**ANALYSIS OF THE LEAK-DETECTION SYSTEM FOR TOP WELDS OF
EBR-II FUEL ELEMENTS**

by

G. D. Hudman and L. C. Walters

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ABSTRACT

An analysis of the leak detector used to check the top welds on EBR-II fuel elements was performed. Data were obtained to allow calculation of volumes of the metering chamber and test chamber at each station of the leak detector. These volumes and a mathematical model were used to calculate decrease in pressure with time for each station.

Calibrated leaks and unknown leak rates were compared with calculated ones. The calculated results for the two calibrated leaks agreed with the observed pressure-time results for the two leaks.

The results of this study show that determining the volumes of each leak-detector station allows the leak-rate sensitivity to be readily calculated for each station. One leak-detector station could not detect a minimum leak rate of 2×10^{-4} std cm³/sec at 40 atm, which is the current specification. The other four stations could meet the specification.

Suggestions are given for periodic calibration of the leak detectors as well as precautions that must be observed to achieve optimum sensitivity when operating the leak detector.

I. INTRODUCTION

The stainless steel cladding of EBR-II driver fuel must contain the fuel, fission fragments, and generated gas. To meet this requirement, each manufactured element must pass a series of inspections. One such inspection is a leak test of the weld of the plug end to the jacket on the completed fuel element. According to specifications in EB-1-51879-C, Rev. E: the "weld at plug to tube must not permit any in-leakage greater than 2×10^{-4} standard cc/sec with a constant external pressure of 40 ± 5 atmospheres of inert gas and atmospheric internal pressure."

The objective of the work described in this report was a detailed examination of the leak-detection system used at EBR-II to determine whether this system can detect a leak that is equal to or less than the minimum leak rate required by the specification. Also, we hoped that the results would lead to a procedure for standardizing the equipment periodically and to recommendations for improving the system.

Because of a misinterpretation of Grunwald's paper* the leak detector was thought to be more sensitive than it is. According to Grunwald: "The sensitivity of the improved equipment is approximately 5×10^{-6} std cm³/sec for a chamber pressure of 600 psig over a 10 minute period." He was referring to std cm³/sec for each atmosphere of pressure drop across the leak. With a 40-atm (600-psi) pressure drop this leak rate would be 2×10^{-4} std cm³/sec. The system can detect this leak rate with a 10-min hold time.

II. DESCRIPTION OF LEAK-TEST SYSTEM

The system used to check for leaks in the EBR-II fuel elements has been described in detail.* An element is placed in a test station, where it is sealed just below the top weld by a seal-gland ring (Fig. 1). This ring is compressed around the jacket by a pneumatic piston. A miniature valve, the equalization valve (an "O" ring), allows helium to flow into a metering chamber, which is then sealed by closing the equalization valve. A valve between the metering chamber and the test chamber, the admission valve (another "O" ring), is then opened, or collapsed. The top weld on the fuel element is included as a boundary for the test chamber (Fig. 1). Should the

*A. P. Grunwald, Leak Testing of EBR-II Fuel Rods, Nucl. Sci. Eng., 12, 419-

Fig. 1. Metering Chamber and Test Chamber
for One Leak-detector Station

top weld have a hole in it, the leak can be detected by a pressure drop with time.

III. VOLUMES OF METERING AND TEST CHAMBERS

Grunwald reported that the combined metering- and test-chamber volume was 0.0081 in.³. He also stated the sensitivity was increased by making the volumes of the metering chamber and test chamber less than that of the plenum volume within a fuel element.

A test was designed to measure the volumes of the metering chamber and test chamber to determine whether the volumes equaled those reported by Grunwald. Each time a transducer or valve is replaced, the chamber volumes can change. As is shown later in this report, the sensitivity of the system depends strongly on the chamber volumes. When the volumes of the metering chamber and test chamber are determined, the leak sensitivity of the test station is readily calculated.

Appendix A is a detailed analysis of the pressure-time behavior of a leak-detection station.

