233
-10-75

Ac-1195

LA-5853-MS
Informal Report

UC-20
Reporting Date: January 1975
Issued: February 1975

MASTER

Toroidal Mode Structure in the Scyllac Full Torus

by

F. C. Jahoda
R. Kristal
W. E. Quinn

los alamos
scientific laboratory
of the University of California
LOS ALAMOS, NEW MEXICO 87544

An Affirmative Action/Equal Opportunity Employer

UNITED STATES
ENERGY RESEARCH AND
DEVELOPMENT ADMINISTRATION
CONTRACT W-7405-ENG. 36

DISTRIBUTION OF THIS DOCUMENT UNLIMITED

In the interest of prompt distribution, this LAMS report was not edited by the Technical Information staff.

Printed in the United States of America Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road
Springfield, VA 22151
Price: Printed Copy \$4.20 Microfiche \$2.25

This report was prepared as an account of work sponsored by the United States Government. Neither the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

TOROIDAL MODE STRUCTURE IN THE SCYLLAC FULL TORUS

by

F. C. Jahoda, R. Kristal, and W. E. Quinn

ABSTRACT

A 360° multiple-slit framing camera (11 viewing slits) has shown that the Scyllac toroidal mode structure is consistent in a statistical sense with the $m=1$ instability predictions of MHD sharp-boundary theory. The first few modes of finite wavelength around the torus have growth rates comparable to the $k=0$ mode, and the developed plasma mode structure approaches cut-off at the theoretical value given by magnetic field bending effects. The data indicate that the $m=1$ instability rather than loss of equilibrium terminates the plasma confinement.

I. INTRODUCTION

The Scyllac full torus experiment is described in the proceedings of the 5th IAEA Conference on Plasma Physics and Controlled Nuclear Fusion Research, Tokyo, November 11-15, 1974.^[1] Measurements of the toroidal mode structure as determined by six simultaneously acquired data points are reported in Ref. 1. The six points could only determine the amplitude of a constant displacement ($n=0$) and the amplitudes and phases of the next two harmonics ($n=1, 2$), where n is the axial mode number defined by the plasma displacement $\xi = \xi_0 e^{i(n\theta - nz/R)}$, with θ and z the azimuthal and toroidal coordinates, respectively, and R the torus major radius. (It is observed that only the $m=1$ mode develops in the minor toroidal cross section. This is attributed to the effects of finite Larmor radius stabilization.^[2]) Statistical tests of the least-squares best fit to the data showed that the plasma displacement must consist of more than the $n=0, 1$, and 2 harmonics.^[1] Randomness of phase between modes causes nonreproducibility of the plasma motion from discharge to discharge and requires $(2n+1)$ simultaneous viewing slits to measure n harmonics. We have subsequently measured 11 positions simultaneously, permitting determinations of up to $n=5$. The more definitive

mode structure measurements thus achieved are the subject of this report.

II. THEORY

The theoretical growth rate, γ , of the $m=1$ instability is given in sharp boundary theory by^[3]

$$\gamma^2(n) = \gamma^2(0) - V_A^2 (2 - \beta) \frac{n^2}{R^2} \quad (1)$$

$$\gamma^2(0) = h^2 V_A^2 \left[-\beta \left(\frac{a}{b} \right)^4 + \frac{\beta(4 - 3\beta)(2 - \beta)}{8(1 - \beta)} h^2 a^2 \delta_1^2 + \frac{\beta(3 - 2\beta)(1 - \beta)}{(2 - \beta)} \delta_0^2 \right] \quad (2)$$

where V_A is the Alfvén velocity, a is the plasma radius, b is the coil radius, h is the wavenumber of the applied $\ell=1$ and $\ell=0$ correction fields, $B_\ell \sim \sin(\ell\theta - hz)$, that establish equilibrium in the toroidal configuration, δ_1 and δ_0 are the corresponding first-order perturbations of the plasma surface $r = a[1 + \delta_1 \cos(\theta - hz) - \delta_0 \cos hz]$, and β is the ratio of plasma pressure to the confining magnetic field pressure.

