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ELASTIC PRECURSOR DECAY AND STEADY WAVE EVOLUTION
IN POROUS ALUMINUM AND POROUS POLYURETHANE

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ABSTRACT

Computer studies of the rate-dependent compaction characteristics of two quite different porous materials were explored in wave propagation studies with the sharp shock code CONCHAS. This code uses the method of characteristics and the shock jump relations to compute the propagation of shock waves in materials, in contrast to other types of codes which require artificial viscosity for such problems. Three features of rate-dependent pore collapse were investigated: (1) the rate of attenuation of elastic precursors (which requires the sharp shock analysis), (2) the evolution of the main compaction wave to a steady wave, and (3) stress relaxation in the material adjacent to an impact surface. The impact condition in these problems was simulated by application of a constant amplitude step increase in velocity to the boundary of a halfspace. Wave propagation solutions were obtained for problems in which various values of the material relaxation time ranging from 0.005 to 0.25 microseconds were assumed. One of the primary conclusions of the investigation is that steady wave analysis still seems to be the most profitable method of studying pore collapse. Steady wave evolution is discussed in detail, and it is proposed that a time equal to 3.5 times the effective steady wave rise time is a good approximation of the steady wave evolution time for the conditions investigated in this study. In the elastic precursor attenuation part of the investigation, significant differences were observed between linear theory predictions, and the nonlinear precursor attenuation characteristics obtained from the computer solutions. In all cases, however, the precursors were observed to attenuate so rapidly that experimental measurement of their amplitudes would be very difficult if not impossible. Finally, the rate of stress relaxation near the impact surface was also so rapid under the conditions investigated, that it is unlikely that this type of experimental measurement can be used to obtain information about rate-dependent pore collapse.

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INTRODUCTION

During the past few years numerous investigations have been conducted on the influence of porosity on the mechanical properties of materials. One of the consequences of these investigations has been further confirmation of the fact that porous materials have properties which are extremely desirable in thermal and mechanical shock environments: shock pulses in a porous material are very rapidly attenuated,¹ and rapid deposition of thermal energy in a porous material does not produce thermal stresses nearly as large as are observed in solids under equivalent conditions.²

The desirable attenuation characteristics of porous materials are related directly to the very large hysteresis observed in their loading-unloading cycles. In fact, a corollary of this observation is that in most cases the larger the change in volume due to irreversible pore collapse, the more effective the material will be as a shock mitigator. Various material models have been developed which permit estimation of the change in volume of the material as a function of shock amplitude. Most of these predictions are based on the assumption that collapse to the equilibrium state is instantaneous. It has been demonstrated that in many situations predictions produced by these models can be used for design calculations with a high degree of confidence.

More recently, as a clearer understanding of dynamic compaction has developed, it has become increasingly apparent that the equilibrium, rate independent models may not be entirely satisfactory for certain very short pulse loadings. The principal reason for concern is that dynamic compaction involves both bulk motion of mass, and very large plastic deformation of the solid, at high strain rates, in the vicinity of the voids. Either of these processes could result in a time dependence of the pore collapse process which could

cause classical rate independent shock wave predictions to be in error. Because of the time required for void collapse, the change in volume of the material during short time loading would be less than indicated by the classical theory, and the conversion of mechanical energy into thermal energy during plastic deformation could be reduced. In other words, thin stress pulses in highly rate-dependent porous materials may attenuate much less rapidly than stress pulses in rate independent materials.

A simple theory of rate-dependent pore collapse was developed recently to examine some of the implications of the above considerations.³ While a great deal more verification and expansion of the theory is still required, its present state is sufficient to estimate the probable influence of rate-dependent effects on various features of compaction. Thus it is now possible to determine just how important the added complication of time dependence to porous material wave propagation calculations really is. This in turn will help to determine how much effort should be devoted to further development of the theory.

The purpose of this paper is to report the results of a systematic parameter study which was initiated to answer some of these questions. This part of the investigation was concerned with the types of porous material response information and measurements that can be obtained during thick pulse wave propagation experiments, i.e., the influence of rate sensitivity on the measurements that are made during air gun plate impacts. The objective of these calculations was to identify the types of measurements which most sensitively reflect the time dependence of pore collapse. This information can then be used for guidance in the formulation and design of future experiments.

MATERIAL PROPERTIES

Wave propagation calculations for two typical porous materials were used to investigate the effects of rate-sensitivity. These materials have characteristic stress-strain curves which represent approximate limits of observed elastic-plastic response. The first material was 0.96 gm/cc rigid polyurethane foam, which is a typical viscoelastic foam produced by addition of a freon blowing agent to the raw material, expanding it to the desired density. This process produces a random array of spherical voids with a mean diameter of 0.2 mm. The assumed equilibrium curve for this material, shown in Figs. 1a and 1c, is quite similar to the idealized elastic-locking model frequently used in porous material calculations. Plate impact experiments have already established that its characteristic relaxation time for void collapse is of the order of 0.05 μ sec.³ The mechanical constants for this material are given in Table 1.

The second material was 2.16 gm/cc 2024 sintered aluminum. This is a typical sintered metal product formed from powder with an average particle size of 16.1 μ . The void structure is irregular with an average void size of 2.5 μ . The assumed equilibrium stress-strain curve of this material is shown in Figs. 1b and 1c, and the mechanical properties are given in Table 1. A characteristic relaxation time has also been estimated for this material, and found to be of the order of several nanoseconds.⁴

The most obvious difference between the two materials described above is that one is a polymeric, viscoelastic material with large voids, and has a matrix with properties that are very highly sensitive to strain rate. The second material is a metal which deforms plastically, has a fairly small void size, is fairly homogeneous, and is made from an alloy whose properties are

known to be fairly insensitive to strain rate. Other differences are apparent in the equilibrium curves. The porous metal exhibits a gradual increase in stress with increased compaction after the yield point is reached, while the viscoelastic material tends to collapse to the solid state with no increase in pressure once the yield point is reached. The differences in compressibility of the two materials, as measured by the maximum difference in pressure between the equilibrium curve and a Rayleigh line,[†] are apparent from Fig. 1a and 1b. The polyurethane is more nonlinear. This nonlinearity can influence the evolution time for steady wave fronts to form in the material. It also follows from the characteristic relaxation times quoted from the two materials that the polyurethane deformation is more strain rate dependent as would be expected from the matrix characteristics and large void size.

