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FAST BREEDER REACTOR
STATIC PHYSICS METHODS DEVELOPMENT
AND ANALYSIS PROJECT

— Quarterly Technical Report —

April - June 1968

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— Quarterly Technical Report —

April - June 1968

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1. INTRODUCTION

In March 1967 Babcock & Wilcox (B&W) initiated a program of analytical methods development applicable to the physics design of fast breeder reactors. One phase of this effort has as its goal the development and utilization of a design-oriented, one-dimensional criticality and depletion code applicable to the physics analysis of fast reactors. This phase of the methods development effort is under full sponsorship of the U.S. Atomic Energy Commission (AEC). Utilization of the code will consist of an investigation in which the design code FARED will be used to relate the sensitivity of certain fast reactor design parameters to the complexity of the physics model used to evaluate them. Finally, several fast reactor critical experiments will be analyzed to verify the applicability of the code and its cross-section libraries. Future topics of study under the FARED project include (1) extension of the criticality and depletion segments to two dimensions with provision for complex fuel management studies, and (2) study of the importance of generating multigroup cross sections using space-dependent spectrum weighting.

In April 1968, a sensitivity study relating to burnup and fuel management in fast reactors was initiated as Task 4 of the FARED project. The objectives of this study are described in the third quarterly progress report (BAW-3867-3).¹

This fourth quarterly progress report for the FARED project covers work done during the period April - June 1968. Major accomplishments during this period are as follows:

1. The ultrafine group option in MC² (Na and O treated as ELMOE materials) was successfully executed.
2. The 68-group (fine) version of MC² was successfully executed.
3. The unresolved resonance routine for REGA was completed, and substantial debugging of the modified STRIP routine was carried out.

4. The RAIM routine is now in the debugging stage. The routine has been formulated so that even with multi-library capability only a little more than 32k storage is required.
5. REDEP formulation is well underway, and initial capability of this depletion segment has been specified.
6. For the burnup calculations, preliminary investigations of the reference library, core and blanket library specification, and B_4C control characteristics were completed. The depletion and fuel management studies for the small core have been completed for the first three cycles.

2. ENDF/B IMPLEMENTATION

2.1. The MC² Code

The RPI revisions of the MC² code, which permit for more efficient handling of resonance and Legendre coefficient data, have been incorporated in our version of MC², and the ANL test case (ANL-7318) has been successfully executed in both fine and ultrafine group modes. These modifications permit the execution of the ultrafine group test problem in about the same time required by the fine group case. On a multiprogram machine the execution of hundreds of short read and write (buffer in and buffer out) statements may result in run times that are many times longer than that of the batch processing machine. The RPI modifications eliminate such inefficiencies in the MC². The following times were observed in our test runs:

	<u>Fine group</u>	<u>Ultrafine group (ELMOE - Na)</u>
CP	322.6	332.4
PP	339.8	557.2

A 68-group version of the new MC² is now in preparation. We expect no difficulty with this task since we have successfully executed a 68-broad group version of the original MC² in the fine group mode.

2.2. Cross-Section Preparation

No production work in cross-section preparation or evaluation was performed this quarter.

3. FARED CODE DEVELOPMENT

3.1. Input Routine (P. N. Colpo)

The FARIN portion of the FARED code performs the functions of reading card input data, organizing the data, creating disk files, and establishing a large part of common storage. The input philosophy adopted for the presentation of punched card data to the FARED code is briefly described below.

The punched card input to FARED that specifies the reactor configuration is handled by the FARIN program and is presented in a free format manner. Items of data are punched on a card as strings of adjacent characters which form a field. Fields are separated by one or more blanks or the end of a card. Data may be punched anywhere on a card. Five types of data fields are recognized as permissible input:

1. Integers — An adjacent string of characters which may include a sign followed by a string of decimal digits. The minus (-) must be punched for a negative integer, but the plus (+) is optional. A blank will terminate the field.

2. Floating Point — An adjacent string of characters which may include a sign followed by a string of decimal digits and decimal point followed by a signed exponent. The presence of a decimal point or an exponent signals that the field is a floating point number. The exponent is not required to be punched if the decimal point is punched. The minus (-) must be punched for a negative number, but the plus (+) is optional. The decimal point may be punched anywhere in the number field. If the decimal point is omitted and the field is judged to be floating, then the decimal point is assumed to precede the first significant digit.

The exponent may be specified as a one- or two-digit integer preceded by a sign, (+) or (-), or the alphabetic character E or the alphabetic character E and a sign. If the sign of the exponent is not punched,

If the foregoing information were presented so that each field were punched somewhere on a separate card and the order of the cards corresponded to the order of the data fields on the foregoing illustration, then the transmission of data in both instances would be identical.

Data as presented to FARIN are identified by category and by sequences of data or individual data items in each category. The categories of data and the word configuration that signals the presence of each category are as follows:

GDATA — general data that are applicable to the reactor design being considered. Included among the data items in this category are reactor geometry parameters, group structure parameters, calculation options to be exercised, and various convergence criteria.

BULK — identifiers and parameters describing bulk materials and zone materials. A bulk material is the basic combination of nuclides that occurs together in a material naturally or in a manufactured state. A zone material, in the initial reactor configuration, is a specified combination of bulk materials. Included in the bulk material specification is an identification of the nuclides making up the bulk material, the number density concentrations of the respective nuclides, an indication of the role of each nuclide in RAIM criticality searches, an indication of the presence or absence of the bulk material in a pin cell, and the identification of all zone materials which are made up of the bulk material described.

More than one bulk material may be described following the data category identifier BULK.

ZONE — identifiers and parameters that specify a zone of the reactor. A zone is some physical volume of the reactor whose identity, location, size, and composition are determined by the data parameters specified.

BLOCK — identifiers and parameters that specify the makeup of the blocks of a reactor. A block is one or more zones in which it is assumed that microscopic cross-section sets for common nuclides are identical. Also provided are the buckling required for the neutron spectrum calculation and identification of the cells that are found in the respective blocks.

CELL — identifiers and parameters that describe the different cells to be considered in the calculation.

To illustrate the method of specifying a reactor configuration, consider the following input example for a three-zone reactor:

```

GDATA CYL 30 15 0 1 1 0 2 0.001 0.001 0.0 1.0 2450. 300.0
BULK BM1 1 FUEL PU239 0.002 PU240 0.0005
      DILUENT U238 0.035
      MIX 016 0.08
      0.5 ZM1
BM2 NA23 0.051
      0.36 ZM2
      0.30 ZM5
BM3 FE54 0.05 CR48 0.04
      0.14 ZM6
      0.10 ZM7
BM4 2 TA82 0.06
      1.0 ZM8
BM5 1 FUEL PU239 0.003 PU240 0.0005
      DILUENT U238 0.034
      MIX 016 0.08
      0.6 ZM9
BM6 1 U238 0.04 016 0.08
      0.6 ZM10
ZONE Z1 CORE 21.0 1.0 0.89 ZM1 ZM2 ZM6 0.11 ZM8
     Z2 CORE 50.0 1.5 0.78 ZM9 ZM2 ZM6 0.21 ZM8
     Z3 BLANKET 70.0 1.75 1.0 ZM10 ZM5 ZM7
BLOCK B1 Z1 Z2 0.005 C1 C2
      B2 Z3 0.007 C3
CELL C1 1 SQ 0.51 2.5 293.16 20.1
     C2 2 SQ 0.60 1.0E10 293.16 1.0E-10
     C3 1 SQ 0.57 2.3 293.16 19.0
ENDATA

```

Card 1

GDATA indicates that general data are to follow. The significance of the data parameter in the category is:

CYL cylindrical geometry
30 broad groups
15 downscattering groups
0 inner boundary condition option
1 outer boundary condition option
1 flux normalization option
0 perpendicular buckling option
2 depletion chain identification

0.001 inner convergence criterion
0.001 outer convergence criterion
0.0 inner radius of the innermost zone
1.0 extrapolation factor
2450. reactor power
300.0 reactor height

Card 2

BULK indicates the presence of one or more bulk material descriptions:

BM1 indicates that the bulk material being described is identified as bulk material 1.

