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Subject Summary Report on the Re-
entry Burnup of Graphite
Particles

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I. INTRODUCTION

A considerable effort has been made since the inception of the Nerva Project to ascertain the amount of aerothermodynamic burnup that could be expected for various sizes of graphite particles during re-entry. The need for such information stems from the safety considerations involved with the operation of a nuclear rocket; in particular, the question of whether or not it is feasible to prevent widespread surface contamination of the earth from ground impact of a highly radioactive core by first fragmenting the core into small particles that would be completely oxidized upon re-entry and remain in the upper atmosphere for long periods of time.

It is well known that graphite is highly resistant to oxidation and melting. Because of this (and other favorable nuclear properties) it is an attractive material for use in nuclear rocket cores. On the other hand, these same properties are a decided disadvantage when one is interested in burning up graphite by aerodynamic heating and oxidation.

Graphite may be classified as a non-ablative material in contrast to more familiar structural materials such as steel and aluminum. It does not melt under ordinary conditions (only at high temperatures and pressures) and its rate of sublimation is insignificant up to temperatures of 3000°K. Since temperatures of this magnitude are not obtained under the re-entry conditions encountered by graphite bodies returning from earth orbit, any detectable mass losses during atmospheric re-entry of a graphite particle can be attributed to chemical reaction with the ambient gases, oxygen and nitrogen. Under the appropriate temperature and pressure, graphite will undergo reaction with both nitrogen and oxygen. However, it has been demonstrated that for surface temperatures below 2800°K, the nitrogen-carbon reaction is nil; ⁽¹⁾ hence from the foregoing information the problem apparently reduces to consideration of mass depletion by reaction with oxygen alone. But it should be pointed out that under the proper aerodynamic conditions, viz, molecular flow, there is the possibility that nitrogen could contribute to the mass loss rate. In formulating the re-entry code, nitrogen-carbon reactions were not considered since (1) the reaction kinetics of a nitrogen-carbon system under hypersonic molecular flow conditions are not

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known well enough to derive a mathematical model for use in the re-entry code; (2) the exclusion of a nitrogen reaction in the code produces conservative results. We are currently considering the nitrogen-carbon reaction kinetics under hypersonic molecular flow in an attempt to evaluate the probability and appropriate analytical model for such a reaction. If the results of this study appear promising, the re-entry code will be revised accordingly and a report issued on the findings. However, it would be unduly optimistic at this time to expect nitrogen to have a pronounced influence on the re-entry burnup of graphite as given in this report.

The problem of determining mass losses during re-entry for a non-ablative body depends on both the aerothermodynamics involved during re-entry as well as the physical chemistry of the oxidation. Thus, in order to evaluate mass loss rates the following aerothermodynamic considerations must be taken into account: (1) is the gas flow viscous, inviscid, or molecular, (2) is a shock wave present, (3) what is the mechanism and rate of mass transport to the body surface. In the realm of chemistry one must ascertain (1) the energy state of gases behind the shock, (2) the rate of reaction between these gases and the solid surface. It then becomes apparent that the re-entry heating and mass loss of graphite is a complex problem, and moreover, that one cannot establish a single analytical model to describe the heating and oxidation throughout the entire spectrum of re-entry aerodynamics.

There are three general classifications of flow regimes in hypersonic re-entry aerodynamics which because of their inherent characteristics determine the heating and mass transfer analysis that can be used to describe these phenomena. These flow regimes* are termed free-molecule, transition and continuum flow and the existence of any one regime is dependent on the dynamics of the gas stream flowing past the body. In particular, the criterion for the existence of a given flow regime is generally recognized as being dependent upon the Knudsen number, which is the ratio of the mean free path of the gas just in front of the body to the characteristic body dimension. The characteristic dimension is some fundamental dimension of the general body shapes; for a sphere it is the radius.