The leak detector includes a small set screw for adjusting the volume of the metering chamber in each test station. By use of the following relationships the determination of the volume change for each turn of the set screw and volume of V_1 and V_2 (metering- and test-chamber volumes) can be evaluated:

$$P_1 V_1 = P_2 (V_1 + V_2) \text{ and} \quad (1)$$

$$P_3 (V_1 + \Delta V) = P_4 (V_1 + \Delta V + V_2). \quad (2)$$

P_1 is the initial pressure of the metering chamber;

P_2 is the expanded-gas pressure in the metering chamber and the test chamber after the admission valve has been opened;

V_1 is the volume of the metering chamber;

V_2 is the volume of the test chamber;

ΔV is a small change in the volume of the metering chamber due to a change in the position of the set screw used to adjust the metering-chamber volume; and

P_3 and P_4 are the pressures of a second pressure drop or gas expansion after a change of ΔV in V_1 .

It follows from Eq. (1) that:

$$\frac{P_1}{P_2} = \frac{V_1 + V_2}{V_1} = 1 + \frac{V_2}{V_1}; \text{ and} \quad (3)$$

$$\frac{P_1}{P_2} - 1 = \frac{V_2}{V_1} = C_1. \quad (4)$$

From Eq. (2):

$$\frac{P_3}{P_4} - 1 = \frac{V_2}{V_1 + \Delta V} = C_2. \quad (5)$$

From Eq. (4):

$$V_2 = C_1 V_1; \quad (6)$$

and from Eq. (5):

$$V_2 = C_2 (V_1 + \Delta V). \quad (7)$$

The test-chamber volume (V_2), should not change when the same dummy element is used for all gas expansions. Thus, from (6) and (7):

$$C_1 V_1 = C_2 (V_1 + \Delta V);$$

$$V_1 = \frac{C_2 \Delta V}{C_1 - C_2}; \text{ and} \quad (8)$$

$$V_2 = C_1 V_1. \quad (9)$$

The constants C_1 and C_2 are calculated from Eq. (4) and (5) after the gas expansions and the metering- and test-chamber volumes are calculated from Eq. (8) and (9), respectively.

A. Calibration of Pressure Transducers

To obtain accurate values for V_1 and V_2 , the following procedure was used to calibrate the pressure transducers. A Heise pressure gauge was installed in line with the system and the output of each transducer was measured at increments of about 100 psi from 0 to 1000 psig. The data are tabulated in Table I.

A linear regression of pressure in psig vs voltage output of the transducer was calculated for the transducer at each station of the leak detector. Table II shows the slopes and intercepts.

Readings of the Heise pressure gauge were compared with those of the recorder of each station. The recorders showed an offset at zero pressure of as much as 20-30 psi, and the scaling was as much as 10% high. The remaining data were taken by use of a digital voltmeter coupled directly to the transducer output leads to avoid any offset or recorder-tracking problems. The data were then converted to pressure, using the slope and intercept for the corresponding transducer.

B. Sample Calculation

The procedure for determining volumes of the metering chamber and test chamber is illustrated by the following example for station No. 2 with pressure-drop data from Table III, in which pressure drops at different stations and with differing numbers of turns of the set screw are shown.

From Eq. (4),

$$C_1 = \frac{1000}{620} - 1 = 0.6129.$$

Table I. Transducer Calibration Data

Station									
1	2	3	4	5	6	7	8	9	10
Pressure, psig	Transducer Output, mV	Pressure, psig	Transducer Output, mV	Pressure, psig	Transducer Output, mV	Pressure, psig	Transducer Output, mV	Pressure, psig	Transducer Output, mV
0	- 1.4555	0	- 1.3613	0	- 1.6583	0	+0.7707	0	- 6.9936
8	0.2559	105	0.3185	100	0.1333	108	2.2891	99	- 5.4450
4	2.1677	200	1.8069	199	1.3134	201	3.6230	200	- 3.8440
0	3.8704	299	3.3534	300	2.8086	300	5.0278	300	- 2.2102
2	5.7460	402	5.0495	400	4.3455	400	6.4684	400	- 0.6094
0	7.4934	500	6.5870	498	5.7931	498	7.8441	502	0.9910
9	9.3174	600	8.2130	601	7.3860	604	9.3542	597	2.3589
8	11.064	699	9.7660	700	8.8498	701	10.738	700	4.1821
8	12.867	800	11.411	797	10.333	797	12.105	800	5.8195
8	14.555	899	13.017	900	11.869	904	13.639	898	7.4259
		997	14.557	995	13.299	998	15.012	1003	9.1021