1 is the cell index for those zone materials identified with bulk material 1.

FUEL indicates that one or more nuclides whose identification and number density concentrations follow are to be classified as fuel material in the RAIM criticality search.

Pu239 0.003 Pu240 0.0005
the identifications and number density concentrations of those nuclides classified as fuel in bulk material 1.

Card 3

DILUENT U238 0.035 - the identification and number density concentrations of those nuclides classified as diluents in bulk material 1.

Card 4

MIX O16 0.08 - the identification and concentrations of the remaining nuclides that make up bulk material 1. Nuclides designated as mix do not enter into the criticality search.

Card 5

0.5 ZM1 the material formed by multiplying each of the number densities of BM1 by 0.5 will be designated as zone material 1 and will be used in combination with other zone materials. The floating point number preceding one or more zone material identifiers is a mixing parameter.

Card 6

- BM2 indicates that the bulk material being described is identified as bulk material 2. The absence of a cell index and nuclide classification work implies that this material is never used in a cell calculation and that all of the nuclides making up the bulk material are classified as mix.
- NA23 0.051 - nuclide identification and concentration of the make-up of bulk material 2.

Card 7

- 0.36 ZM2 - bulk material 2 will be designated as zone material 2 and will be used in combination with other zone materials and will occupy 0.36 of the total volume of the combination.

Card 8

- 0.30 ZM5 - bulk material 2 will also be designated as zone material 5 and will occupy 0.30 of the total volume of the combination with other zone materials.

Cards 9 - 11

- BM3 indicates that the bulk material being described is identified as bulk material 3. The significance of the remaining parameters is similar to that discussed for bulk material 2.

Cards 12 - 13

- BM4 indicates a similar description of bulk material 4. Note that cell index 2 is given, implying that zone material 8 will be used in a cell calculation.

Cards 14 - 17

Bulk material 5 is described in a manner similar to that of bulk material 1.

Cards 18 - 19

Bulk material 6 is described in a manner similar to that of bulk material 4.

Card 20

ZONE indicates the presence of one or more zone descriptions:

- Z1 identifies the zone described as 1.
- CORE type of zone described.
- 21.0 outer radius of zone 1.
- 1.0 mesh interval size for dividing zone 1 for flux calculations.
- 0.89 the volume fraction of zone 1 which will be occupied by the combinations of zone materials 1, 2, and 6.
- 0.11 volume fraction of zone 1 which will be occupied by zone material 8.

Card 21

Z2 identifies the zone described as zone 2. The remaining description is similar to that of zone 1.

Card 22

Z3 identifies the zone described as zone 3.

BLANKET type of zone described. The remaining description is as given above.

Card 23

BLOCK indicates the presence of one or more block descriptions:

- B1 Z1 Z2 the block identified as block 1 is made up of zones 1 and 2.
- 0.005 buckling for the spectrum calculation.
- C1 C2 indicates that cells identified as 1 and 2 will be found in block 1.

Card 24

B2 identifies block 2 which is similarly described as above.

Card 25

CELL indicates the presence of one or more cell descriptions:

- C1 identifies the cell described as cell 1.
- 1 cell index indicates that since cell 1 is in block 1 (zones 1, 2), all zone materials found in zones 1 and 2 and tagged with a cell index of 1 are considered as the make up of cell 1.
- SQ lattice type.

0.51 lump size.
2.5 pitch.
293.16 fuel temperature.
20.1 macroscopic cross section of the outer region of
the cell.

Card 26

C2 identifies the cell described as cell 2. The remaining description is as given above.

Card 27

C3 identifies cell 3 which is similarly described.

Card 28

ENDATA indicates the end of the reactor configuration description.

The foregoing is not intended as a detailed description of the input for a reactor configuration. It is merely intended to convey to the reader the type of input that will be necessary for the completed program.

3.2. The REGA Routine (J. Jacobsen)

REGA prepares a 26-group (or less) microscopic cross-section set for each region in the reactor. Figure 3-1 is a flow chart for the resonance portion of REGA showing the principal units. During the preparation of the routines STRIP and UNRES, it became necessary to change program RETAP which constructs the REGA library tape. Parameters for mass, spin, and scattering length were rearranged, and Parameter $\bar{D}$, the minimum average level spacing for $\ell = 0$ resonances, was included. REGA is approximately 90% complete.

Subroutine STRIP, a component of REGA, computes the average microscopic capture and fission cross sections in the resolved resonance range using information and data from Common and the REGA library tape. STRIP has been changed to provide input and output appropriate to REGA. Modifications are being made to STRIP to provide for passing on to Subroutine UNRES capture and fission average cross sections for the range between $E_k \leq E \leq EULO$ when EULO (lower bound for unresolved resonance region, $\ell = 0$) is located between poly-group boundaries. A similar cross section defined in Subroutine UNRES for

the unresolved range extending from $EULO \leq E \leq E_{k-1}$ will be joined by UNRES to determine the capture and fission cross sections for poly-group k (see BAW-3867-1).² The modification to STRIP consists of changing the fine energy mesh which overlays the entire resolved resonance region, extending from $(EULO)_{MAX}^{\ell}$ to the lowest bound of the last REGA microgroup. The fine energy mesh width is adjusted in the vicinity of each EULO in order that $(EULO)^{\ell}$ for all ℓ corresponds to a boundary of one of the fine mesh intervals.

STRIP is approximately 95% complete. Test runs using the REGA library tape are being conducted.

Subroutine UNRES computes resonant capture and fission cross sections in the unresolved resonance range using information and data from Common and the REGA library tape. It is approximately 99% complete. Figure 3-2 illustrates the major feature of UNRES. Test runs using the REGA library tape are being evaluated.

Figure 3-1. REGA Overlay

Figure 3-2. UNRES Flow Chart

3.3. RAIM and REDEP Routines (J. M. Tilford, C. D. Carmichael)

3.3.1. Status of RAIM Program

The RAIM portion of the FARED code performs the one-dimensional diffusion theory flux calculation with optional criticality searches. A maximum of 30 energy groups are allowed with as many as 15 groups of down scattering. Since the numerical techniques employed in solving the one-dimensional diffusion equations are well developed, appropriate sections of the MACH-I code are used in the RAIM program. However, there are two restrictions in MACH-I to be resolved: (1) a storage requirement of 50 thousand locations, and (2) one microscopic cross section library applicable to the entire reactor.

Modifications have been made in MACH-I to allow the use of multiple microscopic cross section libraries, as many as one for each reactor region. The reactor is divided into blocks, each containing one or more regions. It is not necessary for the regions in a block to be adjacent. Associated with each block are a set of microscopic cross sections and other nuclear data. Subroutine MACRO, in the RAIM program, reads microscopic cross-section data for each isotope in a given block and calculates the corresponding macroscopic cross sections for each isotope in all reactor regions contained in that block. The macroscopic cross sections for each isotope are then summed to obtain the regional macroscopic cross sections. The entire process is repeated for each block in the reactor.

By computing the macroscopic cross sections in this manner, only the microscopic cross sections for a single isotope are contained in core storage at any time. In contrast, the present version of MACH-1 stores the microscopic cross sections for every isotope in BLANK COMMON. This modification permits the use of space-dependent microscopic cross sections and reduces the program's storage requirement by about 10 thousand locations.

Additional modifications are expected to further reduce the storage requirement to approximately 30 thousand locations. These include the elimination of external sources, no additional hydrogen down scattering, removal of certain options and edits, and a more optimized program overlay structure.

The RAIM program is designed to read space-dependent microscopic cross sections and to compute the appropriate regional macroscopic cross sections. These are then used in the flux calculation to determine the group fluxes and corresponding system k_{eff} . Various criticality searches are available for varying certain parameters to obtain a specified value of k_{eff} . The adjoint flux calculation and perturbation analysis will be contained in a separate program. Each of these programs will have an edit routine for calculating and printing various neutronics data.

