EIKONAL MECHANISMS AT HIGH ENERGY[†]

MASTER

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Eikonal Mechanisms at High Energy

These remarks are concerned with two essentially distinct high energy processes -- small angle elastic and wide angle inelastic events in proton-proton scattering -- which may in part be given a common eikonal description, within a field theoretic framework. The main elastic scattering topic covered here is the nature of possible corrections to the Cheng-Wu¹ and Chang-Yan² total cross section estimate $\sigma_T \sim \ln^2 s$, of some relevance in view of the most recent ISR measurements³ which suggest $\sigma_T \sim \text{const.}$ over the range $450 \leq s \leq 2500$ (Gev)². The second topic deals with a prediction of correlations expected in pion production which proceeds by the bremsstrahlung of massive neutral vector mesons (NVM), in an incipient NAL experiment,⁴ with large, measured momentum transfers to the protons. No pretense at mathematical rigor is made; rather, these eikonal descriptions will develop on an intuitive (and possibly correct) physical basis.)

I. Elastic Scattering at Small Angles.

Eikonal descriptions of high energy, close-to-forward pp scattering have been developed by many authors,⁵ who have summed sets of Feynman graphs corresponding to the exchange of mesons between a pair of distinguishable nucleons, as in Figure 1. When the exchanged mesons are NVM, the eikonal approximation has been shown to generate the correct asymptotic scattering amplitude, in the sense that a perturbation expansion of the eikonal amplitude agrees with the corresponding expansion of the leading s-dependence in every order of perturbation theory.⁵ This is not true for simple scalar-meson exchange, and it is not known if the property is in fact true for the more complicated "tower graph" exchanges of massive-photon QED² and φ^3 theory,³ as pictured in

Figures 2 and 3. The validity of the eikonal formalism is here taken as an input assumption, and an attempt is made to assess its predictions. It should be remarked that the Feynman graphs of Figures 2 and 3, as well as all the subsequent, more complicated eikonal graphs to follow, contain a sum over permutations of the ways in which "vertical" exchanged meson lines are attached to the scattering nucleons.

The non-spin-flip, near-forward amplitude has the two-dimensional, impact parameter representation in the limit $s \rightarrow \infty$, $t/s \rightarrow 0$,

$$T(s, t) = \frac{is}{2m^2} \int d^2b e^{i\vec{q} \cdot \vec{b}} [1 - e^{i\chi(s, b)}] \quad (1)$$

where $-t = +\vec{q}^2$, and kinematical factors are defined so that

$\sigma_T \simeq \frac{4M^2}{s} \text{Im} T(s, t=0)$, with M representing the nucleon mass, and $s \gg 4M^2$. The simplest eikonal function corresponding to NVM exchange only (mass m and coupling constant to nucleon g) is given by the s -independent expression⁵

$$i\chi_1 = -\frac{ig^2}{2\pi} K_0(mb) \quad (2)$$

leading to a constant σ_T . Summing the leading $\ln s$ dependence of the tower graphs of Figure 2 leads to

$$i\chi_2 \simeq -a_1 s^{a_2} e^{-a_3 b} \quad (3)$$

with the a_i appropriate constants, while the s -channel iteration of the tower graphs of Figure 3 generates

$$i\chi'_2 \simeq -a'_1 \frac{s^{a'_2}}{\ln s} e^{-a'_2 b^2 / \ln s} \quad (4)$$

with constants a_1' . Both (3) and (4) lead to strong absorption within an impact parameter $b_{\max} \sim \ln S$, and hence to $\sigma_T \sim \ln^2 S$. More complicated graphs containing higher n -point connected, t -channel amplitudes, such as those of Figures 4 and 5, have not been considered in these analyses; and it is the purpose of the model estimates which follow to suggest possible consequences of remedying that omission.

