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THEORY AND APPLICATION OF FRACTURE MECHANICS

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THEORY AND APPLICATION OF FRACTURE MECHANICS

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ABSTRACT

This report describes various approaches to the brittle fracture problem. It is intended not only to provide a brief review of the state of the art, but also to serve as a primer in the use of a modern approach to the combined problems of selection of materials and design against brittle failure.

Included is a generalized description of linear elastic fracture mechanics technology and a comprehensive review of the information required to apply this design concept. Example problems are included to illustrate the methods of designing against brittle fracture and for calculating allowable stresses.

This report is intended to provide the materials and design engineers with the basic information, procedures, and criteria that are necessary to properly coordinate material application and design considerations for the prevention of brittle fracture.

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1. GENERAL APPROACHES TO BRITTLE FRACTURE

Various approaches have been employed to solve the problems associated with brittle fracture. The most widely used approach is the ductile-to-brittle transition temperature approach. However, this approach lacks much of the quantitateness that is required for highly reliable designs and its applicability to higher strength materials is questionable. The more recently developed linear elastic fracture mechanics approach has the potential to provide the desired quantitative answers.

The first section which follows provides a rather brief review of the various types of approaches which fall into the general category of transition temperature approaches. The next section describes approaches based on a more rigorous method of stress analysis including the fracture mechanics approach.

2. THE DUCTILE-TO-BRITTLE TRANSITION APPROACH

The basic philosophy of the ductile-to-brittle transition temperature approach is that a material has a characteristic temperature below which it is susceptible to brittle fracture at a relatively low value of stress in the presence of sharp defects, and above which brittle fracture does not occur. In the presence of a sharp notch or defect the ductility, energy, fracture appearance, or some material property undergoes an abrupt transition from a ductile to a brittle behavior over a narrow range of temperatures. This change forms the basis for defining the transition temperature.

The transition temperatures can be grouped into four general categories as follows:

- 1) The temperature at which the capacity of the material for large plastic deformation in the presence of a very sharp notch or crack rapidly decreases to essentially zero (brittle-to-ductile transition temperature).
- 2) The temperature at which the mode of fracture propagation readily changes with decreasing temperature from a full shear to a flat fracture surface (fracture mode transition temperature).
- 3) The temperature at which the fracture strength of a sharply notched or pre-cracked specimen decreases rapidly from values which are well above the conventional yield stress to values well below it (fracture stress transition temperature).
- 4) The temperature above which a running crack will be arrested (crack arrest transition temperature).

2.1 TESTS FOR DETERMINING TRANSITION TEMPERATURES

Because of its usefulness in evaluating stress for storage tanks, ship plates, and line pipes, the charpy V-notch impact test has been widely accepted as a standard test method for determining transition temperatures and for indicating toughness at specific testing temperatures. Several transition temperatures may be ascertained from any one given set of data obtained from these impact tests since several criteria may be employed. Both the brittle-to-ductile and fracture mode types of transition temperatures are represented in this test.

Figure 2-1 is a schematic representation of the energy absorbed by specimens of various temperature during the charpy V-notch impact test. Also shown are two transition temperatures readily obtained from such tests. T_1 is the transition temperature as determined by some fixed level of impact energy E_1 . E_1 is usually determined by correlations with other types of tests or service performance. The fixed energy criterion is used fairly extensively, particularly for quality control evaluations and acceptance tests of structural steels.

The temperature T_2 , above which the fracture appearance is entirely shear, represents the most conservative criterion in that it yields the highest transition temperature coupled with the maximum energy for fracture. Because of practical limitations, this criterion is seldom used except in development investigations or applications requiring an extremely high degree of toughness.

Another generally known type of transition temperature measurement belonging in the brittle-to-ductile category is the nil-ductility transition temperature, NTD, which is

determined using the "drop-weight" test technique. This test defines the temperature below which the capacity of the material to undergo a small amount of plastic deformation in the presence of a sharp notch is essentially null under impact loading conditions. The relative position of this transition temperature superimposed on a charpy V-notch impact energy curve for the general case is shown in Figure 2-1. At temperatures below the NDT, brittle fracture can initiate and readily propagate at average applied stresses that are only a fraction of the conventional yield strength.

The explosion bulge test has been used to rate materials with respect to the nature and extent of the resulting deformation and cracks when tested over a range of temperatures. The highest temperature where extensive deformation without brittle cracking occurs is referred to as the fracture transition plastic temperature (FTP). The temperature below which the cracking begins to extend beyond the deformed material into the elastically loaded edge regions has been designated as the fracture transition elastic temperature (FTE). The FTE always occurs at a temperature higher ($\approx 60^{\circ}\text{F}$) than the NDT and lower ($\approx 60^{\circ}\text{F}$) than the FTP. The relative positions of these temperatures are also shown in Figure 2-1.

Another category of transition temperature definition is concerned with the temperature at which a running crack will be arrested, commonly referred to as the crack arrest temperature (CAT). Tests to measure the CAT generally involve developing a fast moving brittle crack and determining the stress-temperature conditions necessary to arrest the crack. The arrest temperature is largely dependent on the thickness of the specimen, hence full thickness tests should be used in this type of evaluation. The crack arrest transition temperature type of test, although somewhat bulky, is useful for development, comparison, quality control, and design purposes.

Since the designer is accustomed to working with stresses, this approach has been thought to be particularly attractive because it not only provides him with a transition temperature but also a knowledge of the critical stress for propagation of cracks as a function of temperature. Figure 2-1 shows the relative position of this temperature with the others previously mentioned.

2.2 SIGNIFICANCE AND APPLICATION OF THE TRANSITION TEMPERATURE APPROACH

The applicability of the transition temperature approach to the problem of brittle fracture varies considerably depending upon the criterion that is used and the purpose to which it is applied. While it may be used directly in its simplest form for comparative evaluations of the fracture resistance of materials, its application to design may, in many instances, be quite complex or even impractical. For comparative purposes, almost any one of the various transition criteria may be used directly with a reasonable degree of confidence that the material which exhibits the lowest transition temperature is likely to be the most fracture resistant. In some design situations, the transition temperature may be very useful for establishing design parameters. On the other hand, for some complex application, the selection of an appropriate transition temperature criterion and its subsequent transformation into useful design parameters can be extremely involved. In other applications (e. g., where a material must be used below any of the conventional transition temperatures, or where the material does not exhibit an abrupt ductile-to-brittle transition behavior) the approach is virtually useless as a design tool.

