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ρ^0 Production Density Matrix Elements,
Phase Coherence and Smoothness
of the Ball Amplitudes*

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ABSTRACT

Our experimental observation that $T_{p_{1-1}^{11}} \approx 0$ in an extended range of s and t in $\pi^- p \rightarrow \rho^0 n$ imposes strong constraints on the helicity amplitudes. The Ball invariant amplitudes satisfy these constraints only if $B_1 \approx -B_2$. The last condition, with the smoothness assumption, causes $H_{+-}^{11} - H_{+-}^{-11}$ to vanish at $t \approx -\mu^2$. This results in a zero of $\text{Re } \rho_{10}^{811}$ at $t \approx -\mu^2$ which is clearly not due to phase incoherence as claimed in a recent amplitude analysis.

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A recent claim for a model independent experimental evidence for a possible phase incoherence between p-wave helicity amplitudes at $t \approx -\mu^2$ has revived both theoretical and experimental interest in understanding the origin of this result¹. Correctness of such a claim would have important theoretical consequences for the description of both the single pion production and the extraction of the $\pi\pi$ phase shifts.

Comparison of recent high energy polarized photoproduction² ($\gamma + N \rightarrow \pi + N$) and ρ^0 meson production³ ($\pi^- + p \rightarrow \rho^0 + n$) experiments have given us a clue to find a fundamental dynamical property of helicity one, unnatural parity, single pion production amplitude. In brief, we noted experimentally that the claimed phase incoherence¹, the zero of $\text{Re } t_{\rho_{10}^{11}}$ in ρ^0 production (Fig. 2), the zero of the D_- amplitude⁴ (defined below) and the maximum of the asymmetry ratio in photopion production all occur at the same value of $t \approx -\mu^2$. In addition, we have $t_{\rho_{1-1}^{11}} \approx 0$ experimentally for an extended range of t for ρ^0 production. (Here $t_{\rho_{nn}^{11}}, (t_{\rho_{nn}^{11}})$ denotes the $t(s)$ channel density matrix elements of the dipion system.) The above similarities between the two reactions and their energy independence suggest that we are observing a manifestation of a general result. Although the question of vector meson dominance is outside the scope of this paper we use the mathematical formalism developed for this purpose earlier⁵.

In Fig. 1 we show the density matrix elements $t_{\rho_{1-1}^{11}}$ for $\pi^- p \rightarrow \rho^0 n$ at the pion lab momentum⁷ of 2.7, 4.1, 6, 7, 8, 15 and 17 GeV/c for momentum transfer range of $0 < |t| < 8 \mu^2$. A truly striking fact that $t_{\rho_{1-1}^{11}} \approx 0$ is observed. Note that the zero is non trivial in the sense that it is obtained from non vanishing amplitudes which contribute significantly to $t_{\rho_{11}^{11}} (t_{\rho_{11}^{11}} \approx 0.1)$.

Now we discuss the implications of these results. We introduce the usual s-channel helicity amplitudes $H_{\lambda_n, \lambda_p}^\lambda$ where λ , λ_n and λ_p denote the rho meson, neutron and proton helicities respectively. It is also useful to introduce the amplitudes $G_{\lambda_n, \lambda_p}^m$ by the relation

$$G_{\lambda_n, \lambda_p}^m(s, t) = \sum_{\lambda} d_{m\lambda}^1(\chi) H_{\lambda_n, \lambda_p}^\lambda(s, t) \quad (1)$$

Here $d_{m\lambda}^1(\chi)$ is the usual d function of the rho meson crossing angle χ . For large s and small t

$$\cos \chi \approx \frac{m_\rho^2 - \mu^2 + t}{m_\rho^2 - \mu^2 - t}, \quad \sin \chi \approx \frac{2\sqrt{-t} m_\rho}{m_\rho^2 - \mu^2 - t} \quad (2)$$

It is well known that although G's are not t-channel helicity amplitudes, their products give the t-channel density matrix elements after summing over nucleon helicities. The condition $t_{1-1}^{11} = 0$ then implies that $\text{Re} [G_{++}^{1-} G_{+-}^{1*} + G_{+-}^{1-} G_{++}^{1*}] = 0$.

