MASTER

5 GeV/c Pion-Nucleon Scattering at all Angles*

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Abstract: We fit all $\pi^{\pm}$ p elastic and CEX data at 5 GeV/c on the basis of a simple s-channel model.

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I. Introduction

This paper is a continuation of an investigation into particle scattering processes at all angles, based on a particular s-channel model¹ suggested some time ago by the authors. The model was originally designed to account for the three prominent features that occur in the differential cross-section of π^+p CEX, namely the dips at $t = 0$, $-.6 \text{ (GeV/c)}^2$ near the forward direction, and at $u = -.2 \text{ (GeV/c)}^2$ near the backward direction. By considering expansions of s-channel helicity amplitudes² in terms of d-functions, it was shown¹ how these three features could be correlated if most of the scattering in this process came from a dominant band of angular momentum states centered around $j_c + \frac{1}{2} = kb_c$, where k is the c.m. momentum. In order for the CEX dips to occur at these three places for all (intermediate) energies, the distance b_c had to be energy independent, and it was estimated to be about 1F.

As a first quantitative test of the model, we investigated³ the case of π^+p elastic scattering for momenta $p_{lab} = 2.74, 3.0, 3.5$ and 4.0 GeV/c . At these momenta, the differential cross-sections have been measured at all angles; polarizations have been measured near the forward and backward directions; the total cross-sections and the ratio of the real to the imaginary part of the non-flip amplitude at $t = 0$ are also known. Fitting all of these quantities at any one energy provides a stringent test of the model, and our results were consistent with the general qualitative picture previously outlined for CEX. We refer the reader to Ref. 3 for details. Here we remark only that the scattering was dominated by two main contributions:

- (a) Diffraction, due to the absorption of the lower partial waves.

It was shown that absorption was substantial (about 50%) for all partial

waves corresponding to impact parameter $b \leq 0.8F$, at which distance the absorption fell off rapidly to zero.

(b) Surface effects, at the edge of the interaction region. The angular momenta of these surface waves were indeed found to be centered around $j_c + \frac{1}{2} = kb_c$ with $b_c \approx 0.8F$ at all energies. The fact that b_c occurs at precisely the place where the absorption falls off clearly indicates the consistency of the model.

In the present paper, we extend the model further and examine simultaneously the three coupled reactions π^+p elastic, π^-p elastic and π^-p CEX at 5 GeV/c, since data have recently become available on these processes for scattering at all angles at approximately this energy. In the next section, Section II, we shall discuss our parametrisation, being guided in part by various Regge considerations. In Section III, we list the experimental data used. The actual fits to the data are presented in Section IV, as well as a discussion of how the experimental features are explained in this model. We end with a few brief remarks in Section V.

II. Parametrisation: suggestions from the Regge model

In our previous investigation³ of π^+p elastic scattering, we parametrised the low partial waves by Gaussians in j , and the peripheral band of angular momentum states by a single Breit-Wigner (BW) in j . The latter form is also a simple Regge pole in the angular momentum plane for the s-channel reaction; such a form arises⁴ naturally in all classical considerations of grazing rays. However, we found that a single Breit-Wigner becomes less adequate as the energy is increased. The reason for this lies in the fact that, with a single BW, the sharp valley at $u = -.2 \text{ (GeV/c)}^2$ near the backward direction demands the BW to be narrow; but this in turn leads to diffi-

culties in cancelling the resulting oscillations at intermediate angles in order to make the differential cross-sections small there.

A modified parametrisation is therefore adopted in the present paper, one that is suggested from considerations of the familiar Regge model (with duality). The latter model is of course applicable only at the extreme edges of the scattering region; however we try to extract some essential features and use them in our characterisation of scattering at all angles from an s-channel point of view.

