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**BEMOD, A Code for the Lifetime
of Metallic Fuel Elements**

by

V. Z. Jankus

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BEMOD, A Code for the Lifetime of Metallic Fuel Elements

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ABSTRACT

A computer code (BEMOD) has been developed to describe the behavior of reactor fuel elements during irradiation. BEMOD calculates stresses and strains of a cylindrical sodium-bonded metallic fuel element at several axially distributed points along the element.

Growth of the fuel during irradiation is described as the sum of inexorable swelling and compressible breakaway swelling. Inexorable swelling is a linear function of the fraction of fission gas released. The fuel rod is only allowed to expand radially, and fission-gas release is taken to be an empirical function of the fuel expansion only.

The pressure acting upon the cladding is either the plenum pressure or the pressure exerted by the fuel in direct contact with the cladding. No friction or trapping of bond sodium between the fuel and cladding is allowed. The creep of the cladding is described by an average temperature and an average hoop stress, and irradiation growth of the cladding is neglected.

The behavior of the fuel element is described under steady-state conditions, but the code also calculates the stresses and estimates strain changes for an accidental overpower or flow-coast-down.

As an application, the BEMOD code is used to calculate the influence of several design parameters on cladding strain as a function of burnup of the EBR-II driver fuel. These variables are plenum volume, linear power, smear density, gas release, and cladding strength.

I. INTRODUCTION

The calculations to determine the safe design of fuel elements for the EBR-II reactor have thus far used a cautious approach and conservative assumptions. These relatively simple and analytically manageable calculations cannot predict the actual behavior of the fuel elements, but can provide a conservative limit for safe fuel-element operation in the reactor. The observance of such a limit constitutes a self-imposed burden to the

economical management of reactor fuel. To obtain a more realistic limit, predictions of the behavior of a fuel element over most of its useful life are essential for a reliable indication of potential fuel-element malfunctions and failures. Thus, more elaborate calculations are needed to provide numerical values for the behavior of fuel elements near failure.

Because of the complexities of the problem and the uncertainties associated with material properties, a comparison of predicted and experimentally observed behavior is mandatory. Presently, experiments to determine the strain of the cladding are considered as both convenient and relevant for such a comparison. The results of these nondestructive experiments are expected to reduce the uncertainties in the quantitative and qualitative assumptions underlying the calculations and, eventually, to make the predictions of fuel-element behavior a reliable tool for the design of safe and economical future liquid-metal-cooled fast breeder reactors.

The BEMOD computer code is designed to model the behavior and to estimate the lifetime of metallic fuel elements during irradiation. Metallic fuel elements were chosen because of the large amount of data available on in-reactor performance and the importance of these elements in EBR-II. The BEMOD code is relatively fast and is related to the earlier SWELL code^{1,2} that is intended for the ceramic fuel elements. The relationship concerns both the physical origin and the philosophical attitude used in the codes in choosing the assumptions commensurate with the extent of the experimental data available.

The SWELL program rapidly calculates the lifetime of a ceramic fuel element and the strains and stresses in the cladding near the end of its safe operation. The cladding is assumed to be essentially a thin shell, and the stresses and strains are calculated by assuming that the pressure exerted upon the cladding by the fuel is simply related to the pressure of fission gases. In the SWELL calculations, this pressure is successfully assumed to be either plenum pressure or a fixed fraction of the average pressure of the fission gas still retained in the fuel, whichever is larger. Fission gas is assumed to be released when a temperature (empirically established) is reached for an effective gas release. The fraction of the pressure of retained fission gas to be used in the calculation of the pressure upon the cladding has been determined by fitting the calculations to a set of fuel-element-failure experiments conducted by the British.

In metallic fuel, the gas release cannot be correlated with the temperature of the fuel, but can be related to the amount of volume swelling that occurs when there is an appreciable interconnection of voids. Thus, BEMOD considers the gross expansion of the fuel in a container; BEMOD does not attempt to correlate the average pressure of fission gas remaining in the fuel with the pressure exerted upon the cladding.

Microscopic examination of metallic fuel has clearly demonstrated^{3,4} that the voids observed in low-burnup fuels are not filled with fission gas under high pressure, but are caused largely by the tears in the metallic fuel. The swelling behavior of higher-burnup fuels seems to be largely the result of fission-gas buildup, but the gas release and the force exerted by the fuel presently cannot be quantitatively described from theoretical considerations; therefore, a phenomenological approach is used in BEMOD. Some experimental information of the expansion of the irradiated fuel as a function of pressure is becoming available,⁵ and, in BEMOD, we have endeavored to use this information by describing the fuel as a conforming, nonviscous compressible fluid. This description permits the BEMOD code to calculate the pressure exerted by the fuel on the cladding; in contrast to SWELL, a parameter does not have to be adjusted for the observed failures of the cladding. However, some freedom has been exercised in the interpretation of the basic measurements of the mechanical properties of the irradiated fuel.

The fuel element is represented in the BEMOD code as a long cylindrical rod centrally located in a cylindrical container (see Fig. 1). The

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Fig. 1. Fuel Element Used in BEMOD Calculations $N_t = 6$. The scale in the radial direction is exaggerated; $L_0 = 13.5$ in., $d_0 = 0.174$ in. for fuel-element types Mk IA and Mk IB. Dotted lines exemplify the axial nodes (or midplanes of sections) used in calculations. Every section is considered sufficiently long to neglect axial heat conduction and to use plain strain approximation.

container is filled with sufficient sodium to cover the fuel rod and to provide a thermal contact between the fuel and the container wall. The plenum above the sodium, initially filled with an inert gas (roughly of atmospheric pressure), collects all the fission gas that the fuel releases. When the fuel element is placed in the reactor, the fuel rod expands because of thermal expansion and radiation damage. Since experimental evidence based on EBR-II driver-fuel irradiations suggests that axial swelling of the rod is small compared with radial swelling, the code assumes that the fuel expands only in the radial direction. This expansion squeezes the bond sodium out of the gap and into the plenum and compresses both the inert and the fission gas. During the initial irradiation period, the pressure acting upon both the fuel and the cladding is the same as the plenum pressure. As soon as the fuel comes in contact with the cladding, the local contact pressure becomes larger than the plenum pressure, and the cladding is subjected to varying radial and axial stresses. As a result of these stresses, the cladding is assumed to deform both elastically and plastically. The calculations of strain are

performed at small burnup steps until the stresses found in some axial section are likely to cause rupture, or until a prescribed maximum strain is reached.

II. BEMOD COMPUTATIONS

The BEMOD computations consist of a number of static calculations for increasingly larger values of burnup. The code determines the temperature distribution, the plenum pressure, the direct-contact pressure between fuel and cladding, and the plastic deformation of the cladding. These calculations are performed at each burnup increment for nominal conditions of average linear power, Q_{pb} , and axial coolant-temperature rise, ΔT_c .^{*} These parameters are then varied by prescribed factors to simulate the conditions at overpower and flow-coast-down, and the calculations of stress and strain are repeated. Creep deformations are accumulated at each burnup step, and the computation continues until the calculated failure criteria are reached, or until a prescribed maximum strain, ϵ_{max} , is exceeded.

A. Calculation of Temperature

The calculational model (see Fig. 1) subdivides the fuel element into several axial nodes (N_t) or sections. Axial heat conduction in the cladding and fuel is neglected, and the radial temperature distribution in every section is obtained by using the average coolant temperature as the initial condition. The thermal conductivities of the cladding and the fuel are assumed to be linear functions of temperature.

The calculation of the coolant temperature in BEMOD is the same as in SWELL, which uses the average temperature of the inlet coolant, T_{cin} , the coolant-temperature rise, ΔT_c , and the relative axial power for each of the sections, $Q(N)$. In BEMOD, values of $Q(N)$ are normalized so that their average is unity:

$$\sum_{1}^{N_t} Q(N)/N_t = 1.$$

Assuming constant specific heat of the coolant, the average temperature of each section $T_c(N)$ is found from the preceding section temperature, $T_c(N-1)$, since the temperature rise

$$T_c(N) - T_c(N-1) = (c/2)[Q(N) + Q(N-1)] \quad (1)$$

^{*}We have attempted to call our variables by their FORTRAN names by keeping the first letter intact and relegating the rest to the subscript.

is roughly proportional to the average heat input $[Q(N) + Q(N - 1)]/2$. The proportionality constant c is found from the given total temperature rise

$$\Delta T_c = c \sum Q(N) = c N_t. \quad (2)$$

The temperatures of cladding and fuel are determined by neglecting axial heat conduction. Thus, if the linear heat flux out of fuel-element section N is $Q_{pb}Q(N)$, the temperature (T) at every point (r) of the cladding is obtained from

$$Q_{pb}Q(N) = -2\pi r K_{cl} dT/dr. \quad (3)$$

If the heat conductivity, K_{cl} , of the cladding is a function of the temperature only, the integration of Eq. 3 can be performed after separation of the variables r and T . In particular, if the conductivity of the cladding is linear, then

$$K_{cl} = A_1(1 + A_2T), \quad (4)$$

where A_1 and A_2 are constants, and the integration of Eq. 3 gives a quadratic relation for the cladding temperature at position r :

$$A_1(T - T_{clo})[1 + A_2(T + T_{clo})/2] = [Q_{pb}Q(N)/2\pi] \ln(D_{opp}/2r), \quad (5)$$

where D_{opp} is the outside diameter of the cladding, and T_{clo} is the temperature of the outside surface of the cladding. This temperature is obtained by summing the local average temperature of the coolant to the temperature drop across coolant film.

The fuel temperature at radius r is determined similarly, except that the heat flux across the surface of a unit cylinder of radius r is

$$Q_{pb}Q(N)(r/a)^2 = -2\pi r K_f(T)(dT/dr), \quad (6)$$

where a is the radius of the fuel rod. Again, the conductivity of a solid fuel is assumed to be a linear function of temperature,

$$K_f(T) = A_{28}(1 + A_{29}T). \quad (7)$$

The integration of the differential equation again yields a quadratic equation for T . The calculation of temperature by the SWELL code is somewhat more complicated since the fuel is assumed to consist of two zones of different porosity (and conductivity) that are separated by a prescribed temperature needed for an effective gas release. The code has to resort to a numerical procedure for the determination of the radius of interface and the simultaneous determination of conductivities and temperatures across

the fuel rod. Since only one fuel zone is used in BEMOD, the equations for temperature are quadratic, as Eq. 5, and the temperature is determined by direct solution. However, when the terms tend to cancel each other, a power-series expansion (consisting of three terms) is used. Making a proper choice between these alternate expressions permits the temperature to be calculated with a numerical precision greater than 0.01%.