Some of the complexities arise primarily because of the existence of the different kinds of transition behavior and the many criteria of transition temperatures within any given behavior category. Ideally, if each test and each transition behavior yielded a common transition temperature, there would be no problem. Obviously, this is not the case. Therefore, the choice of a criterion for transition temperature for a particular application involves the consideration of many factors, some of which are: embrittling factors, degree of assurance against crack initiation, degree of assurance against crack propagation, etc.

The concept of using the transition temperature approach for design purposes is that for temperatures above the transition temperature stresses of the order of the nominal yield stress may be tolerated, and below the transition temperature the applied stress must be kept to some unknown lower level. There are two primary steps in these considerations. The first is to select a transition temperature criterion that is appropriate to the needs of

the application and that is truly representative of the embrittling factors that are present. Then, the selected transition temperature criterion must be transformed into terms related to defect size and load-bearing capacity, for ultimately this is the terminology which the engineer can understand and apply.

One sound solution to the general design problem is based on the establishment of a correlation between service performance (preferably failures) and transition temperature obtained from past history. While this type of correlation does not yield direct data concerning the load-bearing capacity, it does, through service experience, provide some insight as to the general stress level that can be sustained above the transition temperature.

Another approach available to the designer involves the use of a transition temperature measurement which also incorporates a direct measurement of the load-bearing capacity. Crack arrest transition tests are good examples. The major limitation is that the results are directly related to the size of the specimen used in the test and have little direct relationship to general shapes and heavy forgings.

The most useful, comprehensive summary of the transition temperature approach is provided in the form of a Fracture Analysis Diagram. This diagram, which is similar to the charpy energy curve with the ordinate expressed in terms of stress, illustrates the general relationships between defect size, applied stress, and temperature. It is applicable to the structural grades of steel which exhibit a pronounced temperature dependent ductile-to-brittle transition behavior and to section sizes of a limited range. It should be noted that the diagram deals with only one dimension (length) of a defect and does not recognize that the critical stress for fracture can vary markedly, depending upon the type of loading and the geometry of the defect. Where applicable, in terms of the material and intended use, the diagram is useful for purposes of making engineering judgements.

Figure 2-1 Schematic Representation of the Relative Positions of Several Transition Temperatures on the Charpy V-Notch Energy Curve

3. THE STRESS ANALYSIS APPROACH

In the previous section we have noted that the general transition temperature approach is most applicable to low strength materials for purposes of comparing materials and for rough design approximations. It is not satisfactory for high strength materials, mainly because these materials do not exhibit a characteristic abrupt ductile-to-brittle transition behavior. Even if it were possible to establish the various transition temperature criteria for all materials, the approach lacks the basic quantitateness that is required to solve specific problems. In addition, the growth of an initial sub-critical size defect to a critical size defect during the operational life time of a cyclic or sustained loaded component is another important consideration which cannot be handled with the transition temperature approach.

The stress analysis approach to the prediction of catastrophic fracture attempts to solve some of these problems. In this approach some property of the stress-strain distribution is used as the prediction criterion. A number of different properties have been used as the criterion and in general they can be placed in one of three categories: (1) stress criteria, (2) strain criteria, and (3) energy release rate criteria. It is required that the stress-strain distribution, or at least a reasonable estimate of this distribution, in the structural component be known. If the component contains notches or sharp flaws, the stress analysis must take this into account. The effects of plastic flow around these stress risers on the stress-strain distribution should also be considered when possible.

Some of the more significant approaches in the stress, strain and energy categories will now be considered.

3.1 THE STRESS CRITERIA

Several stress theories have been proposed for brittle fracture, but, due to the difficulty in experimental verification, these approaches have not been pursued very earnestly. Many theories in their present form are not usable in design considerations.

For general engineering design purposes the stress concentration factor approach is the most applicable of the stress methods. It is possible to express the maximum stress, $\sigma_{\max}$, at the root of a notch or crack in an elastic body as a linear function of the nominal stress, σ_{nom} , in the body containing the notch or crack. The relation between these two stresses is in the form $\sigma_{\max} = K_t \sigma_{\text{nom}}$, where the constant proportionality factor, K_t , is defined as the theoretical elastic stress concentration factor. The stress concentration factor is a function of the geometry of the body containing the notch and of the external load distribution on the body. The magnitude of K_t for a specific geometry and loading condition can be determined theoretically by employing basic principles of the theory of elasticity or experimentally by a number of methods.

For example, an elliptical crack of length $2a$ in an infinite plate subject to a uniform uniaxial tension field perpendicular to the plane of the crack (Figure 3-1) has a theoretical elastic stress concentration factor expressed by the equation

$$K_t = 1 + 2\sqrt{\frac{a}{r}} \quad [3-1]$$

where r is the crack tip root radius.

There are a number of reasons why stress concentration factors are not satisfactory for predicting brittle fracture. First, an attempt to apply them to hairline cracks in any useful manner would be extremely complicated and of doubtful accuracy (as $r \Rightarrow 0$, $K_t \Rightarrow \infty$). Second, they provide no practical way to consider the fracture resistance of the material. Attempts have been made to combine the elastic stress concentration factor concept with the maximum stress concept so that the nominal fracture stress on notched bodies can be accurately predicted. The simplest and most direct method used is to assume that at failure $\sigma_{\max}$ is equal to the uniaxial ultimate strength of the material, σ_u , and that the actual stress concentration factor is equal to the elastic stress concentration factor. This method predicts that the nominal stress will be inversely proportional to the elastic stress concentration factor. When experimental results are compared with those predicted by this method a number of shortcomings of the method become apparent.

- 1) For relatively brittle materials the method underestimates the nominal fracture stress when the elastic stress concentration becomes large.
- 2) For some ductile materials the theoretically predicted nominal fracture stresses are completely misleading (some materials are insensitive to the presence of notches).
- 3) The method does not account for a size effect. Experimental results indicate that for many materials containing sharp notches the size effect is significant.
- 4) Finally, this approach does not predict the difference in fracture stress which occurs in going from a state of plane stress to one of plane strain at the notch tip.

These limitations of the linear elastic stress concentration factor approach are to a large degree the result of two factors. First, since the theory of elasticity which is used in determining the stress concentration factors is valid for a continuum only, deviation would be expected between theory and test results for extremely sharp notches where the use of a continuum becomes invalid. Secondly, due to the high stress concentration at the root of the notch, the material in the vicinity of the notch tip will flow plastically and, therefore, the stresses predicted by elastic theory will be in error. To correct for these deficiencies, various means of modifying the value of the elastic stress concentration factors have been proposed. At present the parameters involved in these correction factors must be estimated or determined by rather extensive testing. With an increasing availability of elastic plastic solutions for notched and cracked geometries, the need for these correction factors may be eliminated.

