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EFFECTS OF LEAKAGE UPON EVALUATION OF FAST-REACTOR MULTIGROUP CROSS SECTIONS FOR REFLECTORS

by

David Meneghetti

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Reactor Physics Division

February 1966

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EFFECTS OF LEAKAGE UPON EVALUATION OF FAST-REACTOR MULTIGROUP CROSS SECTIONS FOR REFLECTORS

by

David Meneghetti

ABSTRACT

Group cross sections for aluminum and nickel reflectors, which include effects of energy-dependent leakages into and out of reflector regions, are considered. Leakages are introduced as absorptions in fine-group ELMOE^{1,2} calculations. Iterations are made between coarse-group regional fluxes from criticality calculations and the leakage-modified ELMOE analyses.

I. INTRODUCTION

The ELMOE^{1,2} code is often used^{3,4,5} to determine composition-homogenized, multigroup-transport and multigroup-elastic-transfer cross sections for fast-reactor cores containing light materials, such as aluminum, steel, nickel, sodium, graphite, etc. For a given composition, the ELMOE code determines the bare-core neutron-flux spectrum (normal mode) by using many hundreds of very-fine-energy groups. The code produces fine-flux-weighted coarse-group cross sections. These are then used in the core region of a subsequent coarse-multigroup, reflected, criticality calculation.

The possible presence of light, resonance-scattering, materials in reflectors introduces the question of the importance of analogously determining coarse-group cross sections for reflector regions. In addition, the possibility of adapting the ELMOE code to enable suitable cross-section averages to be calculated for such nonnormal mode regions is of interest.⁶

II. METHOD

Multigroup cross sections that include considerations of group- and regional-dependent leakages into and out of reflector regions were evaluated.⁶ These were obtained by iterations between coarse-group, multiregion, criticality calculations and modified, fine-group, ELMOE calculations. In the latter, the ELMOE material-bucklings were set at zero and, instead,

the leakage effects in the ELMOE calculations were simulated by insertion of energy-dependent positive and/or negative fictional-capture cross sections. These fictional-capture cross sections were determined from the regional group leakages derived from the criticality calculations. Diffusion-theory, coarse-group, criticality solutions were generally used. The ELMOE code was used in the P1 option and used 0.99133322 as the ratio of successive fine-energy-group boundaries. The cross sections of the first and last groups of the 16-group, coarse, cross-section set were not fine-group averaged. In one case, the neutron-transport DSN code⁷ in S_4 approximation was also used.

For a nonfission region, the fine-group flux, ϕ_j , may be expressed as

$$\phi_j = \frac{\sum_{k=1}^{j-1} \Sigma_{k \rightarrow j} \phi_k}{\Sigma_j + \Sigma_j^L},$$

where the numerator represents the fine-group scattering-down sources and the denominator contains the fine-group removal, Σ_j , and the fine-group leakage-absorption, Σ_j^L . To use the present version of the ELMOE code to obtain the ϕ_j , the flux level in the first group (which is a coarse group) was assumed; i.e., an applied source having neutrons emitted only into this group was assumed.

The following three definitions were used for the fine-group leakage absorption, Σ_j^L , derived from the coarse-group leakage absorptions, Σ_J^L :

$$A. \quad \Sigma_j^L \equiv \Sigma_J^L = \frac{(\text{Net leakage})_J^R}{(\text{Integral flux})_J^R},$$

$$B. \quad \Sigma_j^L = \frac{B_J^2}{3\Sigma_j^{TR}} \equiv \frac{\Sigma_J^L \Sigma_J^{TR}}{\Sigma_j^{TR}},$$

$$C. \quad \Sigma_j^L = \frac{[B_J^2(\text{Smoothed})]_j}{3\Sigma_j^{TR}},$$

where R , j , J , and Σ^{TR} denote the region, fine-group, coarse-group, and transport cross sections, respectively. In Method A, the fine-group leakage-absorptions within a given coarse group are assumed constant and equal to the coarse-group value. Method B allows for fine-energy variation in the leakage-absorption caused by fine structure variations of the transport cross section. The latter are obtained from the ELMOE library.

Method B still retains constant fine-group bucklings equal to the coarse-group bucklings. Method C is similar to Method B, except that the coarse-group bucklings have been smoothed (by a parabolic smoothing in each coarse group). This eliminates the abrupt steps in buckling values at coarse-group boundaries. Comparison of results obtained with the three methods then indicates if these detailed considerations are important in any particular case. For cross sections other than the transport and elastic transfer of aluminum and nickel, the cross sections of YOM⁸ with the conservative $\nu(U^{235})$ values were used.

For comparisons with the iterative results, and to obtain initial values of ELMOE-derived transport and elastic-removal cross sections for use as starting values in the iterative procedure, unmodified ELMOE calculations for the reflector compositions were carried out using the normally employed fission-spectrum source and assuming zero leakage, i.e., buckling zero.

