A STUDY OF THE EFFECT OF COHERENT PRECIPITATES ON THE
SURFACE TOPOGRAPHY PRODUCED BY DEFORMATION OF COPPER-
2 WT.% COBALT ALLOY SINGLE CRYSTALS

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Ph.D. Thesis Submitted to Iowa State University

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A study of the effect of coherent precipitates on the
surface topography produced by deformation of copper-
2 wt.% cobalt alloy single crystals

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Uday Kumar Sinha

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A study of the effect of coherent precipitates on the
surface topography produced by deformation of copper-
2 wt.% cobalt alloy single crystals*

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Copper-2 wt.% cobalt alloy single crystals, solution treated and aged, contain precipitates of 90% Co and 10% Cu in a copper matrix. The precipitates are spherical in shape and coherent with the matrix, and their size depends upon the temperature and time of aging. The hardening of this alloy has been largely attributed to the coherency strain.

In this research, surface topology has been used as the prime method of establishing further data and behaviour trends which will help complete the overall picture of the precipitation hardening mechanism and its effects in this alloy system. Single crystals of the alloy were grown in the easy glide orientation. The crystals were solution treated and aged for various lengths of time in order to get different sizes of precipitate particles. A magnetic method was employed to determine the average particle size and the

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volume fraction of the precipitate. The crystals were successively compressed to approximately 0.3 glide strain and a thorough study of the slip-band step-heights, their separation, and the active slip systems was made after each compression. Interference microscopy was used extensively to measure directly and quantitatively the slip-band step-heights.

I. INTRODUCTION

Copper rich alloys with cobalt have attracted attention largely due to their precipitation hardening behaviour. The alloy of composition Cu-2 wt.% Co when homogenized and aged at 600°C produces precipitates of nearly pure cobalt (90% cobalt and 10% copper). This can be seen from the equilibrium phase diagram shown in Figure 1. The precipitate particles have been observed to be spherical in shape and coherent with the matrix (1). The coherency of the precipitate is retained even up to particle sizes of the order of 500 Å (1,2). It has been shown (3-9) that the particle size of the precipitate is easily and accurately determined by magnetic measurements. Magnetically measured particle sizes correspond quite closely with the particle sizes obtained by electron transmission microscopy (10). The simple nature of the phase diagram of this alloy system, coupled with the ease of particle size measurements by magnetic techniques make this alloy ideally suited for studying the mechanism of precipitate controlled deformation.

The flow stress of this alloy has been found to increase with the particle size of the precipitate. Gleiter (11-13) and Gerold and Haberkorn (14) have proposed theoretical expressions for the increase in flow stress with particle size. Their expressions for the increment in flow stress due to precipitate particles are given below:

$$\Delta\tau = \frac{8.26A \epsilon^{3/2} \mu f^{5/6}}{\alpha^{1/2}} \left(\frac{R}{b}\right)^{1/2} \quad \text{-----Gleiter}$$

$$\Delta\tau = \frac{2.12B \epsilon^{3/2} \mu f^{1/2}}{\alpha^{1/2}} \left(\frac{R}{b}\right)^{1/2} \quad \text{-----Gerold and Haberkorn}$$

In these relations,

f = volume fraction of the precipitate

ϵ = unrelaxed misfit parameter

μ = shear modulus of copper

R = radius of particles

b = Burger's vector for copper

$A = 1.0$ for edge and 0.13 for screw dislocations
respectively

$B = 1.0$ for edge and 0.33 for screw dislocations
respectively

α comes from the expression for line tension of
dislocations

Both theories assume that dislocation motion is impeded by the coherency strain field associated with the particle. The two expressions differ only in their dependence on the volume fraction, which arises due to different assumptions about the flexibility of dislocations. Gleiter (11-13) assumed a completely flexible dislocation whereas Gerold and Haberkorn (14) considered non-flexible dislocations in their derivation. According to their equations, a plot of $(\Delta\tau/\mu)^2$ versus (R/b) should be a straight line passing through the

origin. In practice, however, the increase in flow stress as experimentally determined by Livingston (15), Phillips (1), Koepke and Scott (16), and Olson (17) agrees with the predicted theoretical curve only for small sizes of precipitate particles ($R < 64 \text{ \AA}$). For larger particle sizes the increase in flow stress seems to be controlled by the dislocation bow-out mechanism originally proposed by Orowan (18). The experimental curve at large particle sizes ($R > 64 \text{ \AA}$) follows the theoretical curve predicted by the modified Orowan expression given by Ashby (19,20).

Though there has been a large amount of research carried out on the study of the flow stress, the study of surface topography has been lacking, particularly in the case of this alloy. Surface topography is a means of studying the deformation behavior by measuring the amount of slip, slip-band spacings, and activation of slip systems. Slip is revealed on the surface as a slip trace resulting from the intersection of the slip plane with the surface. Thus a thorough study of surface topography of deformed crystals can provide important information which will aid in the understanding of deformation mechanisms.

When a single crystal is deformed in tension or compression, deformation takes place by the movement of dislocations. In the early stage (called Stage I), dislocations glide on parallel planes causing offsets on crystals which produce a topography similar to an edge view of a sheared deck of play-

ing cards. In general, the displacement occurs parallel to planes having the highest atomic density and in the most closely packed crystallographic directions. Figure 2 shows a sketch of the movement of a parallel set of crystallographic slip planes. In face centered cubic metals, there are four $\{111\}$ close packed planes and three $\langle 110 \rangle$ close packed directions in each plane. Thus there are 12 slip systems in F.C.C. metals. Slip initiates in the slip system which first attains the critical resolved shear stress. The geometric relations for a crystal slip plane, slip direction and stress axis are shown in Figure 3.

The slip system which will operate depends upon the orientation of the stress axis of the crystal in the standard triangle $[001-011-\bar{1}11]$ of a stereographic projection. A standard (001) stereographic projection with the position of the stress axis in the standard triangle is shown in Figure 4. If the stress axis is inside this triangle, slip takes place on the primary system $(111) [\bar{1}01]$, which has the highest critical resolved shear stress. If there was no constraint on the crystal, it would deform by single slip without changing its orientation. But in practice, due to the grips in the tensile test or the platens in the compression tests, the ends of the specimens are kept in line. This causes a rotation of the slip plane towards a direction parallel to the stress axis in tensile tests and towards a direction parallel with the platens in compression tests. Thus the orientation of the crystal

changes with deformation, resulting in a change of the resolved shear stresses on the different slip systems. When the orientation of the stress axis reaches the symmetry line $[001-\bar{1}11]$ of the standard triangle in tensile tests, the conjugate system $(\bar{1}\bar{1}1) [011]$ becomes equally favourable for slip and two slip systems begin to operate. In the case of compression tests, the stress axis moves towards the slip plane normal. When it crosses the symmetry line $[001-011]$, the critical system $(\bar{1}11) [101]$ attains the highest resolved shear stress for slip. As a result, slip which is simple at the beginning of deformation becomes more complex as the deformation progresses.

Slip lines are straight if slip takes place on a single set of slip planes. In the case of F.C.C. metals, slip occurs largely on $\{111\}$ planes and its traces are straight. The surface steps visible in the optical microscope may consist of a number of finer steps only resolvable in the electron microscope. The individual steps or traces of atomic planes on which slip has taken place are called slip lines. If slip takes place on successive atomic planes, it is called homogeneous slip whereas slip on single atomic planes separated by large distances is termed inhomogeneous slip.

In this dissertation surface topology has been used as the prime method for the study of the mechanism of deformation. The extent of slip is easily detected by interference microscopy. Details of interference microscopy techniques are given

in the experimental methods section of this thesis. Extensive use of interference microscopy has been made to measure directly the slip step heights. A complete study of slip step heights, slip-band spacings and active slip systems has been made for specimens with different particle sizes. The particle size of the precipitates has been determined by magnetic measurements. A dislocation model to account for the observed surface topology is proposed.

Fig. 1. Copper-cobalt phase diagram taken from the 1948 edition of the ASM Metals Handbook

Fig. 2. Movement of lamellae of material on a crystallographic slip plane

Fig. 3. The geometry of slip showing the stress axis, slip direction, and the slip plane normal

Fig. 4. Standard (001) stereographic projection showing the position of the stress axis in the standard triangle.

II. LITERATURE REVIEW

The observations recorded in the literature will be discussed under the following topics:

- A. Relation of Slip Lines to Work-Hardening
- B. Homogeneous and Inhomogeneous Slip
- C. Effects of Temperature on Slip Step-Heights
- D. Effects of Stress and Strain on Slip Lines and Their Spacings
- E. Effects of Orientation of the Stress Axis of the Crystal on Slip Step Heights.
- F. Effects of the Size of the Crystal on Slip Step-Heights
- G. Effects of Alloy Additions on Slip Behaviour
- H. Theories of Slip Band Formation

- A. Relation of Slip Lines to the Work-Hardening

The work-hardening curve for face centered cubic metals is well established (21-29). It has been observed that there are three distinct stages of work-hardening. Stage I is characterized by easy glide, since during this stage of deformation, a large proportion of dislocations emitted by sources can slip out of the crystal. As a result, during this stage, long slip lines, uniformly spaced and usually of small step heights are observed.

The end of Stage I coincides with the sudden increase in the generation of secondary sources of dislocations. Several

theories (21,22,25,26,28,29) based on the interaction of dislocations have been proposed to account for the rapid linear hardening during Stage II. Seeger (21,22) postulated that Lomer-Cottrell locks, created by the interaction of dislocations on two slip planes, act as barriers to the movement of dislocations resulting in an increase in the work-hardening rate. However, according to Kuhlmann-Wilsdorf (25,29) at the end of Stage I dislocations are uniformly distributed either in pile-ups or tangles on one or several glide systems. The dislocation density increases, while the average free dislocation length decreases during this stage. The increased stress necessary to bow-out dislocations with decreasing segment length into loops is thought to be the major part of the work-hardening in Stage II. The effective length of slip decreases as a result of the onset of duplex or multiple slip. Slip lines have larger step heights, shorter length, and are less regularly spaced than those in Stage I.

During Stage III for face centered cubic metals there is a decrease in the work-hardening rate. In this part of the curve the stress is a parabolic function of the strain. The barriers to dislocation motion produced by the interaction of primary and secondary slip dislocations are overcome by the onset of cross-slip of screw dislocations from the primary plane. Due to this, Stage III coincides with the beginning of fragmentation of slip bands by the frequent occurrence of connecting slip.