For the Scyllac parameters^[1] $\gamma(0) = 0.7$ MHz and Eq. (1) first becomes negative (the condition for stability) at $n=7$. Furthermore, due to the parabolic dependence on n , $\gamma(n)/\gamma(0)$ is still 0.9 for $n=3$ and only decreases rapidly near the limiting n value.

III. EXPERIMENTAL ARRANGEMENT

The experimental arrangement is indicated in Fig. 1. A single image converter framing camera located at the geometrical center of the torus has 11 horizontal viewing slits, almost uniformly spread in angle over 360° , imaged on its photocathode. The light from each slit is relayed by two plane mirrors, one directly above the slit, and the second a member of a multi-facet pyramid structure also centered on the toroidal axis, to a Cassegrainian telescope ($f/1.3$, focal length 41 cm), which images the slits at 10:1 reduction in an approximately circular array onto the 2.5-cm diameter photocathode.

In order to measure the plasma location within each slit, the slit ends had to be well defined in the image. This was accomplished by a separate long time exposure (in the focusing mode of the camera) of reflecting tape on the slit ends illuminated by individual flood lamps.

An example of the raw data is shown on the left of Fig. 2. The circular arrangement is the somewhat arbitrary result of the alignment procedure, and

while it evokes a mental image of the toroidal geometry, in actual fact, the images are inverted, so that the ends nearest the center of the image are

Fig. 2. Framing camera photograph (left) and radial plasma displacement vs. toroidal angle with least-square best fit curve using four harmonics (right).

actually toward the outside, and the large aspect ratio of the torus is completely distorted.

IV. DATA ANALYSIS

Numerical values of the plasma displacement from the discharge tube axis at each angular position are obtained with the Graf-Pen digitizer under the control of a computer program which then immediately proceeds to a curve fitting routine. The analytical form of the fitting function is chosen to be

$$d = a_0 + \sum_{l=1}^n (a_l \cos l\phi + b_l \sin l\phi). \quad (3)$$

The inputs are the 11 values of displacement d at the 11 different angular positions ϕ and for a specified $n < 5$ the $(2n+1)$ a and b coefficients are determined in a least-square sense with $2(5-n)$ degrees of freedom. The results are displayed in terms of a mode amplitude, $(a_l + b_l)^{1/2}$, and a mode phase angle, $\tan^{-1}(b_l/a_l)$. Also computed is the weighted variance of the fit, i.e., the square root of the sum of the squares of the deviations of the data points from the curve divided by the number of degrees of freedom. Figure 3 illustrates a succession of curves with an increasing number of modes, n , fitted to the data of one particular discharge. Over a collection of 15 such discharges, the weighted variance is a minimum either at $n=3$ or at $n=4$, with almost the same number of times for each.

Fig. 1. 360° framing camera arrangement on Scyllac. The plasma is viewed at 11 different circumferential locations in the toroidal plane.

This means that three modes fit the data pretty well, with the 4th harmonic generally of smaller amplitude aiding the fit only a small amount. On the other hand, using only 2 harmonics, the maximum possible with the earlier data from only 6 positions, the fit is definitely worse. It should also be noted that using 5 harmonics forces an exact fit to the 11 data points, and is statistically meaningless since the curve is forced to pass precisely through the data points, which themselves are determined with less than absolute precision. It is significant, nonetheless, that when 5 harmonics are fitted, the amplitude of the 5th on average is only slightly larger than the 4th, and smaller than the first three, in support of the conclusion that there is a rapid fall-off in amplitude with increasing mode number.

