VERIFICATION OF A MESOSCALE WIND-PREDICTION MODEL
DERIVED FROM A HORIZONTAL VORTICITY THEOREM

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1. INTRODUCTION

A fundamental need in mesoscale prediction of local weather conditions and air pollution episodes is a numerical model capable of simulating the meteorologically significant mesoscale motions of the lower portions of the free atmosphere. This report describes a numerical model which is economical to employ and yet yields a useful product in terms of predicting the three-dimensional mesoscale wind field for operational requirements.

A common and simple approach to predicting the motion of the atmosphere is to assume the conservation of angular momentum (vorticity) while it is advected by the flow field. The normal procedure is to use the vertical component of vorticity ($\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$) for this purpose and derive the flow field for advecting the vorticity from the height field of a constant pressure surface. It follows that a logical approach to predicting the mesoscale winds may also be through the use of vertical vorticity. This approach has several drawbacks when applied to mesoscale flow, namely,

- (1) inaccuracies in the measurement of the height (pressure) field can yield large errors in computing the geostrophic wind field, (Clark, 1969),
- (2) distance between the standard rawinsonde sites does not permit resolution of the free-air mesoscale features in the flow field,

(3) inaccuracies in the measurement of the wind field can yield large errors in computing the relative vertical vorticity. Scale analysis, based on observations, indicates that for mesoscale flow the magnitude of the vertical component of vorticity, ζ , is 10^{-4} to 10^{-5} s^{-1} while the horizontal components of vorticity are of the order of 10^{-2} s^{-1} . This calculation is based on an observed mean horizontal speed gradient of $1 \text{ m s}^{-1}/20 \text{ n mi}$ and a mean vertical speed gradient of $1 \text{ m s}^{-1}/1000 \text{ ft}$. Consequently, horizontal vorticity appears to be about two orders of magnitude greater than the vertical vorticity for mesoscale flow. Thus, it is possible to calculate the horizontal vorticity components on the mesoscale with greater accuracy than the vertical vorticity and it may be possible to assume conservation of horizontal vorticity.

2. DERIVATION OF THE PREDICTION EQUATIONS

The components of horizontal vorticity are defined as

$$\sigma = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \quad \alpha = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}.$$

Scale analysis shows that

$$\frac{\partial w}{\partial x} \ll \frac{\partial u}{\partial z} \quad \text{and} \quad \frac{\partial w}{\partial y} \ll \frac{\partial v}{\partial z}$$

giving,

$$\sigma = \frac{\partial u}{\partial z}, \quad \alpha = -\frac{\partial v}{\partial z}$$

to a very close approximation.

Differentiating the equations of horizontal motion with respect to z and substituting the geostrophic wind for the pressure term gives

$$\frac{\partial \sigma}{\partial t} + \vec{C} \cdot \nabla \sigma - \left[\sigma \frac{\partial v}{\partial y} + \alpha \frac{\partial u}{\partial y} \right] = -f(\alpha - \alpha_g) + F_\sigma \quad (1)$$

$$\frac{\partial \alpha}{\partial t} + \vec{C} \cdot \nabla \alpha - \left[\sigma \frac{\partial v}{\partial x} + \alpha \frac{\partial u}{\partial x} \right] = f(\sigma - \sigma_g) - F_\alpha \quad (2)$$

for incompressible flow. Where, $\sigma = \frac{\partial u}{\partial z}$, $\alpha = -\frac{\partial v}{\partial z}$, f is the Coriolis parameter, $\vec{C}$ is the three-dimensional velocity vector, σ_g and α_g are the geostrophic counterparts of σ and α . The terms in brackets represent deformation and rotational changes in the horizontal vorticity. These equations are simple and easily solved by numerical techniques, except that the ageostrophic terms $(\alpha - \alpha_g)$, $(\sigma - \sigma_g)$ and shear of the friction vector F are difficult to specify on the mesoscale.

