror ratios. In Figure 2, two of the most successful magnetic field configurations are shown. The graph showing the magnitude of the axially directed magnetic field as a function of axial position. The field is actually negative to the left of the cusp. In configuration I, the upstream and downstream mirror ratios were both 1.5. In configuration II,

upstream mirror ratio was reduced to 1.10 and the downstream mirror ratio to 1.4.

The principal diagnostics were magnetic probes which measured the fast diamagnetic signals produced by the circulating electrons. Two probes were positioned on-axis and at the wall, in the vicinity of the magnetic field minimum. Since the wall probe was found to diminish the trapping and shorten the lifetime of the rings, only the axial probe was used in later measurements.

III. Observations

Figure 3 shows the diode current and the diamagnetic signal measured on-axis and at the center of the trap for configuration I. The 20 kA peak diode current produced a layer with a peak diamagnetic signal 185% stronger than the original vacuum magnetic field when injected into 450 mTorr of hydrogen. The field at the center of the trap was reversed for a short time, and the tail of the signal decayed exponentially in about .3 μ s, which is probably the decay of plasma currents excited by the layer electrons. The diamagnetic signal detected on-axis and beyond the downstream mirror indicated large electron losses through this mirror.

Using the magnetic field configuration II of Figure 2, the maximal field changes were reduced, but longer-time trapping of sizable electron rings was achieved. In this case, the magnetic probe recordings are very similar to the probe signals observed in the RECE-Berta experiment (Phelps et. al. (1973)). As example, Figure 4 shows axial probe signal obtained when injecting a 10-kA-350-kV beam into a gas filling of 1100 mTorr hydrogen. After some initial fluctuations, this signal indicates field changes δB of over 630 Gauss using a vacuum field B_0 of 530 Gauss, i. e.

clear field reversal. The signal then decays almost linearly until a value of $\delta B/B_0 \approx 15\%$ is reached. At this point, a rapid final dump of the rings sets in.

This is a typical behavior of the probe signals observed. At smaller pressures the linear decay rate is reduced roughly proportional with pressure. However, the initial strength of the rings also decreases. The maximal life time observed so far is 9.5 μsec .

As in the RECE-Berta experiments, the significance of high-energy electrons in the rings was tested by mixing small partial pressures of a high-Z gas to the basic hydrogen filling. In an alternating series of shots, using a base pressure of 950 mTorr hydrogen and adding 130 mTorr nitrogen, very consistently the normal lifetime of 8 - 9 μsec was reduced to 2 - 3 μsec when the nitrogen was added. On the other hand, the initial ring strength was changed little. As detailed in Phelps et. al. (1973) this increase in decay rate is quite consistent with the enhanced scattering of the fast electrons by the added nitrogen; but, it is clearly larger than the increase in scattering of slow electrons.

Measurements of the ratio of the magnetic field changes on axis and at the wall indicate typical ring radii of 3 - 4 cm. The axial length of the ring appears to be comparable with its diameter.

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FIGURE CAPTIONS

- Figure 1. Experimental arrangement.
- Figure 2. Axial dependence of the magnitude of the magnetic field at the beam radius for configurations I and II. The actual field to the left of the CUSP is negative.
- Figure 3. Diode current (upper trace) and diamagnetic signal measured at the center of the trap (lower trace) for the magnetic field configuration I. The peak diode current is 20 kA and the maximum diamagnetic signal is 185% of the original vacuum field. Pressure of H_2 in trap is 450 mTorr.
- Figure 4. Diamagnetic signal observed on axis for magnetic field configuration II using 1100 in Torr hydrogen filling. (1 μ sec/div, 293 Gauss/div).

I_{DIODE}

ΔB_{TRAP}

2 MICROSEC

Fig. 4

USE OF STRONG RELATIVISTIC ELECTRON RINGS
FOR THE CONFINEMENT OF THERMONUCLEAR PLASMAS*

By

Hans H. Fleischmann

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USE OF STRONG RELATIVISTIC ELECTRON RINGS FOR THE CONFINEMENT OF THERMONUCLEAR PLASMAS

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Summary

Various schemes proposed for the confinement of thermonuclear plasmas are reviewed in which it may be possible to generate and maintain the necessary plasma-internal ring currents by strong self-focussed rings of relativistic electrons or protons. Thus, confinement advantages inherent in some of these schemes may become available for an actual fusion reactor. The results of first experiments injecting very-high current beams of relativistic electrons into a magnetic mirror trap show that such electron rings can be generated and give good indications for their stability.

Introduction

Notwithstanding the recent substantial progress achieved with Tokamak-type machines, the tailoring of magnetic field configurations capable of providing the best or at least sufficient plasma confinement time for a fusion reactor still remains one of the foremost problems in controlled thermonuclear research. Following some of the results of the extended experimental and theoretical research of the last 20 years, it now is generally accepted that field configurations in which the plasma would be confined in the vicinity of minima areas of the absolute magnetic field strength will be able to suppress at least all large-scale instabilities which often led to a rather catastrophic loss of plasma in many earlier experiments. Furthermore, it is widely accepted that this suppression may provide sufficiently good plasma confinement if the respective field lines do not leave these minimum-B areas. The required minimum-B quality can be achieved either by averaging along such closed field lines, so that $\langle 1/B \rangle$ becomes a maximum within a set of nested field surfaces ("average minimum-B") or, better still, if the value of the magnetic field strength increases everywhere outside the confined plasma ("absolute minimum-B" with closed field lines). In both cases, such configurations necessitate the existence of ring currents within the surface of the confined plasma. In average minimum-B configurations, these currents can flow in regions not directly occupied, though fully surrounded by plasma. Thus, it is principally possible to maintain them in closed metallic hoops; however, the cooling and the support of these hoops would pose very serious and hitherto not fully solved engineering problems in an actual fusion reactor. In absolute-minimum-B geometries with closed field lines, these currents would have to flow within the region of plasma confinement itself. Furthermore, the cited stability advantages of this scheme largely are lost if the dynamic motion of these currents is not sufficiently decoupled from the motion of the plasma itself, i.e. if the plasma itself is carrying these currents. In both cases, the respective problems could be avoided if these currents were generated and maintained by rings of highly energetic charged particles, preferentially relativistic electrons.

In this paper, first various proposed schemes in which high-energy particles are used for field shaping are reviewed and analyzed. Then, results of first experiments at Cornell are described in which strong electron rings were generated by injecting electron beams from a very high current accelerator into a magnetic mirror trap. The obtained results also give good

indications concerning the hydrodynamic stability of such rings.

General Considerations

The first confinement scheme ("Astron") to make use of relativistic electrons for field shaping was proposed by Christofilos in 1956¹. As shown in Figure 1a, a cylindrical layer of relativistic electrons was to be trapped between two magnetic mirrors. The superposition of the magnetic mirror field and the field generated by the electron coil leads to a decrease of the field strength inside the electron layer. When the layer becomes sufficiently strong, the field direction at the cylinder axis reverses, and a nested system of closed magnetic field lines with absolute minimum-B character is generated within the electron layer. The electron layer was to be established by consecutive injection of a train of electron pulses.

In the well-known "Levitron"² and "Spherator"³ schemes (see Figure 1b), generally an average minimum-B configuration is generated by a combination of two opposing coil systems, A and B, and an axial current C. In a later publication⁴, Yoshikawa proposes and analyzes the replacement of the inner current loop by a ring of relativistic electrons. Similarly, some stability advantages possibly could be obtained in Tokamak devices if the plasma current could be carried by large-Larmor-radius relativistic electrons.

Neglecting inductive reacceleration of the electrons, in all these cases the slow-down time of the electrons by collisional drag in the fusion plasma

$$\tau_{\text{slow}} = \frac{1}{n \sigma_{\text{ee}} v} \quad (1)$$

Figure 1

Basic Astron (a) and Spherator (b) Configurations

n = plasma density, γ = relativistic mass ratio, $0.25 = 2(1-24)/E_{\text{MeV}}^2$, E_{MeV} beam energy in MeV)

to be larger than the plasma confinement time required by the Lawson criterion, i.e. $n\tau_{\text{slow}} > (n\tau)_{\text{Lawson}}$ for a DT-reactor. According to (1), this requires

$$E_{\text{MeV}} > 12 \text{ MeV}$$

or we obtain

$$\frac{\tau_{\text{slow}}}{\tau_{\text{Lawson}}} \approx \frac{E_{\text{MeV}}}{12}$$

On the other hand, the slow-down time of the electrons to their own synchrotron radiation also has to be larger than the Lawson confinement time. Assuming at the plasma pressure nKT is equal to the energy density of the beam generated magnetic field $B^2/4\pi$, and setting $KT = 10 \text{ keV}$, this condition limits the electron energies to less than about 50 MeV in the original Astron scheme where the beam generated fields determine also the orbital radius of the electrons and, thus, renders this scheme somewhat marginal as actual fusion reactor.

However, this problem can be obviated in several ways: First, when stronger toroidal fields B_0 are applied, as in the Spherator Astron scheme⁴, these toroidal fields help to guide the electrons along the major circumference of the machine and thus allow stronger beam generated fields than in the simple Astron.

Also, this condition possibly may be somewhat relaxed in a conceptual "Bumpy Electron Ring Torus" (see Figure 2)⁵ in which a series of short electron rings is arranged along a toroidal sequence of mirror fields. In this case, the ratio of total volume available for stable plasma confinement to the volume occupied by the electron beams may be significantly increased over the same ratio available in the normal Astron scheme. Results of preliminary experiments⁶ at Cornell indicate that the stability of these electron rings may not be significantly worsened by a canting of the mirror fields. The "Bumpy Torus" presently under construction in Oak Ridge⁷ is similar to this scheme, though the electron radii in this case are significantly smaller than the minor machine radius.

Figure 2

Bumpy Electron Coil Torus

Third, the synchrotron radiation problem could be avoided altogether if corresponding current layers could be generated by high-energy protons or other heavy particles, as first proposed by Christofilos.

