SPECTRUM OF NEUTRONS MODERATED BY HEAVY WATER
AT CRYOGENIC TEMPERATURES

(ASAE-35)

Calculations Performed for
Argonne National Laboratory
Lemont, Illinois

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INTRODUCTION

A calculation has been made, using the monte carlo method, to determine the transmission of neutrons through blocks of deuterium oxide at cryogenic temperatures. Computations were made for cylinders of D_2O ice at four degrees absolute, 60 and 75 centimeters long and 45 centimeters in diameter. The incident neutrons impinged uniformly over one flat surface with an incident intensity proportional to the cosine of the angle between the direction of motion of the particles and the normal to the surface. Their energy distribution was a Maxwellian at a temperature of 300° absolute.

In the initial trials of this calculation, it was found that more than 90% of the incident neutrons were reflected out of the surface of the block at which they had entered. Since in actual use the block will be placed against the thermal column of a reactor, it was clear that the treatment of the incident face as a free surface was not realistic. Such a highly reflecting system would have a marked effect on the flux in the source region, and the material of the thermal column, further, would reflect most of these neutrons back into the D_2O ice. In order to more adequately represent the physical situation, the program was modified so that neutrons attempting to leak out of this surface were optically reflected back into the system.

RESULTS

Figure 1 shows the neutron density in a deuterium oxide cylinder 60 centimeters long and 45 centimeters in diameter as a function of distance from the source. In this case the intensity of the neutron beam reaching the detector is $\exp(-21 \pm 1)$ times the source intensity. The agreement between this figure and the neutron density near the end of the block indicates a strong forward peak in the angular distribution of the neutron flux in this region. Although calculations were made of the angular distribution of emergent neutrons, the statistical precision is not sufficiently good to indicate more than qualitative agreement with this statement.

The rapid attenuation of neutrons indicated in Figure 1 is, of course, the result of leakage out the side of the cylinder. Table I shows the dependence of neutron leakage from the cylindrical side of the block in each of the first four 7.5 centimeter long sections.

TABLE I
NEUTRON LEAKAGE OUT THE CYLINDRICAL SIDE OF THE
45 cm DIAMETER, 60 cm LONG BLOCK IN 7.5 cm SECTIONS

<u>Distance from</u> <u>Source (cm)</u>	<u>% Leakage per Neutron</u> <u>Incident on the Block</u>	<u>% Leakage per</u> <u>Neutron Inci-</u> <u>dent on This</u> <u>Section</u>
0-7.5	97.3	97.3
7.5-15	2.24	82.6
15-22.5	0.404	85.5
22.5-30	0.069	> 90

FIGURE 1 -- NEUTRON DENSITY AS A FUNCTION OF PENETRATION INTO THE BLOCK

Figure 2 is a graph of the neutron density per unit energy as a function of energy, averaged over the entire block of D_2O .

Some indication of the way in which this spectrum varies with penetration into the block may be understood by inspection of Figure 3 in which the neutron density per unit energy for all but the first 7.5 centimeters of the block is plotted as a function of energy.

Beyond the first 15 centimeters, it is difficult to obtain detailed information about the neutron spectrum. Despite the extensive use of the improved sampling techniques described later in this report, the fact that only 0.5% of the neutrons penetrated beyond the first 15 centimeters caused the statistics to deteriorate rapidly at greater penetrations. Figure 4, however, shows the relative fraction of neutrons in each of four broad energy ranges as a function of the penetration. In Figure 5 the dependence of spectrum on penetration is shown in six energy bands. The spectrum of emergent neutrons is described in broad terms by the histogram of Figure 6.

Calculations on a cylinder 75 centimeters long and 45 centimeters in diameter were in agreement with respect to attenuation and spectrum with those on the 60 centimeter cylinder. Within the limits of precision of the calculation the emergent intensity was the same as that computed by extrapolating Figure 1, that is, an attenuation of $\exp(-31 \pm 2)$.

Because of the extremely long computing time required for these calculations as a result of the high attenuation and the program modifications necessary to represent the cross section dependence, the calculations described so far were considered an optimum treatment of the problem. The lengthiness of the computations resulted from the necessity of treating an unusually large number of particles in order to obtain sufficient statistics near the detector, and from the substitution of complex analytic functions for the table look-up processes used in GMC in the mean free path and collision calculations.

