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DEUTERON-STRIPPING REACTIONS
AT HIGH ENERGY **

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The Enrico Fermi Institute
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ABSTRACT

The bootstrap principle suggests that nuclei ($B \geq 2$) should be granted equal status in the nuclear democracy, along with the ordinary ($B \leq 1$) hadrons, simply because they interact strongly. Naively, one might expect nuclei to lie on Regge trajectories, and nuclear reactions to be dual, with resonances in the direct channel building up Regge poles in the crossed channel. With these concepts in mind, we examine in detail the simplest deuteron-stripping reactions, viz. $p + p \rightarrow d + \pi^+$, $p + p \rightarrow d + \rho^+$, and $p + d \rightarrow d + p$. Overall, the most satisfactory model is the One-Pion-Exchange (OPE) model of Yao. We present a revised version of the Yao model, and demonstrate that it gives a good description of the experimental $pp \rightarrow d\pi$ forward differential cross section from threshold up to the highest energies measured. Furthermore, the model quantitatively accounts for the high-energy

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behaviors of the $p + p \rightarrow d + \pi$ and $p + p \rightarrow d + \rho$ forward differential cross sections, which were recently found to have very puzzling high-energy behaviors. Canonical Regge behavior should set in only at much higher energies. The bumps in the resonance region of the $pp \rightarrow d\pi$ cross section do not represent genuine dibaryon resonances, but are merely manifestations of Δ -isobar excitation of one of the nucleons in the deuteron. We conclude that nuclei do not lie on Regge trajectories, although they do couple to non-exotic trajectories. The OPE mechanism can account for the $pp \rightarrow d\pi$ cross section, even at threshold, where the one-nucleon-exchange contribution is greatly suppressed by the p-d-n vertex form factor. To test the general validity of the OPE model, we apply it to $p + p \rightarrow d + p$ at high energy. The model gives an excellent (one-parameter) fit to the experimental differential cross section over a wide range of energies and backward angles. The excited deuteron model of Kerman and Kisslinger is in principle equivalent to the OPE model. The OPE approximation corresponds to writing for the amplitude an unsubtracted dispersion relation in u , and then assuming that the π -N anomalous cut saturates the dispersion integral. In this approach, the absolute scale of the amplitude is calculable, whereas before it was left as a free parameter. The calculated normalization turns out to be somewhat larger than the experimental value, and to improve on this would require a much deeper understanding of the deuteron structure. Since nuclei apparently lie

outside of the hadron bootstrap, we offer the speculation that nuclei may not be hadrons (strongly interacting particles) after all. Their exotic quantum numbers may render them inert to the SU(3)-symmetric strong force, leaving only the SU(3)-breaking medium-strong force through which they can interact.

I. INTRODUCTION

For historical reasons, hadrons with $B \leq 1$ have acquired the misnomer "elementary particles". There are now so many of them, and they can so rapidly transform into one another, that only their collective existence can legitimately be called elementary. The equivalence class of all strongly interacting particles is popularly called the "nuclear democracy" or the "hadron bootstrap". Due to the strength of their interactions, hadrons are believed to be composed of one another and to require each other's existence in a nonlinear but self-consistent way. If a particle interacts strongly, its field has no invariant significance --no more than does a component of a vector. Figuratively speaking, there is no preferred frame in hadron space.

It is commonly believed that nuclei ($B \geq 2$) are composite objects which are not as fundamental as the $B \leq 1$ hadrons. In contradiction, G. F. Chew [1] has argued that nuclei should be granted equal status in the nuclear democracy, simply because they interact strongly. In essence, the uranium nucleus is just as elementary as the proton. It would seem, then, that the division of labor between nuclear physics and elementary particle physics is artificial, for they are simply two different perspectives in hadron space.

Such a heuristic but far-reaching assertion deserves a thorough study. The following are just a few of the questions that must eventually be answered if Chew's interpretation of the bootstrap principle is to be called science:

(1) Do nuclei lie on Regge trajectories with universal slope $\alpha' \approx 1 \text{ BeV}^{-2}$, as do ordinary hadrons? Where can we find the excited nuclei which are the Regge recurrences? We are concerned here with level spacings of $\Delta M^2 \approx 2 \text{ BeV}^2$. This excludes the low-energy rotational spectrum, $\Delta M \sim J(J+1)$,

which moreover is nonlinear.

(2) Do nuclei couple to Regge trajectories? What is the high-energy behavior of a two-body nuclear reaction-- $C^{12}(\alpha, d)N^{14}$, for example? Does the cross section go as $S^{2\alpha-1}$, where α is the intercept of the leading Regge trajectory having the quantum numbers exchanged in the reaction?

(3) Are nuclear reactions dual [2]? That is, do $B \geq 2$ resonances in the direct channel build up Regge poles in the crossed channel, and does Regge behavior at high energy require the existence of $B \geq 2$ resonances at low energy?

(4) Why are nuclei so weakly bound? For instance, the binding energy of the deuteron is 2.23 MeV, which is only about 0.1% of the deuteron rest mass. For other nuclei, the binding energy per nucleon is a smoothly varying function of the charge and baryon number.

(5) In the quark model, nuclei have exotic quantum numbers. Do nuclei and hypernuclei fit into $SU(3)$ multiplets? Does the nuclear force transform as an $SU(3)$ singlet?

(6) Is there something special about $B \geq 2$ states, or do weakly-bound nuclear-like states with $B \leq 1$ also exist?

We initiate the study of these questions by focussing attention on deuteron stripping reactions at high energy. These are reactions of the type $A(d, p) A + 1$, or its inverse. In this paper we will study in detail only the simplest of these reactions: namely, $p + p \rightarrow d + \pi^+$, $p + p \rightarrow d + \rho^+$, and $p + d \rightarrow d + p$.

This work was partly motivated by the recently-observed, anomalous high-energy behaviors of $p + p \rightarrow d + \pi^+$ and $p + p \rightarrow d + \rho^+$ [3]. The exchanged system has the quantum numbers of the neutron (the deuteron has $I = 0$), and therefore the forward (or backward, since the initial protons

are identical) differential cross sections for both reactions should go as $s^{2\alpha-1} = s^{-1.76}$, where $\alpha = \alpha_N(0) = -0.38$ is the intercept of the N_α trajectory, which is the leading trajectory having the quantum numbers of the neutron. However, experimentally it was found [3] that $pp \rightarrow d\pi$ goes as $s^{-2.69 \pm 0.11}$, and $pp \rightarrow d\rho$ goes as $s^{-1.19 \pm 0.19}$. The data, reproduced in Fig. 1, appear to be smooth and linear in the interval $12 \text{ BeV}^2 \leq s \leq 40 \text{ BeV}^2$. The very high-energy data at $s \approx 40 \text{ BeV}^2$ is from Allaby *et al.* [4] and is compatible with a straight-line extrapolation of the lower-energy data of Anderson *et al.* [3].

Naive Regge-pole models cannot cope with these results. Even if something can be contrived to explain the $pp \rightarrow d\pi$ slope, it is hard to understand why $pp \rightarrow d\pi$ and $pp \rightarrow d\rho$ are so different. It can not be argued that the N_Y trajectory, with intercept $\alpha_Y(0) = -0.8$, and implying an energy dependence of $s^{-2.6}$, is the leading trajectory in $pp \rightarrow d\pi$. The N_α trajectory is known to couple strongly to $p-\pi$ and to dominate $\pi^+p \rightarrow p\pi^+$. The N_Y trajectory could dominate if the $p-d-N_\alpha$ coupling were weak, but then $pp \rightarrow d\pi$ and $pp \rightarrow d\rho$ would both go as $s^{-2.6}$. Not even a mixture of N_α and N_Y [5] can explain the slow fall-off of $pp \rightarrow d\rho$.

The difficulty becomes all the more apparent if one draws a quark duality diagram [6] for $pp \rightarrow d\pi$, as done in Fig. 2(a). Note that the $p-d-N_\alpha$ vertex is not the usual Regge vertex encountered in $\pi^+p \rightarrow p\pi^+$, Fig. 2(b). At the $p-d-N_\alpha$ vertex, no quark creation or annihilation takes place. Two three-quark systems just sort of flow together and call themselves a deuteron.

In the quark model, nuclei have exotic quantum numbers, which may somehow account for their weak binding. Nuclei should more appropriately be called "super-exotic", because they are not the usual exotic

states predicted by duality diagrams such as Fig. 2(c)[7]. The diagrams in Fig. 2(b) and (c) are said to conserve D-spin [8], whereas nuclear reactions such as $pp \rightarrow d\pi$ do not. If duality diagrams are faithful representations of hadron dynamics, they seem to imply that nuclei do not couple to Regge trajectories, and that nuclear reactions are not dual. In short, nuclei should not exist.

This view is difficult to maintain considering the fact that $pp \rightarrow d\pi^+$ behaves very much like $\pi^+p \rightarrow p\pi^+$. In Fig. 3 the $pp \rightarrow d\pi$ forward differential cross section is plotted [9] over an extensive energy range. There are dramatic oscillations at low energy which gradually become damped as the energy increases. As in ordinary, dual hadron reactions, the data describe a smooth high-energy behavior which appears to average the low-energy oscillations.

The bumps at $\sqrt{s} \approx 2.2, 3.1, 3.7 \text{ BeV}$ have the quantum numbers of He_2 . These need not be genuine resonances. For that, they must be eigenstates of spin and parity, and be independent of the production mechanism.

Regge asymptotic behavior does not necessarily imply resonances at low energy. In the local-duality theory of Schmid [10], the circles generated in the Argand diagram by partial-wave analysis of the Regge amplitude are rather sensitive to the parametrization of the high-energy data. Slightly different extrapolations from the high-energy to the low-energy region can lead to very different results.

It should be evident by now that conventional hydrodynamic approaches to high-energy nuclear reactions raise more questions than they answer.

There is a model which does give a good description of $pp \rightarrow d\pi$, namely the One-Pion-Exchange (OPE) model represented in Fig. 4. Although this model (or the similar isobar model) has been investigated by many

authors [11-23], the simplest and most useful formulation is that of T. Yao [16]. The basic idea is that at the p-d-n vertex the nucleons must be very nearly on mass shell. Since the deuteron is a weakly bound system, one would intuitively expect the p-d-n vertex form factor to fall rapidly if the nucleons are taken off the mass shell. It is for this reason that the integral over the internal loop converges.

The outline of the paper is as follows: In the next section the Yao model is reviewed in detail along with several important revisions. In section 3 we compare the theoretical predictions for the $pp \rightarrow d\pi$ forward differential cross section with the experimental data. There is qualitative and nearly quantitative agreement at all energies, from threshold to the highest energies measured. The same formula can also be used to compute the $pp \rightarrow d\pi$ forward differential cross section, and again the agreement is good. Canonical Regge behavior sets in only at much higher energies.

The one-nucleon-exchange mechanism, discussed in section 4, gives a negligible contribution to $pp \rightarrow d\pi$, even at threshold.

To demonstrate the general validity of the OPE model, we apply it to $p + d \rightarrow d + p$. This is done in section 4. When compared with the experimental data, the model gives an excellent fit to the p-d backward peak over a wide range of energies and angles. In section 6 we try to understand why the contributions to $pd \rightarrow dp$ from the impulse and rescattering diagrams are apparently suppressed. In section 7 we compare the OPE model with the excited-deuteron model of Kerman and Kisslinger [24].

In section 8 we try to interpret the success of the OPE model in terms of the structure of the p-d-n vertex. In section 9 we demonstrate the equivalence of the Yao approach to the dispersive approach of Chahoud, Russo and Selleri [19].

In section 10 we offer some speculations concerning the role of nuclei with respect to the hadron bootstrap. Our results and conclusions are summarized in section 11. Appendix A contains a brief development of those aspects of the relativistic p-d-n vertex which are needed in the text. Finally, the details of cross section calculations are deferred to Appendix B.

2. YAO MODEL (REVISED)

In view of the success of the OPE model in accounting for $pp \rightarrow pn\pi^+$ in the 1-3 BeV region [25], it was natural for Yao [16] to ascribe $pp \rightarrow d\pi^+$ as due to association of the final p and n to form a deuteron. The Feynman graphs for this mechanism are displayed in Fig. 4. Obviously, the trick is to integrate over the internal hadron loop and get a finite result.