3.3.2. RAIM Edit

All of the neutronics data that are calculated and printed in the RAIM Edit will correspond to flux distributions that are normalized to the total reactor power. The total power includes capture as well as fission power. The total source-normalized reactor power, P'_T , can be written in terms of the fission source-normalized fluxes, $\phi'_g(r)$, according to

$$P'_T = C \int_{V_T} \sum_g \sum_K \left[E_f^K \sigma_{fg}^K(r) + E_C^K \sigma_{cg}^K(r) \right] N^K(r) \phi'_g(r) dv$$

where

E_f^K = MeV/fission for isotope K

E_C^K = MeV/capture for isotope K

$\sigma_{fg}^K(r)$ = microscopic fission cross section for isotope K and energy group g at space point r

$\sigma_{cg}^K(r)$ = microscopic capture cross sections for isotope K and energy group g at space point r

$N^K(r)$ = concentration of isotope K at space point r

V_T = total reactor volume

$C = 1.603 \times 10^{-13}$ (watts per MeV/sec)

The power-normalized group fluxes, $\phi_g(r)$, are then computed in terms of the specified total reactor power, P_T , by

$$\phi g(r) = \frac{P_T \phi'_g(r)}{P'_T}$$

for all groups g . These power-normalized fluxes correspond to the average values in the transverse direction, and all quantities calculated according to these flux distributions represent the transverse average.

The following quantities are to be computed and will appear in the RAIM Edit output:

1. k_{eff}
2. Power-normalized group fluxes.
3. Power distributions —
 - a. Fission power densities.
 - b. Capture power densities.
 - c. Total power densities.
4. Region power —
 - a. Fission power.
 - b. Capture power.
 - c. Total power.
5. Average region power densities —
 - a. Average fission power density.
 - b. Average capture power density.
 - c. Average total power density.
6. Integrated group fluxes by region.
7. Region-averaged fluxes.
8. Reactor-averaged energies —
 - a. Average fission energy.
 - b. Average absorption energy.
 - c. Average flux energy.
9. Neutron balance by region —
 - a. Total region neutron source.
 - b. Total region neutron removal.
 - c. Net region neutron leakage.
10. Region volumes.
11. Reaction rates by isotope and region —
 - a. Fission rate.
 - b. Absorption rate.
 - c. Capture rate.
12. Percentage of total fission, absorption, and capture rates by isotope and region.

13. Ratio of region maximum-to-region average power density.
14. Ratio of region maximum-to-reactor average power density.
15. Conversion ratio by region, core and blanket.
16. Breeding ratio by region, core and blanket.

All quantities that are region-averaged will also be averaged over the entire reactor.

3.3.3. Depletion Methods

Three distinct methods for nuclide depletion calculations are being considered for the REDEP portion of the FARED code. These methods, as listed below, are in order of decreasing complexity.

Type 1 — Point Depletion

In this case, all isotope concentrations are space-dependent both before and after the depletion calculation. The diffusion equations may then use space-dependent macroscopic cross sections or those calculated from region-averaged isotope concentrations.

Type 2 — Point Depletion and Region-Homogenization

For this calculation all isotope concentrations are region-averaged before depletion. A point depletion is then performed, and the isotopic concentrations are homogenized within each region for the next flux-depletion calculation.

Type 3 — Regional Depletion

The isotopic concentrations are considered to be region-homogenized throughout the depletion and flux calculations. The space and time-dependent depletion equation for each isotope can be written in the form

$$\dot{N}^k(r, t) = \frac{dN^k(r, t)}{dt} = Q^k(r, t) - A^k(r, t)N^k(r, t) \quad (1)$$

where

$N^k(r, t)$ = concentration of isotope k at space point r and time t

$Q^k(r, t)$ = production rate of isotope k at space point r and time t

$A^k(r, t)N^k(r, t)$ = loss rate of isotope k at space point r and time t

In order to obtain an analytical solution for $N^k(r, t)$, where $t = t_0 + \Delta t$, from equation 1, it is assumed that Q^k and A^k are time-independent over the interval $(t_0, t_0 + \Delta t)$. In this case, equation 1 can be written as

$$\dot{N}^k(r, t) = Q^k(r) - A^k(r)N^k(r, t). \quad (2)$$

Since the RAIM program uses region-averaged macroscopic cross sections in calculating the neutron group fluxes, the isotopic concentrations must be region-homogenized after a point depletion calculation. The homogenized isotopic concentrations are computed according to

$$N_j^k(t) = \frac{1}{V_j} \int_{V_j} N^k(r, t) dV \quad (3)$$

where

V_j = volume of region j

$N_j^k(t)$ = homogenized concentration of isotope k in region j at time t

Solutions for equations 2 and 3 are given for each type of depletion calculation. The primary criterion for comparison is the region-homogenized isotopic concentration after depletion, $N_j^k(t_0 + \Delta t)$.

Type 1

$$N^k(r, t_0 + \Delta t) = \frac{Q^k(r)}{A^k(r)} \left[1 - e^{-A^k(r)\Delta t} \right] + N^k(r, t_0) e^{-A^k(r)\Delta t} \quad (4)$$

$$N_j^k(t_o + \Delta t) = \frac{1}{V_j} \int_{V_j} \left[1 - e^{-A^k(r)\Delta t} \right] \frac{Q^k(r)}{A^k(r)} dV \\ + \frac{1}{V_j} \int_{V_j} N^k(r, t_o) e^{-A^k(r)\Delta t} dV \quad (5)$$

Type 2

$$N^k(r, t_o + \Delta t) = \frac{Q^k(r)}{A^k(r)} \left[1 - e^{-A^k(r)\Delta t} \right] + N_j^k(t_o) e^{-A^k(r)\Delta t}, \\ \text{for } r \in V_j \quad (6)$$

$$N_j^k(t_o + \Delta t) = \frac{1}{V_j} \int_{V_j} \left[1 - e^{-A^k(r)\Delta t} \right] \frac{Q^k(r)}{A^k(r)} dV \\ + \frac{N_j^k(t_o)}{V_j} \int_{V_j} e^{-A^k(r)\Delta t} dV \quad (7)$$

Type 3

For this case, Q^k and A^k are space-independent, and their region-averaged values are given by

$$Q_j^k = \frac{1}{V_j} \int_{V_j} Q^k(r) dV \quad (8)$$

$$A_j^k = \frac{1}{V_j} \int_{V_j} A^k(r) dV \quad (9)$$

The depletion equation corresponding to equation 2 is now written as

$$\dot{N}_j^k(t) = Q_j^k - A_j^k N_j^k(t). \quad (10)$$

The solution to equation 10 is

$$N_j^k(t_o + \Delta t) = \left[1 - e^{-A_j^k \Delta t} \right] \frac{Q_j^k}{A_j^k} + N_j^k(t_o) e^{-A_j^k \Delta t} \quad (11)$$

An analytical comparison of the values of $N_j^k(t)$ for the three calculational methods is difficult because the r -dependence of $Q^k(r)$ and $A^k(r)$ is not generally known. Therefore, a sample numerical problem was assumed to facilitate a meaningful comparison. The three depletion analyses were applied to an outer radial blanket region composed on only two isotopes, ^{238}U and ^{239}Pu . Three-group microscopic cross sections, fluxes, and initial distributions of ^{238}U and ^{239}Pu were assumed. Tables 3-1 and 3-2 summarize the results corresponding to small time steps, $\Delta t = 10$ days. The flux distributions and microscopic cross sections were assumed constant during all Δt . Tables 3-3 through 3-8 list results corresponding to a three-region blanket zone, the isotope concentrations being homogenized over each individual blanket region. The only difference between the one-region and the three-region case is the volume over which the isotope concentrations are averaged.