Table II. Slopes and Intercepts for Transducer Calibration at Each Leak-detector Station

<u>Station</u>	<u>Slope</u>	<u>Intercept</u>	<u>Correlation Coefficient, γ</u>
1	55.7609	+ 82.3	1.0000
2	62.5545	+ 86.6	1.0000
3	66.4929	+111.0	1.0000
4	70.1465	- 53.0	1.0000
	62.2847	+438.9	0.9999

Pressure (psig) = slope x transducer output (mv) + Intercept

Table III. Pressure drops (psig) Measured at the Five Leak-detector Stations^a

<u>Turns of Set Screw</u>	<u>Station</u>				
	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>
0	1012 511	1000 620	997 615	1007 605	1004 578
-1	1012 502	1000 604	997 602	1006 592	1010 565
-2	1012 491				
-3	1012 482	1000 565	997 575	1005 557	1013 542
-4	1012 471	1000 542			
-4-1/2				1005 528	
-5					1003 508

^aFirst number in each pair is P_1 ; number below it in each pair is P_0 .

From Eq. (5),

$$C_2 = \frac{1000}{604} - 1 = 0.6556$$

For one turn in of the set screw, ΔV was calculated to be $-2.745 \times 10^{-4} \text{ in.}^3$.

From Eq. (8),

$$V_1 = \frac{0.6556 (-2.745 \times 10^{-4})}{0.6129 - 0.6556} = \frac{0.000180}{0.042726} = 0.004212$$

$$V_1 = 0.004212 \text{ in.}^3.$$

From Eq. (9)

$$V_2 = 0.004212 \times 0.6129 = 0.002582 \text{ in.}^3.$$

The set screw was fixed at several positions for each station; thus V_1 assumed several values. From the data listed in Table III, several combinations at each station can be used to calculate V_1 and V_2 . The values listed in Table IV are the arithmetic average of these calculations, which are individually tabulated in Appendix B, Tables 1 - 5.

Only Station No. 1 has a combined volume greater than that reported by Grunwald. Because this station has a ratio of metering-chamber volume to test-chamber volume less than 1.06, at 1000-psi initial helium pressure 515-psi test pressure cannot be obtained. Since the specification calls for $40 \pm 5 \text{ atm}$, 515 psi is the minimum test pressure to meet this specification. Station No. 2 has the smallest volumes and is therefore the most sensitive station, as will be shown by determination of the pressure-time decay characteristics for each station. Station No. 3 has the highest test pressure because the metering chamber is larger by a factor of 1.4 than the test chamber. Another way to increase the test pressure without sacrificing sensitivity is to increase the helium-line pressure.

Table IV. Average Volumes (in.³) of Metering Chamber and Test Chamber for Individual Stations

	Station				
	1	2	3	4	5
Metering Chamber(V ₁)	0.0064	0.0029	0.0045	0.0037	0.0042
Test Chamber(V ₂)	0.0074	0.0025	0.0033	0.0031	0.0041
Combined Metering and Test Chambers	0.0138	0.0054	0.0078	0.0068	0.0083

IV. DETERMINATION OF SENSITIVITY OF SYSTEM TO LEAKS

Once the metering-chamber volume, V₁, and the test-chamber volume, V₂, are determined for each station, the leak sensitivity for each station can be calculated by the following relationship, which is derived in Appendix A.

$$P = \frac{P_1 V_1 + V_2 + V_3}{V_1 + V_2 + V_3} + \left[P_2 - \left(\frac{P_1 V_1 + V_2 + V_3}{V_1 + V_2 + V_3} \right) \right] \exp - KRT \left[\frac{V_1 + V_2 + V_3}{V_3 (V_1 + V_2)} \right] t. \quad (10)$$

P is the final or test-chamber pressure;

P₁ is the initial pressure or helium-line pressure;

P₂ is the pressure in metering chamber and test chamber after the valve between the two is opened;

V₁ is the metering-chamber volume;

V₂ is the test-chamber volume;

V₃ is the volume of the fuel-element plenum;

R is the gas constant, $\frac{\text{atm-liter}}{\text{mole-degree}}$;

T is the temperature in °K;

t is elapsed time in sec; and

K is the leak rate in $\frac{\text{moles}}{\text{sec-atm}}$.

