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DYNAMIC BEAM CLEANING BY A NONLINEAR RESONANCE

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ABSTRACT

The general framework for the dynamic cleaning of a stored proton beam by passing the beam through a nonlinear resonance is developed. The limitations and advantages of this technique are discussed. The method is contrasted with physical beam scraping, which is currently in use at the CERN ISR.

N O T I C E

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I. Introduction

By the action of random scatterings, nonlinear effects, and like processes, real particle beam distributions tend to develop transverse tails or halos. The consequence of this is a comparatively higher level of radiation than would be anticipated from a more localized beam distribution. In the case of storage rings, the halo contributes to higher background levels for experiments, resulting in a decreased statistical accuracy of results. The beam cleaning process is also a process of beam shaping. That is, removal of a vertical halo is equivalent in meaning to a decrease in the vertical beam size. In the case of proton storage rings in which the beams are designed to cross horizontally, the luminosity is inversely proportional to the beam height, but independent of the beam width. Thus, in a current limited situation, one could decrease the vertical size and stack to a larger width, thereby increasing the luminosity without exceeding the current limit. Here, we will not distinguish between the processes of cleaning and shaping. The use of the terms cleaning and halo removal should be taken to include the general process of beam shaping.

The removal of the distribution halo with beam scrapers is an accomplished technique at the ISR.[1] One difficulty here is that a horizontal betatron halo cannot be physically scraped because of the superimposed momentum distribution which is much larger. This may not be a severe limitation in the ISR since the main advantages, i.e., luminosity and background improvements, come from vertical betatron cleaning and from removing the physically present horizontal halo. Physical beam scraping has apparently been successful, although there are certain aspects which are not understood, thus throwing some doubt on its universal application. For example, the application time of the scraper is not firmly established and cleaning is therefore somewhat of an art. In this regard, one wonders whether the scraper might influence the central core under some circumstances and what the role of the central core density might be.

We propose here a different technique of removing beam halo in a stacked coasting beam. By passing a beam through a nonlinear resonance, with appropriate choice of crossing speed and nonlinear detuning, particles with large

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1. P. Bryant, K. Hubner, K. Johnsen, H. Laeger, B. Montague, D. Neet, F.W. Schneider and S. Turner, IEEE Trans. Nucl. Sci. NS-20, p. 799 (1973).

betatron emittances lock into the resonance,[2,3] and are drawn to the physical aperture limit leaving a central betatron core. Vertical beam cleaning can be accomplished this way. In addition, the horizontal beam emittance can be reduced without decreasing the momentum spread. This latter might be useful in a storage ring design with vertical crossing at a zero dispersion collision point. The reduction of horizontal emittance also reduces the danger of "long time" coupling effects in a stacked beam of small vertical size. Although the process is not clearly understood, there is some question as to whether large differences between horizontal and vertical transverse emittances can be maintained in a beam stored for long periods of time.

Direct horizontal betatron cleaning is complicated by the fact that the density distribution is a combination of overlapping betatron and momentum components. Since the desire is, as indicated above, to clean the betatron component and not the momentum component, physical scraping must take place at a point of zero momentum dispersion. But this is a point of high beam density and so an inappropriate place to insert a scraper. Whereas, dynamic resonance cleaning would be ideal here since halo particles could be separated from the beam. Absorbers could be located at a position far removed from the central core and collect the halo.

In general, then,dynamic beam cleaning has the advantage of removing the betatron halo of a beam in both the horizontal and vertical transverse dimensions, and doing so independently. Furthermore, since no scrapers are used, there is no physical contact with the beam core, with absorbers collecting the outgoing halo at a safe distance.

The theory governing the trapping of particles during resonance passage has been previously developed.[3] We will use this theory to determine the characteristic resonance parameters required to accomplish the cleaning: The resonance excitation width, the nonlinear detuning and the rate of tune change in crossing the resonance. Numerical computations will then be used to optimize the cleaning process. Taking the 5th order resonance, the hardware requirements are a set of decapole magnets with the requisite azimuthal harmonic to excite the resonance, a set of octupoles for the purpose of nonlinear

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2. See, for example, A. Schoch, CERN Report, CERN 57-23 (1958).

3. A.W. Chao and M. Month, BNL Report, CRISP 74-9 (1974).

detuning and one or more quadrupole magnets to produce a time dependent ν shift.

