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FLUX DENSITIES IN
A TYPE-II SUPERCONDUCTING SLAB
IN AN INCREASING OR DECREASING
PARALLEL APPLIED MAGNETIC FIELD

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Date Transmitted: July 1975

PREPARED FOR THE U. S. ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION
UNDER CONTRACT W-7405-eng-82

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SLAB IN AN INCREASING OR DECREASING PARALLEL
APPLIED MAGNETIC FIELD*

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ABSTRACT

The time-dependent flux densities in a type-II superconducting slab in an increasing or decreasing parallel applied magnetic field are calculated, taking into account bulk and surface pinning currents as well as induced eddy currents.

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INTRODUCTION

One method for investigating the magnetic properties of a superconducting specimen is the measurement of the time-dependent voltage V in a pick-up coil wrapped around a superconducting slab, which is isothermally subjected to a time-dependent parallel applied magnetic field H_a . In this report we discuss the theoretically predicted voltage V vs H_a when H_a is swept up or down at a constant rate $\dot{H}_a$. This voltage is proportional to the slope of the hysteretic, rate-dependent $\bar{B}$ versus H_a curves. We use in this treatment the critical state formalism for flux flow discussed in Ref. 1.

THEORY

Shown in Fig. 1 is the geometry under consideration. A slab of thickness L_x , width L_y , and length L_z is subjected to a longitudinal applied field $\vec{H}_a = H_a \hat{z}$. We assume $L_x \ll L_y, L_z$. A coil wrapped around the cross section (area $L_x L_y$) picks up an induced EMF, $V = -c^{-1} \dot{\Phi}$, where $\dot{\Phi} = L_x L_y \dot{\bar{B}}$ and $\bar{B}$ is the volume average of the z component of the flux density $\vec{B} = B \hat{z}$. We ignore end effects and assume that $B = B(x, t)$ and $B(-x, t) = B(x, t)$. The fundamental equations governing the flux motion for arbitrary flux density are, from Eqs. (4), (9), and (24) of Ref. 1,

$$\partial H / \partial x = - (4 \pi / c) J, \quad (1)$$

$$\partial E / \partial x + c^{-1} \partial B / \partial t = 0. \quad (2)$$

$$\begin{aligned} E &= \rho_f (J - J_c), \quad J > J_c, \\ &= 0, \quad |J| < J_c, \\ &= \rho_f (J + J_c), \quad J < -J_c, \end{aligned} \quad (3)$$

Fig. 1. Sketch of the superconducting slab under consideration. Measured voltage V senses the time rate of change of flux through the cross section $L_x L_y$.

where H is the z component of the magnetic field $\vec{H}$ in the specimen parallel to $\vec{B}$, J is the y component of the coarse-grained current density $\vec{J}$, and E is the y -component of the effective electric field $\vec{E}$ or, more precisely, the gauge-invariant generalization of the electrochemical potential gradient. We assume the quantities B , H , J , and E are functions of only the variables x and t .

Equations (1) - (3) must be solved subject to the boundary condition

$$H(L_x/2, t) = H_a - (4\pi/c)K_{en} \quad (4)$$

when flux is entering through the surface at $x = L_x/2$, and

$$H(L_x/2, t) = H_a + (4\pi/c)K_{ex} \quad (5)$$

when flux is exiting through this surface. We can also express the surface currents in terms of field steps at the critical entry or exit condition,²

$$\Delta H_{cn} = (4\pi/c)K_{en}, \quad (6)$$

$$\Delta H_{ex} = (4\pi/c)K_{ex}, \quad (7)$$

assumed to be known functions of either the applied field at the surface H_a or the interior magnetic flux density near the surface of the superconductor $B(L_x/2, t)$.

The above equations must be solved using numerical methods for small flux densities where $B \ll H_{c1}$ and $H \sim H_{c1}$, because the connection

between B and H ,

$$B = B_{eq}(H), \quad (8)$$

$$H = H_{eq}(B), \quad (9)$$

is highly non-linear, and the field dependence of ρ_f , J_c , K_{en} , and K_{ex} must also be accounted for. However, in the present paper we consider the solutions of these equations sufficiently far from $H=H_{c1}$ that a linearized version of the above equations holds approximately. That is, we require that the deviation of B about its volume average $\bar{B}$ at all times to be small. We then make the following approximations:

$$\partial B / \partial t = \mu'(\bar{B}) \partial H / \partial t, \quad (10)$$

$$\rho_f = \rho_f(\bar{B}), \quad (11)$$

$$J_c = J_c(\bar{B}), \quad (12)$$

where

$$\begin{aligned} \mu'(B) &= dB/dH = dB_{eq}(H)/dH \\ &= [dH_{eq}(B)/dB]^{-1}. \end{aligned} \quad (13)$$