FIGURE 2 -- NEUTRON SPECTRUM AVERAGED OVER THE WHOLE BLOCK

FIGURE 3 -- NEUTRON SPECTRUM AVERAGED OVER ALL BUT THE FIRST 7.5 CENTIMETERS OF THE BLOCK

FIGURE 4
FRACTION OF NEUTRONS
IN EACH OF FOUR ENERGY
BANDS AS A FUNCTION OF
PENETRATION INTO THE BLOCK

FIGURE 5
FRACTION OF NEUTRON
IN EACH OF SIX ENERGY
BANDS AS A FUNCTION
OF PENETRATION INTO
THE BLOCK

K&E 10 X 10 TO THE INCH 359-5G
KEUFFEL & ESSER CO. MADE IN U.S.A.

THE PHYSICAL MODEL

The calculations were based upon the cross sections of Equations (1) through (4) which were derived by Dr. C. N. Kelber.

$$\begin{aligned} y > 300: \quad \sigma(y \rightarrow y') &= 0.02207 (y' + y) \sqrt{y'/y}; \quad y > y' \\ &= 0; \quad y < y' \end{aligned} \quad (1)$$

$$\begin{aligned} 300 \geq y \geq 150: \quad \sigma(y \rightarrow y') &= (0.02483 \frac{1}{\sqrt{yy'}} (y'y^2 - y'^3); \quad y \geq y' \geq y-1 \\ &= 0; \quad y' < y-1 \end{aligned} \quad (2)$$

$$\begin{aligned} 150 \geq y: \quad \sigma(y \rightarrow y') &= (0.02483) \frac{1}{\sqrt{yy'}} (y'y^2 - y'^3); \quad y \geq y' \geq 0 \\ &= 0; \quad y' > y \end{aligned} \quad (3)$$

$$\sigma(y \rightarrow y') = \sigma_{el} = \frac{2.2072 \left[1 - e^{-0.3y^2} \right]}{y^2} \quad (4)$$

The total cross section given in Equation (4) is plotted in Figure 7. Figure 8 shows the ratio between elastic and inelastic cross sections as a function of energy.

Scattering was assumed to be isotropic in the laboratory system.

FIGURE 7 -- TOTAL CROSS SECTION FOR THE CRYOGENIC D₂O

METHOD OF CALCULATION

A modified version of the GMC⁽¹⁾ code for the IBM 704 electronic computer was used for this calculation. The modifications that were made in order to represent the cross sections of Equations (1) through (4) are described in Figure 9. In outline they consist of the following. Instead of looking up the mean free path in a table dependent on energy group, the program computes it as an analytic function of the energy. When a collision does take place, the program, by selection of random numbers, determines that the collision is to be an elastic one seven-eighths of the time and an inelastic one, one-eighth of the time. Having determined which reaction type takes place on a particular collision, the program adjusts the weight of the particle, that is, the number of neutrons for which it stands, in accordance with the ratio between the elastic and inelastic cross sections. The particle's weight then becomes a function of its history as well as of position. When an inelastic collision takes place, the emergent energy is determined by selection from the probability distributions indicated by the cross sections.

As can be seen from a comparison of the flow charts of Figure 6 with the table look-up methods in the standard GMC code, these modifications result in large increases in computing time. In operation, the selection from probability distributions for the emergent energy constituted an important difficulty because of the extensive computing required. The low efficiency of the standard rejection technique resulting from the strong dependence of probability on emergent energy was eliminated in the high energy range by division of the spectrum into several subranges. The values of the constants indicated in Figure 9 resulting from this subdivision are given in Table II.

(1) Leshan, E. J., "A General Purpose Monte Carlo Program for the IBM 704, Part I: Method", ASAE-2, American-Standard Atomic Energy Division Report, September 1956.