[†]This is a straight line on the pressure volume plane which, for a wave of specified amplitude, connects the yield point with the point representing the final equilibrium pressure in the material. Its significance is that if the wave is steady, all points in the wave front have stress states lying on the Rayleigh line.

CHARACTERISTIC EQUATIONS

The rate-dependent porous material constitutive equation used for the calculations of this investigation has already been described in previous reports.^{3,5} The reader is referred to these references for more detailed information about the model. There are, however, several additional steps in its application which merit consideration. First the rate-dependent constitutive relation will be expressed in a form more suitable for characteristic analysis.

The constitutive equation previously used in WONDY for rate-dependent calculations⁵ was of the form:

$$\left(\frac{\partial \sigma}{\partial t}\right)_x = \frac{\left[\rho \tilde{P}_s \tilde{g}_\alpha + \frac{1}{\rho} \left[\rho_s \tilde{P}_s + \frac{\sigma}{\rho} \tilde{P}_\epsilon\right] \left(\frac{\partial \rho}{\partial t}\right)_x\right]}{\left[1 - \rho \tilde{P}_s \frac{d\alpha_e}{d\sigma}\right]} \quad (1)$$

where t is the time, x is the material coordinate, σ is the stress, ρ is the density, ρ_s is the density of the solid matrix, ϵ is the internal energy, $\tilde{g}_\alpha$ is the pore collapse function, and $\frac{d\alpha_e}{d\sigma}$ refers to the slope of the instantaneous curve through state (σ, α) . The parameter α is the quantity ρ_s/ρ . The assumption was made in the derivation of this equation that shear stresses can be neglected, and therefore that the stress was hydrostatic in nature ($\sigma = P$). In addition, the constitutive equation for the solid was assumed to be of the form $P = \tilde{P}(\rho_s, \epsilon)$, with partial derivatives $\tilde{P}_s = \frac{\partial P}{\partial \rho_s}$ and $\tilde{P}_\epsilon = \frac{\partial P}{\partial \epsilon}$.

For method of characteristic calculations the most familiar form of the constitutive relation is:

$$\frac{\partial \sigma}{\partial t} + A^2 \frac{\partial V}{\partial t} + R = 0 \quad (2)$$

The function A is the accoustical impedance function, R is the stress relaxation function, and V is the specific volume. To relate equation (1) to equation (2) we note that the density ρ in equation (1) is related to the specific volume V by $\rho = 1/V$. Differentiating with respect to time one obtains:

$$\frac{1}{\rho} \frac{\partial \rho}{\partial t} = - \frac{1}{V} \frac{\partial V}{\partial t} , \quad (3)$$

which in turn can be substituted into equation (1). Comparison of like terms in equations (1) and (2) shows the accoustical impedance function is given by:

$$A^2 = \frac{\rho \rho_s \tilde{P}_s + \frac{\sigma}{\rho} \tilde{P}_e}{1 - \rho \tilde{P}_s \frac{d\alpha_e}{d\sigma}} , \quad (4)$$

and the stress relaxation function is given by:

$$R = \frac{\rho \tilde{P}_s \tilde{\epsilon}_\alpha}{\rho \tilde{P}_s \frac{d\alpha_e}{d\sigma} - 1} , \quad (5)$$

The specific equations for $\tilde{g}_\alpha$, $\frac{d\alpha_e}{d\sigma}$, $\tilde{P}_s$, and $\tilde{P}_e$ are given in Appendix A.

For further details of the derivation of characteristic equations see Ref. 7.

NUMERICAL CALCULATIONS

The importance of rate-dependent pore collapse can be determined by investigating three features of dynamic compaction measurable in gas gun impact experiments. These are (1) the rate of attenuation of the elastic precursor, (2) the risetime or width of a steady wave and the distance required for steady wave evolution, and (3) the stress relaxation of material adjacent to the point of application of the load. The effects of various degrees of rate sensitivity on these features were determined from numerical wave propagation solutions obtained with the wave propagation code CONCHAS.⁷ Unfortunately, at the time these computations were made, the CONCHAS code was not sufficiently developed to permit a complete solution for a plate impact problem. Instead, we considered the type of problem in which a step change in particle velocity characteristic of a completely symmetric impact (impacting plate is the same material as the target) is applied to the free surface of a porous half space. While this class of problems does not model the general plate impact problem, it is a special case which can be used to estimate the magnitude of rate-sensitive effects.

Complete wave propagation solutions were obtained for several different impact boundary velocity conditions, and various material relaxation times, for both the porous polyurethane and the porous aluminum. Relaxation times of 0.01, 0.05, and 0.25 μ sec were assumed for the polyurethane, and times of 0.005, 0.01, and 0.025 μ sec were assumed for the aluminum. Although non-dimensional solutions are easily obtained for this type of problem, we chose to represent most of the results in real time and distance, in order to make direct physical comparisons with experimental configurations.

Elastic Precursor Attenuation

The attenuation of elastic precursors in porous materials is one of the more obvious consequences of rate-dependent pore collapse. Unlike corresponding phenomena in solids, the elastic moduli of porous materials are expected to be nonlinear, even at relatively low stress levels, because they reflect both the matrix compression and the change in void volume. A consequence of this nonlinearity, as will be shown later in this section, is that the linear precursor analysis commonly used for solids⁸ is not satisfactory for porous materials.