For technical reasons (ease of calculation) it is convenient to employ a hybrid theory, containing features of both massive-photon QED and φ^3 theory: one introduces a nucleon (ψ), massive NVM (W_μ), and scalar "pion" (π) fields, the latter two of respective masses m and μ , with interaction Lagrangian

$$\mathcal{L}' = ig \bar{\psi} \cdot \sum_\mu \gamma_\mu W_\mu \cdot \psi - \frac{\lambda}{2} \pi \cdot \sum_\mu W_\mu^2. \quad (5)$$

With the aid of straight-forward functional techniques, one may obtain an exact, formal statement⁶ for the eikonal of this problem, corresponding to the exchange of all possible ("vertical") NVM between a pair of scattering nucleons, together with the exchange of all possible scalar ("horizontal") pions between the virtual NVM; it is

$$e^{i\chi} = \exp \left[-\frac{i}{2} \int \frac{\delta}{\delta \pi} D_c \frac{\delta}{\delta \pi} \right] \cdot \exp \left[ig^2 \int \mathcal{T}_I^\mu \cdot \bar{\Delta}_c[\pi] \cdot \mathcal{T}_{II}^\mu \right] \Big|_{\pi=0}, \quad (6)$$

where $\mathcal{T}_{I,II}^\mu(u) = p_{1,2}^\mu \int_{-\infty}^{+\infty} d\xi \delta(u - z_{1,2} + \xi p_{1,2})$; $\bar{\Delta}_c[\pi] = \Delta_c(1 + \lambda \pi \Delta_c)^{-1}$

denotes the relativistic, potential-theory NVM propagator in the presence of a fictitious, c -number scalar field $\pi(u)$; $z_{1,2}$ are configuration-space nucleon coordinates; and $\vec{b} = (\vec{z}_1 - \vec{z}_2)_T$ in the C.M. An alternate expression for

$i\chi + 1$ may be given as the sum of all connected graphs contained in the right-hand-side of (6). Expanding in powers of g^2 , $i\chi = \sum_{n=1}^{\infty} i\chi_n$, $i\chi_n$ denotes that contribution arising from the connected, $nW \rightarrow nW$ t -channel amplitude, as in Figure 6,

$$i\chi_n = \frac{(ig^2)^n}{n!} \int \mathcal{F}_I^{\mu_1}(u_1) \dots \mathcal{F}_I^{\mu_n}(u_n) M_{n,n}^{(c)}(u_1, \dots, u_n, q, \dots, q_n) \mathcal{F}_I^{\mu_1}(v_1) \dots \mathcal{F}_I^{\mu_n}(v_n) \quad (7)$$

and

$$M_{n,n}^{(c)} = \left(\exp \left[-\frac{i}{2} \int \frac{d^4k}{(2\pi)^4} D_c \left(\frac{k}{2} \right) \right] \bar{\Delta}_c(u_1, v_1 | \pi) \dots \bar{\Delta}_c(u_n, v_n | \pi) \right) \Big|_{\pi \rightarrow 0} \Big|_{\text{conn}}, \quad (8)$$

where "connected" here refers to different NVM lines joined by at least one pion line. The final approximation to be made, in (8), is the neglect of all pion self-linkages on the same NVM line.

It is then not difficult to see that the $n = 1$ term reproduces the $i\chi_1$ of (2), while $i\chi_2$ alone produces the analogue of (4). This latter statement may be derived by retaining the leading $\ln s$ dependence of every perturbative contribution to $i\chi_2$, assuming that the sum of these terms is the most important part of the $n = 2$ eikonal. One finds that the leading $\ln s$ dependence comes from pure ladder graphs with ordered pion rapidities, such that for m horizontal pion rungs, there appears the amount $\frac{1}{m!} \ln^m \left(\frac{s}{\mu^2} \right)$. The coefficient of this s -dependence can easily be picked off, and the entire m -pion, $n = 2$ contribution is

$$-\frac{g^4}{2!} (2\pi)^{-2} \int \frac{d^2q}{q^2 + m^2} e^{i\vec{q} \cdot \vec{b}} (2\pi)^{-2} \int \frac{d^2q}{q^2 + m^2} e^{i\vec{q} \cdot \vec{b}} \frac{1}{m!} \left[\frac{\lambda^2}{8\pi} \bar{\alpha}(\vec{q} + \vec{q}_1) \ln \left(\frac{s}{\mu^2} \right) \right]^m, \quad (9)$$

where $\bar{\alpha}(q) = (2\pi)^{-2} \int d^2Q [Q^2 + m^2]^{-1} [(Q-q)^2 + m^2]^{-1} = \frac{1}{4\pi} \int_0^1 dx [m^2 + x(1-x)q^2]^{-1}$.