III. CONVERGENCE CHARACTERISTICS OF THE LEAKAGE-ITERATION METHOD

The spherical-geometry fast systems considered are listed in Table I. System No. 2 has a more degraded core spectrum than System No. 1. Systems No. 1 and 3 have identical core compositions. System No. 1 is similar, though not identical, to the ZPR-9 Assembly No. 1 experiment.⁹ The other two systems have no direct experimental analogue. For analytical purposes, each reflector was considered to be two identical composition regions of 6 and 24 cm thickness, respectively. The criticality-ELMOE iterations were carried out separately but simultaneously for these two regions to ascertain the importance of considering spatial-leakage effects upon the cross-section averaging.

TABLE I. Compositions of Fast Systems with Light, Resonance Reflectors

System No.	Core* Composition, v/o				Reflector**	
	U ²³⁵ (0.048) [†]	U ²³⁸ (0.048) [†]	Carbon (0.0717) [†]	S.S. ^{††} (0.0854) [†]	Material	v/o
1	9.4	70.1	0	9.3	Al (N = 0.0603) [†]	94
2	9.4	0	75.5	9.3	Al (N = 0.0603) [†]	94
3	9.4	70.1	0	9.3	Ni (N = 0.0913) [†]	100

*Core radii = 30-35 cm.

**Inner and outer reflector regions are 6 and 24 cm thick, respectively.

[†]Atoms per cm³ $\times 10^{-24}$ assumed for the full-density isotope.

^{††}Stainless steel.

In general, the need for the leakage-iterative method will depend largely upon the ratio of fine-group leakage-absorption to fine-group removal-absorption. The latter includes capture as well as energy degradation. Since the ELMOE groups are very narrow, the fine-group removal-absorption cross sections are essentially equal to the fine-group transport cross sections. The need for the use of the leakage-iteration methods tends to increase as the absolute value of the ratio of leakage-absorptions to transport cross sections increases. Leakage-absorptions may be both positive and negative.

Listed in Table II are illustrative coarse-group leakage-absorption values and their ratios to the corresponding transport cross-section values. These are for the inner 6 cm of reflector for System No. 1, which has the greater ratios as a function of energy of the systems studied. These values are from S_4 -ELMOE iterative calculations. Negative leakage-absorptions indicate net sources diffusing into the region; positive leakage-absorptions indicate net sources diffusing out of the region. The coarse energy structure is that of YOM.

TABLE II. Aluminum-reflector Coarse-group Leakage-Absorption, Σ_J^L , Inner 6 cm (U^{238} -diluent Core, System No. 1)

Group J	$E_{lower J}$, MeV	Σ_J^L	$ \Sigma_J^L /\Sigma_J^{TR*}$	Group J	$E_{lower J}$, MeV	Σ_J^L	$ \Sigma_J^L /\Sigma_J^{TR*}$
1	3.668	-0.0384	0.49	9	0.067	0.0138	0.10
2	2.225	-0.0270	0.34	10	0.0407	0.0102	0.10
3	1.35	-0.0148	0.14	11	0.025	0.0201	0.46
4	0.825	-0.0131	0.11	12	0.015	0.0155	0.42
5	0.5	-0.0160	0.10	13	0.0091	0.0083	0.14
6	0.3	~0	~0	14	0.0055	0.0149	0.18
7	0.18	0.0053	0.04	15	0.0021	0.0154	0.20
8	0.11	0.0048	0.03	16	0.0005	0.0153	0.20

* Σ_J^{TR} is the homogenized transport cross section for energy group J for the inner 6 cm of the aluminum reflector.

Convergence characteristics of the coarse-group cross sections are given in Tables III and IV for the two regions of the aluminum reflector of the system having the U^{238} -diluent core. The leakage-absorptions were here treated as coarse (i.e., Method A). The ratios of the direct YOM cross sections, the ELMOE-derived coarse-group cross sections assuming no leakage sources, and the leakage-absorption iterative values for the first and also second iterations all relative to the third iteration values are listed. It is noted that transport cross-section values converge with fewer iterations than the elastic-transfer cross sections. Furthermore, even the simple nonleakage ELMOE-derived transport cross sections can result in reasonably satisfactory transport values for most groups.

TABLE III. Convergence of $3\Sigma_J^{TR}$ of Aluminum Reflector by S_4 -ELMOE Iterations* (U^{238} -diluent Core, System No. 1)

Group J	Aluminum Inner-reflector Region, 6 cm					Aluminum Outer-reflector Region, 24 cm				
	$\frac{\Sigma(YOM)}{\Sigma(A3)}$	$\frac{\Sigma(NL)}{\Sigma(A3)}$	$\frac{\Sigma(A1)}{\Sigma(A3)}$	$\frac{\Sigma(A2)}{\Sigma(A3)}$	$3\Sigma(A3), \text{cm}^{-1}$	$\frac{\Sigma(YOM)}{\Sigma(A3)}$	$\frac{\Sigma(NL)}{\Sigma(A3)}$	$\frac{\Sigma(A1)}{\Sigma(A3)}$	$\frac{\Sigma(A2)}{\Sigma(A3)}$	$3\Sigma(A3), \text{cm}^{-1}$
2	1.16	0.98	1.00	1.00	0.235	1.15	0.97	0.99	1.00	0.237
3	1.05	1.00	1.00	1.00	0.323	1.05	1.00	1.00	1.00	0.323
4	1.18	0.99	1.00	1.00	0.359	1.18	0.99	1.00	1.00	0.359
5	1.08	1.00	1.00	1.00	0.469	1.08	1.00	1.00	1.00	0.469
6	1.09	1.00	1.00	1.00	0.492	1.10	1.00	1.00	1.00	0.491
7	1.45	1.01	1.00	1.00	0.448	1.44	1.00	1.00	1.00	0.452
8	2.05	0.98	1.00	1.00	0.416	2.08	0.99	1.00	1.00	0.411
9	2.55	0.93	1.00	1.00	0.398	2.67	0.97	1.00	1.00	0.381
10	1.47	1.04	1.00	1.00	0.322	1.46	1.03	1.00	1.00	0.324
11	8.17	0.83	1.02	1.00	0.132	8.56	0.87	1.00	1.00	0.126
12	1.16	1.10	0.99	1.00	0.110	1.16	1.10	1.00	1.00	0.110
13	1.05	1.02	1.00	1.00	0.183	1.05	1.03	0.99	1.00	0.182
14	1.04	1.03	1.00	1.00	0.246	1.04	1.02	1.00	1.00	0.247
15	0.99	1.00	1.00	1.00	0.235	0.99	1.00	1.00	1.00	0.235