The covariant matrix element corresponding to Fig. 4 (a) is

$$T_a = \int \frac{d^4q}{(2\pi)^4} \bar{u}(p') \sqrt{2} G F(k_1^2) \gamma_5 \left(\frac{i \not{p}_1 + m}{m^2 + M^2 - i\epsilon} \right) \chi$$

$$N F \xi \left(\frac{-i \not{p}_1 + m}{p_1^2 + M^2 - i\epsilon} \right) \frac{m_{\pi N} u(p)}{(k_1^2 + \mu^2 - i\epsilon)}, \quad (2.1)$$

where

$$m_1 = \frac{1}{2}d - g, \quad p_1 = \frac{1}{2}d + g, \quad (2.2)$$

$$k_1 = p' - m_1 = p' - \frac{1}{2}d + q.$$

(2.3)

The on-mass-shell pion-nucleon coupling constant is given by $G^2/4\pi = 14.7$.

The Ferrari-Selleri form factor [26],

$$F(k_1^2) = \left[1 + \frac{(k_1^2 + \mu^2)}{60\mu^2}\right]^{-1},$$

(2.4)

was introduced because the internal pion is off the mass shell.

The relativistic p-d-n vertex, $\Gamma \cdot \xi$, is discussed in Appendix A.

When both nucleons are on the mass shell,

$$\Gamma \cdot \xi = (1 + \frac{\rho}{2}) \gamma \cdot \xi - \frac{3\rho}{4} \frac{\gamma \cdot q \gamma \cdot \xi}{q^2}, \quad (2.5)$$

where ξ is the polarization four-vector of the deuteron ($\xi \cdot d = 0$), $\rho = .0265$ is the D/S ratio of the deuteron wave function, and

$$q = \frac{1}{2}(p_1 - m_1). \quad (2.6)$$

The p-d-n coupling constant is

$$N = \left[\frac{16\pi\alpha}{(1 - \alpha r_t)m(1 + \rho^2)} \right]^{1/2}, \quad (2.7)$$

where $r_t = 1.75F = 8.88 \text{ BeV}^{-1}$ is the triplet p-n effective range,

$\alpha = (m\epsilon)^{1/2} = 45.7 \text{ MeV}$ is the deuteron Fermi momentum, and $\epsilon = 2m - M_d = 2.23 \text{ MeV}$ is the binding energy.

The $\pi N \rightarrow \pi N$ scattering amplitude with one pion off the mass shell

is

$$m_{\pi N} = A + i\alpha B, \quad (2.8)$$

where

$$Q = \frac{1}{2}(k_1 + k), \quad (2.9)$$

and A and B are functions of k_1^2 and

$$S_{\pi p} = -(p + k_1)^2 = -(p_1 + k)^2, \quad (2.10)$$

$$u_{\pi p} = -(p - k)^2 = -(p_1 - k_1)^2. \quad (2.11)$$

The object now is to get the integral

$$\int d^4q (m_1^2 + m^2 - i\epsilon)^{-1} (p_1^2 + m^2 - i\epsilon)^{-1}$$

to converge. This we do by introducing a rapidly falling form factor at the p-d-n vertex. Let us try the form

$$\left(\frac{\lambda^2}{m_1^2 + m^2 + \lambda^2} \right) \left(\frac{\lambda^2}{p_1^2 + m^2 + \lambda^2} \right), \quad (2.12)$$

where λ is a parameter to be determined. From (2.2) we write

$$m_1^2 + m^2 = |g|^2 - g_0^2 + M_d g_0 + \alpha^2, \quad (2.13)$$

$$p_1^2 + m^2 = |g|^2 - g_0^2 - M_d g_0 + \alpha^2,$$

and drop the q_0^2 terms. With this simplification,

$$\int \frac{d^4 q}{(n_1^2 + m^2 - i\epsilon)(p_1^2 + m^2 - i\epsilon)} \frac{\lambda^2}{(n_1^2 + m^2 + \lambda^2)} \frac{\lambda^2}{(p_1^2 + m^2 + \lambda^2)}$$

$$= i \frac{\pi^3 \alpha}{M_d} \zeta, \quad (2.14)$$

where

$$\zeta = 2 \left[(2 \frac{\lambda^2}{\alpha^2} + 4)^{1/2} - (\frac{\lambda^2}{\alpha^2} + 1)^{1/2} - 1 \right] \quad (2.15)$$

is a measure of the OPE coupling strength. Later on, when we compare this theory with the experimental data, we will need $\zeta = 2.9$, or equivalently $\lambda^2 = 31\alpha^2$. In section 9 we will calculate ζ using a dispersive approach, but for the time being we leave it as a free parameter.

The characteristic mass, $\lambda \approx 6\alpha$, of the p-d-n vertex form factor is small compared to the nucleon mass. With such a rapid fall-off it is reasonable to approximate the rest of the integrand in (2.1) by its value at $q = 0$. The pion propagator can be taken out from under the integral, because for the physical situations of interest, $k_1^2 \gg \mu^2$; in which case, the contribution of the pion pole to the integral is negligible. With the above assumptions, (2.1) becomes

$$T_a = \left[\frac{\zeta^2 \epsilon \alpha}{(1 - \alpha r_t)} \frac{G^2}{8\pi} \frac{1}{(1 + \rho^2)} \right]^{1/2} \frac{F(k_1^2)}{(k_1^2 + \mu^2)} \times$$

$$i \bar{u}(p') \gamma_5 (i \frac{\not{p}}{2} + m) \Gamma \cdot \xi (A + i \not{Q} B) u(p), \quad (2.16)$$

where from now on, $d \cdot q = 0$, and

$$k_1 = p_1' - \frac{1}{2} d. \quad (2.17)$$

Summation over the deuteron and proton spins leads to (see Appendix B for the details)

$$\sum_{spins} T_a^\dagger T_a = \frac{\zeta^2}{2} \frac{3 \epsilon \alpha}{(1 - \alpha r_t)} \frac{G^2}{4\pi} \frac{F(k_1^2) k_1^2}{(k_1^2 + \mu^2)^2} \times$$

$$\frac{64 \pi^2}{(2m)^2} 2 S_{\pi p} \frac{d\sigma^{\pi p}}{d\Omega}, \quad (2.18)$$

where

$$\frac{64 \pi^2}{(2m)^2} 2 S_{\pi p} \frac{d\sigma^{\pi p}}{d\Omega} = \text{Tr} \left\{ \frac{(-i \not{p} + m)}{2m} (A^* + i \not{Q} B^*) \frac{(-i \not{p}' + m)}{2m} \times \right.$$

$$(A + i \not{Q} B) \left. \right\} = \frac{1}{2m^2} \left\{ |A|^2 [S_{\pi p} + u_{\pi p} + 2m^2 - 2\mu^2] + \right. \quad (2.19)$$

$$|B|^2 [\mu^4 - (S_{\pi p} - m^2)(u_{\pi p} - m^2)] + 2m(u_{\pi p} - S_{\pi p}) \text{Re} AB^* \left. \right\} \equiv Y^2$$

is the on-mass-shell π -p cross section in the π -p c.m. system. In reality, (2.18) involves the $\pi p \rightarrow \pi p$ cross section for which one of the pions (k_1) is off the mass shell.

The $\pi^+ p \rightarrow \pi^+ p$ amplitude in Fig. 4(a) is pure $I = 3/2$. The contribution of Fig. 4(b) can be taken into account by the replacement

$$\begin{aligned}
 A &\rightarrow A_{3/2} - \frac{1}{\sqrt{2}} \left(\frac{\sqrt{2}}{3} A_{1/2} - \frac{\sqrt{2}}{3} A_{3/2} \right) \\
 &= \frac{4}{3} A_{3/2} - \frac{1}{3} A_{1/2},
 \end{aligned} \quad (2.20)$$

and similarly for B. This leads to the replacement

$$\frac{d\sigma}{d\Omega} \rightarrow \frac{1}{2} \left[3 \frac{d\sigma}{d\Omega}^{\pi^+ p \rightarrow \pi^+ p} - \frac{d\sigma}{d\Omega}^{\pi^+ p \rightarrow \pi^+ p} + 3 \frac{d\sigma}{d\Omega}^{\pi^+ p \rightarrow \pi^0 n} \right] \quad (2.21)$$

in (2.18).

The differential cross section for $pp \rightarrow d\pi$ in the p - p system

is

$$\frac{d\sigma}{d\Omega} = \frac{|d|}{|p|} \frac{(2m)^2}{64\pi^2 S_{pp}} \frac{1}{4} \sum_{\text{spins}} |T - T'|^2, \quad (2.22)$$

where

$$|p|^2 + m^2 = \frac{1}{4} S_{pp}, \quad (2.23)$$

$$|d|^2 = \frac{1}{4 S_{pp}} [(S_{pp} - M_0^2 - \mu^2)^2 - 4 M_0^2 \mu^2], \quad (2.24)$$

$$S_{pp} = -(d+k)^2 = 2 S_{\pi p} + 2 m^2 - \mu^2, \quad (2.25)$$

and T' is obtained from T by interchanging p and p' . The interference term in

$$|T - T'|^2 = T^\dagger T - T^* T' + (p \neq p') \quad (2.26)$$

is spin-summed in Appendix B. The final result is

$$\begin{aligned}
 \frac{d\sigma}{d\Omega}^{\pi^+ p \rightarrow d\pi} &= \frac{\xi^2}{8} \frac{3\epsilon\alpha}{(1-\alpha r_c)} \frac{G^2}{4\pi} \frac{F^2(k^2) k_1^2}{(k_1^2 + \mu^2)^2} \frac{|d|}{|p|} \frac{S_{\pi p}}{S_{pp}} \times \\
 &\left[3(1-J^{(+)}) \frac{d\sigma}{d\Omega}^{(+)} - (1-J^{(-)}) \frac{d\sigma}{d\Omega}^{(-)} + 3(1-J^{(0)}) \frac{d\sigma}{d\Omega}^{(0)} \right] \\
 &+ (p \neq p'),
 \end{aligned} \quad (2.27)$$

where

$$J = \frac{F(k_1^2)}{F(k_1^2)} \frac{(k_1^2 + \mu^2)}{(k_1^2 + \mu^2)} \frac{2}{3} \frac{1}{k_1^2 Y^2} \text{Re}(X^* X' + X^* d X' d), \quad (2.28)$$

$$X_\mu^* = Q_\mu B^*(m + \frac{p \cdot d}{M_d}) - p_\mu (A^* + \frac{d \cdot Q}{M_d} B^*), \quad (2.29)$$

and Y^2 is defined by (2.19). From (2.17) we have

$$k_1^2 = \frac{1}{2} (m^2 - u_{\pi d}), \quad (2.30)$$

and

$$u_{\pi d} = -(d-p')^2 = -(\frac{1}{2}d - k_1)^2 = u_{\pi p}. \quad (2.31)$$

The scattering angle of $pp \rightarrow d\pi$ in the p - p c. m. system is given by

$$u_{\pi d} = m^2 + \mu^2 - S_{pp}^{1/2} (|d|^2 + \mu^2)^{1/2} - 2|p||d| \cos \theta. \quad (2.32)$$

The scattering angle of $\pi p \rightarrow \pi N$ in the π - p c. m. system is given by

$$u_{\pi p} = m^2 + \mu^2 - 2\tilde{p}_0 \tilde{k}_0 - 2(\tilde{p}_0^2 - m^2)^{1/2} (\tilde{k}_0^2 - \mu^2)^{1/2} \cos \delta, \quad (2.33)$$

where

$$\tilde{p}_0 = \frac{1}{2} S_{\pi p}^{-1/2} (S_{\pi p} + m^2 + k_1^2), \quad (2.34)$$

$$\tilde{k}_0 = \frac{1}{2} S_{\pi p}^{-1/2} (S_{\pi p} - m^2 + \mu^2). \quad (2.35)$$

The two angles are related by (2.31-2.33).

This model involves $\pi p \rightarrow \pi N$ cross sections for which one of the pions (k_1) is off the mass shell, and it is ambiguous how this is to be related to the physical cross section. Two natural ways of doing this are:

(1) cos δ fixed. Assume that at fixed $s_{\pi p}$ and $\cos \delta$, the $\pi p \rightarrow \pi N$ amplitude varies slowly with k_1^2 , in which case we must simultaneously solve (2.30-2.35) to relate $\cos \phi$ and $\cos \delta$.

(2) u fixed. Another way is to assume slow variation at fixed $s_{\pi p}$ and $u_{\pi p}$. In this case, we first substitute $k_1^2 \rightarrow -\mu^2$ in (2.34), and then solve (2.30-2.35).