The $m = 0$ term represents the second s -channel iteration of $i\chi_1$, $\frac{1}{2} (i\chi_1)^2$, and does not enter into $i\chi_2$.

$$i\chi_2(s, b) = -2 \left(\frac{g^2}{4\pi} \right)^2 \int d^2q e^{i\vec{q} \cdot \vec{b}} \bar{\alpha}(q) \left[\left(\frac{s}{\mu^2} \right)^{\frac{\lambda^2}{8\pi} \bar{\alpha}(q)} - 1 \right] \quad (10)$$

which, for large s and b , takes the form of (4), thereby re-deriving the Cheng-Wu/Chang-Yan tower graph results.

The intrinsic beauty of the tower graph summations should perhaps be stressed: A multiperipheral calculation of the sum of simple ladder graphs (no s-channel iteration) generates a Regge-like amplitude

$$T(s,t) \sim i \left(\frac{g^2}{2m^2} \right)^2 \bar{\alpha}(g^2) \left(\frac{s}{\mu^2} \right)^{1+\frac{\lambda^2}{8\pi} \bar{\alpha}(g^2)} \quad (11)$$

and a corresponding

$$\sigma_T \sim \left(\frac{s}{\mu^2} \right)^{\frac{\lambda^2}{8\pi} \bar{\alpha}(0)} \quad (12)$$

The trouble with (12), in this field theoretic context, is that λ^2 may be taken arbitrarily large, and the Froissart bound $\sigma_T \lesssim \ln^2 s$ exceeded. Including s-channel iterations, the eikonal model of (10), (4) and (1) produces a result which just saturates the Froissart bound.

There remains the decidedly non-trivial problem of estimating all the remaining $i\chi_n, n \geq 3$. The same sort of perturbation analysis may be attempted, and although one finds the maximum $\ln s$ contributions arising from ordered pion rapidities (apparently generating a factor $\frac{1}{m!} \ln^m \left(\frac{s}{\mu^2} \right)$ for a total of m pions arbitrarily distributed between any number of NVM), the coefficient of this s-dependence is not easily obtained. It is at this point and for this reason that one resorts to a model of such virtual pion processes, in order to be able to assign a b-dependent coefficient to $I_m(s)$. It is simplest to illustrate the particular procedure adopted here by first estimating the $n = 3$ term in the expansion of (6) (including disconnected graphs), rather than $i\chi_3$ itself,

$$\frac{(ig^2)^3}{3!} \sum_{m=0}^{\infty} \frac{1}{m!} \left[-\frac{i}{2} \left(\frac{g}{\mu} \right)^2 D_c \frac{g}{\mu} \right]^m \left(\int \mathcal{F}_I^{\mu} \Delta_c(\pi) \mathcal{F}_{II}^{\nu} \right)^3 \bigg|_{\pi \rightarrow 0} \quad (13)$$

The analysis is carried through by assuming the same $\frac{1}{m!} \ln^m(s/\mu^2)$ factor for any distribution of m horizontal pion lines with ordered rapidities, and a coefficient $\bar{\alpha}_{ij} \equiv \bar{\alpha}(q_i + q_j)$ when a pion links the i th and j th NVM lines. This generates the contribution

$$\frac{(ig^2)^3}{3!} (2\pi)^{-2} \int \frac{d^2 q_1 e^{i\vec{q}_1 \cdot \vec{b}}}{q_1^2 + m^2} (2\pi)^{-2} \int \frac{d^2 q_2 e^{i\vec{q}_2 \cdot \vec{b}}}{q_2^2 + m^2} (2\pi)^{-2} \int \frac{d^2 q_3 e^{i\vec{q}_3 \cdot \vec{b}}}{q_3^2 + m^2} \frac{m!}{n_{12}! n_{23}! n_{13}!} \frac{1}{m!} \left[\frac{\lambda^2}{8\pi} \ln\left(\frac{s}{\mu^2}\right) \right]^m (\bar{\alpha}_{12})^{n_{12}} (\bar{\alpha}_{23})^{n_{23}} (\bar{\alpha}_{13})^{n_{13}} \quad (14)$$

and summing over all $m = n_{12} + n_{23} + n_{13}$ yields

$$\frac{(ig^2)^3}{3!} \left[\prod_{l=1}^3 (2\pi)^{-2} \int \frac{d^2 q_l e^{i\vec{q}_l \cdot \vec{b}}}{q_l^2 + m^2} \right] \left(\frac{s}{\mu^2} \right)^{\frac{\lambda^2}{8\pi}} [\bar{\alpha}_{12} + \bar{\alpha}_{23} + \bar{\alpha}_{13}] \quad (15)$$