*YOM denotes use of Yiftah, Okrent, and Moldauer cross sections.⁸

NL denotes use of ELMOE-derived cross sections, assuming zero leakage.

A1 denotes use of first leakage-iteration, using Method A (see text).

A2 denotes use of second leakage-iteration, using Method A (see text).

A3 denotes use of third leakage-iteration, using Method A (see text).

TABLE IV. Convergence of Σ_J^{ER} of Aluminum Reflector by S_4 -ELMOE Iterations* (U^{238} -diluent Core, System No. 1)

Group J	Aluminum Inner-reflector Region, 6 cm					Aluminum Outer-reflector Region, 24 cm				
	$\frac{\Sigma(YOM)}{\Sigma(A3)}$	$\frac{\Sigma(NL)}{\Sigma(A3)}$	$\frac{\Sigma(A1)}{\Sigma(A3)}$	$\frac{\Sigma(A2)}{\Sigma(A3)}$	$\Sigma(A3), \text{cm}^{-1}$	$\frac{\Sigma(YOM)}{\Sigma(A3)}$	$\frac{\Sigma(NL)}{\Sigma(A3)}$	$\frac{\Sigma(A1)}{\Sigma(A3)}$	$\frac{\Sigma(A2)}{\Sigma(A3)}$	$\Sigma(A3), \text{cm}^{-1}$
2	0.59	0.87	0.96	0.99	0.0155	0.57	0.84	0.96	0.99	0.0160
3	0.65	0.97	1.01	1.00	0.0188	0.63	0.94	1.00	1.00	0.0194
4	0.77	0.88	0.98	0.99	0.0244	0.76	0.87	0.97	0.99	0.0246
5	0.70	0.83	0.96	0.99	0.0327	0.73	0.87	0.98	1.00	0.0313
6	0.93	1.03	1.04	1.02	0.0257	0.83	0.92	0.98	0.99	0.0287
7	1.14	1.10	1.02	1.00	0.0241	1.03	1.00	1.00	1.00	0.0266
8	1.22	1.06	0.99	0.99	0.0293	1.18	1.02	1.01	1.00	0.0303
9	1.64	1.18	1.00	1.00	0.0239	1.49	1.07	1.00	1.00	0.0263
10	1.19	1.24	0.99	0.99	0.0147	1.14	1.20	1.02	1.01	0.0153
11	5.45	1.53	0.95	1.00	0.0075	4.96	1.40	1.00	1.00	0.0082
12	2.12	2.77	0.92	1.03	0.0022	2.23	2.91	1.03	1.00	0.0021
13	1.06	1.44	0.82	1.03	0.0064	1.20	1.61	0.89	0.98	0.0057
14	1.39	1.59	1.03	1.09	0.0066	1.22	1.39	0.92	0.99	0.0075
15	1.66	3.44	1.01	1.03	0.0017	1.39	2.89	0.93	1.00	0.0021

*YOM denotes use of Yiftah, Okrent, and Moldauer cross sections.⁸

NL denotes use of ELMOE-derived cross sections, assuming zero leakage.

A1 denotes use of first leakage-iteration, using Method A (see text).

A2 denotes use of second leakage-iteration, using Method A (see text).

A3 denotes use of third leakage-iteration, using Method A (see text).

Spectral convergence with iterations are shown in Table V. For reasonable spectral convergence, two leakage-absorption iterations suffice for this case. Furthermore, the values of critical radii relative to the leakage-iteration converged values, given at the bottom of Table V, show that the direct use of YOM aluminum cross sections for the reflector results in a ~15% smaller critical volume. Use of nonleakage ELMOE-derived cross sections would overestimate the critical volume by only about 0.3%.