Before going on to compare (2.27) with experiment, we point out how our calculation differs from Yao's [16]: (A) We have explicitly calculated the contribution of the interference term, $\mathcal{I}$, whereas Yao simply assumed it to be small in the kinematic region of interest. (B) Yao's expression for the relativistic p - d - n vertex is incorrect. (C) In the loop integration, Yao took the mean momentum-spread of p_1 and n_1 to be the Fermi momentum, α . Actually, the momentum spread is somewhat larger--about 6α . Contrary to intuition, the rapid fall-off of the p - d - n vertex form factor is not due (at least not directly) to the smallness of the binding energy, but rather to the smallness of the pion mass. We will discuss this later in Section 8.

3. $pp \rightarrow d\pi$, $d\rho$; COMPARISON WITH EXPERIMENT

For over twenty years the reaction $pp \rightarrow d\pi$ and its inverse have been studied in connection with pion-spin and parity determinations, and with tests of isospin invariance and detailed balance. There now exists a large body of data on this process, from threshold up to $s_{pp} \approx 40 \text{ BeV}^2$.

In Fig. 3, which is adapted from Anderson et al. [9], the $pp \rightarrow d\pi$ forward differential cross section is plotted over a wide energy range. The dashed curve was calculated from (2.27), assuming that $\mathcal{I} = 0$ and $\zeta = 2.9$. We have not evaluated $\mathcal{I}$, because it would require a detailed knowledge of the π - N invariant amplitudes, A and B , over an extensive energy range. Only the forward differential cross section is compared with experiment, because here, $\mathcal{I}$ is a minimum. The dashed curve in Fig. 3 was calculated according to the $\cos \delta$ -fixed prescription. For forward scattering, this corresponds to $\cos \theta = \cos \delta = -1$ for the first term in (2.27), and $\cos \theta = \cos \delta = 1$ for the crossed term, $(p \rightarrow p')$. The parameter $\zeta = 2.9$ was chosen so that the theoretical

and experimental curves coincide in the region of the first bump.

Let us now discuss the three energy regions in Fig. 3: high, intermediate, and threshold.

a. High-Energy Region

It is gratifying that the Yao model correctly predicts the high-energy behavior of $pp \rightarrow d\pi$, even though it was originally intended to explain the resonance region. We are hence encouraged to apply the same formula, (2.27), to obtain a prediction for $pp \rightarrow d\rho$.

In order to do this we need $\pi N \rightarrow \rho N$ backward scattering data at high energy. Unfortunately, the only available data is for $\pi^+ p \rightarrow p\rho^+$ at 5.2 BeV/c [27]. The energy dependence is unknown, and so we are forced to assume that it is given by N_α exchange ($\alpha_N(0) = -0.38$):

$$\frac{|k_\pi|}{|k_p|} \frac{d\sigma}{d\Omega_\delta} \pi^+ p \rightarrow p\rho^+ = A_p S_{pp}^{-1.76} \exp(B_p u_{pp}). \quad (3.1)$$

The N_α seems to be the leading trajectory in $\pi^+ p \rightarrow p\pi^+$ (apparently, the Δ -trajectory residue is small) since the parametrization

$$\frac{d\sigma}{d\Omega_\delta} \pi^+ p \rightarrow p\pi^+ = A_\pi S_{\pi p}^{-1.76} \exp(B_\pi u_{\pi p}) \quad (3.2)$$

fits the π -p data. The slope of the backward peak empirically varies as [28]

$$B_\pi \approx [5.0 + 0.6 \frac{S_{\pi p}}{m^2}] \times (\text{BeV}/c)^{-2}. \quad (3.3)$$

At 5.2 BeV/c, the $\pi^+ p \rightarrow p\rho^+$ parameters are [27]:

$$B_p \approx 2.9, \quad A_p/A_\pi \approx 2.0, \quad (3.4)$$

determined for $u_{pp} = u_{\pi p} = 0$ and $s_{pp} = s_{\pi p}$ in (3.1) and (3.2).

Now we must relate the physical $u_{\pi p}^{(P)}$, appearing in (3.2), to the off-mass-shell $u_{\pi p}$, defined by (2.31). Two prescriptions for doing this are:

(1) u fixed. Simply write

$$u_{\pi p}^{(P)} = u_{\pi p} = u_{\pi d}. \quad (3.5)$$

(2) cos δ fixed. From (2.33-2.35) it follows that at $\cos \theta = \cos \delta = -1$,

$$u_{\pi p}^{(P)} = \frac{\tilde{p}}{\tilde{p}} u_{\pi p} + (m^2 + \mu^2) \left[1 - \frac{\tilde{p}}{\tilde{p}} \right] + 2\tilde{k}_0 \tilde{p}_0 \left[\frac{\tilde{p}}{\tilde{p}} - \frac{\tilde{p}_0}{\tilde{p}_0} \right], \quad (3.6)$$

where $u_{\pi p} = u_{\pi d}$ and $\tilde{p}^2 = \tilde{p}_0^2 - m^2$.

To apply these formulae to $pp \rightarrow d\rho$, we need only set $\mu = \mu_\rho$, instead of $\mu = \mu_\pi$. Since only $\pi^+ p \rightarrow p\rho^+$ is known, we calculate the forward differential cross sections for $pp \rightarrow d\rho$ and $pp \rightarrow d\pi$, neglecting the π^- -p cross sections in (2.27), and also the interference term, $\mathcal{I}$, and the crossed term, $(p \rightleftharpoons p')$. The results are plotted in Fig. 5. The absolute scale is fixed by having the $pp \rightarrow d\pi$ cross section calculated according to the cos δ -fixed prescription (dashed curve) coincide with the experimental curve. Of course, the relative magnitude of the two cross sections is

fixed by (3.4). For comparison, the cross sections calculated according to the u -fixed prescription are also plotted (solid curves). The calculated curves appear to be insensitive to extrapolation prescription used. Note that the curves are not straight lines; but they are approximately straight in the region where data exist. The slopes along these segments are about -2.6 for $pp \rightarrow d\pi$, and -1.5 for $pp \rightarrow d\rho$. Compare with Fig. 1. Asymptotically, the two curves have the same slope, -1.76, corresponding to N_α exchange.

The slope obtained for $pp \rightarrow d\rho$ is in satisfactory agreement with the experimental value, $-1.19 \pm .19$, considering the uncertainty in the energy dependence of $\pi N \rightarrow N\rho$, which we assumed to be given by N_α exchange. This assumption may be suspect since the $\pi^+ p \rightarrow p\rho^+$ backward peak does not have a dip at $u \approx -0.15$ (BeV/c)² [27]. The dip is present in $\pi^+ p \rightarrow p\pi^+$, and corresponds to a wrong signature - nonsense zero. The dip is considered as evidence that N_α exchange does in fact dominate the backward peak in $\pi^+ p \rightarrow p\pi^+$.

The reason that the curves in Fig. 5 are not straight lines is mainly due to the factor $k_1^2(k_1^2 + u)^{-2}$ in (2.27), where k_1^2 is slowly approaching its asymptotic limit, $k_1^2 \rightarrow \frac{1}{2}m^2$. At fixed s_{pp} , k_1^2 depends on the mass of the produced meson and leads to a different curvature for $pp \rightarrow d\pi$ and $pp \rightarrow d\rho$. Another important effect comes from the exponential in (3.2), where $B_\pi u_{\pi p}$ is large and positive but approaches zero with increasing energy. $B_\rho u_{\rho p}$ is small, and consequently has little effect.

It is fair to say that the different energy behaviors of $pp \rightarrow d\pi$ and $pp \rightarrow d\rho$ can be quantitatively accounted for in terms of purely kinematic considerations which are already incorporated into the Yao model.³ The lesson is that one should be wary of applying simple Regge phenomenology to nuclear reactions. Only at much higher energies do various kinematic

effects die out and there emerges the simple power behavior characteristic of ordinary hadron reactions. Regge dogma only tells us that Regge behavior should set in at high energy, but it does not say how high.

H. Lee [21] has argued, in a Regge framework, that at high energy the major contribution to $pp \rightarrow d\pi$ should come from the OPE mechanism in Fig. 4, because it has the nearest (to the physical region) u -channel singularity, an anomalous threshold [30] at

$$u_{\pi d} = m^2 + 2\mu(\mu + 2\alpha) \quad (3.7)$$

(and similarly for $t_{\pi d}$). An asymptotic relation was obtained between $pp \rightarrow d\pi$ and $\pi N \rightarrow N\pi$, both of which are given by N_α exchange. This is already manifest in the Yao model, which in addition contains kinematic factors which at non-asymptotic energies distort the simple power behavior of the $\pi N \rightarrow N\pi$ cross section.

b. Resonance Region

In the "resonance" region, the theoretical curve in Fig. 3 makes more drastic swings than does the data. However, the interference term is likely to be important in this region. In the OPE model, the bumps correspond to excitation of Δ isobars at the $\pi N \rightarrow N\pi$ scattering vertex in Fig. 4. That N^* 's do not appear to be excited is a consequence of the $I = 1$ isospin of the exchanged pion, which enhances the $I = 3/2$ $\pi \rightarrow N$ amplitude, as can be seen in (2.20) [14]. The bumps in $pp \rightarrow d\pi$, corresponding to $\Delta(1236, 1950, 2420)$, are predicted by (2.25) to be at $\sqrt{s_{pp}} = 2.19, 3.06, 3.67$ BeV, which are in good agreement with the data [9].

In the OPE model, the intermediate $N\text{-}\Delta$ states are not bona fide resonances; that is, they are not eigenstates of spin and parity. If the process were to proceed by formation of actual He_2^* resonances, as depicted in Fig. 6, we would expect the bumps to be at [13]

$$M_{pp} \approx M_{\Delta} + m, \quad (3.8)$$

minus some small binding energy which is negligible compared to M_{pp} . Equation (3.8) predicts dibaryon resonances at $M_{pp} = 2.18, 2.89, 3.36$ BeV, which closely coincide with the positions predicted by (2.25), except for the last bump, where it is difficult to discern from the data which mass formula is correct.

To be a genuine resonance, a bump must be independent of the production channel. That is, the bumps in $pp \rightarrow d\pi$ and $pp \rightarrow d\pi$ should be at the same energy. This is true for (3.8), but not for (2.25), which depends on the mass of the produced meson. The data appear to favor (2.25) over (3.8) [31].

The OPE model also relates [31] $np \rightarrow d\sigma(310)$ to $\pi N \rightarrow N\sigma(310)$, where $\sigma(310)$ is the "ABC effect" having $I = 0$. Whether or not it is a true resonance is irrelevant -- only that it has $I = 0$. Consequently, only N^* 's can be excited in $\pi N \rightarrow N\sigma$, and therefore in $np \rightarrow d\sigma$. An enhancement corresponding to $N^*(1470)$ excitation is seen in $np \rightarrow d\sigma$. It might also be added that a bump corresponding to $\Delta(1236)$ excitation appears in $np \rightarrow d\gamma$, but not in $dp \rightarrow \text{He}_3 \gamma$ [31].

It is noteworthy that both (2.25) and (3.8) lead to He_2 "trajectories" which differ from the usual Regge trajectories. The square of (3.8) leads to a nonlinear He_2 trajectory due to the $2M_{\Delta}m$ term. The OPE energy relation, (2.25), leads to a linear trajectory,

but with one-half the slope of the Δ trajectory.

In summary, it appears very unlikely that the bumps in $pp \rightarrow d\pi$ are genuine dibaryon resonances, even though the reaction is Regge-behaved at very high energy. In this respect, nuclear reactions seem to differ fundamentally from ordinary hadron reactions.

c. $pp \rightarrow d\pi$ at Threshold

The $pp \rightarrow d\pi$ cross section below the first bump in Fig. 3 can be parametrized in the c. m. frame as [32]

$$\sigma = a \frac{|d|}{\mu} + b \frac{|d|^3}{\mu^3}, \quad (3.9)$$

corresponding to S-wave and P-wave pions. It is a simple matter to evaluate the OPE formula, (2.27), at threshold, and thereby obtain a crude prediction for $\underline{a}$. Threshold for $pp \rightarrow d\pi$ corresponds to threshold for $\pi p \rightarrow \pi p$. Here, the interference term, (2.28), can be evaluated: $\underline{A}(\text{thresh}) = -1/3$. Integrating over the solid angle, multiplying by two to take the $(p \rightleftharpoons p')$ term into account, and inserting the π -N scattering lengths, $2a_1 + a_3 = 0$ and $a_1 - a_3 = 0.27 \text{ u}^{-1}$, we obtain

$$\underline{a}(\text{OPE}) = \zeta^2 \times 24 \mu b. \quad (3.10)$$

To get agreement with the experimental value of Rose [33],

$$\underline{a}(\text{exp}) = 240 \pm 20 \mu b, \quad (3.11)$$

we need $\zeta = 3.2$. This is not far from the value, $\zeta = 2.9$, needed to fit the $pp \rightarrow d\pi$ forward differential cross section in the resonance region. Clearly, OPE is the dominant mechanism of $pp \rightarrow d\pi$, even at threshold.