It is generally considered that free molecule flow occurs whenever the Knudsen number is greater than ten; transition flow for Knudsen numbers between ten and one third and continuum flow for values less than one third. There is some latitude in these rather arbitrary limits for the flow regimes and it is not uncommon to find that some authors will consider free molecule flow occurring for all Knudsen numbers greater than one, transition flow between one and 0.01 and continuum flow at values less than 0.01. This inconsistency does not actually affect the energy and mass transfer calculations to any

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* There are some authors who prefer to compartmentalize the flow regimes further in an attempt to form more rigid analytical models around which they may base their theoretical work in the area of hypersonic flow. However, the classifications as we have given them here are sufficient for our purposes without becoming too pedantic.

great extent overall. Physically, in the case of hypersonic flow, the free molecule flow regime exists whenever the gas stream mean free path is large compared to the body size, to the extent that each gas molecule interaction with the body surface is completely independent of the presence of other gas molecules. In transition flow, the flow field in front of the body becomes dense enough that only a fraction of the incident molecules that could be intercepted by the body behave in the manner of free molecule flow, the rest being impeded prior to contact with the body by either molecules returning from the body surface or other incident molecules. Continuum flow occurs whenever the flow field becomes so dense that a definable shock wave appears in the gas stream just ahead of the body. Here the free stream gas undergoes an adiabatic compression as it passes into the shock wave and consequently is both heated and partly dissociated, with the degree of heating and dissociation depending on the intensity of the shock.

For each of the three flow regimes, analytical models have been constructed to determine the aerodynamic heating and mass loss rates. We will only cover the barest of essentials here, making reference as necessary that more thorough presentations of the re-entry energy and mass transfer considerations are contained in WANL-TNR-039, WANL-TNR-067, and WANL-TNR-072.

Aerodynamic heating of a graphite body in the free molecule regime can be analytically determined by a modified method of the number of body surface-gas molecule collisions occurring in a volume of "stationary" gas swept out by the moving body. The modifications take into account the Maxwellian energy distribution of the gas molecules and the chemical energy input by oxidation of surface carbon atoms. The analytical expressions can be developed starting from the classical analysis made by Stalder and Jukoff (References 2 and 3).

Mass transfer in the free molecule regime is equally as straightforward as the heat transfer analysis. Because of the aforementioned properties of graphite all mass losses during orbital re-entry can be attributed to oxidation of surface carbon atoms; hence mass loss rates are a function of the number of gas molecules (and atoms) colliding with the surface and the probability of reaction. Due to the nature of free molecule flow and the reaction kinetics between oxygen atoms and molecules with a clean graphite surface, our mass transfer model is centered around the hypothesis that under hypersonic flow conditions every atom or molecule hitting the surface will react with a carbon atom. The basis for this rather bold statement has considerable merit from experimental evidence and the full discussion of the free molecule reaction kinetics in the case of the graphite-oxygen atom reaction is presented in WANL-TNR-067. A detailed analysis of the oxygen molecule-graphite reaction in the hypersonic free molecule flow regime will be forthcoming.

In contrast to the free molecule case, the energy and mass transfer phenomena for graphite in the continuum flow regime presents a complicated picture, due primarily to the existence of the shock wave in front of the body and the physical nature of the partially ionized gas layer between the body surface and the shock front. There have been several theories proposed to describe analytically the continuum flow heat and mass

transfer processes for a graphite body, but from the standpoint of basic high temperature gas physics and reaction kinetics, the best of these is that developed by S. M. Scala (Reference 4). Moreover, as shown in WANL-TNR-072, Scala's theory also agrees with experimental evidence on graphite mass loss rates as obtained in a hypersonic plasma wind tunnel.

Scala's theory has definite boundary conditions which limit the applicability of the heat and mass transfer equations for a re-entering body, and it is most important that one be aware of this fact; otherwise, very inconsistent and paradoxical results will be obtained. Briefly, Scala's theory is applicable only for well defined continuum flow, i.e., Knudsen numbers on the order of 0.01 or less and the presence of a strong shock wave. Translated into less esoteric terms, the Scala theory best describes energy and mass transfer as long as the body velocity is above Mach 10 and its surface temperature greater than 1000°R, provided the Knudsen criteria is met. These limitations arise from the mathematical and physical restraints imposed in formulating the theory; a full discussion of these assumptions is presented in WANL-TNR-072 and WANL-TNR-039. It should be pointed out that the very basic difference between free molecule and continuum flow is that mass transfer is reaction rate controlled in free molecule flow and diffusion controlled in continuum flow. Moreover, the free molecule mass and heat transfer mechanisms are the most efficient that can be obtained and therefore represent the upper limit for both processes.

