ANL-7217
Biology and Medicine
(TID-4500)
AEC Research and
Development Report

ARGONNE NATIONAL LABORATORY
9700 South Cass Avenue
Argonne, Illinois 60439

CFSTI PRICES

H.C. \$ 3.00; MN. 65

HEALTH DIVISION
GAMMA-RAY SPECTROSCOPY GROUP
ANNUAL REPORT

July 1964 through June 1965

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RELEASED FOR ANNOUNCEMENT
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A REEXAMINATION OF RETENTION PATTERNS IN
PATIENTS WHO RECEIVED RADIUM BY
MULTIPLE INJECTIONS 33 YEARS EARLIER

Charles E. Miller and Asher J. Finkel

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A REEXAMINATION OF RETENTION PATTERNS IN PATIENTS WHO RECEIVED RADIUM BY MULTIPLE INJECTIONS 33 YEARS EARLIER

Charles E. Miller and Asher J. Finkel

Abstract

Radium retention data have been analyzed in detail for a small series of eight patients who received known amounts of Ra^{226} by weekly intravenous injection in 1931 and who have been measured in a whole-body counter in recent years. When individual power function parameters were computed for each patient, the slopes ranged from -0.191 to -0.634 and the one-day intercepts from 0.029 to 0.977. These parameters are highly correlated ($r = 0.83$) so that the power function does not have two independent parameters. Although the parameters for the "average" power function can be computed in several ways, a reasonable set of mean values gives a slope of -0.4383 and a one-day intercept of 0.3015. The same data starting one year after injection can be fitted equally well if not better with a single exponential term so that retention after one year decreases with a half-time of 14.98 years. About 2.5 percent of the administered radium disappears from the body in accordance with this single exponential term, 97.5 percent having been eliminated within the first year. Retrospective estimates of the maximum radium body burden, which occurred immediately after the last injection of 10 μCi , were computed on the basis of the best-fitting power function for each patient. Although the patients received total amounts ranging from 150 to 300 μCi , the estimates of maximum body radium ranged from 17.6 to 27.8 μCi . The relative merits of the power function and the single exponential function for describing radium retention patterns are discussed in light of the data and the analyses.

Introduction

During the last fifteen years, many persons who have radium deposited in their skeletal tissues have been studied by several groups of investigators in a number of countries. In these studies, the radium contained by the patient has been measured and various types of clinical data have been recorded. The central problem in all of these studies is the fact that observations of clinical importance are correlated with contemporary or terminal body burdens, although the radium may have been present for from 30 to 45 years and the pathological changes may have been in the making for an unknown, and perhaps unknowable, number of years.

On the other hand, persons responsible for radiation safety are not concerned with the current level of radium in any one person but rather with the maximum amount present at any time. The concept of Maximum Permissible Level (MPL) for any radioactive material is based on the amount that a person may safely accumulate and that will not lead to deleterious effects at a later date. It would be exceedingly valuable if the scientific investigator who attempts to determine the dose-response patterns for human subjects were able to calculate the amount of radium present in the body at any time during the exposure period as well as during the remainder of life after the exposure. Since it is not known at this time whether it is the maximum body burden, which usually occurs at the time of the last exposure, or some function of the total accumulated dose to the body over an unspecified period of years that is important in producing clinical changes, the radium retention pattern with time should be reconstructed for correlation studies with the disease processes. For these reasons, it is helpful to have a mathematical expression for the pattern of radium retention in man in order to interpret the clinical effects in persons who have retained the radium for long periods of time. Such an expression could then be employed in the case of single injection, or of multiple injections, or of uniform continuous ingestion for different lengths of time.

The derivation of a mathematical expression for radium retention properly requires many measurements of the same subject spanning the entire era of interest. A limited series of four or five measurements might be fitted approximately with either a single exponential function or a power function, and can be fitted exactly by the sum of two or three exponentials. However, intermediate measurements, which further delineate the true retention curve, may demonstrate that the actual retention pattern deviates from that predicted by a mathematical function that has been fitted to four or five points located at the beginning or the end of the time span. In the case of human subjects, a sufficiently large series of measurements is not available for a precise derivation of a retention expression for radium.

Where a large number of measurements on the same patient do not exist, the validity of the various mathematical expressions can be tested only by the goodness of fit provided by these functions to the limited series of measurements that does exist. In other words, the retention of radium must arbitrarily be assumed to follow a particular mathematical expression. The parameters of the appropriate mathematical terms that provide the best fit to the data can then be determined and evaluated by a variety of criteria for goodness of fit.

In the absence of more suitable data, a meager series of up to eight measurements that are available for each of eleven patients who received supposedly known amounts of radium in 1931 has been analyzed extensively.

The data available for these patients have been fitted with both a power function and a single exponential term. The optimum expression for radium retention based on goodness of fit of this limited number of points should not, however, be used as an explanation of the metabolism of radium in humans. The calculations presented in this paper were made in an attempt to derive the maximum amount of information from the data available for the only series of long-term radium patients whose injection history is reasonably well known and who have been studied by modern, accurate whole-body measurement techniques.

Clinical Material

In 1931, radium chloride was administered intravenously to a group of patients in the Elgin State Hospital (ESH) in Illinois in a study of the therapeutic value of radium. The patients were given a series of from 7 to 45 injections of 10 μ Ci of radium chloride dissolved in 2 cc isotonic sodium chloride solution per injection. The number of injections and the dates of the first and last injection varied from patient to patient, since the weekly treatments were administered on an individual basis.

The radium contained by these patients was first measured by Schlundt et al. during the last week of December, 1931, and again six months later in June, 1932. The results of these gamma-ray and radon breath measurements were given in a publication in which 24 patients were identified by code letters from A to X along with the total amount of radium given to each patient and the elapsed time from the last injection to the first measurement.¹ At about this time, private reports listing the names of the patients, the amount of radium administered to each patient, and clinical changes, if any, were also sent to the supplier of the radium preparation.

In 1950 and subsequently, investigators from Argonne National Laboratory located and compiled all of the available reports and records concerning this group of patients. The key to the original Schlundt code was reconstructed by a comparison of the amount of radium given to each patient in the published paper, in which the patients were listed by code letter, and the values given in the private reports to the commercial supplier, in which the patients were identified by name. Consequently, each patient measured by Schlundt was identified, and the radium body values published by Schlundt could be assigned to particular patients. A search of the numerous case histories of other patients who were in the hospital from 1931 to 1933 led to the discovery of several other patients who had received radium either after the Schlundt measurements in 1931 or 1932, or who were not measured by Schlundt.

In 1951, 14 of the 24 patients measured by Schlundt were located, and their contained radium was determined by whole-body counts and

radon breath analyses with field equipment taken to the hospital. Two other patients who had not been studied by Schlundt were also measured. In 1953, 11 of the original 24 patients were remeasured by the same techniques. The results of the 1951 and 1953 measurements, along with the Schlundt measurements, were analyzed by Norris et al. to determine the parameters of a power function for the retention of radium in humans.² For these analyses, the injection schedule given by Schlundt was approximately employed in the analysis of data for each patient.

Between 1959 and 1964, gamma-ray measurements of those patients who were still alive were made in the whole-body counter at Argonne National Laboratory. Ten of the original 24 patients were remeasured along with 6 patients who had not been measured previously. The whole-body measurements of these patients comprise the largest set of radium retention measurements extant in man over the long span of time required for suitable mathematical analysis. Eight of this group of patients had at least four breath radon and gamma-ray measurements of whole-body radium over the 33-year span, and the presentation and evaluation of these data comprise this report.

Measurement Techniques

The various whole-body measurement techniques employed in the current series of studies from 1959 through 1964 at Argonne National Laboratory will be discussed first. The information obtained by these techniques, especially that relating to the distribution of radium in the body, will be used to evaluate the accuracy of the results obtained by the earlier procedures.

The patients were measured with the 1-meter-arc technique in 1959, the 1.75-meter-arc technique in 1960, the tilting-chair technique on three occasions between 1959 and 1964, and the 7-crystal-position method during 1963-1964. The 1-meter-arc technique was used to determine the absolute amount of Radium B and Radium C and to calibrate the tilting-chair technique for radium measurements. The patient was placed on his side on a bed in a curve so that his posterior surface touched the surface of a thin metal 1-meter arc. The 8-inch diameter by 4-inch thick NaI(Tl) scintillation crystal was placed at the center of the 1-meter arc, and the patient's gamma-ray spectrum was recorded with a 256-channel magnetic memory, pulse-height analyzer. The patient was then curved backward so that his anterior surface touched the metallic arc, and the gamma-ray spectrum was again recorded.

The two spectra obtained from the human subject were averaged in order to make a comparison with the spectrum obtained from a point source in the center of a solid phantom. A 1.0- μ Ci radium source

calibrated by the National Bureau of Standards was placed in the center of a 17-cm-thick Presdwood phantom located on the bed in the center of the position occupied by the human. The thickness of Presdwood between the source and the crystal and the thickness of Presdwood behind the source were varied until the shape of the spectrum obtained from the source in the phantom matched the shape of the average spectrum obtained from the patient. Optimum correspondence was obtained when the phantom was 14 cm thick. The amount of radium present in the patient was then determined from the ratio of the height of the average spectrum from the patient to that from the 1.0- μ Ci source in the phantom. The value thus calculated for each patient was accepted as the absolute quantity.

Each patient was then seated in the tilting chair and his gamma-ray spectrum recorded. Each patient's spectrum was divided by the radium content determined by the 1-meter-arc technique. This new spectrum was the spectrum that would be obtained from 1.0 μ Ci of radium in this patient. The various spectra obtained for the patients were then averaged to obtain a typical calibration spectrum for the tilting chair for 1.0 μ Ci in the body. The patients were then measured in the tilting chair on subsequent visits to the Laboratory, and this calibration spectrum was used to calculate the radium content. The tilting-chair technique was used instead of the 1-meter-arc technique because of the shorter counting times required. The counting rate with the tilting chair per microcurie of radium in the body is four times that obtained with the 1-meter arc.

The patients were also measured on a 1.75-meter arc at a later date. This technique is similar to that employed with a 1-meter arc except that the radius of the arc is oriented vertically instead of horizontally. The 1.75-meter-arc technique, while yielding a result that is less dependent upon the distribution in the body, requires ten times as long a counting time as the tilting-chair technique to yield the same statistical accuracy. It was used principally because it entailed much less discomfort to the patient than did the 1-meter arc.³

In 1963, the patients were measured with the 7-crystal-position technique,⁴ in which the crystal is placed in turn at seven equally spaced positions along the body and the patient's gamma-ray spectrum obtained. These measurements and 49 additional spectra obtained from appropriate phantom measurements are analyzed with the aid of an electronic computer. The individual magnitudes of seven point sources are determined, which, if uniformly spaced along the body, would yield the same radioactive profile as that observed along the patient. The sum of the seven sources yields the total-body content of radium, and the magnitudes of the individual sources are equal to the amount in each one-seventh segment of the body. These measurements disclosed that the radium was not always distributed uniformly along the length of the skeleton and that any particular segment of the body might contain an extraordinarily large

percentage of the total-body radium. This nonuniformity of distribution observed in the body in 1964 will be referred to later in this paper to explain apparent errors in earlier measurements (see Appendix II).

The deck-chair technique, which was also calibrated with the 1-meter-arc technique, was used for most of the measurements in 1951-1953. Four patients were each in turn curved in a 1-meter arc and measured with a 1.25-inch diameter by 2-inch-long crystal placed at the center of the arc. Since the pulses from the crystal were sorted with a single channel analyzer, it was not feasible to obtain the shape of the spectrum at that time. Instead, the patients were counted twice, once with the discriminator adjusted to span the energy band from 25 keV to 450 keV and a second time to span from 450 keV to infinity. After each patient was measured, first facing the crystal and then away from the crystal, a 10- μ Ci source was then counted both one meter from the crystal in the open air and with the 10- μ Ci source placed at several selected positions behind the patient's back. These measurements were then substituted into an appropriate equation⁵ and the amount of radium in the patient calculated. The patient was then seated in a canvas chair and the count rates determined in the same energy bands with the crystal located parallel and 10 cm from the lower spinal column; the top of the crystal was located 60 cm from the floor.

The count rate from each patient was divided by the amount of radium determined by the 1-meter-arc technique. Three of the patients yielded a counting rate when seated in the canvas chair about four times that found when they were counted on the 1-meter arc, while the fourth patient yielded a ninefold value when seated. When these four ratios were averaged, the mean calibration factor for the deck chair obtained in this way was approximately five times the counting rate per microcurie obtained with the 1-meter arc. Since these four patients were the only ones who contained enough radium to be measured on the 1-meter arc with the small crystal available at that time, the other patients, who had considerably smaller amounts, were measured in the deck chair only and the mean calibration factor obtained for the four high-value patients was used to calculate the amount that the other patients contained.

In light of the information recently obtained with the 7-crystal-position technique, it can now be seen that the measurements made with the deck-chair technique may have resulted in sizeable errors. The deck-chair technique will yield a whole-body value that is much too low if a large percentage of the radium is present either in the patient's skull or in his lower limbs. The radium in each of these locations is counted less efficiently than that in the torso, since it is further from the crystal and since in the case of the legs there is also a larger amount of tissue between the radium and the detector. Consequently, a large uncertainty must be assigned to these measurements made in 1951-1953.

The radon content of the breath of these patients was measured during the same period by Stehney et al.⁶ The radon breath measurements contain uncertainties due to two causes. First is the well-known fact that the radon concentration in the breath varies inversely with the respiration rate. Second, the radon breath measurements are variable because some of the patients were not cooperative and in many instances fought the breathing mask. Thus, the breath samples were collected under very adverse conditions and could not be considered to be complete collections in all cases.

Earlier, in 1931-1932, Schlundt measured the patients in the hospital under field conditions. He measured the gamma-rays from Radium B and Radium C by placing a Wulf-Hess electroscope a specific distance behind the seated patient. His electroscope had been cross-calibrated with one belonging to Knowles by measurement of gold radon seeds placed throughout the body of a cadaver.¹ In a prepublication copy of his paper Schlundt listed values for the patients that were 10 percent higher than those ultimately published in the literature. Evidently, Schlundt altered his calibration 10 percent downward between the time of his first draft and his final publication.

Schlundt's gamma-ray measurements undoubtedly contain systematic errors from patient to patient because of nonuniform distribution of radium within the body. If these nonuniform distributions that were present in 1964 existed in 1931, Schlundt's measurement of a patient who contained large amounts in the pelvis or femur would probably be lower than the true amount. His measurement of a patient who contained a large percentage in the head would probably be slightly low, while his measurement of a patient who contained most of the radium in the upper spine would be considerably high. For these reasons, the Schlundt measurements will be reviewed in this paper in the light of current knowledge of specific nonuniformity in radium deposition in the same patients (Appendix II).

Schlundt also made radon breath measurements of the patients. The patients exhaled breath for three to four minutes through a drying train in series with a 2.7-liter volume Lind electroscope. He calculated the amount of radon present in the 2.7-liter electroscope from the rate of discharge of the electroscope observed 10 minutes and 30 minutes after the sample collection. Schlundt assumed a breathing rate of 7.5 liters per minute for all patients, or that the radon in the 2.7-liter volume represented the radon exhaled from the body in 21.6 seconds. In his paper Schlundt stated, "Several patients could not be induced to blow normally and a few refused to blow at all."¹ It is thus obvious that these breath measurements may contain a large error. From recent experience it can be concluded that Schlundt's breath measurements could easily range from a factor of too high to a very small percentage of the true value.

Original and Adjusted Data

The available radium retention data for each patient are compiled in Tables 1A to 11A. Each A table has the same format and contains, in successive columns, the following information: (1) the number of elapsed days between the last injection and each measurement; (2) the amount of radium present in the body that gives rise to the radon that is exhaled in the breath; (3) the amount of Radium B and Radium C determined by gamma-ray measurements; (4) the total radium in the body (sums of the values of columns 2 and 3), which comprises both the radium daughters lost by exhalation and those retained in the body; (5) the retention ratio of radon daughters derived from the earlier measurements and consisting of the ratio of Radium B and Radium C in the body to total-body radium. These first five columns are derived from previously published data along with our more recent measurements in the whole-body counter.

In an effort to secure more accurate data retrospectively, adjusted whole-body radium values have been calculated independently for each patient from the breath radon data and from the Radium B and Radium C gamma-ray measurements. Studies made during the past few years with mice, along with data compiled for the retention of radon daughters in rats, dogs, and man, have demonstrated that the retention of radon in the skeleton follows a power function expression,

$$\text{Radon retention} = c t^d. \quad (1)$$

In this equation, t is the elapsed time in days since the injection, and c and d are parameters that seem to be identical for all species.^{7,8} For mice, and presumably for man, d has a value of about 0.18.⁹ Since radon daughter retention is a function of time, the total-body radium at a specific time can be calculated from either the radon breath measurement or the gamma-ray measurement by the use of this retention equation.

For the data in the present study, the expected mean radon retention ratio was derived for the time of each measurement. A computer was programmed to calculate the amounts of radium and of RaB and RaC present at the time of the measurement from each injection that the patient had received. For example, if a patient had received 15 weekly injections of radium and was measured on the 30th day after the last injection, the amounts of radium present from the injections on the 30th, 37th, 44th, ..., 128th days were calculated and summed. For these calculations, the retention of radium was assumed to follow the power function, $R = 0.479 t^{-0.434}$. These parameters, which differ somewhat from those given by Norris, were obtained in a preliminary analysis of the data discussed in detail in this paper. The fact that the final values obtained by further analysis are somewhat lower than these preliminary values does not introduce more than a tiny error. RaB and RaC were computed for

each injection by multiplying the radium retained from each injection by the expected radon retention on the day of measurement. For this purpose the equation, $\text{Radon retention} = 5.79 t^{0.18}$, was used,⁹ and the RaB and RaC values so obtained were similarly summed. The effective radon retention for the 30th day after the last injection was found by dividing the total RaB and RaC by the total radium. Since the amount of radium contributed to the total by each preceding injection decreases, the radon retention values found by this technique are a few percent lower than the values obtained when the individual radon retention ratios calculated for the 15 different days are averaged directly.

The expected radon retention ratio for each measurement for the appropriate number of injections calculated in this way for each patient is given in column 6, in Tables 1A-11A. The measured breath radon values given in column 2 were divided by $(1 - \text{expected radon retention ratio})$ in order to calculate the adjusted total-body radium from the breath measurement, and these values are listed in column 7. Similarly, the adjusted values in the last column were computed by dividing the measured RaB and RaC values in column 3 by the expected radon retention ratio in column 6. The total-body radium calculated from the Radium B and Radium C gamma-ray measurements would be identical with that computed from breath radon at any one time if the breath and gamma-ray measurements were accurate and if radon retention followed this equation. The differences between the two "adjusted" total-body radium values in this series of measurements are due principally to errors in the measurements.

Analysis of Data by Curve Fitting

A limited series of six or seven measurements of the radium contained by a patient is not sufficient to demonstrate conclusively whether the retention follows a power function, a series of exponentials, or some other mathematical expression. Such a series of measurements can be used only to determine the parameters that provide the best fit to each set of data by an arbitrarily selected function. The parameters computed for each of the various patients can then be used as an indication of the extent of variability of retention within the group.

The power function, $R = 0.54 t^{-0.52}$, proposed by Norris et al.,^{2,10} has been widely used for describing the retention of radium in man. Since Norris and his associates derived the value of these parameters from the very patients discussed in this paper, it is of some interest to reexamine the older data along with more recent measurements of the same individuals. For this purpose the parameters of the power function, a t^b , that provide the best fit to the data for each patient were determined with the aid of an electronic computer. Since the power function is not a linear function with either time or amount, the parameters that provide the best

fit to data obtained for persons who received numerous injections cannot be determined by simple graphical analysis. The amount present, for example, on the 200th day after the last injection in a patient who received 15 injections is the sum of the amounts from each individual injection. The radium contained by a patient who received 15 injections, each of which was assumed to be metabolized in identical fashion in accordance with Norris' values, is depicted in Figure 1. On the first day after the first injection of $10\ \mu\text{Ci}$ the patient would have contained $5.4\ \mu\text{Ci}$ of radium, and the radium content would then have decreased along a straight line to $1.96\ \mu\text{Ci}$ on the seventh day, at which time the patient would have received an additional $10\ \mu\text{Ci}$ of radium. If the patient had not received the second injection, his radium content would have continued to follow a projection of this straight-line segment. The amount present from the second injection, if plotted from the next day onward, would also follow the straight-line section shown for the first injection. However, when it is added to the amount present from the first injection the combined radium content from the 8th to the 15th day follows the curved line shown in the double-log plot in Figure 1. The addition of each injection yields successive curved sections. After the last injection the curve drops quite steeply for a few days and then flattens out, and after a number of years approaches the slope of a single injection. If the quantity present after the last injection is replotted on a graph in which time is specified as days since last injection, the curve will have the shape shown in Figure 2. Here, the curve is quite flat for the first few days and then bends over gradually and again, after several years, approaches the slope of a single injection.

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Fig 1. Double log plot of radium body burden during and after 15 weekly injections of $10\ \mu\text{Ci}$ each calculated in accordance with the power function $R = 0.54 t^{-0.52}$.

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Fig. 2. Double log plot of radium body burden starting one day after a series of 15 weekly injections of $10 \mu\text{Ci}$ each calculated in accordance with the power function $R = 0.54 t^{-0.52}$.

The availability of high-speed electronic computers permitted us to analyze the data in which each radium measurement could be considered as the sum of the individual amounts remaining from each of the 15 to 36 injections administered a specified number of days before the measurements. A computer program, written for the CDC-3600 computer by Dr. Ronald Hamelink, and which employed the Davidon Variable Metric Minimization (VMM) technique,¹¹ calculated the parameters a and b for the power function that would yield the best fit to each set of data for which the number of weekly injections is specified. For each best fit, the program also calculated the amount that each patient contained immediately after the last injection, the amount present on the day of each measurement, and the amount present at any arbitrary time after the last injection.

The parameters of a single exponential function that provided the best fit to the data were also determined. This function has the form,

$$\text{Retention} = a e^{-\lambda t / T_{1/2}}, \quad (2)$$

where a is the zero time intercept, t is time in days, λ is 0.693146, and $T_{1/2}$ is the half-time of the exponential term in days. A computer program that also employed the Davidon VMM function was written by Mr. Donald F. Jordan for this analysis. Since an exponential function is linear along the time axis, the number of injections did not have to be considered here. No attempt was made to fit the data with the sum of a series of exponentials because of the limited number of measurements available, especially at time intervals soon after the injections when the whole-body radium values would have been changing rapidly.

Since the individual measurements for each patient contain errors that are dependent upon physical and physiological conditions at the time of a particular measurement, and which may have been absent on other occasions, the parameters that provide the best fit to the entire series may be unduly influenced by one divergent measurement. The set of parameters that provided the best fit to the greatest series of measurements was considered to represent the retention for the particular patient. The various combinations of measurements that were used, the parameters that provided the best fit to these data, and the resultant discrepancies between calculated and measured values will be discussed separately for each patient, since each case is characterized by its own set of problems.

The same format for presentation of analyses will be followed for each patient. All of the original and adjusted measurements for the patient are given in Table A of each set. Curve-fitting to original data for each patient is given in Table B, while that for adjusted data is presented in Tables C and D.

In each Table B, the first two columns list the number of days since the last injection and the measured values for total radium. Various combinations of these measured values were fitted with a power function and with a single exponential function by the methods described above. The values computed by the best fit in each case is listed for those days where the measured data were used. The appropriate parameters are given for each computation along with two indices of goodness of fit, the mean error sum and the mean percent deviation between calculated and measured data.

The mean error sum is a calculated error function that is the average of the square of the difference between the computed and measured values divided by the computed value. This term is similar to chi square used for frequency distributions. This expression weights more heavily the fit to the larger quantities at earlier times than the fit to the smaller quantities at later times. The mean percent deviation, on the other hand, weights the percent fit to all points equally.

Similar computations are listed in each Table C for whole-body radium values adjusted from radon breath values and in each Table D for whole-body radium derived from gamma-ray measurements. The results for each patient will be discussed in detail seriatim.

Patient 03-101

Clinical Summary. Female. Born 1908. Onset of illness in 1928 when she became depressed and exhibited peculiar behavior. Admitted to ESH in May, 1930 with diagnosis of dementia praecox, paranoid type. Radium chloride, total of 150 μ g reputedly given intravenously in 15 weekly

injections from March to July, 1931,¹ although hospital progress notes indicate that she may still have been receiving weekly radium injections as late as September and November, 1931. Listed as Patient A in previous papers.^{1,2}

Discussion. We have used 30 days as the interval between the last injection and the first measurement even though Schlundt reported 6 months and Norris employed 201 days. An entry on this patient's hospital progress report dated September 8, 1931 states, "Wing Cottage 2, receiving Ra chloride weekly," and an entry on September 28, 1931 states, "Is receiving radium chloride intravenously." Both of these entries were signed by the same resident physician. Another physician on November 17, 1931 wrote, "Wing Cottage, receiving radium chloride." After a recent conversation with the physician who signed the first two entries cited above, we concluded that these notes made by two resident physicians are more reliable than the dates published by persons not directly associated with the day-to-day clinical aspects of this research and who were involved principally with the radium measurements during the last week of December, 1931 and in June, 1932. Accordingly, we have assumed that this patient definitely received weekly radium injections from September 8 through November 17, 1931, or 11 injections during a period of at least 10 weeks. A total of 150 μCi given in 15 injections would have covered a span of 14 weeks. If the injection program had been under way for two weeks on September 8, two more weeks after November 17 would have closed the injection span on December 1. Since the measurements were made during the last week of December, we have arbitrarily chosen 30 days as the interval between the last injection and first measurement. We have also assumed that 150 μCi were injected in accordance with the value given in Schlundt's private reports and publications, since the total amount administered is not given in the hospital records.

