Formulation of Bethe-Salpeter Equation in e-p Deep Inelastic Scattering*

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ABSTRACT

Using perturbation theory for a model of pseudo-scalar (neutral) mesons coupled with the proton, we calculate the hadronic part of an amplitude for a set of dominant ladder diagrams in e-p deep inelastic scattering. We set up a Bethe-Salpeter equation type of formalism and find, in the Bjorken limit and for large ω , that the structure function νW_2 is non-scaling, supporting a previous calculation by Chang-Fishbane.⁽¹⁾ On the other hand, we also find that, by introducing a lower bound $x_{\min}$ in the integral representation of the amplitude, the scaling property of νW_2 can be revived to the same form as that obtained by Drell et al.⁽²⁾ who use a cut-off assumption for the transverse momentum of the mesons.

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I. INTRODUCTION

It is J.D. Bjorken⁽³⁾ who first derives, by a rather general argument, a result that the structure functions $MW_1(\nu, q^2)$ and $\nu W_2(\nu, q^2)$ will respectively become $F_1(\omega)$ and $F_2(\omega)$, which are functions of the ratio $\omega = 2M\nu/-q^2$ only, when the limit $\nu, -q^2 \rightarrow \infty$ is approached with ω being fixed. Since then, under the suggestion that a knowledge of the form of νW_2 or MW_1 may serve as a tool to understand the structure of the nucleon, quite a few authors have been using practical physical processes and various models to study this problem. Despite whatever results those theorists have obtained, experiments⁽⁴⁾ done so far favor Bjorken's predictions, though the kinematic regions of those experiments are still somewhat restricted. On the other hand, there have been arguments⁽⁵⁾⁽⁶⁾ that predicts breaking of scaling in very high energy region.

In this paper, we study a process of e-p deep inelastic scattering as shown in Fig. 1. We adopt the model (i.e., pseudo-scalar coupling and the dominant ladder diagrams) used by the authors in references 1 and 2. But instead of calculating the tensor

$$W^{\mu\nu} \equiv \sum_n \delta^4(q+p-p_n) (2\pi)^3 \langle p | J^\mu(0) | n \rangle \langle n | J^\nu(0) | p \rangle$$

directly by taking the product $\langle p | J^\mu(0) | n \rangle \langle n | J^\nu(0) | p \rangle$ (as the authors in references 1 and 2 do), we write down the T-amplitude of a diagram such as Fig. 2, and determine its absorptive part through the relation

$$\begin{aligned}
 \frac{1}{\pi} \text{Im} T^{\mu\nu} &= \sum_n \delta^4(q+p-p_n) (2\pi)^3 \langle p | J^\mu(0) | n \rangle \langle n | J^\nu(0) | p \rangle \\
 &= \frac{1}{M^2} \left(p_\mu - \frac{q_\mu p \cdot q}{q^2} \right) \left(p_\nu - \frac{q_\nu p \cdot q}{q^2} \right) W_2(\nu, q^2) \\
 &\quad + \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) W_1(\nu, q^2)
 \end{aligned} \tag{1.2}$$

where the second equality is a direct consequence of the conservation of current J^μ . What we have accomplished in this paper can be summarized as follows:

(a) As the n -dimensional integral of a general n -rung ladder diagram is so complicated, both groups of authors in references 1 and 2 introduce some sort of approximations in order to render the integral into a form that is calculable. Based on a totally different and independent method, we offer a formulation that may check into the validity of those approximations, especially those in reference 2 (the first theoretical paper showing a result against scaling!). We obtain, for the case of

n outer rainbows and m inner rainbows, an asymptotic result

$$vW_2^{(n,m)} \approx \left[\left(\frac{g^2}{32\pi^2} \right)^m \frac{\ell_n^m}{m!} \frac{2Mv/M^2}{m!} \right] \left[\left(\frac{g^2}{16\pi^2} \right)^n \frac{\ell_n^n}{n!} \frac{2Mv/M^2}{n!} \frac{\ell_{n-1}^{n-1}}{\omega(n-1)!} \right] \quad (1.3)$$

where g is the coupling constant of the assumed pseudo-scalar coupling with the proton. Equation (1.3) supports the calculations of reference 2, but we can show one more thing: The outer-rainbow part

$$\left[\left(\frac{g^2}{16\pi^2} \right)^n \frac{\ell_n^n}{n!} \frac{2Mv/M^2}{n!} \frac{\ell_{n-1}^{n-1}}{\omega(n-1)!} \right]$$

is actually the absolute upper bound of the unapproximated value.