Let us consider typical Regge terms and partial wave analyse them. For t-channel exchanges, we have

$$-s^{\alpha(t)} = \sum_{l=0}^{\infty} (2l+1) a_l(s) P_l(\cos \theta_s) \quad (1)$$

$$-e^{-i\pi\alpha(t)} s^{\alpha(t)} = \sum_{l=0}^{\infty} (2l+1) c_l(s) P_l(\cos \theta_s) \quad (2)$$

where

$$a_l = -s^{\alpha_0} e^{-y} (-i)^l j_l(iy)$$

$$c_l = -e^{-i\pi\alpha_0} s^{\alpha_0} e^{(ix-y)} (-i)^l j_l(x+iy)$$

and $\alpha(t) = \alpha_0 + \alpha' t$, $x = 2\pi\alpha' k^2$, $y = 2\alpha' k^2 \ln s$ and $j_l(z)$ is a spherical Bessel function. Now in the Veneziano model,⁵ the non-rotating Regge term $s^{\alpha(t)}$ is constructed from the t-, u-channel resonances, while the rotating part $e^{-i\pi\alpha(t)} s^{\alpha(t)}$ corresponds to s-, t channel resonances. Hence it is natural to associate⁶ the $s^{\alpha(t)}$ term with the real non-resonating background in the s channel. In fact, $(-i)^l j_l(iy)$ for fixed large argument, behaves⁷

very much like a Gaussian in l , stressing those partial waves with impact parameter $b = l/k \leq 0.8F$. The rotating Regge part corresponds to s-channel resonances and is naturally associated with an s-channel Breit-Wigner in angular momentum, like the one described above. (For a more detailed investigation of the connection between the Regge model (including cuts) and the present s-channel model, see Ref. 8.)

For u-channel exchanges, the expansions of $s^{\alpha(u)}$ and $e^{-i\pi\alpha(u)} s^{\alpha(u)}$ are similar, apart from an extra factor of $(-1)^l$. The importance of this latter factor becomes obvious when one remembers that in the backward direction the accompanying Legendre polynomials likewise alternate in sign with l . The $(-1)^l$ is therefore significant in building up structure in the backward direction. The presence of a $(-1)^l$ term brings about a distinction between even and odd partial waves, just like an exchange force in potential scattering, and leads to the breaking of "exchange degeneracy" in the s-channel.

With these Regge forms and their s-channel partial wave expansions in mind, we turn to our new parametrisation of the s-channel helicity non-flip and flip amplitudes.²

$$f_{++}^s = \frac{1}{k} \Sigma(j + \frac{1}{2}) T_{++}^j a_{\frac{1}{2}2}^j(\theta_s) \quad (3)$$

$$f_{+-}^s = \frac{1}{k} \Sigma(j + \frac{1}{2}) T_{+-}^j a_{-\frac{1}{2}2}^j(\theta_s) \quad (4)$$

The T-matrix angular momentum elements T_{++}^j , T_{+-}^j were taken in the following form, for each isospin state $I = \frac{3}{2}, \frac{1}{2}$:

$$T_{++}^j = F^R + iF^I + c_1 G_L + (c_2 + ic_3) G_{co} * BW \quad (5)$$

$$T_{+-}^j = c_4 G_L + (c_5 + i c_6) G_{co} * BW \quad (6)$$

We now explain the various pieces.

F^R and F^I are fermi distributions (suggested from our previous $\pi^+ p$ analysis)⁹

$$F = f_1 / [1 + \exp(\frac{b-f_2}{f_3})]$$

with $b = (j + \frac{1}{2})/k$. These represent the contribution from diffraction scattering (and hence are related to Pomeron $I^b = 0$ exchange), and are taken to be the same in both isospin states $I^S = \frac{3}{2}, \frac{1}{2}$. We would anticipate f_2 to be about 0.8F. (F^R, F^I : 6 parameters)

Each coefficient c_i is of the form $c = d \pm (-1)^L e$, a reflection of the expansions of t, u channel Regge terms. ($c_i^{3/2}, c_i^{1/2}$: 24 parameters)

G_L is a Gaussian

$$G_L = \exp(-g_L b^2)$$

and is the real low partial wave contribution corresponding to s^{α} . (As already mentioned, $(-1)^L j_L(iy)$ behaves like a Gaussian as a function of L .)

