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MODEL TESTING REQUIREMENTS

by

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MASTER

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INTRODUCTION

In the course of the development of components involving the flow of fluid, the question of the use of models arises frequently. The ability of a given model to predict the behavior of the prototype is always open to doubt. The discussion in the memorandum considers the various requirements which would have to be met in order to obtain meaningful information.

DISCUSSION

If the Navier Stokes equation is examined, three types of terms become apparent.

Writing this equation as:

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = - \frac{\partial}{\partial x} (p + \rho gh) + \mu \nabla^2 u$$

the inertial or D'Alembert forces are composed of the expanded terms of $\rho \frac{Du}{Dt}$; the pressure forces are $\frac{\partial}{\partial x} (p + \rho gh)$; and the viscous forces, $\mu \nabla^2 u$. Hence in terms of gross parameters $\rho, \Delta P, V, L, h$ (in a given direction):

1. Inertia force $\approx \rho u \frac{\partial u}{\partial x} \approx \rho V \frac{V}{L}$,
2. Pressure force $\approx \frac{\partial p}{\partial x} \approx \frac{\Delta P}{L}$,
3. Viscous force $\approx \mu \nabla^2 u \approx \mu \frac{V}{L^2}$,
4. Gravity forces $\approx \frac{\partial}{\partial x} (\rho gh) \approx \rho g \frac{L}{L} \approx \rho g$

The ratio of the inertial to the viscous force yields the familiar term $\rho \frac{V^2}{L} \div \frac{\mu V}{L^2} = \rho \frac{VL}{\mu}$

easily recognized as the Reynolds number. The ratio of the pressure to inertial force yield the

term $\frac{\Delta P}{L} \div \rho \frac{V^2}{L} = \frac{\Delta P}{\rho V^2}$, which, with the exception of a length to diameter term is proportional to the Fanning friction term. The ratio of inertia forces to gravitational force yield $\rho \frac{V^2}{L} \div \rho g = \frac{V^2}{gL}$ commonly referred to as the Froude number. For gaseous flow, the significance of this term is trivial since variations in pressure usually exceed variations in elevation by orders of magnitude. Compressibility considerations can be obtained from the energy equation

$$u du + C_p dT = 0$$

The ratio of the kinetic term $u du$ to the potential term $C_p dT$ leads to V^2/T which can be interpreted as the Mach number. Hence a three variable reference plane would be required with the axes of Reynolds number, Mach number and the dimensionless pressure drop $(\Delta P/\rho V^2)$.

In order to duplicate the three dimensionless number in two systems denoted by subscripts x and y the following condition must be met:

For Reynolds number:

$$Re = \frac{V_x L_x \rho_x}{\mu_x} = \frac{V_y L_y \rho_y}{\mu_y} \quad (1.00)$$

For Mach number:

$$M = \frac{V_x}{\left(\frac{k_x g R T_x}{m_x} \right)^{1/2}} = \frac{V_y}{\left(\frac{k_y g R T_y}{m_y} \right)^{1/2}} \quad (2.00)$$

For pressure drop:

$$\Pi = \frac{\Delta P_x}{\rho_x V_x^2} = \frac{\Delta P_y}{\rho_y V_y^2} \quad (3.00)$$

Using a general continuity relation that the mass rate of flow $\dot{w}$ is the product of an area represented by the square of a characteristic dimension L , the velocity and the density

$$\dot{w} = L^2 V \rho \quad (4.00)$$

permits the reduction of the dimensionless variables (Re, M, Π) to

$$Re = \frac{\dot{w}_x}{L_x \mu_x} = \frac{\dot{w}_y}{L_y \mu_y} \quad (1.01)$$

which reduced to

$$\dot{w}_x = \left(\frac{L_x}{L_y} \right) \left(\frac{\mu_x}{\mu_y} \right) \dot{w}_y \quad (1.02)$$

$$M = \frac{\frac{\dot{w}_x}{L_x^2 \rho_x}}{\left(\frac{k_x g R T_x}{m_x} \right)^{1/2}} = \frac{\frac{\dot{w}_y}{L_y^2 \rho_y}}{\left(\frac{k_y g R T_y}{m_y} \right)^{1/2}} \quad (2.01)$$

and remembering $\rho = \frac{mP}{gRT}$ and $k_x = k_y$ (if gases having the same number of atoms per molecule are used) the simplification can be made that:

$$\dot{w}_x = \left(\frac{L_x}{L_y} \right)^2 \frac{P_x}{P_y} \left(\frac{m_x}{m_y} \frac{T_y}{T_x} \right)^{1/2} \dot{w}_y \quad (2.02)$$