The units of the specification must be converted from std cm³/sec to moles/sec-atm by use of the conversion, 1 mole = 22,414 std cm³ at 1 atm. Also, the specification requires the test to be at 40 ± 5 atm. Grunwald reported his work at 600 psig, or 40.8 atm. This initial test-chamber

pressure is within the specification limits; therefore 40.8 atm is used in Fig. 2 and Table V. From Station 1, for 600-sec elapsed time, a pressure drop of 6.5 psi (471.7 - 465.2) was calculated from Table V. For the same time from Station 2, the pressure drop was 18.8 psi. These pressure drops are based on the same helium-line pressure. Thus, Station 2 is 2.9 times as sensitive as Station 1. Figure 2 and Table V show that Station 2 has a slope about three times that of Station 1. The calculation of pressure decay with time is shown in Table V and Fig. 2 for each station. The calculations were made by use of Eq. (10). Volumes used in the calculations are those tabulated in Table IV. A leak rate of 2×10^{-4} std cm^3/sec , the limit of the specification, was used in the calculation.

It is sufficient to know only the metering-chamber V_1 and the test-chamber V_2 to calculate the sensitivity of the system. Use of a standard leak to calibrate the system is not necessary. However, to check the calculation and prove that the system would detect a leak as predicted, two calibrated sintered stainless steel flow restrictors, or leaks, were obtained. These leaks were calibrated and supplied by Sandia Laboratories, Livermore, California. They were calibrated with helium and were found to have leak rates of 1.6×10^{-4} and 6.5×10^{-5} std cm^3/sec at a pressure drop of one atmosphere to vacuum. The calibration was estimated to be correct within a factor of two because the leaks were large for the system on which they were calibrated.

The flow restrictors were attached to dummy elements by capillary tubing. The pressure-time information for Station 3 correlates well with the calibration information shown in Figs. 3a and 3b.

In Fig. 3a the first and second trials of the faster-leaking flow restrictor are compared with the error range of one-half and two times the leak rate found for that flow restrictor. Figure 3b shows the same characteristics of the slower-leaking restrictor.

After repeated use, leak standards have plugged and no leakage can be detected. This plugging may be the reason the second trial of both calibrated flow restrictors showed smaller leak rates than did the first trial. After a few tests, the calibrated sintered stainless steel flow restrictors did not show any leakage.

Figure 2. Calculated Change in Pressure with Time for Stations 1 - 5

Table V. Pressures (psig) at Elapsed Time from Opening of Admission Valve
for Leak-detector Stations 1 - 5

Time, sec	Station				
	1	2	3	4	5
0	471.7	543.8	583.1	550.8	513.3
20	471.4	543.2	582.6	550.3	512.9
40	471.2	542.5	582.2	549.8	512.5
60	471.0	541.9	581.7	549.3	512.1
80	470.8	541.3	581.2	548.8	511.7
100	470.6	540.6	580.8	548.2	511.3
120	470.4	540.0	580.3	547.7	510.9
140	470.2	539.4	579.8	547.2	510.6
160	469.9	538.7	579.3	546.7	510.2
180	469.7	538.1	578.9	546.2	509.8
200	469.5	537.5	578.4	545.7	509.4
220	469.3	536.8	577.9	545.2	509.0
240	469.1	536.2	577.4	544.7	508.6
260	468.9	535.6	577.0	544.2	508.2
280	468.7	534.9	576.5	543.7	507.8
300	468.4	534.3	576.0	543.1	507.4
320	468.2	533.7	575.6	542.6	507.1
340	468.0	533.1	575.1	542.1	506.7
360	467.8	532.4	574.6	541.6	506.3
380	467.6	531.8	574.2	541.1	505.9
400	467.4	531.2	573.7	540.6	505.5
420	467.2	530.6	573.2	540.1	505.1
440	466.9	529.9	572.8	539.6	504.7
460	466.7	529.3	572.3	539.1	504.4
480	466.5	528.7	571.8	538.6	504.0
500	466.3	528.1	571.4	538.1	503.6
520	466.1	527.5	570.9	537.6	503.2
540	465.9	526.8	570.4	537.1	502.8
560	465.7	526.2	570.0	536.6	502.4
580	465.4	525.6	569.5	536.1	502.1
600	465.2	525.0	569.0	535.6	501.7
620	465.0	524.4	568.6	535.1	501.3
640	464.8	523.8	568.1	534.6	500.9
660	464.6	523.1	567.7	534.1	500.5
680	464.4	522.5	567.2	533.6	500.1
700	464.2	521.9	566.7	533.1	499.8
720	464.0	521.3	566.3	532.6	499.4
740	463.7	520.7	565.8	532.1	499.0
760	463.5	520.1	565.4	531.6	498.6
780	463.3	519.5	564.9	531.1	498.2
800	463.1	518.9	564.4	530.6	497.9
820	462.9	518.3	564.0	530.1	497.5
840	462.7	517.7	563.5	529.6	497.1
860	462.5	517.0	563.1	529.2	496.7
880	462.3	516.4	562.6	528.7	496.4
900	462.1	515.8	562.1	528.2	496.0