TABLE V. Spectral-flux Convergence in Aluminum Reflector by S_4 -ELMOE Iterations* (U^{238} -diluent Core, System No. 1)

Group J	Aluminum Inner-reflector Region, 6 cm					Aluminum Outer-reflector Region, 24 cm				
	$\bar{\rho}$ (YOM) $\bar{\rho}$ (A3)	$\bar{\rho}$ (NL) $\bar{\rho}$ (A3)	$\bar{\rho}$ (A1) $\bar{\rho}$ (A3)	$\bar{\rho}$ (A2) $\bar{\rho}$ (A3)	$\bar{\rho}$ (A3)	$\bar{\rho}$ (YOM) $\bar{\rho}$ (A3)	$\bar{\rho}$ (NL) $\bar{\rho}$ (A3)	$\bar{\rho}$ (A1) $\bar{\rho}$ (A3)	$\bar{\rho}$ (A2) $\bar{\rho}$ (A3)	$\bar{\rho}$ (A3)
1	0.92	1.01	1.00	1.00	0.0110	0.93	1.01	1.00	1.00	0.0058
2	1.02	1.02	1.00	1.00	0.0198	1.08	1.07	1.01	1.00	0.0119
3	0.99	1.01	1.00	1.00	0.0384	1.08	1.02	1.00	1.00	0.0255
4	0.99	1.05	1.01	1.00	0.0792	1.03	1.10	1.02	1.00	0.0543
5	1.03	1.07	1.01	1.00	0.138	1.13	1.13	1.02	1.00	0.0865
6	0.89	0.98	0.99	1.00	0.185	0.97	1.03	1.00	1.00	0.1351
7	0.87	0.96	0.99	1.00	0.165	0.81	0.96	0.99	1.00	0.1557
8	0.91	0.99	1.01	1.00	0.117	0.76	0.96	0.99	1.00	0.1297
9	0.88	0.94	1.00	1.00	0.0908	0.71	0.93	1.00	1.00	0.1232
10	1.20	0.95	1.00	1.00	0.0870	1.09	0.91	0.99	1.00	0.1370
11	0.81	0.96	1.01	1.00	0.0457	0.65	0.95	1.01	1.00	0.0888
12	2.95	1.13	0.99	1.00	0.0194	2.93	1.11	1.00	1.00	0.0396
13	4.31	2.05	1.02	1.00	0.0029	5.17	2.35	1.04	1.00	0.0049
14	4.39	2.65	0.92	1.00	0.0007	5.24	3.08	0.93	0.99	0.0012
15	5.38	3.19	0.90	1.01	0.0003	5.81	3.34	0.89	1.00	0.0006
16	7.78	9.40	0.86	1.01	0.0000	8.00	9.49	0.84	1.00	0.0001
R (X)/R (A3)	0.951	1.001	1.000	1.000	R = 33.96 cm					

*YOM denotes use of Yiftah, Okrent, and Moldauer cross sections.⁸

NL denotes use of ELMOE-derived cross sections, assuming zero leakage.

A1 denotes use of first leakage-iteration, using Method A (see text).

A2 denotes use of second leakage-iteration, using Method A (see text).

A3 denotes use of third leakage-iteration, using Method A (see text).

R (X) denotes critical radius obtained by use of cross sections denoted by YOM, NL, A1, A2, or A3.

R (A3) denotes critical radius obtained by use of cross sections denoted by A3.

IV. COMPARISON OF CONVERGED, LEAKAGE-ITERATED, CROSS SECTIONS

Comparisons of the converged, leakage-iterated, values of $3\Sigma_{TR}^{A1}$ for the first, 6-cm-thick, reflector region of System No. 1 are listed in Table VI. The group structure is that of the YOM⁸ set. The A values are the cross-section values assuming the coarse absorption-leakage option (Method A). B/A and C/A give the ratios of the coarse buckling-fine transport (Method B) and smoothed buckling-fine transport (Method C) options, respectively,

TABLE VI. Comparison of Converged $3\Sigma_J^{TR}$ Values of Aluminum Reflector* (U^{238} -diluent Core, System No. 1)

Group J	Inner 6 cm				
	3Σ (A), cm^{-1}	Σ (B)/ Σ (A)	Σ (C)/ Σ (A)	Σ (A)DT/ Σ (A)	Σ (A) _{outer} / Σ (A) _{inner} **
1	(0.233) [†]	(1.00) [†]	(1.00) [†]	(1.00) [†]	(1.00) [†]
2	0.235	0.99	0.98	1.00	1.00
3	0.323	1.00	1.00	1.00	1.00
4	0.359	1.00	1.00	1.00	1.00
5	0.469	1.00	1.00	1.00	1.00
6	0.492	1.00	1.00	1.00	1.00
7	0.448	1.01	1.00	1.00	1.01
8	0.416	1.01	1.01	1.01	0.99
9	0.398	1.02	1.01	1.01	0.96
10	0.322	1.00	1.01	0.99	1.01
11	0.132	1.05	1.02	1.11	0.95
12	0.110	1.01	1.01	0.95	1.00
13	0.183	1.01	1.01	1.03	0.99
14	0.246	1.00	1.00	0.99	1.00
15	0.235	1.00	1.00	1.00	1.00
16	(0.233) [†]	(1.00) [†]	(1.00) [†]	(1.00) [†]	(1.00) [†]

* S_4 criticality values were used, except where diffusion theory, denoted by DT, is indicated in column 5.

A denotes use of leakage-iteration, using Method A (see text).

B denotes use of leakage-iteration, using Method B (see text).

C denotes use of leakage-iteration, using Method C (see text).