Such a "p-layer" could be generated either again by stacking of pulse trains of very high energy protons or, considerably easier and cheaper⁵, by injection of a constant H_2^+ beam of only several MeV into a performed electron ring. The H_2^+ would break up by collisions with plasma ions and release protons of 1/2 the energy which would remain in the ring and thus slowly convert it into a p-layer. A constant beam of a few amperes probably would suffice for this conversion. A subsequent compression of the entire ring would increase the proton energy to the required values. An alternate way to use heavy particle rings for confinement may be the usage of fission alpha-particles as proposed by McHally⁸. The main unknowns in either of these schemes that are to be tested experimentally are ways to generate the required rings or layers, and their stability with respect both to rather violent macroscopic instabilities and to small scale two-stream modes. The question of macroscopic stability still is undecided theoretically; comparison with known belt-pinch and similar configurations points to good stability. Experimental and theoretical evidence on possible microinstabilities is somewhat divided with experimental evidence more pointing to stability. More experimental tests of stable parameter ranges are needed.

As a first direct experimental test of his original idea, the ASTRON¹ experiment at Livermore was started by Christofilos. After the solution of considerable technological problems, now a train of up to hundred electron pulses of 5 MeV, 600 A and 300 nsec duration can be injected into a magnetic mirror trap. In most recent experiments⁹ of the group, single-pulse injection led to the generation of electron layers which reducing the magnetic field on the mirror axis by up to 35%, lasting about 1 msec and, with smaller field reduction, can last up to 200 msec. Stacking of several pulses does not increase the achievable layer strength, but is capable of maintaining existing layers exhibiting up to 15% field suppression on axis.

Relativistic Electron Coil Experiment (RECE) at Cornell^{10,11}

The RECE-program at Cornell is aimed at investigating in a mirror configuration the various problems associated with use of charged particle coils for field shaping. In contrast to the Livermore experiment, only a single high-current pulse of electron is injected into a mirror trap (see Figure 3). Presently electron beams of energies of about .5 MeV, currents of about 10 - 20 kA, and a pulse duration of 70 nsec are provided by the Cornell QUIBBLE facility. These beams are injected into a vacuum tank 45 cm in diameter and 2 m long. For space charge neutralization, a hydrogen gas pressure of several hundred micron is maintained in the tank. A magnetic field (see again Figure 3) of about 200 gauss is generated by a set of homogeneous field coils and movable mirror coils energized in series from a capacitor bank. The diagnostic instruments presently include magnetic pick-up loops at various positions along the tank axis and at the tank wall, collimated scintillator X-ray detectors directed at the trap center, a fast electronic framing camera, and time integrated optical cameras.

Figure 4 shows an early sample of the recordings of two magnetic probes (both on axis, one in the trap center, one 30 cm downstream), of the X-ray detector, and of the time-integrated camera. These recordings indicate that current layers persisting for several microseconds (presently up to 40 μsec , and strong enough to reverse the magnetic field on the tank axis) (field changes presently range up to ~ 100% of the

Figure 3

Experimental Arrangement and Axial Field Distribution in the RECE-Berta Experiment

original vacuum field) are generated. The X-ray detector recording which is proportional to the number of electrons hitting the tank wall indicates, after a relatively strong burst at the beginning when most of the electrons are lost, only small losses of fast electrons until the final faster decay, "dump", occurs in the magnetic probe signals when the layer strength has decayed to rather small values. Comparison measurements with two different gas fillings, one

with 800 microns of hydrogen and the other with 400 micron hydrogen and 40 micron of krypton, indicate the usual long layer lifetime in the first case, but a strongly reduced life times of at most 2 μ sec in the second case. Since both fillings are identical in respect to the total ionization cross section of the gas and its scattering cross section for slow electrons, and differ only with respect to their total scattering cross section for relativistic electrons, this result leads to the conclusion that the observed current layers actually consist of relativistic electrons. Thus, by this method, it is possible to generate relativistic electron rings sufficiently strong to reverse the magnetic field in the trap center, i.e. to create an overall magnetic field configuration with closed field lines and absolute minimum-B characteristic.

From the same traces optimistic indications can be obtained with respect to the gross stability of these rings. The relatively slow initial decay agrees quite well with the decay to be expected from the scattering of the fast electrons from the background gas. Only the faster "dump" at the end of the pulse, reminiscent of similar occurrences in the Livermore machine, is indicative of some instability. However, it seems to occur only at a strength level of the layer which is considerably below the level of interest for fusion applications.

To test these points further, the dependence of the negative initial slopes of the magnetic probe signals on the hydrogen gas pressure in the tank were determined. Figure 5 gives some of the results. Except a small offset at small pressures, the decay constant is inversely proportional to the gas pressure as to be expected if the decay is mostly governed by the scattering of the fast electrons in the hydrogen gas.

Figure 4

Sample Recordings from Optical Camera, Magnetic Probes and X-ray Detector

Figure 5

Dependence of Magnetic Probe Slopes on Hydrogen Pressure in Tank

The initial offset is easily explained by the relatively larger scattering cross sections of additional background gas of higher atomic number. Measured background pressures with the hydrogen supply shut off amount to several microns, which easily corresponds to the necessary equivalent of 200 micron hydrogen. Thus it appears that under the present conditions most of the initial decay of the layers is due to simple

cattering which would be seriously reduced in an actual fusion reactor. Additional measurements with varying admixtures of high-Z gases support this conclusion.

The origin of the final dump was the target of other measurements. In a number of respects, it appears to behave similar to the "precessional dump" in the Livermore experiment. In particular, the dump level decreases with decreasing radial gradient of the magnetic field strength, and it can be quenched by the application of poloidal magnetic fields generated by currents driven along the tank axis. A clear discrepancy with the Livermore results, however, exists in its dependence on the initial layer strength (see Fig. 6).

Fig. 6 Dependence of Dump Level on Initial Strength of Electron Ring

The spatial distribution of the magnetic field changes resulting from the electron coil were investigated using seven magnetic probes along the tank axis and three probes at the tank wall. A sample of the resulting spatial plots for various times is shown in Fig. 7. Assuming a radially thin coil with a $\cos k_z$ distribution of the currents, the dimensions of the coil were computed. Quite reproducibly, coil radius around 9 cm and a half-intensity length of around 15 cm, somewhat increasing during the collisional decay, were observed after the initial settling period of 1 - 2 μ sec.

Layer sensitivity against perturbations of the magnetic field were tested using canting of the downstream mirror by angles of up to 7 deg and asymmetries in the poloidal field of up to 10 %. In both cases, no significant change in the decay of our rings was found.

Presently, the time dependence of the energy of the fast electrons is being investigated using X-ray absorption techniques. Preliminary results indicate energy losses roughly compatible with the known collisional losses of fast electrons in gaseous hydrogen.

Conclusions

Overall, the described experiments show that strong rings of relativistic electrons trapped in a magnetic mirror can be rather stable against MHD and micro-instabilities, and against quite pronounced large-scale perturbations. In a next step, these tests have to be extended to other parameter ranges, in particular to longer confinement times, a less collisional neutral background (i.e., smaller hydrogen pressures), and larger poloidal fields (presently up to 40 %). For this purpose

a somewhat larger facility, RECE-Christa, which is presently under construction will be used. Using this facility, it may be possible to obtain also first indications concerning the plasma confinement properties of the electron rings.

Fig. 7 Spatial Distribution of Magnetic Field Changes Generated by the Electron Coil.

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Diffusion Theory of Strong, Stable E Layers

by

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Diffusion Theory of Strong, Stable E Layers

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ABSTRACT

A diffusion theory is presented that treats the long term behavior of strong electron rings (E layers) with and without an applied toroidal field. Loss rates and decay characteristics are predicted that are in good agreement with experimental results from the astron and the Cornell relativistic electron coil experiment. Self consistent density distributions and changes of the E layer energy are also discussed. The average radius of the layer as a function of energy, magnetic fields, and surrounding conducting boundaries is calculated for typical experimental situations.

I. INTRODUCTION

In a previous paper¹, a diffusion theory for weak electron rings (E layers) was developed and compared to experimental results² on the astron.³ A weak electron ring is one having self fields small compared to the magnitude of the externally imposed solenoidal field necessary for radial confinement. Results on strong layers from the Cornell relativistic electron coil experiment⁴ and results on layers in a strong toroidal field from the astron⁵ have become available, prompting extension of the theory.

The electron ring, shown in Figure 1, is assumed to be charged neutralized and monoenergetic. Because the collisional energy loss considered in Section V produces small straggling, an initially monoenergetic E layer will remain so over its lifetime. Although strong E layers (according to the definition mentioned) are considered, they are taken to have low v/γ when viewed as a beam. This is consistent with experiment, and allows treatment of transverse velocities as first order compared to the azimuthal velocity responsible for the ring current. Two limiting cases which allow simple expressions for particle orbits will be considered: 1) $B_\theta/B_z \gg 1$ and 2) $B_\theta/B_z \ll 1$. For the most part a large aspect ratio approximation will be used in which the ring is treated as a beam with toroidal corrections. This procedure is valid since the theory is inherently a large orbit treatment and hence

toroidal effects, such as neoclassical diffusion, are not expected.

II. DIFFUSION ACROSS A STRONG TOROIDAL FIELD

A simple initial case is one in which there is a strong toroidal field present, such that $B_\theta/B_z \gg 1$. This assumption allows neglect of the applied poloidal field (except in its role to provide radial equilibrium) and the self fields if $\xi \approx 1$. ξ is the field reversal parameter, defined as the ratio of the maximum radial change of B_z across the ring to the applied field, B_0 . In Figure 2, transverse particle orbits are projected into a cross section of the ring. The variation of B_θ across the thickness of the ring is neglected. In this case the orbits are circles having a gyroradius, r_g , and a gyrocenter displaced a distance ρ from the beam center. It is also assumed that there is some maximum minor radius, x_b , corresponding to a material obstruction, beyond which particles cannot exist. For the high energy electrons, ρ and r_g are changed by collisions slowly compared to the orbital period. Thus, a particle is lost from the system if $\rho + r_g \geq x_b$. Small angle collisions with a background gas or plasma cause first order changes in the transverse velocity, v_t , but second order changes in the azimuthal velocity, which is assumed constant. Particles take on various values of ρ and r_g , but an attempt to describe diffusion in terms of these variables is complicated by the coupled boundary condition. Instead, the effect of collisions on the variable x (where $x = \rho + r_g$) will be considered.