Figure 1 shows a diagram derived from BOMEX rawinsonde data by Holland and Rasmusson (1973). In this diagram notice that by differentiating with respect to z through layers 33 mb thick ($\approx 1000 \text{ ft}$), no generality is lost in the atmospheric boundary layer by saying

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$$\frac{\partial}{\partial z} \left[g \frac{\partial \tau_*}{\partial p_*} \right] \approx \frac{\partial}{\partial z} \left[f(V-V_g) A \right]$$

and

$$\frac{\partial}{\partial z} \left[\frac{du}{dt} A \right] \approx 0.$$

Above the atmospheric boundary layer (between $p_* = 200$ and 400 mb), we assume

$$\frac{\partial}{\partial z} \left[\frac{f(V-V_g) A}{f(V-V_g)} \right] \approx 0$$

and

$$\frac{\partial}{\partial z} \left[\frac{du}{dt} A \right] \approx 0.$$

Thereby, Eqs. 1 and 2 have been simplified so that, on the average,

$$(3) \quad \frac{\partial \sigma}{\partial t} + \bar{C} \cdot \nabla \sigma - \left[\sigma \frac{\partial v}{\partial y} + \alpha \frac{\partial u}{\partial y} \right] = 0,$$

$$(4) \quad \frac{\partial \alpha}{\partial t} + \bar{C} \cdot \nabla \alpha - \left[\sigma \frac{\partial v}{\partial x} + \alpha \frac{\partial u}{\partial x} \right] = 0.$$

3. THE NUMERICAL MODEL

Equations 3 and 4 were solved numerically over a 200×200 n mi grid with a horizontal grid spacing of 20 n mi. The vertical extent of the model ranges from 5000 to 13000 ft above mean sea level (AMSL). The grid network is centered over the Atomic Energy Commission's Nevada Test Site (NTS) located in the southern part of Nevada. The average terrain elevation is about 4500 ft AMSL and ranges from 2000 ft AMSL in the southern end of the grid to 6000 ft AMSL in the northern end. A 15 min time step is used to obtain predictions out to 6 hrs beyond the initial time.

The advective terms in Eqs. 3 and 4 are calculated by use of a quasi-lagrangian scheme proposed by Winn-Nielsen (1959) and Krishnamurti (1962) and tested by LeBlanc (1972). The derivations in the brackets were approximated by simple centered differences.

The initial wind field is obtained from pilot balloons or radar tracked balloons released from 6-14 sites scattered over the grid field. The wind at each grid point is specified by objective analysis of the u and v components of the observed wind using a technique described by Glahn, et al. (1969). The objectively analyzed wind is then smoothed by applying a nine-point numerical filter. The vorticity (σ and α) fields are then calculated from the smoothed wind field by simple differencing between adjacent vertical grid points.

The following boundary conditions are assumed:

- (1) Windward boundary - σ and α constant;
- (2) Lee boundary - σ and α permitted to flow out;
- (3) Top boundary - σ and α permitted to flow upward and out, but no flux into the model, also, the wind field was advected, i.e., $u = u(x, y, z, t)$, $v = v(x, y, z, t)$;
- (4) Bottom boundary - σ and α are permitted to flow downward and out, but there is no flux upward and into the model.

The vertical velocities for the flow field are calculated from the u - and v - wind components by a method given by Randerson (1970) which uses the incompressible form of the continuity equation and implicitly includes vertical motion induced by the terrain.

At the end of each time step the predicted wind fields are derived from the predicted vorticity fields by integrating downward and by using the advected wind field at 14000 ft AMSL. New vertical motion fields were then calculated from the predicted u - and v - components and the process is repeated until the final prediction is obtained. The input of the data, objective analyses, and predictions take a total of 1.5 min of clock time on a CDC 6400 computer.

4. VERIFICATION

A summary of the mean initial flow for the 33 independent cases used to verify the model is shown in Fig 2 and Table 1. These diagrams show that each direction octant and speed category is fairly well represented in the data. The data cases are randomly scattered throughout the period from June 1966 to July 1973. However, nearly all of the verifications are in the time period between midnight and local noon.

Verification was made over a 120×120 n mi area centered within the model's grid network for nine levels ranging from 5000 ft to 13000 ft AMSL and for hourly intervals from 1 to 6 hrs. Since the horizontal grid interval is 20 n mi, there are 36 grid-points at each level and, therefore, a total of 36^4 grid-points in the volume used for verification. Verification is based on the vector root mean square errors (VRMSE) computed at each of the nine levels and six prediction times. The VRMSE for each level is given by

$$VRMSE = \left[\frac{\sum_{i=1}^6 \sum_{j=1}^6 (u_{i,j}^p - u_{i,j}^A)^2 + \sum_{i=1}^6 \sum_{j=1}^6 (v_{i,j}^p - v_{i,j}^A)^2}{36} \right]^{1/2},$$

where $u_{i,j}^A$, $v_{i,j}^A$ and $u_{i,j}^p$, $v_{i,j}^p$ represent the components of the wind vector at (i, j) for the analyzed and predicted fields, respectively. To test the skill of the model, persistence of the initial flow field was used as a control forecast

for comparison. Verification was made for both the wind and vertical shear (vorticity) fields since the model actually predicts the vorticity field and yields the wind field as a by-product.