Unfortunately, the aerothermodynamics for the transition flow regime is not as amenable to mathematical analysis as either the continuum or free molecule regimes. The basic difficulty here is that transition flow is not a "mixture" of free molecule and continuum flow but a series of separate entities of changing gas flow fields, each with its own physical characteristics. Discontinuities occur in the flow field and the best of the theories to date are generalized approximations. Fortunately, in one respect, a large portion of the transition flow regime is covered by the subregion known as the near-free molecule flow regime, which covers the range of Knudsen numbers less than 10 but greater than one third (approximately). The region between Knudsen numbers of 1/3 and 0.01 is where most of the flow field discontinuities arise, and, fortunately the conditions that give rise to such flow fields are quite unstable and exist only for a very short time compared to the total re-entry time. Therefore, for graphite re-entry, we have considered that near free molecule flow theory adequately describes, within acceptable error limits, mass and heat transfer phenomena in the transition flow regime.

There have been a number of analytical models proposed to describe aerodynamic heating in the near free molecule region. We selected the method developed by Probstein (Reference 5) which is adopted from an earlier work by Willis. Probstein's analysis does not always provide a smooth transition between the near-free molecule and continuum flow heating, with the biggest discrepancies occurring for particle sizes less than 1/8 inch diameter. However, it is the mass loss analysis which is of prime interest to us, rather than aerodynamic heating per se, and here we have found that the near free molecule mass transfer can be approximated by incorporating the very same analysis developed for the free molecule flow regime. This is not to be implied that the actual process is the

same for both regimes, or that the reaction products are identical. Rather, by careful reasoning, we find that the overall effect of mass transfer in both regimes is equivalent. Moreover, we generally find that no large discrepancy exists between the mass transfer rates as postulated for near free molecule flow and that of continuum flow at the transition region between the two regimes.

With this very brief background we are now in a position to move on to a discussion of (1) the WANL digital computer re-entry code (WEREC), (2) the results obtained with this code and (3) a comparison of these results with those obtained by others in previous work on this subject. In order to make a more logical presentation, it is necessary that the following text be arranged chronologically to place the results and conclusions in the proper perspective.

II. PREVIOUS STUDIES ON RE-ENTRY OF NUCLEAR ROCKETS

One of the earliest studies of the potential hazards attendant with the preparation, launching and operation of nuclear rockets was performed by the Martin Company under a contract with the AEC. The results of Martin's efforts were summarized in their report No. MND-2517.

On the subject of graphite re-entry, Martin carried out a parametric study wherein the initial particle size and flight angle (re-entry angle) were varied and the initial re-entry velocity and altitude maintained constant at 25,000 fps and 400,000 feet respectively. The basic mass and heat transfer expressions used by Martin were derived from a simplified free molecule analysis and the Scala analysis, for the free molecule and continuum flow regimes respectively. In order to predict the switchover point from molecular to continuum flow, they used an analysis by Probstein (Reference 5) to predetermine the Knudsen number and thus the altitude at which continuum flow began and Scala's analysis could be employed.

The results of the Martin study indicated that graphite mass losses by ablative processes were nil, and all mass depreciation was the result of oxidation. The most important result, however, was that spherical particles 1/24th inch or less would be completely oxidized during re-entry from 400,000 feet on an initial re-entry angle of -1° or less. This latter point was the stimulus for more exhaustive studies on our behalf to verify the Martin results and gain a better insight into the feasibility of fragmenting the core into small pieces that would burn up during re-entry.

The first WANL studies had a twofold purpose: (1) to provide a base from which we could conduct more rigorous analytical studies if necessary, and (2) to obtain data that would serve as input to an experimental program for verifying Scala's theory on mass loss rates in continuum flow. Like the Martin study the WANL program was a parametric study. However, to simulate re-entry from a typical orbital mission, the initial points were 1.8 million feet altitude, 25,000 fps and flight angles from -5° to -90° . Initial spherical body sizes of 1", 1/2", 1/4", 1/8", 1/16", 1/24", 1/32", and 1/64"

were selected for study. The spherical shape was chosen because it represents the worst geometry for surface-to-mass ratio. A digital computer program was prepared using the appropriate equations of motion, energy balances and Scala's equations for aerodynamic heating and mass transfer. No free molecule heat and mass transfer analysis was included because it was felt that the vast majority of the mass transfer would occur in the continuum flow regimes.

The results of this analytical study and the initial experimental work that was carried out to verify Scala's theory are reported in WANL-TNR-039. Although the above document should be referred to for more explicit details, the most important result was that only spherical particles less than 1/32" diameter would be completely oxidized in contrast to the 1/24" reported by Martin. Moreover, for particle sizes in the 1/16-1/64" range, the mass losses were lower than those obtained by Martin.