B. Homogeneous and Inhomogeneous Slip

Heidenrich and Shockley (30) were the first to observe, by electron microscopy, that the slip bands seen in the optical microscope actually consist of clusters of lamellae of slip. If slip takes place within these clusters, as one unit step on consecutive slip planes, it is called homogeneous slip. Whereas if slip occurs on single atomic planes, separated by large distances, it is known as inhomogeneous slip. A schematic diagram of homogeneous and inhomogeneous slip is shown in Figure 5. The nature of homogeneous or inhomogeneous slip is not completely established for different metals and alloys.

Suzuki and Fujita (31) observed that, in face centered cubic metals including α -brass, homogeneous slip takes place in most cases. But observations made by H. Wilsdorf and J. T. Fourie (32) were different for α -brass. They found that slip occurred on single atomic planes in big steps corresponding to inhomogeneous slip. Inhomogeneous slip was also observed by Charsley and Desvaux (33) in slip line studies on the surfaces of deformed single crystals of α -phase Cu-Al alloys. On similar alloys discrete slip lines were observed by Mitchell et al. (34) using optical microscopy and transmission microscopy of replicas. They also noted a background of plastic deformation on a much finer scale that could not be resolved. Systematic and detailed studies are required to establish fully the nature of slip in different metals and alloys.

Fig. 5. Modes of slip

- (a) Homogeneous slip on a number of successive parallel planes with a total slip distance t
- (b) Inhomogeneous slip of a distance, t , on single atomic planes

C. Effects of Temperature on Slip Step-Heights

Brown (35,36) observed that at higher temperatures the slip bands are spaced further apart and the step heights are greater than at lower temperatures. Similar observations were made by Holden as cited by Brown (35). Holden found on aluminum an average step height of about 2000 Å at room temperature compared to an average step height of 1750 Å at liquid air temperature. Yagamuchi and Togino (37) also observed, by averaging the macroscopic strain over all observed slip bands, that the average slip step-height was about 25% less at liquid air temperature than at room temperature. According to Brown (35) the obstacles to slip are reduced at higher temperatures due to thermal activation resulting in larger step heights.

D. Effects of Stress and Strain on Slip Lines and Their Spacings

Orowan (38) has suggested that the critical resolved shear stress for slip, τ , is related to the spacing of slip planes by the following expression:

$$\tau = \frac{Gb}{2\pi h}$$

Where, h = spacing between the active slip planes

b = Burger's vector

G = shear modulus of the specimen

From the above expression it can be seen that the spacing between the active slip planes decreases with increasing

stress. In fact, this is thought to be a common observation. However, there is a limiting distance to which the slip lines can come together. Kuhlmann-Wilsdorf, Merwe, and Wilsdorf (39) have pointed out that the nearest distance at which two rows of dislocations can pass each other is $A_e/4\pi$ where, $A_e = Gb/2\pi(1-\nu)$, b is the Burger's vector and ν is Poisson's ratio. Thus the spacing between slip bands is restricted by the repulsive interaction of dislocations on parallel slip planes.

Brown (35) observed that the slip bands on all metals increase in number but the lamellae which make up the bands do not apparently change with strain or strain rate. J. T. Fourie (40) in a more recent work found that the average slip step-heights per slip plane increased with the shear strain for α -brass single crystals. He also observed that the slip step-height increased suddenly when a critical stress level was reached.

E. Effects of the Orientation of the Stress Axis of Crystal on Slip Step-Heights

J. T. Fourie (40) studied the developments of slip lines in two α -brass single crystals of different orientations. One crystal was oriented for easy glide whereas the other crystal had an orientation close to the symmetry line $[001-111]$. The slip step-heights on the crystal with an easy glide orientation were found to be greater than those on the crystal which

had an orientation close to the symmetry line. But the rate of increase of step heights with shear strain was slower in the former crystal than the latter. In case of crystals with easy glide orientations, new slip lines were formed with increasing strain resulting in less growth of the initial slip lines. On the other hand, for crystal orientations suited for duplex or multiple slip, many active slip planes were initiated in the early stages of deformation. With increasing strain, deformation took place to a larger extent by the growth of already existent slip lines. Thus crystals with orientations close to the symmetry line had a larger rate of increase of step heights than those crystals with easy glide orientation.

F. Effects of the Size of the Crystals on Slip Step Height

Leibfried (41) has derived the following expression for maximum slip step height.

$$S \approx \frac{\tau L}{G}$$

where S = slip step height in the neighborhood of sources of dislocations

τ = applied shear stress

L = length of the active slip plane

G = shear modulus

A simple calculation for copper, taking $G = 4100 \text{ kg/mm}^2$ and $\tau = 4 \text{ kg/mm}^2$, shows the maximum slip step height to be $1000 \text{ \AA}$ for $L = 0.1 \text{ mm}$ and $10,000 \text{ \AA}$ for $L = 1 \text{ mm}$. The upper limit for

slip appears to be in good agreement with observations made by Kuhlmann-Wilsdorf and Wilsdorf (42). In their experiments with polycrystals having grain diameters from 0.03 to 0.3 mm, they very rarely observed slip step heights on individual slip lines greater than $1000 \text{ \AA}$ as was expected by using the above mentioned equation.

G. Effects of Alloy Additions on Slip Behaviour

It is well known that alloy additions invariably increase the resolved shear stress and alter the form of the work-hardening curve by removing or exaggerating the stages. Stage I is always increased in length with alloy additions for alloys with a wide solubility range. In such alloys solute atoms can be accommodated easily by the matrix. It is expected that solute atoms in such systems will not be strong enough obstacles to limit the slip distance once the critical shear stress has been reached. In case of alloys with narrow solubility range, the slope of Stage I remains the same or is larger than that of the base metal. In such a case the foreign atoms are usually associated with stress fields. According to Seeger (21), the stress fields interfere with dislocation movement and enhance the tendency of slip lines to cluster into bands.

The work-hardening mechanism in Stage II remains more or less the same for pure metals and alloys. In this stage, the lengths of slip lines do not appear to change significantly.

Stage III of the work-hardening curve is dependent on alloy additions to a great extent. The solute atoms can change the stacking fault energy, thereby affecting cross-slip, which is predominant during this stage. In α -brass, the stacking fault energy decreases with increasing zinc content (21). Thus the partial dislocations separate to a larger distance. The partial dislocations have to recombine in order for cross-slip to occur. Since the partials are separated to a larger distance with increased zinc content, there is a corresponding increase in the force needed to recombine the partials. This increase is so pronounced in 72/28 brass (21) that Stage III is not reached in tensile tests at room temperature. It is therefore not surprising that Kuhlmann-Wilsdorf and Wilsdorf (42) did not observe clustering of slip lines into slip bands in α -brass.

Stacking fault energy is not the only effect responsible for the increase in the stress required for the onset of Stage III or the suppression of the clustering of slip lines into bands. Thomas and Nutting (43) have suggested that the difference in the size of atoms is also responsible in promoting or stopping slip band formation. They observed slip bands in pure Al but not in Al-Cu and Al-Mg alloys at room temperature. Though the last two alloys exhibit a clear cut Stage III, no clustering of slip lines into bands was observed in these two alloys. However, they did observe slip bands in Al-Ag alloys

which were similar to those in pure Al. These differences between additions of Cu and Mg on the one hand and Ag on the other hand, were attributed to the differences in the size of atoms. Considering the size factor, Ag atoms fit very well into the Al lattice whereas Cu and Mg introduce mechanical strains. In the latter case the strains interfered with the cross-slip mechanism of slip band formation.

H. Theories of Slip-Band Formation

Several theories (34,35,39,44-46) have been proposed to account for slip band formation. All theories, in general, consider multiplication of dislocations and their propagation.

According to Cottrell (44) once the stress is sufficient to start a Frank-Read source, dislocations move towards the surface. These dislocations are held up by some obstacles until sufficient dislocations have collected to raise the local stress level to the value required to push the lead dislocation through. Slip is spread out over a number of closely spaced parallel slip planes and appears on the surface in the form of a cluster of lamellae. In Cottrell's view the obstacle is produced by slip on another set of planes. Figure 6 shows a schematic representation of the model given by Cottrell.

Brown (35) has proposed an alternative mechanism for the origin of lamellar slip. An avalanche of dislocations is started when the applied stress is sufficient to set off some

process of dislocation multiplication. The moving dislocations are stopped by work-hardening. Since the greatest resistance to slip occurs near a slip band, the most probable site for new slip bands is mid-way between existing slip bands. On the other hand, to account for lamellar slip, i.e., slip occurring very close to existing bands, Brown suggested, without explanation, that the region near slip bands is a region of easy slip due to recovery.

Brown's idea was extended by Mott (45) who suggested that the softening was caused by the diffusion of vacancies formed on the original slip plane by the passage of dislocations. However, this seems unlikely since lamellar formation can occur at low temperatures when diffusion of vacancies is slow.

Diehl, Mader, and Seeger (46) have proposed a double cross-slip mechanism for slip band formation. According to them an extended dislocation is held up in its glide plane by some obstacle. It changes to another $\{111\}$ plane by cross-slip. Once the dislocation has slipped out of the range of the obstacle, it cross-slips back to a plane parallel to the original glide plane. On the cross-slip plane there remains only two short segments of dislocation lines. These two will not move because the resolved shear stress on the cross-slip plane is less than on the primary glide plane. Thus these two segments will be immobilized and the part of the dislocation which has undergone cross-slip twice will act as a Frank-Read

source to produce slip lines parallel to the original one. Repetition of this process produces a slip band.

Kuhlmann-Wilsdorf, Merwe, and Wilsdorf (39,42) have proposed from their observations of fine slip at very low strains in copper, aluminum, and silver that the slip bands develop from the fine slip by a process of annihilation of all dislocations in the immediate vicinity of an expanding slip line.

Mitchell et al. (34,47) have proposed an avalanche mechanism for slip lines seen on the surfaces of α -phase Cu-Al alloy crystals. The avalanche of dislocations are introduced from surface sources when a considerable number of sources on or near the same glide plane are activated as a result of stress concentration due to irregularities in the surface.

In the case of copper-cobalt alloy single crystals, cobalt forms a coherent precipitate with the matrix. The increase in the flow stress has been attributed to the coherency strain field associated with the precipitate particles. It seems that the coherency strain, and particle size and spacing must play an important role in slip band formation.