The strengths of the required magnets provide no practical difficulty in general. The tune shift can be accomplished with a single quadrupole although more might be used for symmetry reasons. The nonlinear detuning provided by octupoles introduces a problem. Because the momentum spread in the beam will be comparatively large due to the momentum stacking of current in p-p storage rings consisting of coasting beams, the octupoles could have the undesired effect of distorting the ν_x-ν_y working line. From an estimate of the octupole strength given in Section II, we conclude that to make dynamic beam cleaning work, the octupoles should be placed at "zero" dispersion points. If placed at such points, the octupoles will produce the required nonlinear detuning and have no influence on the working line, the latter being proportional to the square of the dispersion. Another difficulty relates to the fact that decapoles will excite more than the desired one-dimensional resonance. Connected with this is the required size in the ν_y-ν_x plane of the beam working line (in order to maintain transverse stability of the intense proton beam). To obtain high currents in the ISR, the beam is sometimes stacked across the fan of 5th order resonances right up to the third order resonances along the ν_x-ν_y coupling line. Now, in order to clean dynamically, we must have two beam positions in the ν_x-ν_y diagram, one on either side of the resonance. Thus, it is difficult to see how such a "large" beam (in ν space) could be accommodated. Secondly, stacking a beam along the coupling line means that a simple decapole arrangement will inevitably excite more than one resonance. For example, to clean vertically, we would use a pth azimuthal harmonic to excite the $5\nu_y = p$ resonance. A single such decapole could also excite the $3\nu_y + 2\nu_x = p$ and $\nu_y + 4\nu_x = p$ resonances. If we are restricted to being near the coupling line, all three resonances cannot be avoided. The first of these difficulties is fundamental in that there is only a limited "$\Delta\nu$" that can be tolerated. For dynamic beam cleaning we need twice the tune size needed for beam stability. Thus, for storage rings such as the ISR, operating at high intensities, this technique may not be applicable. However, consider the example of the BNL superconducting design.[4] Here, vacuum and other considerations lead to a much lower current limit than is used at the ISR. At high energy, the requirement on the tune spread is much less

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4. "A Proposal for a Proton-Proton Intersecting Storage Accelerator Project - ISABELLE", H. Hahn and M. Plotkin, Editors, BNL, May 1974.

and the resonance cleaning method could become an attractive scheme. The second difficulty mentioned above can be avoided by introducing a tune split between horizontal and vertical tunes. Such a split actually arises naturally when designing low-β insertions for p-p storage rings.[4] So consider a case where $\nu_y - \nu_x = q$, i.e. stack along the main diagonal, but where the tunes are separated by some integer, q. Then introduce a pth harmonic decapole to excite the $5\nu_y = p$ resonance for vertical cleaning. Now, the harmonics which excite the $3\nu_y + 2\nu_x$ and $\nu_y + 4\nu_x$ resonances intersecting the main diagonal are $p \pm 2q$ and $p \pm 4q$, respectively. Thus, we can isolate the cleaning resonance by an appropriate selection of azimuthal harmonics for the decapole distribution. Of course, if $\Delta\nu$ is sufficiently small, we can move off the main diagonal, where isolation can be achieved as the resonance lines far out. The same reasoning applies to horizontal cleaning with the $5\nu_x = p$ resonance, excited by decapoles skewed with respect to the vertical cleaning decapoles by 18 degrees.

II. Beam Cleaning

To demonstrate the beam cleaning, we consider a beam distribution, D, with an undesired tail of large amplitude particles. This distribution is superimposed on a well-behaved Gaussian beam distribution. For clarity, we take a simple uniform distribution for D:

$$D(\alpha^{\frac{1}{2}})\, d\alpha^{\frac{1}{2}} = \begin{cases} 0.4 & \text{if } \alpha^{\frac{1}{2}} < 2.5 \\ 0 & \text{if } \alpha^{\frac{1}{2}} > 2.5 \end{cases} ,$$

with α = the relative emittance of the particle being considered, while

$$\int_0^\infty D(\alpha^{\frac{1}{2}})\, d\alpha^{\frac{1}{2}} = 1 \quad .$$

Our goal is to remove from D particles with large $\alpha^{\frac{1}{2}}$, say $\alpha^{\frac{1}{2}} \geq \sqrt{2}$, as cleanly as we can.