The coolant-film coefficient, H_C , can be obtained directly from the Nusselt number pertinent to the coolant flow.⁶ The fuel-cladding interface conductance, H_g , used in the SWELL code to calculate the outside temperature of the fuel from the inside temperature of the cladding of a ceramic fuel, depends strongly upon the contact pressure, P_r , between the surfaces

$$H_g = \exp(A_3 + A_4 P_r^3). \quad (8)$$

The same expression is used in BEMOD. However, the presence of a sodium bond makes the dependence of H_g on pressure negligible ($A_4 = 0$), and the equivalent heat-conductance coefficient (A_3) between the cladding and the fuel across the sodium bond should be determined by using the contact conductance⁷ between the sodium bond and the cladding and the thermal conductivity of the sodium.⁸ Since the temperature drop across the sodium bond is not significant, and since only average fuel temperature is needed for calculating the pressure caused by the fuel, BEMOD shirks the complications of a calculation across a variable bond thickness (probably involving coupling between thermal and pressure calculations) and, instead, relegates to the discretion of the user the choice of the bond thickness when determining A_3 . However, the decreasing bond thickness is accompanied by an increase of fuel porosity and a considerable reduction in the conductivity of the fuel. Experimental measurements indicate⁹ that the reduction factor for the fuel conductivity is independent of temperature. Thus, only the input constant A_{28} needs to deviate from the nominal value. The value of A_{28} (Ref. 9) used is for the fully expanded fuel, since this is the condition of interest for further calculations of cladding deformation.

B. The Plenum Pressure

Initially, BEMOD assumes a cylindrical fuel element with an annulus of bond sodium between the fuel and the cladding and a plenum above the fuel containing an amount of inert gas. As the fuel expands, the bond sodium is expelled into the plenum, thus compressing the inert and fission (if any) gases present. To this pressure, we add the saturated vapor pressure of sodium.

The volume of each section of the fuel element is a function of average temperature, burnup, and the ambient pressure (as described in II.C below). The determination of plenum pressure starts with an estimate of

a value. Then, for this pressure, the volumes of each fuel section and the corresponding inside volumes of the cladding are calculated by using an appropriate temperature and by accounting for the creep already suffered. Hence, the volume available for bond sodium is determined and the amount of sodium expelled into the plenum is found (taking into account the variation of the sodium density with temperature). Then, the plenum volume remaining for gas is calculated, and its pressure is determined by using the ideal gas law. The pressure thus obtained, as a rule, does not coincide with the initial estimate.

To obtain coincidence, repeated estimates must be made and each improved by an interpolation procedure. For the interpolation in BEMOD, we have adopted a philosophy that the difference (E) between the reciprocal of calculated pressure and the reciprocal of the initial estimate (i.e., $E = 1/P_C - 1/P_S$) is a bilinear function of the estimated pressure [i.e., $E = (a + bP_S)/(c + P_S)$]. To determine the constants of this bilinear expression, three estimates of pressure (P_1 , P_2 , and P_3) and the corresponding errors (E_1 , E_2 , and E_3) are needed. The pressure at which E vanishes (i.e., $P = -a/b$) can be expressed as

$$P = (W_1P_1 + W_2P_2 + W_3P_3)/(W_1 + W_2 + W_3), \quad (9)$$

where

$$W_1 = (P_3 - P_2) E_3 E_2,$$

$$W_2 = (P_1 - P_3) E_1 E_3,$$

and

$$W_3 = (P_2 - P_1) E_2 E_1.$$

After this new value is determined, we consider it as a new estimate and calculate the appropriate error E , which is then used to further improve the estimate by the bilinear interpolation. If the newly calculated pressure differs from the preceding one by less than $100EPS1$ percent (where $EPS1$ is an input constant), the process is considered to have converged satisfactorily, and the program proceeds to other calculations. If the convergence is unsatisfactory, the program is instructed to repeat the process up to 15 times. If all these calculations are needed, the program is instructed to print a message to that effect and proceed with other business. Such messages occur quite seldom, and, even in such a case, the error in the value of one pressure usually does not affect the validity of succeeding results.

C. Irradiation Growth of the Fuel

The increase of the fuel volume is assumed to be the sum of two terms, thermal expansion and irradiation growth. Thermal expansion of a fuel section is calculated by averaging the local thermal expansion over the cross section of the fuel rod:

$$(V_{ft}/V_{f0}) - 1 = 3 \int_0^a D_1(T) 2rdr/a^2, \quad (10)$$

where V_{ft} and V_{f0} are the volumes of a fuel section at temperatures T and 70°F , respectively, a is the radius of the fuel rod, and $D_1(T)$ is the linear expansion of the fuel from room temperature (70°F) to T . In BEMOD, a short table of values of D_1 (with corresponding temperatures) is supplied in the input, and the code evaluates Eq. 10 by Simpson's integration, interpolating linearly for D_1 (at each calculated temperature) in the supplied table.

The irradiation growth in BEMOD also consists of two terms--the inexorable growth and the breakaway swelling. The inexorable growth presently is assumed to be essentially proportional to the burnup (B_u in percent); that is,

$$I_{ng} = G_{am} B_u [(\bar{T}_f + 460)/(T_b + 460)]^{G_t} + C_{orr} \quad (11)$$

when this expression is positive; otherwise, $I_{ng} = 0$. Here $\bar{T}_f$ is the average temperature of the fuel section (in $^\circ\text{F}$), T_b and G_t are the parameters that are inferred from the observed swelling of EBR-II fuel,¹⁰ and C_{orr} is a small correction constant also observed from EBR-II fuel swelling.¹⁰ The inexorable growth, I_{ng} , is intended to represent the growth due to both solid fission products and small, nearly incompressible fission-gas bubbles and tears. Thus, the proportionality constant G_{am} is somewhat dependent on the volume of the small fission-gas bubbles present in the fuel. In BEMOD, we assume that the proportionality constant G_{am} is equal to G_{m0} (≈ 0.044) if the fuel has not yet released any fission gas. After the release, G_{am} decreases linearly with the fraction of the fission gas released, F_r , to the value of G_{m1} , ≈ 0.02 .

Theoretical calculations¹¹ indicate that metallic fuels develop an appreciable amount of interconnected voids after the fuel volume had increased by a factor of 1.3, and cause an appreciable release of gas present in the fuel. Experimental evidence¹² tends to support this mechanism for fission-gas release. Thus, in BEMOD, the fraction of the fission gas released, F_r , is assumed to be an empirical function of the relative increase of fuel volume, $\Delta V/V_{f0}$. After the fuel fractional volume increase V_1 is reached, we assume that F_r starts rising linearly until the fuel-volume

increase V_2 is reached, at which time $F_r = F_{gr2}$. Afterward, $1 - F_r$ is inversely proportional to the fuel-volume increase. Thus, we assume that

$$F_r(\Delta V/V_{f0}) \begin{cases} = 0, & \text{if } \Delta V/V_{f0} < V_1; \\ = F_{gr2}[(\Delta V/V_{f0}) - V_1]/(V_2 - V_1), & \text{if } V_1(\Delta V/V_{f0}) < V_2; \\ = 1 - (1 - F_{gr2}) V_2/(\Delta V/V_{f0}), & \text{if } V_2 < \Delta V/V_{f0}. \end{cases} \quad (12)$$

This behavior is illustrated graphically in Fig. 2. We assume that the gas is released with some delay.

Fig. 2

Effect of Swelling in Metallic Fuels on Fission-gas Release. The analytical expression (Eq. 12) was used in BEMOD to approximate experimental results.

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The breakaway swelling in EBR-II fuel has been observed when the burnup has exceeded a threshold value Bu^* . The breakaway swelling depends strongly on the composition of the fuel material.¹⁰ In BEMOD, the contribution of breakaway swelling is assumed to be of the form $G_1(Bu - Bu^*)^{G_b}$ whenever $Bu > Bu^*$. Foote¹⁰ found that $G_b \approx 2$ and that G_1 depends strongly on the amount of impurities present in the fuel (silicon particularly). Smith *et al.*⁵ observed that breakaway swelling is quite compressible. The irradiation of pressurized fuel in a thermal reactor has demonstrated that hydrostatic pressure is an effective means to reduce the swelling of the fuel.⁵ The magnitudes of the pressures needed are moderate and quite containable in jackets of the thickness customarily used in fast reactors. Similar behavior has been reported for uranium by Leggett *et al.*,³ McDonnell and Angerman,¹³ and Lehmann *et al.* Since the amount of swelling is influenced by temperature cycling, temperature, and other parameters not well understood, only a simple approximation has been used to represent the breakaway swelling under hydrostatic pressure. We have assumed that at significant pressures, P , the breakaway swelling can be represented by

$$B_{as} = G_2/(1 + P/P_g),$$

where G_2 and P_g are parameters presently known only within a considerable range of uncertainty. Hence, for $B_u > B_u^*$, in BEMOD we assume that the breakaway swelling is

$$B_{as} = G_1(B_u - B_u^*)G_b \text{ or } G_2/(1 + P/P_g), \quad (13)$$

whichever is smaller. In Eq. 13, P is the relevant pressure, and G_1 , B_u^* , G_b , G_2 , and P_g are input constants; G_2 and P_g could easily be made functions of burnup and average temperature if warranted. Data obtained from U-5 w/o Fs irradiations⁵ in CP-5 were used in Fig. 3 to show the effect of hydrostatic pressure on swelling. Quantitative information presented in the figure, however, is subject to further refinement in the interpretation of experimental data.

Fig. 3

Effect of Hydrostatic Pressure on Fuel Swelling. The lines and numerical values have been obtained for a special set of BEMOD input constants and are subject to quantitative modification. Fission-gas release starts at $V_1 = 0.20$.

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Thus, the total volume, V_f , of a section of fuel rod

$$V_f/V_{f0} = (V_{ft}/V_{f0}) + I_{ng} + B_{as} \quad (14)$$

is a function of the pressure P . If this section is not in contact with the cladding, the relevant pressure, P , is assumed to be the plenum pressure. When the fuel section is in contact with the cladding, the pressure relevant for the calculation of fuel volume is taken to be the contact pressure, P_r . To determine the latter, the code simply assumes that contact between the fuel and cladding exists and calculates the contact pressure from equilibrium conditions. If the calculated P_r turns out to be smaller than the known plenum pressure, the assumption of contact is deemed false and the calculated P_r is ignored.

D. Stress and Strain in the Cladding

In BEMOD, the cladding is treated in a thin-shell approximation. Thus, if P is the pressure exerted from the inside of the cladding and P_o is the pressure exerted from the outside, the condition of equilibrium in the radial direction gives the average hoop stress S_p :

$$b_0(P - P_o) = S_p h, \quad (15)$$

where b_0 is the initial inner radius of the cladding and h is its thickness. Similarly, the condition of equilibrium in the axial direction gives the axial stress S_z :

$$\pi b^2 [P_p - P_o(N_t)] = 2\pi b h S_z, \quad (16)$$

where P_p is the plenum pressure, and $P_o(N_t)$ is the outside pressure at the top of the jacket. The condition of compatibility for the hoop strain (u/b_0 , where u is the increase of the radius) in membrane approximation,

$$u/b_0 = (S_p - \nu S_z)/E_1 + \epsilon^t + \epsilon^c + \epsilon^p,$$

states that the hoop strain is composed of the elastic strain $(S_p - \nu S_z)/E_1$, where ν is Poisson's ratio, E_1 is the elastic modulus, ϵ^t is the thermal expansion, ϵ^c is the secondary creep strain, and ϵ^p is instantaneous plastic strain. By substituting S_p and S_z from Eqs. 15 and 16 into the condition of compatibility, we find that the inner radius of the cladding is

$$\begin{aligned} b &= b_0(1 + u/b_0) \\ &= b_0 \left\{ 1 + \frac{b_0}{hE_1} \left[P - P_o - \frac{\nu}{2} P_p + \frac{\nu}{2} P_o(N_t) \right] + \epsilon^t + \epsilon^c + \epsilon^p \right\}, \end{aligned} \quad (17)$$

where b_0 is the initial inside radius of the cladding. The inside volume of a section of cladding, V_c , is assumed to increase as a function of temperature and pressure from an initial value of V_{clo} to

$$V_c = V_{clo} \left\{ 1 + 3\epsilon^t + 2(\epsilon^c + \epsilon^p) + 2 \frac{b_0}{hE_1} \left[P - P_o - \frac{\nu}{2} P_p + \frac{\nu}{2} P_o(N_t) \right] \right\}. \quad (18)$$

To calculate the plenum pressure, we assume that $P = P_p$ is the same in every section. The difference between V_c in Eq. 18 and V_f in Eq. 14 gives the volume available for sodium bond (if any) and allows the determination of the amount of sodium squeezed out of every section into the plenum. The fact that the actual pressure between the fuel and the cladding may be larger than the plenum pressure in some sections does not affect the determination of plenum pressure since, in those sections, all the bond sodium has been squeezed out.