Fig. 6. Cottrell's mechanism of slip band formation (35)

- (a) Formation of the first step
- (b) The step is displaced out of the plane of the source by step in another system
- (c) Slip from the first source producing a second step

III. EXPERIMENTAL METHODS

A. Alloy Preparation

An alloy of nominal composition copper-2 wt.% cobalt was prepared from 99.999% copper and 99.99% cobalt. The copper was in the form of 1/2 inch diameter rods which were cold swaged to 1/8 inch diameter. The cobalt was in strip form with an average thickness of about 0.025 inch. The proper proportions of each metal were melted together in an induction furnace using reactor grade graphite for a crucible. An atmosphere of argon gas was maintained in the furnace throughout the melting period of about 1/2 hour at 1150-1200°C. The resulting ingot, about 5 inches long by 2 inches diameter, was thoroughly cleaned in nitric acid followed by water and alcohol rinses. The surface was machined, polished, and re-cleaned to remove visible defects. The ingot was then rod rolled and swaged to about 1/2 inch diameter. To ensure removal of gaseous impurities, the 1/2 inch diameter rods were cut to 8 inch lengths and remelted in the induction furnace. Hot topping was used during casting to avoid pipe formation. The resulting ingot was devoid of pipe and surface defects. The upper part of the ingot (about 3 inches in length) was discarded and the remaining part was cleaned as described above.

Finally the ingot was sealed in a quartz tube under a vacuum of 10^{-5} Torr, homogenized at 950°C for seven days, and rapidly quenched in ice-brine solution after breaking one end

of the quartz tube. After cleaning the ingot, samples were taken from several random positions for chemical analysis of cobalt, oxygen and nitrogen. The average chemical analysis is given below:

Co = 2.1% by weight

O₂ = 5.1 parts per million

N₂ = not detectable

The cleaned ingot was rod-rolled and swaged down to approximately 7/8 inch diameter. About 7 inches from the top end and about 4 inches from the bottom end were cut off and discarded. The 7/8 inch diameter rod was further reduced to 5/8 inch diameter and cut into half. The two pieces were annealed in a tube furnace in an atmosphere of flowing dry nitrogen gas. The annealed rods were cleaned and swaged to 1/4 inch diameter to be used as the starting material for growing single crystals.

B. Growth of Single Crystals

Single crystals of 1/4 inch square cross-section were grown by a modified Bridgman technique. Several designs of graphite molds were used for holding the polycrystalline alloy and the seed crystal. The best results were obtained with the split graphite mold shown in Figure 7, which includes the overall set up of the furnace. About 10-inch long pieces of rods were cut from the starting stock, thoroughly cleaned, and dried. One piece was put in the square sectioned hole in the

split mold separated by about 1/4 inch from the seed. The seed used was about 1 1/2 inches in length and 1/8 inch in diameter. An accurately oriented copper crystal was used for seeding during the first operation of crystal growing. On subsequent runs small lengths of the grown crystals were used as seeds.

The graphite mold was screwed on to a vertical water cooled stainless steel pedestal attached to a base plate. Thermocouples were inserted at several places on the mold to indicate temperatures. A closed end alumina tube was used to cover the mold assembly. The open end of the alumina tube was fixed rigidly with a water cooled brass flange. The brass flange was screwed to the base plate. During the crystal growing, an atmosphere of flowing helium gas was maintained inside the alumina tube.

The furnace consisted of alumina tube wound with platinum wire contained in a box made of steel plates. Alumina bubble insulating material was packed in the gap between the heating tube and the box to reduce the loss of heat. The furnace was lowered to the level of the bottom of the polycrystalline alloy and the top of the seed. When a part of the top portion of the seed and the bottom part of the polycrystalline alloy melted as indicated by the temperature measurements, the furnace was gradually raised at a rate of 2 1/2 to 3 inches per minute, to continue melting and solidification of the alloy. Each run yielded a crystal about 8 inches long. The grown

Fig. 7. Schematic arrangement of the single crystal growing furnace

crystal was checked for perfection by macro-etching with a concentrated nitric acid solution. If any grain boundary was visible the crystal was discarded. The axial orientation of each grown crystal was determined by Laue back-reflection X-ray pictures of both ends of the crystals. The orientation of the crystal used in this dissertation is shown in Figure 4.

C. Homogenization of Crystals

Homogenization of crystals was important to ensure that the crystals were single phase before aging. The crystals were sealed in a quartz tube under 10^{-5} Torr vacuum and homogenized in a muffle furnace at 1025°C for seven days. The homogenized specimens were quenched in an ice-brine solution to retain the single phase structure. The crystals were cleaned in the manner described previously.

D. Aging Treatment

The aging was carried out in a tube furnace under a flowing dry nitrogen atmosphere. The crystals were placed in holes made in a cylindrical solid graphite mold. All aging treatments were performed at 600°C for different lengths of time varying from 5 minutes to 1000 minutes. After the required aging treatment, the crystals were quenched in ice-brine solution, then bright cleaned with nitric acid solution and finally cleaned with water and alcohol.

E. Specimen Preparation

A bright, plane surface of optical quality is required for interference microscopy work. For the copper-2 wt.% cobalt specimens a combination of (1) mechanical, (2) chemical, and (3) electropolishing gave the desired finish of the surface.

1. Mechanical polishing

Samples of 1/2 inch lengths were spark cut from the aged crystals and hand polished on 240, 400 and 600 grit emery papers. The specimens were held in a specially designed mechanical jig to keep the opposite surfaces parallel and the edges at right angles. Further polishing was done on a piece of cloth spread over a flat glass plate using Extra Prima paste. This polishing effectively removes the scratches resulting from 600 grit emery paper and a bright surface was obtained. Efforts were made to get as good a mechanical polish as possible to reduce the times of subsequent chemical and electropolishing treatments.

2. Chemical polishing

After the final mechanical polishing the surfaces of the crystals were polished on a cotton cloth of brand name Metcloth, spread over a flat piece of Teflon. The Teflon was marked along its length with very small grooves 1/4 inch apart to retain the polishing solution. The following chemical solution suggested by Mitchell et al. (34) was used.

Concentrated nitric acid	20 ml
Concentrated phosphoric acid	40 ml
Concentrated acetic acid	40 ml

The cloth was wetted with the chemical solution and the excess solution was scrapped off with a clean glass slide. This prevented solution from creeping onto the sides of the specimen so that a good edge was maintained during polishing. Each surface was polished for about 1 to 2 minutes. The specimens were thoroughly cleaned in running water and in an ultrasonic bath. The cleaned crystals were further washed with acetone and dried in a fast jet of air.

3. Electropolishing

A short period (~1 minute) of electropolishing in a solution of 60% phosphoric acid and 40% methyl alcohol was found to further increase the quality of the surfaces. The electropolishing was carried out at 0°C and at a current density of approximately $1/2 \text{ amp/cm}^2$.

The electropolished crystals were kept in an evacuated desiccator to reduce oxidation and contamination of surfaces. Effort was made to complete the testing and interference microscope examination as soon as possible after polishing because the quality of surface finish deteriorated with time.

F. Compression Tests

Room temperature compression tests were carried out in an Instron testing machine with a 10,000 pound capacity load cell

at a cross-head speed of 0.002 inch per minute. The specimens were compressed between two anvils that were aligned to the load cell. Teflon sheet was used for lubrication.

The machine deflection and the effects of Teflon were determined by compressing a hardened piece of AISI 4340 steel of the same cross-section as the specimen with the same thickness of Teflon at both ends as lubricant. In order to obtain the true compression values for the specimen, the combined compression of the machine and Teflon was subtracted from the specimen compression data.

The shearing stress and shearing strain for compression tests were calculated by the following equations given by Schmid and Boas (48).

$$\text{Resolved shear stress } \tau_{RSS} = \frac{P}{A_0} \cos \phi_0 \cos \lambda_0$$

Where P = applied load

A_0 = original cross-section area

ϕ_0 = angle between the stress axis and the slip plane normal before compression

λ_0 = angle between the stress axis and the slip direction before compression

The shearing strain, γ , was obtained by solving the following equation.

$$\frac{l_0^2}{l_1^2} = 1 + 2\gamma \cos \phi_0 \cos \lambda_0 + \gamma^2 \cos^2 \lambda_0$$

where, ϕ_0 and λ_0 have the same meaning as before, ℓ_0 and ℓ_1 are the lengths of the specimen before and after the compression respectively.

G. Interference Microscopy

Interference microscopy has the unique advantage of directly resolving very small step heights. Irregularities in the form of steps or recesses, such as cracks, scratches, pores, and the like are easily observed and the differences in levels measured directly.

The principle of the interference microscope is that a beam of light is subdivided into two beams which are made to interfere with each other optically. In its most simple form, the principles of the microscope are shown in Figure 8. A glass plate with a partially transparent mirror coating on the underside is placed between the objective and the specimen surface. The glass plate which forms the comparison surface is positioned inclined to the specimen. The light from the source is reflected downwards through the objective towards the specimen surface. A part of the light is reflected by the partially transparent mirror, while the other part of the light passes through this coating and is reflected by the specimen surface. The lights reflected from these two surfaces pass through the objective and to the eye piece where interference occurs.

At the point where the thickness of the wedge of air

formed between the comparison surface and the specimen surface is equal to d , the difference between the path lengths of the two beams is $2d$. The two beams interfere constructively if $2d$ is a whole number of wavelengths whereas they interfere destructively if $2d$ is an odd multiple of half wavelengths. The specimen surface appears through the eyepiece to be intersected by dark and light interference bands. The spacing between the dark and light interference bands corresponds to an increase in the path length of a whole number and thus an increase in the thickness of the air wedge of half a wave length.

However, if there are irregularities on the surface of the specimen, the interference bands are displaced accordingly. A schematic diagram showing the interference fringe displacement is shown in Figure 9. From a knowledge of interference fringe spacings and interference band displacement, slip step height is easily determined. The ratio of the fringe displacement to the fringe spacing when multiplied by the half-wavelength of the light used gives the step height directly.

In this research a Zeiss Interference Microscope with a multiple beam interference arrangement was used for determining step heights. The interference is produced between the beams reflected from the surface of the specimen and a comparison mirror. A far greater number of beams are formed in this case, since each beam reflected from the specimen surface is split at the reflecting comparison mirror, into one component

which passes through the layer and one component which is again reflected. The use of several beams makes the interference bands narrower and sharper and thus increases the accuracy of measurements. The requirements for multiple beam interference are that the specimen surface and the reference surface must be very highly reflective. For this reason special polishing techniques were employed as mentioned earlier to obtain sharp and clear interference fringes.