3.2 THE STRAIN CRITERIA

The methods involving a strain criteria for the prediction of fracture loads usually use the maximum strain or some critically strained volume as the critical parameter. In general, the strain criteria methods are very similar to the stress methods to predict fracture. An advantage of a strain criteria over a stress criteria is the increased sensitivity of strain compared to stress in the plastic range. For example, for a material which has a very

moderate amount of strain hardening, the strain is a much more meaningful criterion for fracture than the stress in a smooth specimen fracture test. A number of fracture analysis methods involving strain criterion have been introduced in the last few years. Most of these approaches are in the development stage and are not presently suitable for design considerations.

3.3 LINEAR ELASTIC FRACTURE MECHANICS

The theory of linear elastic fracture mechanics provides a means of predicting the fracture load of structures containing sharp flaws of known size and location. The theory can be based on either an energy approach or on a stress intensity approach. Both approaches are closely related and lead to the same results.

3.3.1 Energy Approach

The energy approach to fracture was originally presented by Griffith and later re-emphasized and broadened by Irwin and Orowan. The energy criterion states that crack propagation will occur if the amount of energy which could be supplied to the crack tip during an incremental crack extension is greater than or equal to the energy which would be absorbed at the crack tip during an incremental extension of the crack. This energy balance can be simplified to consider two types of energy--the surface energy of the crack (S) and the strain energy of the system (U). This approach can be demonstrated by applying the theory to a centrally cracked plate (Figure 3-2) subjected to a uniform axial stress σ . The presence of the crack in the body has been shown by Griffith, in a long, involved calculation, to reduce the strain energy of the plate by $\pi \sigma^2 a^2 / 2E$.

The surface energy (S) associated with the two new surfaces of the crack is approximately $2 \gamma a$, where γ is the surface tension of the material. Crack growth can occur without additional work only if the increase in surface energy caused by an incremental increase in crack length δa is compensated by a corresponding reduction in the strain energy (δU) of the system. Thus, the critical condition can be described as

$$\frac{\delta U}{\delta a} = - \frac{\delta S}{\delta a} \quad [3-2]$$

If this expression is simplified, the residual strength of the brittle plate material can be expressed as

$$\sigma = \sqrt{\frac{2E\gamma}{\pi a}} \quad [3-3]$$

Rearranging the above expression gives

$$\sigma \sqrt{a} = \sqrt{2E\gamma/\pi} = \text{const} \sqrt{E\gamma} = \text{material constant} \quad [3-4]$$

This relationship is one of the main contributions of Griffith's theory. It has been experimentally verified using glass specimens.

It can also be shown that the energy release rate, G , of the plate is

$$G = \frac{\pi \sigma_a^2 a}{E} \quad [3-5]$$

The fracture of the plate will occur when

$$\sigma_c = \sqrt{\frac{G_c E}{\pi a}} \quad [3-6]$$

where ideally G_c , the critical value of the energy release rate, can be determined from a fracture test performed on this geometry at some known crack length.

3.3.2 Stress Intensity Criterion

The same equation for the residual strength of a cracked panel can also be obtained by the stress intensity approach. Assume that a crack is a flat, internal, free surface in a linear-elastic stress field. The elastic stress field in the vicinity of the crack tip ($r \Rightarrow 0$) has the general form

$$\sigma_x = \frac{K}{(2\pi r)^{1/2}} \cos \frac{\theta}{2} \left[1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right] + \dots \quad [3-7]$$

$$\sigma_y = \frac{K}{(2\pi r)^{1/2}} f(\theta) \quad [3-8]$$

$$\tau_{r\theta} = \frac{K}{(2\pi r)^{1/2}} g(\theta) \quad [3-9]$$

where the coordinate system (r, θ) is shown in Figure 3-3.

These equations suggest that the opening of the crack is controlled solely by the value of K , a constant which determines how much the stress intensifies at the crack tip (not to be confused with the stress concentration factor, K_t). This stress intensity factor (K) is independent of r and θ and, therefore, gives a single description of the stress intensity at any point near the crack tip. The higher the value of K the greater is the stress required to produce catastrophic propagation and the greater is the resistance of the material to brittle fracture.

The basic assumption in fracture mechanics is that unstable fracture occurs when K reaches a critical value, K_c , commonly called the fracture toughness. It is important to appreciate the difference between K and K_c . The stress intensity factor K is simply a coefficient in an equation describing the elastic stresses in the vicinity of a crack tip. The fracture toughness, K_c , is a particular value of K corresponding to unstable propagation of the crack. The difference between K and K_c is analogous to the difference between stress and strength for a body free of discontinuities. Whereas K varies as a function of load and the geometry of the crack and the part containing the crack, K_c does not. As far as the crack is concerned, any configuration and load distribution that produces a given value of K is equivalent to any other such configuration and load distribution. Thus, K_c is independent of loading conditions and may be considered a material constant. The units of K_c are $\text{psi}-\sqrt{\text{in}}$ and are the same as the material constant developed in Griffith's theory (see Equation 3-4).

Being a material property, the fracture toughness, once properly determined under one set of conditions, is applicable to other geometries and loading conditions at the same temperatures. At present the value of K_c must be determined experimentally. After obtaining the fracture toughness of the material in question for a given temperature and the size of the defect from nondestructive evaluation, it is then possible to evaluate the load-bearing capacity of the structure. Conversely, knowing the toughness and applied stress, it is possible to estimate the critical defect sizes that are required for catastrophic failure.

As an example, the case of the infinite plate containing a finite crack of length $2a$, perpendicular to a uniaxial stress field σ , will be considered again. The stress intensity factor for this geometry is given by $K = \sqrt{\pi a} \sigma$. Therefore, fracture would be predicted at the critical stress $\sigma_c = K_c / \sqrt{\pi a}$, where K_c could be obtained from a small and reasonably simple laboratory specimen.

Since the stress intensity factor describes the elastic stress and strain distribution at the tip of the crack, it would be expected that there should be some relationship between K and the energy release G . It has been shown by Irwin that the following relations exist:

$$K_c = \sqrt{G_c E} \quad (\text{for plane stress})$$

and

$$K_{Ic} = \sqrt{G_{Ic} E / (1 - \nu^2)} \quad (\text{for plane strain})$$

where E is the elastic modulus and ν is Poisson's ratio. These relationships show that the two approaches (stress intensity and energy release rate) are equivalent.

The relationship between the critical stress intensity factor, K_c , and the plane strain critical stress intensity factor for the opening mode of fracture, K_{Ic} , is given in the next section.

