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THE ONSET OF SUPERFLUID FLOW IN THIN HELIUM FILMS

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Abstract

We discuss the onset of superfluid flow in thin He II films on the basis of a recent theoretical model, and show that it is in good agreement with experiment.

In recent years a number of experiments have been carried out on superfluid flow in thin films of He^4 adsorbed on a variety of substrates and a great deal of attention focused on the behaviour of the system near the onset temperature. There are two main types of experiment: (i) The static (or more accurately quasi-static) type in which we class mass flow[1-3], heat transport[4-7] and third sound[8-10]. (ii) The quartz crystal microbalance experiment[11-12] in which the change in resonant frequency of a crystal is studied when a film of He^4 is adsorbed on the surface. Such an experiment is carried out at frequencies around 25MHz and gives zero superfluid density at onset.

All of the first type agree with one another in that the onset of superfluid flow takes place at a finite superfluid density and there is general agreement that, at onset $d/T = \text{constant}$ where d is the thickness of the liquid in the film. It is believed that the first layer is so strongly bound to the substrate that it is effectively solid and plays no role in the superfluidity of the film. Certain experiments[2,8,9,10] also give a measure of the mean superfluid density by measuring either the velocity of third sound[8-10] or the angular momentum of a persistent current[2] and these give the result that, at onset,

$$\frac{\langle\rho_s\rangle d}{T} = \text{constant} \qquad (1)$$

where $\langle\rho_s\rangle$ is the mean superfluid density in the liquid.

There are some uncertainties in the interpretation of the data[13]. The thickness of the film is not completely known. The adsorption isotherm is

$$\log {}^{P_o}/P = {}^{\Gamma}/Td^3$$

where Γ is a constant depending on the substrate and there appears to be some uncertainties in the values of Γ. Recent experiments[7] give $\Gamma = 27$ for glass and $\Gamma = 23$ for CaF_2. However, an error in Γ will affect only the numerical value of the constant and not the form of eq.(2). The expression for the velocity of third sound assumes that the liquid fraction in the film has the same properties as in the bulk. Another unknown quantity is the effect of heterogeneities of the substrate which may affect the effective superfluid density[3].

The most extensive set of experiments on the onset of superfluid flow where the superfluid density is also known are the third sound experiments[8,10] for film thicknesses ranging from 2·1 to 10 atomic layers, and taking $\Gamma = 27$ for a glass substrate they give

$$\frac{\langle\rho_s\rangle d}{T} = 15 \pm 2 \times 10^{21}\ \mathrm{cm}^{-3}\ \mathrm{K}^{-1} \qquad (2)$$

where d is expressed in atomic layers. There seems to be a tendency that the thinner the film the larger the value of $\frac{\langle\rho_s\rangle d}{T}$ but the errors are too large to be definite. It is possible that, since the onset thickness is taken to be the point where all third sound signals disappear and there is a very high attenuation near onset, this tendency is due to the difficulty of detecting very weak signals. One expects that, the thinner the film, the weaker the signal so that the thickness at which the signal disappears is somewhat larger than the true critical thickness. This effect would be more important in very thin films.

Although we cannot make any quantitative statements about the second type of experiment we would like to comment on the available data on the basis of a recently proposed theoretical model[14,15]. The non-existence of superfluid flow above the onset temperature is due, in this model, not to an abrupt drop of the superfluid density to zero but to an instability caused by the appearance of isolated line vortices produced by thermal excitation. That the superfluid density is continuous at onset is indicated by the fact that the specific heat is found, both experimentally[16] and theoretically[15,17], to be perfectly smooth at onset. The isolated vortices allow transitions between one metastable flow state and another which, below the onset temperature, are inaccessible to one another because of an energy barrier which diverges logarithmically with the size of the system[15,18]. The critical velocity above onset is thus zero which will lead to the disappearance of all transport phonomena such as mass flow, heat transport and third sound propagation.

Although this is not a new idea[18], previous authors have omitted to take into account the entropy of the vortex. The energy of an isolated vortex in a system of radius R is

$$E \doteq \frac{\pi\hbar^2\rho}{m} \log {}^{R}/a \qquad (3)$$

and its entropy

$$S = 2k \log {}^{R}/a + O(1) \qquad (4)$$

where a is the radius of the vortex core, ρ is the superfluid number density per unit area and m is the effective mass of a helium atom. The free energy is thus

$$F = \left(\frac{\pi\hbar^2\rho}{m} - 2kT\right) \log {}^{R}/a \qquad (5)$$

At sufficiently low temperatures, the energy dominates so that isolated vortices cannot occur while at high temperatures, the entropy takes over. The onset temperature T_o is then

$$kT_o = \frac{\pi\hbar^2\rho}{2m} \qquad (6)$$

Taking the interatomic separation b to be 3·6 Å and m to be the mass of a helium atom we obtain at onset

$$\frac{\langle\rho_s\rangle d}{T} = \frac{\pi\hbar^2}{2mkb} = 14{\cdot}7 \times 10^{21}\ \mathrm{cm}^{-3}\mathrm{K}^{-1} \qquad (7)$$

which is in remarkable agreement with experiment. The theory is not capable of making any more detailed predictions about the values of $\langle\rho_s\rangle$ and T_o independently of one another.

However, as discussed in a previous paper(15) this number is a lower limit and a more sophisticated theory(15,17) which takes into account the fact that there are a number of interacting vortices in the system gives

$$\frac{\pi\hbar^2\rho}{2mkT_o} - 1 = 2\pi \exp\left\{-{}^{\mu}/kT_o\right\} \qquad (8)$$

where μ is proportional to the energy of the vortex core. From naive arguments one expects that $\mu \propto d\langle\rho_s\rangle$ so that the form of eq.(7) is not changed but the constant is increased.

By making a series of unjustified assumptions about the radius and nature of the core one can estimate that the constant in eq.(7) is increased by 10%-20%, which worsens the agreement with experiment, but such estimates are not to be taken seriously.

The main qualitative feature of the quartz crystal microbalance experiment[11,12] is that the apparent superfluid density tends smoothly to zero at onset. Although quantitative comparison with theory is not possible as the dynamics are unknown, we can construct an argument which indicates that, for such an experiment, such behaviour is to be expected. Within the context of the vortex instability model, the superfluid density does not vanish, but the superfluid flow becomes unstable. Below the onset temperature, the superfluid velocity decays slowly[18] as $(\log t)^{-1}$ because of thermal excitation of bound vortex pairs[15,18], but above it decays rapidly[18], probably as

$$v_s \sim \exp\left\{-{}^{t}/\tau(T)\right\} \tag{9}$$

where $\tau(T) \to \infty$ as $T \to T_o^+$. In such a case we can expect remnants of a superfluid response at temperatures such that $\omega\,\tau > 1$ where ω is the frequency of the experiment. In such a case, low frequency phonomena propagating over long distances (mass flow, heat transport, and third sound) will give a good measure of T_o. The quartz microbalance experiment does not involve propagation since the system is driven at its resonant frequency so that one expects a superfluid response at temperatures above T_o which gradually decreases as the temperature is raised. Thus one expects that the effective $\langle\rho_s\rangle$ will tend to zero at some $T > T_0$. From such considerations we feel that this model may provide an explanation of the behaviour of thin He^4 films, but a dynamical theory and further experiments at different frequencies would be most welcome.

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