At any point, the collisional deflection, $\Delta \underline{v}$, has random direction and a magnitude determined by the probability distribution for coulomb scattering. The components of $\Delta \underline{v}$ perpendicular to $\underline{v}_t$ (as in Figure 2a) and parallel to $\underline{v}_t$ (as in Figure 2b) will be treated separately. In the first case there is a change in ρ only, of magnitude

$$|\Delta \rho| = \Delta v_{\perp} \gamma m_e c / e B_0. \text{ This changes } x \text{ an amount } \Delta x = |\Delta \rho| \cos \alpha.$$

A change in velocity parallel to $\underline{v}_t$ cause a change in r_g , $\Delta r_g = \Delta v_{\parallel} \gamma m_e c / e B_0$, as well as $\Delta \rho$ of magnitude Δr_g . For this type of deflection, $\Delta x = \Delta v_{\parallel} \gamma m_e c / e B_0 (1 - \cos \alpha)$.

Averaging over α , $\Delta v_{\perp}$, and $\Delta v_{\parallel}$, it is found that

$$\begin{aligned} \langle \Delta x^2 \rangle &= \left(\frac{\gamma m_e c}{e B_0} \right)^2 \left(\langle \Delta v_{\perp}^2 \rangle / 2 + 3 \langle \Delta v_{\parallel}^2 \rangle / 2 \right) \\ &= \left(\frac{\gamma m_e c}{e B_0} \right)^2 \langle \Delta v^2 \rangle. \end{aligned} \quad (1)$$

The direction of Δx is random. If the scattering background is uniform $\langle \Delta v^2 \rangle$ is independent of position and hence $\langle \Delta x^2 \rangle$ is independent of the orbital parameters and, on the average, the location of the collision in space. Thus, particle positions in x space fulfill the requirements of a random walk and are therefore described by the diffusion equation

$$D_x \frac{1}{x} \frac{\partial}{\partial x} x \frac{\partial f}{\partial x} = \frac{\partial f}{\partial t}, \quad (2)$$

where f (the distribution of particles in x) equals zero at $x = x_p$. The quantity D_x is given by $D_x = \langle \Delta x^2 \rangle \nu / 2$, where ν is the collision frequency. Estimating $\langle \Delta v^2 \rangle$ for relativistic electrons and substituting into Eq. 1 gives

$$D_x = \frac{1}{2} \left(\frac{\gamma m_e c v_0}{e B_0} \right)^2 \left(\frac{8 \pi c n_g Z(Z+1) r_e^2}{\gamma^2} \right) \ln \left(\frac{b_{\max}}{b_{\min}} \right), \quad (3)$$

where n_g is the density and Z is the atomic number of the background. The quantum mechanically correct impact parameter ratio is $b_{\max}/b_{\min} \simeq \gamma m_e c v_0 / \hbar$ for a gas and $b_{\max}/b_{\min} \simeq \gamma m_e c \lambda_D / \hbar$ for a plasma. The solution of Eq. 2 is

$$f = \sum_n b_n J_0(k_n x) \exp(-D_x k_n^2 t), \quad (4)$$

where $k_n = 2.405/x_b, 5.540/x_b, \dots$. The decay in the fundamental mode is exponential as opposed to the linear decay treated in Section III. This results from the fact that the boundary condition is independent of the self fields. The e-folding time for the fundamental is $t_e = (2.405)^2 / D_x x_b^2$. Experimental results for moderately strong toroidal fields have been obtained on the astron and at Cornell. With B_0/B_z in the range of 0.5, the layer lifetime is extended an amount consistent with theory. Long term loss rates are lessened, giving the decay a more exponential variation as expected.

III. DIFFUSION WITHOUT A TOROIDAL FIELD

A case closer to present experiments in that in which B_0 is weak so that it has a negligible effect on the particle orbits. The particles are constrained by an externally imposed solenoidal field with a shallow mirror, their self fields, and fields caused by currents induced in surrounding conducting walls. The geometry is illustrated in Figure 1.

Azimuthal symmetry is assumed. The total magnetic fields, $B_z(r,z)$ and $B_r(r,z)$, are expanded in a Taylor series about the point $(R,0)$.

$$B_z \cong B_z(R,0) + \left. \frac{\partial B_z}{\partial r} \right|_{R,0} (r - R) + \left. \frac{\partial B_z}{\partial z} \right|_{R,0} z, \quad (5)$$

$$B_r \cong B_r(R,0) + \left. \frac{\partial B_r}{\partial r} \right|_{R,0} (r - R) + \left. \frac{\partial B_r}{\partial z} \right|_{R,0} z.$$

The quantity R , the average radius of the E layer, is calculated self consistently from the equation, $R = \gamma m_e c v_\theta / e B_z(R,0)$. The plane $z = 0$ is considered a plane of symmetry so that $dB_z/dz = 0$ and $B_r = 0$ at $(R,0)$. Also, from the equation $\nabla \cdot \underline{B} = 0$, it is found that $dB_r/dr = 0$. Using Eq. 5 and assuming the low γ/δ limit, the particle equations of motion are

$$\gamma m_e \dot{v}_r = \left(\frac{-ev_\theta}{c} \right) \left[\left. \frac{\partial B_z}{\partial r} \right|_{R,0} + \frac{B_z(R,0)}{R} \right] (r - R), \quad (6)$$

$$\gamma m_e \dot{v}_z = \left(\frac{-ev_\theta}{c} \right) \left(\left. \frac{\partial B_r}{\partial z} \right|_{R,0} \right) z,$$

and v_θ is a constant. These are the equations of uncoupled harmonic oscillators with the solutions

$$v_r = A \cos(\omega_r t + \varphi_1), \quad (7)$$

$$v_z = B \cos(\omega_z t + \varphi_2),$$

where

$$\omega_r = \omega_c \left[\frac{B_z(R,0) r_g}{B_0 R} + \left(\frac{r_g}{B_0} \right) \frac{\partial B_z}{\partial r} \Big|_{R,0} \right]^{\frac{1}{2}},$$

$$\omega_z = \omega_c \left[\left(\frac{r_g}{B_0} \right) \frac{\partial B_r}{\partial z} \Big|_{R,0} \right]^{\frac{1}{2}}.$$

Here, r_g is the gyroradius in the applied field, $r_g = \gamma m_e c v_0 / e B_0$, and ω_c is the single particle gyrofrequency, $\omega_c = e B_0 / \gamma m_e c$.

It has been shown in a previous paper¹ that a collection of particles in a harmonic potential that suffer collisions with a uniform background of scattering centers over long time scales compared to orbital times obeys a cylindrical diffusion equation. Particles are characterized by their velocities at the bottom of the potential, the crossing velocity. If motions in two dimensions are uncoupled, each degree of freedom obeys the diffusion equation and the total distribution function is a product of the two distributions. In the problem in question, particle motions are uncoupled because the magnetic field can be written in the form $B_z = B_z(r)$ and $B_r = B_r(z)$. It will also be assumed that boundary conditions are uncoupled, represented by the maximum excursions, Δz and Δr . Towards the extremities of the E layer these conditions are probably inconsistent with the requirements for a self consistent equilibrium, but they should hold approximately near (R,0) where the majority of particles are located. The distribution of particles in crossing velocities, v_{r0} and v_{z0} , is determined by the equation

$$D \left(\frac{1}{v_{r0}} \frac{\partial}{\partial v_{r0}} v_{r0} \frac{\partial}{\partial v_{r0}} + \frac{1}{v_{z0}} \frac{\partial}{\partial v_{z0}} v_{z0} \frac{\partial}{\partial v_{z0}} \right) f = \frac{\partial f}{\partial t}, \quad (8)$$

where $D = (2/\pi)^2 \langle \Delta v^2 \rangle \nu / 2 = \langle \Delta v_r^2 \rangle \nu / 2$.

Also, $f = f_z(v_{z0}) \times f_r(v_{r0})$, and $f_z = 0$ at $v_{z0} = \Delta z \omega_z$ while $f_r = 0$ at $v_{r0} = \Delta r \omega_r$. If the boundaries are constant, Eq. 8 has the same solutions as Eq. 2. This is not generally the case.

It remains to estimate the magnetic field quantities appearing in ω_z and ω_r : $B_z(R,0)$, dB_z/dr at $(R,0)$, and dB_r/dz at $(R,0)$. Contributions come from 1) applied fields, 2) E layer self fields and 3) fields resulting from currents induced in surrounding conducting walls. The contributions from the first and third satisfy the condition that $dB_z/dr = dB_r/dz$, since $\nabla \times \underline{B} = 0$. Applied fields are almost constant over the length of the layer, $B_z \approx B_0$. There is also a contribution to the gradients, $dB_z/dr \approx -nB_0/r_g$. Typically, n is small compared to unity. Following the treatment of Laslett⁶, the self fields are divided into those that would result if the electron ring were considered as a straight beam and corrections caused by turning it into a torus. The self fields of a beam of uniform density with $\Delta r = \Delta z$ are $B_z(R,0) = 0$, and $dB_z/dr = dB_r/dz = 2I/a^2c$ at $(R,0)$. Here I is the total current of the beam and a its minor radius. In terms of the field reversal parameter, this can be written as $dB_z/dr \approx \xi B_0/2a$. In experimental situations the layer density as a function of minor radius is not uniform, but a rough approximation can be made by taking a equal to the minor radius at half height of the density distribution, $a \sim 0.7 \Delta r$. If toroidal corrections are added, self fields decrease the average solenoidal field by $B_z = -(I/Rc) \ln(8R/a)$, whereas the gradients are negligibly affected. The problem of

image current fields produced by a current ring of small a inside a conducting cylinder has been treated numerically in Ref. 6. If r_w is the radius of a conducting wall, then an approximate expression for the contribution to B_z adapted from this reference is $B_z(R,0) \approx 0.74 (I/Rc)(r_w/R - 1)^{-3/2}$. For reasonable values of r_w/R , image current contributions to the gradients are small. For instance, at $r_w/R = 2$, it is found that $dB_z/dr = 0.2 (I/R^2c)$ at $(R,0)$.