To calculate the contact pressure, $P_r(N)$, we assume that the fuel volume in every section is equal to the inside volume of the cladding,

$$V_f(N) = V_c(N).$$

Since both sides of this equation involve $P_r(N)$ in a rather complicated way, $P_r(N)$ is determined numerically by trial and error. The same

three-point interpolation procedure (see II.B above) is used to improve the estimate until the successive estimates differ by a smaller fraction than EPS1. The value of $P_r(N)$ is assumed to have physical significance and is used in further calculations only when it is larger than the previously obtained plenum pressure, P_p . After determining the pressure inside the cladding, BEMOD calculates S_p (pressure stress) and S_h (thermal stress) by using the elastic theory:¹⁵

$$S_p = (b/h)\{P - P_o[1 + h/b + (1/2)(h/b)^2]\}/[1 + h/2b]$$

and

$$\begin{aligned} S_h &= \alpha E_1 / (1 - \nu) (T_{clo} - \bar{T}_{cl}) \\ &= A_{38} (1 + A_{39} \bar{T}_{cl}) (T_{clo} - \bar{T}_{cl}). \end{aligned}$$

These stresses are used in the following calculations of plastic deformation.

Cladding deformation in BEMOD presently consists of two components: an instantaneous plastic deformation, representing first-stage creep, and a constant-rate process corresponding to second-stage creep. This separation allows a quick estimation of the change of strain and the likelihood of cladding failures at abnormal conditions of short duration. For such conditions (overpower or flow-coast-down), the linear power and coolant-temperature rise are assumed to change drastically, and BEMOD calculates the resulting stresses and estimates changes in strain by assuming that the changes are instantaneous.

In present formulation, the instantaneous plastic strain is computed by

$$\epsilon_P = (S_p + S_h - S_y) / E_1 \quad (19)$$

when the total stress $S_p + S_h$ at the outer surface of the cladding exceeds the yield stress S_y . Equation 19 is based on the reduction of the thickness of the cladding in the elastic range that is still capable of supporting the load and should be adequate as long as $S_p < S_y$. When S_p begins to exceed the yield stress S_y , the elastic modulus rapidly begins to change into a plastic modulus with consequent rapid increase in the plastic strain. In BEMOD, this increase of plastic strain has been approximated by adding the plastic-strain calculated by Eq. 19 to $(S_p - S_y) / E_{pl}$ when the latter expression is positive; E_{pl} is the plastic modulus that is calculated by the subroutine EPLA. Presently E_{pl} is assumed to be a function of the average temperature of the cladding; that is,

$$E_{pl} = A_{54}(1 + A_{55}T). \quad (20)$$

Equations 19 and 20 are negligible if the yield stress S_y is chosen large, e.g., if a large number is used for the input constant A_{43} .

Second-stage creep usually is found proportional to some power of the applied stress, S ; creep magnitude depends strongly on the absolute temperature, T , and the average activation energy, Q ; and it can be calculated satisfactorily in the form

$$\dot{\epsilon}_T^c = AS^n \exp(-Q/RT).$$

The consideration of the total creep strain as a function of the Larson-Miller parameter leads one to believe¹⁶ that the average creep rate (for a given amount of deformation) can also be expressed in a similar form, except that now the exponent of the stress is inversely proportional to the absolute temperature, and the constants A and Q are widely different. To allow for this option in BEMOD, we have generalized the above formula into

$$\dot{\epsilon}_T^c = AS^{n-(Q_1/RT)} \exp(-Q_2/RT).$$

Some empirical and theoretical evidence became available which suggests that in-pile creep at lower temperatures may be enhanced considerably because of high-neutron flux: after a short incubation time, the creep rate at low temperatures becomes proportional to the fast flux.¹⁶ This relationship can be represented by

$$\dot{\epsilon}_\phi^c = BF_{\text{flx}}QS^m,$$

where $\dot{\epsilon}_\phi^c$ is the flux-induced creep, B is a proportionality constant, F_{flx} is the average fast flux, Q is the axial shape factor for fast flux, and m is an appropriate exponent that describes the dependence of this creep on stress S . Temporarily, we have assumed that the stress dependence of flux-induced creep, $\dot{\epsilon}_\phi^c$, is the same as that of out-of-pile creep, $\dot{\epsilon}_T^c$. Thus, the total creep,

$$\dot{\epsilon}^c = \dot{\epsilon}_T^c + \dot{\epsilon}_\phi^c,$$

is represented in the present version of BEMOD by the following expression:

$$\dot{\epsilon}^c = \exp\{A_{40} + [A_{41} + A_{37}/(T + 460)] \ln s\} \{A_{45}F_{\text{flx}}Q + \exp[A_{42}/(T + 460)]\}, \quad (21)$$

where S is the hoop stress, F_{flx} is the average fast flux ($\text{n/cm}^2\text{-sec}$), T is the temperature ($^{\circ}\text{F}$), and A_{40} , A_{41} , A_{37} , and A_{42} are BEMOD input constants corresponding to the constants A , n , Q_1 , and Q_2 , respectively, of the

preceding expression for out-of-pile creep. Equation 21 essentially states that at high temperatures the creep is unaffected by neutron flux, but as the temperature is lowered, the creep reaches a minimum level determined by the neutron flux. The transition temperature seems to be within, or slightly above, the temperature range at which the cladding in EBR-II is operated. The magnitude of the flux-dependent creep is not very well known now, and the present constant A_{45} may turn out to be a function of stress and flux. Equation 21 has the form of the second-stage creep; i.e., the creep rate, $\dot{\epsilon}^C$, is independent of the amount of creep already suffered, ϵ^C .

First-state creep is significant, and attempts have been made to account for it while describing the entire creep by Eq. 21. In the first attempt, the observed second-stage creep rate¹⁶ was simply multiplied by 10, which yielded $A_{40} = -22.339$, $A_{41} = 7$, $A_{37} = 0$, and $A_{42} = -86,059^\circ\text{R}$. In the other attempt, the observed average creep rate¹⁶ during the first percent of deformation was represented by $A_{40} = 41.45$, $A_{41} = 0$, $A_{37} = 8000$, and $A_{42} = 159,707^\circ\text{R}$. Although these two sets of parameters are at large variance, the calculated creep rates at the conditions of interest are not as widely divergent. To calculate the creep rate of the cladding, we are presently using for T the average cladding temperature, T_{clb} , and for S the average pressure stress, S_p .

In BEMOD, the creep of the cladding during a time interval Δt is calculated by using the prevailing pressure at the beginning of the interval. The interior volume is determined from the calculated creep of the cladding, and the pressure of the fuel to be used during the next time interval is calculated from the interior volume. The procedure is satisfactory if the cladding is stronger than the fuel. Usually this is the case until the beginning of appreciable cladding deformation. This order of calculation is inefficient when the fuel becomes much stronger than the cladding. Then small errors in the calculated cladding strain yield magnified errors in the pressure of the fuel, and the calculation has a tendency to lead to oscillations. Precautionary measures have been taken to circumvent wild oscillations: the total plastic strain is never allowed to decrease, and an estimate of the change of strain is made based upon the extrapolation of the preceding calculations. If the calculated strain agrees with the estimate within an input parameter E_{ps2} , the calculation proceeds with the succeeding time interval. If the calculated strain does not agree, a new estimate is made (also by using recently obtained values), the time step is cut (if it is still larger than $T_{26}/10$, where T_{26} is the initial time step), and the calculation is repeated. If this calculation still gives the calculated strain that violates the convergence criterion, another estimate is made without cutting the time step further. If this still does not satisfy the convergence criterion, an arithmetic average of previous too-large and too-small estimates is used, and the procedure is repeated up to ten times. At this time, the fractional error in the estimated strain is expected to be less than 2^{-10} , even if the E_{ps2} convergence criterion has not been satisfied, and the

calculation proceeds with the succeeding time step. This long iterative procedure actually occurs when the fuel is very stiff, and the pressure has to adjust to make the cladding creep at exactly the rate demanded by the incompressible part of the swelling. The time interval, Δt , is never cut by more than one-half at a time; Δt is also increased if the E_{ps2} criterion warrants, but never more than doubled at a time. The time interval is changed with care to include every time value that is an integer of the prescribed initial time interval, T_{26} . In addition to this time step, other intermediate time steps usually are needed when the rate of creep changes appreciably in any fuel section.

III. APPLICATION OF BEMOD TO THE EBR-II DRIVER ELEMENTS*

Several applications of BEMOD are possible for the metal-fueled EBR-II driver fuel elements. The calculations can provide guidance for fuel-element development work, and, coupled with performance data, information can be provided for the model to describe quantitatively the behavior of the metal driver fuel.

The information of primary importance is the probability of fuel-element failure. Since the calculation of the strain to rupture, $EMAX$, is subject to large uncertainties, BEMOD calculates cladding strain and assumes a direct correlation with failure probability. Cladding strain also has the advantage of being directly measurable.

In the results reported here, the stresses due to the temperature gradient have been assumed to make no contribution to the increase in cladding radius, option NO = 2. The cladding creep rate for calculations used in this section has been set equal to ten times the minimum (second stage) creep rate, and the enhancement of creep by fast-neutron flux has been neglected. [In Eq. 21, the first-mentioned set of constants A_{40} , A_{41} , A_{37} , and A_{42} has been used with $A_{45} = 0$, and the primary creep has been neglected ($A_{54} \rightarrow \infty$).] Recently, however, some indications were received that the creep of the stainless steel cladding may be greatly enhanced by the fast-neutron flux; $A_{45} \neq 0$. If this is verified, the conclusions reached in this section are likely to undergo considerable quantitative modifications.

Variables that will be considered in the following discussion are.

1. Plenum volume
2. Linear power
3. Smear density and gas release
4. Effect of cladding strength.

*By P. G. Shewmon.

