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This contract continues to center on non-linear analysis of plasma instabilities in the threshold regime. The emphasis remains on carrying out calculations in realistic geometry and making comparison with experiment. Attention has shifted entirely to fully-ionized plasma. During this year, we completed analysis of the collisional drift-wave instability, made considerable progress in explaining the explosive nature of the mirror-flute instability and in understanding the flute instability in the presence of both density and temperature gradients, and began non-linear analysis of the collisionless drift-wave instability. In addition, we have started to study the application of the threshold method to parametric instabilities in non-uniform plasma. Details of the results and comparison with experiment are given below.

1. Collisional Drift-Wave Instability:

We have completed the analysis of the collisional drift-wave instability including both density and temperature variations. The results have been submitted for publication¹ and also form

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part of the Ph.D. dissertation of Donald A. Monticello (Physics Dept., 1973). Good agreement with the experiment of Hendel, Chu and Politzer² was obtained. This work extended previous calculations by Stix³ and by Hinton and Horton⁴. The pioneering theory of Stix was not entirely self-consistent and yielded a result for the saturation amplitude of the drift-wave which was (as noted by the author) much larger than experiment. Hinton and Horton's calculation was self-consistent and much closer to experiment but neglected the radial variation of perturbed quantities and assumed zero amplitude for the zero-frequency harmonic. Our primary result is the recognition that it is essential to include radial variation of the perturbed quantities, together with the appropriate free-boundary velocity shear condition, to obtain a determinate equation for the amplitude of the zero-harmonic. Indeed, this amplitude was found and showed that flattening of the equilibrium density gradient by the zero-frequency harmonic is the dominant mechanism of non-linear saturation of this instability. [Note: the zero-harmonic, as well as the double-frequency harmonic, are generated by quadratic beating of the growing threshold linear mode with itself]. The effect of the double-frequency harmonic (which was the only one considered in the two earlier studies) was considerably smaller. The result is in good agreement with experiment.

2. Density-Gradient and Temperature-Gradient Flute Instabilities:

Previous work by Simon and Weng⁵ has explained the observed explosive-behavior of the collisionless mirror-flute instability.

For trapped densities very close to the critical onset value (about 10^7 cm^{-3}) which corresponds to $\Delta \ll 1$, ($\Delta \equiv \frac{n-n_c}{n_c}$) we obtained self-saturation of the lower frequency density-mode (the mode which is marginally stable at $\Delta = 0$). For $\Delta \geq \Delta_c$ however, where Δ_c is also small compared to unity, a non-linear instability is triggered in a higher frequency thermal mode by pumping from the lower-mode. This upper mode does not shut itself off and grows to large amplitude, in agreement with experiment⁶.

In an attempt to understand the underlying mechanism of the upper mode's instability, it is necessary to study the general linear theory of flute modes for arbitrary equilibrium density and temperature gradients. We have been able to construct⁷ a complete parameter-space description of the stability surfaces in n' , T' and p^2 . [Here $p^2 \sim n_0/(\text{constant} + k^2)$, and the prime denotes the spatial derivative]. Using the non-linear results of Weng, we can describe the directional evolution, in this space, of a point initially located slightly above the critical surface. We show that the system reaches the stable surface, near its starting point, for $\Delta < \Delta_c$ and misses entirely for $\Delta > \Delta_c$. This result can also be interpreted as the growth of an equilibrium temperature gradient (by mode buildup) sufficient to trigger the upper mode's instability. This work is continuing and will be used to explore the general non-linear threshold behavior of mirror plasmas with arbitrary equilibrium temperature and density gradients.

An unexpected "spin-off" of this research was the resolution of two earlier and contradictory results⁸⁻⁹ on the threshold for the Buneman-Farley instability. We have shown that inclusion of finite ion inertia and finite ion-magnetization (or scale size), although small, yields a threshold for linear onset of this instability at a finite value of k ¹⁰. Omission of inertia predicts onset at $k = \infty$, and omission of finite magnetization (or scale size) predicts onset at $k = 0$. Similar corrections apply to the temperature-gradient instability¹¹ and when included yield an optimum k for onset of this instability. This correction is essential for application of fluid-like equations to the thermal-gradient flute instability and for threshold non-linear calculation.

3. Collisionless Drift Instability:

Preliminary work has begun on the non-linear saturation behavior of the collisionless drift instability. Emphasis is on explaining the experiments of Politzer¹² and of Brossier et al¹³. We are studying the extension of the threshold theory to include the drift-kinetic equation, since this is essential for treatment of the collisionless drift.

4. Personnel:

The principal investigator has devoted 20% of his time to this project during the academic year, and two summer months. Two graduate students were supported by this contract and one other has an NDEA fellowship. Dr. Donald A. Monticello (Physics

Dept.) completed his thesis on the collisional drift instability and now has a post-doctoral appointment at the Institute for Advanced Study. Mr. V. G. Abhyankar (MAS Dept.) is working on the temperature-gradient flute theory and is expected to complete his thesis work sometime in the spring. Mr. Leonard Gross (MAS Dept.) was supported on an NDEA fellowship. He is working on the collisionless drift instability.

Dr. Peter Catto has joined the faculty of the Dept. of Mechanical and Aerospace Sciences. He intends to join the principal investigator in studying the application of threshold methods to parametric instabilities in non-uniform plasma.

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