The pressure drop can be expressed as

$$\Pi = \frac{\Delta P_x \frac{L_x^4}{\dot{w}_x^2}}{\Delta P_y \frac{L_y^4}{\dot{w}_y^2}} = \frac{\Delta P_x}{\Delta P_y} \frac{L_x^4}{L_y^4} \frac{\dot{w}_y^2}{\dot{w}_x^2} \quad (3.01)$$

which can be reduced to

$$\Delta P_x = \left(\frac{\dot{w}_x}{\dot{w}_y} \right)^2 \left(\frac{L_y}{L_x} \right)^4 \frac{P_y}{P_x} \frac{m_y}{m_x} \frac{T_x}{T_y} \Delta P_y \quad (3.02)$$

The elimination of $\left(\frac{\dot{w}_x}{\dot{w}_y} \right)$ between 2.02 and 3.02 yields

$$\frac{\Delta P_x}{\Delta P_y} = \frac{P_x}{P_y} \quad (3.03)$$

while the elimination of $\left(\frac{\dot{w}_x}{\dot{w}_y} \right)$ between 1.02 and 2.02 yields

$$\frac{P_x}{P_y} = \frac{L_y}{L_x} \frac{\mu_x}{\mu_y} \left(\frac{m_y}{m_x} \frac{T_x}{T_y} \right)^{1/2} \quad (3.04)$$

Since the purpose of the model testing is the flow simulation of hydrogen at high pressure and low temperature with a fluid at a pressure level near atmospheric and at a reasonable temperature level, a wide range of fluids can be considered. Ideally a full scale model would be desired since a prototype of the actual component could be used. The last condition corresponds to a scale factor of unity ($L_x/L_y = 1$) and temperature ranges of hydrogen would be between 30°R and 200°R compared to a simulator gas temperature range from 500 to 1000°R . Nominally setting the hydrogen gas temperature at 150°R and designating hydrogen as gas "y", hereafter used with subscript "h", we have for the Reynolds number condition (equation 1.02):

$$\dot{w}_x = \frac{\mu_x}{\mu_h} \dot{w}_h \quad (1.03)$$

For air at 530°R this becomes:

$$\dot{w}_x = 5 \dot{w}_h \quad (1.04)$$

That is, five times the mass flow of air is required to simulate the mass flow of hydrogen. A reduction can be obtained by using cold nitrogen (Table 1) but even with cold nitrogen twice the mass flow is required to simulate the flow of hydrogen. The use of liquids aggravate the problem of high mass flows. From a practical viewpoint very little could be gained by using another fluid (cold N_2 , methane, etc.) when the addition complexities of a closed system are considered.

The numerics for simultaneous simulation of Reynolds and Mach number yield
(equation 3.04):

$$\frac{P_x}{P_h} = 2.47 \quad (3.05)$$

Using this pressure ratio, equation 3.03 reduces to:

$$\Delta P_x = 2.47 \Delta P_h \quad (3.06)$$

A full scale model could then be tested with air under identical Reynolds, Mach and pressure drop dimensionless numbers under the assumed conditions (H_2 at $150^\circ R$, Air at $530^\circ R$), but on examination of the flow and pressures involved (Table 2) equipment of heroic proportions would be required.