Eight measurements are accordingly available for patient 03-101 from the 30th day to the 11,709th day after the last injection (Table 1A). Both breath and gamma-ray measurements were made on the first four occasions, gamma-ray only on the fifth, sixth and eighth, and breath only on the seventh occasion.

Three subsets of these original measurements have been fitted with both a power function and a single exponential function. The parameters for the power function that provide the best fit to all eight measurements are given in column 3 of Table 1B. With this fit, the computed value at 30 days is 10.2 percent low, while that for the 213th day is 42.25 percent high.

When the 213-day measurement was omitted from the sequence and the remaining seven measurements were fitted with a new power function,

the slope for this combination was -0.3368 , which compares fairly well with the slope of -0.3220 found when the second point was included. The value of a , the one-day intercept, was 0.2291 in the latter case, in contrast to 0.1931 when all eight points were included. When the second point was omitted the mean error sum and mean percent deviation were considerably smaller than the corresponding values found when all eight points were included.

The reverse combination was used in which the first value was omitted and the second value included. The slope of the power function for this combination is -0.2160 and the value of a is 0.0756 . The mean error sum and mean percent deviation were practically identical when either the first or the second points were omitted and were much lower than those found when all eight points were included. However, the slopes of the retention curve and the amount retained on the first day differ greatly. Thus, the data for Patient A can be used to demonstrate that the slope of the power function is either -0.2160 or -0.3368 , depending upon which measurement is omitted, or -0.3220 when all the points are included.

In the absence of any justifiable reason for discarding any of these measurements, the parameters obtained for the complete set of eight measurements, designated by an arrow (Table 1B), were entered in the summary table. Similar parameters from each patient will be selected to obtain the best general set of parameters.

The same three combinations of measurements that were fitted with a power function were also fitted with a single exponential. The exponential that provided the best fit to all eight values had a half-time of 16.136 years and a value at day zero of 4.809 . The values computed by this optimum exponential term yielded a value that was 41 percent low for the 30-day measurement and 37.9 percent high for the 213-day measurement.

In view of the results of the many retention studies with animals, one would not expect a single exponential term to fit the data from early times onward. Nevertheless, the parameters for an exponential term were determined that provided the best fit to the seven remaining measurements when the 213-day value was omitted and again when the 30-day value was omitted. The half-time of the exponential for the first combination was 11.128 years, and it was 24.107 years when the 30-day measurement was omitted.

The single exponential provided almost as good a fit to the data from 213 days through $11,876$ days as did the power function. However, the exponential function did not fit the seven points that included the 30-day value and omitted the 213-day value as well as did the power

function, nor did it fit quite as well as did the power function when all eight points were used. The parameters for the exponential term found to fit the data from 213 days onward were entered in the summary Table 16.

The adjusted radium values based on the breath radon measurements are given in column 2 of Table 1C. The parameters for the power function that provided the best fit to these five measurements are given in column 3. The slope of the power function is -0.3419 , in comparison with a slope of -0.3220 when the original values for total radium were used. It was impractical to fit these data with a single exponential since the value obtained for the last measurement on the 11,515th day was equal to the value of the measurement on the 7,118th day, and both values were considerably below that on the 7,878th day.

The adjusted total-body radium calculated from the gamma-ray measurements is given in column 2 of Table 1D. The results of fitting the power function are given in column 3. Here, the calculated values agreed very well with the measured values for the 30-, 213- and 10,264-day measurements and were quite close for the 7,118-, 7,878- and 10,108-day measurements. The power function greatly overestimated the 11,709-day value.

All seven measurements adjusted from gamma-ray measurements were also fitted with a single exponential (Table 1D, column 4). The best fitting exponential had a biological half-time of 12.724 years and a zero-day value of 6.782. The calculated values found with this exponential term underestimated the measured value on the 30th day by 16.7 percent and overestimated the value on the 213th day by 15.2 percent. When the 30-day point was omitted and the remaining six values fitted with a single exponential, the best fitting exponential had a biological half-time of 14.496 years and a zero-day value of 5.760. Significantly, all of the values from 213 days on fit the measurements quite well and had a mean percent deviation of only 5.76 percent. The parameters of this fit were entered in the summary table. Omission of the first two values yielded a somewhat longer half-time without a corresponding improvement in goodness of fit.

When plotted on log-log paper, the first four gamma-ray measurements seemed to follow one fairly flat power function while the last five measurements appeared to follow a much steeper power function. Consequently, the first four and the last six measured values were each fitted with separate power functions as were the first four and last five values derived from gamma-ray measurements. The parameters and calculated values for these combinations are given in Table 1E. While the measured values support power functions of similar slopes, the gamma-ray data can be fitted very well with two extremely different

power functions. Here, the first four measurements fall along a slope of -0.2982 and the last five along a slope of -0.8443 . It was in fact considerations such as these that prompted us to reexamine the single exponential as a description of the later phases of radium retention.

Patient 03-102

Clinical Summary. Male. Born 1905. Onset of illness dated to 1928 when he became seclusive and destitute. Admitted to ESH in October, 1930 with diagnosis of dementia praecox, hebephrenic type. Radium chloride, total of $150\text{ }\mu\text{g}$, reputedly given intravenously in 15 weekly injections from March to July, 1931.¹ Transferred to Manteno State Hospital in 1935. Most recent examination at Argonne National Laboratory on February 18, 1964. Listed as Patient B in previous papers.^{1,2}

Discussion. Radium was never mentioned in this patient's hospital records at ESH. Consequently, we have used Schlundt's value of 183 days (six months) as the elapsed time between the last injection and the first measurement. The fact that this patient received the same number of injections as Patient A (03-101) and that the same injection period was given for both patients might be used to justify the use of the same 30-day interval that we used for patient 03-101. However, in the absence of any supporting entries in the hospital record we have followed Schlundt's reported interval.

The data for patient 03-102 are given in Tables 2A-D. Both breath radon measurements and total-body gamma-ray determinations are available for the first three measurements (Table 2A), while only total-body gamma-ray measurements are available for the last two dates. The values for both the breath radon and the total-body gamma-ray for the measurement 7,261 days after the last injection are obviously incorrect since the sum of the two measurements and the total-body radium calculated from either measurement are both considerably less than the radium values found at two later dates by modern techniques. We have accordingly omitted this point from the analysis.

Table 2B gives the power function parameters for various combinations of these measurements as well as the quantities computed to be present at the time of each measurement by the power function fitting the data best. Although the fit was very good when both the 183- and 7,261-day values were omitted from the computation, the power function should apply to early measurements as well as later ones. For this reason, the slope selected for the final tabulation was -0.5549 , since it was computed from four rather than three points, with only the 7,261-day value legitimately omitted. When the three measurements starting at one year were also fitted with a single exponential term (column 7), the values yielded a biological half-time of 13.55 years.

The radium values adjusted from the radon breath measurements are given in column 2 of Table 2C. The resultant parameters, mean errors and computed values when four combinations of values were fitted with a power function and two combinations with a single exponential are given in the appropriate columns. The parameters are very close to those found when the original measurements were used (Table 2B) since the total-body content as determined by the breath radon measurements is very close to the total-body values.

The radium values adjusted from the gamma-ray measurements are given in column 2 of Table 2D. The resultant parameters, mean errors and computed values when four combinations of values were fitted with a power function and three combinations with a single exponential are given in the appropriate columns. The results obtained with the power function will be discussed first.

As in Table 2A, the 7,261-day value (Table 2D) is obviously in error since it is considerably lower than the later two measurements. When this value is omitted and the remaining four values employed, the values calculated with the optimum power function are low for the first, fourth and fifth measurements (third omitted) and high for the second value. This observation suggests that the second value (356-day) was low, or the first value high. Consequently, the second, fourth and fifth values and the first, fourth and fifth were fitted with a power function. The parameters obtained for the latter set of values agree substantially with those found when five or four values were used but are substantially smaller than those found to fit the original or adjusted breath values. The extremely low value of λ found when the second, fourth and fifth values were used suggests that the 356-day value is in fact low. Nevertheless, the parameters found to fit the set of four will be used in the summary since there is no proof that the first value is not high.

When the four values are fitted with a single exponential term, the calculated value for the 183rd day is 36.6 percent low and the value for the 356th day is 37.8 percent high (column 7). Omission of either the 356-day value (column 8) or the 183-day value (column 9) results in a very good fit to the remaining three values with exponentials of very different half-times. On the basis of the values obtained for either set of three values, there is no reason to select either set of parameters for inclusion in the summary table. In view of the fact that the power function analyses seemed to indicate that the 356-day value was low, one might be tempted to select the parameters obtained when the 183-day value was included. However, it is unrealistic to employ the fit obtained with the one function to justify a fit obtained with the other function if the main purpose of the analysis is to evaluate the two functions. For summary purposes, the intermediate half-time given by the curve for fewer points was selected.

Patient 03-109

Clinical Summary. Female. Born 1904. Onset of illness in 1928 with three brief hospitalizations before admission to ESH in November, 1930 with diagnosis of dementia praecox, hebephrenic type. Radium chloride, total of 180 μ g, reputedly given intravenously in 18 weekly injections from March to July, 1931.¹ No recent examination at Argonne National Laboratory. Death from carcinoma of bladder in September, 1957. Listed as Patient I in previous papers.^{1,2}

Discussion. A notation on the hospital record on May 29, 1931 states, "She received 10 treatments of radium chloride." If this patient received a total of 18 injections as Schlundt states, and if the injections were begun in the middle of March, the tenth injection would have been given within the week preceding the entry in the hospital record, and the 18th injection would have been given about August 25, 1931. Consequently, we have assumed an elapsed time of 155 days between the last injection and first Schlundt measurement instead of the 6-month period stated by Schlundt and used by Norris.

The measured and adjusted radium values for patient 03-109 are given in Table 3A. Only four sets of breath and total-body measurements, two sets by Schlundt and two by Norris, are available on this patient, who died in 1957. The four total-body values were fitted best with a power function that had a slope of -0.6271 and a one-day intercept of 0.8927 (Table 3B). This power function underestimates the 155-day value by 15.8 percent and overestimates the 338-day point by 98.5 percent, a situation that resembles the adjusted gamma-ray data of patient 03-102. When the 338-day value is omitted, the slope of the power function remains almost unchanged but the one-day intercept increases to 1.0655 or greater than one. An intercept value greater than one implies that more radium was present in the patient's body one day after injection than was administered. Since this is impossible, this power function may be used to express radium retention after long intervals but it fails at very short times after injection. When the 155-day value is omitted, the value of b is reasonable but the value of the one-day intercept is extremely small (0.0813). Since neither set of three measurements yielded realistic parameters, those obtained when the total series of four were employed must be chosen as representative for this patient.

When the total series of four measurements were fitted with a single exponential, the calculated 155-day value was less than one-half of the observed value and the calculated 338-day value was 1.57 times the observed value. With the 155-day measurement omitted, the remaining three measurements were fitted with a single exponential, and a half-time of 13.71 years was obtained and, as expected, the curve fit the data quite well since only three points were used.

Similar calculations were made for radium values adjusted from breath radon measurements (Table 3C) and substantially identical results were obtained.

For the adjusted gamma-ray analysis only three values could be used since the 338-day value, which was lower than the 7,243- and 8,003-day values, was undoubtedly incorrect. The parameters for the power function and the single exponential that fit these three points are given in Table 3D.

Patient 03-110

Clinical Summary. Female. Born 1899. Onset of illness in 1929 with hospitalization at private sanitarium. Admitted to ESH in August, 1930 with diagnosis of dementia praecox, paranoid type. Radium chloride, total of 200 μ g reputedly given intravenously in 20 weekly injections from March to July, 1931.¹ However, hospital progress notes indicate that patient was receiving radium treatments in late January, 1931, and a note dated September 28, 1931 states that the patient had received "thirty doses of radium chloride." Most recent examination at Argonne National Laboratory on August 6, 1964. Listed as Patient J in previous papers.^{1,2}

Discussion. We have assumed that the hospital records were probably correct and have recalculated the elapsed time based on 30 injections instead of 20. If this patient received the first injection during late January (assume January 22, 1931) and had received a total of 30 injections, the last injection would have been given on August 18, 1931, or 130 days before Schlundt's first measurement. Consequently, our computations were based on 30 weekly injections of 10 μ g each between the above dates. Schlundt stated that six months had elapsed between the last injection and his first measurement while Norris² employed an elapsed time of 166 days and a total of 20 injections.

The seven measurements available on patient 03-110 are given in Table 4A. Both radon breath measurements and total-body gamma-ray measurements are available for the first four measurements. Only total-body gamma-ray measurements are available for the last three measurements. The parameters and the calculated values for the power function that provide the best fit to these seven measurements are given in Table 4B, column 3, along with the associated error indices. The calculated values agree quite well with the measured values although they are progressively higher than the measured values for the sixth and seventh measurements. This same trend was observed for the gamma-ray measurements of patient 03-101, for whom the measured values after long intervals also fell much faster than that predicted by the power function.

The first four and the last five measured values were fitted with power functions (see columns 4 and 5, Table 4B). A very good fit to each set of data is realized with these two discontinuous, somewhat overlapping power functions. Of particular significance is the fact that the more recent, and probably more accurate, measurements support a power function that has a very steep slope. This power function will undoubtedly fit future measurements much better than that found to fit the series of all seven measurements. Nevertheless, the parameters found to fit all the data will be included in the summary table.

Since a single exponential term could not fit both of the early measurements and also the last terms, the first measurement was omitted and the remaining six values employed (column 6). In addition, the last five measurements from 7,218 days onward were fitted with the exponential. The last five measurements could be fitted quite well with a single exponential term whose half-life was 16.400 years and whose zero-day value is 2.272. This exponential term underestimates the 30-day measured value by a factor of 2.7 and the 313-day value by a factor of 2. Although the retention of radium by this patient from 7,218 through 12,037 days definitely follows a 16.4-year half-time, the value of 10.3 years, which was found when the 313-day value was included, will be used in the summary table.

As found for the directly measured values, the power function fits all the adjusted breath data (Table 4C, column 3) from 130 days through 10,202 days fairly well and then progressively overestimates the last two values. The parameters found to fit the breath data, particularly the value of a , are larger than those found to fit the measured values. Also, as with the measured values, the first four breath (column 3) values can be fitted very well with one power function and the last five (column 5) with a second power function.

The last five values, from 7,218 to 12,037 days, can be fitted with an exponential term of 17.144-year half-time. The breath values at 130 and 313 days are about three and two times, respectively, the value predicted by this single exponential. Again, the half-time of 9.32 years found by using all the data, starting at 313 days, will be used for summary purposes.

The radium values adjusted from the gamma-ray measurements are given in column 2 of Table 4D. The value for the 313th day is obviously wrong since it was less than the four values between the 7,218th day and the 11,806th day. The remaining six gamma-ray values, when fitted with a power function, yielded a slope of -0.2702 and a one-day intercept of 0.0286. This extremely low one-day intercept would suggest that this power function would not be valid at very early times. The last five measurements from 7,218 days through 12,037 days could be fitted with a

power function (Table 4D, column 3) that has a slope of 1.222 and a one-day intercept value of 183.50. Obviously, this retention curve is only valid over the time span of these measurements and cannot be projected back to early times. As a result, these gamma-ray data do not support a power function as an expression for retention.

On the other hand, the six valid gamma-ray measurements can be fitted very well with an exponential term (column 5, of Table 4D) whose half-time is 18.33 years; the computed magnitude at day zero is $2.02 \mu\text{Ci}$. The calculated values with this exponential agree quite well with the measured values for all times including 130 days after injection. The parameters of the exponential that provide the best fit to the last five measurements are fairly close to that found for the six measurements that include the 130-day measurement. Extrapolation of this latter exponential term back to 130 days overestimates the amount present at that time by about 20 percent. In view of the agreement between these two sets of parameters, the parameters based on the set of six that includes the 130-day value will be included in the summary table.

The fact that the analyses of the measured values and of the breath values yield similar parameters for the power function might be interpreted to suggest that these values were correct. Actually, such agreement will exist for any set of data in which the gamma-ray values are low. In the case of this patient, for example, the radium value adjusted from the breath radon on the 130th day would be expected to be 84.7 percent of the total. The omission of the expected gamma-ray value from the measured total would yield a value only 15.3 percent below that calculated from the breath fraction. Disagreement will exist only between the parameters found for the measured and breath values if either the gamma-ray values are extremely high (by a factor of three or more) or if the breath value is appreciably low.

Patient 03-111

Clinical Summary. Female. Born 1909. Onset of illness in 1928 and admitted to ESH in January, 1930 with diagnosis of dementia praecox, hebephrenic type. Radium chloride, total of $200 \mu\text{g}$, reputedly given intravenously in 20 weekly injections between March and July, 1931.¹ A hospital progress note dated May 29, 1931 states that the patient had received 10 injections of radium chloride. Most recent examination at Argonne National Laboratory on December 18, 1963. Listed as Patient K in previous papers.^{1,2}

Discussion. We have assumed that the hospital progress note of May 29, 1931, which states that the patient had received 10 injections of radium chloride, to mean that the patient had received only 10 injections before this date. Consequently, if the patient received 20 weekly

injections, the last injection would have been given by August 7, 1931. The first Schlundt measurement during the last week of December would have occurred about 141 days after the last injection instead of six months as stated by Schlundt, or the 166 days employed by Norris.

The radium contained by patient 03-111 has been measured six times during the past 34 years (Table 5A). Schlundt listed one gamma-ray determination, presumably performed 141 days after the last injection. Evidently, this patient must have been one of the uncooperative ones that he referred to when he said, "Some cannot be induced to blow at all."¹ Norris gave both breath and gamma-ray determinations for the 7,229th and 7,989th days.² On each of these dates, 0.7 μ Ci was found by breath measurement and 0.22 μ Ci by gamma-ray counting for a total-body content of 0.92 μ Ci. The fact that identical values were found on both dates would seem to suggest that these values were fairly accurate. This patient was counted in the Argonne National Laboratory whole-body counter on three occasions subsequently. The first two of the recent gamma-ray measurements yielded higher values than the totals given by Norris or than the whole-body contents calculated by us with either the Norris breath or gamma-ray values. Consequently, only the one Schlundt gamma-ray value and the three recent gamma-ray values are suitable for analysis.

The parameters, calculated values and mean errors, when these four values are fitted with a power function and an exponential, are given in Table 5D (data are not available for Tables B and C). The power function parameters, especially the intercept, seem to be low although the exponential parameters are comparable with those found for other patients.

In view of the limited number of suitable measurements and the two omitted low values, the computed parameters are not included in the summary table but are included here for the sake of completeness in case other investigators study this patient at a later date.

Patient 03-112

Clinical Summary. Female. Born 1899. Onset of illness several years before admission to ESH in January, 1931 with diagnosis of dementia praecox, hebephrenic type. Radium chloride, total of 260 μ g, reputedly given intravenously in 26 weekly injections between March and September, 1931.¹ Transferred to Galesburg State Research Hospital (Illinois) in January, 1956. Most recent examination at Argonne National Laboratory on April 6, 1960. Listed as Patient L in previous papers.^{1,2}

Discussion. Radium is not mentioned in the hospital progress notes for this patient. Consequently, the amounts administered and elapsed times given by Schlundt must of necessity be employed.

The five sets of radon breath and gamma-ray measurements that are available for patient 03-112 are given in Table 6A. The radon breath measurement made on the 307th day is obviously incorrect since both the total-radium content (column 4) and the calculated radium content (column 7) based on it for this day are less than the quantity for the three later measurements.

The parameters that provide the best fit to the four original values are given for both the power function (column 3) and the exponential function (column 4) of Table 6B. If the mean errors between calculated and measured values are used as criteria for goodness of fit, either expression fits the data very well. However, the metabolic patterns suggested by the two functions are entirely different. The power function implies that approximately 80 percent of the radium is eliminated during the first day and the remaining radium is eliminated very slowly during the next 30 years. The exponential expression, on the other hand, has a half-time of 19.114 years, which is very close to that found for other patients. However, the zero-time intercept of the exponential term represents 15.4 μCi , a very much larger value than that found for the other patients. Thus the exponential term suggests that this patient retained a much larger percentage of the injected dose than did the other patients but then eliminated this absorbed quantity at the same rate. Similar parameters are given by the curve-fitting to the total-radium values calculated from the radon breath measurements (Table 6C).

The total-body radium calculated from the gamma-ray measurements is given in column 2 of Table 6D. Five values that included the 307th day measurement were fitted with a power function (column 3) and a single exponential (column 6). The value calculated with the optimum power function for the 124th day is 8.04 percent low, while the value calculated for the 307th day is 16.77 percent high. The slope of the power function was -0.1906 or slightly less than that based on the original total values or on the adjusted breath measurements. The 124-day value calculated from the optimum exponential term was 16.70 percent low, while the 307-day value is 18.18 percent high. If the mean percent deviation is used as a criterion of goodness of fit, the single exponential provides a better fit to the data since it fits the last measurement much better than does the power function. The half-time of the exponential function is 25.904 years.

The 307-day measurement was omitted and the remaining four values fitted with both a power function and a single exponential (columns 4 and 7). The latter, which has a half-time of 20.096 years, fits these four points better than does the power function since the exponential does not diverge from the actual data at long times as the power function does. Similarly, if the 124-day value is omitted, the exponential provides a better fit than the power function to the remaining four values.

The parameters of the exponential that fit the last three measurements are given in column 9, Table 6D. It is interesting and significant that the parameters found when these three values are used are essentially the same as when the 124-day value is included and the 307-day value omitted. Consequently, the parameters obtained for the series of four that include the 124-day measurement will be included in the summary table.

This patient is unique in this small series in having either a small amount of radium retained tenaciously (small a and b in power function) or in having a relatively large proportion of the initially administered radium retained with the same half-time as the rest of the group (large a and average $T_{1/2}$ in the single exponential). Interestingly, this patient resided in Wing Cottage at ESH, in which all patients were housed while they received radium, until at least February 26, 1934. In view of the fact that a search of hospital records has revealed patients who received radium during 1933, this patient may have received additional injections after Schlundt's measurements.

On three occasions during the years 1932-1934, x-rays were made of the sella, and on one occasion of the knee and elbow for epiphyseal lines. No reasons are given in the hospital notes for requiring these x-rays but an endocrinopathy that might have affected growth and development may have been suspected. Although the sella and epiphyseal lines were normal there may have been some extremely high retention of radium by this patient.

Patient 03-114

Clinical Summary. Female. Born 1900. Onset of illness early in 1931 with commitment to ESH in May, 1931 with diagnosis of dementia praecox, catatonic type. Radium chloride, total of 360 μg , reputedly given in weekly intravenous injections between March and July, 1931.¹ On the other hand, a hospital progress note dated September 28, 1931 stated that the patient was receiving 10 μg RaCl_2 intravenously weekly. Most recent examination at Argonne National Laboratory on August 5, 1964. Listed as Patient N in previous papers.^{1,2}

Discussion. We have assumed, rather arbitrarily, that this patient received 30 injections and that 30 days elapsed between the last injection and Schlundt's first measurement even though he gave a 6-month period and Norris used 180 days. In the report on this patient to the radium supplier, Schlundt stated in the first paragraph that this patient was admitted to the hospital on May 8, 1931 and in the second paragraph that she had received 360 μg between March and July, 1931.

The March-to-July injection period is obviously incorrect for several reasons. As verified by the hospital records, the patient was not

admitted to the hospital until May 8, 1931 so that the series of injections could not have started prior to that date. Furthermore, even if the patient had been admitted before March 1st, 36 weekly injections could not have been given in the 22-week-long period from March 1 to July 31. The possibility that the patient received 36 injections twice weekly during the March to July period is refuted by the note on the progress sheet dated September 28, 1931, "Is receiving 10 μg RaCl_2 intravenously weekly." Significantly, this note was signed by the resident physician.

The 360- μg total given by Schlundt is probably also incorrect. If the patient had received weekly injections as stated on the progress sheet and if the series was started within the first week of admission, only 33 injections could have been given before December 25, 1931. From the dates given or surmised for other patients, we have concluded that no injections were given after about December 1. Consequently, this patient could only have received 30 weekly injections and may have received less. The data have been analyzed then on the assumption that 30 weekly injections were administered between May 10, 1931 and November 30, 1931.

Measurements made on eight different dates for patient 03-114 are given in Table 7A. Both radon breath and total-body gamma-ray measurements are available for the first four dates, while only total-body gamma-ray measurements are available for the last four dates. From inspection of the data it is obvious that the radon breath measurement made on the 30th day is incorrect since both the total radium and the adjusted radium values based on it (column 7) are considerably lower than the 213-day total, as well as the adjusted radium calculated from the gamma-ray measurement on the 30th day (column 8). The 213-day radon breath measurement would appear to be slightly high; however, it must be included in the analysis since there is no valid reason for omitting it.

