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Three-Group Adjoint Calculations

by

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Abstract

Adjoint flux calculations are required in the application of perturbation theory to calculate effects on reactivity due to reactor changes resulting from effects of temperature changes or local introduction of fuel or poison. This memo discusses a method by which adjoint fluxes can be readily obtained with the use of the presently existing three-group code, MUG-II.

Temperature coefficient calculations as a function of position in the reactor, employing adjoint fluxes calculated by the above method, are now in progress.

Introduction

The effect of changes in a reactor structure or composition resulting, for example, from temperature changes or localized material substitutions, is most readily determined by means of the perturbation theory (Ref. 1). This theory is being used to predict the temperature coefficient of various reactor designs throughout the life of the reactor. It further allows one to obtain the temperature effect on reactivity due to localized temperature changes as they actually take place when cold water enters the reactor, first through the outer unit cells (first pass), and then through the center cells (second pass). In this manner one can predict the temperature coefficient throughout each unit volume of the reactor without the use of additional multigroup calculations. However, one must obtain a set of "adjoint" fluxes for the same number of energy groups as employed in the calculations of fluxes and power of the reference reactor. Once these adjoint fluxes are available, any other disturbance, such as a local change in poison or fuel, may also be treated without another multigroup calculation. On the other hand, the number of adjoint calculations to be performed is small compared to the many flux calculations required for design. Therefore, the purpose of this report is to show a simple method by which the present "MUC-II" (three-group, two-dimensional), code can be used to produce adjoint fluxes without any change in the coding.

Methods and Calculations

The three-group neutron diffusion equations which are in use at KAPL are as follows.

$$\begin{aligned}
 *D_1 \nabla^2 \phi_1 - \Sigma_{sl_1} \phi_1 + v \Sigma_{f_2} \phi_2 + v \Sigma_{f_3} \phi_3 &= 0 \\
 D_2 \nabla^2 \phi_2 - \Sigma_{sl_2} \phi_2 - \Sigma_{a_2} \phi_2 + \Sigma_{sl_1} \phi_1 &= 0 \\
 D_3 \nabla^2 \phi_3 - \Sigma_{a_3} \phi_3 + \Sigma_{sl_2} \phi_2 &= 0
 \end{aligned} \tag{1}$$

This can be represented as $M\phi = 0$ during steady state. Where M is the appropriate matrix operator and ϕ represents the vector set $(\phi_1, \phi_2, \text{ and } \phi_3)$. In general, since for reactors the elements of the matrix are real numbers, the adjoint operator, M^+ , is obtained by interchanging rows and columns in the matrix. This is explained in detail in Reference 1. Applying this method to the three-group equations one obtains the following set of adjoint equations:

$$\begin{aligned}
 D_1 \nabla^2 \psi_1 - \Sigma_{sl_1} \psi_1 + \Sigma_{sl_1} \psi_2 &= 0 \\
 D_2 \nabla^2 \psi_2 - \Sigma_{sl_2} \psi_2 - \Sigma_{a_2} \psi_2 + v \Sigma_{f_2} \psi_1 + \Sigma_{sl_2} \psi_3 &= 0 \\
 D_3 \nabla^2 \psi_3 - \Sigma_{a_3} \psi_3 + v \Sigma_{f_3} \psi_1 &= 0
 \end{aligned} \tag{2}$$

or using the matrix notation as above, $M^+ \psi = 0$. It is desirable to solve equations (2) with an existing code, since coding is an expensive and time consuming process. The code presently used for S3G/S4G three-group two-dimensional reactor calculations is MUG-II and solves the following set of

*Definition of symbols on Page 10.

general equations which is indeed set (1):

$$\begin{aligned}
 -D_1 \nabla^2 \phi_1 + (\Sigma_{a_1} + \Sigma_{sl_1}) \phi_1 &= \chi_1 S \\
 -D_2 \nabla^2 \phi_2 + (\Sigma_{a_2} + \Sigma_{sl_2}) \phi_2 &= \chi_2 S + \Sigma_{sl_1} \phi_1 \\
 -D_3 \nabla^2 \phi_3 + \Sigma_{a_3} \phi_3 &= \chi_3 S + \Sigma_{sl_2} \phi_2
 \end{aligned} \tag{3}$$

where $S = v(\Sigma_{f_2} \phi_2 + \Sigma_{f_3} \phi_3)$

χ_i = spectrum in group i

$$\sum \chi_i = 1$$

Σ_{a_1} is assumed zero and since it is assumed that all fissions produce fast neutrons only, χ_2 and χ_3 are zero as well.