The computed change in elastic precursor amplitude with propagation distance for polyurethane (impact boundary velocity = .47mm/ μ sec) is shown in Fig. 2a. From the experimental viewpoint, it is a known rule of thumb that the elastic wave front must propagate at least 10 to 20 times the void size (0.2 mm) for a continuum measurement to be well defined. The attenuation curves for the 0.01 and 0.05 μ sec relaxation times in Fig. 2a show that most of the precursor attenuation is complete by the time the wave has traveled 2 to 4 mm. The 0.05 μ sec curve is most characteristic of the real material. Thus, we conclude that measurement of elastic precursor attenuation is likely to be very difficult in polyurethane. Stress levels significantly larger than the equilibrium yield point (3 kbar) will only be observed in very thin samples. Representation of the material as a one dimensional continuum is questionable in such cases, especially if the sample thickness is only an order of magnitude greater than the void size. Although measurements on the .25 μ sec relaxation time material would not be impeded by this difficulty, we are not presently aware of a material with this degree of rate sensitivity.

The curves shown in Fig. 2a were computed using an assumed nonlinear dependence of the elastic constants on pressure.* Herrmann has recently shown

*This dependence is implicitly included in the assumed relation between $\frac{d\alpha_e}{d\sigma}$ and α described in Appendix A.

that neglect of the pressure dependence of the elastic constants can result in overestimation of precursor attenuation in solid metals.⁹ A linearized precursor analysis for porous materials which neglects variation of the elastic constants with pressure is developed in Appendix B. The attenuation curve computed from this linear theory is shown in Fig. 2b. Attenuation was assumed to begin from the same initial stress state as that of the nonlinear calculation. As expected, the linear theory curve lies considerably below the nonlinear theory curve generated with CONCHAS. The reason for this is explained by the fact that the nonlinear attenuation is reduced by a factor which depends on the stress gradient behind the elastic wave front.* Agreement between the two curves can occur only under conditions in which the gradient is very small or zero, a condition that does not apply in this case.

Fig. 3 shows the equivalent precursor attenuation curves for the porous aluminum. These curves are very similar to those for polyurethane with the exception that the distance scale is much shorter. The scale compression is consistent with the shorter assumed relaxation curves for this material. The similarity in shapes indicates that the different compaction characteristics of the two widely different materials are not readily distinguishable in their elastic precursor attenuation curves. It is also clear from the short distances over which attenuation occurs that attenuation experiments on the aluminum are not likely to be successful.

*The sign of the stress gradient determines the position of the nonlinear attenuation curve with respect to the linear estimate. For example, nonlinear attenuation is more rapid in materials exhibiting upper-lower yield point phenomena, where the stress in the material decreases after the initial elastic wave front. Such stress decreases were not observed in these computations.

Steady Wave Evolution

The profile of the compaction wave is another feature of wave propagation that is relatively easy to measure in plate impact experiments. Application of an instantaneous step change in velocity to the boundary of a porous half-space will produce a compression wave which undergoes a change in profile. Initially a step discontinuity, it evolves as an elastic shock followed by a smoothly changing transition to the final wave amplitude. The smooth transition part of the wave will be referred to as the compaction wave. If the final amplitude of the compaction wave is sufficiently high, it may eventually reach a point where the profile no longer changes, and it becomes a steady wave. At this time the dispersive nature of the rate-dependent void collapse is just balanced by the tendency of the wave to shock up as a consequence of the non-linear compressibility of the material. This balance may not be achieved at lower amplitudes, and the wave will continue to spread indefinitely with time.

The evolution of a steady wave in 0.96 g/cc polyurethane with an assumed relaxation time of .05 μ sec is shown in Fig. 4a. Each curve in the figure represents the shape of the wave profile at a certain time during propagation: the vertical lines representing the elastic wave fronts have been omitted in most profiles to better show the elastic precursor attenuation. Therefore, the lowest point of each curve defines the precursor amplitude at a certain time during propagation. One complete curve, including the elastic precursor is shown in Fig. 4a, which represents the shape of the wave profile when it first becomes steady^{*}. Clearly the exact position of this event is somewhat

^{*}The point at which the wave first becomes steady was determined by comparing the last time profile with preceeding profiles. The distance at which a difference in shape was first apparent was considered the steady wave evolution distance.

subject to interpretation. The steady wave evolution time is approximately 3.2 μsec for the solution presented in Fig. 4a, at which time the elastic wave has propagated .71 cm, and the compaction wave has propagated approximately .54 cm.

Wave profiles similar to those in Figure 4a are shown in Fig. 4b for polyurethane with an assumed relaxation time of .25 μsec . The region of transient wave propagation indicated by this solution is much greater because of the increased relaxation time. Although these results were obtained from a CONCHAS code computation, they could also have been obtained from the profiles of Fig. 4a by nondimensional scaling. Scaling is possible in these problems because of the simple form of the relaxation function. The solutions can be nondimensionalized by the independent variables x^* (position) and t^* (time), given by:

$$x^* = \frac{x}{c_0 \tau_0} \quad t^* = \frac{t}{\tau_0}$$

and the dependent variables σ^* (stress), u^* (particle velocity), and V^* (volume), given by:

$$\sigma^* = \frac{\sigma}{\rho_0 c_0^2} \quad , \quad u^* = \frac{u}{c_0} \quad , \quad V^* = \rho_0 V \quad ,$$

where τ_0 is the relaxation time and c_0 is the bulk wave velocity of the solid. These definitions show that the distance scale of the .05 μsec relaxation time profiles must be expanded by a factor of 5 to obtain the .25 μsec relaxation time profiles. This procedure cannot be applied to solutions involving changes in the initial boundary velocity.