Figure 9A -- Calculation of Mean Free Path, λ

Figure 9 B -- Modified Collision Routine

IMPROVED SAMPLING METHODS

Because of the high attenuation of neutrons in the block, it was necessary to modify the sampling techniques in order to obtain reasonable precision. The most important modification was the use of expectation values. This consists in the analytic calculation, at each collision point, of the average contribution per neutron to quantities of interest that would be made by a universe of neutrons each having the coordinates of the particle emerging from the collision. In effect the result of the use of expectation values is that the quantities computed by the program are the integral over all positions, energies, and directions of motion of the product of the collision density and the average contribution per neutron. The statistical precision of quantities evaluated in this way is determined by the number of particles which, during the course of the calculation, had coordinates for which there was a possibility of contributing to the quantity. When expectation values are not used the precision is determined instead, by the number of particles that actually had relevant events.

A second modified sampling technique was the use of importance weighting, that is, splitting and Russian roulette. By assigning an importance to each region, proportional to an estimate of the probability that a neutron in that region will contribute to the quantity of principal interest, the intensity with which each region is studied can be made to depend on its importance rather than on the actual neutron density in that region.

Finally, importance sampling was used in the determination of whether a collision was an inelastic or an elastic reaction. The importance of the use of this technique is illustrated in Figure 8. The high ratio of elastic to inelastic cross sections would have prevented the accumulation of satisfactory statistics on low energy neutrons if the straightforward method had been used. By forcing one-eighth of the collisions to be inelastic and modifying the weights accordingly, the number of low energy particles was increased to a satisfactory value.

TABLE II

CONSTANTS FOR USE IN COLLISION ROUTINE

(See Figure 9B)

m	a_m	m	a_m	m	a_m	m	a_m
1	7.198×10^{-3}	6	0.11182	11	0.3392	16	0.6620
2	2.2272×10^{-2}	7	0.15661	12	0.3951	17	0.7395
3	3.958×10^{-2}	8	0.19603	13	0.4553	18	0.8295
4	5.793×10^{-2}	9	0.2396	14	0.5198	19	0.8782
5	8.985×10^{-2}	10	0.2872	15	0.5886	20	1.0000

THE CALCULATION

The geometry used is described in Figure 10. Zones 1 through 27 are divisions of the cylinder inserted in order to provide a grid for the listing of neutron densities and for the statement of region importances. The large circular annulus consisting of zones 29 through 34 has a radius of 1000 centimeters and is used to obtain the angular distribution of neutrons leaving the cylinder at the far flat surface. Zone 35 is an absorbing zone used to prevent neutrons from reaching the detector zones 29 through 34 if they leak out of the cylindrical surface. The geometry is symmetrical about the incident surface at $Z = 0$ in order to effect reflection of neutrons that would otherwise leak out at this boundary.

Table III is a listing of the input in the form in which it is used by the GMC program.

An example of a listing of the results of the calculation are shown in Table IV in the forms in which they are produced by the program.