The resolution of the step heights depends upon the ability of measurements to determine the ratio of the interference fringe displacement to the fringe spacing. With two-beam interference, it is quite easy to estimate $1/10$ of a band spacing which means an accuracy of about $270 \text{ \AA}$ with thallium light. Since multiple-beam interference bands are narrower and sharper, the accuracy of measurements is considerably increased. According to Tolansky (49) height changes as small as $4 \text{ to } 5 \text{ \AA}$ can be detected.

Interference microscopy was selected for surface step-height measurements because of the high vertical resolution attainable and because the measurements are made directly on the surface being examined. However, it is to be noted that the high resolution and magnification refer only to the direction normal to the surface of observation. Across an object in the plane of observation the magnification corresponds to that of an optical microscope.

Figure 8. Schematic arrangement of an interference microscope

Fig. 9. Schematic representation of deflection of interference bands

H. Particle Size Measurements

The size of the precipitate particles of cobalt was determined by means of a magnetic technique described by Becker (5) and used by several others (6-9,16,17). Cobalt precipitate particles are ferromagnetic and are finely dispersed in the matrix of copper which is very weakly paramagnetic. It is assumed that the particles are of single domains and that their magnetic moments do not interact with each other.

The volume of a precipitate particle in a sample is obtained by the following relation.

$$V = \frac{3kT\chi_i}{M_s M_o}$$

Where, M_s = saturation magnetization of the precipitate material

M_o = saturation magnetization of the sample.

χ_i = initial magnetic susceptibility of the sample

k , T = Boltzmann's constant and absolute temperature respectively.

The volume fraction, f , of the precipitate was easily calculated by the relation, $f = M_o/M_s$.

To determine the size of the particles, the saturation magnetization of the sample at high field and initial susceptibilities must be measured. These were obtained with an oscillating sample magnetometer and a mutual inductance apparatus respectively.

For high field measurements with an oscillating specimen magnetometer, the specimen is vibrated perpendicular to the direction of a static magnetic field. The vibration of the specimen and thus, the magnetic dipole moment associated with the particles, induces an a.c. voltage. This voltage is detected by means of stationary detection coils placed close to the specimen. By comparison of the a.c. signals generated by the samples with those from a standard specimen of known magnetization, magnetic moments are determined. In the present experiment, a pure nickel cylindrical specimen was used as the standard sample. From these high field magnetization measurements the saturation magnetizations of the specimens were determined.

The initial susceptibility of the specimen was measured by means of a mutual inductance apparatus designed by Gerstein (50). The change in the mutual inductance due to the presence of the sample was related to the initial susceptibility by the following relation:

$$\chi_i = \frac{\Delta M}{V\Gamma} \text{ (emu/cm}^3\text{)}$$

Where ΔM = difference in the mutual inductance readings with the specimen in and out of the coil, expressed as turns

V = volume of the specimen (cm^3)

Γ = a geometrical constant determined by a sample of known susceptibility, expressed as $(\text{emu/turns})^{-1}$

The measurements of initial susceptibilities were made at room temperature in a field of approximately 10 gauss. Having thus determined the saturation magnetization and initial susceptibility of the samples, the volume, and consequently, the radius of the precipitate particles were calculated.

I. Determination of the Orientation of a Slip Plane from Its Traces in Two-Surfaces

Two surface trace analysis as described by Barrett (51) was carried out to determine the slip plane operating at every stage of deformation. The angles between the two adjacent surfaces were measured by means of an optical two circle goniometer. The two surfaces and the angles that the traces made with the common edge were plotted on a stereographic projection and the pole of the slip planes causing the slip traces were determined. The indices of the slip planes were fixed by comparison with a standard projection of all likely planes rotated to positions appropriate for the crystal orientation.

IV. EXPERIMENTAL RESULTS

A. Particle Size and Volume Fraction Determination

The results of the magnetic measurements giving the saturation magnetizations, initial susceptibilities, particle radii, volume fraction and weight percent cobalt precipitated are listed in Table I.

The weight percent of cobalt in the precipitate was obtained from the following relation.

$$\text{wt. \% Co} = \frac{M_{\text{Co}} \times \rho_{\text{Co}} \times M_o}{M_{\text{alloy}} \times \rho_{\text{alloy}} \times M_s}$$

where, M_{Co} = mass of cobalt precipitated (gms)

M_{alloy} = mass of the specimen (gms)

M_o and M_s have the same meanings as expressed earlier (p. 35)

ρ_{Co} = density of cobalt = $8.71 \text{ (gms/cm}^3\text{)}$ (52)

ρ_{alloy} = density of Cu + 2 wt.% Co = $8.95 \text{ (gms/cm}^3\text{)}$ (52).

The interparticle spacing, λ , was obtained with the following expression from Kocks (53).

$$\lambda = d \left(\frac{\pi}{6f} \right)^{1/2}$$

where, d = average particle diameter

f = volume fraction of the precipitate

It is clear from Table 1 that within a short period of aging at 600°C , the precipitation is almost completed and the volume fraction of the precipitate remains essentially con-

Table 1. Results of magnetic measurements listing the precipitate particle radius, interparticle spacing, volume fraction and wt.% Co precipitated for specimens solution treated at 1025°C and aged at 600°C

Specimen	Aging time (min)	Initial susceptibility χ_i (emu/cm ³)	Mo (emu/cm ³)	Particle radius (Å)	Inter-particle spacing (Å)	Volume fraction (f)	Wt.% Co
600-5	5	9.067×10^{-3}	20.554	21.7	246.3	.01626	1.428
600-30	30	38.866×10^{-3}	23.216	34	363.2	.01836	1.613
600-60	60	80.112×10^{-3}	24.866	42.3	436.6	.01967	1.728
600-120	120	137.093×10^{-3}	24.850	50.6	522.5	.01965	1.727
600-300	300	276.798×10^{-3}	25.190	63.6	652.3	.01992	1.751
600-1000	1000	232.727×10^{-3}	26.018	90.0 ^a	908.1	.02058	1.808

^aParticle size determined from the extrapolated curve of particle size vs aging time.

stant. Similar observations were made by Koepke (52), Becker (3) and Livingston and Becker (6).

Representative plots of the variation of magnetization with the applied field are shown in Figure 10. It is observed that the magnetization becomes nearly constant as the specimen reaches saturation. The slight drop in the magnetization values corresponding to the magnetic fields greater than 18 Kilogauss is not very well understood. Similar drops in magnetization values were reported by Olson (17) between 10 to 22 Kilogauss. The high field constant value of the magnetization was used as the saturation magnetization of the sample for the evaluation of particle sizes.

The variation of the particle size with aging time is shown in Figure 11. The logarithm of particle size is found to be linearly related to the logarithm of aging time with a proportionality constant of $1/4$ for particle radii up to about $70 \text{ \AA}$. The particle size of $70 \text{ \AA}$ corresponds to an aging time of approximately 400 minutes at 600°C . At longer aging times, there was some deviation from the linear variation; the curve was found to slope downwards slightly. This behaviour results from the fact that at larger particle sizes the superparamagnetic characteristics of precipitate particles do not hold well (3,15). Due to this, the straight line was extrapolated to obtain the particle size radius corresponding to a 1000 minute aging time. A similar $t^{1/4}$ dependence of particle size radius was observed by Koepke (52) for single crystals

Fig. 10. Representative plots of magnetization versus applied field

Fig. 11. Variation of particle radius (Å) with aging time (min)

and Olson (17) for polycrystalline specimens. Koepke however, observed $t^{1/3}$ dependence for polycrystalline specimens which were similar to the findings of Livingston (15). The particle sizes obtained by magnetic measurements are found to be in reasonably good agreement with those obtained by Bonar (10) using transmission electron microscopy.

B. Stress Axis of Compression Crystals

All crystals were grown in the same orientation using the modified Bridgeman technique described earlier. The crystals were grown such that their stress axes were oriented for maximum easy glide during compression. The crystal axis orientation was approximately 31° from $[001]$, 28° from $[011]$ and 24.5° from $[\bar{1}11]$, in the standard stereographic triangle. The approximate orientation of the crystals is shown in Figure 4. Since the samples were spark cut from the same parent crystal, there was very little variation in their orientation.

The rotation of the stress axis was studied by compressing a crystal successively to 99.5, 98, 94, and 90% of its original length. After each compression, a Laue back reflection X-ray picture of the same face was taken. A mechanical jig was used to keep the specimen perfectly flat and the stress axis normal to the X-ray beam. From these pictures, the orientation of the stress axis was determined after each compression and plotted on the same stereographic standard triangle. The rotation of the stress axis with compression

is shown in Figure 12. It is noted that even after 10% compression, the stress axis remained well within the standard triangle which indicates that easy glide should persist for the above compressions.

The rotation in the orientation of the stress axis changes the resolved shear stress on the slip systems. It is seen from the equation given on page 29, that the resolved shear stress is directly proportional to the product of $\cos \phi$ and $\cos \lambda$, where ϕ and λ are the angles that the slip plane normal and the slip direction make with the stress axis. For the orientation of the crystal in the standard triangle $[001-011-\bar{1}11]$, the four slip systems are as follows:

Primary slip system	$(111)[\bar{1}01]$
Cross slip system	$(1\bar{1}1)[\bar{1}01]$
Conjugate slip system	$(\bar{1}\bar{1}1)[011]$
Critical slip system	$(\bar{1}11)[101]$

The results listed in Table 2 indicate that the primary slip system has the maximum resolved shear stress. For the initial orientation the conjugate slip system has the next higher resolved shear stress, closely followed by the critical slip system. It is noted that the resolved shear stress on the critical slip system becomes the second highest (next to the primary system) after only a small amount of compression when the stress axis moves from position 1 to position 3 in Figure 12. For the cross slip system the resolved shear stress remains the least for all positions of stress axis as indicated

- Position 1 - orientation of stress axis before compression
- Position 2 - orientation of stress axis after 3% compression
- Position 3 - orientation of stress axis after 5.78% compression
- Position 4 - orientation of stress axis after 10% compression

Fig. 12. The rotation of stress axis during compression

Table 2. A listing of the variation of $\cos \phi \cos \lambda$ with the orientation of the stress axis during compression^a

Orientation figure	Primary		Cross		Conjugate		Critical	
	(111)	$[\bar{1}01]$	(1 $\bar{1}$ 1)	$[\bar{1}01]$	($\bar{1}\bar{1}$ 1)	[011]	($\bar{1}\bar{1}$ 1)	[101]
No. 1								
Angles with stress axis	58.0	32.5	85.0	32.5	62.5	28.0	24.0	68.0
$\cos \phi_1 \cos \lambda_1$	0.44693		0.07351		0.40770		0.34222	
No. 2								
Angles with stress axis	56.0	34.0	84.5	34.0	64.0	26.0	25.0	66.5
$\cos \phi_2 \cos \lambda_2$	0.46359		0.07946		0.39400		0.36139	
No. 3								
Angles with stress axis	54.5	36.0	84.0	36.0	65.5	25.0	25.5	65.0
$\cos \phi_3 \cos \lambda_3$	0.46979		0.08457		0.37584		0.38145	
No. 4								
Angles with stress axis	51.0	39.0	83.0	39.0	68.0	22.0	27.5	62.5
$\cos \phi_4 \cos \lambda_4$	0.48907		0.09471		0.34733		0.40958	

^a ϕ and λ are the angles between stress axis and slip plane normals and slip directions respectively.

in Figure 12.