corresponding to Regge behavior between each pair of NVM lines. Generalizing the model to n vertical NVM lines produces

$$e^{i\chi} - 1 \rightarrow \sum_{n=1}^{\infty} \frac{(ig^2)^n}{n!} \left[\prod_{l=1}^n (2\pi)^{-2} \int \frac{d^2 q_l e^{i\vec{q}_l \cdot \vec{b}}}{q_l^2 + m^2} \right] \left(\frac{s}{\mu^2} \right)^{\frac{\lambda^2}{8\pi}} \sum_{i < j=1}^n \bar{\alpha}_{ij} \quad (16)$$

a result which contains two features of importance: (a) the i^n factor, providing cancellations between successive terms of the same reality character, and (b) a rapidly growing s -dependence, which suggests that such cancellations may be important. As an example, the further replacement of each $\bar{\alpha}_{ij}$ by $\bar{\alpha}(0)$ generates

$$\sigma_T \rightarrow -\frac{4\pi}{m^2} \text{Re} \sum_{n=2}^{\infty} \frac{C_n}{n!} \left(\frac{ig^2}{4\pi} \right)^n \left(\frac{s}{\mu^2} \right)^{\frac{\alpha_0}{2} n(n-1)} \quad (17)$$

where $\alpha_0 \equiv \frac{\lambda^2}{8\pi} \bar{\alpha}(0)$, and $C_n = \int_0^\infty x dx [2 K_0(x)]^n$.

Equation (17) displays a highly divergent sum, for which one must supply a

proper definition of summability. One such definition produces⁶

$\sigma_T \sim s^{-\alpha_0/8} (\ln s)^{-1/2} \rightarrow 0$, a result not necessarily to be taken seriously, but indicative of the strong cancellations which may be generated by the higher, connected graphs of $i\chi$.

Another way of seeing such cancellation is with the aid of a simple but non-trivial generalization of the recent ABS model,⁷ in which unitary amplitudes are constructed by combinations of the fundamental Feynman graph of Fig. 7. With $G \equiv g^2/4\pi$, $\Lambda \equiv \lambda^2/4\pi$, $\gamma \equiv \ln(s/\mu^2)$, in the context of the interaction (5), these generate an eikonal

$$i\chi_{ABS} = - \frac{\gamma G^2 \Lambda}{(2\pi)^2} \int d^2u \int d^2v \delta(\frac{1}{2} - u - v) K_0^2(mu) K_0^2(mv), \quad (18)$$

a result which may be found by retaining only the linear π -dependence of $\bar{\Delta}_c[\pi]$, and then easily performing the indicated functional differentiation of (6). [One again requires the identification

$$\int dk_0 \int dk_\perp [k_\perp^2 + k_0^2 + \mu^2 - i\epsilon]^{-1} \rightarrow i\pi\gamma,$$

for this integration over the pion $k_{(\pm)} \equiv k_\perp \pm k_0$ variables.] The connected Feynman graph contribution to $i\chi_{ABS}$ is pictured in Fig. 8.

An immediate generalization of this picture is obtained by including, in the construction of the eikonal, all Feynman graphs of form exhibited in Fig. 9, in addition to those of Fig. 7. Arithmetically, this corresponds to retaining up-to-quadratic π -dependence of $\bar{\Delta}_c[\pi]$. The functional operations of (6) are somewhat more complicated, but may still be carried through,⁸ leading to an eikonal⁹ expressed in terms of quadratures over the solution of a linear integral equation,

$$i\chi = i\chi_1 + i\chi'_{ABS} + i\chi_D,$$

where

$$i\chi'_{ABS} = \frac{i}{2} \int f(u) \bar{D}_c(u, v; 1) f(v) \quad (19)$$

$$i\chi_D = \frac{i}{2} \int_0^1 \xi d\xi \int B(u, v) D_c(v-u') B(u', v') \bar{D}_c(v', u; \xi) \quad (20)$$

with

$$\bar{D}_c(x, y; \xi) = D_c(x-y) + \xi \int D_c(x-u) B(u, v) \bar{D}_c(v, y; \xi) \quad (21)$$

and $f(u)$, $B(u, v)$ the respective coefficients of the linear and quadratic $\pi(z)$ dependence of $\bar{\Delta}_c[\pi]$. Here, $i\chi'_{ABS}$ represents the sum of all connected eikonal graphs containing a pair of NVM lines with one-virtual-pion exchange ("f-lines"), and arbitrary numbers of NVM lines with two-pion exchanges ("B-lines"). Graphs with more than one pair of f-lines are disconnected, and cannot appear in $i\chi$. Similarly, $i\chi_D$ represents the sum of all connected graphs involving B-lines only. That graph representing a single B-line corresponds to a radiative correction to $i\chi_1$, and has been omitted from (20).