Curve-fitting data for the original measurements are given in Table 7B. Despite the possibility that the 213-day value is too high and the 30-day value is correspondingly low, the entire set of eight values were fitted with a power function (column 3 of Table 7B). The value of a , the one-day intercept, is smaller than one would expect. Parameters for the power function and for the single exponential that provide the best fit to the seven measurements beginning with the 213th day are given in columns 4 and 7, respectively. The magnitudes of the parameters of the power function found to fit these values seem reasonable. Interestingly, the single exponential provides a considerably better fit to the data than the power function, as demonstrated by either the mean error sum or mean percent deviation. The first two values (Schlundt measurements) were omitted, and the remaining six values fitted with both a power function (column 6) and a single exponent (column 8). When these early values are omitted, the resultant power function has a very

much steeper slope than when one or both of the early measurements were included, while the exponential has a lower slope than if only the 213-day value (column 7) is used to anchor the low end and a slightly steeper slope than if the 30-day value (column 9) were included.

As with the other patients, the single power function (column 4) fits the Schlundt measurements (only one used in this case) very well, underestimates Norris' measurements and then progressively overestimates the later measurements. The second, third and fourth measurements can be fitted almost exactly by a power function, $R = 0.2419 t^{-0.4192}$ (column 5) while the third through eighth measurements can be fitted quite well with a very steep power function, $R = 185.25 t^{-1.1626}$. Consequently, the use of two discontinuous power functions may be an advantage in interpreting the data of this patient.

The results obtained when the radium values calculated from the breath measurements are fitted with a power function or an exponential are practically identical (Table 7C) with those found when the original measured values were analyzed.

The total-body radium values adjusted from the total-body gamma-ray measurements are given in column 2 of Table 7D. The results of fitting these data with a power function and a single exponential term are given in columns 3 and 6, respectively. Again, the single exponential expression provides a better fit to the data than the power function, especially when the 30-day value is excluded. The power function does provide a better fit to the 30-day and the 213-day values than the single exponential but it does not fit the last five measurements well. It underestimates the measurements at 7,118 days and 7,878 days and progressively overestimates the value for the last three measurements. On the other hand, a single exponential fits the last five measurements very well from 7,118 days on, although it underestimates the 30-day measurement and overestimates the 213-day measurement. The seven values from 213 days on can be fitted very well with a single exponential (column 7).

When the first two values are omitted, the remaining six measurements can be fitted very well with a single exponential (column 8). Here, the mean percent deviation between the calculated and observed is only 2.37 percent, and, significantly, the parameters are almost identical to those obtained when the 213-day value was included. Thus the gamma-ray measurements of this patient from 213 days through 11,941 days can be fitted with a single exponential of 12.66 year half-time.

As with the unadjusted measured values, the first four measurements can be fitted with one power function where the slope is fairly flat ($b = -0.3526$) while the last six measurements can be fitted with a second power function where the slope is very steep ($b = -1.3916$).

Patient 03-115

Clinical Summary. Female. Born 1906. Onset of illness six months before admission to ESH in March, 1931 with diagnosis of dementia praecox, catatonic type. Radium chloride, total of 260 μ g, reputedly given intravenously in 26 weekly injections from March to December, 1931,¹ although one hospital progress note dated June 16, 1931 states, "Transferred to Wing Cottage for treatment," and another note dated September 8, 1931 states, "Has received six radium chloride treatments to date." This note implies that injections began late in July, 1931. Most recent examination at Argonne National Laboratory was on August 6, 1964. Listed as Patient O in previous papers.^{1,2}

Discussion. The 23-day elapsed time between the last injection and first measurement as well as the 260 μ Ci total radium injected reported by Schlundt were used in our analyses. These values imply that the first injection was given about June 14 if the patient actually received 26 weekly injections. This date does not agree with either Schlundt's March date or the implied date of late July, but it is not impossible in view of the June 16, 1931 note, "Transferred to Wing Cottage for treatment" (note past tense). If this patient had in fact received only six injections by September 8, 1931 and continued to receive weekly injections until about December 3, 1931, she could have received only 18 or 19 injections. For these reasons we have assumed that 26 weekly injections were given from about mid-June to about December 7, 1931.

All of the measurements available on patient 03-115 are given in Table 8A. Both radon breath and total-body gamma-ray measurements are available for the first four dates and only gamma-ray measurements for the last three times. The 7,871-day breath (and the resultant total) values are slightly higher than the earlier 7,111-day breath and resultant total values. However, from inspection of these data, it cannot be arbitrarily stated that any particular value is incorrect.

Curve fitting to the original total-body radium values is given in Table 8B. Parameters for the power function that provide the best fit to all seven measurements and the mean errors are given in column 3. Calculated values from this optimum power function for the 23rd day are 7.5 percent low while the value for the 206th day is 30.07 percent high. The parameters that provide the best fit to three other combinations of measurements are given in columns 4, 5 and 6. The slope of the power function is not changed greatly by these manipulations. Parameters for the single exponential that provide the best fit to five measurements (with the 23-day and 7,111-day measurements omitted) are given in the last column of Table 8B. These results demonstrate that the measurements for this patient can be fitted quite well from 206 days on with a single exponential of 14.14 year half-time.

Curve fitting to the total-body radium values calculated from the breath measurements are given in Table 8C. Parameters for the power function that provide the best fit to the seven measurements are given in column 3. This power function, which overestimates the 206-day value by 34.81 percent, fits the other six values quite well. The 206-day value was omitted and a very good fit obtained for the remaining six values with a power function (column 4). The 23-day value was omitted and an equally good fit obtained for the six values that included the 206-day value (column 5). The magnitudes of these parameters are considerably smaller than those obtained for the previous two combinations, and the extremely small one-day intercept would seem to indicate that the 206-day value is low. However, in lieu of any proof that the 23-day value is any more accurate than the 206-day value, the parameters found to fit the entire series of seven values will be incorporated in the summary table.

Of the numerous combinations of data fitted with an exponential term, the parameters, calculated values and mean errors for three are given in columns 6, 7 and 8. The best fit is obtained to the series of five that omits the 23-day and 7,111-day values.

Curve fitting to the total-body radium values adjusted from the gamma-ray measurements is given in Table 8D. Parameters for the power function that provide the best fit to the seven measurements are given in column 3. As observed for other patients, the calculated values with this optimum power function underestimates the first measurement, overestimates the second, underestimates the 7,111-day and 7,871-day measurements and progressively overestimates the last three measurements. Omission of the 23-day value (column 4) does not lead to an improved fit. The first four measurements can be fitted fairly well with a power function whose slope was -0.4539 (column 5), and the last five measurements can be fitted very well with a very steep power function (slope = -1.1187) (column 6). In the case of the first four values, the power function overestimates the 206-day value by 17 percent.

Three combinations of the gamma-ray measurements were fitted with an exponential term. A single exponential term cannot fit both of the early values (column 7) nor would one expect a single term to fit them. A single term of 11.437-year half-time does provide a good fit to the last six values (column 8) that includes the 206-day value overestimated by the power function.

The single exponential that provides the best fit to the last five measurements beginning with 7,111 days after the last injection (column 9) has a slightly longer half-time than that found when the 206-day measurement was included.

In summary, the retention of radium in patient 03-115 from 206 days until 11,930 days after the last injection can be fitted somewhat better with a single exponential term than with a power function.

Patient 03-116

Clinical Summary. Female. Born 1908. Onset of illness in 1928; further deterioration led to admission to ESH in February, 1931 with diagnosis of dementia praecox, catatonic type. Radium chloride, total of 250 μ g; reputedly given intravenously in 25 weekly injections from July to September, 1931,¹ although hospital progress notes imply weekly injections from June through November, 1931. Transferred to Manteno State Hospital (Illinois) in July, 1935. Most recent examination at Argonne National Laboratory on February 18, 1964. Listed as Patient P in previous papers.^{1,2}

Discussion. The 30-day value published by Schlundt for the interval between the last injection and the first measurement should not be in error by more than a few days. A note in the hospital records dated November 17, 1931 states, "Receiving radium chloride weekly." Since Schlundt measured these patients the last week in December, the interval could not have exceeded 45 days and may have been less. A note on June 16, 1931 states, "Transferred to Wing Cottage for treatment" and a note on the next day (June 17, 1931) states, "Ra chloride once weekly." If the first injection was given on June 16, 1931, the 25th injection would have been given on December 1, 1931 and the interval would have been about 30 days. The dates for this patient seem to be the most reliable of this series.

Five measurements for patient 03-116 are given in Table 9A. Both radon breath and total-body gamma-ray measurements are available for the first three values and only gamma-ray measurements for the last two values. Although the total-body values from the Radium B and Radium C values are systematically higher than the total-body radium calculated from the radon breath values, none of the values is obviously incorrect. In fact, the extraordinarily high concentration of radium found in this patient's torso with the 7-crystal-position technique at the time of the last whole-body measurement would yield high gamma-ray values when the patient was measured by the technique employed by both Schlundt and Norris.

The parameters of the power function and the single exponential that provide the best fit to the total-radium values are given in Table 9B. If mean percent deviation is used to evaluate the fit, the single exponential provides a better fit to the total series of measurements than does the power function. The power function, which has an extremely low one-day intercept, fits the 30-day and 213-day measurements very well but

progressively overestimates the three later measurements. On the other hand, the single exponential underestimates the 30-day measurement, overestimates the 213-day measurement, but fits the last three measurements quite well. The four measurements beginning with the 213th day can be fitted very well with a single exponential (column 5) but the half-time for this term is slightly longer, 17.434 years versus 15.554 years, than when all five values are used.

Total-body radium calculated from the radon breath measurements is given in column 2 of Table 9C. The parameters for the power function and the single exponential when all five measurements were included are given in columns 3 and 4. The parameters and mean errors are in very good agreement with those given in Table 9B for curve fitting with the original total radium measurements.

Total-body radium values calculated from the gamma-ray measurements are given in column 2 of Table 9D. Parameters for both the power function and the single exponential that provide the best fit to the five measurements are given in columns 3 and 5. Again, the single exponential provides a better fit to the data than does the power function, which fits very early measurements quite well but progressively overestimates the two later measurements. A single exponential underestimates the 30-day measurement and overestimates the 213-day measurement. Both functions underestimate the third measurement on the 7,118th day, so that this measured value of $3.301 \mu\text{Ci}$ based on the gamma-ray measurement may be too high. The radium value for this date found by radon breath measurement, $2.247 \mu\text{Ci}$, was considerably lower and is much closer to the calculated values for this data.

The 7,118-day value was omitted and the four gamma-ray values fitted with both functions (columns 4 and 6). For these measurements, the power function provides a much better fit than the exponential since it fits all four values while the single exponential term cannot fit both early measurements.

Two other permutations of data were fitted with the exponential terms (columns 7 and 8) and approximately the same parameters obtained. In view of the closeness of the two sets of parameters and of the fact that the best fit was obtained with the power function when the 7,118-day value was omitted, the set based on the three measurements will be used in the summary table.

The parameters of the power functions found to fit these data are very interesting. Up to now, a value of a , the one-day intercept, of 0.10 or lower seemed to us to be unreasonably small and was interpreted to demonstrate that some of the measurements were in error. With patient 03-116 values of 0.1020, 0.1024 and either 0.1244 or 0.1718 were

found for a when the total, breath and gamma-ray data, respectively, were fitted with a power function. The fact that the parameters based on the gamma-rays are higher than the other two sets could be interpreted to demonstrate that the other values are low. However, this patient was found to contain 68.6 percent of her total radium in her torso, whereas if the radium were uniformly distributed in the skeleton this section of the body would contain only 42.0 percent. Under these conditions, the first two gamma-ray measurements could be too high and this fact would account for the steeper slope. Significantly, the data for this patient, for whom the injection schedule seems most reliable, yield small values for a and b.

Patient 03-121

Clinical Summary. Female. Born 1909. Onset of illness in December, 1930 when she became depressed and withdrawn. Admitted to ESH in September, 1931 with diagnosis of dementia praecox, catatonic type. Radium chloride, total of 90 μ g, reputedly given in nine weekly intravenous injections between October and December, 1931.¹ Most recent examination at Argonne National Laboratory on December 19, 1963. Listed as Patient U in previous reports.^{1,2}

Discussion. The total number of injections and interval between the last injection and first measurement published by Schlundt for this patient seem to be fairly accurate. Since she was admitted to ESH on September 16, 1931, the first injection was given sometime after that date. A hospital progress note dated November 18, 1931 states, "Annex 4. Given malaria treatment at Wing Cottage and was then returned to Annex 40." Evidently she was transferred from Wing Cottage by November 18, 1931 and the radium treatments probably terminated. Nine weekly injections could have been given in this interval if the first one was given within the first week after she was admitted. If the radium was administered within this period, the interval to the first measurement was 40 to 45 days instead of the 30 days given by Schlundt. We have used the 30-day interval since the use of 40 or 45 days will not change the parameters greatly.

Eight gamma-ray and two breath measurements are available for patient 03-121 (Table 10A). Since Schlundt did not obtain breath measurements on this patient, and since the two Norris breath measurements yield calculated radium body values that are lower than the total-body radium values based on later gamma-ray measurements performed in a whole-body counter, these breath measurements will not be used.

The complete set of gamma-ray measurements were fitted with a power function (Table 10D, column 3) and a single exponential (column 4). For this patient, the power function overestimates the first value (30 days) and underestimates the second value (213 days). This is the opposite effect noted for previous patients. The power function fits the next three values

fairly well but then progressively overestimates the last three values. The exponential function fits the first two measurements quite well, overestimates the next two and then underestimates the last four values. As judged from the mean error, the exponential still provided a better fit to the entire series of measurements than did the power function.

Three additional combinations of data were fitted with the exponential function. These parameters, calculated values and mean errors are given in columns 5, 6 and 7. Although a fairly good fit was obtained to seven measurements (7,118-day omitted), and to the last five measurements, the half-times of the two exponentials are not very different. These values will not be included in the summary table but are presented for the sake of completeness.

Patient 03-126

Clinical Summary. Female. Born 1911. Onset of illness in 1927 with hospitalization in private sanitarium. Admitted to ESH in December, 1930 with diagnosis of dementia praecox, catatonic. Radium chloride, total of 200 μ g, reputedly given intravenously in 20 weekly injections, presumably in 1931.¹ No mention of radium in ESH records. Upper right thigh amputation in 1956 because of complications resulting from attack by another patient. Died in June, 1965 with carcinoma of sphenoid sinus. One examination at Argonne National Laboratory in July, 1960. Listed as Patient Rose D in previous papers.^{1,2}

Discussion. There is no information available at this time that can be used to calculate the interval between the last injection and the first measurement. In the published report in 1933, Schlundt¹ included the name of this patient with the names of a group that had been discharged and not measured. He did state, however, that she had received 200 μ g. In a private report to the radium supplier dated 1932 (no day or month), which antedated the publication of the results of the ESH cases, he gave the results of measurements made on this patient in the office of a private physician. Why, then, were not these values included in the published paper? Since the name of J. Paul Morris, who assisted Schlundt with the ESH measurements, was included in the private report, it would seem that this patient was in fact measured along with the others at the time of the December visit to ESH. In the case of this patient not only is the injection period not known but the date of the measurement is also unknown. The known data and the assumed time intervals are given in Table 11A.

We first assumed a 166-day interval and fitted the data with both a power function and an exponential (Tables 11B, 11C and 11D). The one-day intercept of the power function for these three sets of data ranged from 0.8978 to 0.9828 (column 3 of each table). Since these values seemed

unrealistically high, an interval of 30 days between the last injection and first measurement was employed. The parameters of the power function found to fit the data for this interval are given in the last column of each table. The one-day intercepts ranged from 0.3277 to 0.3428 and the values of b ranged from -0.4036 to -0.4159.

The parameters of the exponential term found to fit the last two measurements, if the first interval was 166 days, are given in each table. These values, in view of the fact that they are based on only two measurements, one of which was corrected for a missing leg, are not of any particular value.

The parameters for this patient are not included in the summary table used to obtain an average function. The measurements are included here for the benefit of those investigators who may study bone samples that are now available for analysis.

Average Power Function

The parameters of the power functions that were adjudged to provide the best fit to the original data, to the adjusted data based on radon breath data, and to the adjusted data based on gamma-ray measurements are given in Table 12. The parameters are included for the eight patients for whom four or more total, breath, and gamma-ray measurements were available. The parameters for the three patients for whom either gamma-ray measurements alone or for whom only three total, breath and gamma-ray measurements were available will be discussed later (Appendix I).

For the original and breath data, the magnitudes of the coefficient, a , ranged from a low of 0.1020 to maxima of 0.8927 and 0.9766, respectively, and the magnitudes of the exponents, b , ranged from 0.2240 to 0.6347. In general, for each patient the value of b found to fit the breath data agreed quite well with that found to fit the original data. For the gamma-ray data, however, the magnitudes of the coefficient, a , ranged from 0.0286 to 0.5922 and the magnitudes of the exponent, b , ranged from 0.1906 to 0.5650. For five of the eight patients, the exponents and coefficients that were found to fit the gamma-ray data were considerably smaller than those found for the original and breath measurements. The good agreement between the parameters for the breath data and the original measurements does not demonstrate that the former were more accurate than the gamma-ray results but rather reflects the fact that the breath measurement contributed 85 to 90 percent of the total measurement at the time of the Schlundt measurements. Consequently, the total unadjusted value (column 4 of each A table for each patient) will be within 10 percent of the adjusted breath value (column 7, A tables) even if the gamma-ray measurement (column 3) is high by a factor of two, and it will be within

TABLE 1A
Original and adjusted data for patient 03-101(a)

Days since last injection	μCi radium			Measured radon retention ratio	Expected radon retention ratio	Total-body radium calculated from	
	Breath radon	RaB & RaC	Total			Breath radon	RaB & RaC
30(b)	7.20	1.005	8.205	0.12	0.1240	8.219	8.105
213(b)	2.50	0.90	3.40	0.26	0.1578	2.968	5.703
7,118(c)	1.04	0.57	1.61	0.36	0.2875	1.460	1.983
7,878(c)	1.16	0.60	1.76	0.34	0.2928	1.640	2.049
10,108(d)		0.495	1.617(e)		0.3061		1.617
10,264(d)		0.521	1.697(e)		0.3071		1.697
11,515(f)	1.00		1.456(f)		0.3134	1.456	
11,709(d)		0.385	1.225(e)		0.3143		1.225

(a) Patient A, Schlundt et al. (1933) and Norris et al. (1955); 15 injections.

(b) Data from Schlundt et al. (1933).

(c) Data from Norris et al. (1955).

(d) Recent measurements made at Argonne in whole-body counter.

(e) Value for total calculated from γ -ray measurement.

(f) Value for total calculated from radon breath measurement.

TABLE 1B
Curve fitting to original data for patient 03-101

Days since last injection	Measured radium values	Values computed by best power function			Values computed by best single exponential		
30	8.205	7.364	8.208	-	4.792	7.745	-
213	3.40	4.836	-	3.414	4.689	-	3.283
7,118	1.61	1.662	1.728	1.668	2.082	2.313	1.906
7,878	1.76	1.608	1.671	1.632	1.904	2.031	1.796
10,108	1.617(a)	1.485	1.537	1.547	1.464	1.389	1.506
10,264	1.697(a)	1.478	1.529	1.542	1.438	1.352	1.488
11,515	1.456(b)	1.424	1.471	1.504	1.241	1.093	1.349
11,709	1.225(a)	1.417	1.463	1.499	1.213	1.057	1.328
N	8	8	7	7	8	7	7
Power function	{ a b T _{1/2} , years	0.1931	0.2291	0.0756			
		-0.3220	-0.3368	-0.2160			
Single exponential	{ a T _{1/2} , years				4.809	7.785	3.338
					16.136	11.128	24.107
Mean error sum		0.0761	0.0106	0.0118	0.2681	0.0843	0.0156
Mean % deviation		12.90	6.83	7.204	19.68	19.68	8.401

(a) Value from γ -ray measurement.

(b) Value from radon breath measurement.

Arrows indicate parameters selected for inclusion in summary tables 12 and 16.

TABLE 1C
Curve fitting to adjusted data for patient 03-101

Days since last injection	Radium adjusted from breath radon measurements	Values computed by best power function
30	8.219	7.274
213	2.968	4.651
7,118	1.460	1.496
7,878	1.640	1.445
10,108	-	-
10,264	-	-
11,515	1.456	1.270
11,709	-	-
N	5	5
Power function	{ a b	0.2074
Mean error sum		-0.3419
Mean % deviation		0.1572
		19.06

Arrow indicates parameters selected for inclusion in summary Tables 12 and 16.

TABLE 1D
Curve fitting to adjusted data for patient 03-101

Days since last injection	Radium adjusted from Y-ray measurements	Values computed by best power function	Values computed by best single exponential		
30	8.105	8.314	6.752	-	-
213	5.703	5.465	6.570	5.602	-
7,118	1.983	1.883	2.346	2.269	2.106
7,878	2.049	1.823	2.094	2.054	1.956
10,108	1.617	1.683	1.502	1.534	1.574
10,264	1.697	1.675	1.467	1.503	1.550
11,515	-	-	-	-	-
11,709	1.225	1.606	1.183	1.244	1.347
	7	7	7	6	5
Power function	{ a b	0.2173			
		-0.3212			
Single exponential	{ a T _{1/2} , years		6.782	5.760	4.214
			12.724	14.496	19.481
Mean error sum		0.0203	0.0676	0.0116	0.0050
Mean % deviation		8.48	10.93	5.76	6.40

Arrows indicate parameters selected for inclusion in summary tables 12 and 16.

TABLE 1E
Additional curve fitting to original and adjusted data for patient 03-101

Days since last injection	Measured radium values	Values computed by best power function		Radium adjusted from Y-ray measurements	Values computed by best power function	
30	8.205	7.408	-	8.105	8.206	-
213	3.40	4.766	-	5.703	5.562	-
7,118	1.61	1.557	1.751	1.983	2.069	2.118
7,878	1.76	1.505	1.684	2.049	2.007	1.946
10,108	1.617	-	1.533	1.617	-	1.578
10,264	1.697	-	1.524	1.697	-	1.558
11,515	1.456	-	1.459	-	-	-
11,709	1.225	-	1.450	1.225	-	1.394
N	8	4	6	7	4	5
Power function	{ a b	0.2071	0.3427		0.1946	25.398
		-0.3372	-0.3807		-0.2982	-0.8443
Mean error sum		0.1306	0.0123		0.0023	0.0096
Mean % deviation		16.92	7.83		2.52	7.264

TABLE 2A
Original and adjusted data for patient 03-102(a)

Days since last injection	μCi radium			Measured radon retention ratio	Expected radon retention ratio	Total-body radium calculated from	
	Breath radon	RaB & RaC	Total			Breath radon	RaB & RaC
183(b)	5.4	0.6	6.0	0.10	0.1544	6.386	3.886
356(b)	2.805	0.3	3.105	0.10	0.1713	3.384	1.751
7,261(c)	0.39	0.105	0.495	0.22	0.2886	0.548	0.364
10,722(d)	-	0.212	0.685(e)	-	0.3094	-	0.685
11,910(d)	-	0.207	0.656(e)	-	0.3154	-	0.656

(a) Patient B, Schlundt et al. (1933) and Norris et al. (1955); 15 injections.

(b) Data from Schlundt et al. (1933).

(c) Data from Norris et al. (1955).

(d) Recent measurements made at Argonne in whole-body counter.

(e) Value for total calculated from γ -ray measurement.

TABLE 2B
Curve fitting to original data for patient 03-102

Days since last injection	Measured radium values		Values computed by best power function			Values computed by best single exponential	
183	6.0	5.412	5.396	6.000	-	4.196	-
356	3.105	3.884	3.941	-	3.105	4.077	3.101
7,261	0.495	0.712	-	-	-	-	-
10,722	0.685	0.568	0.637	0.691	0.687	0.724	0.726
11,910	0.656	0.534	0.601	0.651	0.655	0.594	0.615
N	5	5	4	3	3	4	3
Power function	{ a b	0.8678	0.7335	0.8446	0.3256		
		-0.5854	-0.5549	-0.5614	-0.4593		
Single exponential	{ a $T_{1/2}$, years					4.326	3.259
						11.386	13.551
Mean error sum		0.0678	0.0634	0.0000	0.000003	0.2137	0.0017
Mean % deviation		22.92	13.14	0.52	0.17	19.14	4.14

Arrows indicate parameters selected for inclusion in summary tables 12 and 16.

TABLE 2C
Curve fitting to adjusted data for patient 03-102

Days since last injection	Radium adjusted from breath radon measurements	Values computed by best power function				Values computed by best single exponential	
183	6.386	5.811	5.794	6.386	-	4.539	-
356	3.384	4.139	4.189	-	3.384	4.404	3.379
7,261	0.548	0.730	-	-	-	-	-
10,722	0.685 ^(a)	0.579	0.638	0.691	0.688	0.726	0.729
11,910	0.656 ^(a)	0.543	0.601	0.651	0.654	0.591	0.611
N	5	5	4	3	3	4	3
Power function	{ a b	1.0029	0.8672	0.9802	0.4144		
		-0.5989	-0.5727	-0.5773	-0.4851		
Single exponential	{ a T _{1/2} , years					4.686	3.562
						10.915	12.826
Mean error sum		0.0566	0.0559	0.0000	0.000006	0.2127	0.0019
Mean % deviation		19.44	12.06	0.58	0.251	18.766	4.45

(a) Value from γ -ray measurement.

Arrows indicate parameters selected for inclusion in summary tables 12 and 16.

