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DUAL-AMPLITUDE ANALYSIS OF INCLUSIVE DISTRIBUTIONS*

Kyungsik Kang

Department of Physics

Brown University

Providence, Rhode Island 02912 U.S.A.

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I would like to present a brief summary of the dual-amplitude analyses¹⁻⁵ of the inclusive distributions at high energies! Study of the inclusive processes has been a very popular area in the last few years; experimentally, the single-particle inclusive cross section is only the next level of sophistication above a total cross-section measurement, whereas theoretically, the ingenious hypothesis of Mueller⁶ has enhanced the potential role of the Regge-pole phenomenology in the multiparticle production processes.

As it is well-known by now, the Mueller hypothesis is to relate the inclusive cross-section for $a + b \rightarrow x + \text{anything}$ to a discontinuity⁷ in the missing mass variable $M^2 = (p_a + p_b - p_x)^2$ of the six-point amplitude for $a + b + \bar{x} \rightarrow a + b + \bar{x}$. Furthermore, it can be shown from this hypothesis that a multi-Regge assumption for such discontinuity leads to the hypothesis of limiting distribution or scaling⁸ and ensures a certain factorization properties of the inclusive cross-sections. While this idea can be extended in a straightforward manner to the n -particle inclusive processes, it has been responsible for starting a growing and important phenomenological development of the many-particle production states.

However, it should be noted that factorization of the inclusive cross-sections is not necessarily a consequence of the Regge assumption for the full multiparticle scattering amplitudes. In order to have a certain factorization for the inclusive cross-sections as the Mueller hypothesis suggests, the full multiparticle amplitudes must possess special singularity structure besides being Regge behaved. But we know of no other representations than the dual resonance model⁹ for the multiparticle scattering amplitudes that contain any specific analyticity and multi-Regge behavior: thus the dual amplitude analysis of inclusive cross-sections can not only shed some light

on the connection between the multi-Regge assumption for the full amplitudes and the Mueller hypothesis but also provide an explicit description on the shape of the limiting distribution.

In fact, one can indeed verify the factorization properties of the inclusive distribution from the dual-amplitude analysis in accordance with the Mueller hypothesis (provided, of course, singularities are handled with care in taking the appropriate discontinuity). Moreover, the inclusive cross-sections show the desired limiting properties for all kinematic regions. This leads us to believe that the singularity structure implied by the dual n -point amplitude at high energies is of general validity. As we will see later, the dual-amplitude analysis predicts, among others, the experimentally observed cutoff in transverse momenta, vanishing of the triple Regge vertex for the integral values of the trajectory intercepts, and the nature of the short-range correlation.

A. Single-Particle Distribution¹⁰

The inclusive differential cross-section that one is concerned with is, as explained in Fig. 1, the sum over all possible missing mass channels. The standard dual resonance model is used for the six-point amplitude in the right-hand side of Fig. 1. The distribution function is defined as the inclusive differential cross-section divided by the total cross-section, i.e.,

$$f_{ab}(s_{ab}, M^2, s_{ax}^-, s_{bx}^-) = \frac{2E_x}{\sigma_{ab}^{tot}} \frac{d\sigma_{ab}}{d^3p} \quad (1)$$

where the total cross-section is given from the four-point amplitude by

$$\sigma_{ab}^{tot} = \frac{\pi}{\Gamma(1 + \alpha_v)} (\alpha_{ab})^{\alpha_v - 1}$$

α_v being $\alpha_{aa}^-(t)$, the leading vacuum trajectory in the t-channel and $\alpha_{ab} = \alpha(s_{ab})$. In general, we consider $s_{ab}, M^2 \rightarrow \infty$ limits for the limiting distribution.

Fragmentation:

In either fragmentation region for a and b, the distribution function turns out to be a function of desired variables, i.e., $p_{||}$ and $\vec{p}_\perp$ in the laboratory system or $x = 2p_{||}/\sqrt{s_{ab}}$ and $\vec{p}_\perp$ in the c.m. system of a and b;

$$f_b(x, \kappa = p_\perp^2 + m_x^2) = \iint_0^1 du dv (uv)^{\frac{\kappa}{2} - 1} [(1-u)(1-v)]^{\alpha_v - \alpha_{aax}^- - 1} \quad (2)$$

$$(1-uv)^{\alpha_{aax}^- + \alpha_{aax}^- - 2\alpha_v} \left(1-x \frac{1-uv}{(1-u)(1-v)}\right)^{\alpha_v} \theta\left(1-x \frac{1-uv}{(1-u)(1-v)}\right)$$

Triple Regge Limit:

The triple Regge region is just the phase-space boundary reached from a fragmentation region. For example, from the fragmentation region of b, the phase-space boundary can be reached by taking $s_{ab}/M^2 \sim -\frac{s_{ax}^-}{M^2} \rightarrow \infty$ in addition. The distribution function is then given by

$$f_b(p_{||}, \vec{p}_\perp) = \left(\frac{M^2}{s_{ab}}\right)^{\alpha_v} \left(\frac{s_{ab}}{M^2}\right)^{\alpha_{bx}^-} \left(\frac{s_{ab}^-}{M^2}\right)^{\alpha_{bx}^-} h_b(p_{||}, \vec{p}_\perp) \quad (3)$$

where

$$h_b = \int_0^1 \int_0^1 du dv \theta(1-u-v) u^{-\alpha_{bx}^- - 1} v^{-\alpha_{bx}^- - 1} (1-u-v)^{\alpha_v} \quad (4)$$

$$= \frac{\Gamma(-\alpha_{bx}^-) \Gamma(-\alpha_{bx}^-) \Gamma(1 + \alpha_v)}{\Gamma(1 + \alpha_v - \alpha_{bx}^- - \alpha_{bx}^-)}$$

From (4) one gets the triple Regge vertex function to be

$$g(\alpha_{bx}^-, \alpha_{bx}^-; \alpha_v) = \frac{\pi \Gamma(\alpha_v + 1)}{\{\Gamma(1 + \alpha_{bx}^-) \Gamma(1 + \alpha_{bx}^-)\}^{\frac{1}{2}} \Gamma(1 + \alpha_v - \alpha_{bx}^- - \alpha_{bx}^-)} \quad (5)$$

Note that the triple Regge coupling vanishes for $\alpha_v = \alpha_{b\bar{x}} = \alpha_{\bar{b}x} = 1$ and for all integer values of $\alpha_{b\bar{x}}$ and $\alpha_{\bar{b}x}$ when $\alpha_v = 1$.

Reggeon-particle amplitude:

The triple Regge region relation (3) can also be obtained directly from the six-point dual amplitude B_6 by taking s_{ax}^- , $s_{ax}^- \rightarrow -\infty$ limits. This leads to the concept of the Reggeon-particle amplitude,³ i.e.,

$$B_6 = \frac{\beta_{b\bar{x}} (-\alpha_{ax}^-)^{\alpha_{b\bar{x}}}}{\sin \pi \alpha_{b\bar{x}}} \frac{\beta_{\bar{b}x} (-\alpha_{ax}^-)^{\alpha_{\bar{b}x}}}{\sin \pi \alpha_{\bar{b}x}} R \quad (6)$$

where R denotes the Reggeon-particle amplitude¹¹

$$R = \beta_{b\bar{x}} \beta_{\bar{b}x} \int_0^1 du u^{-\alpha-1} (1-u)^{-\alpha_v-1 + \alpha_{b\bar{x}} + \alpha_{\bar{b}x}} \quad (7)$$

with $\alpha = \alpha(M^2)$ and $\beta_{ij} = (\pi/\Gamma(1 + \alpha_{ij}))^{1/2}$. Then after taking $\alpha \rightarrow -\infty$ limit in (6), and continuing the result to the positive α -values, one can arrive at the triple-Regge-region distribution (3). From (7), we see that the Reggeon-particle total cross-section takes in high energies the expression

$$\sigma_{Rp}^{tot} = \pi \beta_{b\bar{x}} \beta_{\bar{b}x} \frac{(\alpha)^{\alpha_v - \alpha_{b\bar{x}} - \alpha_{\bar{b}x} - 1}}{\Gamma(1 + \alpha_v - \alpha_{b\bar{x}} - \alpha_{\bar{b}x})} \quad (8)$$

The triple-Regge vertex function (5) follows also from (8).

Pionization:

The result for pionization has received good attention primarily because it predicts such phenomenologically observed facts as the transverse momentum cutoff. The distribution function depends only on the scaled variable $\kappa = \alpha_{ax}^- \alpha_{b\bar{x}}^- / \alpha = p_{\perp}^2 + m_x^2$,

$$f(\kappa) = \frac{\Gamma(1 + \alpha_v)}{\pi} \kappa^{\alpha_v} \int_0^1 du u^{-\alpha_{ax}-1} (1-u)^{-\alpha_{ax}-1} \text{Disc}_\lambda V(\alpha_v, \alpha_v; \lambda = \frac{\kappa}{u(1-u)}) \quad (9)$$

where $V(\alpha_1, \alpha_2; \lambda)$ is the usual two-Reggeon-one-particle vertex function,¹²

$$V(\alpha_1, \alpha_2; \lambda) = \int_0^\infty \int_0^\infty dv dw v^{-\alpha_1-1} w^{-\alpha_2-1} \exp \left(-v-w + \frac{vw}{\lambda} \right) \quad (10)$$

