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PHOTOELASTIC STUDY OF STRESS IN PERFORATED HEMISPHERICAL
CLOSURES FOR A PRESSURE VESSEL

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Summary

Photoelastic "frozen stress" tests were made of perforated hemispherical closures for a pressure vessel. Three different hole patterns were investigated. The "circular" and "square" arrangements of holes showed the highest peak stress, a value of stress $\div$ pressure of about 10 being noted in each. In comparison, the "triangular" arrangement of holes yielded a peak value of 8.20.

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Introduction

Little data is presently available to aid the designer in the stress analysis of perforated hemispherical heads for pressure vessels. In a letter to persons interested in obtaining such data (WAPD-CTA (CE)-47) Mr. B. F. Langer, Consulting Engineer at Bettis Site, outlined a series of tests aimed at determining the effect of variations in hole pattern, wall thickness and ligament efficiency. The three tests reported here include variations of hole pattern only: (1) a "circular" pattern with thirteen holes; (2) a "triangular" pattern of nineteen holes; and (3) a "square" pattern of twenty-one holes.

The Photoelastic Models

Figure 1 shows the cross-section of the pressure vessel model with a flanged hemispherical head. The vessel and closure were each machined from solid cylindrical castings of an epoxy photoelastic resin, then joined together by cementing with a quick-setting epoxy. Forty-two steel bolts acting through a like number of steel springs served to provide the required clamping force between the vessel and the head. Typical perforations are shown

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in the head, Figure 1. All the holes were 1.025 in diameter and were sealed off by caps cemented to the hole wall. As shown, the cemented portion of the cap was skewed wherever necessary to bring it roughly into line with the head wall. The purpose for doing this was to insure that forces exerted by the hole caps acted as nearly as possible at the mid-surface of the closure.

Figure 2 is a photograph of the assembled photoelastic model

Figure 2. Photograph of head with circular pattern of holes, bolted to pressure vessel.

incorporating the closure with a circular pattern of holes. When the test on that head had been performed, it was cut from the vessel just above the

cemented joint. The vessel could then be used for the next test after the joint surface was remachined. Pressurizing gas was introduced into the vessel under test through a tapped hole in one of the caps, Figure 2.

Comparison of the three hole patterns is shown by Figure 3. Due to symmetry there are at most five unique hole positions in any of the patterns. Roman numerals identify these, while small letters identify the locations where stress was later found to reach a peak value. Again due to symmetry, there is often more than one point on a hole that is subjected to the same stress and therefore has the same letter symbol. The ligament width between holes and the minimum spacing between hole centers is the same for each hole pattern. As a consequence, each has the same ligament efficiency ($100 \times \text{ligament width} \div \text{hole center spacing}$) of 20 percent.

Stress Freezing and Slicing

Each model was tested in much the same manner as the others. The model was first assembled with spring force intended to provide a ratio of 1.5 between total bolt tension and axial thrust on the head due to internal pressure during the test. Then the model was placed in a large oven and the pressurizing tube connected to it. Temperature of the oven was slowly increased to 163°C. Internal pressure at 6.0 psig was then introduced and held constant throughout the remainder of the test by a Moore "Nullmatic" Series 40-7 regulator. After several hours at the peak, temperature was decreased at the rate of about 3/4 degree per hour until reaching room

Figure 3. Arrangement of holes in three hemispherical heads. Roman numerals identify hole positions; letters indicate locations of highly stressed points.

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temperature. Pressure was released and the model removed from the oven and disassembled. The head was sawed from the vessel just above the cemented joint.

The photoelastic stress patterns were "frozen" into the model by the foregoing process. For examination of these patterns in the polariscope, it is not only possible but necessary to remove thin slices from the head to determine local stress distribution. Due to symmetry of the head and the loads acting on it, it was only necessary to remove and slice a portion of it. Accordingly, a pie-shaped piece was first removed and then cut in two along the mid-surface of the wall. The two pieces were further thinned down by hand to a thickness of only one quarter inch. One slice included the inner surface of the hemispherical head, the other included the outer surface. These will be referred to as surface slices. A third slice was taken from each head, one which was taken radially through the central hole in the head. This will be referred to as the meridional slice.

Photoelastic Stress Patterns

When viewed in the field of circularly polarized light provided by the polariscope, a "frozen stressed" slice of uniform thickness reveals a pattern of light and dark fringes that are locii of points where maximum shear stresses are the same. At the boundary of a hole under tension with pressure acting normal to the hole wall, the relationship between the fringe order and the tensile stress tangent to the hole wall, σ_θ , in a transverse slice viewed normally is:

$$\sigma_\theta + p = \frac{nf}{t} \quad (1)$$

where p numerical value of gas pressure exerted at point of stress, psig
 n fringe order observed
 f photoelastic material constant (psi stress in a one inch thick slice required to produce one fringe)
 t thickness of slice along light path, inches

Figure 4. Viewing curved surface slice with light passing parallel to hole axes.