If one makes the following substitutions in the constants preceding the fluxes of equations (3), one solves equation (2), the adjoint fluxes, without any changes to the existing code. In fact, the UNIVAC (computer presently used) will produce a set of flux calculations which are really adjoint fluxes due to the fictitious input cross sections.

Table 1

In place of these constants of the flux equations	Use these constants for the adjoint equations
Σ_{f_3}	Σ_{f_3}
D_1	D_3
Σ_{a_1}	$(\Sigma_{a_3} - \Sigma_{s_{l_2}})$
$\Sigma_{s_{l_1}}$	$\Sigma_{s_{l_2}}$
D_2	D_2
Σ_{a_2}	$(\Sigma_{a_2} + \Sigma_{s_{l_2}} - \Sigma_{s_{l_1}})$
$\Sigma_{s_{l_2}}$	$\Sigma_{s_{l_1}}$
χ_1	1.0 (must actually use 0.999999)
χ_2	0.1 since $\Sigma_{f_2}/\Sigma_{f_3} = .1$
χ_3	0
D_3	D_1
Σ_{a_3}	$\Sigma_{s_{l_1}}$
These are boundary conditions for the code	Z_3
	Z_2
	Z_3
	0
Σ_{f_1}	0
Σ_{f_2}	0

This method has been used and the adjoint fluxes were obtained in a very short time. These fluxes are now being used in the application of perturbation theory to calculate temperature coefficients. It should be noted that ψ_3 will be calculated as if it were ψ_1 , and vice versa. In other words, the adjoint fluxes of group (1) are really group (3) adjoint fluxes and vice versa. Group (2) is unaffected.

Discussion

Since $\chi_1 + \chi_2 + \chi_3$ in the above set are 1.1 instead of 1.0, the possibility was considered to let $\chi_1 = 10/11$ and $\chi_2 = 1/11$ while χ_3 is always zero. Changing the χ 's results in a change of the normalization constant but it also results in a proportional change of the calculated number of fissions per neutron, v_c , (which should approach 0.4000), and is a check on criticality. Using the χ 's of Table 1 should result in the same v_c for the adjoint fluxes as was calculated for the comparable flux equations. Changing the normalization factor itself is of little importance since in the equations of perturbation theory, fluxes and adjoints appear similarly in both the numerator and denominator (see Reference 2), and any such constant would cancel out.

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List of Symbols

D = diffusion coefficient (cm)

Σ_{sl} = slowing down macroscopic cross section (cm^{-1}).

ν = number of neutrons/fission.

Σ_f = macroscopic fission cross section (cm^{-1}).

Σ_a = macroscopic absorption cross section (cm^{-1}).

ϕ = flux (neutrons/ $\text{cm}^2\text{-sec}$).

ψ = adjoint flux (neutrons/ $\text{cm}^2\text{-sec}$).

$S = \nu (\Sigma_{f_2} \phi_2 + \Sigma_{f_3} \phi_3)$

χ = spectrum, (number of neutrons/unit energy) normalized such

that $\int_E \chi(E) dE = 1$

Subscripts 1, 2, and 3 apply to groups 1, 2, and 3.

References

1. The Elements of Nuclear Reactor Theory by S. Glasstone and M. C. Edlund,
D. VanNostrand Co., Inc. Chapter 13.
2. KAPL-71, Perturbation Methods In Multigroup Calculations by H. Brooks,
(Secret).