To achieve the cleaning, we make use of a mechanism involving a high-order nonlinear resonance. Octupole and decapole magnets are inserted at proper positions. A quadrupole magnet with variable strength is introduced to provide a time changing tune. Before cleaning, the tune is set slightly below some fifth order resonance value $p/5$, then it is increased to cross this resonance at some speed ϵ = the change of tune per revolution. At the moment when the tune crosses the resonance value, five stable islands are produced at the origin of the polar coordinate phase space $(\alpha^{\frac{1}{2}}, \Psi)$ and start to migrate toward larger amplitudes. Particles which are trapped by these islands are then carried to an amplitude outside the original betatron distribution where an absorber removes them.

The dynamical theory of the process just described is treated in Ref. 3. In the following, we summarize the results useful for beam cleaning:

1. Only particles with amplitudes $\alpha^{\frac{1}{2}} \geq \alpha_o^{\frac{1}{2}} = (\epsilon/4\pi\Delta_e\Delta_{NL})^{1/5}$ will be trapped, where more detailed definitions for the nonlinear tune shift Δ_{NL} and the excitation width Δ_e are given in Ref. 3.

2. The fraction of particles with $\alpha \geq \alpha_o$ that are trapped is proportional to $(\Delta_e/\Delta_{NL})^{\frac{1}{2}}$.

3. Those particles having $\alpha^{\frac{1}{2}} \geq \alpha_o^{\frac{1}{2}}$, but which are not trapped, will collapse to smaller amplitudes to fill in the vacancies left over by the trapped particles. This is done in accordance with the Liouville theorem.

4. The ratio Δ_e/Δ_{NL} cannot be made too large (corresponding to say, $\varkappa = 5\Delta_{NL}/4\Delta_e \leq 10$) in order to prevent an early loss of particles.

The above-mentioned properties (1) through (4) make the idea of using a high-order nonlinear resonance to clean the beam feasible and attractive. It can be deduced that in order to obtain good cleaning, one requires that:

(i) $\alpha_o^{\frac{1}{2}}$ should be close to $\sqrt{2}$ so that particles with $\alpha^{\frac{1}{2}} \leq \sqrt{2}$ are kept and particles with $\alpha^{\frac{1}{2}} \geq \sqrt{2}$ are removed;

(ii) Δ_e/Δ_{NL} be as large as possible, i.e. $\varkappa$ should be small and close to the rough limit of the order of 10;

(iii) because of the property (3), $\alpha_o^{\frac{1}{2}}$ should actually be chosen to be less than $\sqrt{2}$;

(iv) ϵ should not be too small so that the trapped particles are carried to the absorber in a reasonable amount of time.

In Fig. 1, we plot a histogram for the case of an ideal cleaning. All particles with $\alpha^{\frac{1}{2}} > \sqrt{2}$ are removed from the uniform distribution D. A trapping efficiency of 43% is needed. The total number of particles is 300. The $\alpha^{\frac{1}{2}}$-bin chosen for this histogram is 0.1.

In Fig. 2, a few sets of parameters are used. Particles trapped by the stable islands are removed by an absorber. A beam cleaning effect is