(b) As far as the question of scaling is concerned, even without getting into involved calculations, our formulation (of integral equation (2.9)) decisively shows that there can be no scaling as long as one uses this model and perturbation theory.

(c) A valuable bonus is obtained in this formulation. We can show, by introducing a fixed lower limit $x_{\min}$ for the integration parameter in an integral equation (cf. (2.9)), that the scaling result of Drell et al. ⁽²⁾ can be reproduced exactly in the same form. Furthermore, we

demonstrate mathematically that $x_{\min}$ is indeed related to the cut-off $k_{1\max}$ of the transverse momentum of the intermediate mesons. Hence the formulation in this paper has provided a mathematical link between the two completely different results of references 1 and 2.

We choose to work in the so-called infinite momentum frame ⁽¹⁾ in which the four-momenta of the proton and virtual photon take the following form

$$P_{\mu} = (P_+, P, P_-) = (2P_0, 0, 0(M^2/2P_0))$$

$$q_{\mu} = (q_+, q, q_-) = (0, \sqrt{-q^2}, 0(M_0/P_0))$$

where $q_{\pm} \equiv q_0 \pm q_3$, q is the transverse component, M of course is the proton mass and the direction of p_3 has been chosen as the z-direction (longitudinal direction). Let s be the total energy square of the proton and incident electron in their c.m. frame. We assume we are always working in the kinematic region such that

$$s \gg 2M^2 \gg -q^2 \gg M^2.$$

Because of $g_{++} = g_{00} + g_{33} = 0$ and $q_+ = 0$, it is easy to see

$$vW_2 = \frac{M^2}{P_+} vW^{++} = \frac{1}{\pi} \cdot \frac{M^2}{P_+} vImT^{++} \quad (1.4)$$

which follows immediately from (1.2). The diagrams corresponding to W^{++} are those whose photon vertices have a γ -matrix $\gamma_+ = \gamma_0 + \gamma_3$ instead of γ_μ and γ_ν which correspond to $W^{\mu\nu}$.

We shall show results (a) and (b), as mentioned above, in section II and leave (c) to be discussed in section III.

II. FORMULATION

(a) Outer-rainbow Ladder Diagram.

In Fig. 2 we assume that in between the proton lines P, P , there exchange pseudo-scalar (neutral charge) bosons of which the mass can be neglected⁽⁷⁾ when compared to the proton mass. In general, any sort of boson lines can be drawn in between the two proton lines. Following Drell et al.,⁽²⁾ only those dominant diagram as shown in Fig. 3 in which there are no crossing boson lines, will be considered. For n boson lines, we define an amplitude $F_n(P)$ through this relation

$$-iT_n(P) = \sum \langle P | F_n(P) | P \rangle = \frac{1}{2} \text{tr} \left\{ \frac{P+M}{2M} F_n(P) \right\} \quad \text{spin average} \quad (2.1)$$

where $T_n(P)$ is the usual T-matrix element (for Fig. 3).

Then with reference to Fig. 3, one can easily write down a recursion relation for $F_n(P)$:

$$F_n(P) = \frac{-ig^2}{(2\pi)^4} \int \frac{d^4k}{(P-k)^2} \cdot \frac{(\not{K}-M)\gamma_5 F_{n-1}(k)\gamma_5(\not{K}-M)}{(k^2-M^2+i\epsilon)^2} \quad (2.2)$$

where g is the coupling constant between the pseudo-scalar boson and the proton. Summing up n we have

$$F(P) = F_0(P) + \frac{-ig^2}{(2\pi)^4} \int \frac{d^4k}{(P-k)^2} \frac{(\not{K}-M)\gamma_5 F(k) \gamma_5 (\not{K}-M)}{(k^2-M^2+i\epsilon)^2} \quad (2.3)$$

where

$$F(P) = \sum_{n=0}^{\infty} F_n(P)$$

Equation (2.3) is the Bethe-Salpeter equation for amplitude $F(P)$. This equation itself alone cannot help us to solve $F(P)$ unless we can write down the structure form of $F(P)$ as in most cases of the method of Bethe-Salpeter equation. We assume

$$F_n(k) = i k_+ \gamma_+ \cdot \int_0^1 dx \int_{\sigma_0}^{\infty} \frac{d\sigma f_n(x, \sigma)}{k^2 + 2x k \cdot q - \sigma + i\epsilon} \quad (2.4a)$$

from which it follows that

$$F(k) = ik_+ \gamma_+ \int_0^1 dx \int_{\sigma_0}^{\infty} \frac{d\sigma f(x, \sigma)}{k^2 + 2xk \cdot q - \sigma + i\epsilon} \quad (2.4b)$$

where $f(x, \sigma) = \sum_{n=0}^{\infty} f_n(x, \sigma)$, and σ_0 is a constant.

