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THE MUELLER-REGGE POLE DESCRIPTION
OF PIONIZATION IN PROTON-PROTON SCATTERING*

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ABSTRACT

The Mueller-Regge Pole formalism is applied to single pion inclusive data in pp scattering. The data is described for $P_{lab} \geq 12$ GeV/c, $y_{lab} \geq 0.65$, and $p_{\perp} = 0.2, 0.4, 0.8$.

The Mueller-Regge pole formalism¹ has proven highly successful in describing and correlating much of the growing body of data on inclusive cross sections and correlations.² In the central plateau at $y=0$ the energy dependence of the data for pion single-particle inclusive cross sections has been shown, in the 'Ferbel Plot',³ to be roughly consistent with the Mueller double-Regge pole formalism.

We study the application of the Mueller double-Regge pole formalism to the two reactions, $pp \rightarrow \pi^\pm + \text{anything}$, for which data is available over a broad range of rapidities and energies. Thus our analysis tests the ability of the formalism to describe both the rapidity and energy dependence of the data. We do not attempt to test the factorization properties of the pure pole model since there is little or no data on πp and Kp scattering above 100 GeV/c.

We make the standard argument^{1,4} that the invariant amplitude $E \frac{d^3\sigma}{dp^3}$ for the reaction, $a+b \rightarrow c + \text{anything}$, is related by unitarity to the imaginary part of the forward amplitude of the reaction, $a\bar{c}b \rightarrow a\bar{c}b$, and that this three-body amplitude is dominated at high energy in the central region by Regge pole exchanges between a and $\bar{c}$ and between $\bar{c}$ and b . We write the Mueller double-Regge formula for the invariant cross section of the reactions, $pp \rightarrow \pi^\pm + \text{anything}$,

as

$$E \frac{d^3\sigma}{dp^3} (pp \rightarrow \pi^\pm + X) = \sum_{i,j} \beta_i^{pp} \tau_i e^{(Y/2+y)(\alpha_i-1)} \gamma_{ij}^{\pi^\pm} (p_\perp) \\ \times e^{(Y/2-y)(\alpha_j-1)} \tau_j \beta_j^{pp} \quad (1)$$

The rapidity difference between beam and target particles is denoted $Y = y_a - y_b$. The $a\bar{c}$ and $\bar{c}b$ systems are assigned the energy variables $\exp[Y/2 - y]$ and $\exp[Y/2 + y]$ with y the center-of-mass rapidity of the pion. The i and j subscripts run over the allowed Regge exchanges. The β_i^{pp} is the three-point coupling and $\gamma_{ij}^{\pi^\pm}(p_\perp)$ the four-point coupling of the Mueller 3-to-3 scattering diagram. The τ_i is the signature of the Regge exchanges, equal to $+1(-1)$ for even (odd) signature.

It is convenient to simplify the notation, taking advantage of the fact that we are dealing just with the reactions $pp \rightarrow \pi^\pm + X$, by defining

$$C_{ij}^\pm(p) \equiv 2\beta_i^{pp} \tau_i \gamma_{ij}^{\pi^\pm} \tau_j \beta_j^{pp} (m_p)^{2-\alpha_i-\alpha_j} \quad (2)$$

We assume $\alpha_p=1$ and $\alpha_f=\alpha_\rho=\alpha_R=0.5$; the effects of lower-lying singularities are approximated with the exchange $\alpha_R=0$. We replace $\exp[Y]$ by s/m_p^2 and then include all mass terms in the $C_{ij}^\pm$ as defined above. Our formula for the inclusive cross section now reads

$$E \frac{d^3\sigma}{dp^3}(pp \rightarrow \pi^\pm + X) = C_{pp} + C_{pR}^\pm s^{-1/4} \cosh(y/2) + C_{RR}^\pm s^{-1/2} + C_{RR}^\pm s^{-3/4} \cosh(y/2) + \dots \quad (3)$$

The C 's are functions of $p_\perp$. We assume $C_{pp}^+ = C_{pp}^- \equiv C_{pp}$ as required by isospin invariance. In general $C_{ij}^+ \neq C_{ij}^-$ in eq.(3) since we have added together even and odd signature exchanges having the same trajectory intercept.