Several other compaction wave problems have been explored. In the first problem, the impact boundary velocity of the polyurethane was lowered to .3 mm/ μ sec, to determine the propagation characteristics of a wave that was closer in amplitude to the equilibrium yield point of polyurethane (3 kbar). These results, for an assumed relaxation time of .25 μ sec, are shown in Figure 4c. The steady wave evolution time was found to be 37.5 μ sec, the elastic wave propagation distance was 8.4 cm, and the compaction wave had propagated 3.9 cm at this time. The steady wave evolution takes much longer than for the previous example. This is partly because the rise time of the compaction wave is longer, as will be discussed later in this section, and partly because the compaction wave velocity is less.

The two remaining problems were concerned with steady wave evolution in porous aluminum. The results of the first computation, which simulated the real material, are shown in Fig. 5a. The experimentally measured p - α parameters and other material constants for this computation are listed in Table I, and the steady wave evolution times and distances are given in Table II. The last computation considered a porous aluminum of identical density, but with an elastic-locking equilibrium curve similar to the equilibrium curve used for the polyurethane. This problem was designed to explore the effects of equilibrium curve variations on the evolution of steady waves. A comparison of the steady wave profiles for the two equilibrium curves is shown in Fig. 5b, and the evolution information is given in Table II.

The information presented so far in this section shows that the duration of the transient part of the wave propagation can vary considerably from computation to computation. Therefore, a simple procedure for estimating when the wave will become steady would be very useful. A possible method of making

such estimates will now be described. It depends on definition of an effective width for the steady wave, in the manner shown in Fig. 6. The value of the effective width is given by:

$$\Delta x_{\text{eff}} = \frac{\Delta \sigma}{\left. \frac{\partial \sigma}{\partial x} \right|_{\text{max}}} ,$$

where $\frac{\partial \sigma}{\partial x}$ is the maximum value of the stress gradient in the steady wave front, and $\Delta \sigma$ is the compaction wave amplitude (the difference in stress between the final amplitude and the equilibrium yield point.). This definition is used in preference to others (for example the distance between two stress points on the wave front) since it is easily computed directly from the porous constitutive relation, and the initial conditions of the problem. Its significance is that it represents a characteristic length of the problem which is a measure of the properties and rate-sensitivity of the material.

The measure of steady wave evolution that we propose is the ratio of the distance required for the wave to become steady, x_s , divided by its effective width, i.e. the value of:

$$\mathfrak{F} = \frac{x_s}{\Delta x_{\text{eff}}} .$$

Values of this ratio, or evolution factor, were computed for each problem and are listed in Table II. We infer from these rather limited results that the evolution factor can be considered approximately a constant, with a value between 3.2 and 3.8, for purposes of estimating evolution distances. Computation 3 is an exception, but this calculation, which was run to a time of 50 μsec , may still not have reached equilibrium even at this late time.

It is true that all the remaining computations had about the same peak stress, and that the evolution factor may be slightly amplitude dependent. In no case, however, is the factor expected to exhibit the wide range of values observed for the other evolution quantities such as the evolution time. We propose, therefore, that the evolution time can be approximately estimated from the effective width by assuming a value of 3.5 for the evolution factor. This procedure must clearly be verified by additional calculations, which, however, are outside the scope of this investigation.

There is an additional basis from a physical point of view for the procedure that we have adopted. First, as mentioned previously, the effective width represents a single parameter that can be considered a characteristic length of the problem. It is dependent on the initial distention, the type of equilibrium curve used, the overstress (and therefore the position of the Rayleigh line), the relaxation time, and the properties of the solid matrix. Second, the distance for the wave to become steady represents the transient portion of the wave propagation: i.e., if the front of the wave is changing, then this change must be mechanically transmitted to the rear of the wave profile (perhaps several round trips of the signal are required) before the wave can become steady. The distance scale of these events must be of the order of the effective width, and therefore the transient portion will be over at a distance which is some multiple of this characteristic length. Thus the evolution factor that we have used is simply the number of effective widths required for evolution to the steady condition.

One further characteristic of the evolution process is apparent from the aluminum computations. The purpose of the simple locking equilibrium curve for Problem 5 was to investigate the consequences of compaction characteristics that were much different from the real material properties used in Problem 4.

The difference between the two solutions is reflected by the effective width values given in Table II. Fig. 1b shows that the maximum overstress is larger for the elastic locking model and therefore this equilibrium curve is more nonlinear than the equilibrium curve for the real material. The net result of these differences is contained in the fact that the effective width for the locking problem is smaller, and steady wave evolution therefore occurs in a significantly shorter distance. In spite of rather large material response differences between the two problems, the evolution factor values are about the same. In other words, we conclude that a change in properties of the material is principally reflected by a change in the effective width of the steady wave, which in turn influences the evolution distance. Direct measurement of the steady wave profile, therefore, is still one of the most sensitive indicators of the properties of porous materials. A difficulty, however, is that so much information is contained in the experimental measurements, that it is almost impossible to resolve the values of the various material constants without additional information about the functional form of the constitutive relation.

Stress Relaxation at the Impact Boundary

The stress history at the boundary was the last feature of porous compaction explored in these computations. For a step increase in velocity applied to the boundary of a rate sensitive material, the initial stress in material adjacent to the boundary is expected to be almost entirely elastic. As time increases, however, the stress relaxes until eventually it reaches the same final equilibrium amplitude as that of the steady wave. It is thus apparent that such relaxation is closely related to the time required for steady wave evolution.