We shall call $f_n(x, \sigma)$ the spectral function which also depends on $k \cdot q$ and q^2 . We can easily see this structure of $F_n(k)$ from the special case of $n = 0$ by observing

$$F_0(k) = \frac{i\gamma_+ (k+q+M)\gamma_+}{(k+q)^2 - M^2 + i\epsilon} = \frac{2\gamma_+ k_+ i}{(k+q)^2 - M^2 + i\epsilon} \quad (2.5)$$

The spectral function of $F_0(k)$ is simply given by

$$f_0(x, \sigma) = 2\delta(1-x)\delta(\sigma + q^2 - M^2)$$

For the general case n , (2.4) serves as a self-consistent assumption, because, by substituting (2.4b) back into (2.3), we find that besides other lower terms, the right hand side of (2.3) reproduces a term of structure (2.4b) which happens to be the most dominant one.⁽⁸⁾

Substituting (2.4b) back into (2.3) and using the

usual parameter representations of Feynman integrals to combine the denominator factors containing k^2 , we have

$$F(k) = F_0(k) + \frac{g^2}{(2\pi)^4} \iint dx d\sigma f(x, \sigma) I(x, \sigma; k) \quad (2.6)$$

where

$$I(x, \sigma; k) = i\gamma_+ k_+ \pi^2 \int_0^1 \frac{yz dy dz}{k^2 + 2xk \cdot q(1-y) - (1-z)^{-1}(\sigma(1-y) + yM^2 + z(1-y)^2 x^2 q^2) + i\epsilon}$$

We can equate the absorptive part on both sides of (2.6) to obtain

$$\begin{aligned} f(x', \sigma') &= 2\delta(1-x') \delta(\sigma' + q^2 - M^2) \\ &+ \frac{g^2}{16\pi^2} \int_0^1 yz dy dz \int dx d\sigma f(x, \sigma) \delta(x' - x(1-y)) \cdot \\ &\cdot \delta\left[\sigma' - \frac{\sigma(1-y) + yM^2 + z(1-y)^2 x^2 q^2}{1-z}\right] \end{aligned}$$

(2.7)

The integration limits for dx and $d\sigma$ are restricted by the δ -functions in (2.7), and by a simple observation, we find that the self-consistent integration limits for dx and $d\sigma$ are respectively given by

$$x' \leq x \leq 1 \quad \text{and} \quad \sigma_0 = -xq^2 + M^2 \leq \sigma \leq \frac{x}{x'}(\sigma' - M^2) + M^2.$$

Furthermore, by making the following transformations

$$\sigma = M^2 - xq^2 \rho, \quad \sigma' = M^2 - x'q^2 \rho',$$

we can write (2.7) as

$$f(x', \rho') = 2(-q^2)^{-1} \delta(1-x') \delta(x'\rho' - 1) + \frac{g^2}{16\pi^2} \int_{x'}^1 dx \frac{(x-x')}{xx'} \int_1^{\rho'} d\rho \frac{(\rho' - \rho)(\rho - x' + t/x') f(x, \rho)}{(\rho' - x' + t/x')^3}$$

where $t = M^2/-q^2$. One can take one more step to simplify this equation by making a transformation for $f(x, \rho)$:

$$f(x, \rho) = 2(-q^2)^{-1} \delta(1-x) \delta(x\rho - 1) + 2c(-q^2)^{-1} \frac{\tilde{f}(x, \rho)}{x(\rho - x + t/x)^3} \quad (2.8)$$

where $c = g^2/16\pi^2$. Then one has

$$\begin{aligned}
\tilde{f}(x', \rho') &= (1-x')(1-x'+t/x')(\rho'-1) \\
&+ c \int_{x'}^1 dx \frac{(x-x')}{x^2} \int_1^{\rho'} d\rho \frac{(\rho'-\rho)(\rho-x'+t/x')\tilde{f}(x, \rho)}{(\rho-x+t/x)^3}
\end{aligned}
\tag{2.9}$$