C. Determination of the Orientation of Slip Planes
by the Two-Surface Trace Analysis

Single crystal specimens of copper-2 wt.% cobalt deformed primarily on the primary slip system. These specimens started showing traces of a secondary system after compressions of the order of 0.3%. The poles of the slip planes causing the slip traces were determined by means of the two surface analysis as described by Barrett (51).

The analysis was carried out for the traces in the central region of the specimens because of end effects. The angle between the two surfaces was measured with a two-circle optical goniometer. The angle was found to vary by about 8 to 10 degrees from a right angle for the range of compressions studied.

A Laue back reflection X-ray pattern of the specimen face was obtained after each compression increment. With the orientation of the face at the center of the stereographic projection, standard $\{111\}$ and $\langle 110 \rangle$ poles were plotted. The trace analysis was carried out on this stereographic projection between the traces on this face and the traces on an adjacent face. The two poles of the traces were found to be very close to the poles of the primary (111) and the critical $(\bar{1}11)$ slip planes. Moreover, the primary and the secondary poles determined by the two-surface trace analysis were found to make

angles of 59° and 23° respectively with the stress axis. From a comparison of the angles made by the stress axis with the slip planes normals, as shown in Table 2, it is apparent that the two traces were caused by the primary and the critical slip systems. A schematic diagram of the traces on the two adjacent faces is shown in Figure 13. The stereographic trace analysis based on the above diagram, is shown in Figure 14.

The resolved shear stress on the critical slip system was found to increase with the rotation of the orientation of the stress axis (Table 2). In fact, it becomes the second highest stressed system after only a small rotation of the stress axis orientation. When the stress axis orientation crosses the symmetry line $[001-011]$, the resolved shear stress on the critical slip system becomes the highest of the four slip systems. Therefore, it is not surprising that the observed secondary traces are caused by the onset of the critical slip system.

D. Slip-Band Step-Height Measurements

The step heights of the primary and secondary slip bands were measured by interference microscopy as mentioned before. The step-height measurements were carried out in the central region of the specimens and away from the influences of the complex stresses developed at the ends.

Two sets of experiments were carried out to determine step heights. In the first set of experiments, specimens aged

Fig. 13. Slip traces on two adjacent surfaces of a specimen aged 1000 minutes at 600°C

Fig. 14. Two-surface trace analysis carried out for a specimen aged 1000 minutes at 600°C after 1.69% compression

at 600°C for 30, 300, and 1000 minutes were successively compressed 5 times to about 87% of their original lengths. Slip band step-heights of single slip bands were measured on the same surface after each compression. In the second set of experiments, similar specimens were compressed three times to a final compression of about 11 to 13%. Slip-band step-heights were measured on the same two adjacent surfaces after each compression. Since the interference microscope only measured the vertical offset of the step, it was essential to convert the measured values of step heights into displacements along the slip plane and in the slip direction.

1. Results of single-surface measurements

The single surface slip-band step-heights were analyzed according to the expression given by Koppenaal and Fine (54). The displacement along the slip plane and in the slip direction is given by the following relation.

$$s = \frac{h}{\sin\phi \sin\theta}$$

where, h = measured slip step-height normal to the plane of observation

ϕ = angle between the slip plane normal and the surface normal

θ = angle between the slip line trace and the actual slip direction

It was established earlier by slip trace analyses that the two observed slip traces conformed to the primary and the critical

systems. The values of θ and ϕ were easily obtained from stereographic projections. The average slip band displacement along the slip plane and in the slip direction is shown in Table 3 for the first set of experiments.

From Table 3 it is observed that the average slip band displacement is larger for specimens aged for longer periods of time. The change in slip-band step-heights with glide strain is shown in Figure 15. From the curves it is apparent that slip band step heights increase with an increase in the glide strain. The curves remain essentially linear up to about 0.12 glide strain, thereafter the rate of increase of the step heights slows down with further increases in the strain. The rate of increase of step heights is larger for specimens aged for longer periods of time. The above observations are true for both the primary and the secondary slip bands.

Figure 16 shows the variation of slip-band step-heights of specimens having different sizes of precipitate particles for the same amount of glide strains. It is observed that the specimens with larger precipitate sizes have larger step heights for the same glide strain. Further, it is also seen that the slopes of the curves, in general, increase with increasing glide strain.

2. Results of two-surface measurements

The analysis of step heights in the slip plane and along the slip direction, from the measurements taken on two adja-

Table 3. Slip-band step-heights obtained from single-surface measurements for specimens homogenized at 1025°C and aged at 600°C

Percent compression	Glide strain	Average slip-band step-heights (Å) in the slip plane and along the slip direction	
		Primary slip band	Secondary slip band
Specimen (600-30)			
0.39	0.0086	1164.3	547.9
2.19	0.0487	1225.9	611.7
5.87	0.1304	1422.2	693.1
9.05	0.2013	1477.4	774.5
13.45	0.3020	1590.6	856.0
Specimen (600-300)			
0.30	0.0067	1445.3	790.1
1.70	0.0378	1589.0	836.0
5.53	0.1229	1834.5	---
9.90	0.2220	2159.3	---
12.90	0.2891	2230.0	1020.2
Specimen (600-1000)			
0.33	0.00719	1462.1	796.0
1.69	0.0375	1630.7	889.4
5.53	0.1226	1847.2	1003.9
9.40	0.2092	2204.5	1087.9
13.47	0.3024	2310.3	1122.4

Fig. 15. Slip-band step-heights of the primary and the secondary traces from single-surface measurements versus glide strain

Fig. 16. Variation of slip-band step-heights of primary and secondary traces with precipitate particle size at different amounts of glide strains

cent faces of the specimen, is given in the Appendix A.¹ The step height in the slip plane and the slip direction is given by the following expression:

$$S_T = [(h_A + h_B \cos \chi)^2 + \{h_A \frac{\cos \alpha}{\cos \beta} + h_B (\frac{\cos \alpha \cos \chi + \cos \gamma \sin \chi}{\cos \beta})\}^2 + (h_B \sin \chi)^2]^{1/2}$$

where, h_A and h_B are step heights measured by the interference microscope on adjacent faces A and B respectively. χ is the angle between the surfaces A and B. α , β , and γ are the angles made by the slip plane normal to the three cartesian coordinates x, y, and z respectively. Axis x was chosen along the normal to face-A and y along the stress axis. It was assumed in the derivation of the above expression that faces A and B were parallel to the axis of compression. This condition was easily achieved experimentally.

Angles α , β and γ were determined by the two-surface trace analyses carried out after each compression of specimens. The angle χ between the two adjacent faces was measured, by means of a two-circle optical goniometer.

The slip-band step-heights calculated from measurements on two adjacent faces are listed in Table 4. The variation of step heights as a function of glide strain is shown in Figure

¹T. E. Scott, Dept. of Metallurgy, Iowa State University, Ames, Iowa. Personal communications, 1974.

Table 4. Slip-band step-heights determined from two-surface measurements for specimens homogenized at 1025°C and aged at 600°C

Percent compression	Glide strain	Average slip-band step-heights (Å) in the slip plane and along the slip direction	
		Primary slip band	Secondary slip band
Specimen (600-30)			
0.56	0.0124	912.9	694.1
6.19	0.1368	1016.4	976.0
13.05	0.2730	1086.4	1056.8
Specimen (600-300)			
0.70	0.0156	1243.2	965.8
5.56	0.1225	1374.3	1278.7
11.74	0.2606	1480.2	1434.4
Specimen (600-1000)			
0.56	0.0122	1295.2	1036.3
5.11	0.1128	1633.1	1358.0
11.77	0.2620	1726.2	1471.3

17. Figure 18 shows the approximate variation of step heights of specimens with different sizes of precipitate particles for fixed glide strains.

The results of measurements on two faces and those obtained from single surface measurements are quite similar. The graphs of Figures 15 and 17 indicate in general a linear increase of step heights for small compressions. The slopes of the curves appear to be greater for specimens aged longer periods of time. A similar linear increase of average glide per slip plane with increasing strain was observed by J. T. Fourie (40) for tensile elongation of α -brass single crystals. The increase in step heights with precipitate particle size (Figure 18) was also found similar to the one-surface measurements (Figure 16).

It is not always possible to make a direct comparison between the results obtained from one-surface and two-surface measurements, since the specimens were compressed to different extents. However, it is obvious that the step heights calculated from single surface analyses are larger than the step heights calculated from two-surface analyses. The variations are larger for primary slip bands than for secondary bands. For single surface measurements the step height in the slip plane and along the slip direction was evaluated from the relation $s = h/(\sin\phi \sin\theta)$, where ϕ is the angle between the surface of observation and the slip plane and θ is the angle between the trace and the actual slip direction. This analy-

sis presupposes the direction of slip. In case of face centered cubic systems, the slip directions are well established for different slip systems. From trace analysis, the two slip planes were found to correspond to the primary and the critical slip systems. The corresponding directions were used to find the angle θ . For the primary slip bands the angles ϕ and θ were within the range of 50 to 70° and 30 to 40°, respectively. An error of $\pm 5^\circ$ in the measurements of angles introduces an error of ± 200 to ± 400 Å in the calculation of step heights, in addition to the errors involved in the measurement of vertical height, h , as measured by the interference microscope. For the case of secondary slip bands, angles ϕ and θ are larger, being in the range of 70 to 85° and 80 to 87°, respectively where $\pm 5^\circ$ error in angle measurement introduces an error of only ± 40 Å in the slip band step heights.