Calculations have been performed with two or three levels of these variables, which have been considered for EBR-II driver fuel elements. The actual predicted values of cladding strain are only approximate, but the relative changes due to these variables should be correctly indicated by the model. It should be emphasized that cladding swelling caused by fast-neutron irradiation is not taken into consideration in BEMOD. Thus, the observed increase in cladding diameter would be the sum of the creep strain plus the void-volume effect in the cladding.

A. Plenum Volume

Plenum volume has been the primary variable in the Mark-I series (Mark I, Mark IA, and Mark IB) of EBR-II fuel elements. The dimensions of the various elements are shown in Table I. The BEMOD predictions of cladding strain (ET) for these three elements are shown in Fig. 4, where the total plastic strain is shown for the axial segment with the largest strain. In all cases, the average linear power $Q = 7.735 \text{ kW/ft.}$ The coolant-temperature rise $D_{eltc} = 240^\circ\text{F.}$ If a cladding strain of 1% is considered, the burnup needed to reach this value increases with each fuel-element design. In the Mark IB, however, the maximum strain no longer occurs at elevation 6 (see Fig. 1), but occurs just above the midplane at elevation 4. Thus, any additional increase in plenum volume greater than that of the Mark-IB design would not increase the element life. In an element with a plenum volume larger than that of Mark IB, the pressure exerted on the cladding by the fuel in the fourth or fifth vertical segments produces more cladding strain than the gas pressure in the plenum in the sixth segment.

TABLE I. Dimensions of EBR-II Fuel Elements

Element Description	Element Type			
	Mk I	Mk IA	Mk IB	Mk II
Fuel diameter, in.	0.144	0.144	0.144	0.130
Fuel length, in.	14.2	13.5	13.5	14.2
Cladding ID, in.	0.156	0.156	0.156	0.150
Cladding OD, in.	0.174	0.174	0.174	0.174
Approximate sodium height above fuel rod, in.	0.645	0.645	0.645	0.645
Gas plenum length, in.	1.597	2.128	2.968	10.559
Gas plenum volume, in. ³	0.0305	0.0407	0.0567	0.1866

Fig. 4

Effect of Plenum Length, L_p , on Maximum Cladding Strain. For inadequate plenum gas volume, cladding starts expanding before fuel touches it. The expansion is largest at the top of fuel rod ($N = 6$), where the cladding is hottest. Mark-I plenum length $L_p = 1.60$ in., Mark-IA plenum length $L_p = 2.13$ in., Mark-IB plenum length $L_p = 2.97$ in. Largest swelling in Mark IB is for $N = 4$. (For $L_p = 2.3$ in., ET(4) has been calculated as undistinguishable from Mark IB.) The dotted line is the calculated ET(4) for Mark IA. (50060)

Figure 5 shows the predicted cladding strain along the length of the Mark-IA and Mark-IB elements after 3.7% burnup at 45 MW (linear power, 7.7 kW/ft; coolant temperature rise, 240°F). Note that the location of the maximum cladding strain shifts on going to the larger plenum volume.

Fig. 5

Inelastic Strain along Mark-IA and -IB Fuel Elements at 3.7 a/o Burnup. The lack of room for bond sodium in Mark IA causes an increase of pressure inside the jacket, which prevents fuel swelling. The cladding is weakest at the hotter end, and thus the expansion of cladding there accommodates the expansion of fuel in cooler sections also.

50058

B. Linear Power

To provide guidance in rating the EBR-II driver fuel for operation at 62.5 MW, calculations of fuel-element strain were performed at linear-power ratings of 7.735 and 10.743 kW/ft; 10.743 kW/ft corresponds to achieving the higher total power through increased linear-power rating with no increase in core diameter. Assumptions were made that the flowrate of the coolant remained the same at these linear-power ratings, and that the coolant temperature rise changed proportionally with changes of the linear-power rating.

Figure 6 compares the maximum cladding strain for the Mark-IA design at these two power levels. Also shown in the figure is the predicted behavior for the same temperature rise at different linear-power levels. This behavior would be attained by appropriately reducing the coolant flow simultaneously with the reduction of the linear-power rating. This gives a slightly longer lifetime because the mean cladding temperature is somewhat lower. The main source of the difference between the two power ratings is that the cladding is weaker at higher temperatures.

Fig. 6

Swelling at the Top of a Mark-IA Fuel Rod, ET(6), for Two Power Ratings. Present operating conditions; average linear power, $Q_{pb} = 7.7$ kW/ft; coolant temperature rise, $\Delta T_{elc} = 240^\circ\text{F}$. Reduction of coolant flow at $Q_{pb} = 7.7$ kW/ft, so that coolant temperature rise is $\Delta T_{elc} = 333^\circ\text{F}$. Increased power rating; average linear power, $Q_{pb} = 10.7$ kW/ft; coolant temperature rise, $\Delta T_{elc} = 333^\circ\text{F}$.

50064

Figure 7 gives a similar plot for the Mark IB; the plenum volume is sufficiently large to accommodate the bond sodium. The highest strain is obtained slightly above the middle of the fuel rod (elevation 4), because of the direct contact between the fuel and the cladding. Also shown by dots in the figure is the strain obtained if the fuel starts to release fission gas after having expanded 20 v/o ($V1 = 0.20$) rather than waiting until 25 v/o expansion is reached ($V1 = 0.25$), as has been assumed in all the other calculations. The increase in volume is now sufficient to allow some release of fission gas, which reduces the "inexorable" fuel swelling and, thus, cladding strain. Since the plenum pressure does not influence the cladding strain, gas release increases the fuel-element lifetime through the reduction in the inexorable swelling.

Fig. 7

Swelling at the Middle of a Mark-IB Fuel Rod, ET(4), for Two Power Ratings. Present operating conditions; average linear power, $Q_{pb} = 7.7$ kW/ft; coolant temperature rise, $\Delta T_{elc} = 240^\circ\text{F}$. Reduction of coolant flow at $Q_{pb} = 7.7$ kW/ft, so that coolant temperature rise is $\Delta T_{elc} = 333^\circ\text{F}$. Increased power rating; average linear power, $Q_{pb} = 10.7$ kW/ft; coolant temperature rise, $\Delta T_{elc} = 333^\circ\text{F}$. Fission-gas release starts when fuel has grown 20% by volume; "inexorable growth" due to irradiation is smaller (indicated by dotted lines).

50059

C. Smear Density and Gas Release

To design a metallic fuel element with a longer lifetime, one should increase the free annular region between the fuel and cladding, i.e., decrease the smear density. This provides more space for the fuel to swell and takes advantage of the fact that metal fuels release a significant fraction of fission gas after they have expanded approximately 25 v/o. The Mark-II design uses the same fuel and cladding as the Mark-I designs, but has a lower smear density and a thicker cladding (see Table I).

Figure 8 shows the maximum cladding strain as a function of burnup for the Mark-IA and Mark-II designs at 45 MW. The following points are apparent from the figure:

1. The burnup necessary to cause 1% strain in the Mark-II cladding is roughly three times that for the Mark IA.
2. The slope of the plastic-strain curve is much smaller for Mark II than for Mark IA. The fuel in the Mark-II design gives about half the rate of inexorable swelling because of the larger amount of gas release.
3. Since the plenum volume is ample in the Mark-II design, the maximum cladding strain occurs in the fourth segment, instead of near the plenum.
4. This model excludes deterioration of the cladding because of the possible interdiffusion of fuel and cladding. The effect of this could be significant at burnups above 6 or 7% at this power level. The extent of such deterioration must and will be checked experimentally.

50062

Fig. 8. Effect of Radial Clearance. The void fractions inside the claddings of Mark IA and Mark II are 0.148 and 0.240, respectively. The "inexorable swelling" in Mark II is reduced because of gas release. The plenum is ample to accommodate all release fission gas, and the cladding is thicker. Fuel elements are operating at average linear power $Q_{pb} = 7.7$ kW/ft. Coolant temperature rise $D_{eltc} = 240^\circ\text{F}$.

D. Effect of Cladding Strength

The effects of neutron irradiation strengthens the cladding, and the creep rate decreases for a given strain. To evaluate the effect of a change in the creep rate, calculations were performed with creep-rate equations that differed from each other by a factor of ten, and the results are shown

in Fig. 9. The effect of changing the creep rate by a factor of ten is not large since the stress in the cladding is initially quite low; at higher burn-ups, the "inexorable" swelling cannot be restrained even with a stronger cladding. The influence of the creep rate may be somewhat larger if the creep of stainless steel is greatly enhanced by the fast flux. Then a lower fuel pressure is needed to sustain the creep, and the compressible break-away swelling is larger.

Fig. 9

Effect of Creep Rate. The dashed lines indicate the minimum (second stage) creep rate for Type 304 stainless steel, out-of-pile measurements. The solid lines indicate accelerated creep rate that is 10 times the minimum creep rate.

50061

IV. PROGRAM OPERATION

The BEMOD program, written in FORTRAN language, started from an early version of the SWELL program¹ and underwent extensive changes in the calculation of pressure exerted by the metallic fuel on the cladding. This calculation now takes into account simultaneous elastic expansion of the cladding. By this method, the original SWELL program has also been preserved; it can be run by using the option number NO = 0 or 1. The input cards have been kept the same, and cards have been added to supply the new information needed. The constants supplied in one input deck (consisting of 17 cards) enable the calculation of the stress and strain of a fuel element until failure. There remains a provision in the input to vary the average linear power, QPB, and void fraction, VO, while keeping the other parameters constant and then repeating the calculations. However, this option should be exercised with caution since, in practical situations, the above changes usually are accompanied by changes in other relevant parameters. Thus, the calculations for a new fuel element or for new operating conditions usually should be accomplished with a completely new input deck. Any number of calculations can be made in one run. The several input decks should be preceded by two cards of comments concerning the entire run and followed by two blank cards before the "END OF FILE" card.

The program can be directed to calculate only for the nominal conditions of average linear power and coolant-temperature drop by choosing another option, NO1 = 0 (effecting considerable savings in running time).

The effects of the two abnormal conditions (overpower and flow-coast-down) on stress and strain can be investigated by choosing $NO1 > 0$. In the latter case, it takes about one-half second of CDC-3600 time per burnup step used. The printing can be limited to the terminal information at the end of the lifetime of the fuel element by choosing $NO2 = 0$. If $NO2 > 0$ is chosen, the program prints out the strains and stresses for every burnup step.

Appendix A contains a detailed explanation of the input deck, the output of BEMOD, and a numerical illustration of the input and output for a Mk-IB fuel element operating at nominal conditions.