This equation is fundamental in our formulation. To relate $f(x, \rho)$ with $W_2(v, q^2)$, we only have to recall (1.2), (1.4), (2.1), and (2.4) from which one can easily obtain

$$vW_2(v, q^2) = Mv \int_0^1 dx f(x, \omega) \tag{2.10}$$

We notice that the inhomogeneous term (i.e., first term on the right-hand side of (2.9)) corresponds to a one-rainbow amplitude, and each iteration will correspond to adding one more rainbow to the amplitude. To test how critically vW_2 depends on q^2 (or t), we can calculate the structure function $vW_2^{(1)}$ for one-rainbow:

$$\begin{aligned}
vW_2^{(1)}(v, q^2) &= Mv \int_0^1 dx f_1(x, \omega) \\
&\simeq \frac{g^2}{16\pi^2} \cdot \frac{1}{\omega} \cdot \ln \frac{\omega}{t}
\end{aligned}
\tag{2.11}$$

where we have picked up the most divergent term in the integral in the limit $t = M^2/q^2 \rightarrow 0$. We have also assumed $\omega \gg 1$ in order that the integral can be somewhat simplified, but the presence of the factor $\ln \omega/t$ is independent of this assumption. Since the integrand of (2.9) is positive definite, each term in the sum $\sum_{n=1}^{\infty} v W_2^{(n,0)} = v W_2$ is positive so that the result in (2.11) is sufficient to show the breaking of scaling in this scheme. Thus $v W_2$ does depend on q^2 sensitively. In fact, by iterating (2.9) $(n-1)$ times, each time taking the most dominant term, we obtain $\tilde{f}_n(x, \rho)$, the spectral function for n -rainbows, in the asymptotic form

$$\tilde{f}_n(x, \rho) \simeq (1-x)(1-x+t/x)(\rho-1) \frac{c^{n-1} \ln^{n-1} 1/x \ln^{n-1} \rho}{[(n-1)!]^2} \quad (2.12)$$

The reason of going into iteration and picking up the dominant term is that we have not yet found a way to solve (2.9) exactly. One can also see that (2.12) is the upper bound of the true $\tilde{f}_n(x, \rho)$, because (2.12) is the exact solution of the integral equation obtained by replacing the kernel of (2.9) by a new one

$$\frac{(1-x')(1-x'+t/x')(\rho'-1)}{(1-x)(1-x+t/x)(\rho-1)} \cdot \frac{1}{x\rho}$$

which is larger than the original one

$$\frac{(x-x')}{x^2} \cdot \frac{(\rho'-\rho)(\rho-x'+t/x')}{(\rho-x+t/x)^3}$$

Using (2.12) and (2.10), we have

$$vW_2^{(n)}(v, q^2) \simeq \frac{c^n}{n!} \ell_n^n(\omega/t) \frac{\ell_n^{n-1} \omega}{\omega^{(n-1)!}} \quad (2.13)$$

Clearly because of the factor $\omega/t = -q_\omega^2/M^2$ inside the logarithm, the result is non-scaling.

(b) Effect of Inner Rainbow.

The simplest diagram that involves the effect of inner rainbow is the one as shown in Fig. 4, in which there are n outer rainbows and one inner rainbow. Of course this is a special case, but it serves the purpose of showing our scheme.

The form of the Bethe-Salpeter equation is the same as in (2.3), except that now $F_0(k)$ of (2.5) should be replaced by

$$\begin{aligned}
F_0^{(1)}(k) &= \frac{i^4 g^2}{(2\pi)^4} \int \frac{d^4 k'}{(k-k')^2} \cdot \frac{\gamma_+ (\not{k}+M) \gamma_5 (\not{k}'+M) \gamma_5 (\not{k}+M) \gamma_+}{(k^2-M^2+i\epsilon)^2 (k'^2-M^2+i\epsilon)} \\
&= i^2 \frac{\gamma_+ (\not{k}+M)}{k^2-M^2+i\epsilon} A^{(1)}(k) \frac{(\not{k}+M) \gamma_+}{k^2-M^2+i\epsilon} \quad (2.14)
\end{aligned}$$

where

$$A^{(1)}(k) = \frac{i^2 g^2}{(2\pi)^4} \int \frac{d^4 k'}{(k'-k)^2} \frac{\gamma_5 (\not{k}'+M) \gamma_5}{(k'^2-M^2+i\epsilon)}$$