The second phase effort was directed at establishing a digital computer program for the free molecule flow in the expectation of being able to reconcile the differences in our first studies and the Martin report. The energy transfer analysis was based on the same considerations outlined in the first section of this paper with a careful analysis of the chemical reaction kinetics being undertaken to place the mass transfer analysis on as rigorous a basis as possible. One salient feature of this program was inclusion of an analysis to continuously determine the Knudsen number during re-entry as a function of the body velocity, size, temperature and altitude. This feature permitted us to determine the altitude at which free molecule flow terminated for any given re-entry conditions. For a more detailed discussion of the considerations involved in establishing our free-molecule analysis reference should be made to WANL-TNR-067.

Several important results were obtained from our free molecule work, which we shall list as follows:

- (1) For body sizes 1" and smaller, free molecule flow exists to altitudes less than 400,000 feet.
- (2) Total percent mass loss in free molecule flow above 400,000 feet is insignificant regardless of re-entry angle, body size or initial altitude.
- (3) For spherical particles 1/16" in diameter and less, almost the entire aerodynamic heating and mass transfer is in the free molecule regime, with most of the mass transfer occurring below 400,000 feet altitude.
- (4) As a corollary to (3), the Scala analysis was applicable either not at all or only in a very small portion of the total re-entry time for particles 1/16" or less.
- (5) The mass loss rates predicted by the Scala theory in the earlier study (WANL-TNR-039) for those altitudes where free molecules or transition flow exist were higher than was predicted by the free molecule flow theory.

The immediate implication from the returns of the free molecule analysis was that, paradoxically, our original study using only the Scala theory produced optimistic results, and instead of getting closer agreement to the Martin study the divergence became more pronounced. Moreover, since free molecule flow produces the most efficient mass and energy transfer that is possible, it then becomes quite clear that the Scala theory cannot be extrapolated very far from its aerodynamic regime without introducing serious errors.

At this point, it was decided that a "bridge" between the termination of free molecule flow and the emergence of continuum flow would be of definite value, and consequently we developed an analysis for the transition regime along the lines of a modified free molecule analysis (near-free molecule flow) using the basic considerations outlined in Section I.

With this task completed, we were then in the position to develop a "unified" digital computer code for graphite re-entry by combining the free molecule, near-free molecule and continuum flow (Scala) analyses into one program. By using the analysis for determining the Knudsen numbers, the termination of one regime and the commencement of another may be determined as a function of the body size, velocity, temperature, altitude and trajectory. Thus the proper energy and mass transfer expressions are used as a function of the flow field.

III. DISCUSSION AND CONCLUSIONS ON GRAPHITE RE-ENTRY

A parametric study was made using the unified re-entry code for spherical graphite particles in the size range of 1" to 1/256" in diameter with initial re-entry angles from 0° to 90°. The results of this study are shown in Table I. The initial conditions other than particle size and re-entry angle were maintained constant at 400,000 feet altitude and 25,000 feet/second velocity. These latter two parameters not only duplicate those specified in the Martin report, but from the standpoint of burnup and heating, the amount of either is insignificant above 400,000 feet as shown in WANL-TNR-067, so that results one would obtain by using higher initial altitudes would be practically identical to those given here.

A cursory examination of the percentage burnup figures reveals that regardless of particle size or re-entry angle, complete (100%) oxidation of graphite does not occur, which is in direct contradiction to the results published in the Martin report and in our earlier work (WANL-TNR-039). Moreover, further examination of these figures indicates that there is an optimum particle size which will produce the maximum percent burnup. This particle size is smaller than 1/16" and greater than 1/64th inch. Note that the 1/32" particle produces the largest percent mass loss of all the sizes that calculations were performed; however, it is important to realize that 1/32" is not necessarily the particle size that will produce the maximum burnup. Particle sizes between 1/16" to 1/32" and 1/32" and 1/64" were not included in this study.

It should be pointed out that instead of there existing an optimum particle size, there is the distinct likelihood that there is a particle size below which a greater percentage of burnup cannot be obtained, e.g., particles less than $1/32$ " all may have maximum burnup of approximately 40%. The reason that there is an optimum size shown in this paper may be due to some inconsistencies in the transition flow heat transfer which effect the flow field and thus the switchover point into Scala's continuum flow analysis. The result may very well be that higher or lower mass loss rates are obtained during a given portion of the re-entry than should be.