The stress histories of a material plane immediately adjacent to the impact surface of rigid polyurethane with various assumed relaxation times is shown in Fig. 7. The boundary velocity for these problems was $0.47 \text{ mm}/\mu\text{sec}$. The general features of the stress relaxation are much the same as the elastic precursor attenuation curves, (Fig. 2a). In addition, nondimensionalization can be used for these curves, in the same manner as for the attenuation curves, so that the distance scales of the three curves in Fig. 7 are displaced from each other by a factor equal to the ratio of the relaxation times. It is also evident from the curve representing the experimentally measured relaxation time, $0.05 \mu\text{sec}$, in Fig. 7, that the front surface stress relaxes to within 5% of its equilibrium value at a time somewhat less than $1 \mu\text{sec}$ after impact. Experimental complications due to impact tilt and large void sizes can make absolute stress determinations in this time regime very difficult. We conclude therefore, that attempts to quantitatively measure stress relaxation at the impact surface of this material will probably be unsuccessful with present experimental techniques.

Similar stress relaxation curves for porous aluminum with an **assumed** impact velocity of $0.3 \text{ mm}/\mu\text{sec}$ are shown in Fig. 8. The general shapes of

those curves are the same as the shapes of the polyurethane curves, so no further comment on them is necessary.

The minima in stress before the final equilibrium levels are reached in Figs. 7 and 8 were unexpected features of the relaxation curves. The sources of these minima have not really been established, although they are believed to be a consequence of the method of formulation of the rate dependent constitutive model. The source of the difficulty may arise from the procedure of defining the overstress in material as the difference between the actual stress and the equilibrium stress at the same value of the compaction parameter α . Under certain conditions, this assumption has the consequence of introducing a discontinuity in the constitutive behavior as the material goes solid*. In support of this proposal, we have established that the minimum is not present when a quadratic $\sigma - \alpha$ relation is used and the boundary velocity is not sufficient to completely eliminate the porosity. These conditions are such that the point of discontinuity at p_s does not enter into the solution. Similarly, the minima are not believed to be a consequence of the finite difference scheme, because the stress histories have been exactly reproduced by WONDY IV solutions, which involve different difference schemes and artificial viscosity. It is evident that this problem should be corrected eventually. At present, however, its effect on the general conclusions of this report is not considered important and additional work to correct it has not been undertaken.

*The elastic-locking equilibrium curve represents a worst possible case of this problem: as long as α is slightly greater than one in value, the equilibrium stress is the same as the elastic limit, with the over stress a maximum in the vicinity of $\alpha = 1$ (see figure 1b). However, as soon as α reaches one, the over stress vanishes. This difficulty could be circumvented by definition, of the overstress at constant density rather than constant α . Such a procedure is more difficult to program into the numerical codes, however, because of the increased computation complexity and logic required in the numerical scheme. It has not been included in the existing codes for these reasons.

SUMMARY & CONCLUSIONS

In summary, the rate dependent response of two quite different porous materials, rigid polyurethane and sintered aluminum, was explored by numerical experimentation. A newly developed numerical wave propagation code CONCHAS was used for these computations. This code is based on the characteristic method of solution of hyperbolic partial differential equations. This method of analysis made possible precise presentation of sharp shock phenomena, such as nonlinear elastic precursor attenuation, which are obscured in numerical computations that employ artificial viscosity.

Three different features of wave propagation were investigated: elastic precursor attenuation, steady wave evolution, and the stress history of the material in the vicinity of the impact surface. In the first part of the investigation, it was demonstrated that the nonlinear response of these materials was large enough to produce significant differences in elastic precursor attenuation from predictions using a linear theory. The rate of nonlinear precursor attenuation was less rapid than the linear prediction, but the elastic precursor amplitude still decreased very rapidly with distance. In fact, this decay was so rapid that experimental measurement of elastic wave attenuation is presently considered unfeasible because of the extreme thinness of the samples that would be required for definitive measurements.

In the second part of the study, development of steady waves in both the polyurethane and the porous aluminum were found to be in agreement with previous WONDY IV calculations.⁵ For the boundary conditions assumed, the time for steady wave evolution was of the order of 3.5 times the effective rise time of the steady wave for both materials. This result strongly suggests that variations in the equilibrium compaction curve are principally reflected in the steady

wave rise time, which in turn determines the evolution time. By considering the time scale of dynamic events in terms of the effective steady wave rise time, it is therefore possible to normalize out rather large individual material differences in compaction response, and observe certain material constants of the specific material under consideration. Another conclusion from these computations is that measurement of the steady wave rise time is still one of the most direct methods of material constant evaluation.

The final part of the investigation explored the nature of the stress relaxation near the impact boundary of the material. Conclusions about the time scale of this part of the determination were very similar to those for elastic precursor attenuation. Again, it is apparent that experimental attempts to measure stress relaxation at the impact surface will probably be unsuccessful because precise measurements at such times are not possible in the relatively complex materials currently available.

With respect to the overall conclusions of these computations, the general features of dynamic compaction of the two materials were observed to be quite similar, in spite of the fact that their equilibrium curves were quite different. In addition, the usefulness of the characteristics type code CONCHAS has been demonstrated for problems in which the correct modeling of sharp shocks is necessary. Finally, these computations show how parameter studies can be used to estimate the probable success of experiments designed to measure complex material response.

APPENDIX A

This appendix describes the material response functions used for the computations described in this report. The equations can be divided into three groups: first, the equations for defining some of the properties of the solid; second, a set of equations which define the instantaneous compaction and release curves; and third, equations for $\tilde{g}_\alpha$ the irreversible pore collapse function. The material property constants for these equations are given in Table I.