** Σ (A)_{outer} and Σ (A)_{inner} denote outer and inner calculated iterated cross sections of outer, 24-cm thickness of reflector and inner, 6-cm thickness of reflector, respectively.[†] Values in parentheses were not ELMOE-averaged.

relative to the coarse-leakage values A. In general, the three methods give similar values for the transport cross sections. Even the values for Group 11, which contains not only a prominent scattering resonance but also a very sharp decrease of the fine flux envelope, do not differ much. This cross-section sensitivity due to rapidly decreasing flux envelope, combined with a prominent scattering resonance, shows up again in the sixth column. Here the values obtained by diffusion theory, iterated with the modified-ELMOE, relative to S_4 , iterated with the modified-ELMOE, are compared. Because of the more rapidly decreasing flux-spectrum envelope obtained in Group 11 by diffusion theory, relative to the S_4 transport approximation, an 11% greater cross-section value results. The last column indicates the spatial-leakage effects upon the transport cross sections. In general, then, even for this relatively leaky reflector, the choice of options is not too important with respect to averaging the transport cross section.

Table VII lists the corresponding values and ratios for the elastic-transfer cross sections in the same format as Table VI. For many of the low-energy groups containing resolved scattering resonances, the B/A and C/A ratios differ considerably from unity. (For Group 2, the deviations are not important and are essentially due to the way of connecting fine groups to coarse Group 1.) The B/A and C/A values indicate that generally the fine transport variations in the leakages can be important in groups containing resonances in combination with a rapid decrease of the

TABLE VII. Comparison of Converged Σ_J^{ER} Values of Aluminum Reflector* (U^{238} -diluent Core, System No. 1)

Group J	Inner 6 cm			$\Sigma (A)^{DT}/\Sigma (A)$	$\Sigma (A)^{outer}/\Sigma (A)^{inner**}$
	$\Sigma (A), \text{cm}^{-1}$	$\Sigma (B)/\Sigma (A)$	$\Sigma (C)/\Sigma (A)$		
1	(0.0067) [†]	(1.00) [†]	(1.00) [†]	(1.00) [†]	(1.00) [†]
2	0.0155	0.86	0.79	0.99	1.03
3	0.0188	0.99	0.99	0.99	1.03
4	0.0244	0.98	1.00	1.00	1.01
5	0.0327	1.00	0.97	1.00	0.96
6	0.0257	1.00	0.97	0.99	1.12
7	0.0241	1.02	1.01	1.00	1.10
8	0.0293	1.00	0.98	0.99	1.03
9	0.0239	0.95	0.97	0.97	1.10
10	0.0147	1.05	1.07	0.96	1.04
11	0.0075	0.88	0.93	0.81	1.09
12	0.0022	1.18	1.27	0.50	0.95
13	0.0064	1.03	1.00	1.48	0.89
14	0.0066	1.05	1.02	0.88	1.14
15	0.0017	1.00	1.00	1.00	1.24

* S_4 criticality values were used, except where diffusion theory, denoted by DT, is indicated in column 5.

A denotes use of leakage-iteration, using Method A (see text).

B denotes use of leakage-iteration, using Method B (see text).

C denotes use of leakage-iteration, using Method C (see text).

** $\Sigma (A)^{outer}$ and $\Sigma (A)^{inner}$ denote outer and inner calculated iterated cross sections of outer, 24-cm thickness of reflector and inner, 6-cm thickness of reflector, respectively.

[†]Values in parentheses were not ELMOE-averaged.

flux envelope. Considerations of the presence of the buckling steps at the coarse-group boundaries are generally not as important. For example, in Groups 11 and 12, where the spectrum is rapidly decreasing, the ratio of the elastic removals by Method C relative to Method B are only 1.06 and 1.08, respectively, whereas, the ratios by Method B relative to Method A are 0.88 and 1.18, respectively. If cases arise in which a particular choice of coarse-group energy boundaries, coupled with a particular composition and geometry, lead to sufficient differences in some groups between the B/A and C/A ratios so that flux spectra are significantly different, it is suggested that such coarse groups be subdivided into two or more coarse groups.

Quite large deviations from unity are noted in the leakage-iterated results based on the diffusion-theory spectrum relative to the S_4 spectrum for this system. (These do not indicate difficulties in the leakage-iteration method but rather that diffusion theory for this core-reflector combination is a poor choice for obtaining the coarse-group spectrum in the reflector. Diffusion theory results in too rapid a decrease in reflector spectra because the combination of a hard-spectra core and a softer-spectra reflector gives rise to diffusion-theory error in core-reflector current in groups where the current direction reverses.) The last column shows that greater differences occur in the elastic-transfer values for the outer-aluminum-reflector region relative to the inner-reflector constants than are determined for the corresponding transport cross sections.

V. CRITICAL VOLUMES

Comparative results of calculated critical radii using YOM cross sections,⁸ nonleakage ELMOE-derived cross-sections, and the cross section obtained from the three leakage-iterative options are shown in Table VIII for the three systems described in Table I. These are diffusion-theory calculations, and the leakage-iteration cases used diffusion theory-ELMOE iteration. For System No. 1, the results of S_4 criticality calculations and ELMOE iterations are also listed (in parentheses).