TABLE 2D
Curve fitting to adjusted data for patient 03-102

Days since last injection	Radium adjusted from γ -ray measurements	Values computed by best power function				Values computed by best single exponential		
183	3.886	3.328	3.323	3.886	-	2.463	3.881	-
356	1.751	2.554	2.606	-	1.751	2.413	-	1.749
7,261	0.364	0.659	-	-	-	-	-	-
10,722	0.685	0.550	0.635	0.686	0.681	0.710	0.733	0.706
11,910	0.656	0.523	0.609	0.655	0.661	0.617	0.607	0.636
N	5	4	4	3	3	4	3	3
Power function	{ a b	0.2817	0.2292	0.2992	0.0656			
		-0.4677	-0.4299	-0.4502	-0.2877			
Single exponential	{ a T _{1/2} , years					2.517	3.994	1.804
						16.082	11.996	21.691
Mean error sum		0.1091	0.0960	0.0000	0.0002	0.1937	0.0023	0.0004
Mean % deviation		36.28	19.551	0.09	0.44	21.01	4.85	2.07

Arrows indicate parameters selected for inclusion in summary tables 12 and 16.

TABLE 3A
Original and adjusted data for patient 03-109(a)

Days since last injection	μCi radium			Measured radon retention ratio	Expected radon retention ratio	Total-body radium calculated from	
	Breath radon	RaB & RaC	Total			Breath radon	RaB & RaC
155(b)	5.90	0.790	6.69	0.12	0.1519	6.957	5.201
338(b)	1.80	0.108	1.908	0.06	0.1704	2.170	0.634
7,243(c)	0.576	0.198	0.774	0.26	0.2885	0.810	0.686
8,003(c)	0.432	0.198	0.630	0.31	0.2937	0.612	0.674

(a) Patient I, Schlundt et al. (1933) and Norris et al. (1955); 18 injections.

(b) Data from Schlundt et al. (1933).

(c) Data from Norris et al. (1955).

TABLE 3B
Curve fitting to original data for patient 03-109

Days since last injection	Measured radium values	Values computed by best power function			Values computed by best single exponential	
155	6.69	5.631	6.691	-	3.110	-
338	1.908	3.783	-	1.910	2.996	1.912
7,243	0.774	0.607	0.719	0.708	0.732	0.735
8,003	0.630	0.571	0.676	0.684	0.626	0.662
N	4	4	3	3	4	3
Power function	{ a b	0.8927	1.0655	0.0813		
		-0.6271	-0.6279	-0.3405		
Single exponential	{ a $T_{1/2}$, years				3.210	2.003
					9.294	13.711
Mean error sum		0.2952	0.0024	0.0034	0.6346	0.0012
Mean % deviation		36.27	4.79	5.72	29.14	3.42

Arrows indicate parameters selected for inclusion in summary tables 12 and 16.

TABLE 3C
Curve fitting to adjusted data for patient 03-109

Days since last injection	Radium adjusted from breath radon measurements	Values computed by best power function			Values computed by best single exponential	
155	6.957	5.914	-	6.959	3.469	-
338	2.170	3.954	2.173	-	3.333	2.176
7,243	0.810	0.621	0.728	0.736	0.740	0.745
8,003	0.612	0.583	0.701	0.692	0.627	0.662
N	4	4	3	3	4	3
Power function	{ a b	0.9766	0.1136	1.1347		
		-0.6347	-0.3749	-0.6323		
Single exponential	{ a T _{1/2} , years				3.5881	2.2932
					8.701	12.223
Mean error sum		0.2620	0.0069	0.0055	0.5947	0.0031
Mean % deviation		31.34 ↑	8.29	7.38	28.72	5.49 ↑

Arrows indicate parameters selected for inclusion in summary tables 12 and 16.

TABLE 3D
Curve fitting to adjusted data for patient 03-109

Days since last injection	Radium adjusted from γ -ray measurements	Values computed by best power function		Values computed by best single exponential
155	5.201	5.201		5.194
338	0.634	-		-
7,243	0.686	0.699		0.747
8,003	0.674	0.661		0.607
N	4	3		3
Power function	{ a b	0.5922		
		-0.5650		
Single exponential	{ a T _{1/2} , years			5.420
				6.937
Mean error sum		0.0001		0.0040
Mean % deviation		1.28		6.33

TABLE 4A
Original and adjusted data for patient 03-110(a)

Days since last injection	μ Ci radium			Measured radon retention ratio	Expected radon retention ratio	Total-body radium calculated from	
	Breath radon	RaB & RaC	Total			Breath radon	RaB & RaC
130(b)	5.90	0.30	6.20	0.05	0.1530	6.966	1.961
313(b)	4.10	0.10	4.20	0.02	0.1713	4.948	0.584
7,218(c)	0.66	0.32	0.98	0.33	0.2886	0.928	1.109
7,978(c)	0.66	0.24	0.90	0.27	0.2938	0.934	0.817
10,202(d)	-	0.22	0.717(e)	-	0.3070	-	0.717
11,806(d)	-	0.1858	0.590(e)	-	0.3151	-	0.590
12,037(d)	-	0.1721	0.544(e)	-	0.3162	-	0.544

(a) Patient J, Schlundt et al. (1933) and Norris et al. (1955); 30 injections.
(b) Data from Schlundt et al. (1933).
(c) Data from Norris et al. (1955).
(d) Recent measurements made at Argonne in whole-body counter.
(e) Value for total calculated from γ -ray measurement.

TABLE 4B
Curve fitting to original data for patient 03-110

Days since last injection	Measured radium values	Values computed by best power function			Values computed by best single exponential	
130	6.20	6.119	6.068	-	-	-
313	4.2	4.326	4.367	-	4.089	-
7,218	0.98	0.867	0.950	0.996	1.147	0.985
7,978	0.90	0.821	0.901	0.894	0.997	0.903
10,202	0.717(a)	0.717	-	0.685	0.662	0.698
11,806	0.590(a)	0.661	-	0.585	0.493	0.580
12,037	0.544(a)	0.654	-	0.573	0.472	0.564
N	7	7	4	5	6	5
Power function	{ a b	0.4086	0.3483	55.675		
		-0.5564	-0.5283	1.093		
Single exponential	{ a T _{1/2} , years				4.332	2.272
					10.306	16.400
Mean error sum		0.0073	0.0020	0.0006	0.0119	0.0003
Mean % deviation		7.90	2.34	2.58	11.30	1.80

(a) Values from γ -ray measurements.
Arrows indicate parameters selected for inclusion in summary tables 12 and 16.

TABLE 4C
Curve fitting to adjusted data for patient 03-110

Days since last injection	Radium adjusted from breath radon measurements	Values computed by best power function			Values computed by best single exponential	
130	6.966	7.050	6.992	-	-	-
313	4.948	4.870	4.915	-	4.799	-
7,218	0.928	0.879	0.961	0.982	1.177	0.969
7,978	0.934	0.829	0.909	0.885	1.008	0.891
10,202	0.717(a)	0.718	-	0.685	0.641	0.696
11,806	0.590(a)	0.659	-	0.588	0.463	0.583
12,037	0.544(a)	0.652	-	0.576	0.441	0.568
N	7	7	4	5	6	5
Power function	{ a b	0.5714	0.4891	55.809		
		-0.5926	-0.5651	-1.0500		
Single exponential	{ a T _{1/2} , years				5.114	2.154
					9.324	17.144
Mean error sum		0.0059	0.0005	0.0018	0.0220	0.0011
Mean % deviation		7.03 ↑	1.82	4.36	14.805 ↑	3.51

(a) Value from γ -ray measurement.

Arrows indicate parameters selected for inclusion in summary tables 12 and 16.

TABLE 4D
Curve fitting to adjusted data for patient 03-110

Days since last injection	Radium adjusted from γ -ray measurements	Values computed by best power function		Values computed by best single exponential	
130	1.961	1.996	-	1.991	-
313	0.584	-	-	-	-
7,218	1.109	0.775	1.042	0.956	1.011
7,978	0.817	0.754	0.923	0.884	0.920
10,202	0.717	0.707	0.686	0.702	0.697
11,806	0.590	0.679	0.574	0.594	0.571
12,037	0.544	0.676	0.561	0.580	0.555
N	7	6	5	6	5
Power function	{ a b	0.0286	183.50		
		-0.2702	-1.222		
Single exponential	{ a T _{1/2} , years			2.018	2.484
				18.330	15.241
Mean error sum		0.0252	0.0038	0.0050	0.0046
Mean % deviation		13.37 ↑	5.84	5.51 ↑	5.87

Arrows indicate parameters selected for inclusion in summary tables 12 and 16.

TABLE 5A
Original and adjusted data for patient 03-111(a)

Days since last injection	μ Ci radium			Measured radon retention ratio	Expected radon retention ratio	Total-body radium calculated from	
	Breath radon	RaB & RaC	Total			Breath radon	RaB & RaC
141(b)	-	0.50	-	-	0.1508	-	3.316
7,229(c)	0.70	0.22	0.92	0.24	0.2884	0.984	0.763
7,989(c)	0.70	0.22	0.92	0.24	0.2937	0.991	0.749
10,219(d)	-	0.38	1.238(e)	-	0.3069	-	1.238
10,443(d)	-	0.319	1.035(e)	-	0.3081	-	1.035
11,823(d)	-	0.2804	0.890(e)	-	0.3150	-	0.890

(a) Patient K, Schlundt et al. (1933) and Norris et al. (1955); 20 injections.

(b) Data from Schlundt et al. (1933).

(c) Data from Norris et al. (1955).

(d) Recent measurements made at Argonne in whole-body counter.

(e) Value for total calculated from γ -ray measurement.

TABLE 5D
Curve fitting to adjusted data for patient 03-111

Days since last injection	Radium adjusted from γ -ray measurements	Values computed by best power function	Values computed by best single exponential
141	3.316	3.321	3.324
7,229	0.763	-	-
7,989	0.749	-	-
10,219	1.238	1.077	1.114
10,443	1.035	1.070	1.087
11,823	0.890	1.033	0.936
N	6	4	4
Power function	$\begin{cases} a \\ b \end{cases}$	$\begin{cases} 0.0761 \\ -0.2866 \end{cases}$	
Single exponential	$\begin{cases} a \\ T_{1/2}, \text{ years} \end{cases}$		$\begin{cases} 3.3761 \\ 17.487 \end{cases}$
Mean error sum		0.0112	0.0043
Mean % deviation		8.16 ↑	5.12 ↑

Arrows indicate parameters selected for inclusion in summary tables 12 and 16.

TABLE 6A
Original and adjusted data for patient 03-112(a)

Days since last injection	μCi radium			Measured radon retention ratio	Expected radon retention ratio	Total-body radium calculated from	
	Breath radon	RaB & RaC	Total			Breath radon	RaB & RaC
124 ^(b)	12.01	2,288	14.3	0.16	0.1507	14.141	15.182
307 ^(b)	3.302	1.794	5.096	0.35	0.1699	3.978	10.559
7,212 ^(c)	4.498	2.158	6.656	0.32	0.2884	6.321	7.483
7,972 ^(c)	4.524	2.262	6.786	0.33	0.2937	6.405	7.702
10,447 ^(d)	3.47	1.73	5.20	0.33	0.3082	5.016	5.613

(a) Patient L, Schlundt et al. (1933) and Norris et al. (1955); 26 injections.

(b) Data from Schlundt et al. (1933).

(c) Data from Norris et al. (1955).

(d) Recent measurements made at Argonne in whole-body counter.

TABLE 6B
Curve fitting to original data for patient 03-112

Days since last injection	Measured radium values	Values computed by best power function	Values computed by best single exponential
124	14.3	14.356	14.231
307	5.096	-	-
7,212	6.656	6.436	7.041
7,972	6.786	6.294	6.529
10,447	5.20	5.922	5.107
N	5	4	4
Power function	$\begin{cases} a \\ b \end{cases}$	0.1816	
		-0.2240	
Single exponential	$\begin{cases} a \\ T_{1/2}, \text{ years} \end{cases}$		14.408
			19.114
Mean error sum		0.0335	0.0085
Mean % deviation		6.21 ↑	2.96 ↑

Arrows indicate parameters selected for inclusion in summary tables 12 and 16.

TABLE 6C
Curve fitting to adjusted data for patient 03-112

Days since last injection	Radium adjusted from breath radon measurements	Values computed by best power function	Values computed by best single exponential
124	14.141	14.188	14.567
307	3.978	-	-
7,212	6.321	6.130	6.533
7,972	6.405	5.990	5.995
10,447	5.016	5.625	4.531
N	5	4	4
Power function	$\begin{cases} a \\ b \end{cases}$	0.1894	
		-0.2343	
Single exponential	$\begin{cases} a \\ T_{1/2}, \text{ years} \end{cases}$		14.772
			16.776
Mean error sum		0.0252	0.0233
Mean % deviation		5.50 ↑	5.61 ↑

Arrows indicate parameters selected for inclusion in summary tables 12 and 16.

TABLE 6D
Curve fitting to adjusted data for patient 03-112

Days since last injection	Radium adjusted from γ -ray measurements	Values computed by best power function				Values computed by best single exponential			
124	15.182	13.961	15.260	-	12.647	15.190	-	-	-
307	10.559	12.330	-	10.664	12.479	-	10.712	-	-
7,212	7.483	7.054	7.174	7.094	7.524	7.778	7.339	7.797	
7,972	7.702	6.923	7.026	6.998	7.117	7.239	7.040	7.247	
10,447	5.613	6.578	6.640	6.742	5.937	5.730	6.148	5.712	
N	5	5	4	4	5	4	4	3	
Power function	$\begin{cases} a \\ b \end{cases}$	0.1479	0.1799	0.0941					
		-0.1906	-0.2108	-0.1392					
Single exponential	$\begin{cases} a \\ T_{1/2}, \text{ years} \end{cases}$				12.762	15.359	10.893	15.605	
					25.904	20.096	34.657	19.726	
Mean error sum		0.1233	0.0594	0.0705	0.1672	0.0104	0.0282	0.0139	
Mean % deviation		11.57 ↑	7.93	8.63	9.76	3.02 ↑	5.37	3.95	

Arrows indicate parameters selected for inclusion in summary tables 12 and 16.

TABLE 7A
Original and adjusted data for patient 03-114(a)

Days since last injection	μ Ci radium			Measured radon retention ratio	Expected radon retention ratio	Total-body radium calculated from	
	Breath radon	RaB & RaC	Total			Breath radon	RaB & RaC
30 ^(b)	1.8	1.116	2.90	0.38	0.1334	2.077	8.361
213 ^(b)	5.69	0.90	6.59	0.14	0.1626	6.794	5.536
7,118 ^(c)	1.152	0.576	1.73	0.33	0.2879	1.616	2.001
7,878 ^(c)	1.188	0.504	1.70	0.30	0.2932	1.681	1.719
10,108 ^(d)	-	0.370	1.207 ^(e)	-	0.3064	-	1.207
10,262 ^(d)	-	0.384	1.250 ^(e)	-	0.3073	-	1.250
11,711 ^(d)	-	0.3226	1.025 ^(e)	-	0.3146	-	1.025
11,941 ^(d)	-	0.2987	0.946 ^(e)	-	0.3158	-	0.946

(a) Patient N, Schlundt et al. (1933) and Norris et al. (1955); 30 injections.

(b) Data from Schlundt et al. (1933).

(c) Data from Norris et al. (1955).

(d) Recent measurements made at Argonne in whole-body counter.

(e) Value for total calculated from γ -ray measurement.

TABLE 7B
Curve fitting to original data for patient 03-114

Days since last injection	Measured radium values	Values computed by best power function				Values computed by best single exponential		
30	2.90	5.953	-	-	-	-	-	2.998
213	6.590	4.292	6.651	6.589	-	6.452	-	-
7,118	1.73	1.581	1.507	1.751	1.816	1.999	1.790	1.610
7,878	1.70	1.532	1.437	1.679	1.616	1.757	1.628	1.507
10,108	1.207 ^(a)	1.418	1.280	-	1.214	1.203	1.233	1.239
10,262	1.250 ^(a)	1.410	1.271	-	1.193	1.172	1.209	1.222
11,711	1.025 ^(a)	1.354	1.196	-	1.024	0.917	1.009	1.077
11,941	0.946 ^(a)	1.346	1.185	-	1.002	0.881	0.980	1.055
N	8	8	7	3	6	7	6	7
Power function	{ a b	0.0873	0.3257	0.2419	185.25			
		-0.3160	-0.4696	-0.4192	-1.1626			
Single exponential	{ a T _{1/2} , years					6.690	4.354	3.005
						11.18	15.198	21.644
Mean error sum		0.3846	0.0198	0.0002	0.0024	0.0095	0.0014	0.0071
Mean % deviation		32.90	11.28	0.82	3.50	6.41	3.04	6.14

(a) Value from γ -ray measurement.

Arrows indicate parameters selected for inclusion in summary tables 12 and 16.

TABLE 7C
Curve fitting to adjusted data for patient 03-114

Days since last injection	Radium adjusted from breath radon measurements	Values computed by best power function		Values computed by best single exponential		
30	2.077	5.874	-	2.210	-	-
213	6.794	4.243	6.843	-	6.605	-
7,118	1.616	1.573	1.486	1.456	1.983	1.710
7,878	1.681	1.524	1.415	1.392	1.737	1.569
10,108	1.207(a)	1.411	1.257	1.221	1.177	1.220
10,262	1.250(a)	1.404	1.248	1.210	1.146	1.199
11,711	1.025(a)	1.348	1.171	1.111	0.890	1.018
11,941	0.946(a)	1.340	1.160	1.096	0.855	0.991
N	8	8	7	7	7	6
Power function	$\begin{cases} a \\ b \end{cases}$	0.0854	0.3619			
		-0.3141	-0.4830			
Single exponential	$\begin{cases} a \\ T_{1/2}, \text{ years} \end{cases}$			2.214	6.855	3.825
				32.22	10.889	16.783
Mean error sum		0.5305	0.0173	0.0152	0.0179	0.0029
Mean % deviation		41.83	9.38	8.85	8.90	3.85

(a) Value from γ -ray measurement.

Arrows indicate parameters selected for inclusion in summary tables 12 and 16.

TABLE 7D
Curve fitting to adjusted data for patient 03-114

Days since last injection	Radium adjusted from γ -ray measurements	Values computed by best power function			Values computed by best single exponential			
30	8.361	8.437	8.238	-	6.789	-	-	8.242
213	5.536	5.528	5.707	-	6.582	5.533	-	-
7,118	2.001	1.546	1.873	1.994	2.053	1.965	1.962	2.158
7,878	1.717	1.484	1.808	1.734	1.806	1.754	1.752	1.869
10,108	1.206	1.344	-	1.231	1.240	1.256	1.256	1.226
10,262	1.248	1.336	-	1.206	1.208	1.227	1.228	1.191
11,711	1.024	1.268	-	1.005	0.946	0.988	0.989	0.905
11,941	0.945	1.258	-	0.978	0.910	0.954	0.956	0.867
N	8	8	4	6	8	7	6	7
Power function	$\begin{cases} a \\ b \end{cases}$	0.1851	0.1432	1556.27				
		-0.4031	-0.3526	-1.3916				
Single exponential	$\begin{cases} a \\ T_{1/2}, \text{ years} \end{cases}$				6.823	5.713	5.668	8.289
					11.250	12.663	12.732	10.037
Mean error sum		0.0393	0.0050	0.0006	0.0636	0.0007	0.0008	0.0073
Mean % deviation		14.04	4.03	2.02	7.866	2.03	2.37	6.33

Arrows indicate parameters selected for inclusion in summary tables 12 and 16.

TABLE 8A
Original and adjusted data for patient 03-115(a)

Days since last injection	μCi radium			Measured radon retention ratio	Expected radon retention ratio	Total-body radium calculated from	
	Breath radon	RaB & RaC	Total			Breath radon	RaB & RaC
23(b)	6.396	1.196	7.592	0.16	0.1288	7.342	9.286
206(b)	2.392	0.702	3.094	0.23	0.1607	2.850	4.368
7,111(c)	0.546	0.390	0.936	0.42	0.2878	0.767	1.355
7,871(c)	0.676	0.338	1.014	0.33	0.2930	0.956	1.154
10,095(d)	-	0.240	0.784(e)	-	0.3063	-	0.784
11,697(d)	-	0.221	0.703(e)	-	0.3145	-	0.703
11,930(d)	-	0.213	0.673(e)	-	0.3157	-	0.673

(a) Patient O, Schlundt et al. (1933) and Norris et al. (1955); 26 injections.

(b) Data from Schlundt et al. (1933).

(c) Data from Norris et al. (1955).

(d) Recent measurements made at Argonne in whole-body counter.

(e) Value for total calculated from γ -ray measurement.

TABLE 8B
Curve fitting to original data for patient 03-115

Days since last injection	Measured radium values		Values computed by best power function			Values computed by best single exponential
23	7.592	7.022	7.602	7.596	7.602	-
206	3.094	4.024	-	-	-	3.060
7,111	0.936	0.889	0.939	0.906	-	-
7,871	1.014	0.848	0.896	-	0.896	1.094
10,095	0.784	0.755	0.797	0.766	0.798	0.812
11,697	0.703	0.706	0.744	0.714	0.744	0.654
11,930	0.673	0.699	0.737	0.708	0.738	0.634
N	7	7	6	5	5	5
Power function	{ a b	0.2189	0.2427	0.2510	0.2425	
		-0.4683	-0.4737	-0.4816	-0.4735	
Single exponential	{ a $T_{1/2}$, years					3.146
						14.140
Mean error sum		0.0426	0.0039	0.0006	0.0047	0.0026
Mean % deviation		9.54	4.84	2.54	5.79	5.02

Arrows indicate parameters selected for inclusion in summary tables 12 and 16.

TABLE 8C
Curve fitting to adjusted data for patient 03-115

Days since last injection	Radium adjusted from breath radon measurements	Values computed by best power function			Values computed by best single exponential		
23	7.342	6.734	7.340	-	4.202	-	-
206	2.850	3.842	-	2.853	4.076	2.679	2.809
7,111	0.767	0.839	0.892	0.865	-	1.065	-
7,871	0.956	0.801	0.850	0.834	1.136	0.962	1.059
10,095	0.784	0.713	0.756	0.761	0.784	0.715	0.798
11,697	0.703	0.665	0.704	0.721	0.600	0.577	0.651
11,930	0.673	0.659	0.698	0.716	0.577	0.560	0.632
N	7	7	6	6	6	6	5
Power function	{ a b	0.2132	0.2382	0.0892			
		-0.4718	-0.4774	-0.3702			
Single exponential	{ a T _{1/2} , years				4.218	2.754	2.883
					11.386	14.209	14.910
Mean error sum		0.0510	0.0054	0.0054	0.3221	0.0290	0.0036
Mean % deviation		7.90	5.83	6.25	22.23	14.82	5.51

Arrows indicate parameters selected for inclusion in summary tables 12 and 16.

TABLE 8D
Curve fitting to adjusted data for patient 03-115

Days since last injection	Radium adjusted from γ -ray measurements	Values computed by best power function			Values computed by best single exponential		
23	9.286	8.904	-	8.792	-	6.101	-
206	4.368	4.993	4.423	5.131	-	5.880	4.329
7,111	1.355	1.044	1.065	1.188	1.259	1.466	1.377
7,871	1.154	0.994	1.019	1.134	1.124	1.258	1.213
10,095	0.784	0.883	0.914	-	0.854	0.804	0.839
11,697	0.703	0.822	0.857	-	0.725	0.583	0.643
11,930	0.673	0.814	0.849	-	0.709	0.556	0.619
N	7	7	6	4	5	7	6
Power function	{ a b	0.2990	0.2070	0.2574	100.00		
		-0.4853	-0.4417	-0.4539	-1.1187		
Single exponential	{ a T _{1/2} , years					6.129	4.479
						9.432	11.437
Mean error sum		0.0379	0.0300	0.0413	0.0033	0.2394	0.0028
Mean % deviation		15.10	16.50	9.21	5.42	17.60	5.20

Arrows indicate parameters selected for inclusion in summary tables 12 and 16.

TABLE 9A
Original and adjusted data for patient 03-116^(a)

Days since last injection	μCi radium			Measured radon retention ratio	Expected radon retention ratio	Total-body radium calculated from	
	Breath radon	RaB & RaC	Total			Breath radon	RaB & RaC
30 ^(b)	5.8	1.10	6.90	0.16	0.1307	6.672	8.416
213 ^(b)	4.20	0.90	5.10	0.18	0.1611	5.006	5.587
7,118 ^(c)	1.60	0.95	2.55	0.37	0.2878	2.247	3.301
10,679 ^(d)	-	0.507	1.639 ^(e)	-	0.3094	-	1.639
11,767 ^(d)	-	0.442	1.404 ^(e)	-	0.3148	-	1.404

(a) Patient P, Schlundt et al. (1933) and Norris et al. (1955); 25 injections.

(b) Data from Schlundt et al. (1933).

(c) Data from Norris et al. (1955).

(d) Recent measurements made at Argonne in whole-body counter.

(e) Value for total calculated from γ -ray measurement.

TABLE 9B
Curve fitting to original data for patient 03-116

Days since last injection	Measured radium values	Values computed by best power function	Values computed by best single exponential		
30	6.90	6.962	5.964	-	-
213	5.10	5.091	5.833	5.146	-
7,118	2.55	2.047	2.512	2.427	2.553
10,679	1.639 ^(a)	1.827	1.627	1.648	1.624
11,767	1.404 ^(a)	1.777	1.424	1.464	1.415
N	5	5	5	4	3
Power function	{ a b	0.1020			
		-0.2840			
Single exponential	{ a $T_{1/2}$, years		5.986	5.266	6.304
			15.554	17.434	14.944
Mean error sum		0.0444	0.0466	0.0022	0.00007
Mean % deviation		11.77 ↑	6.31	2.62 ↑	0.60

(a) Value from γ -ray measurement.