While the distribution function shows a universal cutoff in transverse momentum, i.e.,

$$\lim_{\kappa \rightarrow \infty} f(\kappa) \sim \sqrt{\pi} \Gamma(1 + \alpha_v) 2^{\alpha_{ax} + \alpha_{ax} - 4\alpha_v - 2} (\kappa)^{-\alpha_v - \frac{3}{2}} e^{-4\kappa}, \quad (11)$$

it gives the average transverse momentum to be about 300MeV for $\alpha_v(0) = 0.5$ and $m_x^2 = 0.02 \text{ GeV}^2$.

Average multiplicity:

The average multiplicity defined by

$$\langle n \rangle = \int_{-1}^{+1} dx \int_0^\infty dp^2 f_{ab}(x, \vec{p}) / 2(x^2 + 4\kappa/s)^{\frac{1}{2}} \quad (12)$$

shows logarithmic growth $c \ln s_{ab} + b$. Numerically c ranges around 0.3 for $m_x^2 = 0.02 \text{ GeV}^2$ and $\alpha_v = 0.5$. This is to be compared with the experimental value $c \sim 0.4$ for the charged pion.¹³

B. Two-Particle Distribution

The generalization of the distribution function for the two-particle production is straightforward. We calculate the distribution function

$$f_{ab}^{12} = \frac{4E_1 E_2}{\sigma_{ab}^{tot}} \frac{d\sigma_{ab}}{d^3 p_1 d^3 p_2} \quad (13)$$

from the eight-point dual amplitude. In identifying the inclusive two-particle differential cross-section from the discontinuity in M^2 of the amplitude, one is required to make an analytic continuation in α . Particular care needs to be given in handling singularities. The right procedure is extensively discussed in Ref. 5.

Fragmentation:

The distribution function shows the scaling behavior at high energies, i.e., it depends only on the momenta of the two particles $\vec{p}_1$ and $\vec{p}_2$. In the double fragmentation region where, say, the particle 1 is the fragment of a, while the particle 2 is that of b, the distribution function shows the right factorization and is given by the product of the two single-particle distribution functions. Here, the kinematic relation

$$s_{a\bar{1}\bar{2}} s_{b\bar{1}\bar{2}} / M^2 = s_{\bar{1}\bar{2}} \quad (14)$$

plays an important role to achieve such nontrivial factorization, i.e.,

$\text{Disc}_{M^2} B_8 = (\text{Disc}_{M_1^2} B_6)(\text{Disc}_{M_2^2} B_6)$, M_1^2, M_2^2 being the missing mass variables corresponding to $a\bar{1}\bar{2}$ and $b\bar{1}\bar{2}$ respectively.¹⁴

Di-triple Regge Limit:

This region is arrived by considering s_{ab}/M^2 to be large for fixed values of $s_{a\bar{1}}$ and $s_{b\bar{2}}$. The distribution function is given by a product of

two triple-Regge expressions. Here, the Feynman variables x_1 and x_2 are close to 1 and -1 respectively and

$$f_{ab}^{12} \sim (1-x_1)^{\alpha_v - \alpha_{a1} - \alpha_{a\bar{1}}} (1+x_2)^{\alpha_v - \alpha_{b2} - \alpha_{b\bar{2}}} h_a(x_1, \vec{p}_{1\perp}) h_b(x_2, \vec{p}_{2\perp}) \quad (15)$$

Reggeon-Reggeon Amplitude:

An alternative way of obtaining the distribution function in the di-triple Reggeon region similar to that in the triple-Regge region results an expression for the Reggeon-Reggeon scattering amplitude. According to this representation, the total Reggeon-Reggeon cross section at high energies take the form

$$\sigma_{RR}^{tot} = \frac{\pi}{\Gamma(1 + \alpha_v)} g(\alpha_{a\bar{1}}, \alpha_{a1}; \alpha_{a\bar{1}\bar{1}}) g(\alpha_{b\bar{2}}, \alpha_{b2}; \alpha_{b\bar{2}\bar{2}}) \alpha_v^{\alpha_v - \Sigma - 1} \quad (16)$$

where $\Sigma = \alpha_{a\bar{1}} + \alpha_{a1} + \alpha_{b\bar{2}} + \alpha_{b2}$.