As might be expected from the relative insensitivity of the ELMOE-derived, coarse transport cross sections to the leakage-iterative options, the critical volumes are also insensitive to choice of leakage option. Furthermore, the critical volumes based on the zero-leakage assumption for reflector cross sections differ by less than 1% from the leakage-iterative results. The direct application of the YOM values results in critical volumes ranging from about 12 to 18% less than the leakage-iterated values for these examples.

ZPR-9 Assembly 1 (Ref. 9) differs slightly in composition from the present System No. 1. The primary difference is the presence of ~11% aluminum

instead of the steel in the core region, due to use of aluminum for the structural boxes and drawers of the experimental honeycomb array. Calculation of the ZPR-9 Assembly system by S_4 approximation using the previously determined leakage-iteration aluminum-reflector cross sections of System No. 1 gave a spherical, homogeneous-core, critical mass of 278 kg of U^{235} . The reported⁹ experimental cylindrical geometry, heterogeneous-core, critical mass was about 278.5 kg of U^{235} . Correction of the calculated homogeneous spherical core to an equivalent heterogeneous cylindrical case, for comparison with the experimental value, results in but a relatively small net correction; i.e., an estimated -15 kg correction to heterogeneity is paired with +17 kg due to an estimated 0.94 shape factor.

TABLE VIII. Critical Core Radii and Ratios of Critical Core Radii, Based upon the Various Methods*

System No.	Reflector	Principal Core Diluent	$\frac{R(YOM)}{R(A)}$	$\frac{R(NL)}{R(A)}$	R (A), cm	$\frac{R(B)}{R(A)}$	$\frac{R(C)}{R(A)}$
1	Aluminum	U^{238}	0.954 (0.951)	1.002 (1.001)	35.21 (33.96)	1.000 (1.000)	1.000 (1.000)
2	Aluminum	Carbon	0.934	1.003	31.55	1.000	1.000
3	Nickel	U^{238}	0.959	0.999	29.53	1.000	1.000

*Criticality calculations used multigroup-diffusion theory and ELMOE-diffusion theory leakage-iteration, except values in parentheses, which used S_4 calculations with ELMOE- S_4 leakage-iterations.

YOM denotes use of Yiftah, Okrent, and Moldauer cross sections.⁸

NL denotes use of ELMOE-derived cross sections, assuming zero leakage.

A denotes use of leakage-iteration, using Method A (see text).

B denotes use of leakage-iteration, using Method B (see text).

C denotes use of leakage-iteration, using Method C (see text).

VI. FLUX SPECTRA

The previously shown sensitivity of the elastic-transfer cross sections to reflector leakage suggests that the neutron spectra are sensitive to the use of the cross sections derived from the leakage-iterated method. Table IX lists the ratios of the converged group fluxes obtained by using the various options for the two reflector regions of the system with aluminum-reflected, U^{238} -diluent core. Multigroup-diffusion theory was used for the coarse-group criticality calculation in the comparisons.

TABLE IX. Diffusion-theory Neutron Spectra in Aluminum Reflector Using Reflector Cross-section Options* (U²³⁸-diluent Core, System No. 1)

Group J	Aluminum-reflector Region 1, 6 cm Thick					Aluminum-reflector Region 2, 24 cm Thick				
	$\bar{\phi}$ (YOM)	$\bar{\phi}$ (NL)	$\bar{\phi}$ (A)	$\bar{\phi}$ (B)	$\bar{\phi}$ (C)	$\bar{\phi}$ (YOM)	$\bar{\phi}$ (NL)	$\bar{\phi}$ (A)	$\bar{\phi}$ (B)	$\bar{\phi}$ (C)
	$\bar{\phi}$ (A)	$\bar{\phi}$ (A)		$\bar{\phi}$ (A)	$\bar{\phi}$ (A)	$\bar{\phi}$ (A)	$\bar{\phi}$ (A)		$\bar{\phi}$ (A)	$\bar{\phi}$ (A)
1	0.92	1.01	0.0125	1.00	1.00	0.92	1.02	0.0076	1.00	1.00
2	1.01	1.02	0.0219	1.02	1.03	1.03	1.06	0.0151	1.06	1.08
3	0.98	1.01	0.0415	0.99	0.98	1.03	1.02	0.0310	0.98	0.96
4	0.98	1.05	0.0848	1.01	1.00	0.99	1.10	0.0644	1.01	1.00
5	1.01	1.08	0.148	0.99	1.01	1.09	1.14	0.1028	1.00	1.02
6	0.89	0.99	0.194	1.00	1.01	0.92	1.03	0.1601	1.00	1.03
7	0.86	0.97	0.172	0.99	0.99	0.78	0.96	0.1785	1.00	1.00
8	0.92	0.99	0.120	1.01	1.01	0.78	0.97	0.1411	1.00	1.00
9	0.92	0.94	0.0893	1.02	1.01	0.78	0.93	0.1243	1.02	1.02
10	1.28	0.96	0.0794	0.97	0.96	1.26	0.94	0.1193	0.96	0.95
11	1.25	0.89	0.0290	1.11	1.09	1.27	0.88	0.0456	1.12	1.09
12	5.56	1.36	0.0063	0.95	0.95	6.48	1.43	0.0087	0.93	0.94
13	6.04	2.16	0.0009	1.08	1.12	9.17	2.95	0.0010	1.13	1.20
14	4.03	1.86	0.0003	1.00	1.00	4.99	2.17	0.0004	1.00	1.02
15	4.69	2.22	0.0001	1.03	1.00	4.97	2.32	0.0001	1.03	1.00
16	9.56	8.97	0	1.00	0.93	10.26	9.82	0	1.00	0.91

*YOM denotes use of Yiftah, Okrent, and Moldauer cross sections.⁸

NL denotes use of ELMOE-derived cross sections, assuming zero leakage.