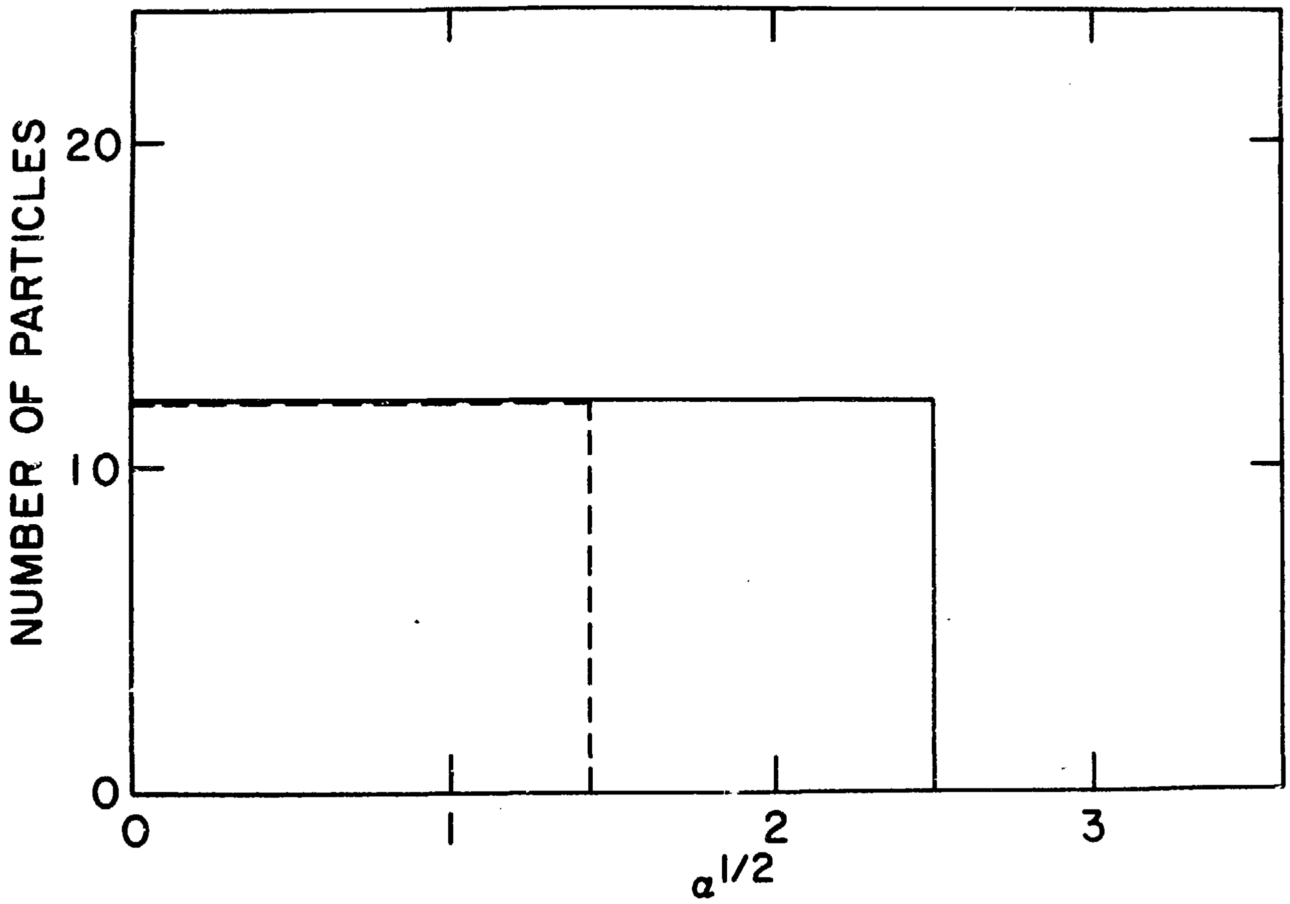

Fig. 1 Histogram for an ideal cleaning. Solid curve is the initial distribution. Dotted curve is the distribution after an ideal cleaning. All particles with $\alpha^{\frac{1}{2}} > \sqrt{2}$ are carried by the stable islands to large enough amplitudes for an absorber to remove them.