APPENDIX A

BEMOD Input-Output Description

1. The Input Deck for BEMOD

Data input for the program is explained with a set of FORTRAN statements concerning the input cards. The input parameters, their symbols, and cards upon which these parameters are introduced are described on the following pages in the order that they must appear in the input deck:

```

804  READ 3001, (TITLE (N), N = 1, 10)
      READ 1000, NT, TCIN, DELTC, DODI, DOPP, HC, XLFPPP, DPPP
      IF (NT) 805, 805, 806

805  STOP

806  READ 2000, AF, FCB, XLPPP, A1, A2, A3, A4, A5
      READ 2000, A6, A9, A12, A13, A14, A15, A25, A26
      READ 2000, A27, A28, A29, A30, A31, A32, A33, A34
      READ 2000, A35, A36, A37, A38, A39, A40, A41, A42
      READ 2000, A43, A44, A45, A46, A47, A48, A49, A50
      READ 2000, A51, A53, A54, A55, A56, A57, A58, A300
      READ 2000, AA20, AA21, AB20, AB21, TFMAX, FFLX
      READ 2000, (Q(N), N = 1, NT)
      READ 2000, (PO(N), N = 1, NT)
      READ 2000, QPBO, QPBD, QPBM, VO, VOD, VOM
      READ 3000, NO, M1, NDL, NO1, NO2, DFDI, XNU, PPO, XLNAO,
1  ALPNA, A401, A402
      READ 2000, TB, GT, CORR, G1, BUST, GB, G2, PG
      READ 2000, GMO, GM1, V1, V2, FGR2, EMAX1, EPS1, EPS2
      READ 2000, (TDL(I), DL(I), I = 1, NDL)

1000  FORMAT (I10, 7E10.4)

2000  FORMAT (8E10.4)

3000  FORMAT (5I2, 7E10.4)

3001  FORMAT (10A8)

```

<u>Card No.</u>	<u>Symbol</u>	<u>Description</u>
1		An alphanumerical identification is introduced for each problem (TITLE) that is printed out in the output.
2	NT	Total number of axial modes in fuel region used in calculation (not larger than 9).
	TCIN	Coolant inlet temperature, °F.
	DELT C	Coolant temperature rise, °F.

<u>Card No.</u>	<u>Symbol</u>	<u>Description</u>
2 (Contd.)	DODI	OD/ID ratio of the cladding.
	DOPP	OD of the cladding in the region of the fuel, in.
	HC	Coolant film coefficient ($\text{Btu hr}^{-1} \text{ft}^{-2} \text{°F}^{-1}$) used to calculate the OD temperature of the cladding from the average temperature of the coolant.
	XLFPF	Length of the axial fuel region, in.
	DPPP	ID of the cladding in the plenum region (in.), which is usually the same as the ID of the cladding in the fuel region.
3	AF	Atoms of fission gas produced per fission.
	FCB	Fissions (cm solid fuel) $^{-3}$ (a/o burnup) $^{-1}$.
	XLPPP	Initial length (in.) of plenum region filled with gas.
	A1 } A2 }	The thermal conductivity of the cladding, $\text{XKCL}(\text{Btu hr}^{-1} \text{ft}^{-1} \text{°F}^{-1})$, is calculated from Eq. 4, $\text{XKCL} = \text{A1} * (1.0 + \text{A2} * \text{T})$, where T is the temperature, °F.
	A3 } A4 }	The effective fuel-cladding interface conductance, $\text{HG}(\text{Btu hr}^{-1} \text{ft}^{-2} \text{°F}^{-1})$, is calculated from $\text{HG} = \text{EXP}(\text{A3} + \text{A4} * \text{PR} ** 3)$, where PR is the contact pressure, psi. A4 does not vanish for ceramic unbonded fuel elements.
	A5	A numerical constant for ideal gas: $[(\pi/4)(22.4 \ell \text{mole}^{-1})(16.39 \text{ cm}^3 \text{ in.}^{-3})]/(6.02 \times 10^{23} \text{ atoms mole}^{-1}) = 4.79 \times 10^{-22}$.
4	A6	A conversion constant from British to metric system: $[(4/\pi)(14.7 \text{ lb in.}^{-2} \text{ atm}^{-1})(1000 \text{ cm}^3 \ell^{-1})]/[(492 \text{°R})(16.39 \text{ cm}^3 \text{ in.}^{-3})] = 2.321$.
	A9	A constant involving energy yield per fission: $[(3.413 \text{ Btu W}^{-1} \text{ hr}^{-1})(12 \text{ in. ft}^{-1})(16.39 \text{ cm}^3 \text{ in.}^{-3})]/[(3.1 \times 10^{10} \text{ fissions W}^{-1} \text{ sec}^{-1})(3600 \text{ sec hr}^{-1})] = 6.015 \times 10^{-12}$.
	A12 } A13 } A14 } A15 }	The constants used in Eq. 22 to calculate the time to failure, TTF (hr), in terms of stress, S, temperature, T, and fluence: $\text{TTF} = (1.0 + \text{A15} * \text{LOGF}(1.0 + \text{A34} * \text{FFLX} * \text{TM} * 3600)) * \text{EXP}(\text{A12} + \text{A13} * \text{LOGF}(\text{S}) + \text{A14}/(\text{T} + 460.0))$, where TM is time (hr) and FFLX is the fast flux.

<u>Card No.</u>	<u>Symbol</u>	<u>Description</u>
4 (Contd.)	A25	Thermal-cycling frequency (hr^{-1}) used to assess the damage due to thermal-cycling fatigue of the cladding in Eq. B.3, if subject to a very large thermal stress.
	A26	The increase of average burnup (a/o) for the next step of steady-state calculation, as assigned initially. This burnup step may be subdivided later by the code if the strain rise is excessive.
5	A27	Not used in BEMOD.
	A28 } A29 }	Thermal conductivity of the fuel, $\text{KKF}(\text{Btu hr}^{-1} \text{ft}^{-1} \text{°F}^{-1})$: $\text{KKF} = \text{A28} * (1.0 + \text{A29} * \text{T})$.
	A30 } A31 }	Linear coefficient of expansion for the cladding: $\text{ALPHA} = \text{A30} * (1 + \text{A31} * \text{T})$.
	A32 } A33 }	Linear coefficient of expansion for the fuel: $\text{ALF} = \text{A32} * (1.0 + \text{A33} * \text{T})$. Used only for ceramic fuels, $\text{NO} \leq 1$. For the metallic fuel with sodium bond, BEMOD determines the thermal expansion in the subroutine PFA by using varying expansion coefficients in given temperature intervals (as given by the last cards of the input deck).
	A34	Used for calculation of time to failure, TTF (see A12).
	A35 } A36 }	Used only for calculation of ceramic fuel elements ($\text{NO} \leq 1$). Pressure on the cladding is less than the average fission-gas pressure PR_1 by $\text{PF} = \text{A35} * (1.0 + \text{A36} * \text{VN}) * \text{PR}_1$, where VN is the volume of retained fission gas, in.^3 .
	A37	A coefficient for the equation of creep (see A40).
	A38 } A39 }	The thermal hoop stress, SH (psi), of the cladding OD is calculated by $\text{SH} = \text{A38} * (1.0 + \text{A39} * \text{TCLB})$ ($\text{TCLB} - \text{TCLO}$), where TCLB is the average cladding temperature, and TCLO is the OD cladding temperature.
7	A40 } A41 } A42 }	The rate of the cladding creep (hr^{-1}) is calculated by using Eq. 21: $\text{EDOT} = \text{EXPF}(\text{A40} + (\text{A41} + \text{A37} / (\text{T} + 460)) * \text{LOGF}(\text{S})) * (\text{A45} * \text{FFLX} * \text{Q}(\text{N}) + \text{EXPF}(\text{A42} / (\text{T} + 460)))$.
	A43 } A44 }	The yield stress, SY (psi), for the cladding is calculated in the subroutine SYA by $\text{SY} = \text{A43} * (1.0 + \text{A44} * \text{T})$, where the OD cladding temperature is used for T.

<u>Card No.</u>	<u>Symbol</u>	<u>Description</u>
7 (Contd.)	A45	Used for calculation of cladding creep rate, EDOT (see A40).
	A46 } A47 }	The elastic modulus, EL (psi), calculated for the cladding in the subroutine ELA by $EL = A46 * (1.0 + A47 * T)$.
	A48	Not used in BEMOD.
	A49 } A50 }	Number of thermal cycles to failure, NF, is calculated in the subroutine NFA by Eq. B.4: $NF = \text{EXPF}((A49 - \text{LOGF}(EP))/A50)$, where the cyclic plastic strain $EP = (SH - 2.0 * SY)/EL$.
8	A51	Not used in BEMOD.
	A53	The temperature for gas release, TG (°F), is calculated in the subroutine TGA by $TG = A53$. TGA is used in the calculation for ceramic fuel ($NO \leq 1$), but not for metallic fuel ($NO > 1$).
	A54 } A55 }	The plastic modulus, EPL (psi), is calculated in the subroutine EPLA by Eq. 20: $EPL = A54 * (1.0 + A55 * T)$.
	A56 } A57 } A58 }	If $NO \leq 1$, the thermal conductivity XKG of the porous fuel is calculated by $XKG = XKS * (2.0 * XKS + XKP - 2.0 * VO2 * (XKS - XKP)) / (2.0 * XKS + XKP + VO2 * (XKS - XKP))$, where VO2 is the void fraction, XKS is the conductivity of the solid fuel, and XKP is the conductivity of pores: $XKP = A56 * (1.0 + A57 * T) + A58 * (T + 460.0) ** 3$. For the metallic fuel ($NO > 1$), the conductivity of the pores can be neglected, and the porous fuel can be simulated by appropriately ⁹ reducing A28 in the expression for the conductivity of the solid fuel.
	A300	Fraction increase in fuel volume due to solid fission products per a/o burnup. Used in the calculation for ceramic fuel elements only ($NO \leq 1$).
9	AA20	The factor by which BEMOD increases the nominal coolant-temperature rise to simulate overpower, abnormal condition 1.
	AA21	The factor by which BEMOD increases the nominal linear power to simulate overpower, abnormal condition 1.

<u>Card No.</u>	<u>Symbol</u>	<u>Description</u>
9 (Contd.)	AB20	The factor by which BEMOD increases the nominal coolant-temperature rise to simulate flow-coast-down, abnormal condition 2.
	AB21	The factor by which BEMOD increases the nominal linear power to simulate flow-coast-down, abnormal condition 2.
	TFMAX	Maximum permissible fuel temperature ($^{\circ}\text{F}$) = solidus temperature for metallic fuel. If this temperature is exceeded at the end of the calculation, a notice is printed out.
	FFLX	Average fast flux ($\text{n cm}^{-2} \text{ sec}^{-1}$) used for the calculations of reduced ductility of the cladding and of enhanced creep (see A12 and A40).
10	Q(N)	The ratio of average linear power in section N to the average linear power of the entire fuel rod: $\Sigma Q(N) = NT$.
11	PO(N)	External pressure on the cladding in section N (psi).
12	QPBO } QPBD } QPBM }	QPBO is the average linear power ($\text{Btu hr}^{-1} \text{ ft}^{-1}$) for the first problem run by BEMOD. The code then increases the average linear power by QPBD without changing any other constants and runs another problem. The process continues until QPBM is exceeded. To use only one linear power, choose $QPBM < QPBO + QPBD$.
	VO } VOD } VOM }	VO is the first void fraction used. After calculating all linear power levels requested, the code increases the void fraction by VOD and runs the next set of problems without changing any other constants. The process continues until VOM is exceeded. To use only one void fraction, choose $VOM < VO + VOD$. $(1 - VO) * \text{FCB}$ is the average number of fissions per a/o burnup per cm^3 in a cross section of a fuel element inside the cladding prior to expansion.
13	NO	An integer is allowed to have four values, which indicate the options chosen to calculate the pressure exerted by the fuel and the creep of the cladding. NO = 0 and 1, used for ceramic fuel, assumes that the pressure exerted by the fuel on the cladding is related to the average pressure of fission gas (see A35).