We see that $F_0^{(1)}(k)$ is the amplitude corresponding to Fig. 5(a). Corresponding to Fig. 5(b), $A^{(1)}(k)$ is the amplitude of a self-energy diagram which is logarithmically divergent. To extract the physically meaningful part from $A^{(1)}(k)$, we follow the usual renormalization procedure and the Feynman regularization method. We can write

$$\frac{1}{(k-k')^2+i\epsilon} = \int_0^{\Lambda+\infty} d\mu \frac{-1}{[(k-k')^2-\mu+i\epsilon]^2}$$

Then, combining k'^2 in the denominator by the usual parametric representation technique, we have

$$A^{(1)}(k) = \frac{ig^2}{16\pi^2} \int_0^{\Lambda+\infty} d\mu \int_0^1 x dx \frac{-x\kappa+M}{k^2 x(1-x) - x\mu - M^2(1-x) + i\epsilon} \quad (2.15)$$

which has the structure

$$A^{(1)}(k) = i \int_{M^2}^{\infty} \frac{(\kappa-M)g^{(1)}(\tau) + h^{(1)}(\tau)}{k^2 - \tau + i\epsilon} d\tau \quad (2.16)$$

We can identify $g^{(1)}$ and $h^{(1)}$ as

$$\begin{aligned} g^{(1)}(\tau) &= \frac{-g^2}{16\pi^2} \int_0^{\Lambda+\infty} d\mu \int_0^1 \frac{x dx}{1-x} \delta\left(\tau - \frac{x\mu + M^2(1-x)}{x(1-x)}\right) \\ &= \frac{-c}{2} [1 - (M^2/\tau)^2] \end{aligned}$$

$$\begin{aligned} h^{(1)}(\tau) &= Mg^{(1)}(\tau) + \frac{g^2}{16\pi^2} \int_0^{\Lambda+\infty} d\mu \int_0^1 \frac{dx}{1-x} M \delta\left(\tau - \frac{x\mu + M^2(1-x)}{x(1-x)}\right) \\ &= \frac{Mc}{2} (1 - M^2/\tau)^2 \end{aligned}$$

We can easily see that $A^{(1)}(k)$ is logarithmically divergent. To extract its finite part, we assume

$$A^{(1)}(k) = \delta m^{(1)} + (k-M) B^{(1)} + A_c^{(1)}(k) \quad (2.17)$$

where both $\delta m^{(1)}$ and $B^{(1)}$ are divergent quantities while $A_c^{(1)}$ is finite and proportional to $(k-M)^2$. From (2.16) we can write

$$\begin{aligned} A_c^{(1)}(k) &= A^{(1)}(k) - [A^{(1)}(k)]_{k=M} - \left[\frac{\partial}{\partial k} A^{(1)}(k) \right]_{k=M} \\ &= i (k-M) [(k^2-M^2) \int \frac{g^{(1)}(\tau) d\tau}{(\tau-M^2)(k^2-\tau+i\epsilon)} + \int \frac{2M h^{(1)}(\tau) d\tau}{(\tau-M^2)^2}] \\ &\quad + i (k^2-M^2) \int \frac{h^{(1)}(\tau) d\tau}{(\tau-M^2)(k^2-\tau+i\epsilon)} \end{aligned} \quad (2.18)$$

When $A^{(1)}(k)$ in (2.14) is replaced by this $A_c^{(1)}(k)$, we have

$$\begin{aligned}
F_0^{(1)}(k) &= i 2 k_+ \gamma_+ \int \frac{d\tau}{(k^2 - \tau + i\epsilon)(\tau - M^2)} \left[g^{(1)}(\tau) + \frac{2M h^{(1)}(\tau)}{\tau - M^2} \right] \\
&= i 2 K_+ \gamma_+ \int \frac{G^{(1)}(\tau) d\tau}{k^2 - \tau + i\epsilon} \quad (2.19)
\end{aligned}$$

where $G^{(1)}(\tau) = \frac{1}{\tau - M^2} \left(g^{(1)}(\tau) + \frac{2M h^{(1)}(\tau)}{\tau - M^2} \right)$