Equations for the Solid

The quantities that are required for computing the values of the acoustical impedance and relaxation functions are indicated by Equations 4 and 5 in the main body of the report. Two of these quantities, the partial derivatives $\tilde{P}_s$ and $\tilde{P}_g$, are related to the equation of state of the solid which is assumed to be the Mie Grüneisen equation:

$$P = P(\rho_s, \mathcal{E}) = P_r + \gamma \rho_s (\mathcal{E} - \mathcal{E}_r) \quad , \quad (\text{A.1})$$

where P_r and $\mathcal{E}_r$ are the pressure and internal energy along a thermodynamic reference path. The normal Hugoniot of the solid was used as the reference curve, with the assumption that the relationship between the shock velocity and the particle velocity was linear. The pressure P_r and internal energy $\mathcal{E}_r$ are therefore given by:

$$P_r = \frac{\rho_{so} c_o^2 \eta}{[1 - s\eta]^2} , \quad (A.2)$$

and

$$\epsilon_r = \frac{P_r}{2\rho_{so}} \eta , \quad (A.3)$$

with

$$\eta = 1 - \frac{\rho_{so}}{\rho_s} . \quad (A.4)$$

The velocity c_o is the bulk sound speed of the solid, and s is the slope of the shock velocity-particle velocity curve. It is evident from Equations (A.2) to (A.4) that P_r and ϵ_r are functions of the solid density only. The product $\gamma\rho_s$ in Equation (A.1) is also a function of the solid density only, and is assumed constant:

$$\gamma\rho_s = \gamma_o\rho_{so} \quad (A.5)$$

where γ_o is the zero pressure Grüneisen parameter value.

The partial derivatives $\tilde{P}_s$ and $\tilde{\epsilon}_\epsilon$ are obtained by differentiating Equation (A.1) to obtain:

$$\tilde{\epsilon}_\epsilon = \frac{\partial P}{\partial \epsilon} = \gamma_o \rho_{so} , \quad (A.6)$$

and

$$\tilde{P}_s = \frac{\partial \tilde{P}}{\partial \rho_s} = \frac{c_o^2 [1 - \eta]^2 [1 + \{s - \gamma_o\} \eta]}{(1 - s\eta)^3} \quad (A.7)$$

Instantaneous Compaction and Release Curves

The term $d\alpha_e/d\sigma$ in Equation (4) is the slope of the instantaneous or elastic curve at point (σ, α) . We assume that the sound speed varies with distention α in the following manner:

$$c = h(\alpha)c_o, \quad (A.8)$$

where c_o is again the bulk sound speed of the material. Herrmann¹ has shown that the slope $\frac{d\alpha_e}{d\sigma}$ is related to the function $h(\alpha)$ (assuming $\sigma = P$) by the relation:

$$\frac{d\alpha_e}{d\sigma} = \frac{\alpha}{\rho_{so}c_o^2} \left[1 - \frac{\alpha}{h(\alpha)^2} \right]. \quad (A.9)$$

The function $h(\alpha)$ was assumed linear in this investigation:

$$h(\alpha) = a\alpha + (1 - a), \quad (A.10)$$

with the constant a defined by the velocity c_o , the velocity c_e of the elastic wave in the uncompacted material, and the initial distention ratio α_o :

$$a = \frac{c_e - c_o}{c_o(\alpha_o - 1)} \quad (A.11)$$

The Function $\tilde{g}_\alpha$

A particularly simple form of $\tilde{g}_\alpha$ is used. The assumption is made that the rate of irreversible pore collapse is dependent on the difference between the actual stress in the material and the equilibrium curve stress:

$$\tilde{g}_{\alpha} = \frac{d\alpha_e}{d\sigma} \frac{[\sigma - \sigma_{eq}]}{\tau_o} , \quad \sigma > \sigma_{eq} \geq p_e$$

with

(A.12)

$$\tilde{g}_{\alpha} = 0 , \quad \sigma < \sigma_{eq}$$

where p_e is the yield point of the material, $d\alpha_e/d\sigma$ is the slope of the instantaneous curve at (σ, α) , and τ_o is the relaxation time. The equilibrium stress, σ_{eq} , is defined by:

$$\sigma_{eq} = \left[\frac{\alpha - 1}{\alpha_y - 1} \right]^{\frac{1}{2}} [p_e - p_s] + p_s . \quad (A.13)$$

The quantity p_s is the pressure required to completely close the voids, and α_y is the distention at the yield point. In general, the value of α_y should be obtained by direct integration of Equation (A.9). However, the measured value of c_e in most experiments is considered a constant. In view of this assumption, a suitable approximation of α_y can usually be obtained by assuming $\frac{d\alpha_e}{d\sigma}$ constant, in which case Equation (A.9) is simplified to:

$$\frac{\Delta\alpha}{\Delta\sigma} = \frac{\alpha_o}{p_{so} c_o^2} \left[1 - \frac{\alpha_o c_o^2}{c_e^2} \right] , \quad (A.14)$$

and α_y is given by

$$\alpha_y = \alpha_o + \frac{\Delta\alpha}{\Delta\sigma} p_e . \quad (A.15)$$

It is important to note in the consideration of Equation (A.13) that an alternate exponential form can also be used for the equilibrium curve. This relation is given by:

$$\sigma_{eq} = \frac{(\alpha_y - 1)}{\alpha'_y} \log_e \left[\frac{\alpha - 1}{\alpha_y - 1} \right] + p_e \quad , \quad (A.16)$$

where α'_y is the slope $d\alpha/d\sigma_e$ at the yield point.

APPENDIX B

Linearized Elastic Precursor Decay Analysis

The attenuation of the initial elastic shock waves in solids has often been described by means of a simple characteristic computation which assumes that the elastic constants of the material are independent of pressure. Recently Herrmann⁹ has shown that this type of calculation poorly approximates elastic precursor attenuation in solids when the stress is high enough to require the use of nonlinear elastic constants. The same type of error is expected for wave propagation in porous materials. In fact, the differences between results derived from the linear modulus theory and the nonlinear theoretical predictions are expected to be much more pronounced for porous materials than the differences observed for solids. The reason for this is that porous materials are inherently more nonlinear; the porous material moduli are influenced not only by the compressibility of the solid matrix, but also by the elastic determination of void structure. Even at low stress levels the elastic moduli of porous materials are expected to be highly nonlinear because of the influence of the void structure.