In order to view the stress pattern at the hole boundaries of the surface slices it was necessary to view them with the light passing through parallel to the hole axis, not normal to the slice surface, as in Figure 4. In that case, only the horizontal component of stress, σ_H , contributes to the fringe order that is seen, and the principal surface stress, σ_θ , must be computed from the observed fringe order and the geometry of the slice:

$$\sigma_H + p = \frac{nf}{t} \quad (2)$$

$$\text{where } \sigma_H = \sigma_\theta \cos^2 \beta - p \sin^2 \beta \quad (3)$$

$$\therefore \sigma_\theta = \frac{nf}{t \cos^2 \beta} - p \quad (4)$$

$$(\text{or}) \quad \sigma_\theta / p = \frac{nf}{pt \cos^2 \beta} - 1$$

Fringe patterns of the two surface slices from each model are shown in Figure 5. No notation of fringe order has been made there. But the fringe

Figure 5. "Frozen stress" fringe pattern in outer and inner surface slices from three heads. Each slice has uniform normal thickness of 1/4 inch.

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order at any given location can easily be counted, starting from zero at any one of the white circular discs surrounded by a black fringe. Each black band is a half order fringe, and the white bands are integral order fringes.

Meridional slice fringe photographs are shown in Figure 6. These are flat slices about 1/4 inch thick, and equation (1) suffices to determine the boundary stresses from the fringe order observed at the inner surface. Taking $p = 0$, equation (1) also is used to determine the outer surface stresses on the meridional slice.

Stresses

Applying equation (4) to the information given by the surface slice fringe patterns, Figure 5, the stress at every point where a peak occurs was determined. The stress so determined is the average stress through the thickness of the slice. To determine values at the surface it was assumed that stress varies linearly from the inner to the outer wall of the vessel, and the slice average stresses were used as a basis for extrapolation to the outer and inner surfaces. The results, in the dimensionless form of σ_0/p are presented in Table I below. All peak stresses were tensile. The locations of the points of peak stress were shown in Figure 3. In Table I, peak ligament stresses are given for the inner and outer surfaces, and the average of the two values is given as well. The reason for presenting the average is that it should correspond generally to the membrane forces on the ligament. The individual surface stresses reflect not only the membrane forces, but in addition any bending or shear effects that exist.

Figure 6. "Frozen stress" fringe pattern in meridional slices from three heads. Slice thickness is $1/4$ inch.

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TABLE I

Peak Stresses, σ_e/p , on Hole Walls at Inner and
Outer Surfaces of Perforated Hemispherical Heads

Point No.	Thirteen Hole Circular Pattern			Nineteen Hole Triangular Pattern			Twenty-one Hole Square Pattern		
	Outer	Inner	Avg.	Outer	Inner	Avg.	Outer	Inner	Avg.
Ia	1.82	5.57	3.70	7.90	3.35	5.62	7.12	6.25	6.62
IIa	1.95	3.78	2.86	8.20	3.20	5.70	6.50	6.52	6.51
IIb	4.83	9.85	7.34	6.90	7.15	7.03	6.40	6.84	6.62
IIc	2.27	6.07	4.17	4.95	6.05	5.50	3.56	6.58	5.28
IIId				3.30	6.30	4.80	--	--	--
IIIa				5.35	5.74	5.55	5.76	7.96	6.86
IIIb				5.62	7.66	6.64	4.06	7.16	5.61
IIIc				5.34	5.84	5.59	--	--	--
IVa				3.10	7.78	5.44	3.81	8.09	5.95
IVb				5.55	6.11	5.83	3.15	10.05	6.60
IVc				4.12	7.78	5.95	2.13	5.04	3.58
Va							4.46	8.34	6.40
Vb							4.20	9.62	6.91
Vc							4.06	8.16	6.11

Stresses on the inner and outer surfaces of the meridional slices are shown in Figure 7, where it is seen that the ligaments are most highly stressed, followed by the flange fillet and the unperforated region of the inner surface of the head, in that order. High peak stresses σ_e are noted in the meridional slices from the circular and triangular patterns because the slice happens to coincide with a ligament there. In the slice from the square pattern closure, no such high peak occurs because the slice passes transverse through all ligaments.

Figure 7. Surface stress, σ_θ/p , on inner and outer surfaces of meridional slices from three hemispherical heads.

Discussion

The relative merits of the three different hole patterns, as revealed by these tests, can be discerned from Table I. The triangular pattern shows the lowest peak surface stress, while the circular and square patterns show peak stresses about 20% higher. On the basis of the maximum value of the average of the inner and outer surface stresses, the square and triangular patterns are about equal in merit, and the circular pattern is about 5 percent higher stressed.

It can be concluded then that: (1) Insofar as resistance to membrane forces are concerned, the hole pattern is of little consequence when ligament efficiency is held constant and (2) Bending effects are apparently the deciding factor in making the triangular pattern superior to the other two patterns.

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Acknowledgement

Possibly the most important prerequisite to a successful "frozen stress" photoelastic test is an accurately machined model with its surfaces free from effects which can easily be introduced by improper machining techniques. The writers are indebted to Mr. John Barcic for machining the vessel and closure models in such a manner that they were highly satisfactory for photoelastic work. In addition, his careful and accurate work in preparing the slices made it possible to obtain the test results by the simplest and most direct procedures.