We are presently considering the use of other transition flow regime (near-free-molecule) analyses in the unified code to replace the Willis-Probstein theory, which, because of mathematical boundary conditions gives rise to discontinuities in the aerodynamic heat flux.

We do not, however, expect any appreciable increase or decrease in the burnup figures stated in Table I that would affect the overall conclusions.

Another interesting aspect is that the smaller the initial particle size, the less the dependence of mass loss on re-entry angle. However, in general, we may also conclude that for particles $1/16$ " and larger, the shallower the initial re-entry angle, the more the burnup. Still further, these results also point out that the burnup obtained is almost independent of re-entry angle for angles less than 5° regardless of particle size.

Now the question arises as to why there is such a discrepancy between both our earlier report (WANL-TNR-039) and the Martin study as compared with these results. First, it will be recalled that the Scala analysis has been developed for use in the well defined continuum flow regime where there is the presence of a shock wave. The Scala treatment requires a well defined shock front since the analysis itself depends on a mathematical representation of the gas behind the shock (Refer to WANL-TNR-072). Since the basic Scala analysis is combined with the equations of state for a normal shock wave, when one attempts to extrapolate this continuum flow analysis beyond both its mathematical and physical boundary conditions, then it is not surprising that erroneous results are obtained. In the case of the Scala analysis, extrapolation of that theory into the transition and free-molecule flow leads to higher mass loss rates than can be theoretically obtained by the most efficient possible process (free-molecule flow). This in a nutshell is the reason that the earlier analytical work as reported in WANL-TNR-039 predicts complete burnup of spherical graphite particles smaller than $1/32$ ". (For a worked out example see Appendix A).

By correlating our findings with the data given in the Martin report, it was noted that particles of about $1/24$ th inch diameter remained in free molecule flow at least during the major portion of, if not the entire, re-entry time. This led to the important clue that the difference in our results and the Martin study could be in the free-molecule flow analysis. A rederivation of the free-molecule mass loss rate equation given in MND-2517 revealed* that through a purely mathematical error, the Martin free molecule flow mass

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*See derivation and comparison in Appendix B

loss rates will predict a rate of approximately 4 times that given in WANL-TNR-067. Since the Martin report made the conservative assumption that CO_2 was the oxidation product, instead of CO as closer analysis shows in WANL-TNR-067, then the difference in mass loss rates becomes a factor of two.

Taking the results listed in Table I for the $1/32$ " particle and recalling from the preceding section that almost the entire mass loss occurs in the free molecule flow regime for particles less than $1/16$ " inch, the predicted burnup figures when multiplied by two would coincide with Martin's findings for shallow re-entry angles. While burnup figures for $1/24$ " particles were not obtained, it is not unreasonable in view of the trend shown by the burnup percentages to expect approximately a 50% mass loss for a $1/24$ " particle, which becomes approximately 100% (or complete burnup) by the Martin analysis.

In summation, the following conclusions may be drawn from the graphite re-entry studies:

1. Shallow re-entry angles produce more burnup than steep angles for all particle sizes investigated.
2. The smaller the particle, the less pronounced becomes the effect of re-entry angle on burnup.
3. There is an initial particle size for which the maximum percent burnup cannot be exceeded. This size is approximately $1/32$ " in diameter for a spherical particle.
4. The maximum burnup obtainable is approximately 50%.
5. The Martin report (MND-2517) has a mathematical error in the free molecule analysis which results both in greater burnup than can be theoretically achieved and the optimistic results they reported.
6. The original WANL analytical study on graphite re-entry burnup as presented in WANL-TNR-039 is incorrect also, because the extrapolation of the Scala continuum flow equations into the transition and free molecule flow regimes is not valid, the results predicting higher mass loss rates and greater total burnup than can be theoretically achieved.