Table 3-1. One-Region Homogenized ^{238}U Concentration
(Concentration 10^{22} nuclei/cc)

<u>Time, days</u>	<u>Type 1</u>	<u>Type 2</u>	<u>Error, %</u>	<u>Type 3</u>	<u>Error, %</u>
0	1.02436	1.02436	0.0	1.02436	0.0
50	1.02260	1.02262	0.00196	1.02262	0.00196
100	1.02085	1.02089	0.00392	1.02089	0.00392
150	1.01911	1.01916	0.00491	1.01916	0.00491
200	1.01737	1.01744	0.00688	1.01744	0.00688
250	1.01563	1.01572	0.00886	1.01572	0.00886

Table 3-2. One-Region Homogenized ^{239}Pu Concentration
(Concentration 10^{20} nuclei/cc)

<u>Time, days</u>	<u>Type 1</u>	<u>Type 2</u>	<u>Error, %</u>	<u>Type 3</u>	<u>Error, %</u>
0	3.36786	3.36786	0.0	3.36786	0.0
50	3.49876	3.48665	-0.3461	3.48676	-0.3430
100	3.62621	3.60342	-0.6285	3.60362	-0.6230
150	3.75031	3.71819	-0.8565	3.71848	-0.8519
200	3.87114	3.83098	-1.0374	3.83137	-1.0315
250	3.98881	3.94184	-1.1775	3.94231	-1.1658

Table 3-3. Three-Region Homogenized ^{238}U Concentration:
Region 1 (Concentration 10^{22} nuclei/cc)

<u>Time, days</u>	<u>Type 1</u>	<u>Type 2</u>	<u>Error, %</u>	<u>Type 3</u>	<u>Error, %</u>
0	1.04711	1.04711	0.0	1.04711	0.0
50	1.04341	1.04342	0.00096	1.04342	0.00096
100	1.03973	1.03975	0.00192	1.03975	0.00192
150	1.03607	1.03609	0.00193	1.03609	0.00193
200	1.03241	1.03244	0.00291	1.03244	0.00291
250	1.02877	1.02881	0.00389	1.02881	0.00389

Table 3-4. Three-Region Homogenized ^{238}U Concentration:
Region 2 (Concentration 10^{22} nuclei/cc)

<u>Time, days</u>	<u>Type 1</u>	<u>Type 2</u>	<u>Error, %</u>	<u>Type 3</u>	<u>Error, %</u>
0	1.02123	1.02123	0.0	1.02123	0.0
50	1.01966	1.01966	0.0	1.01966	0.0
100	1.01808	1.01809	0.00098	1.01890	0.00098
150	1.01651	1.01652	0.00098	1.01652	0.00098
200	1.01494	1.01495	0.00099	1.01495	0.00099
250	1.01338	1.01339	0.00099	1.01339	0.00099

Table 3-5. Three-Region Homogenized ^{238}U Concentration:
Region 3 (Concentration 10^{22} nuclei/cc)

<u>Time, days</u>	<u>Type 1</u>	<u>Type 2</u>	<u>Error, %</u>	<u>Type 3</u>	<u>Error, %</u>
0	1.01003	1.01003	0.0	1.01003	0.0
50	1.00956	1.00956	0.0	1.00956	0.0
100	1.00910	1.00910	0.0	1.00910	0.0
150	1.00864	1.00864	0.0	1.00864	0.0
200	1.00818	1.00818	0.0	1.00818	0.0
250	1.00772	1.00772	0.0	1.00772	0.0

Table 3-6. Three-Region Homogenized ^{239}Pu Concentration:
Region 1 (Concentration 10^{20} nuclei/cc)

<u>Time, days</u>	<u>Type 1</u>	<u>Type 2</u>	<u>Error, %</u>	<u>Type 3</u>	<u>Error, %</u>
0	1.71304	1.71304	0.0	1.71304	0.0
50	2.01090	2.00638	-0.2248	2.00641	-0.2233
100	2.29888	2.29042	-0.3680	2.29048	-0.3654
150	2.57727	2.56541	-0.4602	2.56551	-0.4563
200	2.84637	2.83162	-0.5182	2.83174	-0.5140
250	3.10646	3.08928	-0.5530	3.08943	-0.5482

Table 3-7. Three-Region Homogenized ^{239}Pu Concentration:
Region 2 (Concentration 10^{20} nuclei/cc)

<u>Time, days</u>	<u>Type 1</u>	<u>Type 2</u>	<u>Error, %</u>	<u>Type 3</u>	<u>Error, %</u>
0	3.64494	3.64494	0.0	3.64494	0.0
50	3.74986	3.74878	-0.0288	3.74879	-0.0285
100	3.85316	3.85112	-0.0529	3.85114	-0.0524
150	3.95485	3.94197	-0.0728	3.95200	-0.0721
200	4.05496	4.05136	-0.0888	4.05139	-0.0880
250	4.15351	4.14929	-0.1016	4.14934	-0.1004

Table 3-8. Three-Region Homogenized ^{239}Pu Concentration:
Region 3 (Concentration 10^{20} nuclei/cc)

Time, days	Type 1	Type 2	Error, %	Type 3	Error, %
0	4.36944	4.36944	0.0	4.36944	0.0
50	4.39768	4.39748	-0.00456	4.39748	-0.00456
100	4.42575	4.42538	-0.00836	4.42539	-0.00813
150	4.45367	4.45317	-0.0112	4.45318	-0.0110
200	4.48144	4.48083	-0.0136	4.48084	-0.0134
250	4.50906	4.50836	-0.0155	4.50837	-0.0153

Tables 3-1 and 3-2 show that depletion methods type 2 and 3 yield very nearly the same value for the ^{238}U and ^{239}Pu one-region homogenized concentrations. For ^{238}U these values are nearly identical to that corresponding to method type 1. However, for ^{239}Pu , which exhibits large spatial variations in its concentration, methods 2 and 3 yield significantly different values than does method 1.

The three-region homogenization indicates much smaller variations in the region-homogenized ^{238}U and ^{239}Pu concentrations for the three depletion methods. Tables 3-3, 3-4, and 3-5 show that the ^{238}U concentration is nearly the same for all methods. As shown in Tables 3-6, 3-7, and 3-8, methods 2 and 3 yield very similar values for the region-homogenized ^{239}Pu concentrations and agree much more closely with method 1 than for the single-region homogenization.

Clearly, point depletion is the most accurate method of isotopic depletion. However, the large computer storage requirement for point isotopic concentrations prohibits the practical use of this method of depletion analysis when realistic fuel management options are desired. The regional depletion method will be used in the REDEP portion of the FARED code because it yields results that are almost as accurate as the point depletion and region-homogenization method. As indicated above, regional depletion becomes quite accurate as the number of regions is increased, especially for isotopes whose concentrations are not strongly space-dependent.

3.3.4. Zone Material Depletion Analysis

The RAIM program requires homogenized isotopic concentrations in each distinct reactor zone for the flux calculation. To permit detailed fuel management in the FARED code, zone materials are defined. A zone material is composed of a set of isotopes, each with its homogeneous concentration, and a zone material volume. A zone contains a number of zone materials, and each zone material occupies a specified fraction of the zone volume. The homogeneous concentration of a given isotope in a particular zone is the volume weighted sum of the isotopic concentrations as contributed by each zone material in that zone.

Depletion of isotopes is performed by zone material. That is, each isotope in a zone material is depleted according to the flux distributions and microscopic cross sections applicable to the zone in which the zone material is contained.