For small t, one pion exchange and associated absorptive corrections have been known to be the dominant contributions. It is also known that the ++ amplitudes are down by a factor of 1/s compared to the +- amplitudes in such models. Hence the above relation cannot be satisfied by cancellation between the two terms. Thus we are led to the conclusion that $G_{++}^{1-} \approx 0$. In terms of $H_{\lambda_n, \lambda_p}^\lambda$ these conditions give the following model independent constraints

$$(1 - \cos \chi) H_{++}^{1-} + (1 + \cos \chi) H_{++}^{1+} + \sqrt{2} \sin \chi H_{++}^0 \neq 0. \quad (3)$$

Thus the helicity amplitudes with different helicity flips have to be correlated in a special way. Note that simple angular momentum arguments show that for one elementary pion exchange all $G_{\lambda_n, \lambda_p}^{\pm 1}$ are zero. Hence in such models the extra contributions (ΔH) will satisfy the same constraints. These contributions could be the absorptive corrections in OPE plus absorption models,

nucleon Born terms in gauge invariant Born models, or Regge pole-Regge cut terms in cut-models. Eq. (3) can also be derived directly from the t-channel helicity amplitudes by using the crossing matrix. The constraints (3) should be valid for an extended range of s and at least up to $|t| \approx 8 \mu^2$. A model by Froggatt and Morgan⁸ satisfies these constraints. In the following, however, we attempt to understand them in a more general way.

First we express the helicity amplitudes in terms of the well known Ball invariant amplitudes B_1, B_2, B_3 . Since the general equations are rather complicated, it is more transparent to neglect terms of the order of $1/s$ and obtain¹⁰

$$T_+ = H_{++}^0 = \frac{m_\rho s}{2m} \left[\frac{t + m_\rho^2 - \mu^2}{2m_\rho^2} B_0 + \frac{B_0}{m_\rho^2} - \frac{sB_0}{2m_\rho^2} \right] \quad (4)$$

$$T_- = H_{+-}^0 = -\frac{m_\rho \sqrt{-t}}{m} \left[\frac{t + m_\rho^2 - \mu^2}{2m_\rho^2} B_3 - \frac{sB_2}{2m_\rho^2} \right] \quad (5)$$

$$L_+ = H_{++}^1 + H_{++}^{-1} = \frac{\sqrt{-t/2}}{m} B_0 \quad (6)$$

$$D_+ = H_{++}^1 - H_{++}^{-1} = \frac{\sqrt{-t/2}}{m} sB_0 \quad (7)$$

$$L_- = H_{+-}^1 + H_{+-}^{-1} = -\frac{1}{\sqrt{2}m} (-sB_1 + 2m B_0) \quad (8)$$

$$D_- = H_{+-}^1 - H_{+-}^{-1} = -\frac{1}{\sqrt{2}m} (-sB_1 - 2tB_3 + 2mB_0) \quad (9)$$

In what follows we make use of the fact that $H_{++}^1 \approx -H_{++}^{-1}$. One pion exchange satisfies this relation exactly. Since both the amplitudes have the same net helicity flip it is known that the absorptive corrections are also same^{11,12}. In fitting data on Rho production this condition was also found correct empirically.¹³ This gives $B_0 \approx 0$. We also note that the s and u channel nucleon poles do not contribute to B_0 .

Now assuming that all the Ball amplitudes are independent of k^2 ((off shell vector meson mass)²), Cho and Sakurai¹⁴ and Achasov and Shestakov¹⁵ have derived the following relations between the Ball amplitudes

$$sB_0 - (t - \mu^2) B_8 = 0 \quad (10)$$

$$-2B_8 + sB_6 - (t - \mu^2) B_9 = 0 \quad (11)$$

These relations constitute the smoothness assumption. Note that our constraint equations (3) are different from the equations connecting longitudinal and transverse amplitudes given in Reference 14.

Now it can be easily seen that the relation (3) for ++ amplitudes is automatically satisfied by imposing $H_{++}^1 \approx -H_{++}^{-1}$ and Eq. (11) i.e.

$$+ \sqrt{2} \cos \chi \frac{\sqrt{-t}}{2m} sB_8 + \sqrt{2} \frac{\sin \chi}{4m} m_p sB_9 \approx 0 \quad (12)$$

irrespective of the value of B_9 . Trivial satisfaction of the constraint however will require $B_8, B_6, B_9 \approx 0$. Our numerical experience shows that although the ++ amplitudes are small, they are not negligible specially at low energies and at momentum transfers where other amplitudes vanish. Later on, we will use the fact that D_+ (or B_8) is small compared to D_- , especially at higher energies. In leading order of s , this has the quantum numbers of A_1 exchange. Analyses of the photoproduction experiments are consistent with the smallness of this term. Note that even granting $B_9 \approx 0$, there will be still a question of sB_8 unless the smoothness equations are used. However, because of the smallness of the ++ amplitudes, the ++ constraints cannot be said to imply smoothness although they are mutually consistent.