The loss rate predictions agree with those of Ref. 1 in the limit that $\xi \ll 1$ and the E layer is axially self pinched. In the strong self field limit, an analytic solution cannot be obtained because of the dependence of the boundary condition on ξ . Nonetheless, an approximate solution can be arrived at by recognizing that the boundary changes over a time scale of the order of the fundamental mode decay time whereas the higher order modes decay rapidly in comparison. Thus, the layer will be approximately in the fundamental mode with respect to the instantaneous value of the boundary, and the crossing velocity derivatives can be approximated by

$$\nabla^2 f \simeq (2.405)^2 \left[\frac{1}{\omega_r(t)^2 \Delta r^2} + \frac{1}{\omega_z(t)^2 \Delta z^2} \right], \quad (9)$$

assuming, of course, that the E layer was initially close to the fundamental. Typically, the applied field term in the expression for ω_r is comparable to the self field term. Losses are thus predominantly axial, with $\omega_z \simeq \omega_c \xi(r_g/2a)$. Also taking $N_T(t) \sim \xi(t)$, where $N_T(t)$ is the total number of particles in the E layer, Eq. 8 can be integrated over velocity space and

time to give

$$N_T(t) = N_T(0) - \frac{N_T(0)(2.405)^2 r_g^2 D t}{v_e^2 \xi(0) \Delta z} \quad (9)$$

Under these circumstances, which typify most experiments at Cornell, the E layer decays linearly. Also, since $N_T(0)$ is proportional to $\xi(0)$, the slope is independent of the initial state of the layer as long as it is near the fundamental mode. The decay of high ξ E layers under a variety of conditions is reported in another paper⁷. The results agree with the theory to within the experimental accuracy in measuring the relevant parameters ($\pm 20\%$).

IV. SELF CONSISTENT DENSITY DISTRIBUTIONS

When the orbital periods of E layer particles are short compared to time scales for collisional changes, the layer can be considered to be in a quasi equilibrium with respect to the instantaneous macroscopic fields. Using the velocity distributions arrived at in Section II, it is possible to derive self consistent density distributions such that the resulting magnetic fields are just those needed to produce the density in question. The general two dimensional problem is involved and has been handled numerically.⁸ A simplified one dimensional model that provides some insight into the problem will be considered. The model also has application to the radial equilibrium of E layers of infinite axial extent. Although axially long layers probably

cannot be obtained in diffusion dominated systems with open ends¹, axially closing the layer into a torus removes restrictions on the velocity distributions. Under this circumstance, long layers may be possible.

When the E layer density is not uniform, particles no longer feel a harmonic potential. This in turn affects the distribution of crossing velocities, but the effect is small. Figure 3 shows a phase space plot of one dimensional particle orbits. Collisions cause verticle displacements that change the value of v_{x0} . Collisions near the turning points ($v_x = 0$) have less effect than those at the center of the potential ($x = 0$). A harmonic potential is shown in Figure 3a. If the density goes to zero at the boundary, the potential will be soft, giving orbits such as those in Figure 3b. Near the boundaries, curves corresponding to values of v_{x0} are changed, although the verticle distance between adjacent curves is not greatly changed. This, combined with the fact that collisions near the extremities have a small effect, implies that the distribution in v_{x0} is not strongly affected by this change in the orbits. The most important effect of softening the potential is to change the orbital period so that a particle spends a smaller fraction of its time near $x = 0$ where collisions cause the maximum change in v_{x0} . Softening the potential as in Figure 3b results in a maximum change of only 15% in period (for those particles reaching x_{max})⁹, so that this effect is also small. The distribution in terms of crossing velocities can thus be taken approximately from the solution of Eq. 8.

The geometry of the model is shown in Figure 4. Particles oscillate with first order velocities in x and are confined solely by their self magnetic field, $B_y(x)$, produced by the zero order velocity, $v_z \approx c$. Motions in y are symmetric and make no contribution to the fields. The equation of motion in x can be integrated to give

$$v_x^2 = v_{x0}^2 + \frac{2ev_z A_z(x)}{\gamma m_e c}, \quad (10)$$

where A_z is the vector potential. The density in an element dx is given by the integral of all particles that can pass through dx weighted by the relative amount of time they spend in dx , or dx/v_x . Using Eq. 10, this can be written

$$\begin{aligned} n(x)dx &= \frac{1}{v_{x0\max}} \int_{-\frac{2ev_z A_z}{\gamma m_e c}}^{v_{x0\max}} \frac{f(v_{x0})v_{x0}dv_{x0}}{(v_{x0}^2 + \frac{2ev_z A_z}{\gamma m_e c})^{\frac{1}{2}}} dx \\ &= n(A_z)dx, \end{aligned} \quad (11)$$

where $f(v_{x0})$ is the velocity distribution in terms of crossing velocities. Equation 11 can be solved simultaneously with the equation

$$\frac{d^2 A_z}{dx^2} = \frac{-4\pi ev_z}{c} n(A_z), \quad (12)$$

to find the self consistent vector potential and hence the density. Figure 5 shows the solution for the case in which $f(v_{x0}) = J_0(2.405v_{x0}/v_{x0\max})$. The relative shape of the

density distribution is independent of the magnitude of the self fields and has a thickness determined only by the boundaries. This is in contrast to the one dimensional model developed in Ref. 1 for an axially long, open ended E layer. Here an applied field is necessary for equilibrium, and solutions cannot be found for strong self fields.

The problem of the radial equilibrium of an infinitely long layer can be handled by adding a B_z to bend the sheet of Figure 1 into a ring. The solution is more complicated, and the following features appear: 1) by the correct choice of $v_{x\text{max}}$, solutions can be found that go smoothly to zero at a radial boundary, 2) the relative shape of the density varies with the ratio of the self fields to the applied B_z , 3) the average radius of the layer increases with increasing self fields.

V. ENERGY LOSS

During its lifetime, an E layer can loose energy by inelastic collisions with the background medium, or gain energy through a decrease in the self fields due to layer decay. By changing the average radius of the layer, changes in energy can affect the boundaries for radial diffusion as discussed in Ref. 1. If axial diffusion predominates, the most important effect of a change in energy is through the γ^{-2} term in the diffusion coefficient. If the conditions for linear decay hold and the energy loss rate is slow compared

to higher order mode decay times, the approximate number of particles remaining within the layer is

$$N_T(t) \approx N_T(0) \left\{ 1 + \frac{(2.405)^2 D_0 r_E}{\xi(0) v_0^2 \Delta z} \left[\frac{\gamma_0}{\dot{\gamma} (1 + \dot{\gamma} t / \gamma_0)} - \frac{\gamma_0}{\dot{\gamma}} \right] \right\} \quad (13)$$

A linear time dependence of energy, $\dot{\gamma}$, is assumed and zero quantities are evaluated at $t = 0$.

In present experiments, the effect of energy loss on decay rates is small. With a combination of high γ and high ξ , scattering loss is decreased and energy loss may become more significant. With an estimate of the E layer lifetime in present experimental regimes, the relative energy losses and gains caused by collisions and changing fields can be estimated. Energy is lost collisionally at the rate¹⁰

$$\frac{\dot{\gamma}}{\gamma_0} \approx \frac{4\pi n_g Z r_e^2 \ln(2 \gamma^2 m_e c^2 / I)}{\gamma_0} \quad (14)$$

for a gas where I is the approximate ionization potential. If the average radius of the E layer is approximately equal to r_g and the field reversal, ξ , refers to the average field inside the layer in the plane $z = 0$, then electrons remaining in the layer gain an energy

$$\dot{\gamma} / \gamma_0 \approx \xi / 2 \quad (15)$$

from the decay of the self fields. ξ can be estimated from

Eq. 9. The self field energy gain rate divided by the collisional loss rate is about $\frac{1}{4} Z(Z+1)/\gamma^2$. At the low energies typical of Cornell experiments ($\lesssim 500$ keV), the two processes have about equal effect for a background of hydrogen, so the E layer would be expected to maintain almost a constant energy. At higher γ collisional losses predominate, as has been shown to be the case at Livermore.²

The ratio of the E layer lifetime for diffusion (neglecting energy loss) to the lifetime for energy loss is approximately

$$\frac{t_d}{t_E} \approx \frac{\gamma_0 \xi_0}{4(Z+1)} \quad (16)$$

For $\xi_0 \sim 1$, scattering becomes less important than energy loss in determining the lifetime of the E layer above $\gamma_0 \approx 8$ (for a hydrogen background). In a proton ring, t_d is extended by about a factor of $(m_p/m_e)^2$ while t_E increases by (m_p/m_e) . Thus, scattering would have a negligible effect on a relativistic proton layer.

The initial E layer energy is dependent on the conditions of injection and trapping. These processes are as yet largely unexplained. If the trapping efficiency is high, as on the astron, a significant portion of the particles' injection energy must go into field energy if the layer is strong. At Cornell, where trapping efficiency may be as low as 1%, it is uncertain whether the reversed field configuration is set up at the expense of the trapped particles or the untrapped. At present the initial energy is an empirical quantity. It can be inferred from a measurement of the average layer radius.

In order to facilitate this measurement and to supply values of R for Eq. 5, the average radius, R , is calculated by solving the equation $R = \gamma m_e c v_0 / e B_z(R, 0)$. The magnetic field contributions discussed in Section III are used. In Figure 7, (R/r_g) is plotted versus I/I_0 and (r_w/r_g) . The wall radius is r_w and the thickness of the layer, a , is arbitrarily taken to be one half of R . I is the total layer current and $I_0 = B_0 r_g c / 2$, so that $I/I_0 \approx \xi$. Using Figure 7 in conjunction with an estimate of γ from Eq. 14 and 15, the average E layer radius as a function of time can be estimated.