Zone material depletion is performed according to the regional depletion analysis as described in the preceding section. For this purpose, the following quantities are defined:

$N(I,k,t)$ = homogeneous concentrations of isotope k in zone material I at time t

$V(I)$ = volume associated with zone material I

$N(I,J,k,t)$ = homogeneous concentration of isotope k in zone J at time t as contributed by zone material I

$V(J)$ = volume of zone J

$VF(J,I)$ = volume fraction of zone J occupied by zone material I

$N(J,k,t)$ = total homogeneous concentration of isotope k in zone J at time t

where

$$V(I) = V(J) \cdot VF(J,I)$$

$$N(I,J,k,t) = VF(J,I) \cdot N(I,k,t)$$

$$N(J,k,t) = \sum_I N(I,J,k,t)$$

The general zone-depletion equation for isotope k is

$$\dot{N}(J,k,t) = Q(J,k,t) - \sum_I a(I,J,k,t) \cdot N(I,J,k,t)$$

where

$$Q(J,k,t) = \sum_I Q(I,J,k,t)$$

$Q(I,J,k,t)$ = source term for isotope k of zone material I in zone J at time t

$a(I,J,k,t) \cdot N(I,J,k,t)$ = loss term for isotope k of zone material I in zone J at time t

The depletion equation for each isotope k in zone material I contained in zone J is

$$\dot{N}(I,J,k,t) = Q(I,J,k,t) - a(I,J,k,t) \cdot N(I,J,k,t).$$

The following assumptions will be made to facilitate a suitable solution:

1. During the time interval $(t_0, t_0 + \Delta t)$ the coefficients Q and a are time-independent.

$$Q(I,J,k,t) = Q(I,J,k) \quad \text{for } t \in (t_0, t_0 + \Delta t)$$

$$a(I,J,k,t) = a(I,J,k) \quad \text{for } t \in (t_0, t_0 + \Delta t)$$

2. During the same time interval the value of $N(I,J,k,t)$ is assumed to be constant, with the value

$$N(I,J,k,t) = \frac{1}{2} [N(I,J,k,t_0 + \Delta t) + N(I,J,k,t_0)]$$

3. The derivative term can be approximated by

$$\dot{N}(I,J,k,t) = \frac{1}{\Delta t} [N(I,J,k,t_0 + \Delta t) - N(I,J,k,t_0)]$$

Combining these three basic assumptions, the solution for $N(I,J,k,t_0 + \Delta t)$ has the form

$$N(I,J,k,t_0 + t) = \frac{[2 - a(I,J,k)\Delta t] VF(J,I) N(I,k,t_0) + 2Q(I,J,k)\Delta t}{2 + a(I,J,k)\Delta t}$$

Hence, the depleted zone material concentration of isotope k is given entirely in terms of the coefficients $a(I,J,k)$ and $Q(I,J,k)$ and the initial zone material concentration of isotope k, according to

$$N(I, k, t_0 + \Delta t) = \frac{N(I, J, k, t_0 + \Delta t)}{VF(J, I)}$$

3.4. FARED Fuel Management

The fuel management portion of the FARED code is designed to allow any management policy involving the movement of individual sub-assemblies in a one-dimensional reactor. Actual fuel management is performed by reassigning the location of each zone material within the reactor at desired time intervals. New zone materials may be defined to represent fresh fuel, and all fuel discharged from the reactor is lumped into one out-of-pile zone material. Eventually, out-of-pile fuel management, including reprocessing, fabrication, and fresh fuel make-up, will be allowed.

Figure 3-3 is a diagram of the fuel management control proposed for the FARED code. Each computational and control block is individually discussed below.

3.4.1. Input and REGA Calculation

Microscopic cross-section sets are prepared by the REGA program according to the problem input data. Zone-dependent microscopic cross sections may be used. Various other parameters, such as geometry, mesh spacing and zone designations, are input or computed.

3.4.2. Define Zone Materials

Zone material compositions are defined according to bulk material input specifications:

$N(I, k, t_0)$ = homogeneous concentration of isotope k in zone material I at time t_0

$V(I)$ = available volume of zone material I

Initial zone material volumes are computed according to the zone volume that they occupy.

3.4.3. Define Zones and Volume Fractions

The concentration of isotopes in each zone is computed according to the zone volume fraction occupied by each zone material in the zone:

Figure 3-3. Fuel Management Diagram

$N(I, J, k, t_0)$ = homogeneous concentration of isotope k in zone J at time t_0 as contributed by zone material I

$VF(J, I)$ = volume fraction of zone J occupied by zone material I (input)

$V(J)$ = volume of zone J

$N(J, k, t_0)$ = homogeneous concentration of isotope k in zone J at time t_0

The following quantities are computed from previous input data:

$$V(I) = VF(J, I) \cdot V(J)$$

$$N(I, J, k, t_0) = VF(J, I) \cdot N(I, k, t_0)$$

$$N(J, k, t_0) = \sum_I N(I, J, k, t_0)$$

No zone material may occur in more than one zone.

3. 4. 4. RAIM Flux Calculation

Group fluxes and k_{eff} are calculated by the RAIM program according to the appropriate zone concentration, $N(J, k, t_j)$, and microscopic cross-section sets.

3. 4. 5. RAIM Edit

Various neutronics data are calculated and printed by the RAIM Edit program as described in section 3.3.2.

3. 4. 6. Depletion Control

The depletion time for the $(j + 1)^{th}$ depletion calculation, $t_{j+1} - t_j$, is determined by input, or according to the value of a specified parameter, such as δk_{eff} , maximum or average zone burnup. The number of intermediate time intervals, M , is determined to ensure the accuracy of the depletion approximations. These time intervals are given by

$$\Delta t = \frac{t_{j+1} - t_j}{M}$$

3.4.7. REDEP Zone Material Depletion

The depleted zone material concentrations for each isotope k , $N(I,k,t_j + m\Delta t)$, are computed for $m = 1, 2, \dots, M$ according to the analysis presented in section 3.3.4. The zone material isotopic concentrations at the end of the depletion period, $N(I,k,t_{j+1})$, are obtained.

3.4.8. REDEP Edit

The REDEP Edit calculates and prints various quantities associated with the zone materials, such as depleted zone material isotopic concentrations, isotopic masses, maximum and average burnup.

3.4.9. Final Edit

After the final depletion period, a complete history of each zone material is printed. This final edit will include the following quantities:

1. Zone location of each zone material during every burnup period.
2. Time zone material remained in each zone location.
3. Initial and final zone material isotopic concentrations.
4. Initial and final zone material isotopic masses.
5. Final cumulative maximum and average zone material burnup.

3.4.10. Redefine Zone Materials

There are two cases for expanding the list of zone materials:

1. Insertion of a new zone material into some zone J .
2. Splitting of a zone material into two or more parts and transferring these parts to different zones. A new zone material must be defined for each fraction of the original zone material that goes into a different zone.

3.4.11. Redefine Zones and Volume Fractions

Fuel movement is accomplished by redefining the zone location of each zone material, including the newly created zone materials. For this purpose, new values of $VF(J,I)$ are input. The zone isotopic concentrations are then given by

$$N(J,k,t_{j+1}) = VF(J,I) \cdot N(I,k,t_{j+1}).$$

3.4.12. Out-of-Pile Inventory Edit

A single zone material is used to record the out-of-pile inventory. This zone material represents all fuel that is removed from the reactor. The out-of-pile edit includes the following quantities:

1. Concentration of each isotope.
2. Mass of each isotope.
3. Volume of out-of-pile zone material.
4. Cumulative average burnup.
5. Concentration corrections for decay of radioactive isotopes.

3.4.13. Fuel Management Edit

The fuel management edit lists these quantities:

1. Zone materials located in each zone.
2. Zone isotopic concentration.
3. Zone isotopic masses.

3.4.14. New REGA Calculation

The FARED code will be capable of calculating new zone-dependent microscopic cross sections at the beginning of any depletion period. This feature will also allow temperature and void coefficient calculations to be performed during depletion.

4. FARED BURNUP STUDIES

4.1. Preliminary Investigations (J. M. Stewart)

Before the depletion studies with the ZEUS (BWIG-GAMMIX-SIZZLE) fast reactor design package were begun, three preliminary investigations were carried out as follows:

1. To determine differences in the overall reactor characteristics using the new BWIG cross-section library and the old DHRG3 cross-section library.

2. To determine the effects of using one 18-group cross-section set for the entire reactor (generated for the core composition) instead of using two cross-section libraries generated for the core and blanket compositions separately.

3. To determine the effects on overall reactor characteristics of running reactor depletion calculations while maintaining small excess reactivity instead of maintaining the reactor critical by the addition of the burnable poison B_4C .

The 2000-liter core described in this report was used for all comparisons.