Koltun and Reitan [34] have calculated $\underline{a}$ and have also

found OPE to be the major contributor. However, by their method they obtained $a = 164 - 201 \mu\text{b}$, which is in better agreement with an earlier experimental value, $138 \pm 15 \mu\text{b}$ [35].

4. THE ONE-NUCLEON-EXCHANGE

Whatever became of the One-Nucleon-Exchange (ONE) diagram in Fig. 7? In low energy nuclear physics, deuteron-stripping and pick-up reactions are known to proceed via the ONE mechanism. Here, ONE dominates because the momentum transfer is near the nucleon pole ($t = m^2$). As the energy increases, the exchanged nucleon becomes farther off mass shell, and the ONE contribution should drop rapidly. The cross section is proportional to $(m^2 - t)^{-2}$, where

$$t = M_d^2 + m^2 - 2M_d(q^2 + m^2)^{1/2} \approx m^2 - 2(q^2 + \alpha^2). \quad (4.1)$$

The ONE contribution should start to die out when $|q| > \alpha$. In the case of $pp \rightarrow d\pi$ at threshold, we already have

$$u = m^2 - 2m\mu = 0.7 m^2, \quad (4.2)$$

due to the pion rest mass, which dwarfs the deuteron binding energy. Hence, we would not expect ONE to contribute appreciably to $pp \rightarrow d\pi$ at any energy.

The threshold cross section corresponding to the ONE diagram in Fig. 7 was previously calculated by Schillaci and Silbar [36]. With gradient coupling of the pion, the covariant matrix element is

$$\begin{aligned} T_1 &= \bar{v}(p') N \Gamma \cdot \xi \frac{(-i\not{p} + i\not{k} + m)}{(m^2 - u)} \frac{\sqrt{2}G}{2m} \not{k} \gamma_5 u(p) \\ &= \frac{\sqrt{2}GN}{(m^2 - u)} \bar{v}(p') \Gamma \cdot \xi \left[\not{k} + i \frac{(m^2 - u)}{2m} \right] \gamma_5 u(p) \end{aligned} \quad (4.3)$$

(the full, antisymmetrized amplitude is $T_1 - T_1'$), where now

$$\Gamma_\mu = F_1(u) \gamma_\mu - i F_2(u) \not{p}_\mu \quad (4.4)$$

is the p-d-n vertex for which the neutron is off the mass shell. The form factors, F_1 and F_2 , are related to the Fourier transforms of the non-relativistic S and D-wave functions, and the off-mass-shell form factors, F_3 and F_4 , have been neglected. (See Appendix A for a detailed discussion of the general p-d-n vertex.) The matrix element, (4.3), leads to

$$\begin{aligned} a(\text{ONE}) &= \frac{\mu}{|p|} \frac{(2m)^2}{(M_d + \mu)^2} \frac{G^2}{4\pi} \frac{N^2}{2} \frac{|p|^2}{m^4} F_1^2(u) \\ &= 660 \mu\text{b} \times F_1^2(u). \end{aligned} \quad (4.5)$$

The ONE contribution is greatly reduced by the form factor, $F_1(u)$. For example, the Hulthén wave function (see Appendix A) gives

$$F_1^H(u) = \frac{\lambda_H^2}{(m^2 - u + \lambda_H^2)} \left[1 + \frac{\rho}{\lambda_H^2} - \frac{\rho}{\lambda_H^2} \frac{(m^2 - u)}{2\alpha^2} \right], \quad (4.6)$$

where $\lambda_H^2 = 51.8 \alpha^2$, and $\rho = .0265$. At threshold, $F_1^H(u) = -.044$, and therefore $a(\text{ONE}) = 1.3 \mu\text{b}$, which is peanuts compared to the OPE contribution. Schillaci and Silbar [36] have computed $F_1(u)$ corresponding to more realistic wave functions, and have obtained values corresponding to $a(\text{ONE}) = 22 \div 66 \mu\text{b}$.

The ONE diagram with gradient coupling is the natural result of attaching a soft pion to an external leg of the p-d-n vertex; so it seems strange that $a(\text{ONE}) \ll a(\text{OPE})$. However, the soft-pion technique [37] assumes that the amplitude is effectively unchanged in the limit $\mu \rightarrow 0$. As pointed out by Schillaci and Silbar [36], this condition is not fulfilled by the mechanism in Fig. 7, because the vertex form factor can change by an order of magnitude as the pion mass goes to zero. On the other hand, in the OPE diagram, Fig. 4, the soft pion is attached to the $\pi N \rightarrow N\pi$ scattering vertex. Since the soft-pion technique is known to work for π -N scattering at threshold, it should also be valid for the OPE diagram.

It is not clear why the gradient-coupling prescription used in (4.3) should be valid for $pp \rightarrow d\pi$. What is clear is that ONE is only a small part of the total mechanism. In any case, the ONE amplitude should not be added to the OPE amplitude, because ONE is (at least partially) already included in OPE (see Section 8).

5. $pd \rightarrow dp$ À LA YAO

In comparing the OPE formula (2.27) with the experimental data, we were handicapped by our uncertainty of the interference term. We restricted ourselves only to forward scattering, and there assumed that $\mathfrak{J} = 0$. But $\mathfrak{J} = -1/3$ at threshold, and it is also likely to be important in the resonance region in Fig. 3, where there are discrepancies between the theoretical and experimental curves.

It is desirable to have some other deuteron-stripping reaction which has no interference term. Schillaci and Silbar [38] have applied the Yao model to $K^-d \rightarrow \Lambda n, \Sigma^- p$. In this case, there is no interference term, but unfortunately, there is also no data.

The reaction we seek is p-d backward scattering. All we have to do is change p- π to d-p at the $\pi N \rightarrow N\pi$ vertex in Fig. 4. The calculation is very similar to that in section 2, except that now there is no interference term because the initial particles are distinguishable. The result is

$$\frac{d\sigma^{pd \rightarrow dp}}{d\Omega_\phi} = \frac{\pi^2}{2} \frac{3\epsilon\alpha}{(1-\alpha r_t)} \frac{G^2}{4\pi} \frac{F^2(k_1^2) k_1^2}{(k_1^2 + \mu^2)^2} \frac{S_{pp} |q|}{S_{pd} |k|} \times \frac{3}{2} \frac{d\sigma^{pp \rightarrow p\pi}}{d\Omega_\phi}, \quad (5.1)$$

where

$$S_{pd} = 2S_{pp} + m^2, \quad (5.2)$$

$$k_1^2 = \frac{1}{2}(m^2 - u_{pd}), \quad (5.3)$$

$$\cos \phi = -1 + \frac{2[(M_d^2 - m^2)^2 - u_{pd} S_{pd}]}{[S_{pd} - (M_d + m)^2][S_{pd} - (M_d - m)^2]}, \quad (5.4)$$

and $|p|$, $|d|$ are given by (2.23), (2.24).

As in section 2, $\cos \phi$, in the p-d c.m. frame, is related to $\cos \theta$, in the p-p c.m. frame, according to:

(1) The u-fixed prescription.

$$u_{pd} = u_{\pi d} = m^2 + \mu^2 - S_{pp}^{1/2} (|d|^2 + \mu^2)^{1/2} - 2|p||d| \cos \theta, \quad (5.5)$$

$$k_1^2 = \frac{1}{2}(m^2 - u_{pd}) = \frac{1}{4}(S_{pp} - M_d^2 - \mu^2) + |p||d| \cos \theta. \quad (5.6)$$

(2) The cos θ -fixed prescription. Set $u^2 \rightarrow -k_1^2$ in (5.6), and solve for k_1^2 . The result is

$$k_1^2 = [3S_{pp} + (S_{pp} + 4m^2) \cos^2 \theta + 4S_{pp} \cos \theta \times (1 - \frac{m^2}{S_{pp}} \sin^2 \theta)^{1/2}][q(1 - \frac{4m^2}{S_{pp}})^{-1} - \cos^2 \theta]^{-1}. \quad (5.7)$$

The factor 3/2 in (5.1) comes about as follows: (i) The ratio of the spin degeneracies in the initial states of $pp \rightarrow d\pi$ and $pd \rightarrow dp$ is 4/6. (ii) The $pd \rightarrow dp$ amplitude is the coherent sum of the π^+ and the π^0 -exchange diagrams in Fig. 8(a). Isospin symmetry implies⁴ $G_{\pi^+} = \sqrt{2} G_{\pi^0}$ and $A(\pi^+ d \rightarrow pp) = -\sqrt{2} A(\pi^0 d \rightarrow pn)$, which, along with the antisymmetry of the deuteron isospin wave function, lead to a factor $(1 + 1/2)^2 = 9/4$ in the cross section. (iii) The diagram in Fig.

8(b) should not be counted, since Figs. 8(a) and (b) both contain the diagram in Fig. 8(c).

Craigie and Wilkin [39] have previously applied the Yao model to $pd \rightarrow dp$. However, due to a number of errors they obtained a relation which was low by a factor of 2.5 when compared with the data at a few energies.

Another previous work on $pd \rightarrow dp$ is that of Chahoud, Russo, Mazzanti, and Salvini [40]. Using a dispersive approach, they obtained a broad backward peak at 1.0 BeV, which is about the same order of magnitude as the data.

Equation (5.1) affords a good fit to the p-d backward scattering data over a wide range of energies and angles. The data are best fitted with $\xi = 5.4$, which is nearly double the value, $\xi = 2.9$, required to fit the $pp \rightarrow d\pi$ forward differential cross section. We consider $\xi = 5.4$ to be more reliable, since (5.1) does not involve interference and crossed terms, as does (2.27).

The $pd \rightarrow dp$ and $pp \rightarrow d\pi$ data sources are listed in Table I. From (5.2) it is easy to show that the incident proton kinetic energies in the lab. correspond for the two processes. Up to about $T_p = 1.0$ BeV, the $pp \rightarrow d\pi$ differential cross section can be parametrized as

$$\frac{d\sigma_{pp \rightarrow d\pi}}{d\Omega_0} = \frac{\sigma}{4\pi} \frac{(A + \cos^2 \theta - B \cos^4 \theta)}{(A + \frac{1}{3} - \frac{1}{5} B)}. \quad (5.8)$$

For σ , A, and B, we use the values listed in Table I. At several energies of interest, $pp \rightarrow d\pi$ data does not exist, so we were forced to interpolate from the data at neighboring energies.

In Fig. 9 is displayed almost all the existing p-d backward scattering data from 0.34 to 3.66 BeV. We do not consider lower energies because the $pp \rightarrow d\pi$ threshold is at 300 MeV. The solid curves were calculated from (5.1) according to the u-fixed prescription, and the dashed curves were calculated according to the $\cos\theta$ -fixed prescription. Overall, the agreement between theory and experiment is excellent, considering that there is only one free parameter, ξ .

We now comment on some of the major features in Fig. 9:

(1) The $\cos\theta$ -fixed prescription is better than the u-fixed prescription. It is smoother, and in addition, $\cos\phi = -1$ when $\cos\theta = -1$. The difference between the two curves is an indication of the ambiguity in extrapolating the off-mass-shell amplitude from the physical amplitude. At high energy the ambiguity disappears.

(2) We might have expected the $pd \rightarrow dp$ cross section to be very small since it is second order in the OPE mechanism. The cross section is large because most of the data is in the (3,3)-resonance region.

(3) At 340 MeV, the data is somewhat underestimated, perhaps because we are close to the $pp \rightarrow d\pi$ threshold at 300 MeV, below which ONE may be a more accurate description. Also, the second Glauber term (see the next section) may contribute significantly at low energy.

(4) The theoretical curves at 1.3, 1.5, 1.65, and 2.0 BeV suggest a pronounced dip near $\cos\phi = -1$, but the p-d data do not cover this region. A sharp backward dip has been seen at 1.18 BeV [53], but unfortunately, there are no $pp \rightarrow d\pi$ data corresponding to this energy and angle.

(5) It is surprising that the model works so well, even for k_1^2 up to about $40 \mu^2$. Such high momentum transfers would seem to preclude any justification for OPE dominance. In the dispersive approach (see Sections 8 and 9), the OPE approximation corresponds to considering only the π -N anomalous region. If this region completely saturates the dispersion integrals, it should do so for all values of $k_1^2 = \frac{1}{2}(m^2 - u)$. However, this is very doubtful.