In Grunwald's work, moisture was noted to be a problem in detecting small leaks. In an atmosphere saturated with water and ethanol, leaks were reduced or not detectable at all in known leaking samples. The helium line has no means of trapping moisture that could be in the bottles or lines. A dryer should be included in the helium line to remove any possible moisture to assure best sensitivity.

V. CONCLUSION

Operation of the valve system now used is sometimes marginal. The equalization valve, used to load the metering chamber to the initial 1000 psi, frequently does not collapse and allow the pressure to build rapidly to full pressure. If the operator does not wait until pressure reaches a maximum, some loss in sensitivity can occur. Also the test pressure may not meet specification.

Sometimes the valves leak so that the nitrogen bleeds into the helium system (evidenced by a rise in helium-line pressure to press re. of the nitrogen line) so that the next elements are checked with a helium-nitrogen mixture. This problem can be solved only by a redesign. Capillary tubing could be used in conjunction with ball valves to attain small volumes necessary for sensitivity. This method, however, could result in extensive design problems.

In another approach to leak detection a micrometering valve would be used and pressure buildup, rather than pressure decay, would be measured.

During production of Mark II fuel, two elements were rejected because of leaking top welds. Data recorded at Station 3 were plotted on curves of different leak rates for this station. Results of these calculations show that the element from batch 353 had a leak rate of about 4×10^{-4} std cm^3/sec at 40 atm and the other element, from batch 347, had a leak rate of about $3.5 - 4 \times 10^{-3}$ std cm^3/sec at 40-atm pressure drop. The results of these calculations are shown in Fig. 4.

When a production run is started, and about every three months or 4000 elements, whichever comes first, a complete calibration and system check should be made. The calibration should include connection of a Heise pressure gauge in the helium line. The pressure readings should be checked from

Fig. 3a. Comparison of Calculated Decrease in Pressure with Time with Known Leak Rate: Leak Rate of 1.6×10^{-4} std cm³/sec-atm

Fig. 3b. Comparison of Calculated Decrease of Pressure with Time with Known Leak Rate : Leak Rate of 6.5×10^{-5} std cm³/sec-atm

Fig. 4. Comparison of Leaking Top Welds with Calculated Pressure Decrease
(Leak rates are in std cm³/sec at 40.8-atm. pressure drop)

atmospheric pressure to 1000 psi, the range of the recorders, in step increments and each recorder should be adjusted to be within its stated or best accuracy.

When a valve body is replaced, the station should be checked for valve and solder-seal leaks to insure the helium system is not backfilled with nitrogen. Pressure-drop data should also be compared with the known calculated volumes for each station to assure the valves are collapsing and functioning properly.

When the helium pressure becomes higher than the pressure set point on the helium regulator, nitrogen is leaking into the helium line, either by passing an "O"-ring valve or by the solder seal joint at the ends of the valve body. If this leakage cannot be stopped by tightening the nitrogen-line connectors at the valve-body ends, a new valve body should be installed.

If the pressure continuously builds up after the admission valve is open, either the "O" rings or the valve-body solder seals are leaking. Therefore, the leak check cannot be considered valid. These problems have been prevalent. When a station has been found to be faulty, it has been shut down and the remaining stations used without interference from the defective station.

APPENDIX A

Pressure-time Analysis for a Leak Detector

A pressure-time analysis of a system with the characteristics of those used in the leak detector for EBR-II fuel jackets is given in this Appendix.