<u>Card No.</u>	<u>Symbol</u>	<u>Description</u>
13 (Contd.)		NO = 2 and 3 use experimental data for the swelling of fuel to calculate the pressure exerted on the cladding as described in Section II.C; 1 and 2 use the average temperature of the cladding (TCLB) and the average stress (SP) to calculate the creep rate in Eq. 21 (see A40); 0 and 3 use the OD temperature of the cladding (TCLO) and the calculated OD stress (ST) if this does not exceed the yield stress (SY) to calculate the creep rate (EDOT). Otherwise, the yield stress is used in Eq. 21.
	M1	If NO > 1, BEMOD calculates the thermal expansion of the fuel using Eq. 10. And M1 is equal to one-half the number of intervals used in the evaluation of the integral by the Simpson integration.
	NDL	Number of points for the linear expansion of the fuel supplied by the last cards of the input: $2 \leq \text{NDL} \leq 12$.
	NO1	To skip the calculation at abnormal conditions (over-power and flow-coast-down), NO1 = 0. Otherwise, NO1 > 0.
	NO2	To skip the printing at intermediate burnup steps and receive the detailed information for the terminal burnup only, NO2 = 0. Otherwise, NO2 > 0. If NO2 > 1, the information about the fuel element at nominal conditions is also punched out for every burnup step used. (See Section 2 of this appendix.)
	DFDI	Ratio of initial fuel diameter to ID of the cladding.
	XNU	Poisson's ratio for the cladding used in Eq. 18 for NO > 1.
	PP0	Initial pressure of the inert gas in the plenum at 70°F (psi).
	XLNA0	Initial height of the sodium column above the fuel rod, in.
	ALPNA	Linear coefficient of thermal expansion for bond sodium.
	A401 } A402 }	Saturated vapor pressure for sodium (psi) = $A401 * \text{EXP}(-A402/(T + 460))$.
14	TB } GT } CORR }	The coefficients for calculation of the inexorable growth of fuel in Eq. 11: $\text{XING} = \text{GAM} * \text{BU} * ((\text{TFB} + 460.0)/(\text{TB} + 460.0)) ** \text{GT} + \text{CORR}$. BU is in percent, but CORR is a fraction.

<u>Card No.</u>	<u>Symbol</u>	<u>Description</u>
14 (Contd.)	$\left. \begin{array}{l} G1 \\ BUST \\ GB \\ G2 \\ PG \end{array} \right\}$	The coefficients for calculation of the breakaway swelling in Eq. 13: $BAS = G1 * (BU - BUST) ** GB$, or $BAS = G2 / (1.0 + P/PG)$, where $G1$ and $G2$ are fractions (values in percent divided by 100.0), $BUST$ is in percent, and PG is in psi.
15	$\left. \begin{array}{l} GM0 \\ GM1 \end{array} \right\}$ $\left. \begin{array}{l} V1 \\ V2 \\ FGR2 \end{array} \right\}$	<p>Initial and final values of proportionality constant GAM in Eq. 11: $GAM = GM0 + (GM1 - GM0) * FR$. GAM, $GM0$, and $GM1$ are fractions, and FR is the fraction of fission gas released.</p> <p>Coefficients for calculating the fraction of the fission gas released (FR) in Eq. 12. $V1$ is the fractional volume increase of the fuel ($\Delta V/V_{f0}$) at which gas release starts. $FGR2$ is the fraction of fission gas released at the fractional fuel-volume increase $V2$, when the linear relationship between $\Delta V/V_{f0}$ and FR terminates.</p>
	EMAX1	The total strain (in./in.) at nominal conditions prescribed to terminate the calculation if the termination has not occurred previously because of the failure condition: $ET(N) > EMAX(N)$.
	EPS1	Tolerable fractional error for the numerical determination of pressures (see II.B).
	EPS2	The maximum strain change allowed per burnup step. If the largest increase of strain is small compared with $EPS2$, the program (if warranted) increases the burnup step without jumping over any burnup that is an integer multiple of $A26$.
16, 17	$\left. \begin{array}{l} TDL(I) \\ DL(I) \end{array} \right\}$	The temperature, TDL ($^{\circ}F$), and the elongation of fuel, DL (in./in.), starting from $70^{\circ}F$, used to calculate the average thermal expansion of the fuel by Eq. 10.

2. The Output of BEMOD

For every set of input cards, BEMOD prints out the TITLE of the problem and the input parameters used, with proper identification.

If option $NO2 > 0$ has been chosen the program prints out for every burnup step at nominal: $A20 = 1.0$, $A21 = 1.0$; and abnormal conditions (overpower: $A20 = AA20$, $A21 = AA21$; flow-coast-down: $A20 = AB20$, $A21 = AB21$) the burnup, BU (%), $A20$, $A21$; the plenum pressure, PP (psi); the amount of fission gas released into the plenum, $QLSUM$ (liters at STP);

and the time, TM, hr. On the next line, the computer prints the pressure stress, SP(N) (psi) for every section N. On the following line, the total inelastic strain, ET(N) (in./in.), is printed for nominal conditions; and the creep strain, E(N) (in./in.), for abnormal conditions. The equality $N = NT + 1$ gives the stress and strains of the plenum. The option NO2 > 1 instructs the code to punch out the above information at nominal conditions as follows:

```
PUNCH 5103, BU, TM, PP, QLSUM
PUNCH 5103, SP
PUNCH 5103, ET
PUNCH 5103, E
```

```
5103  FORMAT (10E8.2)
```

Although abnormal conditions are presumed to be uncommon, we may wish to ascertain how much sooner the fuel element may fail if an abnormal condition should occur at some time near the end of the natural lifetime of the fuel element by choosing NO1 > 0. Then, the increase of the plastic strain is calculated at abnormal conditions, and the resulting total inelastic strain is compared with the appropriate calculated strain to failure EMAX(N) and the prescribed EMAS1. When total inelastic strain exceeds EMAX(N) and EMAS1 for the first time, a notice to that effect is printed out, but the calculation disregards the calculated increase of total inelastic strain at abnormal conditions and continues. The notice of excessive strain at abnormal conditions is printed regardless of the value of the printing option NO2, provided that abnormal conditions are considered. If the plastic modulus, EPL, calculated by Eq. 20 is small, the code overestimates the plastic deformation at abnormal conditions, and the indication of possible failure at abnormal conditions may be premature. Also, regardless of the printing option number of NO2, the code prints out a notice when the Miller's criterion¹⁸ for the possible onset of thermal-cycle ratcheting is exceeded for the first time. Since the contribution of thermal-cycle ratcheting calculated by Miller's theory¹⁸ is felt to be excessive and, as indicated by recent calculations,² does not contribute significantly to the total strain, the thermal-cycle ratcheting is ignored in BEMOD, except for the above-mentioned notice.

BEMOD continues calculations of the stress and strain until, at some section of the fuel element under nominal conditions, either EMAX(N) or the prescribed EMAS1 is exceeded by the total inelastic strain ET(N), or until the burnup of 20% is reached. At this time, the code tests whether the maximum allowable temperature, TFMAX, has been exceeded at some section in the center of the fuel rod, and prints a warning if this has happened. The program then prints out the terminal information:

The coolant temperature, XTC(N), °F.

The cladding outside temperature, XTCLO(N), °F.

The cladding inside temperature, TCLI(N), °F.

The fuel outside temperature, $TFO(N)$, °F.

The fuel central temperature, $TF(N)$, °F.

The gas release temperature, $TG(N)$ (relevant only in the calculation of ceramic fuel elements with $NO \leq 1$), °F.

The ratio of the diameter of the cylinder from which the fission gas has been released to the diameter of the fuel (relevant only for $NO \leq 1$).

The amount of the fission gas retained in the fuel section N , $QR(N)$, liters at STP.

The amount of fission gas released from the fuel section $QL(N)$, liters at STP.

The calculated contact pressure, $PR(N)$, psi. This pressure is meaningful only if larger than the plenum pressure, PP .

The average hoop stress of cladding, $SP(N)$, psi.

The thermal hoop stress $SH(N)$, psi.

The total hoop stress for the outside of cladding, $ST(N) = SP(N) + SH(N)$, psi.

The creep strain, $E(N)$, in./in.

The total inelastic strain, $ET(N)$, in./in.

The strain to rupture, $EMAX(N)$, calculated by Eq. B.5, in./in.

3. A Sample Problem: The Input Cards

COMPUTER INPUT DATA FORM

COST CODE _____

PROGRAM										PROBLEM										ORIGINATOR										DATE										PAGE										OF																																																																																									
										1										2										3										4										5										6										7										8																																																											
1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0																																																												
MIKILB										OIPERAT										EIDIAT										NOMINIAL										CONDIT										IONS										DIELIT										CI=240										FI										QIPB										I=17.7										35										IKW/										ETI									
6										7.00										240										1.1										1538										174										20000										13.5										156																																																											
274										361E+										20										21968										9.1										814										591E-										04										1082										4										1791E-										22																													
232116										0115E-										12										301134										1813										822										951E+										05										01085										10416										2																																							
386										00183										81778										E-06										167E-										04										44										46E-										06										169E-										04																																							
72										0										467										2167E-										04										122										339										7										86059																																																											
31000										0144E-										04										2915										E+06										12137E-										04										1154										1										922																																																	
20										000										45										38E+										15										2137E-										04										0118										77										1E-										04										1713E-										10										03									
12										12										2										05										1836										22										2E+										15																																																																					
867										1027										11107										11107										11027										867																																																																																									
295										279										263										247										2311										215																																																																																									
26400										2164E+										04										40										0100										14792										2										2																																																																					
238										011										192308										81										3										147										645										47										7E-										06										68000										00										22400																			
910										0										1007										017										877										48										2										7										150																																																											
044										02										2										4										60										02										00										1										001																																																											
200										1112E-										3										40										00										2175E-										3										600										4										41E-										3										800										6125E-										13									
1000										8175E-										3										111										12										10192E-										3										1195										14										58E-										3										1250										6108E-										13									
7										8										7										8										7										8										7										8										7										8										7										8																													
1										2										3										4										5										6										7										8										9										0																																																	