VI. CONCLUSION

The diffusion models presented provide a good description of electron rings in present experiments. The lifetime of the layer is determined mainly by elastic scattering with a uniform background. For this case, crossing velocity distributions, loss rates, decay behavior, and density distributions have been investigated. If E layers can be used in a regime of plasma confinement, the theory would have to be modified in the following respects: 1) there is a possibility of unstable interactions with an energetic plasma, 2)

there will be a spatial

dependence of scattering centers determined by the confinement properties of the layer, 3) at high layer energies, the lifetime may be determined mainly by energy loss. If the third case holds, over the major portion of the lifetime the total number of particles in the layer should remain approximately constant (assuming there is no material boundary inside the

particle orbits). At some point the energy becomes low enough so that scattering is important, and the layer is lost.

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FOOTNOTES

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FIGURE CAPTIONS

Fig. 1. Electron ring geometry.

Fig. 2. Deflections of transverse particle orbits in a strong toroidal field: (a) Δv perpendicular to $\underline{v}_t$,
(b) Δv parallel to $\underline{v}_t$.

Fig. 3. Particle trajectories in phase space for a harmonic and a soft potential. (a) Harmonic. (b) Soft.

Fig. 4. Sheet of particles confined by its self magnetic fields.

Fig. 5. Self consistent density distribution corresponding to the fundamental diffusion mode of a current sheet.

Fig. 6. Average radius of an electron ring, R , as a function of applied field, B_0 , energy, $\mathcal{E}$, total current, I/I_0 , and conducting wall radius, r_w . $r_g = \gamma m_e c v_0 / e B_0$.

(a)

(b)

(a)

EFFECTIVE
POTENTIAL

PHASE SPACE
ORBITS

(b)

EFFECTIVE
POTENTIAL

PHASE SPACE
ORBITS

OBSERVATIONS OF THE STABLE EQUILIBRIUM AND CLASSICAL DIFFUSION
OF FIELD REVERSING RELATIVISTIC ELECTRON COILS^{*}

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ABSTRACT

Observations of high current relativistic electron coils are presented and interpreted theoretically. Electron coil self magnetic field strengths up to almost twice that of the external magnetic field at the mirror center are observed. Lifetimes up to 40 μ s are related to the classical diffusion of electron coils in a stable equilibrium state. The diffusion mechanism is verified as inelastic small angle scattering of the relativistic electrons from a neutral gas background.

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1. Introduction

Observations are presented of magnetic field reversing¹ electron coils in stable equilibrium obeying classical diffusion in a magnetic mirror. The decay mechanism is identified as diffusion due to inelastic small-angle scattering² of relativistic electrons in a neutral gas background. Experimentally measured axial and radial magnetic field distributions demonstrate that stable donut-like circulating rings of energetic electrons are formed having 8 - 10 cm radius and 15 - 20 cm axial length.

Since Christofilos¹ proposed the Astron fusion reactor concept to create absolute-minimum-B configurations³, his group at Livermore has made a very thorough experimental and theoretical investigation of E-layers trapped in a magnetic mirror⁴. The E-layers they observed, however, never gained sufficient strength to reverse the magnetic field on the mirror axis. With the advent of intense relativistic electron beam accelerators⁵, the attainment of field reversal became possible, as first demonstrated by Andrews et. al.⁶. More recently, evidence of the containment of strong electron coils was reported by Bzura et. al.⁷. This paper includes more details and expands these previous observations on the geometry and decay of these layers, and compares the results with the diffusion theory by Humphries⁸. The new results described herein provide a good base for future experimental studies of plasma confinement and heating in proposed electron coil schemes like the Astron¹ and the Electron-Coil-Spherator proposed by Yoshikawa⁹.

In the following section, design details of the Cornell Relativistic Electron Coil experiment device (RECE-Berta⁷) and the high current water insulated pulse line (QWIBLE¹⁰) are described.

In Section III, the existence of field reversing relativistic electron

coils is established. A description of the magnetic probe, optical, and x-ray diagnostics is given. Various tests are described which prove conclusively that the observed electron coils are comprised of relativistic electrons.

In Section IV, the equilibrium shape of strong relativistic electron coils is discussed. Examples of the axial and radial self-magnetic field distribution of these coils are presented. The conversion to actual current profiles using the Porter-Weiss Green's function solution is described¹¹. The time-resolved development of coil radius, axial center, and axial length at half-maximum is discussed and statistics of the coil location are given.

In Section V, evidence that the coil diffusion is caused by classical small-angle scattering² is given. It is shown that the observed linear rate of decay of the electron coil varies linearly with tank pressure. Different gases are used as the background in order to demonstrate the dependence on a factor $Z \cdot (Z+S)$, where Z is the atomic number, and S is an incoherent electron scattering function which accounts for inelastic scattering from atomic-orbital electrons.

In Section VI, the observed decay of the electron coils is compared with Humphries' theory⁸ on the collisional decay of a strong self-compressed relativistic electron coil. As predicted, the linear decay rate of the coils is found to be statistically independent of the initial coil strength. The experimentally deduced magnitude of the axial diffusion coefficient is in satisfactory agreement with theoretical prediction.

The conclusions of this work are outlined in Section VII.

II. Description of RECE-Berta

The relativistic electron coil experiments to be described were performed on the Cornell RECE-Berta experiment device. As sketched in Figure 1 (a), this device produces an axial magnetic field as for example in Figure 1 (b), which traps the injected electron beam within a copper-lined cylindrical cavity. Electric neutralization is achieved with background H_2 gas fillings of 100-2000 mTorr.

The electron beam source is the Cornell water insulated pulse line QWIBLE¹⁰, which generates a nominally 80 ns pulse, with a peak voltage and current of typically 450 kV and 10 kA. The corresponding waveforms are shown in Figures 2 (a) and (b). Foil damage patterns indicate that the beam cross-section is slightly larger than the 1" diameter tungsten cathode. The anode surface is .25 mil aluminized mylar foil. An in-vacuum anode foil changer¹² makes it possible to fire up to 60 shots before raising the system to air for cleaning and reloading - thereby improving the speed of data acquisition. The electron beam is guided along a soft-iron lined injector tube of 1-3/8" I.D., 1/8" wall thickness and 45" length. The injector wall thickness is sufficient to reduce a 300 Gauss external magnetic field to a negligible level within the tube. Most effective beam transport is achieved when the drift tube is filled with 2 Torr H_2 gas. Measurements at the output end of the tube using a Rogowski coil produce the wave form of Figure 2 (c) and indicate that the main beam pulse is reproducible from shot-to-shot at the output to within 5%, and no second pulse (seen in Figure 2 (a) and related to an unavoidable, but small diode mismatch) appears.

A .001" titanium foil sealed over the end of the drift tube isolates the tube pressure from the typically several hundred micron H_2 main tank

pressure and prohibits electrons from the second diode pulse to enter the tank. This foil does not break between shots. A pin-hole in the foil allows the main tank roughing pump to evacuate the drift tube. As indicated in Figure 1, the injector extends horizontally to the center of the main tank, is displaced vertically 10 cm, and has a 75° angle relative to the main tank cross-section. Other values of injection angle and radius can be selected from a $75\text{--}90^\circ$ and 6-10 cm range.

The vacuum tank consists of a 17-3/4" I.D., 1/2" wall by 5 foot long glass tube and an 18" I.D., 1/8" wall by 1 foot long brass section supporting the injector. As a precaution against breakage of the glass wall by energetic electrons, the glass wall is lined with 1/8" Lexan sheet. A cylindrical copper screen of 16-1/2" I.D. and 6' length, provides a L/R time of about 300 μsec , which is sufficient for the less than 40 μs lifetime experiments observed to date. The ends of the cylindrical copper screen are enclosed by copper screen caps which prevent space-charge build-up on the lucite end flanges of the vacuum vessel. The confinement tank is evacuated using a roughing pump and filled with the desired pressure of H_2 gas through a small micrometer head leak valve.

An axially directed magnetic field ranging from 100-300 G is generated by the superposition of a uniform solenoidal coil and a number of axially moveable mirror coils, as shown in Figure 1 (a). All coils are energized in series from a 50 kJ crowbarred capacitor bank. The 234 μF bank capacitance combines with a ballast inductance of 400 mH to provide a 14 msec quarter-cycle rise time and a current wave form like in Figure 2 (d). The ballast inductance allows the various mirror coils to be changed without significantly affecting the current wave form. To allow time for penetration into

the brass section, the beam injection is delayed 20-24 msec after triggering the bank (note the blip in Figure 2 (d) due to the addition of the Marx erection signal). Figure 1 (b) exhibits the axial magnetic field profile and in most initial experiments this profile reflects a mirror ratio at the end caps of 1.4:1 over the nominal value of $B_0 = 192$ G at the well bottom. Such profiles were measured using a 5000-turn probe with .33 sec RC-integration. Its calculated calibration was checked by comparing its reading within a purely solenoidal field with the magnetic field strength calculated from the current in the coil as determined from the voltage drop across a 1m Ω resistor. Both calibrations agreed to better than 5%. For parameters selected to date, the slight hump (at 21 ms delay) at the 160 cm axial position in Figure 1 (b) improved the beam trapping relative to a smooth case (around 23 ms delay).

III. Generation of Field Reversing Relativistic Electron Rings

In our earlier letter⁷, we reported briefly on some first experiments and tests indicating the generation of field reversing current layers consisting of circulating relativistic electrons. These results have since been improved and expanded, and a more detailed description of the experiments and their related diagnostics is given here. Recent results indicate the trapping of relativistic electron rings, which produce changes of the axial magnetic field in the trap center of up to 190% of the externally applied magnetic field.