Similarly a $\pm 5^\circ$ error in the measurements of angles α , β , γ and χ (Appendix A) for two-surface analysis introduces an error of approximately ± 100 Å for primary and ± 60 Å for secondary slip step heights. Another source of error was introduced in the two-surface analysis by taking an average of slip-band step-heights measured on each surface. This average was then used in the equation (page 56) for determining the step height in the slip plane and in the slip direction. For a more exact analysis, measurements of step heights of the same slip band should be made on both surfaces. In

Fig. 17. Slip-band step-heights of primary and secondary traces from two-surface measurements versus glide strain

Fig. 18. Variation of slip-band step-heights obtained from two-surface measurements with particle radius at different amounts of glide strains

practice, it is very difficult to follow the same slip line from one surface to another. Therefore the measurements were made on several slip bands on both surfaces corresponding to the same wide bands of deformation and an average value was obtained. Though the results of the two-surface measurements seem to differ slightly from the single surface measurements, they indicate a similar variation of step heights with glide strain and particle radius.

3. Comparison of theoretical and measured slip-band step heights

The total amount of slip associated with slip bands was measured by interference microscope. This was carried out for all the slip bands in between two prominent slip bands separated at a larger distance. The total measured value of the step heights was converted into the step height along the slip plane and the slip direction, using the equation given on page 51. The distance between the slip bands on the surface was measured in a direction perpendicular to them. This distance was converted into the normal distance between the two slip bands by using the following relation (Appendix B).

$$W_{\gamma} = W_m \sin \alpha$$

where, W_m = measured distance in a direction perpendicular to the observed traces

W_{γ} = distance normal to the slip bands

α = angle between the slip plane and the surface of observation

The overall glide strain of the specimen was obtained from Schmid and Boas equation (48) as given on page 29. The glide strain when multiplied by the normal distance between the slip bands, gives the expected step height in the slip plane and along the slip direction within the two bands.

The measurements of step heights within fixed bands were carried out after each compression and the results are shown in Table 5. It is observed that for a very small compression the measured values of slip-band step-heights and the calculated theoretical step heights are very close to each other. In the specimen aged at 600°C for 30 minutes, only 12.7% of the theoretically estimated glide step height is unaccounted for. However, for specimens aged for 300 and 1000 minutes, the estimated step heights are accounted for in total by measurements; actually the measured values are slightly larger than the theoretically expected value. This difference could possibly be due to the non-uniformity of the glide strain. With increase in the glide strain, it is noted that a larger percentage of slip step-height is not confirmed by measurements. With increasing deformation, it was observed that larger numbers of secondary slip bands were activated. Part of the step heights not accounted for is due to the step heights associated with the secondary bands. Besides this, another source of error may be in the calculation of the overall glide strain under the assumption that slip occurs on a single slip system only. Since there was a very small

Table 5. A listing of the measured slip-band step-heights and step heights calculated from the overall glide strain in a selected area of the specimen

Percent compression	Glide strain	Width of observed area (μ)	Expected slip-band step-height (μ)	Cumulative measured step height (μ)	Percent unaccounted
Specimen (600-30)					
0.39	0.00867	473.18	4.102	3.579	12.7
2.19	0.0487	295.9	14.41	9.094	36.8
5.87	0.13041	82.3	10.73	4.901	54.3
9.05	0.20136	64.5	12.98	2.953	77.2
13.45	0.30202	97.1	29.30	6.296	78.5
Specimen (600-300)					
0.30	0.0067	499.08	3.344	5.14	none
1.70	0.3782	153.2	5.79	4.4795	22.6
5.53	0.1229	118.9	14.60	5.441	62.7
9.90	0.22206	81.41	18.07	4.773	73.6
12.90	0.28914	100.3	28.9	5.13	82.2
Specimen (600-1000)					
0.33	0.00719	497.29	3.57	4.0	none
1.69	0.03756	158.13	5.939	5.196	12.5
5.53	0.12264	106.32	13.039	7.004	46.3
9.40	0.20922	107.26	22.44	7.476	66.6
13.47	0.30246	367.3	111.10	39.922	64.1

proportion of secondary slip in the initial stages of deformation, the correlation between the theoretically calculated and measured values of step heights was very good. From this observation it appears that a major portion of the slip lines is concentrated into slip bands. Koppenaal and Fine (54) could account for only 42% of the total shear from their slip step height measurements on α -phase Cu-Al alloy crystals. They suggested that an appreciable amount of slip occurred on a fine sub-microscopic scale. In the case of the Cu-2% Co alloy, on the other hand, the slip lines appear to be concentrated into the slip bands.

E. Interferograms

The interferograms of the specimens taken after successive compressions indicate the changes in the slip band step heights with compression. Some representative interferograms are shown in Figures 19 to 21. It is seen from the photomicrographs that secondary slip bands are initiated at a very early stage of deformation. To study the behavior of step heights and slip density, photomicrographs of a selected area were obtained after successive compressions. In Figure 19, photomicrographs of a fixed-area pair, c and d, indicate that there was no additional slip band formation in the primary or in the secondary slip systems between 9.05% and 13.45% compression for specimen aged 30 minutes. Similar observations are made for specimens aged 300 minutes. The fixed-area photo-

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Fig. 19. Interferograms of a specimen aged 30 minutes at 600°C. 500X

- (a) After 0.39% compression
- (b) After 2.19% compression
- (c) After 9.05% compression
- (d) After 13.45% compression, same area as (c)

a

b

c

d

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Fig. 20. Interferograms of a specimen aged 1000 minutes
at 600°C. 500X

- (a) Primary slip bands after 0.33% compression
- (b) Secondary slip bands after 1.69% compression
- (c) Secondary slip bands after 5.57% compression
- (d) Primary slip bands after 9.4% compression

a

b

c

d

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Fig. 21. Interferograms of a specimen aged 300 minutes
at 600°C. 500X

- (a) After 0.3% compression
- (b) After 1.7% compression; same area as (a)
- (c) After 9.9% compression
- (d) After 12.9% compression; same area as (c)

BUTLER'S 100% RAG

a

b

c

d

micrographs, a and b, of Figure 21 also show that no additional slip bands formed between 0.3% and 1.7% compressions. Another fixed-area pair, c and d, of Figure 21 show that no additional slip bands formed between 9.9% and 12.9% compressions. It appears that the formation of slip bands closes somewhere below 0.3% compression. Further deformation takes place by glide on existing slip bands.

F. Slip-Band Spacings

Slip-band spacings were measured at about 640X magnifications on two adjacent faces after each compression of the specimens. The actual spacing, W_r , in a direction normal to the slip plane was obtained by the relation, $W_r = W_m \sin \alpha$, where W_m is the measured spacing in a direction perpendicular to the traces on the surface of observation, and α is the angle between the surface and the slip plane. Angle α was obtained from stereographic projections. The results of slip-band spacings for the primary and secondary traces are shown in Table 6.

It is observed that the secondary slip bands, in general, were spaced at larger distances than the primary slip bands. The spacings of the primary as well as the secondary slip bands decreased with increase in compression. The decrease was most noticeable after the second (~5.6%) compression. The slip-band spacings changed very little after the third (~12.2%) compression. The results indicate that the slip-

Table 6. Slip-band spacings of primary and secondary traces in a direction normal to the slip planes

Percent compression	Glide strain	Slip-band spacings (μ)			Slip-band spacings (μ)		
		Primary traces			Secondary traces		
		Face A	Face B	Avg	Face A	Face B	Avg
Specimen (600-30)							
0.56	0.0124	19.99	17.69	18.84	38.89	45.65	42.27
6.19	0.1368	7.82	7.49	7.65	30.48	37.32	33.90
13.05	0.2730	6.10	7.02	6.56	27.37	35.01	26.19
Specimen (600-300)							
0.702	0.0156	16.38	14.24	15.31	23.86	33.03	28.46
5.57	0.1225	13.52	7.19	10.35	19.52	28.05	23.78
11.74	0.2606	10.86	6.61	8.73	16.19	24.80	20.49
Specimen (600-1000)							
0.557	0.01229	13.78	12.86	13.32	21.03	31.40	26.21
5.11	0.1128	7.92	9.32	8.62	14.28	20.90	17.59
11.77	0.2620	7.16	8.55	7.85	13.76	19.05	16.41

band spacings do not change appreciably after a certain extent of compression. This is consistent with the previous observation that no new slip bands form after a small initial deformation. The slight reduction in the slip-band spacing after the third compression is as a result of the overall reduction in the size of specimens due to the compression.

G. Compression Tests

The change in the resolved shear stress with glide strain of crystals aged 30, 300, and 1000 minutes at 600°C, is shown in Figure 22. Clearly, the yield stress increases with aging time.

The crystals were compressed to a given strain and unloaded for interference microscopic examination. The same crystals were compressed again and the process was repeated. It is observed that the flow stress increased on successive compressions. The reason for this behaviour is not obvious.

The work-hardening curves of the crystals indicate that the extent of the easy glide region decreased with an increase in the aging time. In fact, for crystals aged 1000 minutes, it appears that easy glide has completely vanished. The curves indicate that most of the hardening in this alloy occurred during the first compression. Further it is observed that the work-hardening rate is quite low during the second and the third compressions. It is known that the work-hardening rate during the second stage (linear hardening) remains

quite high and approximately constant (21,25). Thus it appears that in this alloy, the parabolic hardening corresponding to Stage III of the work-hardening curve, starts within a short period of easy glide, or almost instantly at the stress levels required for yielding.

Fig. 22. Resolved shear stress versus glide strain curves for specimens aged 30, 300, and 1000 minutes at 600°C

V. DISCUSSION

Plastic deformation occurs in crystalline solids by the movement of a large number of dislocations through appreciable distances in the lattice. When a dislocation moves out of the surface of the crystal, it produces a step of unit Burger's vector. In this investigation, slip-band step-heights of the order of 700 to 3000 Å (Tables 3 and 4) were observed. This means that about 270 to 1200 dislocations exited the surface in every slip band. Considering that the average initial density of dislocations is very low (35), the generation of dislocations and their propagation play an important role in the formation of these steps. In the following, the results of the two-surface trace analysis, the compression tests, the slip-band step-height measurements, and the observed slip-band spacings are discussed with the above mentioned situation in mind.