4. Mark-IB Fuel Element

MK1B OPERATED AT NOMINAL CONDITIONS, DELTC=240.F, QPB=7.735 KW/FT

NT	TCIN	DELT	DODI	DOPP	MC	XLFP	DPPP	AF	FCB
6	7.0000+002	2.4000+002	1.1154+000	1.7400+001	2.0000+004	1.3500+001	1.5600+001	2.7000+001	4.3600+020
XLFP	A1	A2	A3	A4	A5	A6	A9	A12	A13
2.9680+000	9.1800+000	4.5900+004	1.0820+001	-0.0000+000	4.7900+022	2.3210+000	6.0150+012	3.0134+001	-8.3822+000
A14	A15	A25	A26	A27	A28	A29	A30	A31	A32
9.5000+004	-1.0850+002	4.1600+002	2.0000+001	0.0000+000	3.8600+000	1.8300+003	8.7789+006	1.6700+004	4.4460+006
A33	A34	A35	A36	A37	A38	A39	A40	A41	A42
1.6900+004	1.0000+000	7.2000+001	-0.0000+000	0.0000+000	4.6700+002	-2.6700+004	-2.2339+001	7.0000+000	-8.6059+004
A43	A44	A45	A46	A47	A48	A49	A50	A51	A53
3.1000+004	-4.4400+004	-0.0000+000	2.9500+007	-2.3700+004	0.0000+000	-5.4100+001	9.2200+001	0.0000+000	2.0000+004
A54	A55	A56	A57	A58	AA20	AA21	AB20	AB21	TFMAX
4.5380+015	-2.3700+004	1.1800+002	7.7100+004	1.7130+011	1.2000+000	1.2000+000	2.0000+000	5.0000+002	1.8360+003
FFLX	A300								
2.2000+015	3.0000+002								
Q(N)	8.6700+001	1.0270+000	1.1070+000	1.1070+000	1.0270+000	8.6700+001			
PO(N)	2.9500+001	2.7900+001	2.6300+001	2.4700+001	2.3100+001	2.1500+001			
QPB	QPB	QPB	V0	V0D	V0M				
2.6400+004	2.6400+004	4.0000+004	1.4792+001	2.0000+001	3.0000+001				
NU	M1	NOL	NO1	NO2	DFD1	XNU	PPD	XLNAO	ALPNA
2	3	8	0	1	9.2308+001	3.0000+001	1.4700+001	6.4500+001	4.7700+005
TA	G1	RUST	GT	G2	PG	GB	EPS1	EMAX1	EPS2
9.1000+002	7.8770+002	4.8000+001	0.0000+000	7.0000+001	1.5000+002	2.0000+000	1.0000+003	2.0000+002	1.0000+003
GMU	GM1	V1	V2	FGR2	CORR				
4.4000+002	2.0000+002	2.0000+001	4.0000+001	6.0000+001	-7.0000+003				
TDL(I)=	2.0000+002	4.0000+002	6.0000+002	8.0000+002	1.0000+003	1.1120+003	1.1950+003	1.2500+003	
DL(I)=	1.1200+003	2.7500+003	4.4100+003	6.2500+003	8.7500+003	1.0920+002	1.4580+002	1.6080+002	

QPB= 2.6400+004

V0= 1.4792+001

BU/A26 = 0.00000

BU= 0.0000+000 A20= 1.0000+000 A21= 1.0000+000 PP= 2.7513+001 QLSUM= 0.0000+000 TM= 0.0000+000

SP(N) -4.5785+001-3.1074+001-1.6364+001-1.6527+000 1.3058+001 2.7769+001 3.5124+001

ET(N) 0.0000+000 0.0000+000 0.0000+000 0.0000+000 0.0000+000 0.0000+000 0.0000+000

BU/A26 = 1.00000

BU= 2.0000+001 A20= 1.0000+000 A21= 1.0000+000 PP= 2.7836+001 QLSUM= 0.0000+000 TM= 3.2357+002

SP(N) -4.3134+001-2.8423+001-1.3713+001 9.9826+001 1.5709+001 3.0420+001 3.7775+001

ET(N) 1.3227+059 1.6486+058 1.8169+057 1.5222+035 2.1394+026 8.1907+024 1.4252+023

BU/A26 = 2.00000

BU= 4.0000+001 A20= 1.0000+000 A21= 1.0000+000 PP= 2.9490+001 QLSUM= 0.0000+000 TM= 6.4715+002

SP(N) -2.9578+001-1.4867+001-1.5615+001 1.4555+001 2.9266+001 4.3976+001 5.1332+001

ET(N) 2.6454+059 3.2972+058 3.6337+057 2.1319+027 1.6876+024 1.1627+022 1.3618+022

BU/A26 = 3.00000

BU= 6.0000+001 A20= 1.0000+000 A21= 1.0000+000 PP= 3.1581+001 QLSUM= 0.0000+000 TM= 9.7072+002

SP(N) -1.2447+001 2.2636+000 1.6975+001 3.1685+001 4.6396+001 6.1107+001 6.8463+001

ET(N) 3.9682+059 5.0209+035 7.3773+028 4.9617+025 4.3625+023 1.1973+021 1.0515+021

BU/A26 = 4.00000

BU= 8.0000+001 A20= 1.0000+000 A21= 1.0000+000 PP= 3.5273+001 QLSUM= 0.0000+000 TM= 1.2943+003

0

0

0

0

0

SP(N) 1.7807+001 3.2518+001 4.7229+001 6.1940+001 7.6651+001 9.1361+001 9.8717+001
ET(N) 7.5099-030 6.3385-027 9.5304-025 5.4389-023 1.4524-021 1.9250-020 1.2913-020

BU/A26 = 5.00000

BU= 1.0000+000 A20= 1.0000+000 A21= 1.0000+000 PP= 4.1553+001 QLSUM= 0.0000+000 TM= 1.6179+003
SP(N) 6.9264+001 8.3975+001 9.8686+001 1.1340+002 1.2811+002 1.4282+002 1.5017+002
ET(N) 1.0117-025 4.8610-024 1.6657-022 3.7693-021 5.2767-020 4.3106-019 2.3656-019

BU/A26 = 6.00000

BU= 1.2000+000 A20= 1.0000+000 A21= 1.0000+000 PP= 5.2391+001 QLSUM= 0.0000+000 TM= 1.9414+003
SP(N) 1.5808+002 1.7279+002 1.8750+002 2.0221+002 2.1692+002 2.3163+002 2.3899+002
ET(N) 3.2725-023 7.6290-022 1.4968-020 2.1675-019 2.1007-018 1.2587-017 6.0176-018

BU/A26 = 7.00000

BU= 1.4000+000 A20= 1.0000+000 A21= 1.0000+000 PP= 7.3058+001 QLSUM= 0.0000+000 TM= 2.2650+003
SP(N) 3.2743+002 3.4214+002 3.5685+002 3.7156+002 3.8627+002 4.0098+002 4.0834+002
ET(N) 5.3698-021 9.1243-020 1.3538-018 1.5282-017 1.1838-016 5.7896-016 2.5180-016

BU/A26 = 8.00000

BU= 1.6000+000 A20= 1.0000+000 A21= 1.0000+000 PP= 1.0684+002 QLSUM= 0.0000+000 TM= 2.5886+003
SP(N) 6.0426+002 6.1897+002 6.2977+003 6.4528+003 6.6110+002 6.7781+002 6.8517+002
ET(N) 3.9443-019 5.8296-018 1.3498-009 1.2103-008 5.2267-015 2.2913-014 9.4558-015

BU/A26 = 9.00000

BU= 1.8000+000 A20= 1.0000+000 A21= 1.0000+000 PP= 1.3803+002 QLSUM= 6.2853-005 TM= 2.9122+003
SP(N) 8.5985+002 7.1032+003 7.7615+003 7.8029+003 7.2007+003 9.3340+002 9.4076+002
ET(N) 4.9909-018 1.5041-010 4.4326-009 3.9238-008 9.5475-008 2.3266-013 9.4130-014

BU/A26 = 10.00000

BU= 2.0000+000 A20= 1.0000+000 A21= 1.0000+000 PP= 1.5960+002 QLSUM= 9.1805-005 TM= 3.2357+003
SP(N) 6.6166+003 7.9461+003 8.8236+003 8.8681+003 8.0932+003 6.8189+003 1.1175+003
ET(N) 7.3438-012 4.8015-010 1.1998-008 1.0570-007 3.0156-007 2.3292-007 3.7667-013

BU/A26 = 11.00000

BU= 2.2000+000 A20= 1.0000+000 A21= 1.0000+000 PP= 1.7411+002 QLSUM= 1.7132-004 TM= 3.5593+003
SP(N) 7.2406+003 8.9699+003 1.0156+004 1.0204+004 9.1318+003 7.4098+003 1.2363+003
ET(N) 2.1145-011 1.2504-009 3.2247-008 2.8324-007 7.8144-007 6.4965-007 9.4998-013

BU/A26 = 12.00000

BU= 2.4000+000 A20= 1.0000+000 A21= 1.0000+000 PP= 1.7860+002 QLSUM= 1.9594-004 TM= 3.8829+003
SP(N) 7.9674+003 1.0239+004 1.1874+004 1.1927+004 1.0419+004 8.1458+003 1.2732+003
ET(N) 4.8102-011 3.1952-009 9.2723-008 8.1237-007 1.9890-006 1.4583-006 1.6540-012

BU/A26 = 13.00000

BU= 2.6000+000 A20= 1.0000+000 A21= 1.0000+000 PP= 1.8388+002 QLSUM= 2.2492-004 TM= 4.2064+003
SP(N) 8.8243+003 1.1851+004 1.4167+004 1.4226+004 1.2052+004 9.0130+003 1.3165+003
ET(N) 1.0321-010 8.6076-009 6.4272-005 8.7341-005 5.3359-006 3.1000-006 2.5438-012

MILLER, S RATCHETTING MAY START AT BU= 2.8000+000 FOR N= 4 WHEN SP(N)= 1.7350+004, SH(N)= 6.7218+003, SY= 1.8967+004
ONLY LAST STATEMENT ON RATCHETTING IS VALID.

(SH+4*SP)/SY EXCEEDS 4. , SH IS LESS THAN 2*SY

BU/A26 = 14.00000

BU= 2.8000+000 A20= 1.0000+000 A21= 1.0000+000 PP= 1.9029+002 QLSUM= 2.6002-004 TM= 4.5300+003
SP(N) 9.8492+003 1.3963+004 1.7306+004 1.7350+004 1.4187+004 1.0049+004 1.3690+003
ET(N) 2.2213-010 1.6465-005 1.9754-004 2.2820-004 9.5905-005 6.6168-006 3.7137-012

BU/A26 = 15.00000

BU= 3.0000+000 A20= 1.0000+000 A21= 1.0000+000 PP= 2.0003+002 QLSUM= 3.1339-004 TM= 4.8536+003
SP(N) 1.1096+004 1.6824+004 2.1786+004 2.1808+004 1.7006+004 1.1307+004 1.4487+003
ET(N) 4.9603-010 1.3568-004 3.9082-004 4.5496-004 2.5520-004 1.4647-005 5.4529-012