Arrows indicate parameters selected for inclusion in summary tables 12 and 16.

TABLE 9C
Curve fitting to adjusted data for patient 03-116

Days since last injection	Radium adjusted from breath radon measurements	Values computed by best power function		Values computed by best single exponential		
30	6.672	6.776	5.777	-	-	
213	5.006	4.916	5.648	4.975	-	
7,118	2.247	1.934	2.410	2.334	2.257	
10,679	1.639(a)	1.720	1.554	1.580	1.596	
11,767	1.404(a)	1.673	1.359	1.402	1.435	
N	5	5	5	4	3	
Power function	{ a b	0.1024				
		-0.2909				
Single exponential	{ a T _{1/2} , years		5.798	5.092	4.512	
			15.389	17.312	19.50	
Mean error sum		0.0202	0.0440	0.0014	0.0006	
Mean % deviation		8.29 ↑	8.39	2.06 ↑	1.77	

(a) Value from γ -ray measurement.

Arrows indicate parameters selected for inclusion in summary tables 12 and 16.

TABLE 9D
Curve fitting to adjusted data for patient 03-116

Days since last injection	Radium adjusted from γ -ray measurements	Values computed by best power function		Values computed by best single exponential			
30	8.416	8.238	8.386	6.919	6.843	-	-
213	5.587	5.978	5.629	6.758	6.676	5.777	5.589
7,118	3.301	2.354	-	2.777	-	2.649	-
10,679	1.639	2.094	1.544	1.754	1.621	1.772	1.618
11,767	1.404	2.037	1.492	1.525	1.400	1.567	1.422
N	5	5	4	5	4	4	3
Power function	{ a b	0.1244	0.1718				
		-0.2906	-0.3582				
Single exponential	{ a T _{1/2} , years			6.946	6.871	5.918	5.732
				14.731	14.034	16.807	16.023
Mean error sum		0.1412	0.0028	0.1227	0.1013	0.0412	0.0002
Mean % deviation		22.13	3.29 ↑	14.07	9.89	10.73	0.88 ↑

Arrows indicate parameters selected for inclusion in summary tables 12 and 16.

TABLE 10A
Original and adjusted data for patient 03-121(a)

Days since last injection	μ Ci radium			Measured radon retention ratio	Expected radon retention ratio	Total-body radium calculated from	
	Breath radon	RaB & RaC	Total			Breath radon	RaB & RaC
30 ^(b)	-	0.5	-	-	0.1185	-	4.218
213 ^(b)	-	0.6	-	-	0.1556	-	3.856
7,118 ^(c)	0.387	0.189	0.576	0.328	0.2876	0.543	0.657
7,878 ^(c)	0.360	0.216	0.576	0.375	0.2927	0.509	0.738
10,109 ^(d)	-	0.17	0.555 ^(e)	-	0.3061	-	0.555
10,332 ^(d)	-	0.175	0.569 ^(e)	-	0.3073	-	0.569
11,711 ^(d)	-	0.1387	0.441 ^(e)	-	0.3143	-	0.441
11,942 ^(d)	-	0.1339	0.424 ^(e)	-	0.3154	-	0.424

(a) Patient U, Schlundt et al. (1933) and Norris et al. (1955); 9 injections.

(b) Data from Schlundt et al. (1933).

(c) Data from Norris et al. (1955).

(d) Recent measurements made at Argonne in whole-body counter.

(e) Value for total calculated from γ -ray measurement.

TABLE 10D
Curve fitting to adjusted data for patient 03-121

Days since last injection	Radium adjusted from γ -ray measurements	Values computed by best power function	Values computed by best single exponential			
30	4.218	5.186	4.035	4.085	-	-
213	3.856	2.822	3.884	3.940	3.707	-
7,118	0.657	0.711	0.921	-	0.914	0.721
7,878	0.738	0.682	0.786	0.864	0.783	0.670
10,109	0.555	0.617	0.494	0.555	0.498	0.538
10,332	0.569	0.611	0.471	0.531	0.476	0.527
11,711	0.441	0.581	0.354	0.404	0.360	0.460
11,942	0.424	0.576	0.337	0.386	0.343	0.450
N	8	8	8	7	7	6
Power function	$\begin{cases} a \\ b \end{cases}$	0.2908				
		-0.4064				
Single exponential	$\begin{cases} a \\ T_{1/2}, \text{ years} \end{cases}$		4.061	4.109	3.871	1.447
			9.106	9.586	9.357	19.404
Mean error sum		0.0813	0.0220	0.0052	0.0229	0.0031
Mean % deviation		18.89	15.06	6.62	16.17	6.65

Arrows indicate parameters selected for inclusion in summary tables 12 and 16.

TABLE 11A
Original and adjusted data for patient 03-126^(a)

Days since last injection	μCi radium			Measured radon retention ratio	Expected radon retention ratio	Total-body radium calculated from	
	Breath radon	RaB & RaC	Total			Breath radon	RaB & RaC
166 ^(b)	9.4	1.7	11.1	0.153	0.1541	11.112	11.032
7,254 ^(c)	1.26	0.56	1.82	0.31	0.2886	1.771	1.940
10,575 ^(d)	-	0.379	1.444 ^(e)	-	0.3088	-	1.444 ^(f)

(a) Patient R. D., private correspondence. Schlundt (1932) and Norris et al. (1955): 20 injections.

(b) Data from private correspondence, Schlundt (1932).

(c) Data from Norris et al. (1955).

(d) Recent measurements made at Argonne in whole-body counter.

(e) Value for total calculated from γ -ray measurement.

(f) Corrected for missing leg by adding 15%.

TABLE 11B
Curve fitting to original data for patient 03-126

Measured radium values	Assumed days since last injection	Values computed by		Assumed days since last injection	Values computed by best power function
		Best power function	Best single exponential		
11.1	166	11.103	-	30	11.105
1.82	7,254	1.791	1.820	7,118	1.758
1.444 ^(a)	10,575	1.471	1.444	10,439	1.503
N	3	3	2	3	3
Power function	{ a b	0.9597			0.3428
		-0.5253			-0.4126
Single exponential	{ a $T_{1/2}$, years		3.017		
			27.233		
Mean error sum		0.00033	0.0000		0.0014
Mean % deviation		1.17	0.00		2.51

(a) Corrected for missing leg by adding 15%.

TABLE 11C
Curve fitting to adjusted data for patient 03-126

Radium adjusted from breath radon measurements	Assumed days since last injection	Values computed by		Assumed days since last injection	Values computed by best power function
		Best power function	Best single exponential		
11.112	166	11.112	-	30	11.115
1.771	7,254	1.766	1.771	7,118	1.733
1.444 ^(a)	10,575	1.444	1.444	10,439	1.480
N	3	3	2	3	3
Power function	{ a b	0.9828			0.3482
		-0.5296			-0.4159
Single exponential	{ a T _{1/2} , years		2.766		
			30.875		
Mean error sum		0.0000	0.0000		0.0006
Mean % deviation		0.20	0.00		1.55

(a) Corrected for missing leg by adding 15%.

TABLE 11D
Curve fitting to adjusted data for patient 03-126

Radium adjusted from γ -ray measurements	Assumed days since last injection	Values computed by		Assumed days since last injection	Values computed by best power function
		Best power function	Best single exponential		
11.032	166	11.041	-	30	11.042
1.940	7,254	1.852	1.940	7,118	1.821
1.444 ^(a)	10,575	1.528	1.444	10,439	1.562
N	3	3	2	3	3
Power function	{ a b	0.8978			0.3277
		-0.5141			-0.4036
Single exponential	{ a T _{1/2} , years		3.697		
			21.344		
Mean error sum		0.0029	0.0000		0.0059
Mean % deviation		3.48	0.00		4.79

(a) Corrected for missing leg by adding 15%.

20 percent of the adjusted breath value if the breath value (column 2) is low by a factor of two. Consequently, agreement between the parameters found to fit the adjusted breath values and the adjusted gamma-ray values suggests that both the breath and gamma-ray values were reasonably accurate, while disparity between the two sets of parameters implies that either the early breath or early gamma-ray measurements, or both, were incorrect.

In general, the greater variability of the values of a reflects the fact that for this limited series of measurements the one-day intercept of the power function is affected much more than is the slope by small errors in the early measurements. The slope of the power function is anchored at one end between 30 and 300 days by the Schlundt measurements and at the other end between 7,000 and 12,000 days by the earlier and the more recent Argonne measurements. A few percent reduction in the values between 30 and 300 days would reduce the slope of the power function found to fit these data by about the same percentage but would reduce the value of the one-day intercept by slightly more than twice this percentage. On the other hand, if the later measurements made between 7,000 and 12,000 days were reduced by a few percent, the slope of the power function would be increased by about one-half the percentage change while the one-day intercept would be increased by about the same percentage.

The parameters, particularly the coefficients found to fit the adjusted breath data, for patients 03-102, 03-109, and 03-110 are substantially higher than those found to fit the breath data for the other five patients. Significantly, the parameters found by analysis of the adjusted gamma-ray data of these three patients are considerably lower than those parameters based on breath measurement. These results suggest the possibility that either the early breath measurements are high, or that the number of injections and the reputed elapsed times between the last injection and the first measurements are incorrect. It is of interest that the injection schedule for two of these three patients could not be substantiated. As discussed in the separate section on each patient, radium was never mentioned in the progress notes for patient 03-102, and patient 03-110 received at least 30 weekly injections according to the hospital notes instead of the 20 published by Schlundt; this patient may very well have received more than 30 injections. In the case of patient 03-109, only four measurements are available, all of which were field measurements. Consequently, valid reasons can be advanced for omitting these patients from the group used to calculate the average power function.

The parameters of the so-called "average power function" will vary significantly depending upon whether the power functions found to fit the data for the individual patients are averaged in the linear or log plane, a problem that has been discussed by McWilliams et al.¹² For example, consider the values of two power functions found to fit the data for

patients 03-112 and 03-115, $R_{112} = 0.1816 t^{-0.2240}$ and $R_{115} = 0.2189 t^{-0.4683}$. Of the power functions found to fit the data of the five patients for whom reliable time intervals are available, these two power functions yielded the highest and lowest body burdens, respectively, from 30 days through 30,000 days. Each of these power functions can be used to generate radium values for 1, 3, 10, 30, 300, 1000, 3000, 10,000 and 30,000 days after a single intravenous injection. The two values found for each time can be averaged directly (in the so-called natural plane), or the logarithms of the two values at each time can be averaged and the antilog of the average value determined (so-called log plane). When plotted on log-log paper, the set of arithmetic means found in the natural plane will yield a curved line, concave upward, whose radius of curvature increases with time, while the average found in the log plane will yield a straight line that can be described exactly by a third power function, $R = 0.1994 t^{-0.3462}$. When power functions are averaged in the log plane, as in the latter case, the coefficient of the average power function is the geometric mean of the coefficients (0.1816 and 0.2189) and the exponent (0.3462) is the arithmetic mean of the exponents (0.2240 and 0.4683) of the individual power functions.

On the other hand, when the natural plane arithmetic averages for the times listed for the two functions are employed, the power function that gave the best fit to these averages was found to be $R = 0.1925 t^{-0.3045}$. The coefficient here agrees fairly well with that found in the log plane, but the exponent is significantly smaller. Furthermore, when the values starting at the earliest times were successively omitted from the series and power function parameters were calculated that fit the remaining data best, the values of a and b decreased progressively. For example, the power function, $R = 0.1624 t^{-0.2760}$, was obtained with values from 30 days to 30,000 days and the power function, $R = 0.1459 t^{-0.2624}$, was obtained for the values from 300 to 30,000 days. The addition of a few points at other times between 1 and 100 days will increase the slope over that found to provide the best fit to the original series of times. Conversely, the addition of a few values at later times will decrease the slope of the power function.

In summary, the values of a and b found to provide the best fit to the arithmetic average of two power functions depend both upon the time span of the measurements and upon the relative number of measurements within the various time intervals. This result implies that divergent radium values of two or more patients should not be averaged and the means then fitted with a power function, but rather that the parameters of the power function should be found that provide the best fit to the data for each patient. The parameters of the average power function should then be obtained from the geometric mean of the coefficients and the arithmetic mean of the exponents. This procedure essentially fits the combined data in the log plane by averaging a series of straight lines on a double log plot.

The geometric mean of the coefficients and the arithmetic mean of the exponents for all eight patients, and for five selected patients, are given on the last two lines of Table 12. These means are the parameters of the average power function found when the separate power functions for the patients are averaged in the log plane. As would be expected, the parameters of the average power function based on the original data for all eight patients agree very well with the parameters of the average power function based on the breath data for these patients. The parameters of the average power function based on the gamma-ray data are significantly smaller than those based on the original data or breath data.

When the parameters obtained for patients 03-102, 03-109 and 03-110 are omitted and the means based on the other five patients are calculated, these means based on the original measurement, on the breath measurement and on the gamma-ray measurement all agree very well with the means found for all eight patients based on gamma-ray measurement. Consequently, an average power function, based on analyses of the original measurements, the breath measurements and the gamma-ray measurements for these five patients and of the gamma-ray measurements on all eight patients, is given by $R = 0.1923 t^{-0.3619}$.

When the parameters of the average power function are defined as the geometric and arithmetic mean of the parameters obtained for the individual patients, the power functions for all patients are weighted equally, regardless of the number of measurements employed to calculate the parameters for the individual patient or of the goodness of fit obtained with each power function to the data for each patient. Nevertheless, the mean slope derived in this way from data on the group of eight patients should be used judiciously since the addition of retention data from a few more patients could alter this slope significantly. The slopes computed from this limited number of patients should be used only as an indication of the approximate range of values that may occur.

Another complicating feature results from the impression, suggested by inspection of Table 12, that the power function parameters, a and b , are not independent of each other. In fact, analysis of the data yields correlation coefficients of 0.832, 0.854 and 0.814 between a and b for the original, adjusted breath, and adjusted gamma-ray data, respectively, with $P \leq 0.004$. Similarly, correlation coefficients for $\log a$ and b are 0.885, 0.895 and 0.715, respectively, for the original and adjusted data, again with $P \leq 0.004$. Absolute values of b have been used here for purposes of simplicity.

Regression lines have been computed for the absolute values of a and b , and for $\log a$ and $\log b$. The regression parameters and standard error of estimate are given in Table 13. The regression of b on a gives a much better fit than a on b for all three sets of data, and this situation

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Fig. 3. Plot of power function parameters demonstrating the correlation between the slope (b) and the intercept (a). The straight line is the least squares linear regression of b on a and the interrupted line is the least squares linear regression of $\log b$ on $\log a$.

obtains for the logarithmic transformations as well. A plot of the regression of b on a for the original data is given in Figure 3.

Because of the high degree of correlation between a and b, with coefficients of determination ranging from 0.51 to 0.80, one is tempted to conclude that, for the radium patients studied here, the power function is a 1.2 to 1.5 rather than a 2-parameter system, and that it provides a mathematical pattern of much less flexibility than has hitherto been appreciated.

Another approach to the problem of obtaining an "average" power function weights the data of each patient according to the number of measurements available for that patient and fits these data with a power function directly in the

natural plane. In order to make this computation, the power function found for each patient was used to generate a radium value for the time of each measurement available for that particular patient. These values are the amounts that each of the patients would have had if he or she had received a single injection of $1 \mu\text{Ci}$ on the date that the last actual injection was given. This procedure, in effect, normalizes the data for each patient so that the number of injections can be ignored and smooths the data for each patient in that the value given by the power function is used instead of the actual measured value. These calculated values for all eight patients and also for five selected patients were pooled separately and each set of pooled data was fitted with a power function.

The VMM program, which was used for the computations, adjusts the parameters of the power function successively by a series of iterations to minimize the sum of the errors between the observed values (in this case, pooled and normalized), y_{ob} , and the values calculated by the power function, y_{pf} . The error can be defined either as $(y_{ob} - y_{pf})^2 / y_{ob}$ or as $(y_{ob} - y_{pf})^2 / y_{pf}$. Since the actual quantities (pooled and normalized) that are present in the patient (y_{ob}) and the values that are given by the power function (y_{pf}) are employed in the error function, the parameters of the power function that are obtained are those that provide the best fit to the nontransformed data in the natural plane. Either definition of error will lead to essentially the same parameters for the power function if the points that are being fitted fall quite close together. However, with widely variant

data, the parameters of the power function will differ considerably, depending upon which definition of error is employed.

This point can be demonstrated quite easily with the aid of a simple analysis with the two power functions employed previously. The two power functions obtained for patients 03-112 and 03-115 predict 0.1816 and 0.2189 μCi , respectively, at one day after a single injection of 1 μCi , and 0.2115 and 0.00245 μCi , respectively, 40 years later. For these two sets of values the average power function that yields the smallest

error sum, $\sum \frac{(y_{\text{ob}} - y_{\text{pf}})^2}{y_{\text{ob}}}$, predicts 0.199 and 0.0045 μCi at one day and 40 years, respectively, and is given by $R = 0.199 t^{-0.3951}$. In contrast, the average power function that minimizes the error sum, $\sum \frac{(y_{\text{ob}} - y_{\text{pf}})^2}{y_{\text{pf}}}$, predicts 0.201 and 0.015 μCi at one day and 40 years, respectively, and is given by $R = 0.201 t^{-0.2706}$. The coefficients of these two power functions agree very well because the two one-day values that anchor the power function are very close together. But the exponents are quite different as are the 40-year values predicted by the two "average" power functions. This example highlights the problem of fitting an "average" power function to data of several patients.

Because of these difficulties, the pooled data for eight patients, and also for a selected group of five of these patients, were fitted with an "average" power function by use of both error functions, and the results are given in Table 14. The resultant error sum for each fit is given alongside each set of parameters. As expected from the above discussion of the average of only two power functions, the exponent (b) of the power function

is significantly lower when the error sum $\sum \frac{(y_{\text{ob}} - y_{\text{pf}})^2}{y_{\text{pf}}}$ is minimized than

when the error sum $\sum \frac{(y_{\text{ob}} - y_{\text{pf}})^2}{y_{\text{ob}}}$ is used. However, since in each case the latter error sum is smaller, perhaps the larger exponents should be the ones selected as the valid parameters for the "average" power function.

A third method was also used to fit these data. The amounts and times of all of the individual measurements were converted to logarithms and these log values were fitted with linear regression lines, $y = a + bt$. This is equivalent to $\log R = \log a + b \log t$ where R is the amount, a is the coefficient and b is the exponent of the power function. In this case, the sum of the squared deviations are minimized and the data are fitted in the log plane.¹² The power function parameters derived in this way are given in Table 15 and are very close in most instances to the larger exponents calculated by the pooled, normalized data method discussed above.

For purpose of comparison, the "average" power function parameters computed by each of three sets of methods are listed in Table 15 by method and by source of data. When all eight patients are considered, seven of the nine slopes are in the neighborhood of -0.43 , and the respective intercepts range from 0.18 to 0.33 (with a mean of 0.27). On the other hand, when the data are restricted to five selected patients for whom the early information is more reliable, all nine slopes cluster closely about a mean of -0.3750 , and the mean of the associated intercepts is 0.2068 .

The problem of deciding which parameters should be used for an "average" power function now becomes a matter of selection from among the values arrayed in Table 15. One is tempted simply to use the original data for eight patients for which separate power functions were fitted and the appropriate means were calculated. This selection would yield a slope of -0.438 and an intercept of 0.302 , and we are, in fact, using these parameters in some retrospective computations of maximum body burden in cases of continuous ingestion. However, the more reliable set of data for five selected patients yields a set of parameters that are remarkably uniform and that have the lower values given above. In any case, the selection of any "average" power function is an arbitrary procedure and probably makes only a small difference in the results. It is more important to be aware of the fact that the carefully examined data for eight patients yield a family of power functions whose slopes range from -0.224 to -0.627 , and whose one-day intercepts range from 0.102 to 0.893 . No matter how the "average" power function is computed, its parameters are substantially lower than those given by Norris et al.^{2,10} for earlier data on the same patients, and these lower values may have some important bearing on the retrospective estimates of maximum body burdens when these estimates are based on measurements made 30 to 40 years after the maximum occurred.

Average Single Exponential Function

The parameters of the single exponential term that provided the best fit to the original data, and to the adjusted breath radon and gamma-ray data of each patient are given in Table 16. The magnitudes of the coefficients, a , given in this table are slightly larger (by 0.5 to 2 percent) than those given in Tables B, C and D for the individual patients. The parameters in the latter tables are those that provide the best fit to the radium burdens of each patient when the time of each measurement was defined as the elapsed time since the last injection. This analysis neglected the fact that the patient had received a number of weekly injections and, consequently, that the actual elapsed time between the first measurement and each preceding injection increased seven days per injection. In order to calculate the coefficient applicable to a single injection of

one microcurie of radium, the single exponential derived for each patient for each set of data was projected backward in time to the center of the injection period so that the resulting intercept value is now somewhat larger. Then, the value of each revised coefficient was divided by the total amount of radium administered to the patient to obtain a ratio, a/A , that gives the proportion of the total amount injected that is retained in accordance with the single exponential term.

Paired comparisons of the means for a/A and for $T_{1/2}$ fail to reveal statistically significant differences, although the mean half-time for the adjusted breath data is somewhat lower than that for the other two sets of data. Similarly, the ratio a/A is consistently large for patient 03-112 but in no case does the ratio for this patient exceed 2.4 times the standard deviation for each set of ratios and, hence, it cannot be excluded from the analysis.

In order to obtain the parameters of an "average" single exponential term, the coefficient ratios and half-times, respectively, based on the original, breath and gamma-ray data were pooled and the means and standard errors were determined. The 24 coefficient ratios and half-times yielded a mean a/A of 0.0246 ± 0.0031 and a mean $T_{1/2}$ of 14.984 ± 0.801 years. These data suggest that from approximately one year through 34 years after injection the retention of radium in man follows the single exponential given by

$$R = 0.0246 A^{-0.693t/14.984}$$

where A is the total amount administered, and t is the time of interest. Since the variability of the parameters is less for the exponential terms than it is for power functions, the geometric mean of the 24 half-times (14.507 years) is very close to the arithmetic mean (14.984 years). For this reason, the type of problem that was encountered in averaging power functions is not as important in the case of averaging single exponentials.

The average single exponential term given above implies that 2.46 percent of the injected radium is retained with an average half-time of about 15 years from about one year after injection to 33 years later. In many individual cases, a single exponential term provides a better fit for this time span than does a power function. But the single exponential term does not give any information by which the retention pattern for the injection period and for the first year thereafter can be reconstructed. In other words, the retention pattern for the 97.54 percent of the injected radium that is eliminated during the first year is not available from this analysis. Unlike the case of the power function, there is no significant correlation between the parameters (a and $T_{1/2}$) of the exponential term obtained for each of these patients ($r = 0.42, 0.44$ and -0.14 for the original and the adjusted breath radon and gamma-ray data, respectively). Not

surprisingly, some significant negative correlation exists between the half-time of the exponential term and the magnitude of the slope of the power function found for each patient, when the original and adjusted breath radon data are considered ($r = -0.74$ and -0.83 , respectively, with $P < 0.02$ for the former and $P < 0.01$ for the latter). For the adjusted gamma-ray data, $r = -0.50$, which is not significant statistically. Such a correlation would not be expected a priori since the power function is essentially a one-parameter system with a and b highly correlated, while the exponential function has two independent parameters for each term. On the other hand, a smaller value for b would give a less steep slope as would a larger $T_{1/2}$ in the exponential term, and, hence, some correlation might be expected.

Retrospective Calculation of Amount Administered

As stated in the introduction, a mathematical expression is needed by which the total amount of radium administered to or ingested by patients thirty to fifty years ago can be calculated from a current measurement of total-body radium. Since in each of the eight cases discussed here the amount of radium administered 34 years ago is known and since current measurements are available, the accuracy of the different average power functions, and of the average single exponential can be tested when these functions are used to calculate the total amount administered. It should be emphasized that it is pointless to compute the amount administered to each patient with the specific function calculated for that particular patient since the amount administered was used in fitting the function to the data.

Each measured value obtained during the interval from 28 until 33 years after the last injection (listed in Tables 1-9) was divided, in turn, by the retention predicted by the average power function (1) $R = 0.54 t^{-0.52}$, (2) $R = 0.3015 t^{-0.4383}$, and (3) $R = 0.2068 t^{-0.375}$, as well as by the average exponential function, $R = 0.0246 e^{-0.693t/5472}$. In each equation, t is the elapsed time in days of the particular measurement. In order to facilitate a direct comparison between the results obtained for the various patients who had received different amounts of radium, the computed quantity was divided by the total amount that the patient had received. The ratios of computed to actual values for the three power functions and the single exponential are given in Table 17.

In general, the ratios found with the single exponential function agree very well with those found when the power function $R = 0.2068 t^{-0.375}$ was employed: With either function, the computed value for patient 03-112 was about 3.3 times too high and the value computed for patient 03-110 was about one-third of the true value. The values computed for patient 03-112 were 4.1 and 4.9 times the true value and the values computed for patient 03-110 were about 0.41 and 0.49 of the actual value when the power function $R = 0.3015 t^{-0.4383}$ and $R = 0.54 t^{-0.52}$, respectively, were used.