The first of the preliminary investigations was simply a check for gross inconsistencies in the results obtained using the newly formulated BWIG library (DHRG3 library data plus 68-group ENDF/B cross section and resonance parameters for Na, O, ^{238}U , ^{239}Pu , ^{240}Pu , ^{241}Pu , and fission products, all with the exception of elastic matrices) compared with an identical case which utilized the DHRG3 library. This served as a quick check of the BWIG set so that the study could proceed expeditiously. In addition, the DHRG3 to BWIG library change is typical of the cross-section revisions which will be continually occurring. Thus, a comparison of the core parameters calculated with these two libraries will provide a realistic indication of the sensitivity of core performance to

general cross-section revision. A separate check of the new BWIG library on assembly 48 is in progress, and preliminary indications are that excellent agreement will be attained.

Table 4-1 shows the comparison of selected beginning- and end-of-cycle reactor physics parameters for the cases using the DHRG3 cross-section library and the new BWIG library, respectively. In both cases, separate cross-section sets were used in the core and blankets. As can be seen from the table, agreement was quite good, and it was decided to proceed using the new BWIG library for all subsequent calculations.

Because of certain limitations in the designation of isotopic compositions in the current version of SIZZLE, it was necessary to use only one 18-group cross-section library in the depletion calculations in order to retain the identity of the two fuel exposures in each of the core and blanket zones (a stipulation of the contract). An investigation was carried out to ascertain the effects of this limitation. It was possible to do this on the first fuel cycle, when only one level of fuel exposure is present in each core and blanket zone. Table 4-2 summarizes the calculated changes in physics parameters due to using a constant library (generated for the core composition) instead of using two libraries, one generated for the core composition and one for the blanket composition.

As indicated by Table 4-2, no significant sources of error will be introduced by using the constant library. Thus, libraries generated for the core composition will be used throughout the reactor in all subsequent cases.

The third comparison was made to establish the validity of performing the depletion calculations while maintaining small excess reactivity instead of forcing criticality with a B_4C search at each depletion cycle time step. The deletion of the B_4C -maintained criticality requirement would greatly expedite the completion of the study owing to the computer time and the large number of cases involved. The version of SIZZLE now available allows an isotopic search in only those zones containing the first material. Power flattening requirements and the maintenance of fuel management procedures require the use of two materials in the core. Therefore, the calculation was eliminated to a B_4C search in the first core zone only. For this comparison, the search was performed in core zone 1 with no control in zone two. Table 4-3 shows the

comparison of the lifetime calculation employing the burnable poison search with the calculation that was performed while maintaining small excess reactivity. Each lifetime calculation was performed using the library generated for the core composition throughout the reactor.

As can be seen from the table, agreement is good, especially at the end of the cycle. The beginning-of-cycle agreement would have been more favorable but for the one-zone search limitation. Also, the excess reactivity maintained for the comparison is approximately 1% higher than that maintained in the actual fuel cycles for this core. This would also improve the agreement. Table 4-4 compares isotopic compositions at end of cycle for core zones 1 and 2, axial blanket zone 1, and radial blanket zone 1 for the two cases.

Table 4-4 shows that only small percentage changes are evident in isotopic densities in both core zones at end of cycle between the two cases. The changes are slightly higher in the first axial and radial blanket zones, although quite good agreement is evident when comparing the ^{239}Pu and ^{238}U concentrations. On the basis of the comparisons presented in Tables 4-3 and 4-4, it was decided to perform all subsequent calculations while maintaining small excess reactivity in order to expedite the study.

4.2. Depletion Calculations (J. E. Stewart)

Provision has been made in the BWIG portion of the ZEUS package for multiplying the capture cross sections of all isotopes at all energies by a designated factor FCAP. In addition, for all fissionable isotopes, the fission cross section at all energies may be altered by a factor FFIS. This provision will be used in the depletion calculations to form libraries in which the fission product capture cross section is increased by 50% and, separately, the ^{239}Pu capture cross section is increased by 50%. The two modified libraries will be used along with the reference libraries on both the 2000 and 8000 liter cores in the depletion calculations. Since this method allows the cross-section changes to affect the 68-group spectrum calculation, it will correctly show the effect of changes in multi-group flux spectrum on the broadgroup cross sections used in calculating reference, void, and Doppler core conditions in the 18-group spatial calculation.

Equivalent cylindrical cores were produced for each of the two spherical cores by assuming a core height of 36 inches and an axial blanket thickness of 12 inches for both cores. Since the calculations are limited to spherical geometry, the separate representation of the axial and radial blankets is restricted to a provision for alternate spherical shells surrounding the core. The selection of the thickness and the number of blanket regions is based on the ^{238}U absorption rate spatial distribution in the blankets.

Table 4-5 gives a general description of the two cores, along with a description of the two equivalent cylindrical reactors. For all cases, the feed plutonium isotopic mix used was 76% ^{239}Pu , 19% ^{240}Pu , 3.5% ^{241}Pu , and 1.5% ^{242}Pu .

For each fuel cycle after the first, the feed enrichment is adjusted to obtain the desired initial k_{eff} and a reasonably flat power distribution. The first fuel cycle is designed to terminate at a maximum localized heavy atom burnup of 5%. For each succeeding cycle, the discharge fuel exposure is 10% heavy atom burnup. The axial blanket fuel is cycled in direct proportion to the core fuel. Half of the core and axial blanket fuel is removed at the end of each cycle. For each fuel cycle after the first, two levels of fuel exposure are present in the core and axial blanket. Half of the radial blanket fuel is removed on alternate cycles. Two levels of fuel exposure are present in the radial blanket following cycle 2. Fission product concentrations are cycled in direct proportion to the fuel exposure time. The generalized refueling scheme used in the study is as follows:

	Cycle					
	1	2	3	4	5	6
Core	0 + 0	0 + 1	0 + 1	0 + 1	0 + 1	0 + 1
Axial blanket	0 + 0	0 + 1	0 + 1	0 + 1	0 + 1	0 + 1
Radial blanket	0 + 0	1 + 1	0 + 2	1 + 3	0 + 2	1 + 3

Note: 0 = fresh fuel
 1 = 1 cycle irradiated fuel
 2 = 2 cycle irradiated fuel

At this point, depletion calculations have been carried out on the first three cycles for the small core using the reference library. The first cycle for the large core has been initiated but is not yet complete. Table 4-6 shows beginning-and end-of-cycle power peaking factors for the first three cycles completed on the small core. Table 4-7 gives beginning-and end-of-life average enrichments by zone and for the total core. Also given for each cycle are feed and discharge enrichments for zones and for the total core. Feed and discharge enrichments for core zones 1 and 2 and total core are shown in Figure 4-1 in order to illustrate the approach to equilibrium for this core and library. Equilibrium will result in repeating alternate cycles (i.e., cycles 3, 5, 7 . . . and 4, 6, 8 . . .) instead of repeating successive cycles because of the radial blanket management procedure.

Tables 4-8 through 4-13 show beginning-and end-of-cycle mass balances for the three cycles and also end-of-cycle discharge mass balances.

Table 4-1. Comparison of Selected Parameters Obtained
Using DHRG3 and BWIG Cross-Section Sets

	DHRG3	BWIG	Change, %
<u>Beginning of Cycle</u>			
Core breeding ratio (BR_{core})	0.7723	0.7691	-0.42
Blanket breeding ratio (BR_{blkt})	0.6195	0.6226	+0.50
Total breeding ratio (BR_{total})	1.3918	1.3917	-0.01
Core zone 1 power peaking factor (PF_1)	1.2910	1.2839	-0.55
Core zone 2 power peaking factor (PF_2)	1.2740	1.2764	+0.19
Reactor fissile mass (M_f^R)	1032 kg	1032.2 kg	+0.02
Effective multiplication factor (λ)	1.0261	1.0282	+0.20
Average core power density ($\overline{\rho_P^c}$)	304.6 kW/l	305.3	+0.23
Core relative power (P_R^c)	0.9746	0.9754	+0.08
Blanket relative power (P_R^B)	0.0254	0.0246	-3.3
<u>End of Cycle</u>			
	(521 days)	(528.5 days)	
BR_{core}	0.7310	0.7243	-0.93
BR_{blkt}	0.6386	0.6414	+0.44
BR_{total}	1.3696	1.3657	-0.29
PF_1	1.3780	1.3736	-0.32
PF_2	1.164	1.2050	+3.4
$U_f^{R(a)}$	1163.0 kg	1165.43 kg	+0.21
λ	0.99948	1.0004	+0.09
$\overline{\rho_P^c}$	287.2 kW/l	286.46 kW/l	-0.26
P_R^c	0.9190	0.9163	-0.29
P_R^B	0.0810	0.0837	+3.2
Integrated breeding ratio (IBR) ^(b)	1.3832	1.3841	+0.07
$\Delta\lambda_{cycle}$	-0.02661	-0.02777	+0.42

(a) Includes ^{239}Np correction.

