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PROPOSAL

A Long Confinement Experiment (LCE) Proposal

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B. M. Marder
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A LONG CONFINEMENT EXPERIMENT (LCE) PROPOSAL

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ABSTRACT

By combining the straight theta pinch geometry and the toroidal $k = 1, 0$ geometry it should be possible both to heat and to stably confine a high- β plasma. The benefits of such an approach are discussed and experiments are proposed.

1. INTRODUCTION

Current theta pinch reactor proposals envision plasma containment on a many millisecond time scale. Our present experimental knowledge of theta pinch behavior is restricted to plasmas contained for typically less than 20 μsec . The longer time scales needed for a reactor might introduce a host of phenomena such as anomalous cross field diffusion or slow growing instabilities which are not observable on short confinement time experiments. We feel that a device confining high-quality plasma for hundreds of microseconds may be possible and that it can be built almost entirely using existing, tested technology.

Our concept of an LCE draws heavily on present experience with straight and curved theta pinches. It seeks to combine the two designs in such a manner as to optimize the desirable features of both while eliminating most of their drawbacks. Linear theta pinches are neutrally stable and achieve thermonuclear temperatures by shock heating and adiabatic compression. The containment time of linear theta pinches is, however, limited by plasma flow out the ends. The difficulty with curved theta pinches which achieve equilibrium using $k = 1$ and $k = 0$ fields lies in the incompatibility of the stability and heating requirements, except in staged experiments.¹ For stability a small wall radius to plasma radius ratio, b/a , is desirable. The requirements of shock and compressional heating place lower bounds on this ratio, which, when

stability considerations are included, ultimately result in quite large major radii. Of course, the closed toroidal geometry eliminates end loss.

The LCE design consists of two straight theta pinches combined with two semicircular curved sections to form a racetrack configuration as illustrated in Fig. 1. The heating takes place in the linear sections in which b/a is large. To achieve

Fig. 1. Dual bore racetrack configuration.

stability in the curved sections, the bore diameter is reduced to only twice the plasma diameter; $b/a = 2$. The plasma radius, temperature, and density, as well as the main longitudinal magnetic field remain the same in both sections. The device consists of two regions: neutrally stable

straight heating sections and curved, stable confinement sections. Thus one gains the advantages of conventional theta pinch technology while eliminating end loss through an optimized toroidal-type configuration.

II. EQUILIBRIUM AND STABILITY

In the LCE we wish to use parameters with which we are familiar from previous experiments. In particular, we shall take the plasma radius to be 1 cm throughout the device. In the curved sections the bore will be 4-cm diameter giving a $b/a = 2$. This figure cannot be much smaller without having an unacceptable amount of plasma touching the wall. On the other hand, since, as we shall see, the major radius of the curved sections goes as $(b/a)^6$ we wish to keep it as small as possible. The coil bore in the straight sections will depend on the magnetic field and on the heating requirements since all plasma heating takes place there. No stability restriction affects this dimension.

We wish to derive expressions for the minimum radius necessary to stably confine a plasma in the curved $\ell = 1, 0$ sections. In this region five forces act on the plasma. These are

$$F_{1,0} = \frac{B}{4g_1} B_0^2 h^2 a^3 \delta_1 \delta_0$$

$$F_R = \frac{1}{4R} B B_0^2 a^2$$

$$F_W = \frac{1}{4} \beta \left(\frac{a}{b}\right)^4 \xi B_0^2 \delta_1^2 h^2 a^2$$

$$F_{1,1} = -\frac{1}{4} \beta^2 g_2 \xi B_0^2 \delta_1^2 h^4 a^4$$

$$F_{1,0} = -\frac{1}{4} \beta^2 g_3 \xi B_0^2 \delta_0^2 h^2 a^2$$

where we have adopted the standard scyllac notation. $F_{1,0}$ is the radial force which balances the outward force, F_R . F_W is a wall restoring force produced by dipole image currents. $F_{1,1}$ and $F_{1,0}$ are instability forces associated with the $\ell = 1$ and $\ell = 0$ windings, respectively. ξ is the plasma displacement away from its equilibrium position and

$$g_1 = \frac{2}{3-2\beta}, \quad g_2 = \frac{(4-3\beta)(2-\beta)}{8\beta(1-\beta)},$$

$$g_3 = \frac{(3-2\beta)(1-\beta)}{\beta(2-\beta)}.$$

To achieve equilibrium we equate $F_{1,0}$ and F_R to obtain

$$R = \frac{B_1}{h^2 a \delta_1 \delta_0}. \quad (1)$$

For stability to $m = 1$ modes of both $\ell = 1$ and $\ell = 0$ we balance the destabilizing $F_{1,1}$ and $F_{1,0}$ with the restoring force, F_W . This yields as the criterion for stability

$$\delta_1^2 \left(\frac{a}{b}\right)^4 \geq g_2 h^2 a^2 \delta_1^2 + g_3 \delta_0^2. \quad (2)$$

Since our plasma radius is only 1 cm we know experimentally that higher m modes do not cause trouble. We shall assume that the discharge tube is shaped and lies just inside the grooved, curved coil. No restriction due to the necessity of heating is imposed since no heating is performed in the curved sections.

