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DIGITAL CALCULATIONS OF MAGNETIC FIELDS OWING TO CURRENTS
IN Nb_3Sn RIBBON IN SUPERCONDUCTING QUADRUPOLE LENS

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After the model work on superconducting quadrupole lenses, reported previously,¹ it seemed clear that the construction of full-scale superconducting quadrupole lens systems would not be far in the future and that more detailed calculations of the magnetic fields in such cylindrical quadrupole lenses would serve as a guide and aid in the design of practical, full-scale lens systems. This is especially pertinent since any practical lens will be of finite length, the current sheets will not be infinitely thin, and the current distribution can only be made to approximate $I = I_0 \cos(2\theta)$ by appropriate spacing of the superconducting strip around the circumference of the cylindrical quadrupole.

The full-scale lens, known as Model 4-24, and thought to be one of an identical pair which will make a lens system, will be 4 in. (10 cm) in inside diameter and 24 in. (60 cm) long. This means that the effective length of the current conductor is considered to be 24 inches long. In addition to this first layer of one-half inch wide current conductor, calculations also have been made for a second layer, wound on top of the first layer and having

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1. P. Gerald Kruger, W.B. Sampson, and R.B. Britton, "A Superconducting Quadrupole Lens," BNL Accelerator Dept. Internal Report AADD-104, March 1966.

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an inside diameter of 5.25 in. A combination of the results of the calculations for the first and second layers will give the magnetic field characteristics for such a two-layer superconducting quadrupole lens. Details of the number of turns of one-half inch wide ribbon per $\Delta\theta$ interval are given in Table I. These represent the current distribution, found by digital calculation, to give the closest approximation to $I = I_0 \cos (2\theta)$, for the total number of turns involved. The number of turns in the first 10° , $\Delta\theta$ interval is deduced by dividing the arc for a $\Delta\theta = 10^\circ$ by the nominal strip thickness, namely 0.004. The approximate number of turns per $\Delta\theta$ for other $\Delta\theta$'s is proportional to $\Delta [\sin (2\theta)]$. Thereafter the computations must decide what is the "best" distribution.

TABLE I

Current Distribution in Model 4-24 Superconducting Quadrupole

The number of turns of $\frac{1}{2}$ in. wide Nb_3Sn are given as a function of θ and $\Delta\theta$.

θ	$\Delta\theta$ (degrees)	Number of turns per $\Delta\theta$, first layer	Number of turns per $\Delta\theta$, second layer
10	10	100	131
20	10	88	115
30	10	65	85
40	10	35	45
45	5	0	0
50	5	0	0
60	10	35	45
70	10	65	85
80	10	88	115
90	10	<u>100</u>	<u>131</u>
Total		576	752

The digital computing program calculates the magnetic field components, B_R and B_θ , as well as the total magnetic field at a point P, which is at

some constant radius r , as shown in Fig. 1 and as a function of the azimuthal angle, θ . The magnetic field components are given by:

$$B_R = k\rho (\theta_2 - \theta_1)$$

$$B_\theta = k\rho \ln (r_2/r_1)$$

where $k = \sin \beta_1 + \sin \beta_2$ and β is the angle usually specified in the derivation of the field owing to a long straight wire and resulting in the equation,

$$B = \frac{i}{r_1} (\sin \beta_1 + \sin \beta_2)$$

ρ is the current (in emu/cm) in the strip, along the width w . The other quantities are as depicted in Fig. 1. B_θ is perpendicular to the plane of the current strip and B_R is parallel to the plane of the current strip.

The computer calculates the components B_θ and B_R at the point P owing to each current strip (e.g., 2304 strips in the first layer) around the circumference of the quadrupole, takes the vector sum of these components and finally calculates the resultant magnetic field.

Assumptions or conditions which are implicit or explicit to the calculations are:

a) The current strip is 24 in. long and no end effects are included in the calculation. The finite length is taken into account.

b) The one-half inch width of the current strip is radial and is positioned uniformly in any particular interval $\Delta\theta = 10^\circ$ so that each current strip is separated by an angle interval equal to 10° divided by the number of strips in that angle interval.

c) The current distribution in the strip is uniform.

Figure 2 gives the results of the calculation of B , the magnetic field is in kilogauss per 100 ampere current in the strip, vs. r , the radial distance (inches) from the axis of the quadrupole. The data are given at $\theta = 45^\circ$, which from Fig. 1 is seen to be a pole of the quadrupole. Z is the distance along the axis of the quadrupole, measured from the center of the quadrupole, and for these data $Z = 0$.

Fig. 1. Calculating parameters for superconducting quadrupole.

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Fig. 2. Magnetic field vs. radial distance for Model 4-24 quadrupole.

From these data it is seen that dB/dr is constant to less than 0.07% for $r < 2$ in. This should provide good lens quality.

Also these data give $dB/dr = 7.4$ kG/cm for the first layer only and $dB/dr = 10$ kG/cm for the two layers, assuming $I = 500$ amperes for the two layers and $I = 660$ amperes for one layer alone as seems reasonable from tests previously made¹ with a model lens.