G_{co} is a smooth cut-off function

$$G_{co} = \exp[-g_{co}(b-b_c)^2]$$

for the Breit-Wigner s-channel Regge pole

$$BW = (\Gamma/2) / [k(b-b_c)^2 + \Gamma^2/4]$$

This represents the scattering from the edge of the interaction region

(or, alternatively, the rotating Regge part). G_L, G_{co} and BW are taken to be the same in both isospin states $I^S = \frac{3}{2}, \frac{1}{2}$ and it was found sufficient to take

$g_L = g_{co} = g$. (G_L, G_{co} and BW : 3 parameters).

Altogether there are 33 parameters to describe $\pi^+ p$ elastic, $\pi^- p$ elastic and $\pi^- p$ CEX at all angles. We have not attempted to obtain a fit with some minimum number of parameters - what we have done is to use a parametrisation whose pieces are clearly interpretable.

III. Data

The data used in the fitting were the following.

Differential cross sections ($d\sigma/dt$)

π^+p elastic. Brabson et al. (Indiana), Ref. 10.
 Chandler et al. (Indiana), Ref. 11.
 Rust et al. (Argonne-NAL-Michigan), Ref. 12.
 Coffin et al. (Michigan), Ref. 13.

The π^+p 180° point comes from Kormanyos et al. (Michigan-Argonne), Ref. 14.

π^-p CEX Yvert, Ref. 15.
 Chase et al. (Minnesota), Ref. 16.
 Brockett et al. (Case Western-Oberlin), Ref. 17;
 4.83 GeV/c

Polarisations

π^+p elastic Esterling et al. (Chicago-Argonne), Ref. 18;
 5.15 GeV/c.
 π^-p elastic Esterling et al. (Chicago-Argonne), Ref. 18;
 5.15 GeV/c.
 π^-p CEX Drobnis et al. (Argonne), Ref. 19.

All the amplitudes at $t = 0$ have been calculated by H6hler and Strauss (Ref. 20) from forward dispersion relations. At 5 GeV/c, they estimate

$$\sigma_{\text{tot}}(\pi^+p) = 26.61 \text{ mb},$$

$$\sigma_{\text{tot}}(\pi^-p) = 28.97 \text{ mb},$$

$$(\text{Re}/\text{Im})f_{++}(\pi^+p) = -.29 \text{ at } t = 0,$$

$$(\text{Re}/\text{Im})f_{++}(\pi^-p) = -.15 \text{ at } t = 0.$$

IV. Fits to the data; explanation of experimental features; structure of the amplitudes.

Using the parametrisation described in Section II, we obtained a fit to all the data available for pion-nucleon scattering at 5 GeV/c. The fits to the differential cross-sections at all angles $0 \geq t \geq -8.57 \text{ (GeV/c)}^2$ are shown in Figs. 1, 2, 3, the π^+p polarisations in Fig. 4, and the π^-p CEX polarisation in Fig. 5. Our values of the total cross-sections for $\pi^\pm p$ scattering were 27.2 and 29.6 mb respectively, and the ratios of the real to imaginary parts of the forward non-flip amplitude were -.29 and -.15. The corresponding values of the parameters are listed in Table I.

We now go on to discuss how the main experimental features come about in our fits. For the sake of clarity, we have drawn in Fig. 6, 7, 8 the s-channel helicity amplitudes at all angles for π^+p elastic, π^-p elastic and π^-p CEX scattering respectively. These are related to the s-channel isospin $I^s = \frac{3}{2}, \frac{1}{2}$ and the t-channel isospin $I^t = 0, 1$ amplitudes by

$$\begin{aligned} f(\pi^+p) &= r^{3/2} &= r^0 - r^1 \\ f(\pi^-p) &= \frac{1}{3}(r^{3/2} + 2r^{1/2}) &= r^0 + r^1 \\ f(\text{CEX}) &= \frac{\sqrt{2}}{3}(r^{3/2} - r^{1/2}) &= -\sqrt{2} r^1 \end{aligned}$$

Forward $\pi^\pm p$; $0 \geq t \geq -1.5 \text{ (GeV/c)}^2$

The forward elastic peak and subsequent shoulder are due primarily to diffraction, originating from the absorption of the lower partial waves. In Fig. 9, we have drawn the corresponding Fermi distributions F^R, F^I (which are the same for $I^s = \frac{3}{2}, \frac{1}{2}$ and hence are pure $I^t = 0$) as functions of the impact parameter b . From F^I , we see that the absorption is substantial for all partial waves with $b \leq 0.8F$, at which distance it falls off

quite rapidly to zero.