A most convenient algebraic simplification occurs if one passes to the limit of very large NVM mass between pion emission on any B-line ($\Delta_c(u-v) \rightarrow \frac{1}{m^2} \delta(u-v)$). The price of this simplification is the necessity of introducing a pion transverse-momentum cutoff κ ($\delta^{(2)}(0) \rightarrow \kappa^2$), understood to be limited by m , $\kappa \lesssim m$. But the iterative solutions of (19) - (21) may then be found, and re-summed to give

$$i\chi'_{ABS} = - \frac{YG^2\Lambda}{(2\pi)^2} \int d^2u \int d^2v \delta(\vec{b}-\vec{u}-\vec{v}) K_0^2(mu) K_0^2(mv) \cdot \left[1 + i \frac{Y\Lambda G}{\pi m^2} K_0(mu) K_0(mv) \right]^{-1} \quad (22)$$

and

$$i\chi_D = -\kappa^2 \int d^2u \int d^2v \delta(\vec{b}-\vec{u}-\vec{v}) \left\{ \ln \left[1 + i \frac{Y\Lambda G}{\pi m^2} K_0(mu) K_0(mv) \right] - i \frac{Y\Lambda G}{\pi m^2} K_0(mu) K_0(mv) \right\}. \quad (23)$$

Passing to the limit of very large Y in (22) and (23), one sees how $i\chi'_{ABS}$ becomes independent of Y (and Λ), while the original Y -dependence of $i\chi_{ABS}$ has in effect been transferred to the B-line, double-pion emissions, $i\chi_D$. One has here an example of how the higher, n -point-connected, cross-channel amplitudes generate terms which are, at least, of the same degree of importance as $i\chi_{ABS}$. Incidentally, while the limit of very large NVM mass between pion emissions is not terribly physical, it does permit an exact resolution⁹ of the functional operations of (6), and leads to a constant total cross section.

In summary, within the context of an eikonal picture of high-energy forward scattering, there appears to be no reason to expect higher, cross-channel connected amplitudes to be unimportant. Quite the contrary: if these models have any truth, once one begins to estimate the $i\chi_n$, one cannot stop. The $n = 2$ term generates the Froissart bound, while contributions of higher n provide cancellations away from that upper bound.

II. Wide-Angle Inelastic Scattering.

In contrast to the complicated forward scattering situation, it is possible to apply an intuitive argument at large proton momentum transfers, describing pion production via the decay of ρ_0 or ω NVM which bremsstrahlung from the scattered protons. The hope that such a relatively simple picture may be relevant to high multiplicity events is enhanced by the success of Gaisser's picture¹⁰ of wide-angle pp scattering, in which a simple Born approximation for this process is strongly damped by multiple, virtual, soft NVM exchange between

the external proton legs. By unitarity, the damping of this elastic amplitude is a reflection of NVM bremsstrahlung, with the vector mesons quickly decaying into the observed pions. It is known¹¹ that such a mechanism cannot describe the relatively larger number of pion emissions involving low transverse momentum, $p_{\perp}$, to the protons; and indeed, one only expects such an effect -- which falls as a power¹² of $p_{\perp}^2$ -- to become dominant at larger $p_{\perp}^2 \gtrsim 3(\text{GeV})^2$, when the exponentially decreasing pionization processes are no longer significant.

Predictions of "differential multiplicity" (the number of pions produced as a function of measured proton momenta) have been given¹³ in this model of wide-angle bremsstrahlung, but -- as with pionization models -- the real tests of a particular production mechanism are measurements of its n-particle inclusive correlations. For this purpose it is convenient to employ recent "grand canonical partition function" techniques,¹⁴ which may be traced back to earlier work of Van Hove.¹⁵ The distributions discussed here describe NVMs, not pions directly, and may be obtained from an imminent NAL experiment⁴ in the following manner.