FIGURE 10 -- REPRESENTATION OF THE SYSTEM USED IN THE CALCULATION

TABLE III
INPUT FOR PROGRAM

4	2	1	32	2	6	32	21	1	481	481	511	507828	G I
535					511				52225	5215	5175		G INPT 2
503921					511		5095863	5082642	5162992	5052002	5040967		G INPT 3
502407	5026706				5022889		5019032	5115178	5011393	409914	498468		G INPT 4
497064	405713				494429		402229	492135	401608	40118	4877		G INPT 5
4842	48358				48299		43242	48191	40143	481	4761		G INPT 6
4779					522		521	516667	515	512333	512667		G INPT 7
512	511667				511333		511167	511	502333	506667	506		G INPT 8
505333	504667				514		503333	502667	502	501667	501333		G INPT 9
501	40667				496		49522	49467	494	49322	49267		G INPT 10
492	49133				511		5009987	51099	5029663	50992	5098438		G INPT 11
50973	5095713				50936		5090888	51575	5033363	50784	5172538		G INPT 12
50657	5057812				50488		5028588	50271	5014263				G INPT 13
495	501				5015		502	525	503	5035	504		G INPT 14
5045	505				5055		506	5065	507	5075	508		G INPT 15
5085	509				5095		511						G INPT 16
26 32 28	2 25 21 22 25				1								SPEC-FXD
511	711				711		491	532	541				SPEC-FLD
1													ISUBI M1
18													ISUBI M2
511													NSUBI M1
511													NSUBI M2
511	511												RHO M1-8
1 1 2	1 2				14 35		12 10	1			8 28		ZONE 1
1 2 3	2 3				14 35		12 11	1 1 1 1			9 28		ZONE 2
1 3 4	3 4				14 35		12 12	1 2 2 1			17 28		ZONE 3
1 4 5	4 5				14 35		12 13	1 3 3 1			18 29		ZONE 4
1 5 6	5 6				14 35		12 14	1 4 4 1			26 28		ZONE 5
1 6 7	6 7				14 35		12 15	1 5 5 1			27 28		ZONE 6
1 7 8	7 8				14 35		12 16	1 6 6 1			28 29		ZONE 7
1 8 28	8 28				14 35		13 9	1 7 7 1			28 30		ZONE 8
1 8 28	8 28				13 8		12 17	1 7 7 1			28 31		ZONE 9
1 1 11	1 11				12 1		10 19	1			28 32		ZONE 10
1 2 12	2 12				12 2		10 20	1 1 10 1			28 33		ZONE 11
1 3 13	3 13				12 3		10 21	1 2 11 1			28 34		ZONE 12
1 4 14	4 14				12 4		10 22	1 3 12 1			1 35		ZONE 13
1 5 15	5 15				12 5		10 23	1 4 13 1			2 35		ZONE 14
1 6 16	6 16				12 6		10 24	1 5 14 1			3 35		ZONE 15
1 7 17	7 17				12 7		10 25	1 6 15 1			4 35		ZONE 16
1 8 28	8 28				12 9		11 18	1 7 16 1			5 35		ZONE 17
1 8 28	8 28				11 17		10 26	1 7 16 1			6 35		ZONE 18
1 1 20	1 20				10 10						7 35		ZONE 19
1 2 21	2 21				10 11		1 19	1			8 35		ZONE 20
1 3 22	3 22				10 12		2 20	1			19 20		ZONE 21
1 4 23	4 23				10 13		3 21	1			20 21		ZONE 22
1 5 24	5 24				10 14		4 22	1			21 22		ZONE 23
1 6 25	6 25				10 15		5 23	1			22 23		ZONE 24
1 7 26	7 26				10 16		6 24	1			23 24		ZONE 25
1 8 28	8 28				10 18		7 25	1 8 27 1			24 25		ZONE 26
1 8 28	8 28				9 26		7 25	1			25 26		ZONE 27
2 15 35	1 8 28				1 16 20						19 10		ZONE 28
2 16 28	1 16 28				1 17 36		18 30				10 1		Z/NE 29
2 16 28	1 16 28				1 17 36		18 29	1 10 31			20 11		Z/NE 30

TABLE III (Continued)

INPUT FOR PROGRAM

2 16 28	1 16 28	1 17 36	19 30	1 20 32	11 2	ZONE	31
2 16 28	1 16 28	1 17 36	20 31	1 21 33	20 19	ZONE	32
2 16 28	1 16 28	1 17 36	21 32	1 22 34	11 10	ZONE	33
2 16 28	1 16 28	1 17 36	22 33	1	2 1	ZONE	34
15 28	15 28	14 1	1 8 28		1 2	ZONE	35
17 29	1 17 29	1					
5110000	511	511		511	525625	L-I	1
5110000	513	511		511	53225	L-I	2
5110000	519	511		511	5350625	L-I	3
5110000	5227	511		511	539	L-I	4
5110000	5281	511		511	5410463	L-I	5
5110000	53243	511		511	542025	L-I	6
5110000	53729	511		511	5427563	L-I	7
5110000	542187	511		511	5436	L-I	8
5110000	54328	511	511		5214063	L-I	9
5110000	513	511	511		525625	L-I	10
5110000	519	511	511		5312656	L-I	11
5110000	5227	511	511		53225	L-I	12
5110000	5281	511	511		5335156	L-I	13
5110000	53243	511	511		5350625	L-I	14
5110000	53729	511	511		53529	L-I	15
5110000	542187	511	511	511	571	L-I	16
5110000	546561	511	511	511	571002	L-I	17
5110000	549841	511	511	-49718		L-I	18
5110000	519	511	511	-503333		L-I	19
5110000	5227	511	511	-509098		L-I	20
5110000	5281	511	511	-512999		L-I	21
5110000	53243	511	511	-5213923		L-I	22
5110000	53729					L-I	23
5110000	542187					L-I	24
5110000	546561					L-I	25
5110000	5519683					L-I	26
5110000	553					L-I	27
5110000	511					L-I	28
5110000	511					L-I	29
5110000	511					L-I	30
5110000	511					L-I	31
5110000	511					L-I	32
5110000	511					L-I	33
5110000	511					L-I	34
5110000						L-I	35
511	511	511	511	511	511	I-J	1-8
511	511	511	511	511	511	I-J	9-16
511	511	511	511	511	511	I-J	17-24
511	511	511	511	511	511	I-J	25-32