Figure 3-1 Elliptic Crack in an Infinite Sheet
Subjected to Normal Uniaxial Stress

Figure 3-2 Infinite Sheet with Center Crack
Subjected to Normal Uniaxial Stress

Figure 3-3 Coordinates Measured from the Leading
Edge of a Crack and the Stress Components in the Crack Tip Stress Field

4. LIMITATIONS OF FRACTURE MECHANICS

Although linear elastic fracture mechanics is very useful in material selection and other design considerations, it does have a bounded area of use. At present the bounds of the areas of application have not been distinctly determined, but in applying the theory one should be aware of its general limitations.

4.1 EFFECT OF PLATE THICKNESS ON FRACTURE TOUGHNESS

The terms plane stress and plane strain are also commonly encountered in the subject of fracture mechanics. Figure 4-1 illustrates the effect of plate thickness on the fracture toughness and physical appearance of the fracture. A fracture in a relatively thin sheet usually consists of a certain amount of slant and flat (or square) types of fracture surfaces. Plane stress conditions in centrally cracked panels refer to a panel which is thin enough to allow deformation in the thickness direction. The resulting plane stress fracture surface normally approaches a condition of 100% slant fracture and suggests that fracture was accompanied by a considerable amount of plastic deformation. For plane stress conditions the critical stress intensity factor is denoted by K_{Ic} .

In general, as the specimen becomes thicker, the percentage of slant fracture becomes less and the fracture becomes more brittle in appearance. The transition in the fracture mode and decrease in K_{Ic} with increasing plate thickness have been qualitatively related to the change in the size of the plastic zone at the crack tip. At some critical thickness the crack propagation is governed by plane strain conditions and the stress intensity factor reaches a minimum designated as K_{Ic} . K_{Ic} is particularly pertinent in materials evaluation because it is a constant which is essentially independent of specimen dimensions. From a practical standpoint, the initiation of slow crack growth is also governed by plane strain conditions so that K_{Ic} not only characterizes total failure in relatively thick sections where plane strain conditions prevail, but it also controls the stress at which cracks extend in thinner sections. Even under conditions of combined plane strain and plane stress, the use of K_{Ic} values will normally result in a conservative design.

4.2 MODES OF FRACTURE

The mode of fracture refers to the direction of the relative motion between the two corresponding crack surfaces during the fracture process. The relative crack surface movements can be resolved into three basic modes. In mode I, the opening mode, the crack surfaces move opposite and perpendicular to each other as shown in Figure 4-2(a). In mode II, the forward sliding mode as shown in Figure 4-2(b), the two crack surfaces move in approximately the same plane and in a direction perpendicular to the line of the crack tip. Mode III, the tearing mode as shown in Figure 4-2(c), is the mode in which the two corresponding crack surfaces move in approximately the same plane and in a direction which is parallel to the line of the crack front. These three modes are sufficient to describe all forms of crack surface displacements. Most fracture mechanics studies are predominately mode I, although some limited studies of mode II and mode III have been performed.

The crack tip elastic stress fields, of all bodies which are loaded such that the crack surfaces have a mode I type of displacement, differ by a multiple constant only, K_I . The displacement fields also differ by on the same multiple constant. Similarly, the stress and displacement fields around the tip of cracks, whose surfaces have the mode II or mode III type of relative displacement, will also differ by only multiple constants. These constants are defined as the mode II stress intensity factor, K_{II} , or the mode III stress intensity factor, K_{III} .

4.3 PLASTIC FLOW AT THE CRACK TIP

The theory of elasticity is used in the determination of the stress intensity factor for various geometries and loading conditions. This fact places rather significant restrictions on the use of a critical stress intensity factor as a criterion for failure of an initially cracked body. Due to extremely high stress concentration at the crack tip, plastic flow will occur in a zone around the crack tip. If the zone of plastic flow is small compared to the area around the crack tip in which the elastic stresses are mathematically described, then the stress intensity factor will adequately represent the elastic-plastic stress and strain conditions at the crack tip. Therefore, the requirement is imposed that the size of the plastic zone at fracture must be very small compared to the total volume of the cracked body.

The size requirements needed to obtain valid K_c measurements at various thicknesses can be relaxed somewhat by use of a plastic zone correction factor. The effect of a limited amount of plastic flow at the crack tip can be corrected for by adding a fictitious incremental length, r_y , to the physical crack length a_o , and then using this corrected crack length, $a = a_o + r_y$ in calculating K_c . This correction is determined for the point at which the stress on the cracked panel is equal to the yield strength of the material. For plane stress conditions the correction term is $r_y = K_c^2 / 2\pi \sigma_{ys}^2$ and for plane strain conditions the term is $r_{ly} = K_{Ic}^2 / 6\pi \sigma_{ys}^2$. The use of the plastic zone correction factors is valid when the amount of yielding at the crack tip is small. These corrections are often neglected, and the result is a conservative estimate of fracture toughness. If gross yielding occurs, then the use of the correction factor is of no aid, and linear elastic fracture mechanics should not be applied.

4.4 EFFECT OF CRACK TIP ROOT RADIUS

It has also been shown that the value of K_c is sensitive to the size of the crack tip root radius. For many materials the value of K_c decreases in proportion to the decrease in the square root of the root radius until the root radius reaches some minimum value (Figure 4-3). As the root radius decreases below this value, the magnitude of K_c remains constant. To eliminate the effect of root radius, K_c is often determined by using a test specimen having a fatigue crack extending beyond the machined crack. This may not be necessary for very brittle materials such as graphite and beryllium.

4.5 EFFECT OF TEMPERATURE AND STRAIN RATE

The two variables of temperature and strain rate are the predominant mechanical variables which can have a strong influence on K_{Ic} --- the plane strain fracture toughness parameter for the opening mode of fracture. For high strength materials the variation of K_{Ic} with temperature and strain rate is not well defined. In this case it is recommended that the temperatures and strain rates employed in the determination of K_{Ic} correspond to those prevailing in the intended application. As a first approximation, however, those materials which exhibit an increase in yield strength with decreasing temperature or

increasing strain rate can be expected to exhibit a decrease in K_{Ic} for those conditions which produce an increase in yield strength. Stated more clearly, for materials which have a strong strain rate and temperature sensitivity, K_{Ic} generally decreases with decreased temperature and increased strain or loading rate.

4.6 SLOW CRACK GROWTH DURING CYCLIC LOADING

While the termination of the life of a structure or a component may be based on the critical flaw size for catastrophic failure, it must be recognized that the total useful life of a cyclic-loaded component is dependent upon the rate of growth of flaws from a subcritical size to a critical size. Therefore, both an understanding of the critical combination of stress and defect size for catastrophic fracture and the measurement of crack growth characteristics of the material under application conditions are essential to determining the useful life of a component.