As Table 2 indicates, the flow and pressure requirements which respectively simulate the Reynolds and Mach number (equations 1.03 and 2.03 respectively) would require very large air supplies at high pressures, but would result in very reasonable pressure drops (equation 3.03). If compressibility effects are not expected to be serious, lower pressure ratios than that required to satisfy 1.03 and 2.03 simultaneously can be used. In the event

that compressibility effects are small so that they can either be neglected or that suitable corrections* can be used, the pressure drop term can be written as (numeric substitution in 3.02)

$$\Delta P_x = 6.10 \frac{P_h}{P_x} \Delta P_h \quad (3.07)$$

The pressure drop can be scaled arbitrarily by the choice of the pressure ratio. This approach does permit an additional degree of freedom in the design of simulation tests.

Reducing the scale factor ($L_x/L_h < 1.0$) reduces the flow requirements but increases the required pressure level. Table 3 presents the flow and pressure requirements for a one tenth scale model ($L_x/L_y = 0.1$). The comparison of quantities required and pressure levels expected under such a scaled condition with a full sized model is not encouraging. The

*The pressure drop term ΔP could probably be replaced by its compressible equivalent

$\Delta P / \left(1 + \frac{M^2}{4}\right)$ which stems from the general series:

$$\frac{\Delta P}{\rho V^2} = 1/2 \left[1 = \frac{1}{4} M^2 + \frac{2-k}{24} M^4 + \frac{(2-k)(3-2k)}{192} M^6 \dots \right]$$

(See H. W. Liepman and A. E. Puckett "Aerodynamics of a Compressible Fluid" p. 27;

Wiley 1947)

pressure level requirement will be extremely difficult to meet, so the Mach effects will be difficult to model.

From the considerations of similitude it becomes apparent that best simulation would be accomplished with the largest possible air supply at the highest reasonable pressure. The degree of compromise required by economic considerations will determine the degree of compromise in simulation.

TABLE 1

<u>FLUID</u>	<u>TEMPERATURE °R</u>	<u>VISCOSITY MICROPOISE</u>	<u>μ_x/μ_h</u>
----- Gases -----			
H ₂	150	36	1.00
Air	530	180	5.00
	1000	263	7.30
Argon	530	221	6.14
CO ₂	530	148	4.11
Cl	530	133	3.70
He	530	194	5.39
CH ₄	530	108	3.00
Steam	672	125	3.47
N ₂	180	68	1.89
----- Liquids -----			
Methyl Alcohol	530	6,000	167.0
Acetone	530	3,200	88.8
Glycerine	530	1.49×10^9	4.1×10^7
Water	530	10,000	277.8

TABLE 2
For Full Scale Model ($L_x/L_h = 1$)

<u>Hydrogen Flows ($\dot{w}_h$)</u>	<u>Air "Simulator" Flows ($\dot{w}_x$)</u>
1 pps	5 pps
10 pps	50 pps
100 pps	500 pps
<u>Hydrogen Pressure Level (P_h)</u>	<u>Air Pressure Level</u>
15 psia	37 psia
450 psia	1111 psia
1500 psia	3705 psia

TABLE 3
For Small Scale Model ($L_x/L_h = 0.10$)

<u>Hydrogen Flows ($\dot{w}_h$)</u>	<u>Air "Simulator" Flows ($\dot{w}_x$)</u>
1 pps	0.5 pps
10 pps	5.0 pps
100 pps	50.0 pps
<u>Hydrogen Pressure Level (P_h)</u>	<u>Air Pressure Level</u>
15 psia	370 psia
450 psia	11,111 psia
1,500 psia	37,050 psia

NOMENCLATURE

C_p	=	Specific heat at constant pressure
g	=	Gravitational constant
h	=	Height
k	=	Ratio of specific heats
L	=	Length
m	=	Molecular weight
M	=	Mach number
P	=	Pressure
ΔP	=	Pressure difference
R	=	Universal gas constant
t	=	Time
T	=	Temperature
u	=	Velocity component in x direction
v	=	Velocity component in y direction
V	=	Mean velocity
w	=	Velocity component in z direction
x	=	Cartesian coordinates
y	=	Cartesian coordinates
z	=	Cartesian coordinates
$\dot{w}$	=	Mass flow rate
μ	=	Viscosity
Π	=	Dimensionless pressure drop

Subscripts

h	=	Pertaining to hydrogen
x	=	Pertaining to gas x
y	=	Pertaining to gas y