Combining Eqs. (1) and (2) we find

$$\frac{R}{a} \geq \frac{g_1 g_2 \delta_1}{\delta_0} \left[\delta_1^2 \left(\frac{a}{b}\right)^4 - g_3 \delta_0^2 \right]^{-1}. \quad (3)$$

This expression is minimized with respect to δ_0 by taking

$$\delta_0^2 = \frac{1}{3g_3} \left(\frac{a}{b}\right)^4 \delta_1^2. \quad (4)$$

Substituting this into Eq. (3) yields

$$\frac{R_{\min}}{a} = \frac{3g_1 g_2 \sqrt{3g_3}}{2 \delta_1^2} \left(\frac{b}{a}\right)^6. \quad (5)$$

Notice that the minimum radius is a monotonic function of both δ_1 and b/a . If we take $a = 1$ cm, $b/a = 2$ and $\delta_1 = 1$ we arrive at the following table.

β	δ_0	ha	$R_{\min}$ (cm)
0.2	0.06	0.093	14.70
0.4	0.10	0.134	5.07
0.6	0.16	0.159	2.74
0.8	0.27	0.166	1.92

An important feature of the proposed equilibrium is that, although linear theta pinches are only neutrally stable and thus allow lateral motion, the addition of the wall stabilized curved sections renders the entire plasma stable and determines a unique, stable equilibrium.

III. DYNAMIC PHASE

We have presented a plasma equilibrium configuration of reasonable size which has the potential of being hot, closed, and stable. In this section we propose a method by which such an equilibrium configuration might be achieved. Several preliminary experiments will suggest themselves in the course of the discussion.

We envision a sequence in which hot, high- β plasma is injected from the linear sections into the closed ends. After the initial filling of gas the magnetic field in the curved coil is slowly raised to its final value. This produces a solenoidal field in the un-ionized gas. After this small coil field is established, the main linear field is quickly ($\approx 3 \mu\text{sec}$) brought to the same value, imploding and compressing the plasma column in a time short compared to the injection time into the curved section. This creates a hot, linear theta pinch plasma of 1-cm radius which injects along the field lines into the curved end sections. The linear sections should be sufficiently long so that they continue to inject until a plasma of uniform density, temperature, and radius is established throughout the device. No compression is performed in the curved sections.

There are, of course, questions raised as to the feasibility of this method. Will the plasma tend to form unstable filaments? Even more fundamentally, is the wall stabilization sufficient if the b/a does remain small? While theory and simulation can provide some insight, these questions can be definitively answered only experimentally. Fortunately, only modest experiments need be performed.

A crucial necessity of achieving the desired equilibrium is that the junction between the large and small bore coils must not cause unacceptable plasma distortions. This transition region can be made smooth by extending the small bore coil as a straight section in which the $L = 1$ field is

slowly brought to full strength. Since $b/a = 2$ this causes no stability problems. When the $L = 1$ field is fully applied, the section is then curved and the $L = 0$ field is added. Such a configuration is shown schematically in Fig. 2. Experimentation should indicate if this extension is actually needed.

An alternate approach which was considered dispenses with the restriction of uniform plasma radius and envisions instead a bore of uniform radius in which the field is weakened in the curved sections causing the plasma to expand to the desired $b/a = 2$. This method appears less practical for two reasons. The first is that the nonuniform field produces an unstable mirrorlike configuration at the junction. The second is that it would decrease the achievable β in the straight sections since β goes inversely as the square of the magnetic field. That is, if $\beta = 1$ in the low field curved regions, the maximum β in the linear sections would be B_c^2/B_L^2 .

Fig. 2. Gradual development of $L = 1,0$ fields.

IV. A LONG CONFINEMENT EXPERIMENT USING THE SCYLLAC FACILITY

As an example of what might be done using these ideas, let us consider a future experiment using the Scyllac bank. Such an experiment might be considered after measurements are completed on the full Scyllac torus. The parameters, while not optimized, are consistent with previous experimental results and available capacitor banks.