Fatigue (slow) crack propagation is a localized phenomenon dependent upon the temperature, environment, and stress conditions at the crack front. For given conditions, the stress intensity factor K_I is a function of the applied load (stress) and the crack length; that is, $K_I = f(\sigma, a)$. As the crack grows under constant load cycling, the stress intensity increases since both "a" and σ are increasing. Eventually, the crack grows to a sufficient length that the stress intensity K_I increases to a level equivalent to the material characteristic K_{Ic} , the critical stress intensity factor. At this point, brittle fracture (rapid crack propagation) occurs.

Consideration of the ratio K_{Ii}/K_{Ic} (where K_{Ii} is the initial stress intensity factor) provides a technique for estimating how close conditions are to fracture. The data are quite useful when shown in the form illustrated in Figure 4-4. For example, knowing the ratio K_{Ii} to K_{Ic} , the number of cycles to failure can be predicted. Conversely, knowing the desired life of the component and K_{Ic} of the material, the maximum allowable initial stress intensity factor K_{Ii} can be determined. Since K_{Ii} is a function of the applied stress and the flaw size, it is then possible to compute either the initial allowable flaw size for a given stress or, conversely, the allowable stress for some given initial flaw size.

Figure 4-1 Dependence of K_c and Fracture Appearance
 (In Terms of Percentage of Square Fracture) on Thickness of Plate Specimens

Figure 4-2 Basic Failure Modes of Fracture Mechanics
 (a) Opening, (b) Sliding, (c) Tearing

Figure 4-3 Effect of Crack Tip Root Radius on the Stress Intensity Factor

Figure 4-4 Typical Cyclic Flow Growth Data

5. FRACTURE TOUGHNESS TESTING

Fracture toughness testing is a relatively recent development which requires the testing of rather large, expensive specimens; as a result, the amount of experimental data available to confirm existing concepts is limited and as more data become available for examination the recommended testing procedures may become modified.

Although the plane stress stress intensity factor, K_{Ic} , is assumed to be a material property which varies inversely with yield strength, it is obvious that this assumption is applicable only to materials of the same metallurgical history. The effect of metallurgical parameters upon yield strength and other conventional strength criteria is much less than the effect of the same parameters upon fracture toughness. Yield strength measurements are affected only by those variables which result in a change in the degree of material yielding, whereas fracture toughness is basically an energy measurement at fracture instability and is affected by all variables which affect the micromechanisms of failure. As a result, fracture toughness measurements must be conducted with test specimens representative of the material to be used in component fabrication. The chemistry, heat treatment, working history and test orientation must be nearly identical to that of the component in order to insure satisfactory toughness measurements.

Tabulated K_{Ic} data arranged in handbook form for design reference must provide sufficient information to adequately describe the material tested. All variables known to affect K_{Ic} measurements must be included. Only then can the data be utilized for design against brittle failure.

5.1 LABORATORY TEST SPECIMEN

Several factors must be considered when selecting a particular type of specimen for a fracture toughness test program. Foremost among these are: the expected toughness of the material to be tested, the size and shape of the available test material, the loading capacity of the available test facilities, the economical usage of test material, and the accuracy desired in the toughness measurements.

Various methods of testing have been suggested, each with a preferred area of application. There are five fairly common methods which are being used to determine plane-strain fracture toughness: (1) tensile test on a circumferentially notched round specimen, (2) tensile test with a surface-cracked sheet or plate specimen, (3) tensile test on a center-notched sheet specimen, (4) notch bend test, and (5) tensile test on a single-edge-notch specimen. If performed properly, all of these tests (as well as others which are available using double cantilever beam specimens, wedge opening loading specimens, compact tension specimens, etc.) should yield similar fracture toughness values.

Test specimens are usually pre-cracked to insure accurate determination of the fracture toughness. When specimens are not cracked, the measured fracture toughness is usually high. Tests on materials which are difficult to pre-crack, such as graphite and beryllium, have shown that accurate values of fracture toughness can be obtained with sharp, carefully machined notches.

After the basic form of test specimen has been determined, the next problem is deciding how large the specimen should be. The most current general rule that can be used in estimating the minimum size of specimen that will provide plane strain conditions is that the thickness and crack length should be $\geq 2.5 (K_{Ic}/\sigma_{ys})^2$. For very brittle materials such as graphite and beryllium, the minimum size is even smaller. Once these dimensions are determined, the rest of the specimen dimensions can be established by direct proportion.

When the crack geometry and applied stress are known, K_{Ic} is determined from simple equations. For example, for a through-the-thickness crack of length $2a$ in an infinite plate, the stress intensity factor is $K_{Ic} = \sigma (\pi a)^{1/2}$ where σ = gross stress remote from the crack. The expressions for K_{Ic} for laboratory test specimens are slightly more complicated. Since the specimens have finite dimensions, the influence of proximate boundaries on the crack stress field must be taken into account. The geometrical proportions and the corresponding stress intensity relationship is shown for a center-cracked plate test specimen in Figure 5-1.

$$5 < W/B < 10$$

$$K_I = \frac{P a^{1/2}}{B W} \left[1.77 + 0.227 \left(\frac{2a}{W} \right) - 0.510 \left(\frac{2a}{W} \right)^2 + 2.7 \left(\frac{2a}{W} \right)^3 \right]$$

for range $\frac{2a}{W} = 0$ to 0.7

(a)

(b)

Figure 5-1 (a) Proportions for Symmetrical
Center-Cracked Plate Test Specimen, (b) Fatigue Crack Starter

6. APPLICATION TO DESIGN

In general the expression for K takes the form $K^2 = Q \sigma^2 \pi a$, where Q is a function of the crack geometry.

The number of geometries and loading conditions for which solutions for the stress intensity factors is known is rather limited. Figures 6-1 to 6-30 include several stress intensity solutions given for various fracture mechanics specimens that may be useful in design considerations. The following examples illustrate the use of fracture mechanics in design.

6.1 PRESSURE VESSEL EXAMPLE

Compute the pressure at failure for the tank shown in Figure 6-31, which contains a crack of length 0.15 in. oriented as shown. The tank is fabricated from a steel of ultimate strength $\sigma_u = 300,000$ psi, and $K_{Ic} = 34,000$ psi- $\sqrt{\text{in.}}$

Assume thin wall approximations. From conventional strength-of-materials theory, the stress across the crack is obtained from $\sigma_1 = Pr/t$, where σ_1 = nominal hoop stress, P = internal pressure, r = tank radius, and t = tank wall thickness.