A. Two-Surface Trace Analysis

The two-surface trace analysis of the copper-2 wt.% cobalt alloys revealed that the major amount of deformation occurred on the primary slip system. This was expected from the orientation of the stress axis (Figure 4). However, the critical slip system was also found to be operative at very small strains. The reason for the initiation of the critical slip system can be partly understood from Table 2. The table lists the variations of the resolved shear stress on the

primary, conjugate, cross, and critical slip systems with the change in the orientation of the stress axis of the crystals. It is observed that on the primary slip system initially the resolved shear stress is maximum while it is somewhat less on the conjugate and the critical slip systems. The cross-slip system has the least resolved shear stress for all positions of the stress axis orientation. However, the stress on the critical slip system increases with change in orientation. In fact, the resolved shear stress on the critical system becomes the highest of the four systems as the stress axis orientation crosses the symmetry line $[001-011]$ in Figure 12. The critical slip system becomes operative at an early stage of deformation since the stress level in this alloy is already high and because there is an increasing tendency for slip on this particular system.

B. Resolved Shear Stress-Glide Strain

The resolved shear stress-glide strain curves (Figure 22) for the aged copper-2 wt.% cobalt alloy clearly show the increase in flow stress with aging time. The easy glide region appeared to decrease as the aging time of the specimen was increased. For example, the easy glide region seems to have vanished completely for a specimen aged 1000 minutes. The curves in Figure 22 show that the major portion of hardening takes place during the first compression. On further compression the work-hardening rate appears to be rather low. Since

the stress level is quite high, there is a chance for secondary slip to become operative. It is not surprising that slip traces corresponding to the critical system were found by two-surface trace analysis.

C. Slip-Band Step-Height Measurements

Results of measurements of slip-band step-heights (Table 5) by interference microscopy of a prescribed area compared very closely, at small deformations ($\gamma = 0.008$), to those calculated from the overall glide strain. It is known that the interference microscope measures the step heights of slip bands and not the individual slip lines which constitute the bands. The reason for this is that the magnification of the interference microscope across an object in the plane of observation, is just like an ordinary optical microscope. Therefore, the good correlation of measured step heights and the calculated step heights indicate that all the slip lines are confined to the slip bands. This observation is different from the one made for solid solution, single phase alloys (34,55) where a large amount of slip was attributed to micro-slip or fine slip. However, in the present investigation, at deformations greater than $\gamma = 0.037$, quite large variations (Table 5) of the measured step heights from the calculated step heights were observed. Perhaps the variations arise from the initiation of a large number of secondary slip bands. The secondary slip bands were neglected in the measurements

of cumulative step-heights. Moreover, the glide strains were evaluated from the Schmid and Boas (48) equation which considers only the primary slip.

The interferograms of the prescribed area of the specimens of copper-2 wt.% cobalt [Figure 19 (c and d), Figure 21 (a and b, c and d)] reveal that no new slip bands are formed after a certain strain is reached. It appears that the formation of new slip bands stopped somewhere below 0.3% compression. Probably subsequent deformations were accompanied by glide on the existing slip bands.

The variation of slip-band step-heights with the glide strain is shown in Figures 15 and 17. It is obvious that the step-heights increase with increased glide strain. Further it is noted that the step-heights increase at a faster rate for small glide strains as compared to large glide strains. In general, as the strain increases the curves tend to become flatter. Similar observations can be made with regards to the variation of slip-band step-heights with the precipitate particle sizes (Figures 16 and 18). It is seen that the specimens with larger precipitate particle sizes invariably have larger step-heights for the same glide strain.

D. Slip Band Spacings

The slip-band spacings for copper-2 wt.% cobalt alloy crystals were found to be larger (Table 6) at the beginning of deformation. With increased deformation, the spacings were

found to decrease to nearly a constant value for both the primary and the critical slip bands. Slip bands of the critical slip system were observed to have wider spacings than the primary slip bands. The slight decrease in the slip-band spacings, from the second to the third compression (Table 6), appears to be due to the overall decrease in the length of the specimen by compression. The result that the slip-band separation becomes nearly constant after a small deformation verifies the observations made earlier on the interferograms of the prescribed area. The separation of slip bands apparently is determined by the interaction of dislocations on parallel glide planes (39). Besides this, on either side of the plane containing dislocations released from a source there is a corridor (39) within which no new source can start to operate. Hence, the formation of slip bands in this alloy takes place until an optimum separation is reached.

E. Conclusions

The main conclusions from the results are that (1) the slip lines are concentrated into slip bands, and (2) the formation of slip bands ceases after a small deformation. As a result of the second conclusion the slip bands spacings remain nearly constant on further compression.

The above observations and conclusions must be discussed in the light of dislocation generation and propagation mechanisms. A large number of dislocations are required to account

for the observed slip-band step-heights generated by dislocations exiting the surface in addition to those left on the slip planes. There must be some dislocation multiplication mechanism operating during the deformation. According to Ansell (55), in the case of precipitation hardened alloys, the stress required to move dislocations through appreciable distances (large compared to the interparticle spacing) is larger than the stress required to generate dislocations. Obviously a large number of dislocations are generated, but the mechanism of the dislocation generation is not well established for this alloy. The usual Frank-Read source mechanism of generation is not possible in this alloy because the stress required in its operation is higher than the observed flow stress of the crystals (17). The stress required to initiate the Frank-Read sources decreases with an increase in the precipitate particle size or the interparticle spacing. However, in these specimens, even for precipitate of radius $90 \text{ \AA}$, the stress for activating Frank-Read sources remains higher than the flow stress of the material (17).

It is possible, however, that the dislocations may be generated at or near the surface. According to Hollomon as cited by Kuhlmann-Wilsdorf and Wilsdorf (42) the stress required to activate such sources is about half of the stress needed for the initiation of a Frank-Read source within a crystal. This amount of stress is still slightly larger (17)

than the observed flow stress for the specimens used in this research. Yet, discontinuities at the surfaces could produce a local stress concentration sufficient to activate dislocation sources. Mitchell et al. (34) have employed this argument for α -phase copper-aluminum alloys.

The flow stress of the solution treated alloy is much lower than the age-hardened alloy (52). It is possible that sources may be operative at a stress less than the flow stress of the alloy and the dislocations are piled-up against the particle stress field barrier. Once the applied stress is sufficiently high the dislocations can cut through the particles and spread out in closely spaced slip planes or bands.

Moreover as a dislocation cuts through the particle, the effective size of the particle is reduced. With the reduction in the size of the particle, its stress field is reduced correspondingly. As a result, once a dislocation cuts through the particle barrier, further movement of dislocations becomes easier. Consistent with this, is the fact (Figure 22) that, once the flow stress is reached, the work-hardening rate does not increase very rapidly. The small observed work-hardening may be due to dislocation-dislocation interactions.

Another possible means for dislocations to circumvent particles is the Orowan bow-out mechanism. Figure 23 taken from Ref. (16) shows the hardening expected from the theories of particle cutting proposed by Gleiter (11), and Gerold and

Haberkorn (14), and the behaviour expected from the Orowan bow-out mechanism. The increase in the hardening as observed in this investigation is also plotted. The results indicate that the cutting mechanism is the main source of hardening for particles of radius less than approximately $64 \text{ \AA}$. The Orowan bow-out mechanism appears to become the predominant mechanism for specimens with larger particles ($R > 64 \text{ \AA}$).

Fig. 23. Comparison between the experimental data for Cu-2.0 wt.% Co aged at 600°C and theoretical values for precipitation hardening (Line D) according to Gleiter (11) and Gerold and Haberkorn (14). [Curves A, B, and C represent the theoretical expressions for a modified Orowan mechanism (16)]

VI. SUMMARY

The surface topography of copper-2 wt.% cobalt alloy single crystals was studied as a means to better understand the mechanisms of deformation and precipitation hardening in this alloy system. The sizes of the precipitate particles were measured magnetically. The particle size obtained by the magnetic measurements agreed very well with previous work.

The single crystals were deformed in compression to various degrees using an Instron testing machine. It was observed that the majority of slip occurred on the primary slip system. This was in accordance with the position of the stress axis in the standard triangle of the stereographic projection. However, the two-surface trace analysis revealed that in addition to the primary slip system, the critical slip system also became operative at an early stage of deformation. The high stress level and the tendency toward increasing resolved shear stress on the critical system with the rotation of the stress axis during compression were thought to be the reason for the early operation of this system.

The slip bands appearing on the surfaces were examined by interference microscopy, which gives a direct and quantitative measurement of step heights. Two sets of experiments were performed to determine the step heights. In one set of experiments specimens were compressed successively five times to about 0.3 glide strain. After each compression,

measurements of step heights were made on the same surface every time. In the second set of experiments, similar specimens were compressed three times to approximately the same maximum glide strain. Measurements of step heights were made on two adjacent surfaces. It is known that the interference microscope measures the step heights in a direction perpendicular to the plane of observation. Therefore, to get meaningful results, it was necessary to convert the measured step heights into displacements parallel to the slip plane in the slip direction. For the first set of experiments, a simple relation was used which presupposed the direction of slip. However, for the two-surface measurements, a mathematical analysis was developed to get the step heights in the slip plane and in the slip direction. The results of the two sets of experiments were found to be within reasonable agreement, considering the experimental errors. It is also seen that the errors involved in the evaluation of step heights from the two-surface measurements are considerably less than those involved in single-surface measurements. The slip-band step heights were found to increase with the glide strain. It was also found that with increasing strain the rate of increase of the step heights decreased. A similar variation of step heights with particle size was observed for constant glide strain. The increase in the step heights with glide strain or with precipitate particle size was found invariably to be

larger for specimens with larger particles.

Cumulative step heights were determined by measurements of all the slip-band step-heights within a given area of the specimen. These were compared with the theoretically evaluated step heights for the same area. There was very good agreement between the two results for small strains. It was concluded that the slip in this alloy was confined within the slip bands.

The study of the interferograms of a prescribed area obtained after successive compressions revealed that the formation of slip bands stopped after 0.3% compression. Further deformation occurred by glide on the existing slip bands. A similar conclusion was drawn from the measurements of slip-band spacings. It was found that the separation of the slip bands became nearly constant after a minimum strain level was reached.