As we have seen above for $\beta = 0.8$, $\delta_1 = 1$, and $b/a = 2$ one finds for marginal stability $R = 2$ m, $\delta_0 = 0.27$, and $ha = 0.16$. Given a radius of 2 m, we propose two linear portions of 6 m each, so that the total linear and curved portions are roughly equal as shown in Fig. 1.

We want a final plasma column of 1-cm radius and we ask what size bore is required in the linear region to produce this. We can relate the final column size after injection to the linear region column size before injection by using a simple energy conservation argument:

$$\left\{ \begin{array}{l} \text{Thermal energy in} \\ \text{linear column} \\ \text{before injection} \end{array} \right\} + \left\{ \begin{array}{l} \text{Magnetic work} \\ \text{done as linear} \\ \text{column compresses} \end{array} \right\} = \left\{ \begin{array}{l} \text{Final total} \\ \text{thermal} \\ \text{energy} \end{array} \right\} + \left\{ \begin{array}{l} \text{Magnetic work} \\ \text{done as curved} \\ \text{column expands} \end{array} \right\}$$

Define L_s = length of straight regions

L_c = length of curved regions

$k = L_s/L_c$

a_1 = plasma radius in linear region before injection

a_2 = final plasma radius

Recall $p = nkT$, $u = \frac{3}{2} nkT$ = thermal energy/volume
so thermal energy/length = $\frac{3}{2} p = \frac{3}{2} \frac{B^2}{8\pi}$ for a $\beta = 1$ plasma.

Finally,

$$\frac{3}{2} \frac{B^2}{8\pi} \pi a_1^2 L_s + \frac{B^2}{8\pi} \pi (a_1^2 - a_2^2) (L_s) = \frac{3}{2} \frac{B^2}{8\pi} \pi a_2^2 (L_s + L_c) + \frac{B^2}{8\pi} \pi (a_2^2) L_c$$

which yields

$$a_2 = \left[\frac{k}{k+1} \right]^{1/2} a_1$$

We have assumed that the magnetic field is constant in both regions during the injection which is consistent with the bank parameters listed below. This simple model also suggests that the temperature will remain constant during the injection since the energy expended displacing the field in the curved region equals the compressive work done on the plasma in the linear region. To see this one calculates the final density by conservation of particles:

$$\begin{aligned} n_1 \pi a_1^2 L_s &= n_2 \pi a_2^2 (L_s + L_c) \\ n_2 &= n_1 \frac{a_1^2}{a_2^2} \left[\frac{L_s}{L_s + L_c} \right] \\ &= n_1 \left[\frac{k+1}{k} \right] \left[\frac{k}{k+1} \right] = n_1 \end{aligned}$$

For a $\beta = 1$ plasma this implies $T_2 = T_1$.

To determine the linear bore size we need one final result. Using a simplified model for implosion heating¹ we find for $\gamma = 2$

$$\begin{aligned} a_1 &= x_0 b = x_{SH} b \left(\frac{B_{SH}}{B} \right)^{1/2} \\ B_{SH}^2 &= \frac{10^8 v_s^2 m_i^{1/2} n_A^{1/2}}{\pi^{1/2} b} \end{aligned}$$

Then,

$$a_1 = \text{const} \frac{v_s^{1/4} b^{3/4} N_o^{1/8}}{B^{1/2}} \sim b^{5/4}$$

Since the capacitor bank is of fixed size, B is assumed to vary as $1/b$. We shall take:

$$a_2 = 1 \text{ cm}$$

$$k = 1$$

$$a_1 = a_2 \sqrt{2} = 1.414 \text{ cm.}$$

In the 5-m-linear Scyllac experiments in which the

*Strictly speaking one should include the effects of gas initially in the curved region but this represents only 8% of the volume and can be ignored.

bore was 10.5 cm, a plasma radius of 1.0 ± 0.1 cm was obtained. Using the above scaling we conclude

$$2b = (10.5) \times \left(\frac{1.414}{1.0} \right)^{4/5} = 13.75 \text{ cm.}$$

The following bank parameters are obtained assuming the linear 6-meter section is driven by 5 Scyllac racks and the toroidal section is driven by part of the Zeus bank.