000099

TABLE IV
OUTPUT FOR PROGRAM

REJECT 46	I 71	P LEAK 53	N LEAK 52	P IV 53	N IV 53	P CENS 50
.524691645	.100015938	.065999909	.419387246	.209000001	.082555514	.000000000
N CENS 50	N SZ1 47	N SZ2 50	EX LK 53	EX SZ 44	EX FIS 50	EX INL 50
.000000000	.49603928	.000000000	.078844392	.130655284	-.269367712	.000000000
ALPHA1 31	ALPHA2 1	ALPHA3 31	P SZ T 51	NET CG 50	NET BG 51	NET VG 24
-.183962008	-.199968125	-.199968119	.699999999	.000000000	-.679625826	.338911445

TABLE IV (cont.)
OUTPUT FOR PROGRAM

	DIST 1 54	DIST 2 52	DIST 3 53	DIST 4 52	DIST 5 49	DIST 6 49	DIST 7 46
01	.00046789	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
02	.000621721	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
03	.001465575	.000126203	.00000000	.00000000	.00000000	.00000000	.00000000
04	.000982886	.000819362	.000015073	.000003802	.000068666	.000029403	.000002120
05	.001553136	.005262353	.000025370	.000005821	.000185346	.000115263	.000185392
06	.000463576	.000749945	.00000816	.000002194	.000039815	.000021628	.000059188
07	.003501129	.05821401	.000020888	.000009184	.000419532	.000267093	.000089755
08	.000873308	.011182409	.000070394	.000000350	.000085102	.000079924	.000377229
09	.003468220	.000876289	.000007353	.000002260	.000415098	.000204990	.001597721
10	.000773113	.021600051	.000000420	.000000136	.000122917	.000124383	.000177923
11	.000498310	.060799147	.000227505	.000027490	.0004642106	.070776675	.042304627
12	.000041140	.060200702	.000023986	.000035929	.085137708	.018760304	.0205843156
13	.000965690	.062750219	.001708471	.000075693	.000516111	.018818325	.000193294
14	.000057852	.000002134	.000000015	.000000008	.0000000920	.000000196	.000008764
15	.087176766	.410677236	.066610605	.314393890	.490392400	.156948400	.044192341
16	.000228499	.000481003	.000000081	.0000001480	.0000812145	.000188661	.000252482
17	.000669608	.056847005	.000240360	.000142085	.001469374	.043031403	.000506424
18	.005202544	.00117135	.000222431	.000086930	.002252811	.001113835	.0011338241
19	.000030673	.014305803	.00176143	.000329663	.000036538	.005577018	.000203067
20	.000053629	.000661340	.000002336	.0000021784	.000018230	.000004003	.000164663
21	.000627095	.003140780	.000001225	.000966338	.004381901	.002204941	.018953290
22	.000000000	.014235585	.000911945	.000009839	.000046077	.000046910	.000008693
23	.000000465	.000329144	.000331085	.000331219	.026856018	.001518526	.000081849
24	.000000000	.000000000	.000000012	.000000003	.000000002	.000000476	.000000945
25	.000000000	.000000000	.000000004	.000000048	.0000003683	.000015011	.000000338
26	.000000000	.000000000	.000000019	.000000011	.000000057	.000000846	.000001628
27	.000000000	.000000000	.000000001	.000000002	.000000000	.000000032	.000000362
28	.000000858	.000034734	.000000000	.000000000	.000000003	.000000003	.000000000
29	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000
30	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000
31	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000
32	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000
01	DIST 8 43	DIST 9 44	DIST 10 53	DIST 11 53	DIST 12 53	DIST 13 52	DIST 14 50
02	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000
03	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000
04	.000046972	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000
05	.002764038	.003310609	.012107363	.000352777	.000000491	.000000064	.000000026
06	.000765397	.003310609	.017316877	.004926296	.000113361	.000026313	.000050449
07	.555980087	.0033601241	.027943210	.002922860	.000096407	.000011786	.000013345
08	.004584361	.000125865	.051646894	.005730852	.000552292	.0008687543	.002353843
09	.003050720	.015258263	.074196722	.033395592	.000433996	.000060672	.000182694
10	.000001094	.003031316	.024131018	.003222288	.000151382	.000169349	.000217563
11	.253780058	.000077733	.049033698	.003086425	.000018157	.000039921	.003923203
12	.191861352	.428489110	.013469309	.041381589	.015117910	.000059488	.001718338
13	.000006586	.001250293	.004098242	.004161946	.030152163	.004668915	.006622265
14	.000000000	.000000000	.004098242	.003387173	.000289184	.000089277	.003746626
15	.222514600	.081492114	.000823088	.000040715	.000001802	.000001171	.000141356
16	.000717227	.00021033	.016769200	.002432405	.002400804	.020497205	.003593948
17	.000042392	.000006409	.012632817	.000459707	.000017311	.000464628	.000341340
18	.001731110	.001155431	.084190934	.003135658	.003881288	.000136663	.002054852
19	.006204575	.000853492	.003565053	.000297252	.000196139	.000312523	.000213063
20	.000107620	.000003188	.000048392	.00052936	.000121006	.000018572	.000227713
21	.004180890	.008547615	.003120241	.000084867	.000025397	.001527371	.006311276
22	.000029797	.000014848	.000141643	.000041726	.000085640	.000018542	.000544781