From the array of final charged particle tracks in a typical pp bubble chamber picture, one selects those with two outgoing protons only,¹⁶ corresponding to a fixed set of (six) invariants constructed from the proton 4-momenta. One pairs each of the charged pion tracks with another of opposite charge, determining in this manner which pions have originated in the decay of a ρ_0 . Those pions which cannot be so identified, arise from ω (or other resonance) decay, or from multiperipheral mechanisms. One must also neglect finite-width corrections of the NVM-pion decay, in order to avoid introducing correlations between decay pions of like charge. If the bremsstrahlung mechanism is valid, one expects the fractional number of such inferred ρ_0 to

increase with nucleon momentum transfer. Each pair of so-labeled pions defines a parent p_0 of appropriate 4-momentum k ; and a sufficient number of such events permits the introduction of one- and two-particle inclusive distribution functions, $\rho(k)$ and $\rho(k_1, k_2)$, constructed at fixed nucleon momenta.

A "total" cross section (two-proton inclusive distribution) summed over all emitted mesons, for fixed final nucleon momenta, is given by

$$\tau(p, p') = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^3k_1 \dots \int d^3k_n |\Gamma(p_1, p_2; p'_1, p'_2, k_1, \dots, k_n)|^2 \delta^{(4)}(Q - \sum_i k_i) \quad (24)$$

where $d^3k = d^3k/\omega$ and $Q = p_1 + p_2 - p'_1 - p'_2$. A class of "limited correlation" models may be defined by the replacement

$$|\Gamma(p_1, p_2; p'_1, p'_2, k_1, \dots, k_n)|^2 \rightarrow |\Gamma_{cl}(p_1, p_2; p'_1, p'_2)|^2 f(k_1) \dots f(k_n), \quad (25)$$

where $f(k)$ depends upon p_1, p_2, p'_1, p'_2 in some model-dependent way; e.g., de Groot¹⁴ uses (in the CM) $f = f(k_{\perp}, k_{\parallel}/p_0)$, while the bremsstrahlung model may be defined by $f = \frac{g^2}{(2\pi)^3} \left(\frac{p_1}{k \cdot p_1} + \frac{p_2}{k \cdot p_2} - \frac{p'_1}{k \cdot p'_1} - \frac{p'_2}{k \cdot p'_2} \right)^2$. The "limited" correlations of such models are those due only to energy-momentum conservation.

Inclusive meson distributions are most conveniently written in terms of a "grand partition function",

$$C(Q) = (2\pi)^{-4} \int d^4z e^{iQ \cdot z} \exp \left[\int d^3k f(k) e^{-ik \cdot z} \right] \quad (26)$$

when the implicit p_1, p_2, p'_1, p'_2 dependence of $f(k)$ is also understood to be present in $C(Q)$. For fixed nucleon momenta, and therefore fixed Q , one has

$$\frac{1}{\tau} \frac{d\tau}{d^3k} \equiv \rho(k) = f(k) C(Q-k) / C(Q) \quad (27)$$

and

$$\frac{1}{c} \frac{d^2 \tau}{dk_1 dk_2} \equiv \rho(k_1, k_2) = f(k_1) f(k_2) c(Q - k_1 - k_2) / c(Q). \quad (28)$$

Using (27) to eliminate the $f(k)$ dependence of (28), there results

$$\frac{\rho(k_1, k_2)}{\rho(k_1) \rho(k_2)} = \frac{c(Q) c(Q - k_1 - k_2)}{c(Q - k_1) c(Q - k_2)} \quad (29)$$

which is the simplest sort of correlation function to measure.

For soft NVM bremsstrahlung, and in the limit of high multiplicities, it is possible to approximate the ratio of (29) and obtain¹²

$$\frac{\rho(k_1, k_2)}{\rho(k_1) \rho(k_2)} \approx \exp \left[-k_1^\mu \psi_{\mu\nu} k_2^\nu \right] \quad (30)$$

where $\psi_{\mu\nu}$ is a symmetric matrix depending only upon measured, fixed nuclear coordinates. Equation (30) may be most simply tested by observing those ρ_0 of small transverse momentum, $k_\perp \sim 0$, where upon (30) reduces to

$$\exp \left[-a k_1 k_2 + \frac{1}{2} b (k_1 \omega_1 + k_2 \omega_1) - c \omega_1 \omega_2 \right], \quad (31)$$

where the k_i of (31) refer to longitudinal ρ_0 momenta, and a, b, c denote specific functions of nucleon momenta.