TABLE II

	Each Linear Section	Each Toroidal Section (R = 2 m)
Length	6 m	6.3 m
Coil bore	13.75 cm	4.0 cm
L_{coil}	22.0 nH/m	1.6 nH/m
Estimated L_{source}	11.0 nH/m	10.0 nH/m
L_{total}	33.0 nH/m	11.6 nH/m
α	0.67	0.14
Capacitance	1942 μF (5 racks)	8500 μF (12 shelves of Zeus)
Bank voltage	50 kV	14.2 kV
$\tau/4$	5.1 μsec	6.4 μsec
E_{θ}	673 V/cm @ 6 cm	192 V/cm @ 2 cm
F_z	60.8 kG	60.8 kG
Bank energy	2.43 MJ	0.86 MJ (1.7 MJ @ 20 kV)
Coil magnetic energy	1.46 MJ	116 kJ

These numbers were obtained using

$$\tau = 2 \pi \sqrt{LC}$$

$$L_{\text{coil}} = \frac{\mu_0 \pi b^2}{\ell(1 + b^2/4\ell)^{1/2}}$$

$$a = \frac{L_{\text{coil}}}{L_{\text{coil}} + L_{\text{source}}}$$

$$E_{\theta} = \left(\frac{\alpha V}{2\pi b} \right) \left(\frac{r_{\text{wall}}}{b} \right)$$

$$\left(\frac{B^2}{8\pi} \right) \text{Vol.} = (0.9) \alpha \frac{1}{2} cV^2$$

where in the last formula the factor (0.9) allows

for crowbar losses.

Note that only 10% of the energy is invested in the curved region and this fraction decreases as K increases. The toroidal bank is based on the use of Zeus capacitors having 715 μF and 30 nH per shelf (including cable inductance).² Spark gap switches and an ignitron-type crowbar could be used if the crowbar could be extended to 200 μsec .

The ion temperature in the straight region before injection can be estimated from the recent 5-m linear experiment using the simple scaling

$$kT_i = \text{const} \frac{E_{\theta}^{1/2} B}{N_0^{3/4}}$$

TABLE III

	Results 5-m linear (45 kV)	Proposed 6-m Section (50 kV)
E_{θ}	857 V/cm	673 V/cm
B	60 kG	60.8 kG
N_0	10 mTorr	10 mTorr
kT_i	2.75 keV	2.5 keV

This scaling can be derived from either the above mentioned perfect bounce treatment by Ribe or a more conservative snowplow model.

It bears emphasizing that the equilibrium discussed here is created by injection which is not a very well understood phenomenon. Preliminary experiments such as the one described in the next section would be mandatory.

Another experimental feature of some significance in our assumption is that a shaped helical quartz tube must be procured. It would undoubtedly be expensive and time consuming to build, but the considerable savings in machine size would offset the cost. It seems worthwhile to begin exploring the use of helically shaped discharge tubes in all Scyllac-type configurations.

V. A PRELIMINARY EXPERIMENT USING THE PROPOSED LINEAR SCYLLA IV-P MACHINE

To test wall stabilization in the presence of $\ell = 1$ fields and injection of high- β plasma requires a relatively simple linear experiment. Theoretical work underway on the stability of diffuse plasmas in helical $\ell = 1$ systems will be very useful in the final design of an experiment but we

can sketch a proposal as follows.

Imagine two conventional straight bore theta pinch machines, 2.0 m long, on each side of a 1-m helical small-bore section. In this instance $k = L_s/L_c = 4$ so the radius reduction would only be $\sim 11\%$ due to injection although the radius would continue to fall due to end-losses. Assume one had the equivalent of one Scyllac rack to drive two meters. The two regions would then have approximately the following characteristics:

TABLE IV

	Each Linear Region	Helical Region
Length	2 m	1 m
Coil Bore	12 cm	4 cm
L_{coil}	7.1 nH	1.6 nH
Est. L_{source}	5.0 nH	10 nH
L_{total}	12.1 nH	11.6 nH
α	0.59	0.14
Capacitance	388 μF	2145 μF
Bank voltage	50 kV	10 kV
$\tau/4$	2.6 μsec	7.8 μsec
B_z	53.5 kG	53.5 kG
E_θ	700 V/cm ($r = 5$ cm)	110 V/cm ($r = 2$ cm)
Initial kT_i	2.2 keV	-
a	1.2 cm	-
Bank energy	485 kJ	107 kJ
Coil magnetic energy	258 kJ	15 kJ

While there is no final choice of δ , h_a , and coil bore in the helical portion, we show in Fig. 3 a sketch of the configuration near the straight-helical junction assuming the following parameters:

$$\begin{aligned} b &= 2 \text{ cm} \\ a &= 1 \text{ cm} \\ \delta_1 &= 1 \\ \lambda &= 25 \text{ cm} \\ h_a &= 0.25 \end{aligned}$$

This system, which would be very unstable without the close wall, would provide dramatic evidence for the existence of wall stabilization, if such exists, and would also determine the feasibility of high- β injection.