6. HADRON-DEUTERON ELASTIC SCATTERING

To understand why OPE should be the dominant mechanism in p-d backward scattering at high energy, we present a brief survey of hadron-deuteron elastic scattering at all energies and angles.

The forward diffraction peak of $pd \rightarrow dp$ is well described by the semi-classical optical model of Glauber [54], which is equivalent to the diagrams in Fig. 10 [55, 56]. For forward scattering, the impulse diagram in Fig. 10(a) is dominant, because the scattering amplitude has a t-channel anomalous threshold at $t=16m\epsilon$, which is very close to $t=0$. In the impulse approximation, the π -d total cross section is simply the sum of the π -p and π -n cross sections. Of course, the bumps observed in the π -d cross section do not represent genuine dibaryon resonances.

With increasing momentum transfer, the nucleons coupled to the deuterons are driven off the mass shell. Since the deuteron momentum-space wave function falls off rapidly with momentum transfer (see Appendix A), so must the impulse contribution. Eventually, the double-scattering (or "shadow") term in Fig. 10(b) takes over, because the second scattering allows the nucleons comprising the deuteron to remain more nearly on mass shell. Interference between the impulse and the double-scattering term leads to a dip in the differential cross section. At very large momentum transfers, where the impulse contribution is negligible, the double-scattering term gives a broad backward peak, which at 1 BeV is an order of magnitude too low [56].

This interpretation of the p-d backward peak should already have been disqualified on the grounds that the double-scattering term does not have a Regge cut, which is considered necessary for its persistence at high energy [55]. On the other hand, the impulse diagram does have a Regge pole. At very high energy, the impulse diagram should be the sole survivor in the Glauber series for forward elastic scattering.

A rough estimate of the energy at which the double-scattering term should become invalid was made by Landshoff and Gribov [57], and runs essentially as follows: The double-scattering diagram for X-d elastic scattering, where X is any hadron, is shown in Fig. 11. Energy-momentum conservation at the $XN \rightarrow NX$ vertices leads to

$$X''^2 = \frac{1}{2}(X'^2 + X^2) + \frac{1}{4}t + (X' + X) \cdot \Delta + \Delta^2, \quad (6.1)$$

where $\Delta = \delta' - \delta$ and $t = -(X' - X)^2$. We consider only the case $X'^2 = X^2$.

The important feature of (6.1) is that although the $(X' + X) \cdot \Delta$ term averages to zero, its mean value can be very large. At $t = 0$, the average $\langle X''^2 \rangle \approx X^2$. In the lab. frame at high energy, the mean deviation from the average is $\langle |X| |\Delta| \rangle \approx X_0 \alpha$, where we have used $\Delta_0 \ll |\Delta|$ and $\langle |\Delta| \rangle \approx \alpha$. In effect, the random oscillations at the deuteron vertices are amplified by the large, driving X momentum. The amplitude of oscillation grows linearly with incident energy, so that at very high energy, X'' spends most of its time far off mass shell. In general, the $XN \rightarrow NX''$ amplitude should decrease when X'' is taken off the mass shell, and therefore, the double-scattering contribution should shrink with energy. With an $XNN-X''$ form factor having a characteristic mass squared of about 1 BeV^2 , the critical energy at which the

double-scattering term should begin to shrink is $X_0 \approx 1 \text{ BeV}^2/\alpha \approx 20 \text{ BeV}$, at $t = 0$.

For large t , the average $\langle X''^2 \rangle$ is also far off the mass shell, and so the critical energy should be much smaller. It is not surprising, then, that the double-scattering contribution to p-d backward scattering appears to be negligible.

The double-scattering term was doomed anyway, because it is not Regge-behaved at high energy. For $X = p$, the diagram in Fig. 11 has a π trajectory-elementary-N cut in the u channel (assuming π -trajectory exchange in $pn \rightarrow np$). However, Regge theory tells us that N_α -trajectory exchange should dominate, and only the OPE diagram in Fig. 8 fulfills this requirement.

It is natural to wonder, what is the high-energy behavior of $\pi d \rightarrow d\pi$, which is the simplest $B = 2$ exchange reaction? The double-scattering diagram in Fig. 11 must be disqualified because it has an N_α trajectory-elementary-N cut in the u channel. Next, consider the diagram in Fig. 12. This mechanism is ideal, because the two internal pions can both remain on mass shell, and at the same time satisfy (6.1). At high energy and $u \approx 0$, the two-pion-exchange diagram in Fig. 12 has a $\Delta - \Delta$ Regge cut.

It would be nice to have some experimental data on π -d backward scattering at intermediate and high energies. We expect considerable structure due to resonance excitation of the nucleons in the deuteron, as occurs in $pp \rightarrow d\pi$ and $pd \rightarrow dp$.

7. N*'S IN THE DEUTERON?

The currently popular interpretation for the p-d backward peak at high energy is the excited-deuteron model of Kerman and Kisslinger [24]. Realizing that ONE is inadequate at these momentum transfers, they added the $N^*(1688)$ -exchange diagram in Fig. 13. In this model, the deuteron is not composed of just a proton and a neutron, but there is also a small probability that it will be in an excited state, d^* , consisting of N and $N^*(1688)$. With a d^* admixture of 0.5-2.5%, they were able to fit the p-d backward peak at 1.0 BeV [24] and 0.58 BeV [43].

The success of the d^* model is hardly surprising, considering its similarity to the OPE model. As sketched in Figs. 14(a) and (b), $pd \rightarrow dp$ is related to $\pi N \rightarrow N\pi$, which by duality can be represented either as N_α -trajectory exchange [Fig. 14(c)] or as Δ production in the direct channel [Fig. 14(d)]. Since N and $N^*(1688)$ are the first two resonances on the N_α trajectory, what Kerman and Kisslinger have essentially done is to generate a Regge pole by exchanging a tower of resonances [58]. In this sense, Figs. 13 and 14(c) are equivalent, and to add them would be double counting. In fact, the 1% d^* probability, which was needed to fit the p-d backward peak at 1.0 BeV, was estimated via an OPE transition potential for $pn \rightarrow pn^*$.

The trouble with the d^* model is that the d^* probability must change with energy. The 1.0 BeV data could be fitted with 1.0% d^* [24], but 2.5% was needed to fit the 0.58 BeV data [43]. Coupling constants should not change with energy. In the OPE model, the interpretation is simply that we are going over the (3,3)-resonance bump.

The $N^*(1470)$ resonance, even though it is lighter than the $N^*(1688)$, should have a much smaller probability of virtual excitation in the deuteron, because the 1470 lies on a much lower-lying trajectory

than does the 1688. Sharma, Bhasin and Mitra [59] have extended the model of Kerman and Kisslinger to include the exchange of other N^* ($I = \frac{1}{2}$) resonances besides the $N^*(1688)$. The fit to $pd \rightarrow dp$ does not markedly improve.

Nath, Weber, and Kabir [60] have suggested the reaction $\pi^- d \rightarrow p \Delta^-$, which is pure $I = 3/2$ exchange, as a probe of the Δ - Δ content of the deuteron. For this reaction they emphasized the $\Delta(1236)$ -exchange and the OPE-triangle diagrams, and claimed that at high energy, the latter is included in the former. One might expect the OPE mechanism to give a complete account of this reaction. However, $\pi^- d \rightarrow p \Delta^-$ has no t-channel anomalous singularities, as does $\pi d \rightarrow pp$. Consequently, OPE dominance is uncertain (see sections 8 and 9). The $\pi^- d \rightarrow p \Delta^-$ cross section is a measure of the contribution of normal t-channel singularities to the short-distance structure of the deuteron, and is likely to be much smaller than that predicted by Nath et al. [60].

8. STRUCTURE OF THE DEUTERON

The question remains: Does the deuteron consist solely of protons and neutrons, or is there a small admixture of N- N^* ? In Fig. 14(c) the deuteron only couples to p-n. The N^* does not couple directly to the deuteron, but only by first converting into a nucleon via an OPE interaction with the other nucleon.

Let us examine the structure of the deuteron in more detail. The renormalized p-d-n vertex is analyzed in Fig. 15 as an OPE-triangle diagram, a two-pion-exchange-triangle diagram, etc. Consider the case in which the proton is on mass shell and the neutron has mass u . The singularity structure in the dimensionless variable η , where

$$u = m^2 + \eta \alpha^2, \quad (8.1)$$

is sketched in Fig. 16. The most important singularities are:

- (i) the nucleon pole at $\eta = 0$; (ii) a π -N anomalous cut starting at $\eta \approx 2\mu(\mu + 2\alpha)/\alpha^2 = 30.6$; (iii) a $\pi\pi$ -N anomalous cut starting at $\eta \approx 8\mu(\mu + \alpha)/\alpha^2 = 98$; (iv) a normal π -N threshold at $\eta = (2m_\mu + \mu^2)/\alpha^2 = 134$.

We are interested in the p-d-n vertex form factor, so we must subtract out the singularities coming from the nucleon propagator. The form factor does not have a nucleon pole at $\eta = 0$; it is the deuteron wave function that has the pole (see Appendix A). The p-d-n vertex form factors satisfy once-subtracted dispersion relations [61] in η :

$$F_{1,2}(\eta) = F_{1,2}(0) + \frac{\eta}{\pi} \int_{30.6}^{\infty} d\eta' \frac{\text{Im} F_{1,2}(\eta')}{\eta'(\eta' - \eta)}. \quad (8.2)$$

In section 2 we used the experimental OPE coupling strength, $g = 2.9(5.4)$, to determine the characteristic mass, $\lambda^2 = 31(78)\alpha^2$, of the p-d-n vertex form factor. This mass turns out to be around the characteristic mass, $\lambda_H^2 = 51.8\alpha^2$, of the form factor corresponding to the Hulthén wave function. The major contribution to the dispersion integral, (8.2), seems to come from the π -N anomalous cut, which is closest to the physical region, $\eta \leq 0$. The vertex form factor falls rapidly, not because the deuteron is weakly bound, but due rather to the smallness of the pion mass.

The ONE amplitude should not be added to the OPE amplitude, because the former is (at least partially) already included in the latter. In fact, ONE and OPE are complementary descriptions of the

p-d-hadron vertex. Near the nucleon pole, the ONE approximation is accurate; whereas, far from the pole, the OPE description is more appropriate since other channels besides the neutron can couple significantly to p-d.

What we have essentially done in section 2 is to crudely evaluate the p-d-n vertex for when the neutron is far off the mass shell. The general form of the vertex is

$$i\Gamma_\mu = [F_1(n^2)i\gamma_\mu + F_2(n^2)q_\mu] + [F_3(n^2)i\gamma_\mu + F_4(n^2)q_\mu] \frac{(i\not{x} + m)}{2m}. \quad (8.3)$$

The $pp \rightarrow d\pi$ matrix element, (2.16), is proportional to

$$\bar{u}(p') \Gamma \cdot \xi (i\not{x} + m) \gamma_5 (A + iQB) u(p), \quad (8.4)$$

where now, $\Gamma \cdot \xi$ is the on-mass-shell vertex, (A1) or (2.5). Using $k_1 + p = k + d/2$, $n = p - k$, and keeping only the neutron-pole term in $(A + iQB)$, we rewrite (8.4) as

$$\bar{u}(p') \Gamma \cdot \xi [(i\not{x} + m) + i\not{k}_1] \left(\frac{-i\not{x} + m}{m^2 + m^2} \right) \gamma_5 u(p) + \dots \quad (8.5)$$

In addition to a contribution to the off-mass-shell form factors, F_3 and F_4 , (8.5) contains a second term, involving $\not{k}_1$, which has no analog in the general p-d-n vertex, (8.3). Therefore, (2.16) can only be an approximation to the actual $pp \rightarrow d\pi$ matrix element, and should not be taken too seriously.