The objective in this section is to derive a linearized relation for the attenuation of elastic precursors in porous materials. We wish eventually to compare predictions from this linearized model with the elastic precursor predictions of the nonlinear theory.

The first step in the derivation of the attenuation equation is based on the fact that no irreversible void collapse occurs in the elastic

precursor, i.e., void collapse is completely defined by the instantaneous component α_e . This observation requires that the term $d\alpha_e/d\sigma$ is obtained directly from the definition $\alpha = \rho_s/\rho$:

$$\frac{d\alpha_e}{d\sigma} = \frac{d\alpha}{d\sigma} = \frac{1}{\rho} \frac{d\rho_s}{d\sigma} - \frac{\rho_s}{\rho^2} \frac{d\rho}{d\sigma} \quad , \quad (\text{B.1})$$

We also consider a strictly mechanical theory which neglects sheer stresses ($\sigma = P$, $P = \hat{P}(\rho_s)$) so that:

$$\frac{d\sigma}{d\rho_s} = \frac{d\hat{P}}{d\rho_s} \approx \frac{\partial \tilde{P}}{\partial \rho_s} \equiv \tilde{P}_s \quad (\text{B.2})$$

When these simplifications are substituted back into the equations for A^2 and R (Equations 4 and 5 of the main text), the results are:

$$A^2 = \rho^2 \frac{d\sigma}{d\rho} \quad (\text{B.3a})$$

$$R = - \frac{\rho^2}{\rho_s} \frac{d\sigma}{d\rho} \tilde{g}_\alpha \quad (\text{B.3b})$$

The function $\tilde{g}_\alpha$ in Equation (4) has been set equal to zero, as consistent with the assumed mechanical theory restriction.

Next the specific form of the function $\tilde{g}_\alpha$ used for the nonlinear numerical predictions is considered:

$$\tilde{g}_\alpha = \frac{d\alpha_e}{d\sigma} \frac{[\sigma - \sigma_{eq}]}{\tau_o} \quad (\text{B.4})$$

The stress σ_{eq} is the equilibrium stress and τ_o is the characteristic relaxation time. Substitution of this equation into Equation B.3b gives

$$R = \left[1 - \frac{\rho \frac{d\sigma}{d\rho}}{\rho_s \tilde{P}_s} \right] \frac{[\sigma - \sigma_{eq}]}{\tau_o} \quad (\text{B.5})$$

The term $\rho \frac{d\sigma}{d\rho}$ in the above equation is characteristic of the bulk modulus of the porous material, and $\rho_s \tilde{P}_s$ is equivalent to the bulk modulus for the solid.

Integration of the characteristic equation defining the propagation of the shock front is the final step in the derivation. The characteristic equation is:

$$d\sigma = A du = - R dt \quad (B.6)$$

At the shock front

$$\sigma = \rho_o U_s u \quad , \quad (B.7)$$

where U_s is the shock velocity. The differential form of this equation:

$$d\sigma = \rho_o U_s du + \rho_o u dU_s \quad (B.8)$$

is used to express du in terms of dP in the characteristic equation:

$$\left(1 + \frac{A}{\rho_o U_s}\right) d\sigma + \frac{Au}{U_s} dU_s = - R dt \quad (B.9)$$

This result can be integrated if we assume, as in the elastic analysis of solids, that the shock velocity U_s remains constant. This assumption also requires that A remain constant so that Equation (B.9) is reduced to an ordinary differential equation:

$$\frac{d\sigma}{dt} = - \frac{R}{2} \quad (B.10)$$

A specific form for R has already been given in Equation (B.5). If the final restriction that σ_{eq} is a constant is imposed, as was assumed for the polyurethane results described in this report, then the integrated form of Equation (B.10) is:

$$\sigma(t) = \sigma_{eq} + (\sigma(0) - \sigma_{eq}) \exp -\beta \frac{t}{\tau_0} \quad (B.11)$$

where $\sigma(0)$ is the stress at $t = 0$, and

$$B = \frac{1}{2} \left[1 - \frac{\rho \frac{d\sigma}{d\rho}}{\rho_s \tilde{\rho}_s} \right] \approx \frac{1}{2} \left[1 - \frac{\rho_o c_e^2}{\rho_{so} c_o^2} \right] . \quad (B.12)$$

The velocity c_e in the definition of β is the elastic wave velocity, ρ_{so} is the unstressed density of the solid matrix, and c_o is the bulk sound speed in the solid.

TABLE I

Material Constants

(Except as noted, these constants are defined in Appendix A)

<u>Constant</u>	<u>Actual Value</u>	<u>Non-Dimensional Value</u>	<u>Constant</u>	<u>Actual Value</u>	<u>Non-Dimensional Value</u>
<u>Solid Aluminum</u>			<u>Solid Polyurethane¹</u>		
ρ_{so}	2.71 gm/cc	1.0	ρ_{so}	1.1904 gm/cc	-
c_o	0.535 cm/ μ sec	1.0	c_o	0.246 cm/ μ sec	-
s	1.37	1.37	s	1.43	-
γ_o	2.1	2.1	γ_o	1.00	-
ν^*	0.333	0.333			
Y_o^*	0.0032 Mbar	0.004125			
<u>2.168 gm/cc Porous Aluminum^{**}</u>			<u>0.96 gm/cc Polyurethane Foam¹</u>		
α_o	1.25	1.25	α_o	1.24	-
p_e	0.0008 Mbar	0.001031	σ_{eq}	0.0030 Mbar(constant)	-
p_s	0.006 Mbar	0.007735	c_e	0.223 cm/ μ sec	-
c_e	0.41 cm/ μ sec	0.7644	τ_o	0.05 μ sec	-
τ_o	To be determined	1.000	* ν is Poissons ratio, Y_o is the yield point.		