$$\simeq g^{(1)}(\tau) / \tau \quad \text{if } \tau \gg M^2.$$

Replacing $F_0(k)$ by $F_0^{(1)}(k)$, k by $k + q$ and using (2.6), we obtain

$$\begin{aligned}
f^{(1)}(x', \sigma') &= -\delta(1-x') G^{(1)}(\sigma' + q^2) \\
&+ \frac{g^2}{16\pi^2} \int_0^1 yz dy dz \int dx d\sigma f^{(1)}(x, \sigma) \delta(x' - x(1-y)) \cdot \\
&\cdot \delta \left[\sigma' - \frac{\sigma(1-y) + yM^2 + z(1-y)^2 x^2 q^2}{1-z} \right]
\end{aligned}$$

The superscript over f is to indicate that the effect of

one inner rainbow is involved. We can do the same transformation as before and obtain

$$f^{(1)}(x', \rho') = - (1-x') G^{(1)}(M^2 + q^2 - \rho' q^2) \\ + c \int_{x'}^1 dx \frac{(x-x')}{xx'} \int_1^{\rho'} d\rho \frac{(\rho' - \rho)(\rho - x' + t/x') f^{(1)}(x, \rho)}{(\rho' - x' + t/x')^3}.$$

Now if we iterate this integral equation n times, again each time taking the most dominant term, we find

$$f_n^{(1)}(x, \rho) \simeq \frac{c}{2} \cdot \ln \frac{\rho}{t} f_n(x, \rho)$$

where we have used the asymptotic form of $G^{(1)}(\tau)$ and have assumed $\rho \gg 1$. Similarly, for m inner rainbows, we find

$$f_n^{(m)}(x, \rho) \simeq \left(\frac{c}{2}\right)^m \frac{\ln^m \rho/t}{m!} f_n(x, \rho) \quad (\rho \gg 1) \quad (2.20)$$

Thus we see that the effect of including the inner rainbows

is simply to multiply to the outer rainbow spectral function by a factor $(c/2)^m \cdot \ln^m(\omega/t)/m!$. Using this result and (2.12), one can show that the structure function corresponding to n outer rainbows and m inner rainbows is given by

$$v_{W_2}^{(n,m)} = \left(\frac{c}{2}\right)^m (\ln \omega/t)^m \cdot \frac{c^n}{(n-1)!n!} \cdot \frac{\ln^{n-1} \omega}{\omega} \cdot \ln(\omega/t)^n \quad (2.21)$$

where $n = 1, 2, \dots$; $m = 0, 1, 2, \dots$

Summing over m and n , we have

$$v_{W_2}(v, q^2) = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} v_{W_2}^{(n,m)} = \left(\frac{\omega}{t}\right)^{c/2} \cdot \frac{\ln \frac{\omega}{c}}{\omega \sqrt{c \ln \omega \ln \frac{\omega}{t}}} I_1(z_0) \quad (2.22)$$

where $z_0 = 2 \sqrt{c \ln \omega \ln \frac{\omega}{t}}$ and $I_1(z_0) = -i J_1(iz_0)$

is an associated Bessel function of first degree.

We may add a remark here: We have dropped all terms with $n = 0$ since these correspond to pure inner rainbow contributions which are lower in order of magnitude by a

factor $\ln \omega/t$ when compared to those from the same number of outer rainbows. As an example, one can verify this fact by comparing $f_1(x, \rho)$ with $-G^{(1)}(M^2 + q^2 - \rho q^2)\delta(1-x)$. We assert that this has been found true in general.

An immediate result one can derive from (2.21) is the multiplicity of boson produced:

(i) From inner rainbow

$$\bar{n}^{\text{in}} = \sum_{m,n} m \nu W_2^{(n,m)} / \nu W_2 = \frac{c}{2} \cdot \ln \frac{\omega}{t}$$

(ii) From outer rainbow

$$\bar{n}^{\text{out}} = \sum_{m,n} n \nu W_2^{(n,m)} / \nu W_2 \simeq c \ln \omega \ln \frac{\omega}{t}$$

Result in (i) is identical with that by Chang-Fishbane⁽¹⁾, while (ii) looks very much like $c \ln \omega \ln[1+k_{\perp \text{max}}^2/M^2]$ obtained by Drell et al.⁽²⁾