9. DISPERSIVE APPROACH

Several authors [18-21] have calculated $pp \rightarrow d\pi$ using a dispersive approach. The simplest treatment is that of Chahoud, Russo and Selleri [19]. Instead of directly integrating over the OPE loop, they wrote an unsubtracted dispersion relation in u for the amplitude,

$$T(u_{nd}) = \frac{1}{\pi} \int du \frac{\text{Im} T(u)}{(u - u_{nd})}, \quad (9.1)$$

and assumed that the major contribution to the integral comes from the π -N anomalous cut, since it is the closest singularity (other than the nucleon pole which occurs in the $\pi N \rightarrow N\pi$ part of the amplitude) to the physical region, $u < m^2$. The discontinuity across the anomalous cut (the imaginary part of the amplitude) follows from the Cutkosky [62] prescription,

$$\frac{1}{(m_1^2 + m^2 - i\epsilon)} \rightarrow 2\pi i \delta(m_1^2 + m^2) \theta(m_{10}), \quad (9.2)$$

and similarly for the other two poles in (2.1). The discontinuity,

$$2i \text{Im} T = \int d^4q (2\pi i)^3 \delta(m_1^2 + m^2) \delta(p_1^2 + m^2) \delta(k_1^2 + m^2) \theta(m_{10}) \theta(q_{10}) \theta(k_{10}) \times$$

$$\frac{\sqrt{2}GN}{(2\pi)^4} \bar{u}(p') \gamma_5 (i\not{p}_1 + m) \Gamma \cdot \xi (-i\not{p}_1 + m) (A + i\not{A}B) u(p), \quad (9.3)$$

involves only on-mass-shell quantities (sans the Ferrari-Selleri form factor). This explains why, for direct integration in section 2, we needed only the on-mass-shell p - d - n vertex.

In this picture, the deuteron couples directly only to protons and neutrons. There can be no direct N^* - d coupling if the anomalous region completely saturates the dispersion integrals, (8.2) and (9.1). In this approximation, the N^* couples to the deuteron only by first changing into $\pi + N$. There is direct N^* - d coupling only if the π -N normal cut contributes to the dispersion integrals.

Also, it can be argued [19] that the discontinuity across the $\pi\pi$ -N anomalous cut ($98 \leq n < 134$) should be small because of the absence of π - π resonances with mass below 400 MeV.

We now calculate the OPE coupling strength, ζ , using the dispersive approach. In the deuteron rest frame,

$$k_1^2 + \mu^2 = (p_1^2 - \frac{1}{2}d^2) + \mu^2 + q_1^2 - q_0^2 + M_d q_0 - 2kp'_d |q_0 + 2kp'_d |q| \cos\psi, \quad (9.4)$$

and

$$d^4q = \frac{1}{4M_d |p'_d|} d|q|^2 d(M_d q_0) d\varphi d(2kp'_d |q| \cos\psi). \quad (9.5)$$

Inserting (2.13), (9.4) and (9.5) in (9.3) gives

$$2i \text{Im} T = \frac{2\pi^3}{iM_d |p'_d|} \int d|q|^2 d(M_d q_0) d\varphi d(2kp'_d |q| \cos\psi) \times \\ \delta(|q|^2 + \alpha^2) \frac{1}{2} \delta(M_d q_0) \delta\left(\left[p'_d - \frac{1}{2}d\right]^2 + \mu^2 - \alpha^2 + 2kp'_d |q| \cos\psi\right) \times$$

$$\frac{\sqrt{2}GN}{(2\pi)^4} \bar{u}(p') \gamma_5 \left(i\frac{p}{2} - i\frac{q}{2} + m \right) \Gamma \cdot \xi \left(-i\frac{p}{2} - i\frac{q}{2} + m \right) \times$$

$$(A + i\epsilon B) u(p). \quad (9.6)$$

The non-delta-function part of the integrand can be approximated by its value at $q = 0$. With this simplification, the factor

$$2i \operatorname{Im} I(u) = \frac{2\pi^4}{iM_d |p_d'|} \quad (9.7)$$

in (9.6) corresponds to the integral over the nucleon poles in (2.14).

Using (8.1),

$$|p_d'| = \left[p'^2 + \frac{(p' \cdot d)^2}{M_d^2} \right]^{1/2} \approx i\alpha \left(\frac{\eta}{2} \right)^{1/2}. \quad (9.8)$$

Then assuming that the dispersion integral is saturated by the π -N anomalous cut,

$$\begin{aligned} I(u_{\pi d}) &= \frac{1}{\pi} \int du \frac{\operatorname{Im} I(u)}{(u - u_{\pi d})} \\ &\approx \frac{i\pi^3 \alpha}{M_d} \int_{30.6}^{134} \frac{d\eta (2\eta)^{-1/2}}{(k^2 + \frac{1}{2}\eta\alpha^2)} \approx \frac{i\pi^3 \alpha}{M_d} \times \frac{8.5}{k^2}, \end{aligned} \quad (9.9)$$

where k_1^2 is now defined by (2.30).

The k_1^{-2} pole in (9.9) effectively corresponds to the pion pole in (2.16). On comparing (9.9) with (2.14), we identify $\xi = 8.5$. This result is somewhat larger than the experimental values, 2.9 and 5.4. In order to get 2.9 (5.4), we would have to cut off the integral at $\eta = 57(88)$. Apparently, the π -N anomalous cut does not saturate the dispersion integral, and there must be cancellations with the $\pi\pi$ -N anomalous cut, and perhaps with the normal π -N cut. A similar situation obtains in dispersive calculations of the p-d-n vertex form factors. On considering only the π -N anomalous region, F. Gross [63] obtained form factors which fall off too slowly with momentum transfer [64]. To improve on our result, (9.9), is tantamount to having a complete, quantitative theory of the deuteron; and that does not yet exist.

10. THE NATURE OF NUCLEI

Besides their exotic quantum numbers, the characteristic that distinguishes nuclei from the ordinary hadrons is their relatively weak binding energies. The binding energy of the deuteron, for instance, is related to the depth and range of the interaction potential between proton and neutron. Although the effective potential can be simulated by a series of meson exchanges, it is not unique. There is as yet no convincing calculation of the deuteron binding energy based on fundamental considerations.

Another distinguishing characteristic is that nuclei apparently do not lie on Regge trajectories, although they do couple to non-exotic trajectories. Nuclei seem to constitute a distinct and separate class in

the hadron bootstrap, the class of "super-exotics" which violate D-spin [8].

In fact, we question whether nuclei are really hadrons at all. In view of their super-exotic quantum numbers it is tempting to hypothesize that nuclei may be inert to the SU(3)-symmetric part of the strong interaction. Then, the only force left to bind nuclei would be the medium-strong, SU(3)-breaking component of the strong interaction. The interaction strength is just right to account for the scale of nuclear binding energies, which is weaker than strong but stronger than electromagnetism. Admittedly, this is speculative.

A more modest question is whether the nuclear force is SU(3) symmetric. The isospin symmetry of nuclear reactions, and the existence of isobaric analog resonances are well established. But what about "strangeness analog resonances" [65, 66]? Such resonances have been observed in $K^- + C^{12} \rightarrow \pi^- + B^{11} + p$ and $K^- + N^{14} \rightarrow \pi^- + C^{13} + p$ [67]. Detailed studies of strangeness analog resonances might reveal how the nuclear interaction potential transforms under SU(3). If the nuclear force is really the medium-strong force, then the interaction potential should transform as the eighth component of an octet. Not enough is known about low-energy hyperon-nucleon interactions to decide this question at present [68].

Do nuclei and hypernuclei belong to SU(3) multiplets [69]? The deuteron has $I = 0$, $Y = 2$, and only the $\overline{10}$ representation of $8 \otimes 8$ contains this element. Where are the other members of the $\overline{10}$ multiplet? There have been reports of a 2130 MeV $\Lambda - p$ enhancement in $K^-d \rightarrow \Lambda p \pi^-$ [68,70], but it is likely to be a kinematic effect. In any event, the relevance of unitary symmetry to nuclear and hypernuclear states is uncertain, since the $\Sigma - \Lambda$ mass difference is consider-

ably larger than the binding energy per baryon [65].

Do nuclear-like states with $B \leq 1$ also exist? One candidate is the controversial $Z_0^*(1830)$ enhancement [71], a loosely bound (?) $K^*(890)$ -N system observed in the phase-shift analysis of K-N scattering.

A possible mechanism responsible for the Z_0^* is pictured in Fig. 17. In effect, the K-N scattering amplitude can be driven resonant by the rapid opening of the K^* -N channel [72]. At the K^* -N threshold, the OPE amplitude for $KN \rightarrow K^*N$, iterated through unitarity, can generate an effective attractive potential.

Obviously, this effect can not occur below the K^* -N threshold, because the pion can not couple to K-K, but only to K-K* or K*-K*. This might explain why K-nuclei (nuclei containing a bound K^0 or K^+) apparently do not exist. K-nuclei could decay only via the weak interaction; whereas K^* -nuclei (e.g., the Z_0^*) can decay rapidly via the strong interaction -- making them very difficult to study.

It is interesting to note that the $Z_0^*(1830)$ has the same I , Y , and nearly the same mass as the deuteron. Also, since the $K^*(890)$ mass is close to the nucleon mass, the singularity structure of the $N-Z_0^*-K^*$ vertex is similar to that of the $p-d-n$ vertex.

11. SUMMARY AND CONCLUSIONS

(1) The OPE model gives a good description of the $pp \rightarrow d\pi$ forward differential cross section, from threshold up to the highest energies measured. In the resonance region, the bumps are all in the right places, but the detailed fit is not so good. However, the interference term, I , which was assumed to be zero, is likely to be significant in this region. Also, the crossed term, $(p \rightleftharpoons p')$ in (2.27), makes

an appreciable contribution. The first term in (2.27) involves backward ($\cos \delta = -1$) π -N scattering, and k_1^2 is a minimum; whereas the crossed term involves forward ($\cos \delta = 1$) π -N scattering, and k_1^2 is a maximum. The validity of this model is doubtful at large k_1^2 .

(2) The OPE mechanism can adequately account for the $pp \rightarrow d\pi$ threshold cross section. The ONE contribution is small, because even at threshold the momentum transfer is sufficiently large so that the rapidly falling p-d-n vertex form factor greatly suppresses the ONE contribution.

The ONE and OPE amplitudes should not be added. To the extent that the π -N anomalous region completely dominates the p-d-n vertex form factor, ONE is completely contained in OPE. However, it is unlikely that the overlap is complete. For momentum transfers close to the nucleon pole, ONE is an accurate approximation; whereas, far from the pole, OPE is more appropriate.

(3) The OPE formula, (2.27), correctly accounts for the high-energy behaviors and the relative magnitude of the $pp \rightarrow dp$ and $pp \rightarrow d\pi$ forward differential cross sections. At very high energy, $pp \rightarrow dp$ and $pp \rightarrow d\pi$ should both go as $s^{-1.76}$, as suggested by Regge theory. But at non-asymptotic energies there are kinematic factors in (2.27) which distort the canonical Regge behavior of the $\pi N \rightarrow N\pi$ scattering cross sections and lead to different energy behaviors for $pp \rightarrow dp$ and $pp \rightarrow d\pi$. Moreover, these kinematic effects are still important at energies where all resonant structure has completely died out. The moral: Beware of Regge fits to nuclear reactions.

(4) The bumps in the $pp \rightarrow d\pi$ cross section do not represent genuine dibaryon resonances, but rather, non-resonant N- Δ states, the masses of which are correctly predicted by the OPE model. The N- Δ "trajectory" has one-half the slope of the Δ trajectory. Moreover, the N- Δ states are not eigenstates of spin and parity, because the N and the Δ are separated by a large distance. Although nuclear Regge trajectories may still exist, all the major features of the data can be simply understood without them. The Schmid analysis [10], in which resonances at low energy are a consequence of Regge asymptotic behavior, is foiled by kinematic distortions which blur the connection between the high and low-energy regions.

Nuclei appear to be composite objects, whose hadronic constituents can individually couple to Regge trajectories, but which can not interact collectively to form resonances. As suggested by the quark model, nuclear reactions are not dual. In general, anomalous singularities preclude duality.

(5) $pd \rightarrow dp$ is ideal for testing the OPE model, because there are no interference or crossed terms, and only one input cross section is required. The theoretical curves, calculated from (5.1) with $\zeta = 5.4$, fit the data over a wide range of energies and angles. The $pd \rightarrow dp$ data clearly indicate $\Delta(1236)$ -excitation of one of nucleons. It will be interesting to see how the model fares above 1.5 BeV, where the higher Δ isobars should be excited, and where the data are sparse.

In the d^* model of Kerman and Kisslinger, the large p-d backward peak is interpreted as due to a 0.5-2.5% admixture of N-N* (1688) state in the usual n-p non-relativistic deuteron wave function. Although it is in principle equivalent to the OPE model, the d^* model is inadequate

because the p - d - N^* coupling constant has to change with energy.

(6) In the OPE model, the only free parameter is the OPE coupling strength, ζ . We needed $\zeta = 5.4$ to fit the $pd \rightarrow dp$ differential cross section, whereas $\zeta = 2.9$ was required to fit the $pp \rightarrow d\pi$ forward differential cross section. Since the $pp \rightarrow d\pi$ case involves uncertainties in the interference and cross terms, we consider $\zeta = 5.4$ to be more trustworthy.