Initial Conditions:

$$P_2 = 1 \text{ atm}$$

$$P_3 = 1 \text{ atm}$$

Valve 2 closed, Valve 1 open, Metering Chamber at P_1 :

$$N_1 = \frac{P_1 V_1}{RT}; \quad N_2 = (1) \frac{V_2}{RT}; \quad N_3 = (1) \frac{V_3}{RT}. \quad (1)$$

At $t = 0$ Valve 1 closed, Valve 2 open:

Let P_2 = initial pressure in $V_1 + V_2$ when Valve 2 opens

$$\frac{P_2 (V_1 + V_2)}{RT} = N_1 + N_2 = \frac{P_1 V_1 + V_2}{RT};$$

$$P_2 = \frac{P_1 V_1 + V_2}{V_1 + V_2}. \quad (2)$$

If a leak exists at $t = \infty$, the final equilibrium pressure, P_f , is:

$$P_f = \frac{(V_1 + V_2 + V_3)}{RT} = N_1 + N_2 + N_3 = \frac{P_1 V_1 + V_2 + V_3}{RT};$$

$$P_f = \frac{P_1 V_1 + V_2 + V_3}{V_1 + V_2 + V_3}. \quad (3)$$

Let P be the pressure as a function of time in the metering and test chamber.

Let N_c be the number of moles of gas in the metering and test chamber after valve 2 is opened.

Let $V = V_1 + V_2$.

Let P_3 be the pressure in the plenum volume as a function of time.

At all times:

$$(N_1 + N_2 + N_3)_{\text{initial}} = N_c + N_3 = \text{constant} = \frac{PV}{RT} + \frac{P_3 V_3}{RT}. \quad (4)$$

From Eq. (1),

$$\text{Constant} = \frac{P_1 V_1 + V_2 + V_3}{RT}.$$

Therefore, from Eq. (4),

$$PV + P_3 V_3 = P_1 V_1 + V_2 + V_3; \text{ and}$$

$$P_3 = \frac{P_1 V_1 + V_2 + V_3 - PV}{V_3}. \quad (5)$$

Also from Eq. (4),

$$\frac{dN_c}{dt} = - \frac{dN_3}{dt}. \quad (6)$$

Now we assume that the leak rate is proportional to the pressure drop across it.

$$\frac{dN_c}{dt} = -k(P - P_3); \quad (7)$$

$$N_c = \frac{PV}{RT}; \quad \frac{dN_c}{dt} = \frac{V}{RT} \frac{dp}{dt}. \quad (8)$$

Substituting equations (8) and (5) into equation (7) gives the required differential equation:

$$\begin{aligned} \frac{V}{RT} \frac{dp}{dt} &= -K \left[P + \frac{PV}{V_3} - \left(\frac{P_1 V_1 + V_2 + V_3}{V_3} \right) \right]; \\ \frac{V}{RT} \frac{dp}{dt} &= -K \left[P \left(1 + \frac{V}{V_3} \right) - \left(\frac{P_1 V_1 + V_2 + V_3}{V_3} \right) \right]. \end{aligned} \quad (9)$$

Integration gives:

$$\begin{aligned} \int_{P_2}^P \frac{dp}{\left(1 + \frac{V}{V_3} \right) P - \left(\frac{P_1 V_1 + V_2 + V_3}{V_3} \right)} &= \frac{-KRT}{V} \int_0^t dt; \\ \left(1 + \frac{1}{V} \right) \ln \left[\left(1 + \frac{V}{V_3} \right) P - \left(\frac{P_1 V_1 + V_2 + V_3}{V_3} \right) \right] \Big|_{P_2}^P &= \frac{-KRT}{V} t; \\ \frac{\left(1 + \frac{V}{V_3} \right) P - \left(\frac{P_1 V_1 + V_2 + V_3}{V_3} \right)}{\left(1 + \frac{V}{V_3} \right) P_2 - \left(\frac{P_1 V_1 + V_2 + V_3}{V_3} \right)} &= \exp \frac{-KRT}{V} \left(1 + \frac{V}{V_3} \right) t; \end{aligned}$$