III. DISCUSSION

From what we have done in the previous section, there is no doubt about the fact that, within the domain of this model, scaling is badly broken. However, because of the striking similarity between our value of multiplicity $\bar{n}^{\text{out}}$ and that of Drell et al.⁽²⁾, it is worthwhile to examine how the results of the structure function differ from each other. In a different notation, the result of Drell et al.⁽²⁾ is

$$F_{2(n\pi^0)}(\omega) = \frac{\xi_0}{\omega} \cdot \frac{1}{(n-1)!} (\xi_0 \ln \omega)^{n-1}$$

where

$$\xi_0 = \frac{g^2}{16\pi^2} \ln (1 + k_{\perp \text{max}}^2 / M^2) .$$

They obtain this result by assuming the existence of a transverse cut-off momentum $k_{\perp \text{max}}$ for all emitted particles. Our result, from (2.13), can be written as

$$v_{W_2}^{(n)}(v, q^2) = \frac{\eta_0}{\omega} \cdot \frac{1}{n!} \cdot \frac{(\eta_0 \ln \omega)^{n-1}}{(n-1)!}$$

where
$$\eta_0 = \frac{g^2}{16\pi^2} \ln(2Mv/M^2).$$

The difference between the two results is that $F_{2(n\pi^0)}$ has a factor ξ_0^n which is scaling, while $vW_2^{(n)}$ contains $\eta_0^n/n!$ which breaks scaling. Now suppose we can set a fixed lower limit $x_{\min}$ ($x' < x_{\min} < 1$) for the x -integration in (2.9), namely, instead of having (2.9), we write

$$\begin{aligned} \tilde{f}(x', \rho') &= (1-x')(1-x' + t/x') (\rho'-1) \\ &+ c \int_{x_{\min}}^1 dx \frac{x-x'}{x^2} \int_0^{\rho'} d\rho \frac{(\rho'-\rho)(\rho-x' + t/x')}{(\rho-x + t/x)^3} \tilde{f}(x, \rho). \end{aligned}$$

Then $(n-1)$ times of iterations (again, picking the most dominant term for each iteration) will produce a factor (which is from the x -integration; the other factor $(1-x')$ $(1-x'+t/x')(\rho'-1) \ln^{n-1} \rho'$ remains unchanged as in (2.12))

$$c^{n-1} \left[\int_{x_{\min}}^1 \frac{dx}{x} \right]^{n-1} = \left(\frac{g^2}{16\pi^2} \right)^{n-1} \ln^{n-1}(x_{\min}^{-1}).$$

(3.1)

instead of $c^{n-1} \ln^{n-1}(x'^{-1})/(n-1)!$ which we obtain previously from (2.9). Suppose we can further truncate the lower limit of the x -integration in $\int_0^1 dx f_n(x, \omega)$ to

$\int_{x_{\min}}^1 dx f_n(x, \omega)$ *. By so doing and using (3.1), we have this factor

$$\left(\frac{g^2}{16\pi^2} \ln \frac{1}{x_{\min}} \right)^n \quad (3.2)$$

instead of $\eta_0^n/n!$ in the expression of $vW_2^{(n)}$. By formally setting

$$x_{\min} = (1 + k_{\perp \max}^2/M^2)^{-1} \quad (3.3)$$

we immediately see that (3.2) is just ξ_0^n which is the factor in the expression of $F_2(n\pi^0)(\omega)$ of Drell et al. Hence introducing a $x_{\min}$ is equivalent to introducing a cut-off $k_{\perp \max}$. To demonstrate that the number $k_{\perp \max}$ really corresponds to the cut-off of the transverse

* $f_n(x, \omega) = 2c (-q^2)^{-1} \tilde{f}_n(x, \omega)/x(\omega - x + t/x)^3$ which follows (2.8) with the elastic scattering term excluded.