TABLE IV (contd)

OUTPUT FOR PROGRAM											
	DIST29 42	DIST30 44	DIST31 41	DIST32 40	DIST33 32	DIST34 50	DIST35 50				
15	.000200968	.004806071	.054881613	.031362654	.041614987	.000289172	.002536109				
16	.006183256	.001205837	.000757279	.000095435	.000732182	.000002528	.000004412				
17	.000233736	.000211096	.002397296	.001665767	.000000381	.000000167	.000000167				
18	.000976170	.030152943	.017806864	.002057608	.004162907	.000703880	.000069209				
19	.000018829	.000009213	.000041172	.000000977	.000000122	.000000000	.000015096				
20	.000009692	.003215614	.001141728	.000023047	.000044197	.000003066	.000000583				
21	.000092288	.012445500	.003677952	.020454947	.000382900	.000276417	.000169795				
22	.000007264	.000189750	.000283057	.00012470	.00010838	.000001143	.000000759				
23	.000346734	.202085369	.000256889	.000256889	.000098395	.000075395	.000004788				
24	.000000000	.000000024	.000000032	.000000000	.000000000	.000000000	.000000000				
25	.000020527	.000074665	.000009578	.000125230	.000217559	.000000000	.000000885				
26	.000000049	.000000013	.000000010	.000000073	.000000129	.000000000	.000000004				
27	.000000000	.000000137	.000000321	.000000363	.000000006	.000000000	.000000000				
28	.000000064	.00013811	.000000061	.000000000	.000000000	.000000000	.000000000				
29	.000000007	.000008112	.000014226	.000000000	.000000000	.000000000	.000000000				
30	.108200759	.258964150	.02572511	.000000000	.000000779	.000000000	.000006322				
31	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000				
32	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000				
01	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000				
02	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000				
03	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000				
04	.000187891	.000000008	.000000000	.000000000	.000000000	.000000000	.000000000				
05	.004898848	.000003410	.000000000	.000000000	.000000000	.000000000	.000000000				
06	.000062809	.000001211	.000000000	.000000000	.000000000	.000000000	.000000000				
07	.262281921	.02716023	.390174991	.000000000	.000000000	.000000000	.000000000				
08	.000214879	.000000306	.000001170	.000000000	.000000000	.000000000	.000000000				
09	.003259448	.000003823	.054637491	.000000000	.000000000	.000000000	.000000000				
10	.019475799	.000000002	.001486755	.000000000	.000000000	.000000000	.000000000				
11	.011522341	.000059539	.001173985	.000000000	.000000000	.000000000	.000000000				
12	.088765892	.000000000	.189000000	.000000000	.000000000	.000000000	.000000000				
13	.000245831	.000015601	.000132369	.004065777	.000000000	.000000000	.000000000				
14	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000				
15	.009686979	.002415706	.002410906	.161387140	.000000000	.000000000	.000000000				
16	.000375701	.000000047	.0000000159	.000000000	.000000000	.000000000	.000000000				
17	.000000000	.000000097	.000000023	.000000000	.000000000	.000000000	.000000000				
18	.004854326	.000019856	.000099095	.000000000	.000000000	.000000000	.000000000				
19	.000000218	.000000026	.014934578	.000000104	.000000000	.000000000	.000000000				
20	.000039629	.000000185	.000000287	.000000000	.000000000	.000000000	.000000000				
21	.000545173	.000020580	.143109607	.000000000	.000000000	.000000000	.000000000				
22	.000019236	.000000429	.000135376	.000000000	.000000000	.000000000	.000000000				
23	.000043272	.000000433	.000014118	.000000157	.000000000	.000000000	.000000000				
24	.000000000	.000000001	.000000236	.000000000	.165137047	.000000000	.000000000				
25	.000085524	.000000021	.000005462	.000000051	.000000000	.000000000	.000000000				
26	.000000392	.000000000	.000000000	.000000063	.000000000	.000000000	.000000000				
27	.000000015	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000				
28	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000				
29	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000				
30	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000				
31	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000				
32	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000				
01	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000				
02	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000				
03	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000				
04	.000002133	.000000000	.003165504	.000000000	.000000000	.000000000	.000000000				
05	.000059226	.000000000	.000006216	.000000000	.000000000	.000000000	.000000000				
06	.000002079	.000000000	.000000000	.000000000	.000000000	.000000000	.000000000				