A denotes use of leakage-iteration, using Method A (see text).

B denotes use of leakage-iteration, using Method B (see text).

C denotes use of leakage-iteration, using Method C (see text).

In general, the larger spectral discrepancies due to use of nonleakage ELMOE cross sections occur in the lower energy portion of the reflector spectrum, where the inelastic scattering sources are negligible and where the fine flux envelope is rapidly decreasing with decreasing energy. Because coarse Group 1 is not ELMOE-subdivided, the fine-flux envelope in coarse Group 2 is not well delineated. Therefore, deviations in the coarse Group 2 flux (and elastic removal) by the various options may not be correct. Fine-group subdivision also of coarse Group 1 will rectify this uncertainty. The close similarity of the values in the columns $\bar{\phi}$ (B)/ $\bar{\phi}$ (A) and $\bar{\phi}$ (C)/ $\bar{\phi}$ (A) show that the smoothing of the coarse energy-dependent buckling is not significant. The deviations of both $\bar{\phi}$ (B)/ $\bar{\phi}$ (A) and $\bar{\phi}$ (C)/ $\bar{\phi}$ (A) from unity, however, indicate that considerations of fine structure variations of transport cross sections occurring in the leakage terms are important in those coarse groups in which the flux-spectrum envelope is rapidly decreasing with decreasing energy and which also contain resolved scattering resonances.

Corresponding aluminum-reflector flux spectra are listed in Table X for the system having a lower-spectrum carbon-diluent core. In this case, because the core contains carbon instead of U²³⁸, the core spectrum does not decrease as rapidly with decreasing energy. The reflector spectrum therefore also does not decrease as rapidly. The use of nonleakage ELMOE-derived cross sections therefore does not lead to as severe errors in flux spectra as in the case having the U²³⁸-diluent core. The need for leakage-derived, elastic-removal cross sections is, however, still evident. Again the $\bar{\phi}$ (B)/ $\bar{\phi}$ (A) and $\bar{\phi}$ (C)/ $\bar{\phi}$ (A) columns do not differ much.

TABLE X. Diffusion-theory Neutron Spectra in Aluminum Reflector Using Reflector Cross-section Options* (Carbon-diluent Core, System No. 2)

Group J	Aluminum-reflector Region 1, 6 cm Thick					Aluminum-reflector Region 2, 24 cm Thick				
	$\bar{\phi}$ (YOM)	$\bar{\phi}$ (NL)	$\bar{\phi}$ (A)	$\bar{\phi}$ (B)	$\bar{\phi}$ (C)	$\bar{\phi}$ (YOM)	$\bar{\phi}$ (NL)	$\bar{\phi}$ (A)	$\bar{\phi}$ (B)	$\bar{\phi}$ (C)
	$\bar{\phi}$ (A)	$\bar{\phi}$ (A)		$\bar{\phi}$ (A)	$\bar{\phi}$ (A)	$\bar{\phi}$ (A)	$\bar{\phi}$ (A)		$\bar{\phi}$ (A)	$\bar{\phi}$ (A)
1	0.89	1.01	0.0278	1.00	1.00	0.90	1.02	0.0165	1.00	1.00
2	1.00	1.02	0.0239	1.02	1.03	1.03	1.05	0.0298	1.05	1.07
3	0.98	1.02	0.0899	0.99	1.00	1.05	1.04	0.0638	0.98	0.99
4	0.90	0.99	0.122	1.00	1.00	0.89	0.99	0.1055	1.00	1.01
5	0.96	1.02	0.127	1.00	1.00	0.98	1.01	0.1156	1.00	0.99
6	0.89	0.99	0.132	1.00	1.00	0.90	0.99	0.1331	1.00	1.00
7	0.89	1.00	0.116	0.99	0.99	0.84	0.99	0.0128	0.99	0.99
8	0.94	1.01	0.0883	1.01	1.01	0.84	1.01	0.0989	1.00	1.00
9	0.95	0.99	0.0805	1.01	1.02	0.81	0.99	0.0946	1.02	1.02
10	1.19	0.97	0.0790	0.98	0.98	1.21	0.96	0.1039	0.97	0.97
11	1.19	0.91	0.0396	1.07	1.03	1.00	0.90	0.0524	1.08	1.04
12	2.37	1.08	0.0193	0.99	0.99	2.92	1.12	0.0223	0.98	0.98
13	1.71	1.18	0.0110	1.03	1.04	2.13	1.35	0.0101	1.06	1.08
14	1.21	1.03	0.0081	1.00	1.02	1.26	1.04	0.0080	1.00	1.03
15	1.09	0.98	0.0100	1.00	1.01	1.13	0.96	0.0109	1.00	1.02
16	0.93	1.08	0.0055	1.00	1.00	0.91	1.11	0.0065	1.00	0.97

*YOM denotes use of Yiftah, Okrent, and Moldauer cross sections.⁸

NL denotes use of ELMOE-derived cross sections, assuming zero leakage.