The applicable stress intensity factor, taken from Figure 6-31, is $K_I = \sigma \sqrt{\pi a}$. Therefore,

$$\sigma_1 = \frac{Pr}{t} = \frac{K_{Ic}}{\sqrt{\pi a}} \quad \text{or} \quad P = \frac{K_{Ic}}{\sqrt{\pi a}} \left(\frac{t}{r} \right) = 61.6 \frac{t}{r} \text{ ksi}$$

The limiting tank dimensions, in terms of pressure, are $t/r = 0.0163 P$. If the problem is to determine the minimum permissible resolution of non-destructive testing equipment, the maximum critical flaw size can be found from

$$a = \frac{1}{\pi} \sqrt{\frac{K_{Ic}^2}{Pr}}$$

Suppose the crack is circumferentially oriented. The nominal longitudinal stress is given by $\sigma_2 = \frac{Pr}{2t}$. From fracture mechanics (with K_I again taken from Figure 6-1)

$$P = \frac{2K_{Ic}}{\sqrt{\pi a}} \left(\frac{t}{r} \right) = 123.2 \frac{t}{r} \text{ ksi}$$

As a comparison, the limiting pressure can be calculated from the conventional distortion energy theory

$$\sigma_f = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2} = .863 P \left(\frac{r}{t} \right)$$

or

$$P = 348 \text{ t/r ksi, for } \sigma_y = 300,000 \text{ psi}$$

Thus the limit determined by fracture mechanics is much smaller than that of the conventional failure criterion, which neglects the effect of the crack.

6.2 SURFACE CRACKED THICK WALL EXAMPLE

Suppose we wish to investigate the design of a thick wall structure with fracture mechanics methods. Assume a surface flaw such as that shown in Figure 6-32 is present. To predict the critical crack length (2b) or flaw size for such a structure, we can use the model given by Figure 6-29,

$$K_{Ic} = 1.1 \sigma \sqrt{\pi a_c / Q} \quad \text{for } \frac{a}{b} < \frac{1}{2}$$

where

$$Q = \left[\phi^2 - (0.212) \left(\sigma / \sigma_{ys} \right)^2 \right]$$

and σ is the gross applied stress, a_c is the crack depth, 2b is the crack length at the surface of the sheet, σ_{ys} is the 0.2% yield strength, and ϕ is an elliptic integral whose values are given in Table 1 as a function of a/b .

Suppose we plan to design the structure of 1/2 in. thick, 18% Ni, Grade 300 maraging steel. In the longitudinal grain direction, this alloy has a K_{Ic} value of about 87,000 psi- $\sqrt{\text{in}}$. We know from experience that .25 is a good estimate for the surface flaw ratio of this alloy. If our operating stress is to be 80% of the material yield strength (240,000 psi), we find that for $a/2b = 0.25$, Q is approximately 1.34. Rearranging the equations for a_c and K_{Ic} so that

$$\frac{a_c}{Q} = \frac{K_{Ic}^2}{1.21\pi \sigma^2}$$

and substituting the above values, we calculate that $a_c = 0.047$. Therefore, since $a/2b = 0.25$, the critical crack length $(2b) = 0.188$ in. If inspection techniques cannot detect cracks this small, the design must be based on the smallest crack which can be detected.

Suppose, for example, that the smallest detectable surface flaw is 0.1 inch deep and 0.4 inch long. For the same properties as above, we find that by trial and error $Q = 1.5$. Thus,

$$\frac{a_c}{Q} = \frac{0.1}{1.5} = \frac{K_{Ic}^2}{1.21\pi\sigma^2} = \frac{(87,000)^2}{1.21\pi\sigma^2}$$

and σ , the maximum safe operating stress, is found to be 132,000 psi.

If we have fatigue loading, the K_{Ii} factor must first be determined for the required number of life cycles. This factor is then used in place of K_{Ic} to determine the smallest detectable flaw size at start of life or maximum safe operating stress through life.

Figure 6-1 Cracked Sheet with Uniform Normal (or Biaxial) Stress at Infinity

Figure 6-2 A Crack in an Infinite Sheet Subjected to Centrally Applied Wedge Forces

Figure 6-3 An Edge Crack in a Semi-Infinite Sheet Subject to Tension

$$K_I = \frac{P}{(2\pi a)^{1/2}}$$

$$K_{II} = \frac{Q}{(2\pi a)^{1/2}}$$

Figure 6-4 An Infinite Sheet
With a Semi-Infinite Crack Subjected to a Concentrated Force

$$K_I = \frac{2P}{(2\pi a)^{1/2}}$$

Figure 6-5 An Infinite Sheet
With a Semi-Infinite Crack Subjected to Wedge Forces

$$K_I = \frac{P(c^2 - a^2)^{1/2}}{2(\pi a)^{1/2}(c - a)}$$

$$K_I = \frac{P(c^2 - a^2)^{1/2}}{2(\pi a)^{1/2}(c + a)}$$

(Left crack tip)

(Right crack tip)

For K_{II} , replace P by Q in above relations

Figure 6-6 An Infinite Sheet
With Colinear Semi-Infinite Cracks with a Concentrated Force

(Right crack tip)

$$K_I = \frac{P_\infty}{(\pi a)^{1/2}} + \frac{2 M_\infty}{a^{3/2} \pi^{1/2}}$$

$$K_{II} = V_\infty / (\pi a)^{1/2}$$

Figure 6-7 An Infinite Sheet
With Load Transmitted Across a Neck Between Two Semi-Infinite Colinear Cracks

$$K_I = \frac{P_\infty \pi^{1/2}}{a^{1/2}} \left(\frac{4\pi - 12}{\pi^2 - 8} \right)$$

$$+ \frac{M_\infty \pi^{1/2}}{a^{3/2}} \left(\frac{4\pi - 8}{\pi^2 - 8} \right)$$

Figure 6-8 Semi-Infinite Notch
Approaching the Free Edge of a Half Plane

$$K_I = \sigma (\pi a \psi)^{1/2}$$

$$\text{where } \psi = \frac{2b}{\pi a} \tan\left(\frac{\pi a}{2b}\right)$$

$$K_{II} = K_{III} = 0$$

Figure 6-9 Periodic Array of Cracks
Along a Line in a Sheet, Uniformly Stressed at Infinity

$$K_I = \frac{P}{\left(2b \sin \frac{\pi a}{2b} \cos \frac{\pi a}{2b}\right)^{1/2}}$$