The results are shown in Figs. 3-7. In Figs. 3-6 the VRMSE of the prediction model winds and vertical shear ($\frac{1}{2}$) versus time is presented for comparison with persistence VRMSE. In the upper left corner of each figure, the standard deviations of the vector mean square errors (VMSE) are given. The VMSE standard deviations are useful in determining whether the model's VMSE are significantly less than persistence VRMSE. No discussion or diagram is presented for the 5000- and 6000-ft levels because the model VRMSE for these levels were no different than persistence VRMSE.

Verification statistics for a level near the bottom of the model is presented in Fig. 3. This figure is typical in showing that the average VRMSE increases with time for both the model and persistence predictions. Using a two-sided t-test for 1188 degrees of freedom ($df = 36 \text{ grid-points} \times 33 \text{ cases}$) and for the 95% confidence level ($\alpha = .05$) it was found that there is no significant difference between the model wind predictions and persistence at the 7000-ft level. In those cases where the VRMSE of the model's predictions are less than those of persistence, a useful test for statistical significance is the one-sided t-test. Using the one-sided t-test ($df=1188, \alpha = .05$), the 7500-ft shear predictions from the model had significantly less VRMSE than persistence.

Figure 4 represents the average VRMSE occurring near the middle of the model. For this level, all the VRMSE of the model are significantly less than persistence VRMSE except the wind VRMSE at 1 hr.

Figure 5 illustrates the VRMSE for the top level of the model. For this level, all the model VRMSE are significantly less than persistence VRMSE. This figure also shows that the VRMSE in the predicted winds tend to diverge from those of persistence with increasing time. Also, Figs. 3, 4, and 5 indicate that the VRMSE for the model also departs increasingly from that of persistence for increasing elevation.

An average of all the VRMSE for the seven levels between 7000 and 13000 ft AMSL is given in Fig. 6. Notice in this figure, that all VRMSE for the model are less than persistence VRMSE except the wind VRMSE for 1 hr. In this diagram the one-sided t-test ($df=1188 \times 7, \alpha = .05$) is applied to test for statistical significance. The circles plotted above the persistence VRMSE curve for the wind in Fig. 6 represent the temporal change in the magnitude of the wind vector as a function of the square root of time. Reed (1958) and others have found that the wind changes are proportional to the square root of the time interval up to 24 hrs. Reed's findings are of particular interest since he analyzed the 3-, 6-, 12-, and 24-hr changes in

the upper wind observations taken on the NTS during the period 1951-1957. Using the persistence VRMSE for the wind at 1 hr, the resulting changes in the wind with time were derived. The derived values, being slightly greater than the corresponding persistence VRMSE, may indicate that the 33 cases used in the verification of the model are more persistent than normal because of the spatial averaging of the wind variations.

Figure 7 shows the average predicted and observed temporal values of vertical speed shear between 7000 and 14000 ft AMSL. This figure illustrates that the model is capable of simulating the general trend in the observed speed shear.

Table 2 summarizes the percent frequency of occurrence that the model VRMSEs for shear were less than those of persistence. Ties were counted as a "no". In general, it may be seen that the model can beat persistence about 60% of the time except at the 5000 and 6000-ft levels.

Table 3 summarizes the percent frequency of occurrence that the model VRMSEs for wind were less than those of persistence. Comparison of Tables 2 and 3 shows that, in general, the model is not as successful in out-performing wind persistence as it is for shear persistence.

5. SUMMARY

The sole test of the validity of any theory is experimental verification. Furthermore, there is a quantitative difference between models that "appear" to yield "reasonable" results and those that have been verified against a representative sample of actual observations. We have selected persistence as the control forecast because experience has shown us that persistence can be a formidable opponent in forecasting mesoscale wind systems. Experience gained in using persistence leads us to suggest that it is a useful tool in objectively assessing the skill of models.