momentum of the bosons, we carry out a direct momentum integration for the amplitude of one outer-rainbow (cf. Fig. 5(c)):

$$F_1(P) = \frac{i^4 g^2}{(2\pi)^4} \int \frac{d^4 k}{(k^2 + i\epsilon)} \frac{\gamma_5 (\not{P} - \not{k} + M) \gamma_+ (\not{P} - \not{k} + \not{q} + M) \gamma_+ (\not{P} - \not{k} + M) \gamma_5}{((P-k+q)^2 - M^2 + i\epsilon) [(P-k)^2 - M^2 + i\epsilon]^2}$$

where k is the 4-momentum of the neutral pion. From (1.2), (1.4), and (2.1), one can write

$$v_{W_2}^{(1)}(v, q^2) = \frac{1}{\pi} \frac{Mv}{4P_+^2} \text{tr} \{ (\not{P} + M) \text{Im} (iF_1(P)) \} ,$$

and

$$\text{Im} (iF_1(P)) = \frac{(-\pi^2) g^2}{(2\pi)^4} \int d^4 k \frac{N}{D} \delta_+(k^2) \delta_+((P-k+q)^2 - M^2) \quad (3.4)$$

$$\text{where} \quad \delta_+(k^2) = \theta(k_0) \delta(k^2) = \frac{1}{2k_0} \delta(k_0 - |\vec{k}|) ,$$

$$N = \gamma_5 (\not{P} - \not{K} + M) \gamma_+ (\not{P} - \not{K} + M) \gamma_+ (\not{P} - \not{K} + M) \gamma_5$$

$$\begin{aligned} D &= [(P-k)^2 - M^2]^2 = 4 (P \cdot k)^2 \approx 4 [P_0 (\sqrt{k_3^2 + k_\perp^2} - k_3) + \frac{M^2}{2P_0} k_3]^2 \\ &= (P_0/k_3)^2 [k_\perp^2 + M^2 (k_3/2 P_0)^2]^2 \end{aligned}$$

($k_3 \gg k_\perp$ from the second δ -function in (3.4))

By expanding[†]

$$\text{tr} \{ (\not{P} + M) N \} = -16 k_+ (P_+ - k_+) P \cdot k \approx -8 P_+^2 k_+ k_- = -8 P_+^2 k_\perp^2,$$

we find, on the average,

$$v W_2^{(1)} \approx \frac{g^2/16\pi^2}{\omega} \cdot \int_0^{(\sqrt{2M\nu} + \sqrt{-q^2})^2 \rightarrow 2M\nu} \frac{dk_\perp^2}{k_\perp^2 + \bar{M}^2} \quad (3.5)$$

[†] From the δ -functions in $\int m(iF_1(P))$, k_+ (or k_3) is found to be at most a fraction of P_+ . Thus $\text{tr} \{ (\not{P} + M) N \} \propto P_+^2 k_\perp^2$ is always correct and only the factor -8 may be changed if one wants the exact trace.

where $k_{\perp}$ is the transverse momentum and $\bar{M}$ is some mass parameter comparable to the nucleon mass. Expression (3.5) corresponds to

$$\frac{g^2/16\pi^2}{\omega} \int_0^1 \frac{dx}{x + t/\omega} \quad (t = M^2/-q^2) \quad (3.6)$$

which is the first line of (2.11) (if one retains only the dominant terms in $f_1(x, \omega)$). Thus replacing the upper limit $(\sqrt{2M\nu} + \sqrt{-q^2})^2$ by $k_{\perp \max}^2$ in (3.5) is equivalent to replacing the lower limit 0 by $x_{\min}$ in (3.6), provided that they are connected by

$$x_{\min} = \left(1 + \frac{k_{\perp \max}^2}{\bar{M}^2}\right)^{-1}$$

Allowing $x_{\min} \rightarrow 0$ is equivalent to letting $k_{\perp \max}^2 \rightarrow 2M\nu$. Hence we have shown and provided mathematically the link between scaling and non-scaling depending on the assumption of a cut-off $k_{\perp \max}$ or not.

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FIGURE CAPTIONS

- Fig. 1: e-p deep inelastic scattering with one photon (virtual) exchanged.
- Fig. 2: Compton scattering of virtual photon.
- Fig. 3: A dominant outer rainbow ladder diagram with n rainbows for the amplitude $F_n(P)$.
- Fig. 4: Diagram of n outer rainbows plus one inner rainbow. A general case would be n outer rainbows plus m inner rainbows which corresponds to $f_n^{(m)}(x, \rho)$ and $v W_2^{(n,m)}$ in (2.20) and (2.21) respectively.
- Fig. 5(a): Diagram of one inner rainbow.
- Fig. 5(b): Self-energy diagram of a meson (neutral) emitted and re-absorbed on a proton line.
- Fig. 5(c): Diagram of one outer rainbow.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5(a)

Fig. 5(b)

Fig. 5(c)