It should be noted²¹ that the imaginary part of f_{++}^s , which dominates in the forward peak itself, has a zero at $t = -.7 \text{ (GeV/c)}^2$, the resulting hole in $d\sigma/dt$ being filled up by the real part of f_{++}^s . This is quite different from the usual Regge parametrisation of diffraction where the Pomeron²² is taken to be structureless, i.e., no zeros. Also in Harari's recent "dual-absorptive" model,²³ Pomeron diffraction is taken to be structureless, the sudden change of slope in $d\sigma/dt$ at $t \approx -.8 \text{ (GeV/c)}^2$ being due in his model to interference with direct channel resonances.

This point as to whether the imaginary part of the non-flip amplitude has a diffraction zero at $t \approx -.7 \text{ (GeV/c)}^2$ or not is an important distinction between the two types of model. In particular, the explanation of the double zero of the π^+p polarisations at $t = -.6 \text{ (GeV/c)}^2$ (see below) is different.

Cross over at $t = -.2 \text{ (GeV/c)}^2$

The crossover point at $t = -.2 \text{ (GeV/c)}^2$ of the π^+p elastic differential cross-sections arises from the zero in the imaginary part (see Fig. 8a) of the s-channel helicity non-flip amplitude in the CEX reaction. Regge fits also have a zero here in this amplitude,²² but it is written in initially as a multiplicative factor in the parametrisation. In the present model, it is easy to see how it comes about naturally from the BW term in (5) and the property that $d_{11}^c(\theta_s) \approx J_0(b_c \sqrt{-t})$ has a zero at $t \approx -.2 \text{ (GeV/c)}^2$ for $b_c \approx 0.9F$.

CEX dips at $t = -.6 \text{ (GeV/c)}^2$ and $u = -.2 \text{ (GeV/c)}^2$

The qualitative correlation of these dips provided the original motivation¹ of the model, as discussed in the Introduction. Our present

fits support the original proposal, namely that they come about in a unified way primarily from the peripheral angular momentum band (parametrised here by a BW) in the flip amplitude. To illustrate this point further, we show in Fig. 10 the contribution of the s-channel helicity flip CEX BW, and in Fig. 11 the partial wave decomposition of this BW. The BW is centered around $b_0 \approx 0.9F$, and clearly gives rise to the experimental structures at $t = -.6$ and $u = -.2 \text{ (GeV/c)}^2$, as anticipated.

The $u = -.2 \text{ (GeV/c)}^2$ dip in backward π^+p elastic scattering originates in a similar way. In backward π^-p elastic scattering, the precise shape is not well determined, due to the lack of good data in this region.

Polarisation at $t = -.6 \text{ (GeV/c)}^2$

The dips in the elastic $\pi^\pm p$ polarisations around $t = -.6 \text{ (GeV/c)}^2$ suggest a double zero. In the present model, one zero comes from the vanishing of the imaginary part of the s-channel helicity non-flip amplitude (see Figs. 6a, 7a) at $t \approx -.7 \text{ (GeV/c)}^2$ (the same zero that produces the first diffraction zero in the elastic differential cross-sections). The other zero comes from a zero in the real part of the s-channel helicity flip amplitude (see Figs. 6b, 7b) at $t \approx -.5 \text{ (GeV/c)}^2$. The approximate mirror symmetry of the forward $\pi^\pm p$ polarisations comes from the fact that the real parts of the $\pi^\pm p$ flip amplitudes are of opposite sign (in Regge language, this corresponds to saying that the real part of this flip amplitude is dominated by the $I^t = 1$ exchange contribution).

We note that the explanation of these polarisation structures is different from the usual Regge explanation. There, the double zero comes from the real part of the signature term for ρ exchange (the Pomeron being structureless). In the present approach, it is more natural to have just

a single zero in the real part of the flip amplitude (arising from the oscillations of the $d_{-22}^1(\theta_s)$ function).