$$1 + \frac{V}{V_3} P - \left(\frac{P_1 V_1 + V_2 + V_3}{V_3} \right) = \left[\left(1 + \frac{V}{V_3} \right) P_2 - \left(\frac{P_1 V_1 + V_2 + V_3}{V_3} \right) \right] \left[\exp - \frac{KRT}{V} \left(1 + \frac{V}{V_3} \right) t \right];$$

$$\left(1 + \frac{V}{V_3} \right) P = \frac{P_1 V_1 + V_2 + V_3}{V_3} + \left[\left(1 + \frac{V}{V_3} \right) P_2 - \left(\frac{P_1 V_1 + V_2 + V_3}{V_3} \right) \right] \left[\exp - \frac{KRT}{V} \left(1 + \frac{V}{V_3} \right) t \right]; \text{ and}$$

$$P = \frac{P_1 V_1 + V_2 + V_3}{V_1 + V_2 + V_3} + \left[P_2 - \left(\frac{P_1 V_1 + V_2 + V_3}{V_1 + V_2 + V_3} \right) \right] \left[\exp - KRT \left(\frac{V_1 + V_2 + V_3}{V_3 (V_1 + V_2)} \right) t \right].$$

Check boundary conditions at $t = 0$

$$P = P_2.$$

At $t = \infty$

$$P = \frac{P_1 V_1 + V_2 + V_3}{V_1 + V_2 + V_3}$$

Try numbers from Grunwald:

$$P = \frac{P_1 V_1 + V_2 + V_3}{V_1 + V_2 + V_3} + \left[P_2 - \left(\frac{P_1 V_1 + V_2 + V_3}{V_1 + V_2 + V_3} \right) \right] \left\{ \exp - KRT \left[\frac{V_1 + V_2 + V_3}{V_3 (V_1 + V_2)} \right] t \right\}$$

$$V_1 = 0.065 \text{ cm}^3;$$

$$V_2 = 0.065 \text{ cm}^3;$$

$$V_3 = 0.540 \text{ cm}^3 = \text{plenum volume of Mark IA fuel element};$$

$$P_1 = 1200 \text{ psi} = 82 \text{ atm}; \text{ and}$$

$$P_2 = \frac{82(0.065) + 0.065}{0.130} = 41.5 \text{ atm.}$$

Grunwald claims a sensitivity of $5 \times 10^{-6} \frac{\text{std cm}^3}{\text{sec}}$ for a chamber pressure of 600 psig over a 10-min. period:

$$5 \times 10^{-6} \frac{\text{std cm}^3}{\text{sec}} \times \frac{\text{mole}}{2.24 \times 10^4 \text{ std cm}^3} = 2.23 \times 10^{-10} \frac{\text{mole}}{\text{sec}} .$$

From Eq. (7),

$$K = 2.23 \times 10^{-10} \frac{\text{mole}}{\text{sec}} \times \frac{1}{40.8 \text{ atm}} = 5.47 \times 10^{-12} \frac{\text{mole}}{\text{sec-atm}} .$$

What is $(P_2 - P)$, i.e., the pressure drop after $10_{\text{min}} = 600_{\text{sec}}$?

$$KRT \left[\frac{(V_1 + V_2 + V_3)}{V_3(V_1 + V_2)} \right] t =$$

$$5.47 \times 10^{-12} \frac{\text{mole}}{\text{sec-atm}} \times 82.1 \frac{\text{cm}^3 - \text{atm}}{\text{mole-deg}} \times 298 \text{ deg} \times 600 \text{ sec} \left(\frac{0.67}{0.070} \right) \frac{1}{\text{cm}^3}$$

$$-KRT \left[\frac{V_1 + V_2 + V_3}{V_3(V_1 + V_2)} \right] t = -7.68 \times 10^{-4} .$$

$$P = 0.9992 (P_2) + \left(\frac{P_1 V_1 + V_2 + V_3}{V_1 + V_2 + V_3} \right) 7.68 \times 10^{-4} ;$$

$$\frac{P_1 V_1 + V_2 + V_3}{V_1 + V_2 + V_3} = \frac{82(0.065) + 0.065 + 0.54}{0.13 + 0.54} = 8.86;$$