As a consequence of the above results, a fitting procedure with all mode numbers $n=0$ to $n=4$ was selected for the best representation of the data. The tabulated chi-square distribution for 2 degrees of freedom fits the distribution obtained from the sums of squares of the deviations if the experimental accuracy σ is chosen to be 0.15 cm. Repeated readings of the data indicate that this relatively large uncertainty is not unreasonable. The phase angles of the modes appear randomly distributed, and the relative amplitudes also vary from discharge to discharge. In consequence, the plasma column assumes a different shape every discharge, as illustrated by comparing the 4 harmonics case in Fig. 3 with the right side of Fig. 1. This is just what would be expected for a superposition of several

Fig. 3. Radial plasma displacement vs. toroidal angle for one discharge successively fitted with an increasing number of harmonic terms.

instability modes, and all further inferences must be of a statistical nature.

The amplitudes of the various modes, $n=0$ through $n=4$, were averaged over 15 discharges and are plotted in the lower half of Fig. 4. To compare with theory, we use the calculated $n=0$ growth rate of 0.7 MHz and the relative growth rates for the other n numbers, as indicated by the upper half of Fig. 4, as well as the actual time, $t = 4.6 \mu s$, when data were obtained. The relative displacements of the various modes are then calculated on the "white noise" assumption of equal initial amplitudes. The absolute scale value is determined by fitting to the observed data point for $n=1$. This sets the coefficient of the growing exponential $x = x_0 e^{1t}$ at $x_0 = 0.023$ cm. The resultant calculated displacements are plotted as crosses in the lower half of Fig. 4, and connected by the solid line.

The scale is not fitted to $n=0$ because it is a somewhat special case which is only a constant shift term in position vs. angle and can be confused with small errors in setting up the correct equilibrium conditions due, for instance, to slightly wrong initial fill pressure. There should in fact be an initial 6-mm inward equilibrium shift since all the viewing slits were in groove locations, and using this or some other value instead of the discharge

Fig. 4. Spectrum of theoretical growth rates (upper) and average displacement vs. mode number (lower).

tube center would slightly change the observed value for $n=0$.

V. CONCLUSIONS

The data presented are entirely consistent with sharp boundary calculations for the expected $m=1$ instability modes. A priori, instability might be expected to develop with equal likelihood in the vertical plane. Streak camera pictures, however indicate much less vertical than horizontal motion. This could well be explained by slight force imbalances in the horizontal plane providing the initial perturbation from which the instability grows primarily in the toroidal plane. These could arise from small differences in the main magnetic field around the torus. We must then distinguish instability motion resulting from an initial perturbation due to force imbalance from the total motion being the result of the longer term action of this force imbalance.

Simple arguments can be invoked to show that a force imbalance of only 2% can establish the initial perturbation of 0.02 cm in about 0.4 μ s, i.e., a time short compared to when the position determination is made, so that the instability calculation can actually be applied from essentially $t=0$. On the other hand, the same 2% force imbalance continuing to act in the absence of instability would give a trajectory which is too slow to explain the observed limit on plasma confinement. This reinforces our interpretation of the data.

This argument breaks down if the force imbalance is assumed appreciably larger, and the actual value that might have existed is unknown. A significant experiment would have been to measure at two instants of time during the same discharge, to determine whether the separate n modes grow at different rates. Neither the available light level nor the exigencies of scheduling permitted such a measurement. It must also be realized that the plasma parameters are continually changing in time, especially up to peak compression, and we have used a very simple model independent even of the observed axial flow over the first few microseconds. With these qualifications in mind, the $m=1$ MHD instability theory is the most likely interpretation of the observations.

ACKNOWLEDGMENTS

We gratefully acknowledge our indebtedness to the entire Scyllac operating crew and especially thank F. L. Ribe and R. E. Siemon for clarifying discussions.

REFERENCES

1. E. L. Cantrell, W. R. Ellis, H. W. Harris, F. C. Jahoda, R. Kristal, M. D. Machalek, J. R. McConnell, W. E. Quinn, F. L. Ribe, G. A. Sawyer, F. T. Seibel, and R. E. Siemon, Paper CN-33/E 1-2, Proc. 5th Int. Conf., Tokyo, (1974) IAEA (to be published).
2. J. P. Freidberg, Phys. Fluids 15, (1972) 1102.
3. F. L. Ribe and M. N. Rosenbluth, Phys. Fluids 13, (1970) 2572.