Pionization:

Again, we find that the distribution function shows the exponential cutoff in both transverse momenta. For example, in the case of finite invariant mass s_{12} , the two particles are correlated and behave like a single particle of effective mass $\sqrt{s_{12}}$

$$f^{12} \sim f(\kappa_{12}) (2\kappa_1 \kappa_2)^{\frac{\alpha_{12}}{2} + \frac{\alpha_{\bar{1}\bar{2}}}{2}} \left\{ \Gamma\left(-\frac{\alpha_{\bar{1}\bar{2}}}{2}\right) + \text{cr}\left(-\frac{\alpha_{\bar{1}\bar{2}}-1}{2}\right) \right\} \quad (17)$$

$$\left\{ \Gamma\left(-\frac{\alpha_{12}}{2}\right) + \text{cr}\left(-\frac{\alpha_{12}-1}{2}\right) \right\}$$

where $\kappa_1 = s_{a\bar{1}} s_{b\bar{1}\bar{2}}/M^2 \sim m_1^2 + p_{1\perp}^2$, $\kappa_2 = s_{a\bar{1}\bar{2}} s_{b\bar{2}}/M^2 \sim m_2^2 + p_{2\perp}^2$,

$\kappa_{12} = s_{a\bar{1}\bar{2}} s_{b\bar{1}\bar{2}}/M^2 \sim s_{12} + (\vec{p}_{1\perp} + \vec{p}_{2\perp})^2$ and $f(\kappa_{12})$ is given by (9).

In passing we note from (17) that the probability of producing a high-mass particle for fixed transverse momentum decreases with the mass exponentially.

In the case of finite value of the relative rapidity y defined by

$$s_{12} = m_1^2 + m_2^2 + 2(m_1^2 + p_{1\perp}^2)^{\frac{1}{2}} (m_2^2 + p_{2\perp}^2)^{\frac{1}{2}} \cosh y - 2p_{1\perp} p_{2\perp} \cos\phi \quad (18)$$

with arbitrary azimuthal angle ϕ defined to be ($0 \leq \phi \leq \pi$), the transverse momentum cutoff is of the form

$$f^{12} \sim e^{-4(p_{1\perp}^2 + p_{2\perp}^2 + 2p_{1\perp} p_{2\perp} \cos\beta)} \quad (19)$$

where $\cos\beta$ depends on y and ϕ , and has the property $\cos(y, \phi) \geq \cos\beta(y, \pi)$. Note thus that the two particles are likely to move opposite to each in the transverse momentum plane as long as y is finite.

Two-particle correlation function:

The pionization region has the obvious advantage to study the short-range correlation effect.¹⁵ By defining the correlation function by

$$C(y, p_{1\perp}, p_{2\perp}, \cos\phi) = f^{12}(y, p_{1\perp}, p_{2\perp}, \cos\phi) / f(\kappa_1) f(\kappa_2) - 1 \quad (20)$$

we find to the e^{-y} order

$$C(y, p_{1\perp}, p_{2\perp}, \cos\phi) \simeq -\frac{32}{\alpha_v} (p_{1\perp} p_{2\perp})^2 \left(\frac{p_{1\perp}^2 + p_{2\perp}^2}{2p_{1\perp} p_{2\perp}} + \cos\phi \right) e^{-y} \quad (21)$$

so that the correlation length in y is of the order 1. This can be understood as coming from the contributions of the leading vacuum trajectory and its first daughter, i.e., $\exp[(\alpha_v - 1) - \alpha_v]y$. However, if there exists an additional vacuum trajectory α_v , whose intercept lies between those of α_v and $\alpha_v - 1$, then the correlation length would be $(\alpha_v - \alpha_v)^{-1}$ instead.

However, as far as the correlation in $\cos\phi$ is concerned, (21) is the minimal form that has to exist in two-particle production as α_v , would factorize. Of course, as $y \rightarrow \infty$, $C(y, p_{1\perp}, p_{2\perp}, \cos\phi) \rightarrow 0$ so that $f^{12} \rightarrow f(\kappa_1) f(\kappa_2)$ and we achieve the uncorrelated pionization of two-particle production.