$$P = 0.9992 (P_2) + 6.80 \times 10^{-3}$$

$$P_2 - P = P_2(1 - 0.9992) - 6.80 \times 10^{-3};$$

$$\Delta P = 0.0264 \text{ atm} = 0.39\text{-psi drop in 10 min.}$$

APPENDIX B

Volume Measurements for Stations 1-5

Tables VI - X show the volumes of the metering chambers and test chambers at Stations 1 - 5 respectively. Each possible combination of pressure-drop data was used. The volume of the metering chamber, V_1 , is reduced by the ΔV for that calculation for each new reference point. For example, in Table VI the first block of results was calculated by use of the initial setting of the set screw as a reference. In the next block, the reference was taken with the set screw one turn in from the initial setting; therefore, the metering chamber will be smaller by one turn of the set screw, or $2.745 \times 10^{-4} \text{ in.}^3$, than that of the first block of data. The third block of calculations will have a V_1 smaller than the first block by two turns, etc.

Table VI. Calculated Volumes of Metering and Test Chambers (in.^3) for Station 1

<u>$V_1 + V_2$</u>	<u>V_2</u>	<u>V_1</u>	<u>Turns of Set Screw^a</u>	<u>$V_1 + \Delta V$</u>
0.015555	0.007701	0.007854		0.007854
0.014301	0.007080	0.007221		0.007221
0.015050	0.007451	0.007599		0.007599
0.014850	0.007352	0.007499		0.007499
0.013001	0.006552	0.006449	+1	0.006724
0.014548	0.007332	0.007217		0.007492
0.014371	0.007243	0.007129		0.007404
0.016165	0.008322	0.007843	+2	0.008392
0.014850	0.007645	0.007205		0.007754
0.013500	0.007070	0.006430	+3	0.007254
Mean	0.007375			0.007519
Deviation	± 0.000467			± 0.000440

After Tests $V_1 = 0.006421$

Table VII. Calculated Volumes of Metering and Test Chamber (in.³) for Station 2

$V_1 + V_2$	V_2	V_1	Turns of Set Screw	$V_1 + \Delta V$
0.006794	0.002582	0.004212		0.004212
0.006513	0.002475	0.004038		0.004038
0.006447	0.002450	0.003997		0.003997
0.006123	0.002425	0.003699	+1	0.003974
0.006083	0.002409	0.003674		0.003949
0.005466	<u>0.002378</u>	<u>0.003088</u>	+3	<u>0.003912</u>
Mean	0.002453			0.004014
Deviation	± 0.000071			± 0.000106

After Test $V_1 = 0.002916$

Table VIII. Calculated Volumes of Metering and Test Chambers (in.³) for Station 3

$V_1 + V_2$	V_2	V_1	Turns of Set Screw	$V_1 + \Delta V$
0.008341	0.003196	0.005145		0.005145
0.008688	0.003329	0.005359		0.005359
0.008581	<u>0.003400</u>	0.005181	+1	<u>0.005456</u>
Mean	0.003308			0.005320
Deviation	± 0.000103			± 0.000159

After Test $V_1 = 0.004497$

Table IX. Calculated Volumes of Metering and Test Chamber (in.³) for Station 4

<u>V₁ + V₂</u>	<u>V₂</u>	<u>V₁</u>	<u>Turns of Set Screw</u>	<u>V₁ + ΔV</u>
0.009165	0.003659	0.005506		0.005506
0.007883	0.003147	0.004736		0.004736
0.007773	0.003103	0.004670		0.004670
0.007147	0.002941	0.004206	+1	0.004481
0.007227	0.002974	0.004253		0.004528
0.006773	<u>0.003019</u>	0.003754	+3	<u>0.004578</u>
Mean	0.003141			0.004750
Deviation	±0.000266			±0.000382

After Test V₁ = 0.003652

Table X. Calculated Volumes of Metering and Test Chamber (in.³) for Station 5

<u>V₁ + V₂</u>	<u>V₂</u>	<u>V₁</u>	<u>Turns of Set Screw</u>	<u>V₁ + ΔV</u>
0.007424	0.003150	0.004274		0.004274
0.009419	0.003996	0.005422		0.005422
0.009786	0.004152	0.005634		0.005634
0.010478	0.004617	0.005861	+1	0.006136
0.010239	0.004511	0.005728		0.006003
0.009485	<u>0.004410</u>	0.005075	+3	<u>0.005899</u>
Mean	0.004139			0.005561
Deviation	±0.000536			±0.000681

After Test V₁ = 0.004189

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