(b) Based on burnup rate of 1.05 g/MWd.

Table 4-2. Comparison of Case Using Library Generated for Both Core and Blanket¹ With That Using Core Library Only²

	Case 1	Case 2	Change, %
<u>Beginning of Cycle</u>			
BR_{core}	0.7691	0.7704	+0.17
BR_{blanket}	0.6226	0.6220	-0.10
BR_{total}	1.3917	1.3924	+0.05
PF_1	1.2839	1.2861	+0.17
PF_2	1.2764	1.2772	+0.06
M_f^R	1032.2 kg	1032.2 kg	--
λ	1.0282	1.0282	--
$\overline{\rho}_P^c$	305.3	305.4	+0.03
P_R^c	0.9754	0.9757	+0.03
P_R^B	0.0246	0.0243	-1.22
<u>End of Cycle (528.5 days)</u>			
BR_{core}	0.7243	0.7314	+0.98
BR_{blanket}	0.6414	0.6443	+0.45
BR_{total}	1.3657	1.3757	+0.73
PF_1	1.3736	1.3784	+0.35
PF_2	1.2050	1.2050	--
$M_f^{R(a)}$	1165.43	1167.23	+0.15
λ	1.0004	0.99971	-0.07
$\overline{\rho}_P^c$	286.46	288.15	+0.59
P_R^c	0.9163	0.9216	+0.58
P_R^B	0.0837	0.0784	-0.3
$IBR^{(b)}$	1.3841	1.3893	+0.38
$\Delta\lambda_{\text{cycle}}$	-0.02777	-0.02849	+0.26

(a) Includes ^{239}Np correction.

(b) Based on burnup rate of 1.05 g/MWd.

Table 4-3. Comparison of Excess Reactivity Lifetime
With B₄C-Maintained Criticality Lifetime

	B ₄ C	No B ₄ C	Change, %
<u>Beginning of Cycle</u>			
BR _{core}	0.7596	0.7704	+ 1.42
BR _{blanket}	0.6504	0.6220	- 4.37
BR _{total}	1.4100	1.3924	- 1.25
PF ₁	1.2946	1.2861	- 0.66
PF ₂	1.3858	1.2772	- 7.84
M _f ^R	1032.2	1032.2	--
λ	0.99995	1.0282	--
$\overline{\rho}_P^c$	283.55	305.4	+ 7.7
P _R ^c	0.9743	0.9757	+ 0.14
P _R ^B	0.0257	0.0243	- 5.45
<u>End of Cycle (528.5 days)</u>			
BR _{core}	0.7317	0.7314	- 0.04
BR _{blanket}	0.6473	0.6443	- 0.46
BR _{total}	1.3790	1.3757	- 0.24
PF ₁	1.3752	1.3854	+ 0.74
PF ₂	1.2064	1.2050	- 0.12
M _f ^{R(a)}	1170.53	1167.23	- 0.28
λ	0.99908	0.99971	--
$\overline{\rho}_P^c$	287.71	288.15	+ 0.15
P _R ^c	0.9202	0.9216	+ 0.15
P _R ^B	0.0798	0.0784	- 1.75
IBR ^(b)	1.3988	1.3893	- 0.68

(a) Includes ²³⁹Np correction.

(b) Based on a burnup rate of 1.05 g/MWd.

Table 4-4. Comparison of End-of-Cycle Isotopic Densities for the Case With B₄C-Maintained Criticality¹ and the Case With Excess Reactivity²

Isotope	Core zone 1			Core zone 2		
	Case 1	Case 2	Change, %	Case 1	Case 2	Change, %
²⁴¹ Pu	0.3979-4	0.4009-4	+0.75	0.6802-2	0.6815-4	+0.19
²⁴⁰ Pu	0.2657-3	0.2676-3	+0.71	0.4240-3	0.4243-3	+0.07
²³⁸ U	0.6521-2	0.6512-2	-0.14	0.5896-2	0.5899-2	+0.05
²³⁹ Pu	0.9134-3	0.9132-3	-0.02	0.1378-2	0.1380-2	+0.14
²⁴² Pu	0.1817-4	0.1824-4	+0.38	0.3072-4	0.3076-4	+0.13
²³⁹ Np	0.2520-5	0.2536-5	+0.63	0.1402-5	0.1399-5	-0.21
B _{nat}	0.1030-3	--	x	--	--	x
C	0.3535-4	--	x	--	--	x
O	0.1637-1	0.1637-1	--	0.1644-1	0.1644-1	--
Na	0.9283-2	0.9283-2	--	0.9283-2	0.9283-1	--
304 SS	0.1723-1	0.1723-1	--	0.1723-1	0.1723-1	--
FF	0.7811-3	0.7950-3	+1.7	0.7849-3	0.7750-3	-1.3

Isotope	Axial blanket zone 1			Radial blanket zone 1		
	Case 1	Case 2	Change, %	Case 1	Case 2	Change, %
²⁴¹ Pu	0.8674-7	0.8313-7	-4.3	0.9976-7	0.9541-7	-4.6
²⁴⁰ Pu	0.5246-5	0.5096-5	-2.9	0.6197-5	0.6012-5	-3.1
²³⁸ U	0.7870-2	0.7874-2	+0.05	0.1010-1	0.1010-1	--
²³⁹ Pu	0.2201-3	0.2170-3	-1.4	0.2692-3	0.2652-3	-1.5
²⁴² Pu	0.1050-8	0.9914-9	-5.9	0.1162-8	0.1094-8	-5.3
²³⁹ Np	0.1547-5	0.1539-5	-0.52	0.1882-5	0.1872-5	-0.53
B _{nat}	--	--	x	--	--	x
C	--	--	x	--	--	x
O	0.1628-1	0.1628-1	--	0.2084-1	0.2084-1	--
Na	0.9283-2	0.9283-2	--	0.7043-2	0.7043-2	--
304 SS	0.1723-1	0.1723-1	--	0.1723-1	0.1723-1	--
FF	0.6822-4	0.6602-4	-3.3	0.7616-4	0.7364-4	-3.4

Table 4-5. General Description of Spherical Cores

	Core	Axial blanket	Radial blanket	Average blanket	Reflector
<u>Small Core</u>					
Volume, l	2000	1333	4492	5825	4465
Outer diam, cm	156.32	--	--	246.32	286.32
No. of regions	2	5	5	--	1
Steel, type	304 SS	304 SS	304 SS	--	304 SS
Fuel, % TD	90	90	90	--	--
Fuel temp, K	1644	1366	1366	--	--
Volume fraction:					
Fuel	0.37	0.37	0.47	0.45	--
Na	0.43	0.43	0.33	0.35	0.50
Steel	0.20	0.20	0.20	0.20	0.50
<u>Equivalent Cylindrical Reactor</u>					
Volume, l	2000	1333	4492	--	4465
Outer diam, cm	166.88	166.88	273.43	--	--
Height, cm	91.44	30.48 × 2	121.92	--	--
<u>Large Core</u>					
Volume, l	8000	5333	6910	12,243	8067
Outer diam, cm	248.14	--	--	338.14	378.14
No. of regions	2	5	5	--	1
Steel, type	304 SS	304 SS	304 SS	--	1
Fuel, % TD	90	90	90	--	--
Fuel temp, K	1644	1366	1366	--	--
Volume fraction:					
Fuel	0.37	0.37	0.51	0.45	--
Na	0.43	0.43	0.29	0.35	0.50
Steel	0.20	0.20	0.20	0.20	0.50
<u>Equivalent Cylindrical Reactor</u>					
Volume, l	8000	5333	6910	--	8067
Outer diam, cm	333.76	333.76	428.44	--	--
Height, cm	91.44	30.48 × 2	121.92	--	--