A denotes use of leakage-iteration, using Method A (see text).

B denotes use of leakage-iteration, using Method B (see text).

C denotes use of leakage-iteration, using Method C (see text).

The third system had the U^{238} -diluent core reflected by a 30-cm thickness of nickel. The transport cross sections of nickel are considerably greater than those of aluminum, especially in the lower-energy groups. The necessity for leakage-iterated cross sections are therefore correspondingly less. This is evident in Table XI where the flux-spectra comparisons for this case are listed. If the thickness of nickel reflector were decreased, the need for a leakage-iterations procedure would increase.

TABLE XI. Diffusion-theory Neutron Spectra in Nickel Reflector Using Reflector Cross-section Options* (U^{238} -diluent Core, System No. 3)

Group J	Nickel-reflector Region 1, 6 cm Thick					Nickel-reflector Region 2, 24 cm Thick				
	$\bar{\phi}$ (YOM)	$\bar{\phi}$ (NL)	$\bar{\phi}$ (A)	$\bar{\phi}$ (B)	$\bar{\phi}$ (C)	$\bar{\phi}$ (YOM)	$\bar{\phi}$ (NL)	$\bar{\phi}$ (A)	$\bar{\phi}$ (B)	$\bar{\phi}$ (C)
	$\bar{\phi}$ (A)	$\bar{\phi}$ (A)		$\bar{\phi}$ (A)	$\bar{\phi}$ (A)	$\bar{\phi}$ (A)	$\bar{\phi}$ (A)		$\bar{\phi}$ (A)	$\bar{\phi}$ (A)
1	0.98	1.00	0.0065	1.00	1.00	1.03	1.01	0.0008	1.00	1.00
2	1.01	1.01	0.0132	1.01	1.04	0.99	1.06	0.0026	1.04	1.14
3	1.06	1.00	0.0313	1.01	1.00	1.16	1.07	0.0115	1.04	1.06
4	1.16	1.05	0.0681	0.99	0.99	1.53	1.14	0.0300	0.98	0.97
5	1.06	1.03	0.133	1.00	1.01	1.32	1.10	0.0690	1.00	1.02
6	0.96	0.99	0.158	1.00	1.00	1.05	1.03	0.0981	1.00	1.01
7	1.00	1.00	0.127	1.00	1.00	1.06	0.99	0.0864	1.00	1.00
8	0.90	0.95	0.115	1.00	1.00	0.89	0.98	0.0933	1.00	1.00
9	0.98	0.98	0.0729	0.99	0.98	0.93	0.95	0.0734	1.00	0.99
10	0.96	0.98	0.0694	1.00	0.99	0.89	0.95	0.0805	0.99	0.99
11	1.38	1.02	0.0268	1.01	1.00	1.35	0.98	0.0339	1.00	0.99
12	1.08	1.04	0.0124	1.01	1.00	1.07	0.98	0.0160	1.00	0.99
13	0.85	1.00	0.0167	1.01	1.01	0.82	0.98	0.0227	1.00	0.99
14	1.22	0.99	0.0181	1.00	0.99	1.22	0.98	0.0276	1.00	0.99
15	1.41	1.01	0.0173	1.01	1.00	1.42	0.98	0.0293	1.00	0.99
16	0.83	1.02	0.114	1.00	1.00	0.84	1.00	0.3245	1.00	0.99

*YOM denotes use of Yiftah, Okrent, and Moldauer cross sections.⁸

NL denotes use of ELMOE-derived cross sections, assuming zero leakage.

A denotes use of leakage-iteration, using Method A (see text).

B denotes use of leakage-iteration, using Method B (see text).

C denotes use of leakage-iteration, using Method C (see text).

VII. CONCLUSIONS

Nonleakage ELMOE calculations give coarse-group values for transport cross sections closely similar to values obtained by the leakage-iterative procedures. These noniterative ELMOE calculations were carried out by assuming zero buckling and a fission-source energy distribution of applied sources.

Leakage-iterative methods are necessary to obtain the approximate, coarse-group, elastic-transfer cross sections. Furthermore, the detailed variations of the fine-group leakage (Method B) due to fine-group, scattering-resonance variations of the transport cross sections can be important in those coarse groups in which the envelope of the spectral distribution is rapidly decreasing (or increasing). It is noted, however, that for many calculational purposes the use of the coarse leakage-iterative absorption (Method A) may suffice.

The use of Method C containing also the intra-coarse-group energy smoothing of the buckling generally did not affect the spectra much differently than Method B. The use of this smoothing option (Method C) is suggested to test if smoothing is important for particular coarse groups in a given situation. If there is significant sensitivity of results to the smoothing, then subdivision of that coarse group into two or more finer coarse groups is suggested.

The effects upon regional coarse-group cross sections of different fine-structure flux variations on either side of an interface has not been considered in this study. Preliminary idealized calculations¹⁰ suggest that these effects might affect the averaging of the elastic-transfer cross sections near interfaces.

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