Magnetic probes were the most important diagnostic. The magnetic field changes produced by the trapped electron coil we've measured using up to nine axially movable magnetic pick-up probes arrayed along the axis and the wall of the machine (see Figure 1 (a)). All probes were directed axially, and their signals were integrated with a RC-time constant of 300 μ sec. They were calibrated to better than 5%. As an example, Figure 2 (e) shows the

record of a wall probe near the coil center. After the initial injection transient of a few hundred nsec duration (during which all plasma currents decay), the probe signals often indicated axial movement of the coils for a brief phase of a few μ sec, followed by a theoretically predicted⁸ linear decay phase, and ending in a rapid final dump which will be described in more detail elsewhere¹³. The initial layer strength varied from shot-to-shot by factors of up to two. However, the general shape remained.

Figure 3 shows the recording of the axial probe at the trap center for the strongest layer observed so far, indicating a magnetic field change of 360 Gauss, i.e. from a vacuum field of $B_0 = +190$ Gauss to a reversed field of -170 Gauss or $\zeta = (\delta B/B_0)_{\text{Axis}} \approx 190\%$. In contrast to the initial layer strength, the slopes in the linear part of the traces generally reproduced quite well (see Section V).

In addition, the light output from the collisionally excited background gas was measured. For this purpose, an 8" diameter lens focused the light emitted along the tank axis from a radial sector of the coil onto the object slit of a spectograph. A lead shielded PM-tube was placed in the image plane of a strong line (H_β for example), and traces such as given in Figure 2 (f) were observed. After a very strong initial spike at injection, the light signal decayed in a manner quite similar to that of the signal from an axial magnetic probe located in the trap center. At that stage, the size of the light signals varied roughly proportional to the size of the magnetic signals. A similar observation was reported¹⁴ from the Livermore experiment. It supports the conclusion that the field changes in the trap center were directly proportional to the total number of trapped electrons.

A fast liquid scintillator-PM tube combination was used to detect x-rays

coming from the confinement tank. To avoid saturation of the PM tube during injection, the tube was shielded with a 2" thick lead collimator which allowed it to see only the central portion of the magnetic well bottom where the layers settled. After a strong initial spike at injection, the x-ray monitor detected a smaller and constant x-ray level until the electron coil dumped, yielding a final spike always coincident with the rapid final cut-off in the light and magnetic probe signals. The flat region can be identified with a constant radial loss of electrons which is consistent with observations of the linearly decaying relativistic electron coil. For electron coils with similar magnetic probe signals, an average radiation of 20% in the height of the final x-ray spike was seen when a 1/4" aluminum absorber was placed in front of the PM tube and then removed. This result indicates that the major part of the detected signal was due to x-rays having energies greater than 50 keV. Additional more recent absorption measurements with lead indicate electron energies of several hundred keV, which is consistent with the peak diode voltage levels (see Figure 2 (a)).

Using these diagnostics, various tests concerning the character of the observed current rings were performed as described briefly earlier. For a series of at least 10 shots in each test; the downstream mirror was alternately removed, a mechanical rake was alternately inserted, and a high-Z gas (Krypton) was alternately added to the H_2 background.

When the mirror coil downstream from the injector was removed, helically propagating beams resulted with the magnetic probe signals exhibiting field changes of 20-40% of the vacuum field, rise times of about 50-100 nsec, and decay times of 200-300 ns. These decay times are consistent with estimates of the collisional decay of plasma currents in the background gas, as

described in more detail elsewhere^{6,15}. The pitch distance of the helical beams was roughly the same as their 20 cm diameter, or somewhat longer than estimates based on the injector pitch angle and adiabatic axial acceleration along the upstream mirror field. Magnetic probe signals beyond a few hundred nsec were observed only when the downstream mirror coil was inserted.

The probe signals again decayed in 200-300 ns when a lucite strip (2 cm wide and 25 cm long in the radial direction) was moved into the trap region. In a similar experiment, a .7 mil diameter tungsten wire was placed radially in the ring center. In this case, a decay time of about 20 μ sec would be expected for a relativistic electron ring of 20 cm length and a gyro period of 2 nsec. The observed lifetime decrease of roughly a factor 2 for strong layers (normally about 20 μ s long) was consistent with such additional losses.

In another series of experiments, tank fillings of 800 μ H₂ were compared with fillings of 400 μ H₂ and 40 μ Krypton. The two fillings were roughly equivalent with respect to ionization processes and the scattering of slow electrons. However, fast electrons were scattered faster by an order of magnitude when the Krypton is present. This is borne out by the probe signal duration, which decreased sharply to 1-2 μ s, as for example in Figure 2 (h). These tests were performed without other changes in machine or probe parameters. This result clearly indicates that the observed ring currents are carried by trapped high-energy electrons. As pointed out in Section IV, this conclusion is corroborated by the ratio of time integrated x-ray signals during the injection and during the decay and dump phase of the rings. Also, more recent measurements of the absolute x-ray intensity emitted by a .0007" tungsten wire spanned across the larger radius is consistent with this conclusion.

IV. The Equilibrium Shape of Relativistic Electron Coils

In this section axial profiles of the electron coil self-magnetic field along the tank axis and at the wall are presented. Examples of the development and statistics of the actual coil radius and length derived from this data are given.

At incremental times in the layer development, axial profiles were reduced from the signals of a series of magnetic probes placed at 2-1/2" spacings on axis and at the wall. An example is displayed in Figure 4. During the injection phase, the magnetic field changes generally seemed to extend over a rather large axial length. Then, the coil edges collapsed axially and the center built up as the plasma currents decayed, setting up a strong coil which then decayed in time. In other cases, the coil first moved down into the well-bottom over the first 1-3 μsec and then decayed in time. Conversion from field profiles to current profiles were made using a computer program developed by Porter and Weiss¹¹ and adapted to the present parameters. This program calculates the Green's function solution for a unit current ring, assuming an idealized radially thin layer with a $\cos^2 kz$ axial distribution of the current. To facilitate data conversion, the graphs given in Figure 5 (a), (b) and (c) were calculated. They give the mean radius and axial length (full-width-half-maximum, or FWHM) of the current distribution as a function of the signal ratio of two centrally located probes on-axis and at the wall, as a function of the axial length (FWHM) of the on-axis or the wall magnetic field profiles. As shown in Figure 5 (d), the fit of the recalculated field distribution with experimental data is quite satisfactory, at least over the axial length at half maximum of the coil.

The time development of the coil radii, axial centers, and axial length have been deduced from several examples and are shown in Figure 6. Note that the radius tends to vary little with time about an average value, that the coil settles repeatably into the same axial location, and that the axial length tends to remain constant for sufficiently strong self magnetic fields, and then diffuses axially at some later time as the self-field weakens. In particular, for two coils of the same initial strength, the higher pressure background causes an earlier lengthening of the coil, while for two coils with the same background pressure, the weaker coil generally is initially longer and lengthens earlier than the stronger coil. This observation is expected because of the dominant axial pinching forces of strong self-magnetic fields.

In Figure 7, some shot-to-shot statistics of the coil geometry are displayed versus hydrogen gas pressure. Rather independent of the ambient pressure, the mean layer radius reproduces to within about 10%, and the ring center settles in very closely identical axial positions. The initial lengths (FWHM) of the strong coils $\zeta_0 \gtrsim 1$ range from 7-10 cm, as compared with 10-13 cm for $\zeta_0 \lesssim 1$. In both cases, the length increases to around 16 ± 4 cm just before the dump (see Figure 7), with originally strong layers again generally exhibiting the smaller values. The exhibited fluctuations in radius and axial lengths of the established electron coils are larger than the estimated uncertainties in the data reduction, and probably relate to shot-to-shot variations in the initial trapping mechanism of the coils. More detailed investigations on correlations between various parameters are in progress.

Total trapped beam currents determined from the above computer program

range up to 5 kA. This combines with the average cyclotron period of 2 nsec for the fast electrons, to suggest that presently a few percent of the electrons injected into the tank are trapped. As mentioned earlier⁷, this finding agrees quite well with a ratio of about 10 between the time-integrated x-ray intensities observed during the first injection burst and that observed during the decay and dump phase of the rings. At present, the trapping mechanism can only be conjectured and first investigations have not yet led to definite clues. However, in the Livermore Astron experiment, enhancements of the trapping efficiency have been observed earlier⁴ for above normal gas fillings of several mTorr of hydrogen, and more recently¹⁶ for gas fillings of up to 100 mTorr.

V. Decay of Electron Rings Due to Classical Scattering

In this section, evidence is presented that during the linear-decay phase, the decay of the electron rings is dominated by classical scattering of the fast electrons in the background gas, rather than by any machine related mechanisms or by plasma instabilities.

As described in more detail elsewhere^{2,8}, the velocity diffusion coefficient of relativistic electrons due to multiple scattering in the background gas can be expressed as

$$D = \frac{16}{\pi} \frac{c^2 r_e^2}{\beta^3 \gamma^2} \sum_j f_j n_j Z_j (Z_j + S_j) \ln \frac{a_0 \gamma \beta^2}{2 Z_j^{4/3} r_e} \quad (1)$$

where c is the speed of light, r_e is the classical electron radius, $\beta = v/c$, $\gamma = (1 - \beta^2)^{-1/2}$ (v is the mean velocity of the electrons in the coil), a_0 is the Bohr radius (the impact parameter is chosen as the shell size of the dominant neutral gas background), and f_j , n_j , Z_j , and S_j denote the form factor, density of scattering centers, atomic number, and incoherent

scattering function of the j^{th} species. The form factor f_j represents a Born approximation to the atomic electron screening effects², and is unity to within 4% for our experimental conditions of 400 keV electron energy, and background gases from hydrogen to krypton. The incoherent scattering function S represents a correction for inelastic scattering by the bound electrons. S is unity for hydrogen and helium to within 10% for our experimental conditions, but it drops to lower values for heavier gases (where its contribution is less than 5% anyway). The effects of finite nuclear size are negligible.