Results of the verification of the proposed numerical model, derived from a horizontal vorticity theorem, has shown that this simple model exhibits skill over persistence in simulating the mesoscale flow in the free air. The model is more skillful in predicting vertical wind shear than in predicting winds. In the atmospheric boundary layer (5000-7000 ft AMSL) no significant difference was detected between the errors in the persistence forecast and those in the model.

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(FIGURES ON FOLLOWING PAGE)

FIGURE 1. VERTICAL PROFILES OF TOTAL ACCELERATION, $\frac{d^2z}{dt^2}$; THE EXCESS OF CORIOLIS OVER PRESSURE GRADIENT FORCE PER UNIT MASS, $f(v-v_g)$; AND THE FRICTIONAL FORCE IN THE DIRECTION OF THE MEAN SURFACE WIND, $\frac{dU}{dt}$, WHERE THE A IS A GRID-SCALE AREA AVERAGE ON A p^* SURFACE AND $p^* = 0$ AT SEA LEVEL (AFTER HOLLAND AND RASMUSSEN, 1973).

FIGURE 2. MEAN VECTOR DIRECTION DISTRIBUTION FOR THE 33 CASES USED IN THE VERIFICATION OF THE MESOSCALE WIND-PREDICTION MODEL.

	SPEED CATEGORIES (KTS)						
	0-5	6-10	11-15	16-20	21-25	26-30	31-35
VECTOR SPEED	10	8	10	3	1	0	1
SCALAR SPEED	3	11	13	3	2	0	1

TABLE 1. MEAN WIND SPEED DISTRIBUTION FOR THE 33 CASES USED IN THE VERIFICATION OF THE MESOSCALE WIND-PREDICTION MODEL.

FIGURE 3. WIND VECTOR ERROR FOR THE 7000-FT LEVEL AND THE WIND SHEAR VECTOR ERROR FOR THE 7500-FT LEVEL VS. TIME FOR 33 CASES.

FIGURE 4. WIND VECTOR ERROR FOR THE 10000-FT LEVEL AND THE WIND SHEAR VECTOR ERROR (Φ) FOR THE 10500-FT LEVEL VS. TIME FOR 33 CASES.

FIGURE 5. WIND VECTOR ERROR FOR THE 13000-FT LEVEL AND THE WIND SHEAR VECTOR ERROR (Φ) FOR THE 13500-FT LEVEL VS. TIME FOR 33 CASES.

FIGURE 6. WIND VECTOR ERROR AND THE WIND SHEAR VECTOR ERROR (Φ) OVER A 6x6x7 GRID VOLUME FOR 33 CASES. THE CIRCLES REPRESENT THE TEMPORAL CHANGE IN THE OBSERVED WIND VECTOR AS A FUNCTION OF THE SQUARE ROOT OF TIME.

FIGURE 7. TEMPORAL CHANGES IN THE OBSERVED AND PREDICTED MEAN SCALAR SPEED SHEAR OVER A 120 X 120 SQ. N.M.L. AREA THROUGH A 7000-FT LAYER SITUATED BETWEEN 7000 AND 10000 FT MSL FOR 33 CASES.

		PERCENT											
		LEVEL (X 10 ³ ft)											
TIME (HR)		5	6	7	8	9	10	11	12	13		%	
1		9	24	70	70	79	67	79	79	88		63	
2		42	33	73	76	64	79	88	70	73		66	
3		39	21	55	70	61	61	76	70	88		60	
4		48	42	58	52	58	61	76	64	76		59	
5		48	33	58	52	64	45	61	61	70		55	
6		58	52	58	55	61	52	70	73	73		61	
		41	34	60	62	64	61	75	70	78			

TABLE 2. PERCENT OF THE TIME THAT THE MODEL SHEAR VECTOR PREDICTIONS BEAT THOSE OF PERSISTENCE.

		PERCENT											
		LEVEL (X 10 ³ ft)											
TIME (HR)		5	6	7	8	9	10	11	12	13		%	
1		27	30	39	52	39	52	52	64	61		46	
2		15	36	36	48	61	52	64	55	64		48	
3		39	48	42	48	67	52	55	61	64		53	
4		42	45	45	48	61	61	52	61	67		54	
5		52	64	48	45	48	64	55	64	61		56	
6		52	58	58	48	48	52	61	67	61		56	
		38	47	45	48	54	56	57	62	63			

TABLE 3. PERCENT OF THE TIME THE MODEL WIND VECTOR PREDICTIONS BEAT THOSE OF PERSISTENCE.