In our fit, the $\pi^- p$ CEX polarisation has a simple zero around $t = -.5 \text{ (GeV/c)}^2$. (Recent unpublished CERN polarisation measurements²⁴ at 5 GeV/c are inconclusive on this point, but their 8 GeV/c data are consistent with having only a single zero, somewhere between $t = -.5$ and -1.0 (GeV/c)^2 .)

V. Concluding Remarks

We have demonstrated that it is possible to obtain a reasonable fit to all the pion-nucleon scattering data at 5 GeV/c. We find that the data at all angles can be correlated on the basis of a straightforward s-channel picture,¹ that of scattering from an absorbing interaction region of radius about $0.8 - 0.9 F$ and at the edge of which there are strong edge effects.

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Table I

All parameters are in units of ($\sqrt{m_0}$ GeV), except f_2 , b_c which are in Fermis and $\Gamma/2$ is dimensionless.

	f_1	f_2	f_3
F^R	-0.21	0.63	0.80
F^I	0.33	0.75	0.71

BW	g	b_c	$\Gamma/2$
	0.21	0.93	3.0

Coefficients c	d_1	e_1	d_2	e_2	d_3	e_3	d_4	e_4	d_5	e_5	d_6	e_6
$I^S = \frac{3}{2}$	-0.101	-0.038	0.054	0.002	-0.026	-0.005	-0.001	-0.001	0.021	0.002	-0.025	0.008
$I^S = \frac{1}{2}$	-0.019	-0.006	0.084	0.007	0.049	-0.003	0.012	-0.012	-0.028	0.003	0.028	-0.003

Figure Captions

- Fig. 1: π^+p differential cross-section; see Section III for data.
- Fig. 2: π^-p differential cross-section; see Section III for data.
- Fig. 3: π^-p CEX differential cross-section; see Section III for data.
- Fig. 4: π^+p forward elastic polarisations; see Section III for data.
- Fig. 5: π^-p CEX forward polarisation; see Section III for data.
- Fig. 6: Real (---) and imaginary (—) parts of π^+p s channel (a) helicity non-flip, (b) helicity flip, amplitudes (in $\sqrt{mb}$). In the forward directions, the amplitudes have been reduced by the factors indicated.
- Fig. 7: Real (---) and imaginary (—) parts of π^-p s-channel (a) helicity non flip, (b) helicity flip, amplitudes (in $\sqrt{mb}$).
- Fig. 8: Real (---) and imaginary (—) parts of π^-p CEX s channel (a) helicity non-flip, (b) helicity flip, amplitudes (in $\sqrt{mb}$).
- Fig. 9: Partial wave composition (in $\sqrt{mb}$ GeV) of (a) F^R , (b) F^I against impact parameter b in Fermis.
- Fig. 10: Contribution of peripheral Breit Wigner in s-channel helicity flip amplitude to CEX differential cross-section.
- Fig. 11: Partial wave composition (in $\sqrt{mb}$ GeV) of (a) real part, (b) imaginary part, or the s-channel helicity flip BW in CEX, against impact parameter b (F).

Fig. 1

$\pi^+p \rightarrow \pi^+p$ (5 GeV/c)

Fig. 4 (Chen et al.)

Fig. 2

$\pi^- p \rightarrow \pi^- p$ (5 GeV/c)

Fig. 2 (Continued)

Fig. 5

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$\pi^- p \rightarrow \pi^0 n$ (5 GeV/c)

Fig. 3 (Chen and *et al.*)

Fig. 4 (Chen & Hensley)

Fig. 5. (Chu & Hardy)

Fig. 6(a) (Chen et al.)

Fig. 6 (b) (Ch. 4 Rodig)

Fig. 7(a) (chiral theory)

Fig. 7(b) (Chen model)

Fig. 8(a) * (Chen, Pandey)

Fig. 8(b) (Chau & ndhy)

Fig. 9 (Continued)

Fig. 10

Fig. 10 (Chiu & Hendry)

Fig. 11 (Chen & Hendry)