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IX. APPENDIXES

Appendix A. Analysis of Slip Vector by Interference Microscopy - Treatment of Measurements on Two Surfaces

A schematic geometry of slip in a single crystal is illustrated by Figure 23. The slip step heights are measured on Face A and Face B, which are adjacent to each other and make an angle χ . The coordinate system has been defined consistent with measurables.

Let the unit vectors along the x, y, and z right handed cartesian coordinate system be $\hat{i}$, $\hat{j}$, and $\hat{k}$. $\hat{i}$ is considered to be along the normal to the Face A, $\hat{N}_A$, while $\hat{j}$ is along the stress axis, T.A. The unit vector $\hat{k}$ is defined by $\hat{k} = \hat{i} \times \hat{j} = \hat{N}_A \times \text{T.A.}$, which is normal to plane of T.A. and $\hat{N}_A$.

Let the slip plane normal $\overline{\text{SPN}}$ make angles of α , β and γ with the x, y, and z coordinates respectively.

$$\text{Thus } \overline{\text{SPN}} = \cos \alpha \hat{i} + \cos \beta \hat{j} + \cos \gamma \hat{k}$$

$$\text{and } \overline{\text{SPN}} \times \hat{k} = \cos \beta \hat{i} - \cos \alpha \hat{j} = \bar{S}_A$$

$$\text{where } |\bar{S}_A| = (\cos^2 \alpha + \cos^2 \beta)^{1/2}$$

where $\bar{S}_A$ is a vector lying in the glide plane because it is normal to $\overline{\text{SPN}}$. It is also in the T.A. - $\hat{N}_A$ plane since it is normal to $\hat{k}$ which is normal to the T.A. - $\hat{N}_A$ plane. Thus, the vector $\bar{S}_A$ lies along the intersection of the slip plane and the plane in which h_A is measured. h_A is the step height normal to the Face A measured by interferometry.

Let the angle between $\bar{S}_A$ and $\hat{i}$ be ϕ_A

$$\begin{aligned}\bar{S}_A \cdot \hat{i} &= |\bar{S}_A| |\hat{i}| \cos \phi_A \\ &= (\cos^2 \alpha + \cos^2 \beta)^{1/2} \cos \phi_A\end{aligned}$$

$$\text{Also } \bar{S}_A \cdot \hat{i} = (\cos \beta \hat{i} - \cos \alpha \hat{j}) \cdot \hat{i} = \cos \beta$$

$$\therefore \cos \phi_A = \frac{\cos \beta}{(\cos^2 \alpha + \cos^2 \beta)^{1/2}}$$

$$\text{Now } \hat{S}_A = \frac{\bar{S}_A}{S_A} = \frac{\cos \beta \hat{i} - \cos \alpha \hat{j}}{(\cos^2 \alpha + \cos^2 \beta)^{1/2}}$$

Let a be the magnitude of displacement parallel to $\hat{S}_A$.

$$\therefore (a \hat{S}_A) \cdot \hat{i} = a \cos \phi_A = h_A$$

$$\text{or } a = \frac{h_A}{\cos \phi_A} = h_A \frac{(\cos^2 \alpha + \cos^2 \beta)^{1/2}}{\cos \beta}$$

$$\therefore a S_A = h_A \frac{(\cos \beta \hat{i} - \cos \alpha \hat{j})}{\cos \beta} \dots \quad (1)$$

Now since the plane A and plane B are not necessarily at right angles, it is assumed that $\bar{N}_B$ and $\hat{N}_A$ make an angle χ with each other.

$$\bar{N}_B = \cos \chi \hat{i} + \sin \chi \hat{k}.$$

The coefficient of the $\hat{j}$ term is zero since it is assumed that both faces A and B are parallel to the T.A.

$$\hat{N}_B = \frac{\bar{N}_B}{|\bar{N}_B|} = \bar{N}_B = \cos \chi \hat{i} + \sin \chi \hat{k}$$

Furthermore:

$$\begin{aligned}\hat{N}_B \times \hat{T.A.} &= (\cos \chi \hat{i} + \sin \chi \hat{k}) \times \hat{j} \\ &= -\sin \chi \hat{i} + \cos \chi \hat{k}\end{aligned}$$

Let us define $\hat{N}_B \times \hat{T.A.}$ by a vector $\bar{M}$, which is normal to plane of $\hat{N}_B$ and $\hat{T.A.}$.

$$\begin{aligned}\bar{SPN} \times \bar{M} &= (\cos \alpha \hat{i} + \cos \beta \hat{j} + \cos \gamma \hat{k}) \times (-\sin \chi \hat{i} + \cos \chi \hat{k}) \\ &= \cos \beta \cos \chi \hat{i} - (\cos \alpha \cos \chi + \cos \gamma \sin \chi) \hat{j} + \\ &\quad \cos \beta \sin \chi \hat{k} \dots\end{aligned}\quad (2)$$

$$\text{Let } \bar{S}_B = \bar{SPN} \times \bar{M}$$

It is easily seen that the vector $\bar{S}_B$ lies in the slip plane and the plane defined by $\hat{N}_B$ and $\hat{T.A.}$. Therefore, it lies at the intersection of the slip-plane and the plane containing $\hat{T.A.}$ and $\hat{N}_B$, i.e., along the slip plane and the plane in which h_B is measured by interferometry.

From Equation 2,

$$|\bar{S}_B| = (\cos^2 \beta + (\cos \alpha \cos \chi + \cos \gamma \sin \chi)^2)^{1/2}$$

Let the angle between $\bar{S}_B$ and $\hat{N}_B$ be ϕ_B

$$\bar{S}_B \cdot \hat{N}_B = |\bar{S}_B| |\hat{N}_B| \cos \phi_B$$

$$\text{Also } \bar{S}_B \cdot \hat{N}_B = \cos \beta \cos^2 \chi + \cos \beta \sin^2 \chi = \cos \beta$$

$$\text{or } \cos \phi_B = \frac{\cos \beta}{(\cos^2 \beta + (\cos \alpha \cos \chi + \cos \gamma \sin \chi)^2)^{1/2}}$$

$$\text{Now } \hat{S}_B = \frac{\bar{S}_B}{|\bar{S}_B|} = \frac{\cos \beta \cos \chi \hat{i} - (\cos \alpha \cos \chi + \cos \gamma \sin \chi) \hat{j} + \cos \beta \sin \chi \hat{k}}{(\cos^2 \beta + (\cos \alpha \cos \chi + \cos \gamma \sin \chi)^2)^{1/2}}$$

Let b = magnitude of displacement parallel to $\hat{S}_B$.

$$\therefore (b \hat{S}_B) \cdot \hat{N}_B = b |S_B| |N_B| \cos \phi_B = b \cos \phi_B = h_B$$

Where h_B is the vertical distance measured on Face B by interferometry.

$$\text{or } b = \frac{h_B}{\cos \phi_B} = \frac{h_B (\cos^2 \beta + (\cos \alpha \cos \chi + \cos \gamma \sin \chi)^2)^{1/2}}{\cos \beta}$$

$$\therefore b \hat{S}_B = h_B \frac{[\cos \beta \cos \chi \hat{i} - (\cos \alpha \cos \chi + \cos \gamma \sin \chi) \hat{j} + \cos \beta \sin \chi \hat{k}]}{\cos \beta} \quad (3)$$

Since the total displacement

$$\bar{S}_T = a \hat{S}_A + b \hat{S}_B$$

$$= [(h_A + h_B \cos \chi) \hat{i} - (\frac{h_A \cos \alpha}{\cos \beta} + \frac{h_B (\cos \alpha \cos \chi + \cos \gamma \sin \chi)}{\cos \beta}) \hat{j} + h_B \sin \chi \hat{k}] \quad (4)$$

$$\therefore |\bar{S}_T| = [(h_A + h_B \cos \chi)^2 + (\frac{h_A \cos \alpha}{\cos \beta} + \frac{h_B (\cos \alpha \cos \chi + \cos \gamma \sin \chi)}{\cos \beta})^2 + (h_B \sin \chi)^2]^{1/2} \quad (5)$$

When the Faces A and B are at right angles i.e., $\chi = 90^\circ$ the Equation 4 and 5 are reduced as follows:

$$\bar{S}_T = h_A \hat{i} - (h_A \frac{\cos \alpha}{\cos \beta} + h_B \frac{\cos \gamma}{\cos \beta}) \hat{j} + h_B \hat{k} \dots \quad (6)$$

$$|\bar{S}_T| = [\sqrt{h_A^2 + (h_A \frac{\cos \alpha}{\cos \beta} + h_B \frac{\cos \gamma}{\cos \beta})^2 + h_B^2}]^{1/2} \dots \quad (7)$$

In the Equations 4 to 7, all the variables are easily determined. h_A and h_B are measured interferometrically, while the angles α , β and γ are determined from the two-surface trace analysis. The angle χ is obtained by two circle optical goniometry. Thus all quantities required to calculate $\bar{S}_T$ are measurables.

Appendix B. Correction of Slip-Band Width Measured on a Surface Inclined to the Slip Plane

A schematic representation of slip traces on the surfaces of a deformed crystal is shown in Figure 25(a). W_m is the normal distance between the two traces measured on the surface of observation.

The relation between the surface normal, normal to the slip traces in the plane of observation, and the slip plane normal is shown in Figure 25(b). It is obvious that

$$W_r = W_m \cos \rho$$

where, W_r = slip band width along the direction of slip plane normal

ρ = angle between the slip plane normal and the normal to the slip trace in the place of observation.

Further, the slip plane normal ($\overline{SPN}$), the normal to the surface ($\hat{N}_A$), and the normal to the slip trace all lie on a common great circle. Therefore, from the geometry of Figure 24(b), it is seen that,

$$\rho = 90^\circ - \alpha$$

or $\cos \rho = \sin \alpha$

$$W_r = W_m \sin \alpha$$

A similar relation for surface B will be as follows:

$$W_{r_B} = W_{m_B} \sin \psi$$

where ψ = angle between slip plane normal and normal to surface B. Since the angle between the slip plane normal and the normal to the surface is easily obtained from the stereographic projection, the slip-band width is easily obtained using the above equations.

Fig. 25. Analysis of slip-band width measurements
 (a) Schematic representation of slip traces on the surfaces of a deformed crystal
 (b) The geometric relation between the slip plane normal, surface normal, and the normal to slip traces in the plane of observation