The approach to the scaling limit is predicted by eq.(3) to go as $a+bs^{-1/4} \cosh(y/2)$ at any fixed transverse momentum⁵. This relation appears approximately valid at fixed y .^{3,6} We tested the predicted rapidity dependence by plotting the data⁷ versus $\cosh y/2$ at fixed s and $p_{\perp}$ ($=0.2, 0.4, 0.8$); one such plot is exhibited in Fig.1. It can be seen that the data is roughly linear in $\cosh(y/2)$ for $y_{lab} = \frac{Y}{2} - y \geq 0.65$ (the solid curves in the plot terminate at $y_{lab} = 0.65$).^{*}

This linear behavior in $\cosh(y/2)$ is encouraging but one should also check the predictions of the Mueller-Regge pole formalism for the energy dependence of the parameters of these straight lines. For fits of the form $A^{\pm} + B^{\pm} \cosh y/2$ to $E \frac{d^3\sigma}{dp^3} (pp \rightarrow \pi^{\pm} + X)$ at fixed s and $p_{\perp}$, we expect

$$\begin{aligned} A^{\pm} &\cong C_{PP} + C_{RR}^{\pm} s^{-1/2} \\ B^{\pm} s^{1/4} &\cong C_{PR}^{\pm} + C_{RR'}^{\pm} s^{-1/2} \end{aligned} \quad (4)$$

at fixed $p_{\perp}$. We plot $A^{\pm}$ and $B^{\pm} s^{1/4}$ (obtained from the data⁷ at $y_{lab} > 0.65$) versus $s^{-1/2}$ in Fig. 2. For clarity we show the data only at those energies ($P_{lab} = 12, 24, 500, 1500$ GeV/c) for which A and B are well-determined.

* Our data cut at fixed y_{lab} is roughly equivalent to a requirement that both subenergies in the $a\bar{c}b \rightarrow a\bar{c}b$ amplitude be above a fixed minimum value.

We see from Fig.2 that the data are consistent with the linear behavior in $s^{-1/2}$ of $A^\pm$ and $B^\pm s^{1/4}$ indicated in eq.(4). The data are also consistent with the expectation that A^+ and A^- should have a common intercept at $s^{-1/2} = 0$, since $C_{pp}^+ = C_{pp}^-$. It is important to remember that the error estimates on $A^\pm$ and $B^\pm s^{-1/4}$ shown in Fig.2 reflect only the quoted statistical errors in the experimental data and do not include systematic errors. The marked energy variation in $B^\pm s^{-1/4}$ for all three values of $p_\perp$ considered should be noted; it is this feature of the data that necessitates the inclusion of lower Regge effects, simulated by the R' pole. We conclude that the Mueller-Regge couplings included in eq.(3) are the minimum number needed to adequately describe the data⁷.

Our formula, eq.(3), for the single particle inclusive cross section at fixed transverse momentum is linear in the unknown coupling coefficients. It is straightforward to perform multiple linear regressions using eq.(3), yielding both the couplings and their errors resulting from the statistical errors in the data. Guided by our graphical analysis we cut the data⁷ at $y_{lab} = 0.65$ and use just the PP, PR, RR, and RR' couplings of eq.(3). The fit for $p_\perp = 0.4$ is shown as the solid lines in Fig.1 and the parameters are given in Table 1.

We have studied several sources of uncertainty in this analysis: the data cut at $y_{lab} = 0.65$, the number of terms used in eq.(3), and the choice of energy variable. The choice of data cut does not

significantly change the quality of the fit as long as $y_{\text{cut}} \gtrsim 0.5$, as our graphical analysis indicated; the PP and PR couplings change by a few percent, the others are much less stable. By varying the number of terms retained in eq.(3) we have found, as suggested in our graphical analysis, that PP, PR, RR, and RR' couplings are the minimal set of couplings needed to obtain a reasonable fit; as one adds more couplings the PP coupling changes by 5-10% and the PR coupling by about 50%. The energy variables $p_a \cdot p_c$ and $p_b \cdot p_c$ and the equivalent energy variables $\cosh(Y/2-y)$ and $\cosh(Y/2+y)$ will lead to successful Mueller-Regge Pole descriptions for $y_{\text{lab}} \gtrsim 0.65$; * the variables $(p_a - p_c)^2 = t$ and $(p_b - p_c)^2 = u$ change sign for $y_{\text{lab}} \sim \log(m_p/p_{\perp})$ which makes them unsuitable except near 90° or at large $p_{\perp}$.