Assuming a conservative $dB/dr = 7.0$ kG/cm for a one layer lens, the length of the lens 60 cm, the distance between the ends of two identical lenses in a lens system 25 cm, and particles of momentum 30 BeV/c, then the approximate focal length of such a system would be about 8.7 meters. However, if each lens had two layers of Nb_3Sn the focal length of the lens system would be 4.3 meters.

Data from Fig. 2 also allow one to make an estimate of the energy stored in the magnetic field of the system. For one full-scale lens with 660 amperes in one layer of Nb_3Sn this is found to be about 35 kilojoules, so that a two-lens system would have about 70 kilojoules of stored energy. Correspondingly, a full-scale two-lens system with each lens having two layers of Nb_3Sn and 500 amperes, the total stored energy would be about 220 kilojoules. This is not a large amount of stored energy but also it is not trivial.

It may also be noted from Fig. 2 that 8 inches from the axis of the lens the magnetic field is down to 130 gauss per 100 ampere current in the coil. Thus at 660 amperes the fringing field at this point would be 860 gauss, which in some cases may be ignored and in any case would not present too difficult a shielding problem.

It is of interest to calculate B vs. r at various values of Z so that one can see how dB/dr varies with Z . Such data are presented in Figs. 3 and 4 with $\theta = 45^\circ$ for all such calculations. This value of θ is chosen because it corresponds to a pole where no current strips exist. Note that different current values are given for the two-layer case. This is due to the Nb_3Sn critical current limitation. For a one-layer superquadrupole it is expected that 660 amperes may be used in the G.E. one-half inch strip. But because of the larger field obtained when two layers are used, only about 500 amperes will be useful in a two-layer lens. Figure 4 gives a curve of the total B vs. Z for the single-layer case.

Fig. 3. Field gradient vs. axial distance for Model 4-24 quadrupole.

Fig. 4. Magnetic field vs. axial distance for Model 4-24 quadrupole.

Figure 5 shows the calculated variation of B with θ , the azimuthal angle measured about the long axis of the quadrupole (see Fig. 1). It is noted that the variation of B from the average value for the first layer is about $\pm 0.05\%$, whereas for the second layer it is only about $\pm 0.03\%$. This is owing to the fact that for the first layer $r = 1.5$ in. is 75% of the radial distance to the coil winding, whereas for the second layer it is only 57% of the distance. For both layers the variation is less than for the first layer, owing to some smoothing for one layer by the other. In any case, the variations are small enough to provide adequate optics for such a lens system. In fact, $\pm 0.2\%$ variation may be considered adequate for many purposes.

Initial design studies, which are intended to lead to construction, are now in progress for a Model 4-24 two-lens system, with one layer of one-half inch wide strip for each lens. As now planned, the geometric aperture will be 4 in. inside diameter, the insulation being provided by a good vacuum in the 4 in. I.D. pipe on which the quadrupole coils will be mounted. Cold, thin windows will provide entrance and exit ports for 30-BeV protons.

G.K. Green has suggested that larger inside diameter lenses may be considered in the future for collecting and focusing secondary particles in beam handling systems. Accordingly, we will take a "look into the future" here.

To do this, consider a Model 10-40, i.e., a lens 40 in. long and 10 in. inside diameter. At present this is thought to provide an 8 in. clear I.D., leaving $\Delta r = 1$ in. for super insulation so that the lens could be used in air at room temperature.

Table II gives a proposed turn distribution for a Model 10-40 lens. Digital calculations for one layer of one-half inch ribbon indicate that B (at $r = 4$ in.) = 3.6195 kG per 100 amperes so that $dB/dr = 0.356$ kG/cm per 100 amperes.

Recent measurements on the critical current of a 0.090 in. wide straight sample of RCA developmental type R60-261 ribbon give the critical current as 375 amperes at 80 kG. If one extrapolates these data to a one-half inch wide strip one gets a critical current of about 2000 amperes. This would predict $dB/dr = 7.1$ kG/cm for the Model 10-40.

Fig. 5. Magnetic field variations vs. azimuthal angle for Model 4-24 quadrupole.

TABLE II

Proposed Turn Distribution for a Model 10-40 Lens (Future)

One layer of RCA No. R60-261 ribbon assumed $\frac{1}{2}$ in. wide and 0.005 in. thick.

$\Delta\theta$ (degrees)	Number of turns per $\Delta\theta$
10	174
10	153
10	114
10	60
5	0
5	0
10	60
10	114
10	153
<u>10</u>	<u>174</u>
90	1002

Accordingly, for two identical lenses separated by 25 cm and comprising a simple lens system, the focal length of the system for 20 BeV/c secondary particles would be about one meter. This would make a useful component for a future beam handling system.

The optical properties also would be good. This is seen from Fig. 6, where $B(\text{kG})$ vs. θ is presented for calculated values of B at $r \cong 10$ cm (4 in.). The variation of B with θ is about $\pm 0.05\%$.

On the basis of these speculations for the future, one lens of the type of Model 10-40 would require about 16,000 feet of one-half inch wide ribbon which, if the cost were \$1.50 per foot, would cost about \$25,000 for the Nb_3Sn for one lens. This plus the engineering costs make such a system seem feasible, one to three years in the future.

Fig. 6. Angular magnetic field variations for Model 10-40 superconducting quadrupole.