Note that $\mu' > 1$ for all values of B in the range $0 < B \leq H_{c2}$ and that $\mu' \rightarrow \infty$ as $B \rightarrow 0$. With the above approximations, flux motion into the specimen is governed by a diffusion equation, obtained by combining Eqs. (1) - (3),

$$\partial H / \partial t = D_f(\bar{B}) \partial^2 H / \partial x^2, \quad (14)$$

where

$$D_f(B) = c^2 \rho_f(B) / 4 \pi \mu'(B) \quad (15)$$

is the flux-flow magnetic diffusivity and is usually of the same order of magnitude as the normal-state magnetic diffusivity,

$$D_n = c^2 \rho_n / 4 \pi. \quad (16)$$

Note that $D_f \ll D_n$ when $B \ll H_{c1}$, where $\rho_f \ll \rho_n$ and $\mu' \gg 1$, and that $D_f < D_n$ at $B = H_{c2}$, where $\rho_f = \rho_n$ and $\mu'(H_{c2}) > 1$. Note also that, with the above approximations, B obeys the same diffusion equation as H , Eq. (14).

APPLIED FIELD CHANGING SLOWLY

If the applied field changes slowly enough, then $|E| \ll \rho_f J_c$, and the field distribution is given by the static critical state model. For $x > 0$, Eqs. (1) and (3) yield

$$\partial H / \partial x = \pm (4 \pi / c) J_c(\bar{B}), \quad (17)$$

where the upper (lower) sign holds for $\dot{H}_a > 0$ ($\dot{H}_a < 0$). Upon applying the boundary conditions, Eqs. (4) and (5), we obtain

$$H(x, t) = H(L_x/2, t) \pm (4 \pi / c) J_c(\bar{B}) (x - L_x/2) \quad (18)$$

and

$$\bar{H}(t) = H(L_x/2, t) \pm \pi L_x J_c(\bar{B})/c. \quad (19)$$

Defining

$$H_b^*(\bar{B}) = (4\pi/c) J_c(\bar{B}) (L_x/2)$$

as the difference in H that can be supported by bulk pinning between the center of the slab and the surface, we obtain

$$\bar{H}_+ = H_a - \Delta H_{en}(H_a) - H_b^*(\bar{B}_+)/2, \dot{H}_a > 0, \quad (21)$$

$$\bar{H}_- = H_a + \Delta H_{ex}(H_a) + H_b^*(\bar{B}_-)/2, \dot{H}_a < 0. \quad (22)$$

The graphical solution for $\bar{B}_+$ ($\bar{B}_-$) for the field increasing (decreasing) case, where $\dot{H}_a > 0$ ($\dot{H}_a < 0$), is outlined in Fig. 2. For a given applied field H_a , the average flux density $\bar{B}_+$ in the slab for increasing field is less than that ($\bar{B}_-$) for decreasing field.

The voltage V divided by $V_0 = -c^{-1} L_x L_y \dot{H}_a$, measured in the absence of the specimen, is

$$V/V_0 = \dot{\bar{B}}/\dot{H} = \mu'(\bar{B})\dot{\bar{H}}/\dot{H}_a. \quad (23)$$

If we regard $\bar{B}_+$ and $\bar{B}_-$ in Eqs. (21) and (22) as functions of H_a , then we may write

$$V_+/V_0 = \mu'(\bar{B}_+)\partial\bar{H}_+/\partial H_a, \quad \dot{H}_a > 0, \quad (24)$$

$$V_-/V_0 = \mu'(\bar{B}_-)\partial\bar{H}_-/\partial H_a, \quad \dot{H}_a < 0. \quad (25)$$

Fig. 2. Construction for the determination of the volume-averaged magnetic fields $\bar{H}$ and flux densities $\bar{B}$ at a given applied field H_a slowly increasing (subscripts +) or slowly decreasing (subscripts -). See Eqs. (21) and (22) of the text. H_{en} is the critical entry field, H_{eq} is the equilibrium field, and H_{ex} is the critical exit field.