The ratios found for patient 03-112 are about ten times that found for patient 03-110 when either of the power functions or the exponentials were employed.

Except for patient 03-102, the ratios found with each power function for each patient decrease as the elapsed time of the measurements increases. This effect would be expected in view of the fact that the individual power functions found to fit the data of each patient successively underestimated the later measurements. On the other hand, the various ratios found for each particular patient with the single exponential, while exhibiting some scatter, do not demonstrate this trend. However, the spread in ratios found for any particular patient with the power function is of no practical importance in view of the large spread in ratios among the various patients.

These calculations have failed to demonstrate that any one of these three power functions or that the single exponential term yields a more reliable retrospective value for the total amount administered. These results are included here specifically to point out the uncertainties that exist when the total amount administered or ingested is calculated retrospectively by the application of an "average" power function or an "average" exponential term.

Retrospective Estimates of the Pattern of Radium Retention with Time

Since between 97 and 98 percent of injected radium is usually eliminated during the first year after administration, and since studies with mice have suggested that it is the radiation delivered soon after injection of a radionuclide that is the effective carcinogenic dose,¹³ it is of some interest and importance to attempt a retrospective reconstruction of the radium retention pattern. Such a reconstruction for each patient would provide an estimate of the maximum body burden, as well as estimates for selected times during and after the injection period.

The computer program that was used to fit the power function to the individual sets of data also calculated the body burdens for arbitrarily selected intervals after the last injection, as well as the maximum radium burden that occurred immediately after the final injection of 10 μCi was administered. The results for various sets of data are given in Tables 18, 19 and 20. For the original data, as well as for those derived from the adjusted measurements, the maximum radium burdens, with one low exception of 12.6, ranged from 16.8 to 27.8 μCi (with mean values of 19.7 to 22.6 μCi) despite the fact that totals of 150 to 300 μCi had been injected. One day after the last injection these average values dropped to 12 and to 16 μCi , respectively, and continued to fall at progressively slower rates.

thereafter. By one year the mean values fell to levels about three times the contemporary values that are now being found at 12,000 days after the last injection.

The estimated body burdens during and after a series of injections are shown in several figures, similar to Figure 1, which depicted the values computed from Norris' parameters. Figure 4 illustrates the stepwise increase of body radium and its slow release for patient 03-112 who appears to have retained radium tenaciously (small value for $-b$; relatively high whole-body value after 30 years) although she eliminated all but 18 percent of the injected radium within 24 hours (small value for a , the one-day intercept). The figure gives details for the first 15 injections as well as the profile of retention after the 26 injections that she is known to have received. By way of contrast, the retention pattern given by the power function parameters for the adjusted data of patient 03-109 is shown in Figure 5. Here, where again for convenience only 15 injections are shown although the patient received 18, the patient retained 89.3 percent of the radium injected 24 hours earlier but lost radium in accordance with a relatively steep slope ($b = -0.627$). Similar representation of an "average" curve for the eight patients ($a = 0.3015$, $b = -0.4383$) is given in Figure 6.

194-40

Fig. 4. Double log plot of radium body burden starting one day after a series of 15 weekly injections of $10 \mu\text{Ci}$ each calculated in accordance with power function $R = 0.18 t^{-0.22}$ (patient 03-112). Additional line shows the course of retention after 26 injections, which this patient received.

194-101

Fig. 5. Double log plot of radium body burden during and after 15 weekly injections of $10 \mu\text{Ci}$ each calculated in accordance with power function $R = 0.89 t^{-0.627}$. (patient 03-109)

194-100

Fig. 6. Double log plot of radium body burden during and after 15 weekly injections of $10 \mu\text{Ci}$ each calculated in accordance with a newly-derived average power function $R = 0.30 t^{-0.438}$.

While all three curves are similar in shape, they are sufficiently different to alter the retention after 30 years, as the superimposition of the curves in Figure 7 demonstrate. These differing curves also emphasize the importance of the range of values that power function parameters may assume and warn against naive dependence upon "average" values.

194-99

Fig. 7. Double log plot of radium retention curves during and after a series of 15 weekly injections of 10 μCi each calculated in accordance with several power functions. The solid line represents the newly-derived average power function, $R = 0.30 t^{-0.438}$ (Figure 6). The line of dashes represents the most shallow curve, that of patient 03-112 (Figure 4). The dot-dash line represents the steepest retention curve in our series, that of patient 03-109 (Figure 5).

Although indices such as cumulative μCi -days or cumulative rad-days will not be used here for the evaluation of a dose-response curve, principally because in the absence of knowledge of the length of latent periods one does not know how long to continue the cumulation, this retrospective reconstruction will enable such computations to be made. The more important point that emerges is that the computed maximum radium burden at no time exceeds 17.2 percent of the total radium injected in 10- μCi weekly amounts, and may be as low as 4.2 percent (if the reconstruction for patient 03-110 from the adjusted gamma-ray data is correct).

This type of reconstruction that is possible with power functions cannot be done for single exponentials, as explained in a preceding section. With the single exponential function, we are on sure grounds only after the first year after the last injection has elapsed, and in the absence of knowledge of short-term exponentials the existing data cannot be used to estimate maximum body burdens by summing exponential terms. One is forced to use "average" power function reconstructions in order roughly to estimate the maximum body burden in those cases whose injection history is known and who have been recently studied but who were not part of the group originally measured by Schlundt. When the final pathological findings are tabulated, such cases may prove to be valuable additions to the series described in this paper.

Retrospective Calculation of Radium Buildup and Cumulative Microcurie-Days

The three so-called "average" power functions used in a previous section to calculate the amount administered to the patients were also used to calculate both the buildup and retention of radium by a hypothetical patient as well as the cumulative microcurie-days this patient would have received. It was assumed that such a patient received 15 weekly injections of $10 \mu\text{Ci}$ each and that the retention was identical for each injection, i.e., the retention pattern for any injection was not altered by the previous injection. The buildup and retention patterns found for the three power functions (Figure 8) do not differ as significantly as might have been expected. The body radium builds up somewhat faster to a slightly higher value if retention follows the power function $R = 0.54 t^{-0.52}$ than if it follows the other two power functions, and with the former curve it also declines somewhat faster to a slightly lower value at 12,000 days (32.85 years). At 150 days after the first injection, or 52 days after the last injection, the radium content predicted by the steeper power function ($R = 0.54 t^{-0.52}$) is about 42 percent higher than that predicted by the power function with the flattest slope, $R = 0.2068 t^{-0.3750}$. Conversely, at 12,000 days the latter power function predicts about 34 percent more than the former power function. However, between 800 and 900 days, all three power functions predict about the same amount.

194-98

Fig. 8. Double log plot of radium retention curves during and after a series of 15 weekly injections of $10 \mu\text{Ci}$ each calculated in accordance with several "average" power functions. The solid line represents the newly-derived average power function, $R = 0.30 t^{-0.438}$ (Figure 6). The line of dashes represents the equation proposed by Norris et al.^{2,10} $R = 0.54 t^{-0.52}$ (Figure 1). The dot-dash line represents another "average" power function computed from the data on five patients, $R = 0.21 t^{-0.375}$.

Each of the three power functions was integrated and the cumulative microcurie-days predicted by each function is plotted in Figure 9. Of the three power functions, $R = 0.54 t^{-0.52}$ yields higher cumulative microcurie-day values at early times and only slightly lower values at 12,000 days. The other two "average" functions show the reverse. But all three functions yield about the same integrated value from 4000 to 5000 days.

194-103

Fig. 9. Double log plot of cumulative microcurie-days after 15 weekly injections of $1.0 \mu\text{Ci Ra}^{226}$ calculated in accordance with three "average" power functions (cf. Figure 8).

These analyses are included in this report to emphasize the fact that all three so-called "average" power functions yield similar total-body values and very close cumulative microcurie-day values at long times and, more important, as mentioned previously, the difference between patients due to biological variability completely overshadows these differences between "average" functions.

Discussion and Conclusions

Several questions remain unanswered after this rather exhaustive analysis of the radium data, some of which were originally used by Norris to arrive at the power function, $R = 0.54 t^{-0.52}$. Is the power function a valid expression for the retention of radium in humans over the time span of 30 days through 33 years covered by these measurements? Does the retention of radium in humans follow this power function from the first day on, or from perhaps only the 20th or 100th day on? What is the significance of the correlation found between a and b in the power function for these patients? Are there perhaps better mathematical descriptions of the pattern of radium retention?

It is impossible to demonstrate conclusively from any analysis of these data whether the retention of radium in humans follows either the power function or the single exponential. The schedule and number of injections given to the patients are simply not known with complete assurance. The early measurements are not accurate enough, and measurements that could be used to establish a distinction between these two functions are not available for the critical time of about 3,000 days.

The retention of radium in these patients is definitely below that predicted by the power function for the time span from 21 to 33 years after the last injection. Also, for several patients the power function consistently underestimates the radium content of the patient on the day of the first Schlundt measurement and overestimates it on the occasion of the second Schlundt measurement. These trends suggest that the power function may not fit very well at either end of this time span, although the new average function, $R = 0.3015 t^{-0.4383}$, provides a better fit to these data than did the previous power function, $0.54 t^{-0.52}$.

The validity of the power function as an expression for the retention of radium at short times cannot be demonstrated by a direct analysis of these data but can be evaluated only by an examination of the retention patterns predicted for earlier times. The retrospective calculations of the amounts present during the course of the injections, made by use of the various power functions derived for the individual patients, demonstrate that the rate of accumulation during the course of injection is inversely related to the amounts present after long times.

For a constant number of hypothetical injections at uniform intervals, the person with the largest $-b$ value (steepest loss slope) would also have the highest one-day intercept, a , and would thus have the fastest buildup and the highest body burden after the last injection. No dependable data exist in the literature to support or disprove this view.

The high degree of correlation found to exist between the power function parameters a and b might be due to the fact that the uptake in patients is inversely related to the elimination. On the other hand, this correlation between a and b may reflect some factor other than retention in the human. The most obvious explanation is that the early Schlundt measurements happened to be very low for those people who exhibit small values of a and high for those people who have large values of a . Yet this explanation seems unlikely in view of the fact that both the breath measurements and the gamma-ray measurements, which are technically independent of each other, yield equivalent values of a and b .

What, then, are the relative merits of the power function and the single exponential function in describing radium retention? The detailed analyses set forth here have shown that:

- 1) An improved version of the power function can be derived, namely, $R = 0.3015 t^{-0.4383}$, for a series of eight patients for whom radium measurements are available for a 33-year span.
- 2) For individual cases there is a wide range of values for the power function parameters a and b .
- 3) These parameters, a and b , are highly and significantly correlated, especially if a logarithmic transformation is used.
- 4) While the power function does not fit the known data over the time span from one to 30 years as well as a single exponential term, it can be used to calculate retrospectively the day-by-day course of radium retention during the perhaps crucial period during which the injections occurred, and for the first year thereafter. The accuracy of these computations cannot be established from currently-available data, but one may presume that they are approximately correct.

The single exponential function, on the other hand, also has certain advantages and limitations:

- 1) It provides an excellent fit to the data from one to 33 years after administration and surpasses the power function in this respect for the period from 21 to 33 years.
- 2) Although the single exponential cannot be used to describe radium retention during the course of injections and the first year thereafter, it can provide an estimate of the total amount injected that may be more accurate than that predicted by the power function after the first year.

- 3) The initial amount of radium retained in accordance with a single exponential term after the first year ranges from 1.1 to 5.9 percent of the total amount administered, and the overwhelming proportion of the radium is excreted on a short-term basis, for which there are no data at the present time on which to base a mathematical description.
- 4) The half-time values found for the best single exponential fit to the data on the patients average 15 years and are very close to those found for 14 radium dial painters who ingested radium 40 years ago and who have had at least two whole-body radium determinations during the past eight years (unpublished observations).

TABLE 12

Summary of power function parameters giving best fit to original and adjusted data

Patient	Number of injections	Original data		Breath data		γ -Ray data	
		a	b	a	b	a	b
03-101	15	0.1931	-0.3220	0.2074	-0.3419	0.2173	-0.3212
03-102	15	0.7335	-0.5549	0.8672	-0.5727	0.2292	-0.4299
03-109	18	0.8927	-0.6271	0.9766	-0.6347	0.5922	-0.5650
03-110	30	0.4086	-0.5564	0.5714	-0.5926	0.0286	-0.2702
03-112	26	0.1816	-0.2240	0.1894	-0.2343	0.1479	-0.1906
03-114	30	0.3257	-0.4696	0.3619	-0.4830	0.1851	-0.4031
03-115	26	0.2189	-0.4683	0.2132	-0.4718	0.2990	-0.4853
03-116	25	0.1020	-0.2840	0.1024	-0.2909	0.1718	-0.3582
Mean ^(a) (for eight patients)		0.3015	-0.4383	0.3327	-0.4527	0.1817	-0.3779
Mean ^(a) (03-102, 03-109, 03-110 omitted)		0.1911	-0.3536	0.1988	-0.3644	0.1982	-0.3517

(a) The geometric means are given for a and the arithmetic means are given for b.

TABLE 13
Linear regression of power function parameters

Regression ^(a)	Data	Regression parameters	S.E. estimate
a on b	Original	$a = -0.3331 + 1.6316 b$	0.1704
	Adjusted breath	$a = -0.4302 + 1.9137 b$	0.1871
	Adjusted γ -ray	$a = -0.1922 + 1.1202 b$	0.1029
b on a	Original	$b = 0.2758 + 0.4253 a$	0.0870
	Adjusted breath	$b = 0.2865 + 0.3811 a$	0.0834
	Adjusted γ -ray	$b = 0.2419 + 0.5920 a$	0.0748
log a on log b	Original	$\log a = 0.1184 + 1.6714 \log b$	0.1877
	Adjusted breath	$\log a = 0.2108 + 1.8729 \log b$	0.1969
	Adjusted γ -ray	$\log a = -0.2399 + 1.6256 \log b$	0.3060
log b on log a	Original	$\log b = -0.1627 + 0.4219 \log a$	0.0943
	Adjusted breath	$\log b = -0.1824 + 0.3878 \log a$	0.0896
	Adjusted γ -ray	$\log b = -0.2465 + 0.2624 \log a$	0.1229

(a) Absolute values used for simplicity

TABLE 14
Parameters for average power functions computed
from pooled, normalized data by minimizing an error function

	Original data			Adjusted breath radon data			Adjusted γ -ray measurements		
	a	b	Error sum	a	b	Error sum	a	b	Error sum
Eight patients									
Error sum defined by $\Sigma(y_{ob} - y_{pf})^2/y_{pf}$	0.1824	-0.3251	0.2067	0.2003	-0.3399	0.1893	0.1614	-0.3078	0.2561
Error sum defined by $\Sigma(y_{ob} - y_{pf})^2/y_{ob}$	0.2428	-0.4281	0.1474	0.2429	-0.4228	0.1393	0.1970	-0.4207	0.2128
Five selected patients									
Error sum defined by $\Sigma(y_{ob} - y_{pf})^2/y_{pf}$	0.1640	-0.2952	0.1484	0.1730	-0.3078	0.1460	0.1602	-0.2860	0.1548
Error sum defined by $\Sigma(y_{ob} - y_{pf})^2/y_{ob}$	0.2013	-0.3863	0.1167	0.2048	-0.3937	0.1143	0.2155	-0.3890	0.1081

TABLE 15
Comparison of power function parameters obtained by
various methods for averaging

Eight patients	Original data		Adjusted breath radon data		Adjusted γ -ray measurements	
	a	b	a	b	a	b
Means of individual power function parameters	0.3015	-0.4383	0.3327	-0.4493	0.1817	-0.3779
Pooled, normalized data ^(a)	0.2428	-0.4281	0.2429	-0.4228	0.1970	-0.4207
Least squares fit in log plane	0.2742	-0.4297	0.2968	-0.4382	0.1847	-0.3832
Five selected patients						
Means of individual power function parameters	0.1911	-0.3536	0.1988	-0.3644	0.1982	-0.3517
Pooled, normalized data ^(a)	0.2013	-0.3863	0.2048	-0.3937	0.2155	-0.3891
Least squares fit in log plane	0.2164	-0.3787	0.2251	-0.3895	0.2099	-0.3684

^(a)Computed by minimizing the error sum $(y_{ob} - y_{pf})^2 / y_{ob}$.

TABLE 16
Summary of parameters for the single exponential terms
that give the best fit to original and adjusted data

Patient	Original data			Adjusted breath data			Adjusted γ -ray data		
	a	a/A	T _{1/2} , yrs	a	a/A	T _{1/2} , yrs	a	a/A	T _{1/2} , yrs
03-101	3.351	0.0223	24.107	-	-	-	5.798	0.0386	14.496
03-102	3.282	0.0219	13.551	3.588	0.0239	12.826	1.812	0.0121	21.691
03-109	2.020	0.0112	13.711	2.314	0.0129	12.223	-	-	-
03-110	4.414	0.0147	10.306	5.221	0.0174	9.324	2.039	0.0068	18.330
03-111	-	-	-	-	-	-	3.419	0.0171	17.487
03-112	14.607	0.0562	19.114	15.004	0.0577	16.776	15.570	0.0599	20.096
03-114	6.807	0.0227	11.180	6.978	0.0233	10.889	5.801	0.0193	12.663
03-115	3.205	0.0123	14.140	2.934	0.0113	14.910	4.583	0.0176	11.437
03-116	5.343	0.0214	17.434	5.168	0.0207	17.312	5.824	0.0233	16.023
03-121	-	-	-	-	-	-	4.133	0.0461	9.586
Mean		0.0228	15.443		0.0239	13.466		0.0268	15.757
Standard deviation		0.0143	4.557		0.0157	2.988		0.0176	4.056
Standard error		0.0051	1.611		0.0059	1.130		0.0059	1.352
Geometric mean			14.901			13.175			15.267

TABLE 17
Tests of retrospective calculations of total radium administered
when different mathematical functions are used

Patient	Total amount injected, μCi	Recent whole-body measurement		Ratio of computed to actual amount of Ra^{226} administered if retention follows			
		Days elapsed since last injection	Amount, μCi	Power function ^(a)	Power function ^(b)	Power function ^(c)	Single exponential ^(d)
03-101	150	10108	1.617	2.414	2.035	1.655	1.576
		10264	1.697	2.553	2.150	1.747	1.687
		11709	1.225	1.974	1.644	1.324	1.463
03-102	150	10722	0.685	1.054	0.885	0.717	0.722
		11910	0.656	1.067	0.888	0.714	0.804
03-110	300	10202	0.717	0.538	0.453	0.368	0.354
		11842	0.590	0.478	0.398	0.320	0.358
		12073	0.544	0.445	0.370	0.298	0.340
03-111	200	10219	1.238	1.394	1.174	0.954	0.918
		10443	1.035	1.178	0.991	0.804	0.790
		11823	0.890	1.081	0.900	0.724	0.809
03-112	260	10447	5.613	4.917	4.134	3.356	3.295
03-114	300	10104	1.207	0.901	0.759	0.618	0.588
		10264	1.252	0.942	0.793	0.644	0.622
		11707	1.025	0.826	0.688	0.554	0.612
		11936	0.946	0.770	0.640	0.515	0.581
03-115	260	10095	0.784	0.674	0.569	0.463	0.440
		11697	0.703	0.653	0.544	0.438	0.484
		11930	0.673	0.632	0.525	0.423	0.477
03-121	90	10109	0.555	1.382	1.165	0.947	0.903
		10332	0.569	1.433	1.206	0.979	0.952
		11711	0.441	1.185	0.987	0.796	0.878
		11942	0.424	1.152	0.958	0.771	0.870
03-125	200	10575	1.444	1.654	1.390	1.127	1.120

(a) Power function $R = 0.54 t^{-0.52}$.

(b) Power function $R = 0.3015 t^{-0.4383}$.

(c) Power function $R = 0.2068 t^{-0.3750}$.

(d) Single exponential $R = 0.0246 e^{-0.693t/5472}$.

TABLE 18
Estimates of radium body burden based on best power function fit to original data

Patient	a	b	Total μCi injected	Estimated body burdens						
				Maximum, after last injection	1 day later	10 days later	100 days later	300 days later	1000 days later	10,000 days later
03-101	0.1931	-0.3220	150	18.35	10.17	8.48	5.84	4.40	3.09	1.49
03-102	0.7335	-0.5549	150	24.16	21.13	13.72	6.98	4.28	2.32	0.66
03-109	0.8927	-0.6271	180	24.93	23.43	14.21	6.87	4.03	2.04	0.50
03-110	0.4086	-0.5564	300	21.63	15.50	11.24	6.69	4.41	2.49	0.72
03-112	0.1816	-0.2240	260	27.77	19.48	18.04	14.79	12.45	9.86	5.99
03-114	0.3257	-0.4696	300	23.06	16.13	12.81	8.38	5.90	3.64	1.29
03-115	0.2189	-0.4683	260	18.08	10.14	7.93	5.06	3.52	2.16	0.76
03-116	0.1020	-0.2840	250	17.59	8.54	7.65	5.89	4.72	3.51	1.86
	Mean			21.94	15.57	11.76	7.56	5.46	3.64	1.66
	Standard error of mean			1.31	1.97	1.29	1.09	1.03	0.91	0.64

TABLE 19
Estimates of radium body burden based on best power function fit to breath radon data

Patient	a	b	Total μCi injected	Estimated body burdens						
				Maximum, after last injection	1 day later	10 days later	100 days later	300 days later	1000 days later	10,000 days later
03-101	0.2074	-0.3419	150	18.36	10.32	8.46	5.68	4.21	2.89	1.33
03-102	0.8672	-0.5727	150	25.77	24.01	15.22	7.55	4.57	2.42	0.66
03-109	0.9766	-0.6347	180	25.91	25.21	15.12	7.23	4.22	2.11	0.51
03-110	0.5714	-0.5926	300	24.14	19.55	13.58	7.75	4.97	2.70	0.73
03-112	0.1894	-0.2343	260	27.79	19.57	18.03	14.64	12.23	9.58	5.68
03-114	0.3619	-0.4830	300	23.76	17.17	13.46	8.68	6.04	3.69	1.26
03-115	0.2132	-0.4718	260	17.76	9.77	7.61	4.84	3.35	2.05	0.72
03-116	0.1024	-0.2909	250	17.41	8.37	7.46	5.71	4.55	3.36	1.75
	Mean			22.61	16.74	12.37	7.76	5.52	3.60	1.58
	Standard error of mean			1.46	2.32	1.42	1.08	1.00	0.88	0.60

TABLE 20
Estimates of radium body burden based on best power function fit to γ -ray data

Patient	a	b	Total μCi injected	Estimated body burdens						
				Maximum, after last injection	1 day later	10 days later	100 days later	300 days later	100 days later	10,000 days later
03-101	0.2173	-0.3212	150	19.42	11.47	9.57	6.59	4.98	3.49	1.69
03-102	0.2292	-0.4299	150	16.79	8.95	6.75	4.05	2.78	1.73	0.65
03-109	0.5922	-0.5650	180	22.29	17.91	11.86	6.22	3.85	2.08	0.58
03-110	0.0286	-0.2702	300	12.59	2.86	2.61	2.08	1.70	1.29	0.71
03-112	0.1479	-0.1906	260	26.61	18.01	16.93	14.32	12.37	10.14	6.63
03-114	0.1851	-0.4031	300	19.71	11.44	9.61	6.73	4.99	3.30	1.35
03-115	0.2990	-0.4853	260	20.33	13.15	10.11	6.33	4.34	2.62	0.89
03-116	0.1718	-0.3582	250	19.51	11.12	9.49	6.77	5.12	3.52	1.58
	Mean			19.66	11.86	9.62	6.64	5.02	3.52	1.76
	Standard error of mean			1.43	1.72	1.44	1.24	1.13	0.99	0.71

TABLE 21
Power function parameters giving best fit to selected data

Patient	Original data		Breath data		γ -Ray data		Patient	Original data		Breath data		γ -Ray data	
	a	b	a	b	a	b		a	b	a	b	a	b
03-102 ^(a)	0.2454	-0.4334	0.2597	-0.4373	0.1562	-0.3939	03-121	-	-	-	-	0.2908	-0.4064
03-111	-	-	-	-	0.0761	-0.2866	03-126	0.9597	-0.5253	0.9828	-0.5296	0.8978	-0.5141

^(a)Recalculated on assumption that interval between last injection and first measurement was 30 days.

APPENDIX I. DATA FROM ADDITIONAL PATIENTS

For the sake of completeness, the parameters of the power function that provide the best fit to the data of three additional patients, as well as to data for patient 03-102 when a different injection schedule was used, are given in Table 21. The parameters for patients 03-111 and 03-121 were not included in the previous tables, since only gamma-ray measurements were available. The exponents for both patients and the coefficient for patient 03-121 are very comparable to those found for the eight patients discussed in detail. The parameters for patient 03-126 are based on only three rather questionable measurements. The date of the Schlundt measurement could not be ascertained and the last measurement, which in itself was quite accurate, was arbitrarily increased by 15 percent to correct for a missing limb. These parameters, which are considerably higher than the mean values, might reflect erroneous elapsed times instead of a higher than average retention pattern.