It will be most interesting to analyse these forthcoming NAL experiments with Eqs. (30) and (31) in mind. If the Gaisser mechanism¹⁷ is really correct for large nucleon momentum transfers, this correlation ratio should be satisfied. One hopes the experiments will provide a definitive answer in the very near future.

References

1. H. Cheng and T. T. Wu, Phys. Rev. Letters 24, 1456 (1970).
2. S-J. Chang and T-M. Yan, Phys. Rev. Letters 25, 1586 (1970).
3. P. Strolin, "ISR Results on pp Total Cross Sections and Elastic Scattering", presented at the Fourth International Conference on High Energy Collisions, Oxford Univ., England, April 1972.
4. A. Shapiro, private communication concerning a scheduled bubble chamber and hybrid system NAL experiment.
5. For example, M. Lévy and J. Sucher, Phys. Rev. 186, 1656 (1969); G. Tiktopoulos and S. Treiman, Phys. Rev. D2, 805 (1970); E. Eichten and R. Jackiw, Phys. Rev. D4, 439 (1971).
6. R. Blankenbecler and H. M. Fried. "Eikonal Estimates and Cancellations at High Energies", SLAC preprint, 1972.
7. R. Aviv, R. L. Sugar and R. Blankenbecler, "Unitary Models for Multiparticle Amplitudes", SLAC preprint, 1972.
8. H. M. Fried, "Functional Methods and Models in Quantum Field Theory", The M.I.T. Press, Cambridge, 1972.
9. H. M. Fried, "Eikonal Cancellations in a Soluble Model", Division de Physique Théorique (Tour 16, Paris) preprint, 1972.
10. T. K. Gaisser, Phys. Rev. D2, 1337 (1970).
11. H. M. Fried and T. K. Gaisser, Phys. Rev. D (in press).
12. H. M. Fried and T. K. Gaisser, "Tests in Hadronic Bremsstrahlung Production at Large Nucleon Momentum Transfer", Brown preprint, 1972.
13. H. M. Fried and T. K. Gaisser, Phys. Rev. D4, 3330 (1971).
14. A. Bassetto, M. Toller and L. Sertorio, Nucl. Phys. B34, 1 (1971);
E. H. de Groot, "The Grand Partition Function in the Uncorrelated Jet Model", Oxford preprint, 1972.

15. L. VanHove, Rev. Mod. Phys. 36, 655 (1964).

16. Extraction of this information from the data entails for more complicated procedures than suggested by the cavalier treatment presented here.

Typically one expects to see one slow and one fast proton in the midst of a pion spray. If the slow proton has momentum $\lesssim 1.5$ Gev it can be identified as a proton, while the fast proton is frequently assumed to be the most energetic of the outgoing cluster of charged particles. This discussion assumes that there will be an arbitrarily large amount of data, from which the small fraction of processes at large nucleon momentum transfer are selected and used to construct ρ_0 distributions.

17. Until now, Gaisser's picture of wide-angle pp scattering has been the only one available using two parameters -- determined from other experiments -- to fit the s, t dependence of $d\sigma/dt$ over many orders of magnitude.

Recently, a parton/quark analysis has been given by J. F. Gunion, S. J. Brodsky and R. Blankenbiller, "Composite Theory of Large Angle Scattering and New Tests of Parton Concepts", SLAC preprint, 1972, which depends upon a single parameter, and produces fits to the data virtually indistinguishable from Gaisser's. Correlation tests of inelastic amplitudes could serve to distinguish between them.

Figure Captions

- Fig. 1. Meson exchange between nucleons.
- Fig. 2. Closed-fermion-loop exchange.
- Fig. 3. Tower-graph exchanges in φ^3 theory.
- Fig. 4. Graphs not included in QED tower-graph sums.
- Fig. 5. Graphs not included in φ^3 -theory tower-graph sums.
- Fig. 6. Cross-channel connected amplitude contribution to $i\chi_n$.
- Fig. 7. Basic Feynman graph of the ABS model.
- Fig. 8. The eikonal graph of the ABS model.
- Fig. 9. Additional basic graph of the generalized ABS model.

FIGURES