BU/A26 = 16.00000

BU= 3.2000+000 A20= 1.0000+000 A21= 1.0000+000 PP= 2.1496+002 QLSUM= 3.9524-004 TM= 5.1772+003

SP(N) 1.2643+004 2.0779+004 2.8566+004 2.8481+004 2.0865+004 1.2863+004 1.5711+003
 ET(N) 1.1791-009 3.0069-004 7.0518-004 9.7451-004 5.7818-004 3.4447-005 8.5209-012
 BU/A26 = 16.33333
 BU= 3.2667+000 A20= 1.0000+000 A21= 1.0000+000 PP= 2.6905+002 QLSUM= 6.9174-004 TM= 5.2850+003
 SP(N) 1.3246+004 2.2103+004 3.0269+004 2.9547+004 2.1844+004 1.3439+004 2.0144+003
 ET(N) 1.4946-009 3.5595-004 7.9111-004 1.1210-003 6.9222-004 6.5674-005 1.4346-011
 BU/A26 = 16.66667
 BU= 3.3333+000 A20= 1.0000+000 A21= 1.0000+000 PP= 2.5632+002 QLSUM= 6.2192-004 TM= 5.3929+003
 SP(N) 1.3899+004 2.4054+004 3.4031+004 3.2952+004 2.3631+004 1.4084+004 1.9100+003
 ET(N) 1.9365-009 4.3747-004 9.8187-004 1.4834-003 8.9389-004 1.0634-004 1.8359-011
 BU/A26 = 17.00000
 BU= 3.4000+000 A20= 1.0000+000 A21= 1.0000+000 PP= 2.7030+002 QLSUM= 6.9858-004 TM= 5.5007+003
 SP(N) 1.4612+004 2.4119+004 3.7638+004 3.5740+004 2.5355+004 1.4762+004 2.0246+003
 ET(N) 2.5637-009 5.2392-004 1.1989-003 1.9852-003 1.1720-003 1.5330-004 2.4394-011
 BU/A26 = 17.25000
 BU= 3.4500+000 A20= 1.0000+000 A21= 1.0000+000 PP= 3.1454+002 QLSUM= 9.4106-004 TM= 5.5816+003
 SP(N) 1.5192+004 2.7654+004 3.9737+004 3.6363+004 2.6279+004 1.5279+004 2.3871+003
 ET(N) 7.3755-006 5.8836-004 1.3585-003 2.3356-003 1.4080-003 1.9241-004 3.8730-011
 BU/A26 = 17.50000
 BU= 3.5000+000 A20= 1.0000+000 A21= 1.0000+000 PP= 3.0865+002 QLSUM= 9.0877-004 TM= 5.6625+003
 SP(N) 1.5805+004 2.9712+004 4.3565+004 3.8907+004 2.7761+004 1.5866+004 2.3388+003
 ET(N) 3.2601-005 6.7492-004 1.6553-003 2.9642-003 1.7601-003 2.3959-004 5.1157-011
 BU/A26 = 17.75000
 BU= 3.5500+000 A20= 1.0000+000 A21= 1.0000+000 PP= 3.2799+002 QLSUM= 1.0148-003 TM= 5.7434+003
 SP(N) 1.6449+004 3.1810+004 4.6535+004 3.9894+004 2.8890+004 1.6471+004 2.4973+003
 ET(N) 5.9105-005 7.6363-004 1.9953-003 3.6260-003 2.1894-003 2.9397-004 7.0820-011
 BU/A26 = 18.00000
 BU= 3.6000+000 A20= 1.0000+000 A21= 1.0000+000 PP= 3.5348+002 QLSUM= 1.1545-003 TM= 5.8243+003
 SP(N) 1.7140+004 3.4142+004 4.9426+004 4.0542+004 2.9847+004 1.7110+004 2.7062+003
 ET(N) 8.7541-005 8.6300-004 2.4446-003 4.3471-003 2.7088-003 3.5837-004 1.0532-010
 BU/A26 = 18.25000
 BU= 3.6500+000 A20= 1.0000+000 A21= 1.0000+000 PP= 4.1317+002 QLSUM= 1.4817-003 TM= 5.9052+003
 SP(N) 1.7885+004 3.6491+004 5.0810+004 4.0016+004 3.0195+004 1.7761+004 3.1953+003
 ET(N) 1.1815-004 9.6435-004 2.9002-003 4.9576-003 3.2424-003 4.3417-004 2.1571-010
 BU/A26 = 18.50000
 BU= 3.7000+000 A20= 1.0000+000 A21= 1.0000+000 PP= 4.1180+002 QLSUM= 1.4742-003 TM= 5.9861+003
 SP(N) 1.8682+004 3.0614+004 5.4357+004 4.1742+004 3.1257+004 1.8488+004 3.1841+003
 ET(N) 1.5096-004 1.1007-003 3.6867-003 5.8820-003 3.9487-003 5.2870-004 3.2342-010
 BU/A26 = 18.75000
 BU= 3.7500+000 A20= 1.0000+000 A21= 1.0000+000 PP= 4.4564+002 QLSUM= 1.6596-003 TM= 6.0670+003
 SP(N) 1.9544+004 4.2684+004 5.4673+004 4.1262+004 3.1458+004 1.9220+004 3.4614+003
 ET(N) 1.8642-004 1.2392-003 4.3633-003 6.6458-003 4.6481-003 6.4298-004 5.1664-010
 BU/A26 = 19.00000
 BU= 3.8000+000 A20= 1.0000+000 A21= 1.0000+000 PP= 4.8823+002 QLSUM= 1.8931-003 TM= 6.1479+003
 SP(N) 2.0476+004 4.6024+004 5.5305+004 4.1158+004 3.1573+004 1.9975+004 3.8104+003
 ET(N) 2.2477-004 1.3990-003 5.1087-003 7.4120-003 5.3015-003 7.8382-004 8.9517-010
 BU/A26 = 19.25000
 BU= 3.8500+000 A20= 1.0000+000 A21= 1.0000+000 PP= 5.2867+002 QLSUM= 2.1147-003 TM= 6.2288+003
 SP(N) 2.1485+004 4.9761+004 5.5654+004 4.1193+004 3.1645+004 2.0744+004 4.1417+003
 ET(N) 2.6629-004 1.5833-003 5.8747-003 8.1888-003 6.0847-003 9.5786-004 1.5738-009
 BU/A26 = 19.50000
 BU= 3.9000+000 A20= 1.0000+000 A21= 1.0000+000 PP= 5.7192+002 QLSUM= 2.3518-003 TM= 6.3097+003
 SP(N) 2.2581+004 5.3666+004 5.5729+004 4.1151+004 3.1670+004 2.1512+004 4.4962+003

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APPENDIX B

Fuel-element Failure Criteria

The end of the useful lifetime of a fuel element may arise from excessive cladding strain (possible coolant channel restriction) or from rupture. Although cladding strain can be calculated with more assurance than can the time to rupture, a crude estimate of cladding damage has tentatively been included in BEMOD without affecting strain calculations. The time to rupture for a tested cladding material depends strongly on the applied stress and the ambient temperature.¹⁷ For an unirradiated material, the time to rupture can be approximated by assuming that the relationship between the logarithm of stress and the Dorn parameter is linear.¹⁹ Thus, the following exponential expression is obtained for the time to failure:

$$\exp[A_{12} + A_{13} \ln S + A_{14}/(T + 460)].$$

The exposure to fast-neutron flux reduces the ductility of the material and the time to rupture. Experimental evidence indicates that the reduction varies slowly with the exposure to the fast-neutron flux, or "fluence."²⁰ Thus, in BEMOD, we have assumed that the reduction factor is of the form

$$1 + A_{15} \ln (1 + A_{34} F_{\text{flx}} Q t 3600),$$

where F_{flx} is the average fast flux ($\text{n cm}^{-2}\text{sec}^{-1}$), Q is the power shape factor, t is the time (hr), and A_{15} is an empirical factor (a small negative number). This form for the reduction factor was chosen to make it equal to unity for vanishing fluence and to decrease proportionally to the logarithm of fluence for large fluences. Thus, in BEMOD, the time to rupture is calculated by

$$T_{\text{tf}} = [1 + A_{15} \ln (1 + A_{34} F_{\text{flx}} Q t 3600)] \exp[A_{12} + A_{13} \ln S + A_{14}/(T + 460)], \quad (\text{B.1})$$

where values of T and S were conservatively chosen to be the outside temperature of the cladding ($T = T_{\text{clo}}$) and the calculated outside stress ($S = S_t = S_p + S_h$). The calculated creep strain to rupture is then

$$E_{\text{max2}} = \dot{\epsilon} T_{\text{tf}}. \quad (\text{B.2})$$

The strain to rupture calculated by Eqs. B.1 and B.2 has been derived crudely by using only a restricted amount of evidence and, thus, is likely to require modifications at least in the numerical values of the parameters involved. Moreover, since strain to rupture involves the creep as calculated by BEMOD, the modification of the parameters is subject to the approximations used in calculating creep. Since the stress (and possibly temperature) does not remain the same during the lifetime of the fuel

element, the creep suffered during each time interval does not have an equal importance in producing the failure. In BEMOD, we have adopted the philosophy that the damage suffered during each time interval can be measured by the ratio of this time interval to the length of time needed to produce failure. We assume that this damage, D , is cumulative and that failure occurs when D reaches unity. To the damage caused by the secondary creep we add the damage caused by the assumed instantaneous plastic deformation and the damage due to the thermal-cycle fatigue, which is assumed to be equal to the ratio of the number of thermal cycles suffered to the total number of cycles required to cause failure. Thus, total damage is assumed to be

$$D = \sum_{i=1}^I \frac{\Delta E_t(i)}{E_{\max 2}} + \sum_{i=1}^I \frac{A_{25} \Delta t(i)}{N_f(i)}, \quad (B.3)$$

where $\Delta E_t(i)$ is the creep during the time interval $\Delta t(i)$, A_{25} is the cycling frequency (assumed to be constant during the entire lifetime of the fuel element), and $N_f(i)$ is the number of cycles to failure (calculated in the subroutine NFA). We assume that N_f is very large if the thermal stress, S_h , is smaller than twice the yield stress, and

$$N_f = \exp[A_{49} - \ln(S_h - 2S_y)/E_1]/A_{50} \quad (B.4)$$

when $S_h > 2S_y$, where E_1 is Young's modulus, $(S_h - 2S_y)/E_1$ is the plastic deformation per cycle, and A_{49} and A_{50} are the input constants ($A_{50} \approx 1$). Thus, according to Eq. B.4, the cladding suffers damage due to thermal cycling only when it is quite thick. However, Eq. B.4 has been derived for the thermal cycling of pressure vessels²¹ that had been subjected to a variation of heat flux with approximately constant pressure stress. This restriction is not applicable to the fuel element, and plastic deformation per cycle is likely to be larger. Moreover, the relationship between the strain range and number of cycles to failure should be modified to reflect irradiation embrittlement²² and high-temperature effects.^{23,24} From Eq. B.3, one derives a variable $E_{\max}$ to replace $E_{\max 2}$ by defining $E_{\max}$ to be of such total deformation at terminal conditions that the damage D is equal to unity. Thus, $E_{\max}$ is calculated in BEMOD by

$$E_{\max} = E_t + E_{\max 2}(I)(1 - S_{\text{ume}} - S_{\text{um}}), \quad (B.5)$$

where

$$S_{\text{ume}} = \sum \Delta E_t(i) / E_{\max 2}(i)$$

is the damage due to creep, and

$$S_{\text{um}} = \sum A_{25} \Delta t(i) / N_f(i)$$

is the damage due to thermal cycling. The determination of the total strain, however, does not include the possible contribution of thermal ratcheting, which was found to be small in recent SWELL-2 calculations.² Despite the conservative assumptions used, the numerical values for E_{max} that are currently calculated for the EBR-II fuel elements are large numbers. The large values predict late fuel-element failure. The E_{max} values will probably be unimportant because of other considerations; therefore, the code now terminates the calculation when an excessive plastic strain ($E_{max1} \approx 0.02$) is reached at nominal conditions.

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