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APPENDIX A

The continuum flow oxidation rate equation of Scala is as follows:

$$\dot{M}_s = r^{-1/2} (C) (10)^a (V)^b \text{ lbs/ft}^2\text{-sec} \quad (\text{Eq. 1})$$

where

$\dot{M}_s$ = stagnation point mass loss rate,

r = body radius, ft

C = a variable dependent on altitude and body wall temperature
(see below)

a = a variable dependent on altitude (see below)

V = velocity of body

b = a constant = 0.875

The relationship between the stagnation point mass loss rate and the observed surface mass loss rate can be shown to be:

$$\dot{M}_b = 0.130 \dot{M}_s = 0.13 r^{-1/2} (C) (10)^a (V)^b \quad (\text{Eq. 2})$$

where

$\dot{M}_b$ = surface oxidation rate, lb/ft²-sec

Also "C" and "a" have the following relationship to altitude and body temperature:

$$C = 0.508 + 1.4725 \times 10^{-4} T_w; (T_w < 3000^\circ\text{R}) \quad (\text{Eq. 3A})$$

$$C = 1.17 - 1.045 \times 10^{-6} Y - (7.41 \times 10^{-5} - 2.85 \times 10^{-10} Y) T_w; (T_w > 3000^\circ\text{R}), \quad (\text{Eq. 3B})$$

$$a = -(4.6759 + 9.16 \times 10^{-6} Y) \quad (\text{Eq. 4})$$

where

T_w = body wall temperature, °R

Y = altitude, ft.

The molecular flow oxidation rate equation as derived in WANL-TNR-067 is:

$$\frac{dM_b}{dt} = 1.9 \times 10^{-1} (R_b)^2 \left(\frac{\rho_\infty}{\rho_o} \right) U (\mathcal{S} + \mathcal{v}), \frac{\text{lbs}}{\text{sec}} \quad (\text{Eq. 5})$$

where

M_b = mass of body, lbs

R_b = radius of body, ft

ρ_∞ / ρ_o = ratio of ambient air density to sea level air density

U = body velocity, ft/sec

$\mathcal{S} + \mathcal{v}$ = atom fraction oxygen in ambient gas.

Now $\dot{M}_b$ for molecular flow is:

$$\dot{M} = \frac{1}{A} \frac{dM_b}{dt} = \frac{1.9 \times 10^{-1}}{4\pi} \left(\frac{\rho_\infty}{\rho_o} \right) U (\mathcal{S} + \mathcal{v}), \frac{\text{lb}}{\text{ft}^2 \text{-sec}} \quad (\text{Eq. 6})$$

Let us now compare the mass transfer rates as calculated by the Scala analysis and the molecular flow analysis for a 1/64" diameter particle at approximately 300,000 feet.

From computer data at 301,000 feet using a continuum flow analysis (Scala) the following parameters are obtained:

$$V = 13,750 \text{ ft/sec}$$

$$Y = 301,000 \text{ ft}$$

$$T = 3587^\circ\text{R}$$

Using Equations 3B and 4 to calculate "C" and "A" respectively we obtain:

$$C = 0.90$$

$$a = -7.436$$

Now $r^{1/2} = 1.54 \times 10^{-2}$, hence from Equation 2.

$$\dot{M}_b = 0.13 \left(\frac{1}{1.54 \times 10^{-2}} \right) (0.90) (10)^{-7.436} (13,750)^{0.875}$$

$$\therefore \dot{M}_b = 115 \times 10^{-5} (\text{lb/ft}^2 \text{-sec}), \text{ continuum flow,}$$

If we now use data from a molecular flow computer print out at 300,000 ft altitude, the following parameters are obtained:

$$U = 14,000 \text{ ft/sec}$$

$$P_{\infty}/P_0 = 2.552 \times 10^{-6}$$

$$(S+V) = 0.21$$

Therefore using Equation 6,

$$\dot{M} = \frac{1.9 \times 10^{-1}}{4\pi} \times (2.552 \times 10^{-6}) \times (1.4 \times 10^4) \times (0.21)$$

$$\dot{M} = 11.7 \times 10^{-5} \text{ lb/ft}^2\text{-sec, molecular flow}$$

It can be shown that at 300,000 feet altitude with a velocity of 14,000 ft/sec, a 1/64" diameter body will be in molecular flow using the Knudsen criterion. Therefore the $\dot{M}$ calculated by molecular flow formula is the correct value to be used. More important it can be seen that the continuum flow analysis gives an $\dot{M}$ value 10 times that of the molecular flow value. Since molecular flow mass transfer is the theoretical maximum obtainable, then it becomes obvious that the continuum flow analysis leads to erroneous and optimistic results when used to calculate mass transfer rates outside the continuum flow regime.