We tried to calculate ζ by means of an unsubtracted dispersion relation in u . Taking only the π - N anomalous region into account, the result is $\zeta = 8.5$. Apparently, the π - N anomalous cut does not saturate the dispersion integral. The same is true for dispersive calculations of the p - d - n vertex form factors.

(7) For high-energy nuclear reactions we propose the following rules: (i) The asymptotic behavior is specified by Regge theory. This is the simplest way of ensuring that unitary bounds on scattering amplitudes are respected. (ii) The amplitude factorizes in the t -channel. (iii) At lower energies (e.g., the resonance region), the main contribution comes from diagrams which are Regge-behaved at high energy. This follows from analyticity or smoothness. It would be very unsmooth for diagrams to suddenly change their relative importance. This is a weak form of duality which may still be applicable to nuclear reactions. (iv) At energies below, say, the threshold for pion production, diagrams which are not Regge-behaved (e.g., ONE) may become more important.

Accordingly, the double-scattering diagram for $pd \rightarrow dp$ should not be dominant, because it is not Regge-behaved. The same should be true for $\pi d \rightarrow d\pi$. In this case, the leading diagram should have a Δ - Δ Regge cut in the absence of dibaryon Regge trajectories which couple to nuclei.

(8) Since nuclei seem to differ fundamentally from ordinary hadrons, a speculative scheme for segregating nuclei from the hadron bootstrap is that perhaps they are not really hadrons. Due to their super-exotic quantum numbers, nuclei may be inert to the $SU(3)$ -symmetric part of the strong interaction. They could then bind together and interact only through the $SU(3)$ -breaking medium-strong force. That is to say, the nuclear force may really be the medium-strong force. Nuclear-like states with $B \leq 1$ may also exist. One candidate is the $Z^*_0(1830)$, which except for spin and baryon number is very similar to the deuteron.

Plenty of work remains to be done. Some obvious directions to pursue are: polarization, extension to light nuclei, $np \rightarrow d\gamma$, $\pi^- d \rightarrow p\Delta^-$, $\pi d \rightarrow d\pi$, and $pA \rightarrow Ap$. It is hoped that this investigation stimulates interest in the development of a fundamental theory of the deuteron, and of nuclei in general.

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APPENDIX A. THE RELATIVISTIC p-d-n VERTEX

The literature [61, 63, 64, 73-75] dealing with this subject is somewhat confusing. No two references are alike, and there are also errors in some. In the following we develop those aspects of the relativistic p-d-n vertex which are needed in the text.

On-mass-shell

The relativistic p-d-n vertex when both nucleons are on the mass shell is

$$\bar{v}(p) i \Gamma \cdot \xi u(n) = \bar{v}(p) [F_1(-m^2) i \gamma \cdot \xi + F_2(-m^2) g \cdot \xi] u(n), \quad (A1)$$

where ξ is the polarization four-vector of the deuteron, and

$$g = \frac{1}{2}(p-n). \quad (A2)$$

In general,

$$g^2 = \frac{1}{2}(p^2 + n^2) - \frac{1}{2}d^2, \quad (A3)$$

$$g \cdot d = \frac{1}{2}(p^2 - n^2). \quad (A4)$$

When both nucleons are on the mass shell, $q \cdot d = 0$, and $q^2 = -\epsilon^2 = -\epsilon m$, where ϵ is the binding energy.

In the deuteron rest frame, $\xi_0 = 0$; and when both nucleons are on mass shell, $q_0 = 0$. Then by defining

$$F_1(-m^2) = (1 + \frac{\rho}{2}),$$

(A5)

$$F_2(-m^2) = \frac{3\rho}{4} \frac{m}{\alpha^2} + \frac{1}{2m} (1 + \frac{\rho}{2}),$$

the on-mass shell vertex, (A1), becomes

$$\begin{aligned} \bar{v}(p) i \Gamma \xi u(m) &= \chi_p^\dagger \left\{ (1 + \frac{\rho}{2}) \underline{\sigma} \cdot \underline{\xi} - \frac{3\rho}{4} \underline{\sigma} \cdot \underline{\hat{q}} \underline{\hat{q}} \cdot \underline{\xi} \right\} \chi_n \\ &= \chi_p^\dagger \left\{ \underline{\sigma} \cdot \underline{\xi} - \frac{\rho}{4} [3 \underline{\sigma} \cdot \underline{\hat{q}} \underline{\sigma} \cdot \underline{\hat{q}} - \underline{\sigma}(\underline{\sigma} \cdot \underline{\xi}) \underline{\sigma}] \right\} \chi_n \end{aligned} \quad (A6)$$

in the non-relativistic limit.

In (A6), the first term is the S-wave coupling, and the term proportional to ρ is the D-wave coupling since it involves the Fourier transform of the dipole operator,

$$S_{pm}(\hat{r}) = 3 \underline{\sigma}_p \cdot \underline{\hat{r}} \underline{\sigma}_n \cdot \underline{\hat{r}} - \underline{\sigma}_p \cdot \underline{\sigma}_n. \quad (A7)$$

In (A6), the $\sqrt{8}$ is needed to normalize the dipole operator, since in the triplet state,

$$\int \frac{d\Omega}{4\pi} S_{pm}^2(\hat{r}) = 8. \quad (A8)$$

Hence, ρ is the D/S ratio of the deuteron wave function.

Another popular form for the p-d-n vertex is

$$\bar{v}(p) i \Gamma \xi u(m) = i \bar{v}(p) \left[F_1(-m^2) \underline{\gamma} \cdot \underline{\xi} - \frac{\alpha^2}{m} F_2(-m^2) \frac{\underline{\gamma} \cdot \underline{\hat{q}} \underline{\hat{q}} \cdot \underline{\xi}}{q^2} \right] u(m), \quad (A9)$$

which is identical to (A1) when both nucleons are on the mass shell. The form (A9) can accurately be approximated by

$$\Gamma_p = (1 + \frac{\rho}{2}) \underline{\gamma} - \frac{3\rho}{4} \frac{\underline{\gamma} \cdot \underline{\hat{q}} \underline{\hat{q}}}{q^2}. \quad (A10)$$

We employ this form for calculations in Appendix B.

We now determine the p-d-n coupling constant, N , following the method of Sakita and Goebel[74]. Near the deuteron pole, the np $\rightarrow$ np matrix element (assuming $\rho = 0$ for simplicity),

$$\frac{N^2}{(M_d^2 - S_{np})} \bar{v}(p') \underline{\gamma}_\mu u(m') \bar{v}(p) \underline{\gamma}_\mu u(m), \quad (A11)$$

leads to

$$\sigma_{np} = \frac{3m^2}{64\pi} \frac{N^4}{(q^2 + \alpha^2)^2}. \quad (A12)$$

On the other hand, σ_{np} involves singlet and triplet scattering in the combination

$$\sigma_{np} = \frac{3}{4} \sigma_t + \frac{1}{4} \sigma_s, \quad (A13)$$

where in terms of the S-wave phase shift,

$$\sigma_t = 4\pi |q + i g \cot \delta_t|^{-2}. \quad (A14)$$

In the effective range (shape-independent) approximation,

$$g \cot \delta_x = -\frac{1}{a_x} + \frac{1}{2} r_x g^2, \quad (A15)$$

which at the deuteron pole ($q = i\alpha$) gives

$$-\alpha = -\frac{1}{a_x} - \frac{1}{2} r_x \alpha^2. \quad (A16)$$

Eq. (A13) now becomes

$$\sigma_{mp} = 3\pi |(q - i\alpha)[1 + \frac{1}{2} r_x (q + i\alpha)]|^{-2} + \frac{1}{4} \sigma_s. \quad (A17)$$

Finally, equating the residues of the $q = i\alpha$ poles in (A12) and (A17) yields

$$N = \left[\frac{16\pi\alpha}{(1 - \alpha r_x)m(1 + \rho^2)} \right]^{1/2}. \quad (A18)$$

Off-mass shell

When the proton is on the mass shell, and the neutron is slightly off the mass shell, the p-d-n vertex has the general form

$$i\Gamma_\mu = [F_1(m^2)i\gamma_\mu + F_2(m^2)g_\mu] + [F_3(m^2)i\gamma_\mu + F_4(m^2)g_\mu] \frac{(i\not{x} + m)}{2m}, \quad (A19)$$

where

$$m^2 + m^2 = 2(1q^2 + \alpha^2). \quad (A20)$$

Neglecting the off-mass shell form factors, F_3 and F_4 in (A19), the deuteron wave function is

$$\left[\frac{m^2}{2d_0 p_0 m_0} \right]^{1/2} N \bar{u}(p) i\Gamma \cdot \xi \frac{(-i\not{x} + m)}{m^2 + m^2} u(m) \approx \frac{m^{1/2} N}{2(1q^2 + \alpha^2)} \chi_p^\dagger \{ F_1(m^2) \underline{\sigma} \cdot \underline{\xi} + [F_2(m^2) - \frac{1}{2m} F_1(m^2)] \frac{1q^2}{m} \underline{\sigma} \cdot \hat{q} \hat{q} \cdot \underline{\xi} \} \chi_n. \quad (A21)$$

We want to compare (A21) with the usual non-relativistic wave function in momentum space,

$$\begin{aligned} & \left(\frac{m}{8\pi} \right)^{1/2} N \left(\frac{d^3r}{4\pi} e^{-i\mathbf{q} \cdot \mathbf{r}} \chi_p^\dagger \left\{ \frac{u(r)}{r} \underline{\sigma} \cdot \underline{\xi} - \frac{\rho}{\sqrt{8}} \frac{w(r)}{r} \times \right. \right. \\ & \quad \left. \left. [3 \underline{\sigma} \cdot \hat{r} \underline{\sigma} \cdot \underline{\xi} \underline{\sigma} \cdot \hat{r} - \underline{\sigma}(\underline{\sigma} \cdot \underline{\xi}) \underline{\sigma}] \right\} \frac{1}{\sqrt{2}} \chi_n \right. \\ & \quad \left. = \frac{m^{1/2} N}{2} \chi_p^\dagger \left\{ \tilde{u}(q) \underline{\sigma} \cdot \underline{\xi} - \frac{\rho}{\sqrt{2}} \tilde{w}(q) \times \right. \right. \\ & \quad \left. \left. (3 \underline{\sigma} \cdot \hat{q} \hat{q} \cdot \underline{\xi} - \underline{\sigma} \cdot \underline{\xi}) \right\} \chi_n, \right. \end{aligned} \quad (A22)$$

where

$$\begin{aligned} \tilde{u}(q) &= \int_0^\infty r dr u(r) j_0(qr), \\ \tilde{w}(q) &= - \int_0^\infty r dr w(r) j_2(qr), \end{aligned} \quad (A23)$$

and

$$\begin{aligned} j_0(x) &= \frac{\sin x}{x}, \\ j_2(x) &= \left(\frac{3}{x^2} - \frac{1}{x}\right) \sin x - \frac{3}{x^2} \cos x, \end{aligned} \quad (\text{A24})$$

are spherical Bessel functions. On comparing (A21) and (A22),

$$\begin{aligned} F_1(m^2) &= (|q|^2 + \alpha^2) [\tilde{u}(q) + \frac{\rho}{\sqrt{2}} \tilde{w}(q)], \\ F_2(m^2) &= (|q|^2 + \alpha^2) \left[-\frac{3\rho}{\sqrt{2}} \frac{m}{|q|^2} \tilde{w}(q) \right] + \frac{1}{2m} F_1(m^2). \end{aligned} \quad (\text{A25})$$

The D/S ratio, $\rho = .0265$, was determined [64] from the electric quadrupole moment of the deuteron. The D-state probability is

$$P_D = \frac{\rho^2 \int dr w^2(r)}{\int dr [u^2(r) + \rho^2 w^2(r)]}. \quad (\text{A26})$$

We have $P_D = \rho^2$ only if $u(r) = w(r)$, but this is not the case. The currently popular value is $P_D = .07$.