^{**}From unpublished data.

TABLE II: Computational Results

Problem	Material	Type	Wave Amplitude	τ_0	Effective Steady Wave Width Δx_{eff}	Evolution Elastic Wave Position	Distance Compaction Wave Position x_s	Evolution Factor $\mathcal{F} = x_s / \Delta x_{\text{eff}}$
			kbar	usec	cm	cm	cm	
1	Polyurethane	$p_s = p_e$	8.3	0.05	0.16	0.71	$0.54 \pm .04^\dagger$	3.3
2	Polyurethane	$p_s = p_e$	8.3	0.25	0.82	3.70	$2.70 \pm .2$	3.3
3	Polyurethane	$p_s = p_e$	4.5	0.25	1.73	8.4	$3.85 \pm .2$	2.25
4	Aluminum	$p_s > p_c$	9.6	0.025	1.3	3.59	1.3 ± 0.08	3.37
5	Aluminum	$p_s = p_e$	9.6	0.025	1.0	3.75	1.0 ± 0.08	3.75

[†]These estimated errors represent line width variations in the wave profiles.

LIST OF REFERENCES

1. Herrmann, W., "Constitutive Equations for Compaction of Porous Materials," presented at the Symposium on Applied Mechanics Aspects of Nuclear Effects in Materials, ASME Winter Meeting; Washington, D. C., November 28-December 3, 1971.
2. Ng, R., and Butcher, B. M., "The Dynamic Response of Heated Porous Aluminum," SC-RR-710790, Sandia Laboratories, Albuquerque, N. M., April 1972.
3. Butcher, B. M., "The Description of Strain-Rate Effects in Shocked Porous Materials," Shock Waves and the Mechanical Properties of Solids, (Syracuse University Press, Syracuse, New York, 1971), pp. 227-244.
4. Butcher, B. M., Sandia Laboratories, Albuquerque, N. M. - Unpublished Data.
5. Butcher, B., "Numerical Techniques for One-Dimensional Rate-Independent Porous Material Compaction Calculations," SC-RR-710112, Sandia Laboratories, Albuquerque, N. M., April 1971.
6. Courant, R., and Friedrichs, K. O., Supersonic Flow and Shock Waves, (Interscience Publishers, Inc., New York, 1948), p. 80.
7. Hicks, D. L., and Holdridge, D. B., "The CONCHAS Wave Code," SC-RR-72-0451, forthcoming Sandia Laboratories report.
8. Malvern, L. E., "Plastic Wave Propagation in a Bar of Material Exhibiting a Strain-Rate Effect," Quart. Appl. Math. 7-8, 405-411, (1949-1951).
9. Herrmann, W., Hicks, D. L., and Young, E. G., "Attenuation of Elastic-Plastic Stress Waves," Shock Waves and the Mechanical Properties of Solids, (Syracuse University Press, Syracuse, New York, 1971), pp. 23-63.

Figure 1a: Equilibrium pressure-volume curve for 0.96 g/cm^3 rigid polyurethane.

Figure 1b: Equilibrium pressure-volume curve for 2.16 g/cm^3 aluminum.

Figure 1c: P- α curves for porous rigid polyurethane and aluminum.

Figure 2a: Elastic precursor attenuation in rigid polyurethane with an impact boundary velocity of $0.47 \text{ mm}/\mu\text{sec}$. The parameter τ_0 is the assumed relaxation time for void collapse.

Figure 2b: A comparison of the computed nonlinear elastic precursor attenuation with theoretical predictions based on a linear theory. The value of τ_0 for these computations was 0.25 μsec .

Figure 3: Elastic precursor attenuation in porous aluminum with impact boundary velocity of 0.3 mm/ μ sec. The parameter τ_0 is the assumed relaxation time for void collapse.

Figure 4a: Wave evolution in 0.96 g/cm^3 rigid polyurethane. The impact boundary velocity was $0.47 \text{ mm}/\mu\text{sec}$, and the relaxation time was $0.05 \mu\text{sec}$.

Figure 4b: Wave evolution in 0.96 g/cm³ rigid polyurethane. The impact boundary velocity was 0.47 mm/ μ sec, and the relaxation time was 0.25 μ sec.

Figure 4c: Wave evolution in 0.96 g/cm^3 rigid polyurethane. The impact boundary velocity was $0.3 \text{ mm}/\mu\text{sec}$, and the relaxation time was $0.25 \mu\text{sec}$.

Figure 5a: Wave evolution in 2.16 g/cm³ aluminum. The impact boundary velocity was 0.3 mm/μsec, and the relaxation time was 0.025 μsec.

Figure 5b: A comparison of steady wave profiles computed with an elastic-locking equilibrium curve ($p_s = p_e$), and a quadratic $P-\alpha$ equilibrium curve ($p_s > p_e$).

Figure 6: Definition of the effective steady wave width.

Figure 7: Porous polyurethane impact boundary stress histories. The assumed impact boundary velocity was $0.47 \text{ mm}/\mu\text{sec}$. The vertical arrows show the minimum points in the 0.01 and $0.05 \mu\text{sec}$ curves. The minimum in the $0.25 \mu\text{sec}$ curve is not shown in this figure because it occurs much later in time.

Figure 8: Porous aluminum impact boundary stress histories. The assumed impact boundary velocity was 0.3 mm/μsec. The vertical arrows show the minimum points in the 0.005 and 0.010 μsec curves. The minimum in the 0.025 μsec curve is not shown in this figure because it occurs much later in time.

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