Figure 6-10 Periodic Array of Cracks
Along a Line in a Sheet Subjected to Concentrated Loads

$$K_I = \sigma \sin^2 \alpha (\pi a)^{1/2}$$

$$K_{II} = \sigma \sin \alpha \cos \alpha (\pi a)^{1/2}$$

Figure 6-11 A Crack in an Infinite Sheet
Subjected to Uniform Tension at an Arbitrary Inclination

$$K_I = \frac{P}{2(\pi a)^{1/2}} \alpha + \frac{Q}{2(\pi a)^{1/2}} \beta$$

$$K_{II} = \frac{-P\beta}{2(\pi a)^{1/2}} + \frac{Q\alpha}{2(\pi a)^{1/2}}$$

where $\alpha \equiv \left(\frac{a+b}{a-b} \right)^{1/2}$

$$\beta \equiv \left(\frac{k-1}{k+1} \right)^{1/2}$$

where $k = 3.4 \nu$ (for plane strain)

Figure 6-12 Concentrated Force on the Surface of a Crack in an Infinite Sheet

$$K_{I,II} = \frac{\sigma (\pi R)^{1/2}}{(1 + \sin^2 \frac{\alpha}{2})}$$

$$\left(\frac{\sin \alpha (1 \pm \cos \alpha)}{2} \right)^{1/2}$$

(use + for K_I , - for K_{II})

Figure 6-13 A Curved Crack in an Infinite Sheet Subjected to Biaxial Tension

$$K_I = \left[1 + 0.12 \left(1 - \frac{a}{b} \right) \right]$$

$$\frac{\sigma (\pi a)^{1/2}}{\phi} \left(\frac{2t}{\pi a} \tan \frac{\pi a}{2t} \right)^{1/2}$$

$$K_{II} = 0$$

$$K_{III} = \frac{\tau (\pi a)^{1/2}}{\phi} \left(\frac{2t}{\pi a} \tan \frac{\pi a}{2t} \right)^{1/2}$$

where ϕ is the complete elliptic integral of the second kind given in Table 1

Figure 6-14 Semi-Elliptical Surface Crack in a Plate Subjected to General Extension

For point on crack edge determined by angle β

$$K_I = \frac{\sigma (\pi a)^{1/2}}{\phi} \left(\sin^2 \beta + \frac{a^2}{b^2} \cos^2 \beta \right)^{1/4}$$

$$K_{II} = K_{III} = 0$$

ϕ = elliptic integral defined and tabulated in Table 1

Figure 6-15 Elliptical Crack in an Infinite Body Subjected to Uniform Tension

$$K_I = 2\sigma \left(\frac{a}{\pi} \right)^{1/2}$$

$$K_{II} = K_{III} = 0$$

Figure 6-16 Circular Crack in an Infinite Body Subject to Uniform Tension

$$K_I = K_{III} = 0$$

$$K_{II} = \tau (\pi a)^{1/2}$$

Figure 6-17 Infinite Cracked Sheet with Uniform In-Plane Shear

$$K_I = K_{II} = 0$$

$$K_{III} = \tau (\pi a)^{1/2}$$

Figure 6-18 Infinite Sheet with Tunnel Crack Subjected to Out-of-Plane Shear

Figure 6-19 Periodic Array of Cracks Along a Line in a Sheet, Uniform In-Plane Shear at Infinity

Figure 6-20 Periodic Array of Cracks Along a Line in a Sheet, Uniform Out-of-Plane Stress at Infinity

$$K_I = K_{II} = 0$$

$$K_{III} = \tau(\pi a)^{1/2}$$

Figure 6-21 Edge Crack in a Semi-Infinite Body Subjected to Shear

$$K_I = \sigma (L\pi)^{1/2} f\left(\frac{L}{r}\right)$$

$$K_{II} = 0$$

where $f\left(\frac{L}{r}\right)$ is obtained from Table 2.

Figure 6-22 A Crack (or cracks) from a Circular Hole in an Infinite Sheet Subjected to Uniform (or biaxial) Stress

$$K_I = \sigma (\pi a)^{1/2} f\left(\frac{a}{b}\right)$$

where $f\left(\frac{a}{b}\right)$ is obtained from Table 3.

Figure 6-23 Central Crack in a Strip of Finite Width Subjected to Uniform Tension

$$K_I = \sigma (\pi a)^{1/2} g(a/b)$$

$$K_{II} = K_{III} = 0$$

where $g(a/b)$ is obtained from Table 4.

Figure 6-24 Single-Edge Notch in Sheet of Finite Width Subjected to a Uniform Stress

$$K_I = \sigma (\pi a)^{1/2} \left[\frac{2b}{\pi a} \tan \frac{\pi a}{2b} \right]^{1/2} h\left(\frac{a}{b}\right)$$

$$K_{II} = K_{III} = 0$$

where $h(a/b)$ is obtained from Table 5.

Figure 6-25 Double-Symmetric Edge Cracks in a Finite Plate Subjected to Uniform Tension

$$K_I = \frac{2P}{(\pi a)^{1/2}} \alpha_2$$

where:

$$\alpha_2 = \left[1 + \left(\frac{a}{w} \right)^2 \left(12 \cosh^{-1} \left(\frac{w}{a} \right) + 2 \right) \right]^{1/2}$$

Figure 6-26 Single-Edge Notch in Sheet of Finite Width Subjected to an Edge Loading

$$K_I = \frac{6M}{d^{3/2}} G(a/h)$$

$$K_{II} = K_{III} = 0$$

where $G(a/h)$ is obtained from Table 6.

Figure 6-27 An Edge Crack in a Strip Subjected to In-Plane Bending

$$K_I = \sigma_{net} (\pi D)^{1/2} f(d/D)$$

$$K_{II} = K_{III} = 0 ; \sigma_{net} = \frac{4P}{\pi d^2}$$

where $f(d/D)$ is obtained from Table 7.

Figure 6-28 Round Bar in Tension with Circumferential Notch

$$K_I^2 = \frac{1.21 \pi P^2 a}{w^2 B^2} \left[\frac{1}{\phi^2 - 0.212 \left(\frac{P^2}{w^2 B^2 \sigma_{ys}^2} \right)} \right]$$

where σ_{ys} is the yield strength of the material and ϕ is the elliptic integral given in Table 1

Figure 6-29 Surface Cracked Plate Subjected to Uniform Tension

$$K_I = \frac{6 M a^{1/2}}{B w^2} Y$$

where $Y = A_0 + A_1 \left(\frac{a}{w} \right) + A_2 \left(\frac{a}{w} \right)^2 + A_3 \left(\frac{a}{w} \right)^3 + A_4 \left(\frac{a}{w} \right)^4$

	<u>A_0</u>	<u>A_1</u>	<u>A_2</u>	<u>A_3</u>	<u>A_4</u>
$2L/w = 8$	1.96	-2.75	13.66	-23.98	25.22
$2L/w = 4$	1.93	-3.07	14.53	-25.11	25.80