For the (+-) amplitudes we find the constraint to be

$$(1 - \cos \chi)(-sB_1) + \sin \chi \frac{\sqrt{-t}}{m_\rho} (-sB_2) \\ + \left[2t \cos \chi + \frac{\sin \chi \sqrt{-t}}{m_\rho} (t + m_\rho^2 - \mu^2) \right] B_3 \approx 0. \quad (13)$$

The last term is readily seen to vanish. Cancellation of the first two terms requires a dynamical condition $B_1 \approx -B_2$ for the range of s and t under consideration. Note that the smoothness conditions have not been used yet.

If we do use Eq. (10) however, the condition ($B_1 \approx -B_2$) has a very interesting consequence for the amplitude D_- . Since $B_3 \approx 0$, and $B_1 \approx -B_2$, Eq. (10) gives $D_- \approx \frac{1}{\sqrt{2}m} (\mu^2 + t) B_3$. This shows that irrespective of the value of B_3 , D_- will have a zero at $t \approx -\mu^2$. Now the density matrix element $\text{Re } s_{\rho_{10}}^{11}$ is given by

$$6 \text{Re } s_{\rho_{10}}^{11} = \text{Re} [D_+ T_+^* + D_- T_-^*] \quad (14)$$

If D_+ is small, Eq. (14) implies that $\text{Re } s_{\rho_{10}}^{11}$ passes through zero at $t \approx -\mu^2$. This is in remarkable agreement with the experimental data plotted in Fig. 2 at several energies.¹⁵ In a recent amplitude analysis Estabrooks and Martin¹ found a solution in which this zero arose from the phase incoherence between the H_{+-}^1 and H_{+-}^0 amplitudes. Our discussion here relates the zero in $\text{Re } s_{\rho_{10}}^{11}$ to a zero in the amplitude D_- in a much more fundamental manner without requiring any phase incoherence whatsoever. (It is clear that the same zero arises in $\text{Re } s_{\rho_{10}}^{10}$ and in photoproduction.

As for the ratio of density matrix elements we have

$$\frac{s_{\rho_{1-1}}^{11}}{s_{\rho_{11}}^{11}} = \frac{|L_+|^2 + |L_-|^2 - |D_+|^2 - |D_-|^2}{|L_+|^2 + |L_-|^2 + |D_+|^2 + |D_-|^2} \quad (15)$$

Clearly if $|D_+| \approx 0$ this ratio becomes ≈ 1 at $t \approx -\mu^2$. Again this is confirmed by the data.^{8,17} This ratio in the case of photoproduction is the

asymmetry parameter for the isovector photons and is also found⁶ to be ≈ 1 at $t \approx -\mu^2$. These latter facts have been discussed for the case of photo-production and Rho production by Ross et al.¹¹, Cho and Sakurai¹², Contogouris¹³ et al, and Williams¹⁴, using various models. Their implications for the phase coherence have been discussed recently by the present authors¹⁵.

In conclusion we wish to emphasize that the condition $B_1 \approx -B_2$ is demanded by the experimentally observed fact that $t_{\rho_{11}} \approx 0$ for a wide range of incident beam momenta (2.7 - 17 GeV/c) and for at least $0 < |t| < 8\mu^2$. Primary use of the smoothness assumption has been made for the region $|t| \sim \mu^2$ in producing a zero in $\text{Re } s_{\rho_{11}}$. Thus upto $|t| \sim \mu^2$ the smoothness assumption is experimentally correct. Our model independent constraints (Eq. (3)) must be satisfied by any theoretical model (e.g. absorptive model, gauge invariant Born model, Regge cut model, Veneziano model, etc.) in the energy and momentum transfer range under consideration.

We wish to thank A. Contogouris for many illuminating discussions and F. J. Loeffler for his generous support. L. G. would like to thank F. Turkot and Brookhaven National Laboratory for a summer appointment. K. V. wishes to acknowledge award of an I.U. Summer Research Fellowship. Our thanks are also due to our colleagues at the Universities of Colorado, Pennsylvania, Toronto, Wisconsin, Notre Dame and Stanford Linear Accelerator Center for participating in the data compilation.

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FIGURE CAPTIONS

- Fig. 1 The experimentally determined density matrix elements ρ_{1-1}^{11} .
 As indicated in the graph the data were taken at 2.7, 4.1, 6, 7, 8, 15, and 17 GeV/c incident momenta. The z axis is along the direction of the incident pion in the dipion rest frame (t channel helicity frame).
- Fig. 2 The experimentally determined density matrix elements $\text{Re } \rho_{10}^{s11}$.
 As indicated in the graph the data were taken at 4.1, 6, 15 and 17 GeV/c momenta. The z axis is opposite to the flight of the recoiling nucleon in the dipion rest frame. (s channel helicity frame.)

Fig. 1

Fig. 2