C. Epilogue

The material that I have presented here represents only a small part of the massive calculations. The mathematical details as well as the kinematics for the single-particle and the two-particle distributions can be found in Ref. 1 and Ref. 5 respectively. I have not described the case of the fixed missing mass where the standard two-body Regge phenomenology would apply. It is found that¹ the assumptions of duality, two-body Regge behavior and existence of a limiting distribution imply the triple-Regge behavior. Another topic that³ I have not discussed here is the phenomenology based on the finite energy sum rules for the Reggeon-particle and Reggeon-Reggeon amplitudes. In the former case, one gets relations between the usual Regge residues and the triple Regge vertices while for the latter one reduces to the relationship between the two-Reggeon-one-particle couplings and the triple Regge vertices. Another interesting problem is to study the inclusive cross sections for the high-spin particles.⁵ The dual-resonance-amplitudes approach has a simple way of obtaining such distributions. For example, the residues at the pole of $\alpha_{\overline{12}} = \ell$ in the single-fragmentation and the correlated pionization regions give the spin- ℓ -particle distribution functions for fragmentation and pionization respectively. For the case of $\ell = 1$ and $m_x^2 = 0.59(\text{GeV})^2$, the multiplicity grows like $c \ln s_{ab} + d$ again but c is around 0.03 for $\alpha_v = 0.5$. The correlation effects between the rapidities of the two produced particles and between the relative rapidity and the azimuthal angle or the two transverse momenta are not described either.

To sum up, the dual-amplitude analyses have helped us to gain some insight into the inclusive distributions. We have seen that the universal cutoff in transverse momenta, the logarithmic increase of the multiplicity,

the structure of the triple Regge vertex and the nature of short-range correlations in rapidity and azimuthal angle are successfully predicted. Thus I believe that the dual amplitudes have perhaps the generally valid singularity structure for the multiparticle dynamics and therefore one can continue to rely on them to guide the future multiparticle phenomenology.

References and Footnotes

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11. Finite energy sum rule for this amplitude can be constructed as outlined in Ref. 3.
12. K. Bardakci and H. Ruegg, Phys. Rev. 181, 1884 (1969).
13. N. F. Bali et al., Phys. Rev. Letters 24, 557 (1970); M.-S. Chen et al., Phys. Rev. Letters 26, 280 (1971). A crude estimate for $m_x^2 = 0$ gives

$$C = \frac{\pi}{2} \frac{(\Gamma(\alpha_v + 1))^2}{(\alpha_v + 1) \Gamma(2\alpha_v + 1)}$$

14. B. Hasslacher et al., Phys. Rev. D4, 3089 (1971) have considered the two-particle distributions in the dual resonance model for the uncorrelated pionization only. But their interpretation of the factorization in the discontinuity, for example, their Eq. (15), does not agree with our results in References 2, 4, and 5. This is due to their wrong handling of kinematic relations in their Eq. (2); they use the relations $s_{a1} s_{12}/s_{a12} = \kappa_1 = p_{11}^2 + M^2$, $s_{b2} s_{21}/s_{b21} = \kappa_2 = p_{21}^2 + M^2$ throughout the paper. Of course, the correct relations should be $\kappa_1 = p_{11}^2 + m_1^2$ and $\kappa_2 = p_{21}^2 + m_2^2$.
15. See Ref. 4 for more details.

$$\begin{aligned}
 S_{ab} \frac{d\sigma_{ab}^x}{d^3p/E_x} &\sim \sum_i \left| \begin{array}{c} a \\ b \end{array} \rightarrow \text{Disc}_{M^2} \rightarrow \begin{array}{c} \bar{x} \end{array} \right|_{M^2, i}^2 \\
 &= \sum_i \left| \begin{array}{c} a \\ b \end{array} \rightarrow \text{Disc}_{M^2} \rightarrow \begin{array}{c} \bar{a} \\ \bar{b} \end{array} \right|_{M^2, i}^2 \\
 &= \text{Disc}_{M^2} \left(\begin{array}{c} a \\ \bar{x} \\ b \end{array} \right) \\
 S_{ab} E_{x_1} E_{x_2} \frac{d\sigma_{ab}^{x_1 x_2}}{d^3p_{x_1} d^3p_{x_2}} &\sim \sum_i \left| \begin{array}{c} \bar{x}_1 \bar{x}_2 \\ a \\ b \end{array} \rightarrow \text{Disc}_{M^2} \rightarrow \begin{array}{c} \bar{x} \end{array} \right|_{M^2, i}^2 \\
 &= \sum_i \left| \begin{array}{c} \bar{a} \\ \bar{b} \end{array} \rightarrow \text{Disc}_{M^2} \rightarrow \begin{array}{c} \bar{x}_1 \bar{x}_2 \end{array} \right|_{M^2, i}^2 \\
 &= \text{Disc}_{M^2} \left(\begin{array}{c} \bar{a} \\ \bar{b} \end{array} \right) = \text{Disc}_{M^2} \left(\begin{array}{c} \bar{a} \ x_1 \ x_2 \ \bar{b} \\ a \ \bar{x}_1 \ \bar{x}_2 \ b \end{array} \right)
 \end{aligned}$$

FIG. 1. The generalized optical theorem to relate the inclusive cross sections with disc_{M^2} .