If the magnitudes of $\partial \Delta H_{en}/\partial H_a$, $\partial \Delta H_{ex}/\partial H_a$, and $\partial H_b^*(\bar{B}_\pm)/\partial H_a$ are small by comparison with unity, then

$$\partial \bar{H}_+/\partial H_a \approx \partial \bar{H}_-/\partial H_a \approx 1, \quad (26)$$

and the value of $V_\pm/V_0$ at a given value of H_a is simply the slope of the B_{eq} vs H curve at the corresponding flux density $\bar{B}_\pm$.

More generally, however, we have

$$\begin{aligned} \partial \bar{H}_+/\partial H_a = & 1 - (4\pi/c)\partial K_{en}(H_a)/\partial H_a \\ & - (\pi L_x/c)\partial J_c(\bar{B}_+)/\partial H_a, \end{aligned} \quad (27)$$

$$\begin{aligned} \partial \bar{H}_-/\partial H_a = & 1 + (4\pi/c)\partial K_{ex}(H_a)/\partial H_a \\ & + (\pi L_x/c)\partial J_c(\bar{B}_-)/\partial H_a. \end{aligned} \quad (28)$$

Since we expect $K_{en}(H_a)$, $K_{ex}(H_a)$, and $J_c(\bar{B}_\pm)$ usually to be monotonically decreasing functions of H_a , we expect that

$$\partial \bar{H}_-/\partial H_a < 1 < \partial \bar{H}_+/\partial H_a. \quad (29)$$

Moreover, if the pinning currents in the specimen exhibit a peak effect near H_{c2} , then V_+/V_0 and V_-/V_0 should have corresponding structure near H_{c2} .

APPLIED FIELD CHANGING RAPIDLY

When $|\dot{H}_a|$ is sufficiently large, we must take into account the induced eddy currents. To estimate the effect of these currents upon the values of $\bar{B}_+$ and $\bar{B}_-$, we replace $\partial B/\partial t$ in Eq. (2) by $\mu'(\bar{B})\dot{H}_a$. Since $E=0$ at $x=0$, this immediately leads to

$$E(x,t) = -c^{-1}\mu'(\bar{B})\dot{H}_a x. \quad (30)$$

For $x > 0$, Eqs. (1) and (3) then yield

$$\partial H/\partial x = \pm (4\pi/c)J_c(\bar{B}) + \dot{H}_a x/D_f(\bar{B}), \quad (31)$$

from which we obtain

$$\begin{aligned} H(x,t) = & H(L_x/2,t) \pm H_b^*(\bar{B}) (x-L_x/2)/(L_x/2) \\ & + \dot{H}_a (x^2 - L_x^2/4)/2D_f(\bar{B}) \end{aligned} \quad (32)$$

and

$$\begin{aligned} \bar{H}(t) = & H(L_x/2,t) \pm H_b^*(\bar{B})/2 \\ & - \dot{H}_a L_x^2/12D_f(\bar{B}). \end{aligned} \quad (33)$$

The effect of the eddy currents is represented by a term

$$\Delta H_{ed}(\bar{B}) = |\dot{H}_a| L_x^2/12D_f(\bar{B}), \quad (34)$$

which can become very large for small $\bar{B}$. We obtain for $\dot{H}_a > 0$,

$$\begin{aligned}\bar{H}_+ &= H_a - \Delta H_{en}(H_a) - H_b^*(\bar{B}_+)/2 \\ &\quad - \Delta H_{ed}(\bar{B}_+),\end{aligned}\tag{35}$$

and for $\dot{H}_a < 0$,

$$\begin{aligned}\bar{H}_- &= H_a + \Delta H_{ex}(H_a) + H_b^*(\bar{B}_-)/2 \\ &\quad + \Delta H_{ed}(\bar{B}_-).\end{aligned}\tag{36}$$

The graphical determination of $\bar{B}_+$ and $\bar{B}_-$ is outlined in Fig. 3.