Table 4-6. Small Core Reference Library Power Peaking Factors

	<u>Beginning of cycle</u>	<u>End of cycle</u>
<u>Cycle I</u>		
Core zone 1	1.2876	1.3637
Core zone 2	1.2781	1.2194
(PF) ₁ /(PF) ₂	1.0074	1.1183
<u>Cycle II</u>		
Core zone 1	1.3196	1.3752
Core zone 2	1.2454	1.1992
(PF) ₁ /(PF) ₂	1.0590	1.1468
<u>Cycle III</u>		
Core zone 1	1.3010	1.3597
Core zone 2	1.2547	1.20710
(PF) ₁ /(PF) ₂	1.3069	1.1264

Table 4-7. Small Core Reference Library Enrichments

$$\text{enrichment}^{(a)} = \frac{\text{mass}(^{239}\text{Pu} + ^{241}\text{Pu})}{\text{mass}(\text{Pu} + \text{U})}$$

	<u>Enrichment</u>			
	<u>Feed</u>	<u>Beginning of cycle</u>	<u>End of cycle</u>	<u>Discharge</u>
<u>Cycle I</u>				
Core zone 1	0.1156	0.1156	0.1206	0.1206
Core zone 2	0.1966	0.1966	0.1865	0.1865
Core average	0.1562	0.1562	0.1537	0.1537
<u>Cycle II</u>				
Core zone 1	0.1205	0.1206	0.1241	0.1242
Core zone 2	0.2024	0.2024	0.1852	0.1783
Core average	0.1615	0.1577	0.1548	0.1514
<u>Cycle III</u>				
Core zone 1	0.1171	0.1205	0.1241	0.1268
Core zone 2	0.2025	0.1972	0.1874	0.1828
Core average	0.1599	0.1589	0.1558	0.1549

(a) Includes ²³⁹Np correction.

Table 4-8. Small Core Reference Library Mass
Balance — Cycle 1

	<u>Core</u>	<u>Axial blanket</u>	<u>Radial blanket</u>	<u>Total</u>
<u>Beginning-of-cycle mass, kg</u>				
²³⁹ Pu	968.81	--	--	968.81
²⁴⁰ Pu	242.20	--	--	242.20
²⁴¹ Pu	44.62	--	--	44.62
²⁴² Pu	19.12	--	--	19.12
Total Pu	1274.75	--	--	1,274.75
²³⁸ U	5213.21	4287.79	18,495.27	27,996.27
Total (U + Pu)	6487.96	4287.79	18,495.27	29,271.02
<u>End-of-cycle mass, kg</u>				
²³⁹ Pu	916.39	46.15	112.19	1,074.73
²⁴⁰ Pu	264.19	0.52	0.99	265.70
²⁴¹ Pu	42.82	0.00	0.01	42.83
²⁴² Pu	19.30	0.00	0.00	19.30
Total Pu	1242.70	46.67	113.19	1,402.56
²³⁸ U	4998.98	4236.98	18,374.18	27,610.14
Total (U + Pu)	6241.68	4283.65	18,487.37	29,012.70

Table 4-9. Small Core Reference Library Mass
Balance — Cycle 1

	<u>Core</u>	<u>Axial blanket</u>	<u>Radial blanket</u>	<u>Total</u>
<u>End-of-cycle discharge mass, kg</u>				
²³⁹ Pu	458.20	23.08	--	481.28
²⁴⁰ Pu	132.10	0.26	--	132.36
²⁴¹ Pu	21.41	0.00	--	21.41
²⁴² Pu	9.65	0.00	--	9.65
Total Pu	621.35	23.34	--	644.69
²³⁸ U	2499.49	2118.49	--	4617.98
Total (U + Pu)	3120.84	2141.83	--	5262.67

Table 4-10. Small Core Reference Library Mass Balance — Cycle 2

	<u>Core</u>	<u>Axial blanket</u>	<u>Radial blanket</u>	<u>Total</u>
<u>Beginning-of-cycle mass, kg</u>				
²³⁹ Pu	959.28	23.22	112.19	1,094.69
²⁴⁰ Pu	267.37	0.26	0.99	258.62
²⁴¹ Pu	44.49	0.00	0.01	44.50
²⁴² Pu	19.54	0.0	0.00	19.54
Total Pu	1280.68	23.48	113.19	1,417.35
²³⁸ U	5085.12	4262.39	18,374.18	27,721.69
Total (U + Pu)	6365.80	4285.87	18,487.37	29,139.04
<u>End-of-cycle mass, kg</u>				
²³⁹ Pu	905.09	66.76	216.40	1,188.25
²⁴⁰ Pu	276.81	1.23	3.73	281.77
²⁴¹ Pu	43.32	0.02	0.06	43.40
²⁴² Pu	19.67	0.00	0.00	19.67
Total Pu	1244.89	68.01	220.19	1,533.09
²³⁸ U	4883.00	4211.93	18,252.84	27,347.77
Total (U + Pu)	6127.89	4279.94	18,473.03	28,880.86

Table 4-11. Small Core Reference Library Mass Balance — Cycle 2

	<u>Core</u>	<u>Axial blanket</u>	<u>Radial blanket</u>	<u>Total</u>
<u>End-of-cycle discharge mass, kg</u>				
²³⁹ Pu	433.91	43.79	108.20	585.90
²⁴⁰ Pu	140.71	0.98	1.87	143.56
²⁴¹ Pu	21.20	0.00	0.03	21.23
²⁴² Pu	9.70	0.00	0.00	9.70
Total Pu	605.52	44.77	110.10	760.39
²³⁸ U	2400.27	2093.44	9126.42	13,620.13
Total (U + Pu)	3005.79	2138.21	9236.52	14,380.52

Table 4-12. Small Core Reference Library Mass
Balance — Cycle 3

	<u>Core</u>	<u>Axial blanket</u>	<u>Radial blanket</u>	<u>Total</u>
<u>Beginning-of-cycle mass, kg</u>				
²³⁹ Pu	967.02	22.97	108.20	1,098.19
²⁴⁰ Pu	260.07	0.25	1.86	262.18
²⁴¹ Pu	44.96	0.00	0.03	44.99
²⁴² Pu	19.76	0.0	0.00	19.76
Total Pu	1291.81	23.22	110.09	1,425.12
²³⁸ U	5074.95	4262.38	18,374.05	27,711.38
Total (U + Pu)	6366.76	4285.60	18,594.23	29,246.59
<u>End-of-cycle mass, kg</u>				
²³⁹ Pu	911.31	66.46	212.68	1,190.45
²⁴⁰ Pu	279.41	1.21	4.46	285.08
²⁴¹ Pu	43.77	0.02	0.10	43.89
²⁴² Pu	19.88	0.00	0.00	19.88
Total Pu	1254.37	67.69	217.24	1,539.30
²³⁸ U	4874.09	4212.01	18,252.96	27,339.06
Total (U + Pu)	6128.46	4279.70	18,470.20	28,878.36

Table 4-13. Small Core Reference Library Mass
Balance — Cycle 3

	<u>Core</u>	<u>Axial blanket</u>	<u>Radial blanket</u>	<u>Total</u>
<u>End-of-cycle discharge mass, kg</u>				
²³⁹ Pu	443.64	43.54	156.20	643.38
²⁴⁰ Pu	144.80	0.96	3.98	149.74
²⁴¹ Pu	21.88	0.02	0.09	21.99
²⁴² Pu	10.02	0.00	0.00	10.02
Total Pu	620.34	44.52	160.27	825.13
²³⁸ U	2384.60	2093.48	9066.55	13,544.63
Total (U + Pu)	3004.94	2138.00	9226.82	14,369.76

Figure 4-1. Discharge and Feed Enrichments for Small Core-Reference Library

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