The parameters given for patient 03-102 are quite interesting. No dates could be found for the administration of radium to this patient. The fact that Schlundt listed the same total quantity, injection schedules and elapsed time for patient 03-102 as he did for patient 03-101, along with his listing of these patients serially as "A" and "B," led us to re-analyze the data on the supposition that 15 weekly injections had been received by both patients between about August 20, 1931 and November 26, 1931 (the schedule used for patient 03-101 based on hospital notes). The curve-fitting for various combinations of the original, breath, and gamma-ray data with the power function and the single exponential are given in Tables 22A-D. These parameters agree extremely well with the average values given for all eight patients in Table 12. If these parameters of Table 22 are substituted for those for patient 03-102 in Table 12, then only the parameters for patients 03-109 and 03-110 would seem out of line with the values for the other patients.

TABLE 22A

Additional calculations for data on patient 03-102
based on the assumption that the first measurement
was 30 days after the last injection^(a)

Days since last injection	μCi radium			Measured radon retention ratio	Expected radon retention ratio	Total-body radium calculated from	
	Breath radon	RaB & RaC	Total			Breath radon	RaB & RaC
30 ^(b)	5.4	0.6	6.0	0.10	0.1240	6.164	4.839
213 ^(b)	2.805	0.3	3.105	0.10	0.1578	3.331	1.901
7,118 ^(c)	0.39	0.105	0.495	0.22	0.2875	0.547	0.365
10,579 ^(d)	-	0.212	0.687 ^(e)	-	0.3084	-	0.687
11,767 ^(d)	-	0.207	0.658 ^(e)	-	0.3144	-	0.658

(a) Patient B, Schlundt et al. (1933) and Norris et al. (1955); 15 injections.

(b) Data from Schlundt et al. (1933).

(c) Data from Norris et al. (1955).

(d) Recent measurements made at Argonne in whole-body counter.

(e) Value for total calculated from γ -ray measurement.

TABLE 22B

Curve fitting to original data for patient 03-102

Days since last injection	Measured radium values	Values computed by best power function			Values computed by best single exponential	
30	6.0	5.894	5.856	-	4.202	-
213	3.105	3.231	3.309	3.105	4.076	3.101
7,118	0.495	0.711	-	-	-	-
10,579	0.687	0.594	0.662	0.687	0.726	0.728
11,767	0.658	0.566	0.632	0.658	0.596	0.617
N	5	5	4	3	4	3
Power function	{ a b	0.2717	0.2454	0.1980		
		-0.4560	-0.4334	-0.4062		
Single exponential	{ a T _{1/2} , yrs				4.223	3.190
					11.402	13.580
Mean error sum		0.0204	0.0045	0.000000	0.2126	0.0017
Mean % deviation		15.41	3.91	0.20	19.09	4.13

Arrows indicate parameters selected for inclusion in summary tables 12 and 16.

TABLE 22C
Curve fitting to adjusted data for patient 03-102

Days since last injection	Radium adjusted from breath radon measurements	Values computed by best power function			Values computed by best single exponential	
30	6.164	6.132	6.094	-	4.440	-
213	3.331	3.354	3.427	3.331	4.303	3.326
7,118	0.547	0.735	-	-	-	-
10,579	0.687 ^(a)	0.614	0.676	0.688	0.728	0.731
11,767	0.658 ^(a)	0.584	0.645	0.658	0.594	0.614
N	5	5	4	3	4	3
Power function	{ a	0.2843	0.2597	0.2355		
	{ b	-0.4574	-0.4373	-0.4248		
Single exponential	{ a				4.463	3.431
	{ T _{1/2} , yrs				11.071	12.980
Mean error sum		0.0133	0.0010	0.0000	0.1937	0.0019
Mean % deviation		12.43	1.93	0.06	18.22	4.39

(a) Value from γ -ray measurement.

Arrows indicate parameters selected for inclusion in summary Tables 12 and 16.

TABLE 22D
Curve fitting to adjusted data for patient 03-102

Days since last injection	Radium adjusted from γ -ray measurements	Values computed by best power function			Values computed by best single exponential	
30	4.839	4.424	4.401	-	2.798	-
213	1.901	2.546	2.624	1.901	2.732	1.899
7,118	0.365	0.632	-	-	-	-
10,579	0.687	0.535	0.608	0.683	0.714	0.711
11,767	0.658	0.512	0.583	0.663	0.612	0.636
N	5	5	4	3	4	3
Power function	{ a	0.1754	0.1562	0.0588		
	{ b	-0.4201	-0.3939	-0.2760		
Single exponential	{ a				2.809	1.937
	{ T _{1/2} , yrs				14.651	20.034
Mean error sum		0.0799	0.0657	0.0000	0.3071	0.0005
Mean % deviation		31.96	17.50	0.46	24.20	2.391

Arrows indicate parameters selected for inclusion in summary Tables 12 and 16.

APPENDIX II. EFFECT OF NONUNIFORMITY OF RADIUM DEPOSITION IN THE SKELETON

When the various combinations of data were fitted with either a power function or a single exponential, each Schlundt and Norris measurement was either assumed to be correct and was used directly, or if it was obviously incorrect, it was omitted from the analysis. No attempt was made to adjust any of these measurements to correct for variations in detector sensitivity that resulted from the distribution of radium in a nonuniform fashion in the various patients. If we assume that the same relative radium distribution existed in the patient's body in 1931-1932 and 1951-1952 as in 1964, we can estimate the relative error introduced in each case into the earlier sets of measurements by the nonuniformity of radium distribution in the body.

The percentage of the total-body radium that was present in each seventh segment of the body of each patient in 1964 is given in Table 23. These results were obtained by recent measurements of the patients by the single-crystal, seven-position technique.⁴ In addition, the percentage of the skeletal weight that is computed to be present in each seventh segment of the body is also given in the table. If the radium were uniformly distributed throughout the skeleton, the percentage of body radium present in each segment would be identical to the percentage of the total skeletal weight in that segment. In fact, a previously published study¹⁴ of the distribution of radium in about 40 former dial painters demonstrated that the average distribution for the group, as well as that in many individual patients, was the same as the distribution of skeletal proportions given here.

Although Schlundt's chamber is apparently no longer in existence, we can still estimate the relative sensitivity of this detector to radium in different segments of the body. Since the sensitivity of the counter to radium in different segments of the body is mainly a function of the distance to the radium and the interposed body mass between the radium deposit and detector, the sensitivity of Schlundt's counter to radium in the different segments will not be greatly different from that of a NaI(Tl) crystal located at the same position.

Schlundt placed his chamber fairly close to the patient's back at about shoulder height. If we arbitrarily assign a sensitivity of one to the radium in our segment 2, then his counter would have about one-half this sensitivity to radium in segments 1 and 3, and about one-fourth the sensitivity in segment 4. Segments 5, 6 and 7 were so shielded from the chamber by the patient's body and were so far away that we can neglect the radium present in these volumes.

For the 1951-1953 measurements, a small NaI(Tl) crystal was placed behind and 10 cm from the seated patient's hips. In this case, if we assume that the radium in our segment 4 was counted with a sensitivity of one, the radium in segment 3 was counted with about one-half this efficiency, the radium in segment 2 with about one-fourth, and the radium in segment 1 with about one-eighth. The radium in the legs was shielded from the counter to such a degree that we can neglect the component for these calculations.

For Schlundt's measurements, the sum of the percentage of radium in segment 2, one-half the percentage in segments 1 and 3, plus one-fourth the percentage in segment 4, can be compared to the sum of the fractions of skeleton in these sections. In the case of patient 03-101, Schlundt's counter would have a relative sensitivity of 0.415 compared to an expected value of 0.354 based on skeletal distribution. Consequently, his measurement was probably 17 percent high for this patient.

For Norris' measurements, the sum of the percentage of radium in segment 4 can be added to one-half that of segment 4, one-fourth of that in segment 2 and one-eighth of that in segment 1. Again, in the case of patient 03-101, the counter employed during 1951-1953 would have a relative sensitivity to the radium present of about 0.305 compared to an expected sensitivity based on skeletal distribution of about 0.275. Thus these measurements would also be about 11 percent high.

These errors of 17 and 11 percent are relatively unimportant, for if the measured values were corrected by these amounts, the parameters of the power function would be reduced only slightly. Since the parameters for this patient based on the original gamma-ray measurements agree quite well with those based on breath measurements, the former are probably reasonably correct.

By a similar analysis one can conclude that the Schlundt gamma-ray measurement on patient 03-102 was probably about 35 percent low while the Norris measurement was about 20 percent low. In terms of the curve-fitting to the set of data for this patient, the Norris measurement was at least 50 percent low. This conclusion is consistent with the fact that the power function parameters found to fit the gamma-ray data are very much smaller than those found to fit the breath values.

Although patient 03-110 contains a relatively large percentage of her total radium in her skull, both the Schlundt and Norris measurements yielded values that were about 10 percent high. The extremely low radium values found by Schlundt cannot be explained by the distribution present in this patient's body today unless 1) the radium in segments 1 and 3 was counted with less relative sensitivity with respect to the radium in segment 2 than is assumed for these calculations, or 2) the radium was not concentrated in the skull of this patient in 1931.

Both patients 03-114 and 03-115 contain a considerably higher than average percentage of total-body radium in positions 3 and 4 and a smaller than average percentage in the first segment. Based on the assumed sensitivities, Schlundt's measurement would be about 15 percent high while Norris' measurement would be about 75 percent high. It is significant that the gamma-ray data of both of these patients supported two discontinuous power functions (Tables 7D and 8D). This apparent break in the retention curve can now be explained by the fact that the 7,111 and 7,871-day measurements were too high.

Patient 03-116 contains higher than average amounts of radium in segments 2, 3 and 4, and less than average in the other four segments. Schlundt's and Norris' gamma-ray measurements were probably about 24 and 17 percent high, respectively. These high measurements could account for the fact that the power function parameters based on the gamma-ray measurements were larger than those based on breath radon.

Five of these six patients contain more than the average amount of radium in the upper torso. Consequently, Schlundt's gamma-ray measurements for these patients could have ranged from 10 to 24 percent high. However, in the absence of any information concerning his absolute calibration, we are not justified in reducing all or any of his values by an arbitrary amount. Nevertheless, the fact is inescapable that either his gamma-ray value for patient 03-102 is considerably low or his values for the other patients are very high. The discrepancy between the parameters found to fit the breath and gamma-ray data for patient 03-102 can now be explained.

A somewhat greater variability due to nonuniformity of radium distribution in the body exists in the Norris measurements. If we assume that his absolute calibration was correct, then his published value for one patient was low, the values for three patients were 10 to 15 percent high and the values for two patients were about 175 percent of the true value.

Acknowledgement

The authors are pleased to acknowledge the assistance of Dr. Ronald A. Hamelink, Mr. Burton S. Garbow, and Mr. Donald F. Jordan of the Applied Mathematics Division, who provided considerable assistance with the computer programs used in the preparation of this paper. Mr. Henry F. Lucas kindly provided us with radon breath analyses for some of the patients. Mr. John B. Corcoran and Mr. Carter A. Priess provided technical assistance with the whole-body measurements. Mrs. Marilyn Langler, R.N., Miss Regina Wall, R.N., and Mrs. Edna Fadden, R.N. helped with the handling of patients. The interest and cooperation of Dr. Werner Tuteur of the Elgin State Hospital and Dr. R. J. Graff of the Manteno State Hospital made the recent studies possible.

TABLE 23
Summary of nonuniformity of radium distribution in skeleton

Patient	Percentage of total count in each segment							Comments
	1	2	3	4	5	6	7	
03-101	24.3	21.4	6.4	19.0	17.1	7.6	4.2	Less radium in lower legs and feet than expected. Expect Schlundt's γ -ray values to be somewhat too high; Norris' values less in error.
03-102	18.0	8.8	2.5	16.1	16.4	13.5	24.7	Less radium in neck and thorax; much more in feet than expected. Expect Schlundt's and Norris' γ -ray values to be low.
03-110	31.3	12.4	12.7	16.9	9.7	10.0	7.2	Radium higher in head than expected; generally lower elsewhere except thorax. Expect Schlundt and Norris to be low, especially Norris.
03-114	5.4	21.8	16.2	36.8	9.9	6.2	3.7	Radium unusually high in trunk. Expect Schlundt and Norris to have high γ -ray values.
03-115	15.3	17.5	12.2	34.2	6.4	8.2	6.2	Radium higher in trunk and less in legs. Expect Schlundt and Norris to be high on γ -rays.
03-116	8.6	23.4	19.5	25.7	8.1	9.1	5.6	Same as patient 03-114.
Estimated proportion of skeletal mass per segment	19.4	18.3	6.4	17.3	13.4	13.2	12.0	

Note: Schlundt--detector over upper thoracic spine; would tend to exaggerate counts from head and upper trunk and to decrease counts from lower extremities.
Norris--detector over lumbar spine; would exaggerate counts from trunk and upper thighs and decrease counts from head and lower extremities.

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EVALUATION OF THE IMPROVED POWER FUNCTION

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Speckman and Norris¹ and Decker, Kasper and Norris² have demonstrated that the retention of Sr^{85} in mice, rats and dogs injected at any age can be fitted more accurately with the expression, $R = a(t+d)^{-b}$ than with the simple power function, $R = at^{-b}$. They state 1) that the optimum value of d required to provide the best fit to the experimental data is a function of the age of the animals, 2) that the value of b , which may vary from species to species, is constant for animals of all ages within any species if the optimum value of d is employed, and 3) that the value of b found with the improved power function is the same value that fits the long-term measurement with a simple power function. They propose that once the equation for d as a function of age and the value of b are determined for a particular isotope for a given species, it will then be possible to calculate the buildup and retention of this isotope in the animal under the conditions of constant ingestion.

In a recent paper,³ we fitted retention data for radium in mice both with a series of discontinuous power functions and with the sums of a series of exponential terms. These analyses demonstrated that the retention of radium in one mouse (No. 52), for example, could be approximated by one power function (slope = -0.2046) from 250 through 20,000 minutes, and with a second power function (slope = -0.2919) from 20,000 through 628,000 minutes. Similarly, the retention by another mouse (No. 54) could be fitted by one power function (slope = -0.2531) from 250 through 39,000 minutes, by a second power function (slope = -0.4458) from 39,000 through 100,000 minutes, and by a third power function (slope = -0.2916) from 100,000 through 586,000 minutes (see Figure 10). It was also shown that the sum of four exponential terms fitted the experimental data almost exactly from the time of injection, instead of from 250 minutes on, until the death of the animal. These mouse data, which are very accurate, have been used to study the improved power function, $R = a(t+d)^{-b}$.

Methods and Results

Application of the Improved Power Function to Mouse Data

In order to be able to evaluate the improvement in fit provided by incremental changes in d and to employ an existing computer program that fitted the data with the simple power function, $R = at^{-b}$, the following procedure was employed. Arbitrarily selected values of time, d , such as 50, 100, 200 minutes, etc., were added in turn to the time of each measurement made on a particular mouse. Each complete series of measurements

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Fig. 10. Double logplot of radium retention by two mice (Nos. 52 and 54) after intravenous administration. The data have been fitted with discontinuous power functions.

with adjusted time values was then fitted with a simple power function, at^{-b} , with the aid of an electronic computer and the Variable Metric Minimization program.⁴ The resultant error sums were examined and larger

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Fig. 11. Mean error sums for various values of increment d in the improved power function plotted against d . Computations for data on four mice demonstrate that a minimum error sum, and hence an optimum value of d , exists for each mouse.

values of time were added to the actual time of each measurement if the minimum error sum had not been reached, or intermediate values of time were added if the increments employed had straddled the minimum. This procedure, in effect, fitted the data with the expression, $R = a(t+d)^{-b}$, where d was arbitrarily set at different values.

The mean error sums when various values of time, d , were added to the data for four mice are plotted in Figure 11. Here, where all the measurements starting immediately (i.e., within 20 or 28 minutes) after the injections were employed, each minimum is fairly well defined. The values of a and b for the simple power function, and of a , b and d for the modified power function that provide the best fit to the data for these four mice are given in Table 24, along with the mean error sums. As the latter indicate, the modified

power function does provide a significantly better fit to the data. However, the coefficients of variation for each set demonstrate that the values of b exhibit about the same variability for the improved as for the simple power function, while the values of a have a greater variability. Consequently, since the values of b do not converge to the same value when optimal values of d are used, this modification of the power function does not provide a constant value of b within a species.

We have reported in the preceding paper that a high degree of correlation exists between the a and b parameters of the power function found to fit the retention of radium in humans. That is, a large value of a is always associated with a large value of b . For the four pairs of parameters given in Table 24 a correlation coefficient of 0.983 ($P \cong 0.02$) exists between a and b of the single power function and a value of 0.938 ($P \cong 0.06$) for the modified power function. These results would argue against the improvement in flexibility purported to result from the modified power function.

TABLE 24

Parameters that provide best fit to all data
starting 20 minutes after injection

Simple power function ($R = at^{-b}$)			
Mouse No.	a	b	Mean error sum
52	2.5174	0.1950	0.00869
53	2.1476	0.1778	0.00407
54	2.9823	0.2374	0.00493
55	2.9653	0.2273	0.00429
Mean	2.6532	0.2094	
Standard deviation	0.3999	0.0278	
Coefficient of variation	15.09%	13.28%	

Improved power function [$R = a(t+d)^{-b}$]				
Mouse No.	d	a	b	Mean error sum
52	432	5.1468	0.2662	0.00040
53	175	3.1918	0.2181	0.00054
54	210	5.0757	0.2922	0.00057
55	275	5.2298	0.2866	0.00045
Mean		4.6610	0.2658	
Standard deviation		0.9815	0.0337	
Coefficient of variation		21.06%	12.68%	

The reason for the improvement in fit when arbitrary increments of time are added can be seen by examination of the data plotted in Figure 12. Here, the measured radium values for mouse 54 are plotted against 1) the actual elapsed times between injection and measurement, 2) elapsed time plus 210 minutes, and 3) elapsed time plus 1000 minutes. Only the actual elapsed times are shown after 10,000 minutes since the addition of 210 minutes or even 1000 minutes will not shift the horizontal position of the data significantly. The addition of 210 minutes shifts the early measurements to the right so that they fall closer to this line. The addition of 1000 minutes obviously shifts the early measurements too far to the right so that these points do not fall close to the line that fits the later measurements. The fact that the later measurements fell along the line when 1000 minutes had been added suggested that the optimum value of d might depend upon the time and number of early measurements.

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Fig. 12. Double log plot of radium retention by mouse No. 54 against time for the original data (•), for data adjusted by addition of $d = 210$ minutes (X), and for data adjusted by addition of $d = 1000$ minutes (o)

Dependence of d on Time Span of Measurements

In order to evaluate the dependence of d upon the time span of the measurements employed, the same procedure was performed on the data for mice 52 and 54 when only the measurements made after the first 250, 1440 and 12,000 minutes were included, the earlier measurements in each set being omitted. The mean error sums that result when different values of d are added to the data starting at 0, 250, 1440 and 12,000 minutes for mice 52 and 54 are shown in Figures 13 and 14, respectively. The minima are less well defined as additional early measurements are omitted, and the start of the time span of measurements is thereby advanced.

The values of d , a and b that provided the best fit to the data for mice 52 and 54 for these four arbitrary time spans are given in Table 25. The best fit was obtained when 432, 600, 1000 and 2500 minutes, respectively, were added to the measurements made on mouse 52, and when 210, 210, 2500 and 5000 minutes, respectively, were added to the time of the measurement of mouse 54. Significantly, the optimum value of d did not increase in the same manner for the two mice. Consequently, the optimum value of d required to fit the data for a mouse does vary with the time span of the measurement but not as a direct function of it. The variability shown by these data make it difficult to prove the hypothesis that the optimum value of d is directly related to the age of the mouse at the time of injection since the time span selected for analysis may introduce an overriding influence on the optimum value of d .

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Fig. 13. Plot of the mean error sums obtained when different values of d were added to the data of mouse 52, starting at 0, 250, 1440, and 12,000 minutes after injection

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Fig. 14. Plot of the mean error sums obtained when different values of d were added to the data of mouse 54, starting at 0, 250, 1440, and 12,000 minutes after injection

TABLE 25

Dependence of the parameters of the modified power function on arbitrary changes in time span of measurements

Measurements from t minutes until death of animal	d	a	b	Mean error sum
Mouse 52				
0	432	5.1468	0.2662	0.000398
250	600	5.6146	0.2738	0.000350
1500	1000	6.1590	0.2816	0.000393
12,000	2500	7.7100	0.2999	0.000290
Mouse 54				
0	210	5.0757	0.2922	0.000568
250	210	5.0170	0.2911	0.000611
1500	2500	8.5979	0.3380	0.000191
12,000	5000	11.1923	0.3594	0.000144

The proponents of the modified power function also state that the value of b (slope) when the optimum value of d is employed is identical to the b of the simple power function found to fit the long-time measurements. The statement is true only if the slopes of the early section of the retention

curve are never steeper than the long-term segment. As can be seen in Table 25, the value of b found for both mice increased progressively as the measurements made during the first 250, 1400 and 12,000 minutes were omitted. The later two values of b (0.2816 and 0.2999) found to fit the data for mouse 52 starting at 1500 and 12,000 minutes, respectively, do agree quite well with the value of 0.2919 found to fit the last portion of the data with the simple power function (Figure 10). Moreover, for mouse 54 the values of b (0.2922 and 0.2911) found to fit all of the data or those from 250 minutes onward, respectively, do agree with the value of b (0.2916) for the last portion of the data fit by the simple power function found to fit. However, the values of b (0.3380 and 0.3594) found to fit the data from 1500 and 12,000 minutes onward were much higher than the final slope due to the steep middle section. It thus appears that the use of the modified power function $[R = a(t+d)^{-b}]$ may or may not yield a value of b which is the same as the long-term value or the same value for all animals of the same species.

Interdependence of a on b and b on d

From a visual inspection of the values of a , b and d given in Table 25 for either mouse, one would suspect that these terms are highly correlated with each other but not with the time span of the measurements employed. By computer analysis, correlation coefficients of 0.997 and 0.993 ($P < 0.01$) were found between a and b , and of 0.978 and 0.979 ($P \approx 0.02$) between d and b for mice 52 and 54, respectively. This high degree of correlation simply means that a and b increase in a predictable manner as d is increased.

The thesis has been put forth that the introduction of the d term into the power function permits the a term to assume magnitudes different from the actual one-day value.¹ Consequently, the modified power function supposedly has one more degree of freedom than the simple power function. Actually, the value of a is still almost completely defined by the value of b and is, in effect, the one-day value of the simple power function required to fit data that have been translated d amount of time to the right along the x axis. In practice, the fact that the a of the modified power function has a value different from the actual one-day value only complicates the problem since now the magnitude of a varies over a wide range.

This situation can be demonstrated with the results obtained when the data of mice 52 and 54 from 12,000 minutes onward were fitted with the modified power function. The values of a , b and the resultant mean error sums found for different values of d from 0 to 10,000 minutes are given in Table 26. Practically the same fit to the data, i.e., mean error sum, was obtained for any value of d from 0 to 10,000. For both mice and for this range of d , the value of b increased about 18 percent while the value of a doubled. Although this change in b might be considered to be insignificant,

it becomes very important when the dependence of a on b is considered. Correlation coefficients of 0.996 and 0.993 were found to exist between the values of a and b given in Table 26 for mice 52 and 54, respectively. Comparable correlation coefficients were found to exist between the a and b terms found when variable values of d were added to all the data, the data from 250 minutes onward, and the data after 1500 minutes. When the values of a in Tables 26 and 27 were plotted against b on a semilog grid, the resulting lines were in fact quite straight (Figure 15). Linear regression equations of $\log a$ on b were obtained for each set of data and are given in Table 28. In effect, a is still highly dependent upon b . Thus the fact that a can assume values other than the actual one-day value has not introduced a second degree of freedom but has further complicated the problem since now it is necessary to have a table relating a to b in order to be able to choose the appropriate a .

TABLE 26
Parameters that provide best fit to data starting 12,000 minutes after injection

d	a	b	Mean error sum	d	a	b	Mean error sum
Mouse 52				Mouse 54			
0	6.5098	0.2865	0.0003006	0	7.5287	0.3277	0.0002016
500	6.7418	0.2893	0.0002959	1000	8.1918	0.3345	0.0001781
1000	6.9763	0.2920	0.0002925	2000	8.8829	0.3410	0.0001620
1500	7.2261	0.2948	0.0002958	3000	9.6319	0.3475	0.0001515
2000	7.4626	0.2974	0.0002949	4000	10.3935	0.3535	0.0001455
2500	7.7100	0.2999	0.0002895	5000	11.1923	0.3594	0.0001438
3000	7.9617	0.3025	0.0002905	6000	12.0546	0.3653	0.0001455
4000	8.4802	0.3075	0.0002950	8000	13.8861	0.3764	0.0001568
6000	9.5740	0.3170	0.0003122	10,000	15.9193	0.3872	0.0001762
8000	10.7526	0.3261	0.0003379				
10,000	12.0211	0.3348	0.0003696				

TABLE 27
Parameters that provide best fit to data starting 250 minutes after injection

d	a	b	Mean error sum	d	a	b	Mean error sum
Mouse 52				Mouse 54			
0	3.5360	0.2318	0.0016818	0	4.0011	0.2694	0.0008651
100	3.9155	0.2413	0.0011084	50	4.2697	0.2757	0.0007350
150	4.0976	0.2454	0.0009205	100	4.5184	0.2811	0.0006629
200	4.2737	0.2493	0.0007684	150	4.7693	0.2863	0.0006241
250	4.4484	0.2529	0.0006556	200	5.0170	0.2911	0.0006113
300	4.6183	0.2563	0.0005618	250	5.2613	0.2955	0.0006180
350	4.7882	0.2595	0.0004922	300	5.5074	0.2998	0.0006395
400	4.9558	0.2627	0.0004396	400	5.9946	0.3077	0.0007152
450	5.1220	0.2656	0.0004012	500	6.4820	0.3150	0.0008212
475	5.2046	0.2670	0.0003866	600	6.9717	0.3217	0.0009473
500	5.4509	0.2712	0.0003747	700	7.4658	0.3280	0.0010870
600	5.6147	0.2738	0.0003504	800	7.9648	0.3339	0.0012370
700	5.9400	0.2787	0.0003562	900	8.4700	0.3395	0.0013940
800	6.2641	0.2834	0.0003844	1000	8.9819	0.3448	0.0015560
900	6.5877	0.2878	0.0004302				
1000	6.9114	0.2909	0.0004894				
1200	7.5631	0.2998	0.0006419				
1500	8.5486	0.3104	0.0009133				
2000	10.2354	0.3257	0.0014304

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Fig. 15. Semilog plot of the values of a against b (Tables 26 and 27). The data from Table 26 are indicated by X while the data from Table 27 are shown by dots.