Figure 6-30 Single Edge-Cracked Plate Bend Specimen

Figure 6-31 Thin Wall Pressure Vessel with Wall Crack

Figure 6-32 Surface Cracked Plate

Table 6-1 Values of the Complete Elliptic Integral ϕ^2 for a/b Ranging from 0.00 to 1.00

$\frac{a}{b}$	ϕ^2	$\frac{a}{b}$	ϕ^2	$\frac{a}{b}$	ϕ^2
0.00	1.00	0.53	1.51	0.81	2.02
0.06	1.02	0.54	1.53	0.82	2.04
0.12	1.04	0.56	1.55	0.83	2.06
0.16	1.06	0.57	1.57	0.84	2.08
0.18	1.08	0.58	1.59	0.85	2.10
0.20	1.10	0.59	1.61	0.86	2.13
0.23	1.12	0.60	1.63	0.87	2.16
0.25	1.14	0.61	1.65	0.88	2.18
0.27	1.16	0.62	1.67	0.89	2.20
0.29	1.19	0.64	1.69	0.90	2.22
0.31	1.21	0.65	1.71	0.91	2.24
0.32	1.23	0.66	1.73	0.92	2.26
0.34	1.25	0.67	1.75	0.93	2.29
0.36	1.27	0.68	1.77	0.94	2.32
0.38	1.29	0.69	1.79	0.95	2.34
0.39	1.31	0.70	1.81	0.96	2.36
0.41	1.33	0.71	1.83	0.97	2.38
0.42	1.35	0.72	1.85	0.98	2.41
0.44	1.37	0.73	1.87	0.99	2.44
0.45	1.39	0.74	1.90	1.00	2.46
0.46	1.41	0.76	1.92		
0.48	1.43	0.77	1.94		
0.49	1.45	0.78	1.96		
0.50	1.47	0.79	1.98		
0.52	1.49	0.80	2.00		

Where:

$$\phi = \int_0^{\pi/2} (1 - k^2 \sin^2 \theta)^{1/2} d\theta$$

$$\text{and } k^2 = 1 - \left(\frac{a}{b}\right)^2$$

Table 6-2 Values of $f(L/R)$ for Cracks from a Hole in an Infinite Sheet, Figure 6-22

L/R	— ONE CRACK —		— TWO CRACKS —	
	$f(L/R)$		$f(L/R)$	
	<u>Uniaxial Stress</u>	<u>Biaxial Stress</u>	<u>Uniaxial Stress</u>	<u>Biaxial Stress</u>
0	3.39	2.26	3.39	2.26
0.1	2.73	1.98	2.73	1.98
0.2	2.30	1.82	2.41	1.83
0.3	2.04	1.67	2.15	1.70
0.4	1.86	1.58	1.96	1.61
0.5	1.73	1.49	1.83	1.57
0.6	1.64	1.42	1.71	1.52
0.8	1.47	1.32	1.58	1.43
1.0	1.37	1.22	1.45	1.38
1.5	1.18	1.06	1.29	1.26
2.0	1.06	1.01	1.21	1.20
3.0	0.94	0.93	1.14	1.13
5.0	0.81	0.81	1.07	1.06
10.0	0.75	0.75	1.03	1.03
∞	0.707	0.707	1.00	1.00

Table 6-3 Values of $f(a/b)$ for a Central Crack
 in Strip of Finite Width, Figure 6-23

$\frac{a}{b}$	$f(a/b)$
0.074	1.00
0.207	1.03
0.275	1.05
0.337	1.09
0.410	1.13
0.466	1.18
0.535	1.25
0.592	1.33

Table 6-4 Values of $g(a/b)$ for a Single Edge Notch
 in a Finite Width Sheet, Figure 6-24

$\frac{a}{b}$	$g(a/b)$
0	1.12
0.1	1.15
0.2	1.20
0.3	1.29
0.4	1.37
0.5	1.51
0.6	1.67
0.7	1.88
0.8	2.13
0.9	2.45
1.0	2.84

Table 6-5 Values of $h(a/b)$ for a Double-Edge Notch in a Finite Width Plate, Figure 6-25

a/b	$h(a/b)$		
	$L/b=1.00$	$L/b=3.00$	$L/b=\infty$
0.1	1.13	1.12	1.12
0.2	1.13	1.11	1.10
0.3	1.14	1.09	1.09
0.4	1.16	1.06	1.06
0.5	1.14	1.02	1.02
0.6	1.10	1.01	1.01
0.7	1.02	1.00	1.00
0.8	1.01	1.00	1.00
0.9	1.00	1.00	1.00

Table 6-6 Values of $G(a/h)$ for a Notched Beam in Bending, Figure 6-27

a/h	$G(a/h)$
0.05	0.36
0.1	0.49
0.2	0.60
0.3	0.66
0.4	0.69
0.5	0.72
0.6	0.73
> 0.6	0.73

Table 6-7 Values of $f(d/D)$ for a Round Bar
With a Circumferential Crack, Figure 6-28

$\frac{d}{D}$	$f(\frac{d}{D})$	$\frac{d}{D}$	$f(\frac{d}{D})$
0	0	0.70	0.240
0.1	0.111	0.75	0.237
0.2	0.155	0.80	0.233
0.3	0.185	0.85	0.225
0.4	0.209	0.90	0.205
0.5	0.227	0.95	0.162
0.6	0.238	0.97	0.130
0.65	0.240	1.00	0

7. STATISTICAL CONSIDERATIONS AND RELIABILITY

The K_{Ic} value determined from experimental tests will be statistically distributed, much the same as any other material property. The interaction between a structural strength function and a load function may be visualized as shown in Figure 7-1. This pictorialization remains meaningful regardless of the strength criterion chosen or the types of distributions involved. The diagram is usually plotted with the abscissa scale in terms of stress but may just as well be expressed in terms of stress intensity ($\text{psi} \cdot \sqrt{\text{in.}}$). The material strength is then represented by the fracture toughness and the load by the linear elastic stress intensity. If the distributions are normal the reliability can be determined from the standard stress-strength interference theory (see DFM: 1325, November 11, 1968, "Structural Reliability Analysis Techniques and Reflector Example", by G. J. DeSalvo). In order to obtain the allowable stresses, the stress intensity must be converted to stress through the use of an appropriate fracture mechanics model and crack size. It is also possible to first convert the fracture toughness of the material to a stress distribution and then combine this with the load stress distribution in order to find the reliability and allowable stresses. Since fracture mechanics is based on linear theory, either method should give the same results.

Figure 7-1 Stress and Strength Distributions for Reliability Calculations

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