APPENDIX B

The free molecule mass loss rate equation presented in the Martin report is as follows:

$$\frac{d(R_b)}{dy} = \frac{1.2}{16} \frac{(p_g)e^{-bY}}{(p_b) \sin \phi} \quad (\text{Eq. 1})^*$$

Starting with the basic equation shown in MND-2517,

$$\frac{d(M_b)}{dt} = A_c \sqrt{p_{o_2}} \times \xi \quad \text{we shall proceed to derive Equation 1 above.}$$

$$M_b = \frac{4}{3} \pi R_b^3 p_b$$

$$\frac{dM_b}{dR_b} = 4\pi R_b^2 p_b = 4(A_c) p_b$$

$$\frac{dM_b}{dt} \times \frac{dR_b}{dM_b} = A_c \sqrt{p_{o_2}} \times \xi \times \frac{1}{4(A_c)p_b} = \sqrt{p_{o_2}} \left(\frac{\xi}{4}\right) \times \left(\frac{1}{p_b}\right)$$

$$p_{o_2} = (0.20) (p_g) e^{-bY} = \frac{1}{5} (p_g) e^{-bY}$$

$$\sin \phi = \frac{V_r}{V} \longrightarrow V_r = V \sin \phi$$

$$V_r = \frac{dR}{dt} = \frac{d}{dt} (R_e + Y) = \frac{d(R_e)}{dt} + \frac{dY}{dt} = \frac{dY}{dt}$$

$$\therefore \frac{dY}{dt} = V \sin \phi$$

$$\frac{dR_b}{dt} \times \frac{dt}{dY} = V (p_{o_2}) \left(\frac{\xi}{4}\right) \times \left(\frac{1}{p_b}\right) \times \left(\frac{1}{p_b}\right) \times \frac{dt}{dY}$$

$$\frac{dR_b}{dY} = V \left[\frac{1}{5} (p_g) e^{-bY} \right] \left(\frac{\xi}{4}\right) \left(\frac{1}{p_b}\right) \frac{1}{V \sin \phi} = \frac{1}{5} \left[\frac{(p_g) e^{-bY} \left(\frac{\xi}{4}\right)}{(p_b) \sin \phi} \right]$$

Using the assumption that $\xi = \frac{12}{32}$ (presumes formation of CO_2 as reaction product), then the above equation becomes:

$$\frac{dR_b}{dY} = \frac{1 (p_g) e^{-bY}}{5 (p_b) \sin \phi} \left(\frac{12}{32} \right) \times \frac{1}{4},$$

*See end of Appendix for nomenclature

or

$$\frac{d(R_b)}{dY} = \frac{1.2}{16} \frac{(\rho_g)e^{-bY}}{(\rho_b)\sin \phi} \times \frac{1}{4}$$

The above equation is identical with that shown by Martin except for the 1/4 factor and has been derived from the basic equation used by Martin. Thus the equation given in MND-2517 (Eq. 1) will predict mass loss rates four times greater than actually can be obtained.

NOMENCLATURE

R_b	=	body radius
Y	=	altitude
ϕ	=	flight angle (re-entry angle)
ρ_b	=	Body density
ρ_g	=	sea level atmospheric density
b	=	altitude scale factor to convert sea level atmospheric density to ambient density
M_b	=	mass of body
A_c	=	cross sectional area
V	=	velocity
ρ_{O_2}	=	density of oxygen in ambient atmosphere
ξ	=	stoichiometric ratio of mass of carbon oxidized to mass of oxygen used
R	=	distance from center of earth to body
R_e	=	radius of earth
t	=	time
V_r	=	radial component of velocity

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TABLE I

Per Cent Burnup vs Initial Particle Size and R-entry Angle

Angle	Initial Particle Size								
	1"	1/2"	1/4"	1/8"	1/16"	1/32"	1/64"	1/128"	1/256"
0°	7.0%	12%	20%	33%	40%	52%	43%	42%	41%
-1°	6.9%	12%	20%	33%	40%	53%	43%	42%	41%
-3°	6.0%	10%	19%	33%	40%	52%	43%	42%	41%
-5°	5.2%	10%	17%	33%	40%	52%	43%	42%	41%
-10°	4.3%	8%	14.5%	25%	38%	49%	43%	42%	41%
-30°	3%	7%	10%	18%	34%	43%	43%	42%	41%
-60°	2.3%	7%	8%	15%	25%	41%	43%	42%	41%
-90°	2.2%	7%	7.4%	14%	25%	40%	43%	42%	41%

Initial Conditions:

Altitude = 400,000 ft

Velocity : 25,000 FPS