Fig. 3. Region of connection between a linear and toroidal coil and shaped discharge tube showing how b/a varies in the two regions.

A. Dividing the Torus into Smaller Segments

The parameters presented here are optimal in the sense that neutral stability was assumed. Small changes in β or b/a can result in a theoretically unstable plasma. It is possible to leave a margin for safety, by increasing the major radius, but even then a serious problem may remain. The small value of b/a in the curved sections exaggerates the difficulties associated with the β dependence of the equilibrium in that small column displacements (comparable to the plasma radius) would allow plasma to come in contact with the wall. Simple force balance considerations show that the wall stabilization term is insufficient to balance the force resulting from an incorrect β even if R is made considerably larger than the minimum value. It is therefore clearly desirable that the plasma achieve the prescribed β in a time short compared to the nonequilibrium drift time. This criterion suggests reducing the length of the curved regions into which plasma is injected which in turn requires that the curved region be segmented into more than two parts.

Consider a stable small bore torus configuration divided into n equal sectors. We have considered $n = 2$ but we will now allow n to be larger so that the injection time can be reduced. The curved segments are connected by straight sections for heating. A simple model for selecting n is the following. We take the toroidal radial drift time to be

$$\tau_r = \left(\frac{2 b \rho}{F(\beta)} \right)^{1/2}$$

where ρ and F are mass and force per unit length of column, respectively. Neglecting the instability and wall terms we can approximate $F(\beta)$ as follows

$$F(\beta) = F_R - F_{1,0} = F_R(1 - F_{1,0}/F_R),$$

$$F(\beta) = \frac{\beta B_o^2 a^2}{4R} f(\beta),$$

and

$$f(\beta) = \frac{(3-2\beta)}{4(1-\beta/2)(1-\beta)} hR \left(\frac{\delta B_{\ell=1}}{\beta_o} \right) \left(\frac{\delta B_{\ell=0}}{B_o} \right).$$

The graph in Fig. 4 shows a typical variation of $F(\beta)$ for β at equilibrium of 0.7. Writing $\beta = p/(B^2/8\pi)$ and $v_{th} = \sqrt{2kT/m}$ we can rewrite τ_r as

$$\tau_r = \sqrt{\frac{2bR}{f(\beta)}} \frac{1}{v_{th}}.$$

Roughly speaking, the injection time, τ_{inj} , is

$$\tau_{inj} = \frac{1}{2} \frac{\text{curved length}}{\text{thermal velocity}} = \frac{\pi R}{n v_{th}}.$$

For $\tau_{inj} = \tau_r$ we obtain

$$n \sim \pi \sqrt{f(\beta) R/2b}.$$

Typical numbers might be $f(\beta) \sim 0.5$, $R/2b \sim 25$ and

Fig. 4. Radial force vs β .

we obtain

$$n \sim 11.$$

Thus, segmenting can be used to facilitate rapid plasma formation much as the implosion phase does in present experiments.

Referring to Fig. 5 we find that the resulting machine size is

$$R' = R \left(1 + \frac{k}{n \tan(\pi/2h)} \right)$$

where again k = length of straight/length of curved. Depending upon the need, one might choose an LCE configuration using the Scyllac bank as shown in Fig. 6. Consistent dimensions for $k = 1$ would be

$$R = 2.02 \text{ m}$$

$$R' = 4.00 \text{ m}$$

$$L_s = 1.06 \text{ m}$$

$$L_c = 1.06 \text{ m}.$$

Fig. 5. Quantities used for discussing a segmented geometry in which R is the radius of curvature for the curved region and R' is the overall radius of the device.

Segmented Scyllac $n=12$

Fig. 6. Possible LCE configuration using 12 sections.

Twelve of the Scyllac racks could provide the field for the straight sections, and Zeus capacitors as described above could provide field for the curved sections. As the sketch clearly shows, the shape is particularly well adapted to the Scyllac experimental area.

VI. CONCLUSIONS

We may summarize the main points in our thinking as follows:

1. The combination of a straight theta pinch for heating and a curved helical system for confinement offers promise as a method to stably contain high- β plasma.

2. The additional uncertainty and complexity introduced by a) the interface between straight and helical fields and b) the injection process can be studied at reasonable cost of money and manpower.

3. If the injection process is found feasible, the LCE (Long Confinement Experiment) may be the quickest route to high- β plasma experiments on the millisecond time scale, and ultimately, to a reactor design.

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