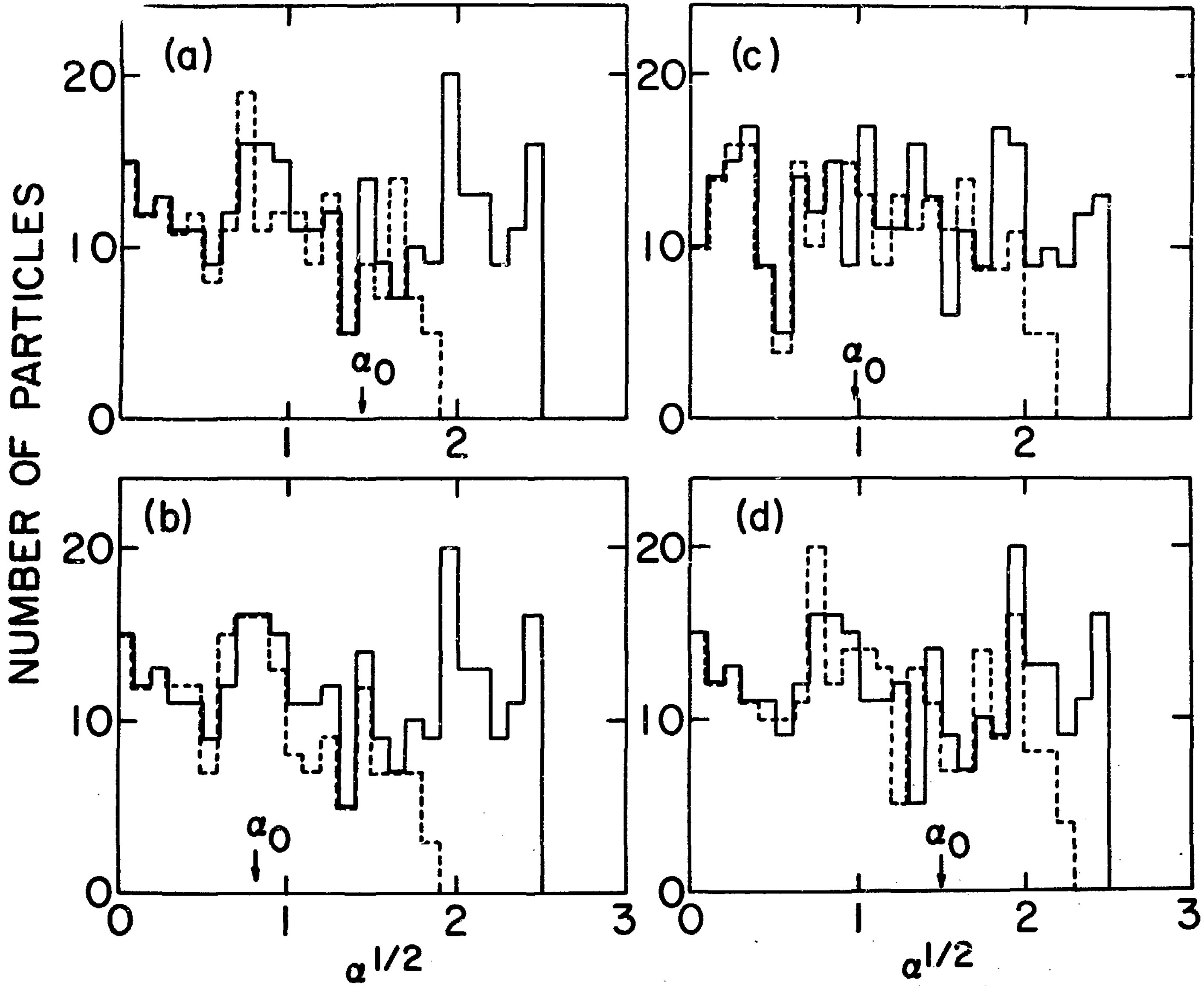

Fig. 2 Actual beam cleaning. Solid curves are initial distributions. Dotted ones are after the cleaning procedure. The parameters for the four cases correspond to: (a) $\epsilon = 2 \times 10^{-6}$, $\Delta_e = 7 \times 10^{-5}$, $\Delta_{NL} = 10^{-3}$, (b) $\epsilon = 5 \times 10^{-7}$, $\Delta_e = 7 \times 10^{-5}$, $\Delta_{NL} = 10^{-3}$, (c) $\epsilon = 1.5 \times 10^{-6}$, $\Delta_e = 5 \times 10^{-5}$, $\Delta_{NL} = 2.5 \times 10^{-3}$, and (d) $\epsilon = 1.5 \times 10^{-6}$, $\Delta_e = 2 \times 10^{-5}$, $\Delta_{NL} = 2 \times 10^{-5}$. The best cleaning is obtained for case (b). The $\alpha^{\frac{1}{2}}$-bin chosen is 0.1.

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evident. Comparing the individual graphs of Fig. 2, we find that the optimal parameter choice is

$$\Delta_{NL} \approx 10^{-3}$$

$$\Delta_e \approx 7 \times 10^{-5}$$

$$\epsilon \approx 10^{-6}$$

corresponding to a choice of reference emittance, $\epsilon_o = 1.6 \times 10^{-6}$ rad.m. These parameters can easily be realized in practice. The number of octupoles, decapoles and quadrupoles, together with their strengths are tabulated in Table I.

Table I. Magnet Strengths for Dynamic Beam Cleaning

Magnet	Number	Length (m)	Pole Tip Field at 4 cm (kG)*	Comments
Quadrupole	1	1.0	1.6	Required rate of field change is 17.5 kG/sec.
Octupole	4	1.0	2.6	
Decapole	4	1.0	7.8	

*The field strengths are for the following parameters: Particle momentum $p = 200$ GeV/c; machine radius $R = 428.3$ m; and tune $\nu \approx 20.4$.

III. Conclusions

We have developed the general framework for the dynamic cleaning of a stored proton beam by passing the beam through a nonlinear resonance. In particular, we have studied the use of a 5th order resonance for this purpose. We have discussed both the limitations and advantages of this technique, contrasting it with the physical beam scraping method currently in use at the CERN ISR.

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