Thus, according to Eq. (1), the time scale of a purely collisional decay due to multiple scattering of the fast electrons should be inversely proportional to

$$\frac{1}{\beta^3 \gamma^2} \sum_j n_j Z_j (Z_j + 1) \ln \frac{a_0 \gamma \beta^2}{22^{4/3} r_e} \quad (2)$$

This behavior was to be tested experimentally. For this purpose, the dependence of the negative slope of the linear part of the magnetic probe signals on the pressure and composition of the background gas was determined. As pointed out later in more detail, this slope proved to be quite independent of the initial strength of the rings and shot-to-shot statistics for given conditions appeared quite small ($\leq 10\%$). Also in view of the good reproducibility of the coil location, this slope seemed to provide a reliable diagnostic for the investigation.

First, several shots were made with hydrogen and helium gas background at pressures from 100 - 2000 mTorr, and with the peak diode voltage and external magnetic field strength and other conditions held constant to within 5%. The results are shown in Figure 8, including sample traces at 940 mTorr

and 110 mTorr hydrogen, and a plot of the resulting decay slopes. For hydrogen, the slopes seem to vary linearly with pressure as required by Eq. (2). The indicated straight line fits the data within the experimental statistics. The apparent offset of this line at the origin is most likely caused by the enhanced effect of the measured tank base pressure of a few mTorr of unknown composition, but probably air, with water vapor etc. For the monoatomic helium gas ($Z = 2$) the decay clearly is faster for identical pressures. The indicated straight line is calculated from the mentioned hydrogen line according to Eq. (2). It appears quite consistent with the observed data.

To get additional information on the atomic number dependence, three high- Z gases were used: argon, nitrogen and krypton. These gases cannot be used directly as the background gas, because the electrons are scattered too severely at the pressures needed to provide sufficient trapping. Therefore, a small percentage of high- Z gas was mixed with a 400 mTorr hydrogen reference pressure. The experimental procedure was first to establish a certain pressure of the high- Z gas in the tank with the hydrogen inlet shut off. Then the original hydrogen flow (400 mTorr) was reopened and the measurements performed. In the evaluation, the high- Z pressure readings had to be corrected to obtain the actually lower partial pressure of the admixed high- Z gas in the presence of the larger hydrogen pressure, because of the sweeping effect of the higher-pressure, lower-viscosity hydrogen. Table I presents a few examples of these corrections as calculated from Dushman¹⁷. In the first column, the high- Z gas pressures measured in the absence of hydrogen are recorded. The second column exhibits an effective pumping speed derived from the composite viscous-molecular flow through

the connecting hose and the rated pumping speed. After normalizing these effective speeds to that of 400 mTorr hydrogen, pressures given in the last column were obtained. For larger partial pressures, these corrected values agreed to within 15% with values derived from a direct measurement of the pressure difference with hydrogen only and with high-Z gas added.

Several shots were made with different impurity gases and partial pressures up to 80 mTorr. The straight lines shown in Figure 9 again were derived from the best-fit line for hydrogen (Figure 8) and the calculations in Table II of the predicted ratio of scattering times in hydrogen and in the other gases, assuming identical pressure and electron energies. As required in Eq. (2), account is made of the number of scattering centers per molecule, the factor $Z(Z+1)$ of each atom and the logarithmic -correction for the range of impact parameters.

The experimental data fit these relative predictions to within the experimental errors and statistics. In addition to the possible errors in the pressure measurements and in the slope determination, discrepancies could arise from day to day variations in diode characteristics, i.e. energy of the trapped electrons, etc.. Furthermore a systematic difference could result from differences in the collisional energy losses of the fast electrons in the various gas fillings, and corresponding differences in the gain in electron energy from the decaying self-field (betatron acceleration). The latter effects presently are under more detailed investigation, and will be published soon. Overall, we estimate that the sum of the experimental errors should not contribute more than 20-30% of the indicated pressure dependencies.

The inverse variation of the electron coil decay with the square of the energy could be verified on only a small range of energy. Statistical data

on the energy dependence over a wide range of energies is difficult to obtain at present because of the need to simultaneously change the external field strength, profile and other crucial parameters. A more detailed analysis of the energy dependence has to wait for the results of the mentioned energy measurements.

All the data in Figures 8 and 9 were obtained from wall probes located in the plane of the electron coil. This is sufficient for relative data for the various gases. For a comparison with theoretical predictions, the decay rates on the axis are of interest. For this purpose, the slopes obtained from on-axis probes are compared in Figure 10 with those of the wall probes, for the data of Figures 8 and 9. On the average, the slopes of the on-axis probes measures 4.7 times that of the wall probes.

VI. Comparison with Theory

In a recent theoretical analysis⁸, Humphries investigated the decay of self-focussed rings of relativistic electrons due to a slow collisional diffusion of these electrons. He proposed the model of a ring of electrons circulating at a radius R in a homogeneous magnetic field and having a maximum axial length of $2\Delta z$ and radial thickness of $2\Delta r$ (both defined by material boundaries). Furthermore, he assumed $\Delta z \sim \Delta r \ll R$ and a roughly constant current density over a cross sectional radius $a \approx 0.7 \Delta z$. The material boundaries Δr and Δz are assumed independent. Under these assumptions, which appear reasonably self-consistent, the electrons are expected to execute uncoupled simple harmonic oscillations in the axial and radial direction. Consequently, the diffusion of these oscillating electrons, due to small-angle multiple scattering in the neutral gas background, is expected to obey uncoupled cylindrical diffusion equations.

Assuming that the electrons are constrained radially by the external magnetic field B_0 and the image forces at the wall, he investigated the axial diffusion of the electrons against the self-magnetic field of the ring. Finally, he assumed that, after a short initial transient phase in which the higher modes decay, the axial and radial shape of the ring remains constant in time.

Under these assumptions, Humphries obtained his Eq. (9) for the time dependence of the total number $N_T(t)$ of electrons in the ring,

$$N_T(t) = N_T(0) - N_T(0) \frac{(2.405)^2 r_g^2 D}{\zeta(0) \Delta z} t \quad (3)$$

where $N_T(0)$ is the number of electrons at $t = 0$, r_g is the single-electron gyroradius in the external magnetic field and D is the diffusion coefficient defined in Eq. (2). The initial strength parameter is defined as $\zeta(0) = 4 I(0)/acB_0 = N_T(0) 2Be/\pi aRB_0$, where $I(0)$ is the initial circulating current in the ring. Thus, the decay of the number of electrons in the ring becomes linear in time and the decay rate, $dN_T(t)/dt$, is independent of the initial ring strength. Since the ring geometry is assumed time independent, the same results should be observed at any point in space for the self magnetic field generated by the ring.

Good qualitative agreement between these predictions and our experimental findings is apparent. As pointed out earlier, the various magnetic probe recordings indicate a quite linear decay. To check the independence of dN_T/dt on $N_T(0)$, the hydrogen data of Figure 8 was renormalized to the equivalent slope for a 400 keV electron coil in an external field of $B_0 = 190$ Gauss, and a gas background of $500 \mu H_2$. The base pressure offset (approximately $180 \mu H_2$, as seen in Figure 8) has been added into the data before renormalization. The wall probe data of Figure 8 is converted to

on-axis probe data using the 4.7 conversion factor deduced from Figure 10, and plotted versus the parameter $\zeta_0 = (\zeta_0 B(0)/B_0)_{\text{Axis}}$ defined in Section III. The data exhibits scatter about an average value of 7 Gauss/ μs , but still tends to be ^{sufficiently} independent of ζ_0 . The relation between ζ_0 and $\zeta_0(0)$ ^{from Eq. (1)} depends on the coil geometry, as discussed in more detail later. These observations agree with the theoretical predictions in spite of the observations presented in Figure 6 that the rings are not constant in length (as the theory assumes) but expand axially during their decay.

For a quantitative comparison, the theoretical model has to be adapted to the existing experimental situation. First, if the axial length of the observed rings was constant in time, then the decay rates of various probes at different positions would be directly proportional to the total number of electrons in the coil, and Eq. (3) could be directly rewritten to give the time dependence of the magnetic field generated by these rings,

$$\delta B(z) = \delta B(0) - \delta B(0) \frac{(2.405)^2 \tau_D}{\zeta_0(0) \Delta z} z \quad (4)$$

The following analysis will be performed for the self-magnetic field changes on the tank axis in the center of the rings, as subject to the conditions of 400 keV electrons in a 500 μH_2 gas background and a self-field equal in magnitude to an external field of $B_0 = 190$ Gauss (or $\zeta_0 = 1$). From Figure 11, these conditions imply that $\frac{1}{\zeta_0 B(0)} \frac{d}{dt} \zeta_0 B(A)$ has a mean value of 37 μs^{-1} to within 25%. For the sake of comparison, the experimental conditions leading to this number will be used in the rest of the section. Due to the actual axial lengthening of the experimentally observed rings during their decay, the decay rate predicted by Eq. (4) should be somewhat smaller than

the experimentally observed rate. Furthermore, the quantities r_g , D , ζ and Δz have to be identified with experimental observations. For the above experimental conditions, $r_g \approx 13$ cm, and a straight forward calculation of Eq. (2) yields the coefficient $D = 4.2 \text{ ms}^{-1}$.

In the Humphries model, Δz denoted the axial length limitation of the rings. As shown in Figures 1 (a) and (b), no such axial limitations exist close to the wall bottom where the rings are trapped. Material objects (walls, injector, etc.) are far removed from the axial extension of the rings, and even the axial changes of the externally applied mirror field are small compared with changes in the induced self-field over the ring dimensions. On the other hand, there is some evidence that contradicts the theoretical assumptions of a parabolic spatial variation of the self-field (Figure 4 suggests a faster axial change - note dashed line) and a large aspect ratio (coil radius to radial ratio) (from Figure 6, $R = 9$ cm, and initial observations with a TRW framing camera indicate radial widths exceeding 3 cm). Considering the long decay times in question (up to 10,000 orbits), the linearized equations of motion may not be proper, and higher order coupling of the axial and radial motion is possible. This possibility is enhanced by the results of x-ray measurements discussed in Section III, which exhibit a constant radial loss of electrons to the wall. Thus, the effective limitations of the rings should be somewhere between one half of the radial space (i.e. about 10 cm) and the axial length of the rings (i.e. 10-15 cm). In the following discussion, it will be assumed that $\Delta z = 12$ cm.