According to this treatment, the normalized induced voltages $V_{\pm}/V_0$ are given by Eqs. (24) and (25), where $\bar{B}_+$ and $\bar{B}_-$ are to be determined from Eqs. (35) and (36), and

$$\begin{aligned}\partial \bar{H}_+ / \partial H_a &= 1 - (4\pi/c) \partial K_{en}(H_a) / \partial H_a \\ &\quad - (\pi L_x/c) \partial J_c(\bar{B}_+) / \partial H_a \\ &\quad - \partial \Delta H_{ed}(\bar{B}_+) / \partial H_a,\end{aligned}\tag{37}$$

$$\begin{aligned}\partial \bar{H}_- / \partial H_a &= 1 + (4\pi/c) \partial K_{ex}(H_a) / \partial H_a \\ &\quad + (\pi L_x/c) \partial J_c(\bar{B}_-) / \partial H_a \\ &\quad + \partial \Delta H_{ed}(\bar{B}_-) / \partial H_a.\end{aligned}\tag{38}$$

Since $D_f(\bar{B})$ is a monotonically increasing function of $\bar{B}$, ΔH_{ed} is a monotonically decreasing function of H_a , and we expect that Eq. (29) holds also when the effect of the eddy currents is included.

Fig. 3. Construction for the determination of the volume-averaged magnetic fields $\bar{H}$ and flux densities $\bar{B}$ at a given applied field H_a to H_a rapidly increasing (subscripts +) or rapidly decreasing (subscripts -). The effect of the induced eddy currents is included in the term ΔH_{ed} . See Eqs. (35) and (36) of the text.

DISCUSSION

From Eqs. (24), (25) and (35-38) we see that the basic shape of plots of $V_{\pm}/V_0$ vs H_a is determined by the shape of μ' . In the absence of pinning and at extremely low field sweep rates we obtain

$$V_{\pm}/V_0 = \mu'(\bar{B}) = [dH_{eq}(\bar{B})/d\bar{B}]^{-1}, \quad (39)$$

sketched as curve a in Fig. 4, where $H_{eq}(\bar{B}) = H_a$ or $\bar{B} = B_{eq}(H_a)$. When pinning and eddy currents are present, however, there are several effects: First, the value of $\bar{B}$ at which μ' is to be evaluated is shifted to a lower (B_+) or higher (B_-) value for a given value of H_a , depending upon whether H_a is increasing or decreasing. (See Figs. 2 and 3.) Thus, the peak in $V_{\pm}/V_0$, which occurs for $\bar{B} \sim 0$, is shifted to higher values of H_a when H_a is increasing and to lower values of H_a when it is decreasing, as sketched by curves b and c in Fig. 4. Second, the amplitudes $\partial \bar{H}_{\pm}/\partial H_a$ depend upon whether H_a is increasing or decreasing and are sensitive to the derivatives of K_{en} , K_{ex} , J_c , and ΔH_{ed} with respect to H_a . We expect that $\partial \bar{H}_+/\partial H_a > 1$ and $\partial \bar{H}_-/\partial H_a < 1$, such that the peak in V_+/V_0 is enhanced and that in V_-/V_0 is suppressed. In addition, specimens showing a peak effect near H_{c2} should exhibit corresponding structure in $V_{\pm}/V_0$. Third, the linearization procedure breaks down when $\bar{B}$ is too close to zero and the structure in $V_{\pm}/V_0$ at $\bar{B} = 0$ predicted by Eqs. (24) and (25) is then smeared out and rounded, as indicated by the dashed portions of curves b and c in Fig. 4.

Fig. 4. Sketch of the theoretically expected behavior of V_+/V_0 (H_a increasing) and V_-/V_0 (H_a decreasing) versus H_a , (a) in the absence of pinning with a slow rate of change of H_a and (b) and (c) in the presence of pinning with a rapid rate of change of H_a .

We emphasize that $\bar{B}_+(H_a, \dot{H}_a)$ and $\bar{B}_-(H_a, \dot{H}_a)$ are the average flux densities in the slab in the field increasing and field decreasing cases ($|\dot{H}_a| = \text{constant}$) and that the signal $V_{\pm}/V_0$ simply measures the derivatives of these quantities with respect to H_a :

$$V_{\pm}/V_0 = \partial \bar{B}_{\pm}(H_a, \dot{H}_a) / \partial H_a. \quad (40)$$

The above theoretical treatment explains qualitatively several of the features of experimental values of $V_{\pm}/V_0$ vs H_a in a superconducting Nb specimen, shown in Fig. 3 of Ref. 3. An attempt now should be made to test the theory more stringently by applying the theory to these and similar experiments and determining whether not only qualitative but also quantitative agreement is obtained.

ACKNOWLEDGMENTS

The author is grateful to Dr. L. J. M. van de Klundert, Mr. E. A. Gijsbertse, and Dr. D. M. Kroeger for stimulating discussions and to Professor G. Eilenberger for his hospitality at the Institut für Festkörperforschung of the Kernforschungsanlage Jülich, where part of this work was carried out.

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