As an instructive example, consider the Hulthén wave function,

$$u_H(r) = e^{-\alpha r} - e^{-\beta r}, \quad (\text{A27})$$

$$w_H(r) = \alpha r h_2^{(1)}(i\alpha r) - \frac{\beta^2}{\alpha^2} \beta r h_2^{(1)}(i\beta r),$$

where $\beta = 5.18 \alpha$, and

$$h_2^{(1)}(ix) = \frac{e^{-x}}{x} \left(1 + \frac{3}{x} + \frac{3}{x^2} \right) \quad (\text{A28})$$

is a spherical Hankel function of the first kind. Eqs. (A27) into (A23) yield [75]

$$\tilde{u}_H(q) = \frac{(\beta^2 - \alpha^2)}{(|q|^2 + \alpha^2)(|q|^2 + \beta^2)}, \quad (\text{A29})$$

$$\tilde{w}_H(q) = -\frac{|q|^2}{\alpha^2} \tilde{u}_H(q).$$

APPENDIX B. SPIN SUMMATION OF THE OPE AMPLITUDE

We want to sum the OPE amplitude, (2.16), over the proton and deuteron spins. The term of interest is

$$M = \bar{u}(p') \gamma_5 (i \frac{\not{p}}{2} + m) \Gamma \cdot \xi (A + i \not{Q} B) u(p), \quad (B1)$$

where

$$\Gamma \cdot \xi = (1 + \frac{\not{p}}{\sqrt{2}}) \gamma \cdot \xi - \frac{3p}{\sqrt{2}} \frac{\gamma \cdot \not{p} \gamma \cdot \xi}{p^2}, \quad (B2)$$

and

$$Q = \frac{1}{2}(k_1 + k_2) = p' + \frac{1}{2}p - \frac{3}{4}d. \quad (B3)$$

First we sum the diagonal term, $|M|^2$:

$$\begin{aligned} \sum_{\text{spins}} M^\dagger M &= \sum_{\xi} \text{Tr} \{ \bar{u}(p) \gamma_4 (A + i \not{Q} B)^\dagger (\Gamma \cdot \xi)^\dagger (i \frac{\not{p}}{2} + m)^\dagger \gamma_5 \gamma_4 \times \\ &\quad u(p') \bar{u}(p') \gamma_5 (i \frac{\not{p}}{2} + m) \Gamma \cdot \xi (A + i \not{Q} B) u(p) \} \\ &= k_1^2 \sum_{\xi} \text{Tr} \{ \frac{(-i \frac{\not{p}}{2} + m)}{2m} (A^* + i \not{Q} B^*) \Gamma \cdot \xi^* \Gamma \cdot \xi \times \\ &\quad \frac{(-i \frac{\not{p}}{2} + m)}{2m} (A + i \not{Q} B) \} \\ &= 3(1+p^2) k_1^2 \text{Tr} \{ \frac{(-i \frac{\not{p}}{2} + m)}{2m} (A^* + i \not{Q} B^*) \frac{(-i \frac{\not{p}}{2} + m)}{2m} \times \\ &\quad (A + i \not{Q} B) \}. \end{aligned} \quad (B4)$$

The trick in evaluating the interference term in $|M - M'|^2$ is to sum $M^* M'$ instead of $M^\dagger M$. M' can be obtained from M by interchanging p and p' . Using $u^* = \gamma_2 v$ and $\bar{v}^* = -\bar{u} \gamma_2$, we obtain

$$\begin{aligned} \sum_{p \text{ spins}} M^* M' &= \text{Tr} \{ \bar{v}(p') \gamma_5 (i \frac{\not{p}}{2} + m)^* (\Gamma \cdot \xi)^* (A + i \not{Q} B) \times \\ &\quad u^*(p) \bar{u}(p) \gamma_5 (i \frac{\not{p}}{2} + m) \Gamma \cdot \xi (A' + i \not{Q}' B') u(p') \} \\ &= \text{Tr} \{ (i \frac{\not{p}}{2} + m) \Gamma \cdot \xi^* (A^* + i \not{Q} B^*) \frac{(-i \frac{\not{p}'}{2} + m)}{2m} \times \\ &\quad (i \frac{\not{p}}{2} + m) \Gamma \cdot \xi (A' + i \not{Q}' B') \frac{(-i \frac{\not{p}'}{2} + m)}{2m} \} \\ &= (1+p^2) \text{Tr} \{ \gamma \cdot \xi^* i \gamma \cdot X^* \frac{(i \not{p} + M_d)}{2M_d} \gamma \cdot \xi i \gamma \cdot X' \frac{(i \not{p} + M_d)}{2M_d} \}, \end{aligned} \quad (B5)$$

where we have inserted (B2) and

$$\begin{aligned} &\frac{(-i \not{p} + M_d)}{2M_d} \frac{(-i \frac{\not{p}}{2} + m)}{2m} (A^* + i \not{Q} B^*) \frac{(-i \frac{\not{p}'}{2} + m)}{2m} \frac{(i \not{p} + M_d)}{2M_d} \\ &= \frac{(-i \not{p} + M_d)}{2M_d} i \gamma \cdot X^* \frac{(i \not{p} + M_d)}{2M_d}, \end{aligned} \quad (B6)$$

$$X_\mu^* = Q_\mu B^* (m + \frac{p \cdot d}{M_d}) - p_\mu (A^* + \frac{d \cdot Q}{M_d} B^*), \quad (B7)$$

and then (separately) averaged q and q' over the solid angle. Finally, summing over ξ , we arrive at the result,

$$\sum_{\text{spins}} M^* M' = 2(1+p^2) \left(X^* X' + \frac{X^* \cdot d X' \cdot d}{M_G^2} \right).$$

(B8)

FOOTNOTES

1. We are unable to verify this with a Finite-Energy Sum Rule (FESR), because the imaginary part of the amplitude is not known. An FESR merely requires that a smooth extrapolation of the high-energy data somehow averages the low-energy data.
2. Under the interchange $p \rightleftharpoons p'$, $u_{\pi d} \rightleftharpoons u'_{\pi d} = t_{\pi d}$. Furthermore, $s'_{\pi p} = s_{\pi p}$. But note that $u'_{\pi p} \neq t_{\pi p}$.
3. I hear that D. J. Brown and co-workers at Carleton University (Ottawa) have also investigated $pp \rightarrow d\rho$ in the Yao model. Also, B. Kayser [29] has noted that at the same angle and energy, $u_{\pi p}$ and $u_{\rho p}$ differ, thereby giving rise to different energy behaviors for $pp \rightarrow d\pi$ and $pp \rightarrow d\rho$. We have already taken this effect into account.
4. The convention is: $|\pi^+\rangle = (\phi_1 + i\phi_2)/\sqrt{2} = -|I = I_3 = 1\rangle$. Associating a minus sign with $|\pi^+\rangle$ permits the use of the Clebsch-Gordan coefficients.

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TABLE I

pd $\rightarrow$ dp and pp $\rightarrow$ d π Data

T_p (lab) BeV	pd $\rightarrow$ dp Ref.	pp $\rightarrow$ d π Ref.	σ μb	A	B
0.340	41	32,50	250	0.22	0.00
0.450	42	32,50	1330	0.22	0.10
0.580	43	50	3150	0.22	0.00
0.660	44	50	2790	0.23	0.40
0.715	45,46	50	2200	0.28	0.37
0.750	46,47	50	1690	0.30	0.38
1.00	45-49	50,51	452	0.27	0.59
1.30	49	51			
1.50	49	51			
1.65	46,47	51			
2.00	46	51			
3.66	45,46	52			

FIGURE CAPTIONS

- Fig. 1 The center-of-mass forward ($\cos \theta = \pm 1$) differential cross sections for $p + p \rightarrow d + \pi^+$ and $p + p \rightarrow d + \rho^+$ as a function of the total center-of-mass energy squared. The 0° cross sections were extrapolated from data measured at 5° in the lab. The data are from Refs. 3 and 4. This figure was adapted from an unpublished graph by the Chicago group (Ref. 3).
- Fig. 2 Quark duality diagrams for: (a) $p + p \rightarrow d + \pi^+(\rho^+)$, (b) $\pi^+ + p \rightarrow p + \pi^+$ and (c) $B + \bar{B} \rightarrow B + \bar{B}$.
- Fig. 3 The center-of-mass forward ($\cos \theta = \pm 1$) differential cross section for $p + p \rightarrow d + \pi^+$ as a function of the incident proton kinetic energy in the lab., and of the total c.m. energy. This graph was adapted from Anderson *et al.* (Ref. 9). The "other published data" are cited in Ref. 9. The dashed curve was calculated from Eq. (2.27) according to the cos δ -fixed prescription (see text). The OPE coupling strength, $\zeta = 2.9$, was chosen so that the theoretical and experimental curves coincide at low energy.
- Fig. 4 The one-pion-exchange diagrams for $p + p \rightarrow d + \pi^+$ involving: (a) π^+ exchange, and (b) π^0 exchange. The total amplitude also involves the corresponding graphs in which the initial protons are interchanged.
- Fig. 5 The center-of-mass forward ($\cos \theta = \pm 1$) differential cross sections for $p + p \rightarrow d + \pi^+$ and $p + p \rightarrow d + \rho^+$, as calculated from Eq. (2.27) according to the cos δ -fixed prescription (dashed curves)

Figure Captions (continued)

- and the u-fixed prescription (solid curves). In the regions where data exist, the curves are approximately straight. The slopes along these segments are about -2.6 for $pp \rightarrow d\pi$, and -1.5 for $pp \rightarrow d\rho$.
- Fig. 6 Formation of a dibaryon resonance in $p + p \rightarrow d + \pi^+$.
- Fig. 7 The one-nucleon-exchange graph for $p + p \rightarrow d + \pi^+$. The total amplitude also involves the corresponding graph in which the initial protons are interchanged.
- Fig. 8 One-pion-exchange graphs for $p + d \rightarrow d + p$. In (a) are displayed the π^+ and π^0 exchange graphs. Graph (b) should not be counted, because (a) and (b) both include (c).
- Fig. 9 The center-of-mass differential cross section for $p + d \rightarrow d + p$ as a function of the c.m. scattering angle, ϕ , the incident proton kinetic energy in the lab. (0.340 to 3.66 BeV), and the k_1^2 of the internal pion in units of the pion mass. The data were taken from the various sources listed in Table I. The proton lab energy in $p + d \rightarrow d + p$ corresponds to the same proton lab. energy in $p + p \rightarrow d + \pi^+$. The theoretical curves were calculated from the OPE relation, Eq. (5.1), according to the cos θ -fixed prescription (dashed curves) and the u-fixed prescription (solid curves). The data are fitted with $\zeta = 5.4$.

Figure Captions (cont.)

- Fig. 10 The first two terms in the Glauber series for $p + d \rightarrow p + d$:
(a) the impulse approximation, and (b) the double-scattering or shadow term.
- Fig. 11 The double-scattering diagram for $X + d \rightarrow d + X$, where X is any hadron. This is simply a generalization of Fig. 10(b) untwisted.
- Fig. 12 Feynman diagram for $\pi + d \rightarrow d + \pi$ in which two internal pions propagate between the nucleons comprising the deuteron.
- Fig. 13 The excited deuteron model of Kerman and Kisslinger (Ref. 24), in which $p + d \rightarrow d + p$ proceeds by neutron exchange and also by $N^*(1688)$ exchange.
- Fig. 14 Equivalent representations of $p + d \rightarrow d + p$ in the OPE approximation. First there is the relation (a) between $pd \rightarrow dp$ and $\pi d \rightarrow pp$. Omitting the crossed diagram, (b) $\pi d \rightarrow pp$ is in turn related to $\pi N \rightarrow N\pi$. By duality, $\pi N \rightarrow N\pi$ can be represented either as (c) N_α -trajectory exchange or as (d) Δ -resonance formation in the direct channel.
- Fig. 15 The renormalized p-d-n vertex analyzed in terms of an OPE-triangle diagram, a two-pion-exchange-triangle diagram, etc.
- Fig. 16 Singularity structure of the p-d-n vertex where the proton is on mass shell and the neutron is off mass shell with mass $(m^2 + n \alpha^2)^{1/2}$. In the n plane the leading singularities are:
(i) the nucleon pole at 0, (ii) the π -N anomalous threshold at 30.6, (iii) the $\pi\pi$ -N anomalous threshold at 98, and (iv) the π -N normal threshold at 134.

Fig. Caps. (continued)

- Fig. 17 A possible mechanism (see Ref. 72) for Z_0^* -resonance production in $K + N \rightarrow K + N$. At the $K^*(890)$ -N threshold, the OPE amplitude for $K + N \rightarrow K^* + N$, iterated through unitarity, can generate an effective attractive potential.

Fig.1

(a)

(b)

(c)

Fig.2

Fig.3

Fig.4

Fig.5

Fig.6

Fig.7

Fig.8

Fig.9

Fig.10

Fig.11

Fig.12

Fig.13

Fig.14

Fig.15

Fig.16

Fig.17