In Humphries model, $\zeta(0) = 4I/acB_o = \delta B_{out} - \delta B_{in}/B_o$, δB_{out} equal to the difference in magnetic field strength between the outer and inner edge of

the ring normalized to the external applied magnetic field at the ring center. In the experiments the quantity $\zeta_0 = (\delta B/B_0)_{\text{Axis}}$ is measured. The connection between these two parameters depends critically on the coil geometry. For a large aspect ratio ring, $\zeta = 2 I/cR$, so that $\zeta(0) = 2 R/a$ $\zeta_0 = 2.25 \zeta_0$ where the mean coil radius given in Figure 6, $R = 9$ cm., and the theoretical assumption, $a = .7\Delta z \approx 8$ cm. has been used. Thus, Eq. (4) implies that $\frac{1}{\delta B(0)} \frac{d}{dt} \delta B(t) = 12 \text{ ms}^{-1}$, which is roughly a factor of three below the experimental number. This agreement is satisfactory in view of the idealizations of the theory. Note that $R/a \approx 1.1$, which is clearly not large enough to justify that approximation.

In an effort to obtain better agreement with experiment and also to relate more closely with earlier data reduction methods, the homogeneous donut model is relaxed to the Porter-Weiss current distribution¹¹ used in Section IV. The electron coil is assumed to be radially thin (an assumption which implies no radial diffusion and a large aspect ratio) but with a $\cos^2 kz$ axial current profile, as exhibited in Figure 5 (d). If the signal ratio of the on-axis magnetic probe to the wall probe is constrained to fit the mean value of 4.7 deduced from Figure 10 and the mean coil radius is 9 cm as in Figure 7, then Figure 5 (a) and (b) suggest that $L \sim 15 \sim 20$ cm, which is a little longer than the range of values in Figure 6. Using the Porter-Weiss theory, it follows that $\zeta(0) \sim 1.6 \zeta_0$. For the experimental conditions defined earlier, the 17 ms^{-1} , which is a little less than half the experimental number.

An electron coil geometry which may be a better approximation to actual experimental distribution has been developed by Beal et. al.⁴. The coil electrons are assumed to have a $\cos^2 Kz$ axial current distribution and a

$J_1(k_0 r)$ radial distribution, where $k = \frac{2\pi}{L}$. For the special case that the coil extends radially from the axis to the wall, or $k_0 r_w = 3.83$, then the radial diffusion can not occur, and the peak of $J_1(k_0 r_w)$ occurs at $R = 9.6$ cm (in good agreement with Figure 7, and Lovelace has shown that $\zeta(0) = 1.4 \frac{4L^2 + 170}{4L^2 + 85} \zeta_0 = 1.5 \zeta_0$ for $L \sim 15$ cm. Under the experimental conditions defined earlier, the normalized decay rate is 18 ms^{-1} , or roughly half the experimental number. A more thorough investigation of this model will be deferred until current efforts to measure radial distribution are completed.

In light of the simplifications necessary to theoretically describe the electron coil diffusion, the agreement between theory and experiment is satisfactory, and combines with the good experimental verification of the small scattering theory (in Section V) to insure that the observed electron coil diffusion is predominantly classical.

VII. Conclusions Summary

Observations of high current relativistic electron coils with magnetic field changes up to twice that of the external magnetic mirror field have been presented. Corroboration of magnetic probe, optical, and x-ray diagnostic have combined with various basic tests (changes in the mirror field, a lucite strip obstacle, and high-Z gas perturbations) to prove conclusively that the observed electron coils are comprised of relativistic electrons.

The time-development of the self-magnetic field profiles have been analyzed. The Porter-Weiss computer program for the Green's function solution and an assumed $\cos^2 kz$ axial current distribution has been used to obtain information about the coil average radius and axial length at half-maximum. The time-resolved development of the coil radius, axial center, and axial length has been discussed along with statistics on the coil location.

Evidence that the electrons in the coil are subject to classical small angle multiple scattering has been presented. The observed linear decay of the electron coils has been shown to vary linearly with tank pressure. Data on different admixtures of hydrogen and various high-Z gases has verified the predicted $Z(Z + S)$ dependence on atomic number. Indications that the scattering times are inversely proportional to the square of the energy have been mentioned.

The observed decay of the electron coils has been compared with Humphries theory on the collisional decay of strong self-compressed electron rings. Generalizations of the Humphries geometry to the Porter-Weiss geometry, and the geometry of Beal et. al. have been discussed. Good qualitative agreement has been shown, in that the theory predicts the experimental observations of linear electron coil decay at a rate which is independent of the initial strength of the coil. Satisfactory quantitative agreement has been demonstrated, in that the normalized decay rates predicted by the three coil geometries studied are from one-third to one-half the experimentally deduced number. The main reasons for this discrepancy have been identified with experimental observations (beyond the scope of the theoretical models) of lengthening of the coil axial distributions during decay x-ray measurements of the loss of electrons radially to the walls, and first observations of radial distributions which indicate small aspect ratios.

In conclusion, clear evidence has been presented that self-compressed field reversing relativistic electron coils can exist in a stable equilibrium. The loss mechanism has been well established as small angle multiple scattering of relativistic electrons in the coil. The experimentally

observed decay rates are in satisfactory enough agreement with theoretical guidelines to insure that the diffusion of the electron coils is predominantly classical.

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FIGURE CAPTIONS

- Figure 1. (a) Schematic diagram of RECE-Berta experimental device.
(b) Typical on-axis magnetic field profile.
- Figure 2. (a) Diode voltage (copper-sulfate voltage divider).
(b) Diode current (annular shunt of carbon resistors).
(c) Net current at point of injection into mirror (Rogowski coil).
(d) External magnetic field coil current (1 m Ω shunt).
(e) Electron coil self-magnetic field trace measured by a wall probe located near the coil center.
(f) Intensity of coil-produced light emitted along the axis from a radial sector of the coil (PM tube).
(g) X-ray intensity emitted from coil (PM tube-scintillator).
(h) Electron coil self-magnetic field trace measured by an on-axis probe for a tank filling of 400 μH_2 and 40 μK_r .
- Figure 3. Record of the strongest electron coil observed to date. The mirror configuration shown in Figure 1 (b) was used, along with a 420 keV, 8.3 kA beam, a 650 μH_2 background gas pressure, and 190 Gauss external magnetic field. The maximum value of ζ was 190%.
- Figure 4. Electron coil axial magnetic field profiles at selected times during a typical event.
- Figure 5. (a) Estimate of electron coil mean radius.
(b) Estimate of electron coil length at half maximum current (FWHM) from axial magnetic field data.
(c) Estimate of coil length from off-axis field data.
(d) Comparison of experimental magnetic field profile with that calculated for a cosine-squared axial current distribution.
- Figure 6. Examples of the time development of the electron coil geometry.
- Figure 7. Statistical location of electron coil radius, axial center and axial length of half the maximum current (FWHM).
- Figure 8. Results of the scattering experiments with low-Z background gases.
- Figure 9. Results of the scattering experiments with a mixture of high-Z background gases and a 400 μH_2 reference background.
- Figure 10. Relation between the self-magnetic field decay level measured by on-axis and wall magnetic probes which are located near the statistical center of the electron coils.
- Figure 11. The self-magnetic field decay rate versus the strength of electron coils with an average energy of 400 KV in a 500 μH_2 background.

TABLE I

PUMPING SPEED CORRECTIONS			
Gas	Measured Pressure (microns)	Pumping Speed (lit./sec.)	Corrected Pressure (microns)
H ₂	400	7.0	400
N ₂	30	4.0	20.5
A	28	3.6	14.3
A	15	3.0	6.3
Kr	15	2.6	5.6

TABLE II

CLASSICAL SCATTERING CORRECTIONS

GAS	N	Z(Z+1)	$\log \frac{10^5 E \text{ (MeV)}}{4 Z^{4/3}}$	H ₂ RATIO
H ₂	2	2	9.2	1
He	1	6	8.4	1.36
N ₂	2	56	6.6	20.1
A	1	342	5.4	50.4
Kr	1	1332	4.4	160

Fig. 1(a)

o.k.

Fig. 1(n)

Fig. 2

(a) 200 KV/div.
200 ns/div.

(b) 7.3 KA/div.
200 ns/div.

(c) 3 KA/div.
200 ns/div.

(d) 122 A/div.
5 ms/div.

(e) 29 Gs/div.
2 μ s/div.

(f) 50 mV/div.
2 μ s/div.

(g) 20 mV/div.
2 μ s/div.

(h) 29 Gs/div.
2 μ s/div.

5 divisions

108 cm

On-axis probes

113 cm

Fig. 3

E-LAYER MAGNETIC FIELD PROFILE (Gauss)

SHOT: 19
DATE: 4-17-72
STANDARD MIRROR
CONFIGURATION
 $700 \mu\text{H}_2$
 $B_0 = 192$ Gauss
PROBE CALIBRATION
3.2 Gauss/mV

FIELD
REVERSAL

ON-AXIS PROBES

OFF-AXIS PROBES

POSITION (cm)

$\times 10^{-8} \cdot S(0)$

Figure 3(a)

Fig. 4(c)

Fig. 2

+12.1

Fig. 8

940 μH_2
2 $\mu\text{s}/\text{cm}$

95 cm
Wall Probes
 $\int B dt$
27 Gauss/cm
107 cm

110 μH_2
2 $\mu\text{s}/\text{cm}$

Fig. 9,

400 μH_2 plus 8 μAr
2 $\mu\text{s/cm}$

$\int \dot{B} dt$
11 Gauss/cm
107 cm
wall probes
 $\frac{\partial \theta}{\partial t}$
2 V/cm

400 μH_2 only
2 $\mu\text{s/cm}$