TABLE IV (contd)

OUTPUT FOR PROGRAM

07	.034117531	.000000000	.000000004	-.000000000	.000000000
08	.000002776	.000000000	.000000000	-.000000000	.000000000
09	.000102919	.000000000	.000000000	-.000000000	.000000000
10	.000221830	.000000000	.053849123	-.000229000	.000000000
11	.000198923	.000000000	.024513322	-.076548328	.000000000
12	.092759010	.000000000	.058663558	-.126517784	.000000000
13	.000021025	.000000000	.003222093	-.000898766	.076442261
14	.000000000	.000000000	.000042509	-.000025055	.001983430
15	.002861239	.000000000	.000086922	-.000006264	.000357826
16	.000005199	.000000000	.000000329	-.000000000	.000060966
17	.000000188	.000000000	.000000001	-.000000000	.000000051
18	.000078008	.000000000	.000000000	-.000000000	.000000145
19	.000016919	.000000000	.029709791	-.001851696	.000000000
20	.000000658	.000000000	.005191042	-.019651583	.000000000
21	.000191222	.000000000	.002685938	-.026414374	.004357860
22	.000000856	.000000000	.001244819	-.015192276	.002848756
23	.000005393	.000000000	.000070228	-.000363606	.000313984
24	.000000002	.000000000	.000077106	-.000036186	.000209274
25	.000000997	.000000000	.000000052	-.000000000	.000072824
26	.000000004	.000000000	.000000000	-.000000000	.000000291
27	.000000000	.000000000	.000000000	-.000000000	.000000000
28	.000000000	.000000000	.000000011	.000000004	.006972608
29	.000000000	.000000000	.000000000	.000000000	.027617745
30	.000007145	.000000000	.000000000	.000000000	.002754451
31	.000000000	.000000000	.000000000	.000000000	.010384081
32	.000000000	.000000000	.000000000	.000000000	.000887082
33			.000000000	.000000000	.014263704
34			.000000000	.000000000	.003707964
35			.000000000	.000000000	.008496763