It may easily be determined that the couplings obtained from the 'Ferber Plot' for the reactions $pp \rightarrow \pi^\pm + X$ are incompatible with our findings, which suggests strong $s^{-1/2}$ terms should be present. From Table 1 we find the ratio $C_{\text{PR}}^\pm / C_{\text{PP}} \sim -2$ or -4 whereas in the 'Ferber Plot'³ the ratio is ~ -1 . This change is due to the increased difficulty of fitting both s and y dependence - requiring RR and RR' terms.

We have found that $\gamma_{\text{pf}}^- \sim -3\gamma_{\text{pp}}^-$ by use of eqs.(1-3), Table 1,

* This was tested by making plots of the data at fixed s and p versus $\frac{1}{\sqrt{\cosh(Y/2-y)}} + \frac{1}{\sqrt{\cosh(Y/2+y)}}$; the plots are linear for $y_{\text{lab}} \geq 0.65$.

and the estimates of β_f^{PP} and β_p^{PP} given in ref.[8].

The p dependence of C_{PP} , C_{PR}^+ , and C_{PR}^- for $0.2 \leq |t| \leq 0.8$ is very nearly $ae^{-6p_\perp}$ (judging by semilog plots of the couplings in Table 1) just as the data itself is. Adding several more terms to eq.(3) does not change this trend.

A possibly important reservation about this fit is that much of the chi-squared arises in the high s , small y region covered by the British-Scandinavian data, where the Regge description is most likely to be valid. The present ISR data show a rise at small y which our fit does not reproduce. Consequently we have considered the possibility that the Mueller double-Regge Pole formalism is only valid in the high s , small y region covered by the British-Scandinavian data and have made a fit to this data alone. The fit is shown with dashed lines in Fig.1 and the couplings are given in Table 1. We have used only the PP, PR, and RR terms of eq.(3) since further terms do not improve the fit significantly. The conclusions of the previous three paragraphs remain basically unchanged.

We conclude that the data on the reactions $pp \rightarrow \pi^\pm + X$ is compatible with the Mueller double-Regge Pole formalism for $P_{lab} \geq 12$ GeV/c, $y_{lab} > 0.65$ and $p_\perp = 0.2, 0.4, 0.8$, and requires seven double-Regge couplings: C_{PP} , $C_{PR}^\pm$, $C_{RR}^\pm$, $C_{RR}^\pm$. The PP and PR couplings have an exponential dependence on $p_\perp$ for $0.2 \leq p_\perp \leq 0.8$.

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CAPTIONS

Fig.1

Mueller-Regge Pole fits to $E \frac{d^3\sigma}{dp^3} (pp \rightarrow \pi^\pm + X)$ at $p_\perp = 0.4$.

The British-Scandinavian data is shown with solid triangles.

Fig.2

The straight-line (in $\cosh y/2$) coefficients of eq.(4) as a function of $s^{-1/2}$.

Table 1

The coefficients of eq.(3) resulting from Mueller-Regge Pole fits to the data of ref.[7].

TABLE 1

	All Data with $y_{lab} > 0.65$			British-Scandinavian Data		
	$p_{\perp}=0.2$	$p_{\perp}=0.4$	$p_{\perp}=0.8$	$p_{\perp}=0.2$	$p_{\perp}=0.4$	$p_{\perp}=0.8$
C_{RR}^{+}	-128 ± 24	-135 ± 7	-21.6 ± 0.9	0	0	0
C_{RR}^{+}	121 ± 13	91 ± 4	12.9 ± 0.5	765 ± 75	185 ± 16	9.8 ± 9.5
C_{PR}^{+}	-116 ± 2	-34.5 ± 0.5	-3.73 ± 0.07	-298 ± 16	-71 ± 3	-5.4 ± 3.0
C_{PP}	63.1 ± 0.4	16.9 ± 0.1	1.632 ± 0.016	75.8 ± 1.3	19.9 ± 0.2	1.88 ± 0.23
C_{PR}^{-}	-126 ± 2	-40.0 ± 0.4	-4.28 ± 0.05	-325 ± 18	-71 ± 3	-6.0 ± 2.9
C_{RR}^{-}	31 ± 6	86 ± 3	8.1 ± 0.4	908 ± 91	163 ± 15	9.1 ± 9.6
C_{RR}^{-}	45 ± 12	-117 ± 6	-10.2 ± 0.8	0	0	0

Fig.1

Fig.2