TABLE 28

Parameters for regression of logarithmic transformation of a on b

Time span	Log a
Mouse 52	
All data	$= -0.5074 + 4.5887 b$
250 minutes on	$= -0.5954 + 4.9162 b$
1500 minutes on	$= -0.6786 + 5.2097 b$
12,000 minutes on	$= -0.7667 + 5.5143 b$
Mouse 54	
All data	$= -0.6071 + 4.5044 b$
250 minutes on	$= -0.6573 + 4.6666 b$
1500 minutes on	$= -0.7679 + 5.0447 b$
12,000 minutes on	$= -0.9161 + 5.4685 b$

Even though a is highly correlated with b , the modified power function could still have one more degree of freedom than the single power function if the value of d were not highly correlated with b . To test this, the correlation coefficients between b and d found for the four time spans of data for both mice were obtained. When all data starting immediately after injection were employed, correlation coefficients of 0.953 and 0.967 were found between b and d for the 21 and 17 arbitrary values of d employed for mice 52 and 54, respectively. These values of d ranged

from 0 to 1000 minutes. For the three other time spans, the comparable correlation coefficients were about 0.99. Since all of these correlation coefficients are highly significant statistically and since b is accordingly highly dependent upon d , the modified power function does not have a larger number of degrees of freedom than the single power function.

Conclusions

It is probably true that the actual retention data for these mice, or for any animal, can be fitted more accurately with a single modified power function, $R = a(t+d)^{-b}$, than they can be with a single simple power function,

$R = at^{-b}$. The addition of optimum value of d effectively transposes the early measurements to appropriate times either longer or shorter than the actual times so that they fall closer to the straight line prescribed by the simple power function that fits the long-term measurement.

However, it is doubtful that the constants for a generally improved power function that fits all animals of different ages within a species can be derived. The magnitude of d required to provide the best fit to a particular set of retention data is highly dependent upon the time schedule of measurements after the injection. Since this value of d does not vary systematically for all animals as the elapsed time between injection and the first measurement is increased, the value of d cannot be related only to the age of the animal at the time of injection.

The magnitude of both a and b depends both upon the time span of the measurement and the value of d employed. In addition, even if the long-term retentions in animals of all ages were identical, the value of b would converge to the value of the simple power function required to fit the long-term retention only if the slope of the retention curve during earlier time were never steeper than the final section.

The substitution of the modified power function for the simple power function destroys the usual arguments put forth in behalf of the simple power function over the sum of a series of exponentials, i.e., the simplicity of the function and the fact that there are only two constants whose magnitudes are fairly well known and which can be easily remembered. Now, instead, there are three interrelated constants and the magnitude of a , in particular, varies over a wide range.

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CESIUM-137 RETENTION IN MICE OF DIFFERENT AGES

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Some question exists as to the biological half-time of Cs^{137} in man. At the international symposium on Assessment of Radioactivity in Man held in Heidelberg, Germany, May, 1964, a number of papers were devoted to a review of the results obtained by investigations in several countries.¹ Some investigators reported half-times as short as 10 days for newborn humans, 30 or 40 days for children and 60 to 150 days for adults. Since this age effect is opposite to that we had observed for the retention of radium in mice, we undertook a study of the retention of Cs^{137} in mice as a function of age.

Cs^{137} chloride was injected intravenously into mice of varying ages: 21 days, 30 days, 3 months, 11 months, 22 months and 32 months. Each injection of 0.1 ml contained about 0.2 μCi Cs^{137} , which gave about 50,000 counts per minute in the mouse with our counting system. Each age group consisted of six female mice except for the oldest group, which consisted of five mice. Each mouse was placed in a thin, perforated holder and counted under the face of an 8-inch-diameter by 4-inch-thick NaI(Tl) crystal that was located in the human whole-body counter.

The Cs^{137} content of each mouse was measured 2 minutes after injection and again about 400 minutes later. Subsequent measurements were made every day for the next 15 days and then less frequently until the Cs^{137} content had decreased to 0.1 percent of the amount injected (35th to 65th day).

Results and Analysis of Data

The average Cs^{137} content of the mice in each age group and the elapsed time between injection and measurement are given in detail in Tables 36 to 41 of the Appendix. The Cs^{137} content is given as a fraction of the quantity present at the time of the first measurement immediately after injection.

The results obtained for the six age groups form a rather consistent pattern. In general, the younger the mice, the more rapidly they lost cesium (Table 29). After the mice were one year old the retention values tended to cluster together and the very old mice appeared to retain cesium somewhat less avidly than did the mice that were 11 and 22 months old at the time of injection.

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TABLE 29
Cs¹³⁷ retention by mice as a function of age

Age of mice at injection	Days required to decrease to percentage of injected dose					
	50%	25%	10%	1.0%	0.5%	0.1%
21 days	1.597	3.854	7.361	19.306	24.167	35.417
30 days	2.292	5.069	9.722	24.792	29.861	43.611
3 months	2.986	6.215	13.403	32.639	38.611	54.722
11 months	2.917	7.222	14.306	35.764	42.917	64.722
22 months	4.722	9.375	17.188	38.333	45.000	60.764
32 months	4.722	9.618	16.319	37.083	43.264	60.069

Since the purpose of this study was to determine the dependence of the biological half-time of Cs¹³⁷ in mice upon the age of the animal at the time of injection, the data obtained for each age group were analyzed extensively. The retention of soft tissue-seeking radioisotopes, such as Cs¹³⁷, is traditionally expressed as either a single exponential term or as the sum of two, or perhaps three, exponential terms. This curve fitting, which is frequently accomplished by graphic analysis, leads to parameters for the exponential terms that provide an approximate fit to the data. Once published, these parameters are often accepted as if they provided an exact fit to the data, and, consequently, the values of these parameters are assumed to describe the metabolic mechanism of the animal. Our analyses were performed not only to obtain the best possible fit by a sum of exponentials but, in addition, to determine whether a unique fit to the data could be obtained with some combination of continuous or discontinuous exponential terms.

The retention data for each age group have been fitted either (1) with the sums of two, three and four exponential terms, (2) with the sums of exponential terms for a segment of the time span and a discontinuous term over the remaining section, or (3) with several discontinuous terms. This curve fitting has been performed with the aid of an electronic computer and a program called Variable Metric Minimization (VMM) developed by Davidon.² The basic input data are the time and the quantity of Cs¹³⁷ for each measurement in the series, the number of exponential terms to be used, and approximate values of the parameters of the exponential terms. The computer then successively alters the parameters to minimize an error function between the actual and computed values. The error function used in this analysis was defined as $\sum (\text{obs} - \text{calc}) / \text{calc}$ for all the points in a series of measurements. Consequently, a 1.0 percent difference between the calculated and observed values when the mouse has only 0.1 percent of the injected dose produces the same contribution to the error sum as a 1.0 percent difference between the calculated and observed values when the mouse contains 100 percent of the

injected dose. It was necessary to define the error sum in this way if the small values at long times were to contribute a large enough component to the error function to enable the computer to fit these small values with a long half-time term.

This program includes one feature that can be of importance for data fitting. The program cannot employ more exponential terms than specified on the input format but it can and will, for all practical purposes, eliminate terms by converging the half-time of two or more exponents to the same value if the data can be best fitted by fewer terms than specified. If a sufficient number of accurate measurements are available, the program will arrive at the unique solution of, for instance, three exponential terms, even though the observations span a relatively small lapse of time.

For the time of each measurement the computer output lists the observed values, the values calculated by use of the parameters that provided the best fit, and the percentage difference between actual and computed values. In addition, the computer gives the parameters for the best fit and the total error sum. These quantities, particularly the percentage differences, can be visually scanned to evaluate the fit and to determine whether the set of parameters provide a unique fit (i.e., computed values depart from the observed values within the limits of counting errors) or only the best fit with a given function.

It turns out that the retention data for Cs^{137} injected intravenously into mice of various ages cannot be fitted with the same number of exponential terms by permitting only the parameters to vary. Consequently, the analysis of the data for each age group will be discussed separately.

21-Day-Old Mice

When plotted on semilog paper, all of the retention data of the 21-day-old mice from 2 minutes after injection through 55,000 minutes (38 days), when they contained less than 0.1 percent of the amount injected, fell along a continuous curve which approached a straight line at long times (Figure 16). This entire set of data can be fitted very closely with the sum of three exponential terms. Although measurements (on the 11.2 and 32nd days) fell about 6 percent above and one measurement (on the 16.9th day) fell about 5 percent below the values predicted by the sum of three exponentials, most of the actual values fell within 1 percent of the computed values.

The parameters obtained for the sum of a series of two, three and four terms and the resultant error sums are given in Table 30. The sum of two exponential terms did not provide as good a fit to the data as did the sum of three exponentials. When four terms were employed, the computer reduced two exponents to the same value and effectively returned to the same values that were found when only three terms were employed. The sum of three exponentials can be accepted as a reasonably exact fit to the data.

Fig. 16

Cesium-137 retention by mice after administration by intravenous injection at age of 21 days. Smooth curve is the sum of three exponential functions.

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TABLE 30
Parameters of exponential terms to fit retention of Cs^{137} in 21-day-old mice

No. of exponentials	Time span of measurements, minutes	No. of measurements	a_1	$T_1^{(a)}$	a_2	$T_2^{(a)}$	a_3	$T_3^{(a)}$	a_4	$T_4^{(a)}$	Total error sum	Mean error sum
2	2-54719	29	0.7288	2693	0.1469	7087					0.062060	0.002140
3	2-54719	29	0.2766	731	0.6193	3090	0.1187	7423			0.016814	0.000580
4	2-54719	29	0.2764	730	0.6194	3089	0.1178	7423	0.00099	7350	0.016814	0.000580

^(a) T_n = half-time in minutes of nth exponential term.

The retention pattern and analyses obtained with these data can be employed to illustrate the shape of the curve defined by the sum of exponential terms. This discussion will aid in the explanation of the analyses of data obtained with the older mice.

Attention is directed to the fact that when plotted on a semilog grid these retention data do not follow a straight line at early times. However, the retention pattern does approach a straight line after 25 days since the first two terms do not contribute to this later segment. In general, a single exponential term will yield a straight line on a semilog grid and the sum of two exponential terms whose coefficients and exponents are very different in magnitude will yield a pattern that has two straight-line segments. As the magnitude of the two half-times approaches the same value, the two straight-line segments will change into a single curved line and then into a single straight line. Accordingly, the sum of two exponential terms of similar half-times will not yield a curve with two straight-line segments of slightly different slopes but will give rise to a single straight-line segment.

30-Day-Old Mice

The retention data of the 30-day-old mice followed a slightly more complicated pattern than that of the 21-day-old mice. When plotted on semi-log paper, the data from 2 to 24,000 minutes could be fitted with a gently sloping curve that became tangential to a straight-line section from 28,930 through 44,674 minutes (Figure 17). The points from 44,674 through 64,971 minutes were considerably above the projection of this straight-line segment but appeared to fall along a second straight line. These two straight-line segments can be fitted separately by graphic methods with exponentials having half-times of 7400 and 8600 minutes, respectively.

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Fig. 17. Cesium-137 retention by mice after administration by intravenous injection at age of 30 days. Smooth curve is the sum of three exponential functions that fit the data well up to 31 days after injection.

The parameters of the exponential terms obtained with the aid of the VMM program when all of the data were fitted with two, three and four terms are given on the first three lines of Table 31. The last two terms of the series of four have about the same half-times (9651.1 and 9623.8 minutes, respectively) as the third term of the series of three (9650.6 minutes). Also, the sum of the coefficients of the third and fourth terms is identical to the coefficient of the third term of the series of three. The fact that the computer reduces a series of four terms to a series of three terms when all of the data are employed demonstrates that this retention curve for 30-day-old mice does not follow the sum of four exponentials. It also casts doubt on the practical wisdom of the often heard statement that a curve can be fitted with the sum of a series of any mathematical ex-

pression and that the fit improves as more terms are employed in the series. On the other hand, the fact that the computer returns to a series of three terms

TABLE 31
Parameters of exponential terms to fit retention of Cs137 in 30-day-old mice

No. of exponentials	Time span of measurements, minutes	No. of measurements	a_1	$T_1^{(a)}$	a_2	$T_2^{(a)}$	a_3	$T_3^{(a)}$	a_4	$T_4^{(a)}$	Total error sum	Mean error sum
2	2-64971	28	0.7489	3060	0.2221	7968					0.052694	0.001882
3	2-64971	28	0.5081	2028	0.4264	5141	0.0827	9651			0.014715	0.000526
4	2-64971	28	0.5081	2027	0.4264	5140	0.0813	9651	0.0014	9624	0.014715	0.000526
3	2-44674	25	0.5134	2072	0.3711	4914	0.1314	8614			0.009919	0.000354
1	44674-64971	4							0.1513	8680	0.003083	0.000110

(a) T_n = half-time in minutes of nth exponential term.

does not prove that the sum of three terms represents a unique fit to the data but only that the data can be fitted as well with three terms as with four terms.

The presence of two straight-line segments by graphic analysis prompted us to analyze the data in another fashion. The data from 2 through 44,674 minutes were fitted with the sum of three continuous exponential terms and the remaining data from 44,674 through 64,971 minutes were fitted with a single exponential term (the value at 44,674 was common to both groups). The parameters of the sum of three terms and of the single term, as well as the resultant error sums, are given in the lower portion of Table 31. The mean error sum for the 29 points is 0.000464 when the data are divided into two sections and fitted in this fashion compared with a mean error sum of 0.000526 when the entire set is fitted with the set of three exponential terms. This improvement may or may not be significant.

The parameters of the third term ($a = 0.1314$, $T_{1/2} = 8613.9$) found to fit the data from 2 to 44,674 minutes and the parameters of the single term ($a = 0.1513$, $T_{1/2} = 8680.2$) found to fit the data from 44,674 to 64,971 minutes are interesting when compared to each other and when compared with the parameters of the third term ($a = 0.0827$, $T_{1/2} = 9650.6$) found to fit the whole series. The relative close agreement between the former two terms would suggest that the third term of the set when all data were employed would fall midway between these two values. Instead, because of the interaction of terms in the entire series, the half-time of the third term is appreciably longer and the coefficient is considerably smaller. This difference emphasizes the danger of using a retention equation that fits a given time span to predict the amount present at later times. The value predicted for 200,000 minutes (139 days) by the three-term expression based on all the data is 2.8 times that predicted by the single term that fits the last segment.

3-Month-Old Mice

The retention pattern exhibited by the 3-month-old mice is in turn different from that of the 21-day and 30-day-old mice. When plotted on a large semilog grid (Figure 18), the values from 2 through 5790 minutes fall on a smooth curve that becomes tangential to a straight line over the segment from 7129 through 20,181 minutes ($T_{1/2} = 6850.6$ minutes). The values from 23,051 through 50,696 minutes fall along a second straight line ($T_{1/2} = 8610.4$ minutes), while those from 66,452 through 86,649 fall along a third straight-line segment ($T_{1/2} = 10,818$ minutes). The value at 60,745 minutes falls slightly above both of these straight-line segments and suggests a smooth transition from one segment to the next. The value at 82,322 minutes is too high and has been omitted from the analysis.

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Fig. 18. Cesium-137 retention by mice after administration by intravenous injection at age of 3 months. Data have been fitted with the sum of two exponential functions up to 14 days, and with two discontinuous exponential functions thereafter.

Although the data were analyzed in many ways, only the more significant combinations will be discussed. The parameters, total error sums and mean error sums for these analyses are given in Table 32. When all of the data, with the exception of the errant value at 82,322 minutes, are

TABLE 32
Parameters of exponential terms to fit retention of Cs^{137} in 3-month-old mice

No. of exponentials	Time span of measurements, minutes	No. of measurements	a_1	T_1 (a)	a_2	T_2 (a)	a_3	T_3 (a)	a_4	T_4 (a)	Total error sum	Mean error sum
2	2-86649	31(b)	0.6324	3557	0.3423	9290					0.047395	0.001529
3	2-86649	31(b)	0.6324	3557	0.3453	9290	-0.0029	9303			0.047395	0.001529
4	2-86649	31(b)	0.5076	2651	0.3277	7282	0.1691	9694	0.000899	37146	0.011082	0.000358
2	2-20181	17	0.4226	2166	0.5907	7412					0.004239	0.000249
3	2-50646	26	0.2948	1991	0.3576	4332	0.3588	9058			0.005929	0.000228
4	2-86649	31(b)	0.5076	2651	0.3277	7282	0.1691	9694	0.000899	37146	0.110820	0.000358
1	2-5790	7	0.9972	4459							0.001593	0.000228
1	7129-20181	10	0.6856	6851							0.002233	0.000223
1	23051-50646	9	0.4400	8610							0.000483	0.000054
1	66452-86649	4(b)	0.1572	10818							0.001447	0.000362

(a) T_n = half-time in minutes of nth exponential term.

(b) 82322-minute value omitted.

fitted with the sum of three exponentials, the parameters of these terms are almost exactly reduced to those obtained when the data are analyzed as the sum of two terms. Four terms, on the other hand, yield a somewhat different array of parameters.

The data from 2 through 20,181 minutes, which span the curved section plus one straight-line segment, could be fitted with the sum of two terms but not by three terms since three terms were always reduced to two by the computer. The data from 2 through 50,646 minutes could be fitted by the sum of three terms but not by the sum of four terms. The entire set of data from 2 through 86,649 minutes (value at 82,322 minutes omitted) could be fitted with the sum of four terms but not by three since the computer always reduced the set of three terms to the same set of two obtained if the analysis employed only two. The observation that the data in the time span from 2 through 20,181 minutes would only support a series of two terms and that the time span from 2 through 50,646 minutes would support only three terms while the entire set of data supported four terms suggests that the sum of exponentials provides an appropriate fit to the data but not a unique fit.

The data for each of four time segments were fitted separately with single exponential terms. The time span of the measurements, along with the parameters and errors, are also given in Table 32. By an examination of these mean error sums, it is not possible to state unequivocally that retention follows either the sum of exponentials or the discontinuous single exponential terms. The mean error sums are 0.000249 and 0.000225, or about equal, when the data from 2 through 20,181 minutes were fitted with the sum of two terms and with two discontinuous terms, respectively. The comparable values are 0.000228 and 0.000166 when the data from 2 through 50,646 minutes are fitted with the sum of three terms and with three discontinuous curves, respectively. In this latter case, discontinuous terms provide a slightly better fit. However, the goodness of fit obtained for the third section of data with a single term is probably significant. Eight of the nine measurements between 23,051 and 50,646 minutes fall within 1.0 percent and the ninth measurement falls within 1.43 percent of the line described by the single exponential ($T_{1/2} = 8610.4$) found to fit this span.

This degree of goodness of fit to the data over this time span could be obtained with the sum of two exponentials only if the first had a half-time fairly close to 8610 minutes and the second term had a half-time that was several times this value. The sum of these two terms cannot yield the change of slope that occurs between 50,646 to 66,452 minutes and the straight-line segment ($T_{1/2} = 10,818$ minutes) from 66,452 to 86,649 minutes.

These data suggest that the retention of Cs^{137} simply changes with time and that the pattern is not readily describable mathematically.

11-Month-Old Mice

The retention of Cs^{137} by 11-month-old mice is, in general, similar to the retention of Cs^{137} by 3-month-old mice. When plotted on semilog paper (Figure 19), the data follow a slight curve from 2 through 5670 minutes and then three separate straight-line segments from 7110 through 17,275 minutes, from 19,100 through 52,230 minutes and from 56,520 through 72,354 minutes. The three values between 80,886 and 92,381 minutes could be interpreted to suggest the presence of still another straight-line segment.

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Fig. 19. Cesium-137 retention by mice after administration by intravenous injection at age of 11 months. Data have been fitted with the sum of two exponential functions up to 11 days, and with two discontinuous exponential functions thereafter.

In an attempt to find a unique fit to these data, various segments of the data were fitted with one to four exponential terms (Table 33). The 18 measurements from 2 through 15,790 minutes could be fitted with the sum of two exponential terms but not with three. The 34 measurements from 2 through 52,230 minutes could be fitted with a series of three terms but not four. Interestingly, the parameters of the first and third terms of the series of three exponentials for the data from 2 through 52,230 minutes agree quite well with the parameters of the two terms found to fit the data from 2 through 15,790 minutes. Nevertheless, if the parameters of these

three terms were specified and the data from 2 through 15,790 minutes analyzed for this time span, the series was always reduced to two terms, a result that demonstrated that two terms are adequate.

TABLE 33
Parameters of exponential terms to fit retention of Cs¹³⁷ in 11-month-old mice

No. of exponentials	Time span of measurements, minutes	No. of measurements	a_1	T_1 (a)	a_2	T_2 (a)	a_3	T_3 (a)	a_4	T_4 (a)	Total error sum	Mean error sum
2	2-15790	18	0.5205	2341	0.4919	9098					0.002159	0.000120
3	2-15790	18	0.3691	2386	0.1522	2244	0.4911	9106			0.002161	0.000120
3	2-52230	34	0.4049	2288	0.1574	3460	0.4474	9411			0.007170	0.000211
2	2-72354	38	0.5749	2775	0.4246	9632					0.202080	0.000532
3	2-72354	38	0.5398	2503	0.4681	9196	0.00065	196436			0.009622	0.000253
4	2-72354	38	-0.0169	124	0.5448	2468	0.4698	9186	0.00065	227065	0.009409	0.000248
3	2-92381	41	0.6502	3770	0.3127	10642	0.000037	26768			0.258643	0.006308
4	2-92381	41	0.5769	2645	-1.1124	7861	1.5414	8357	0.00032	-84510	0.017867	0.000436
1	2-5670	12	0.9984	4321							0.002370	0.000198
1	7110-17275	7	0.6692	7332							0.000430	0.000061
1	19100-52230	15	0.4643	9313							0.003722	0.000248
1	56520-72354	4	0.3144	10360							0.001553	0.000389
1	80886-92381	3	0.1909	22367							0.003464	0.001155

(a) T_n = half-time in minutes of nth exponential term.

The 38 measurements from 2 through 72,354 minutes were fitted with the sums of both three and four exponential terms. Again, the parameters of the first and second terms of the series of three and the parameters of the second and third terms of the series of four are relatively close to each other and to the parameters of the series of two found to fit the data from 2 through 15,790 minutes. The last terms of both of the longer series suggest the presence of a small amount that is retained with a long half-time. This term may or may not have some biological significance. The first term of the series of four (note the negative coefficient) does not represent a very significant proportion of the quantity injected into the mouse. Thus, for all practical purposes, the two series of terms are identical.

The parameters of the series of three and four terms that provide the best fit to the entire series of 41 measurements were also determined. The mean error sums are considerably larger than for other combinations of data, and the parameters differ substantially between these two sets and also from those obtained for shorter spans of data.

From analyses of the results obtained above, one of two conclusions is possible. Either the entire retention pattern does not follow the sum of a series of exponential terms, or the data are not accurate enough to support the analysis. The second possibility seems unlikely in view of the fact that when plotted the data fall very close to straight-line segments with very little scatter. For this reason the data were fitted with discontinuous single exponential terms. The time span from 2 minutes through 15,790 minutes can be fitted closer with the sum of two terms than with two discontinuous terms. However, any larger span of measurements can be fitted better by employing the sum of the two terms for the first 15,790 minutes and then the appropriate discontinuous functions for the measurements at later times.

22-Month-Old Mice

When plotted on semilog paper (Figure 20), the retention data of the 22-month-old mice follow three separate straight-line segments and do not demonstrate the curved section found at early times for the younger mice. The eleven measurements from 2 through 12,908 minutes fall on a line defined by a half-time of 6806.1 minutes, the twelve measurements from 14,345 to 36,183 minutes (value of 21,542 minutes omitted) yield a half-time of 8412.6 minutes while the nine measurements from 40,454 through 85,276 minutes indicate a half-time of 9789.4 minutes (Table 34). The values

194-96

Fig. 20. Cesium-137 retention by mice after administration by intravenous injection at age of 22 months. Data have been fitted with three discontinuous exponential functions.

TABLE 34
Parameters of exponential terms to fit retention of Cs^{137} in 22-month-old mice

No. of exponentials	Time span of measurements, minutes	No. of measurements	a_1	$T_1^{(a)}$	a_2	$T_2^{(a)}$	a_3	$T_3^{(a)}$	Total error sum	Mean error sum
1	2-12908	11	0.9992	6806					0.001958	0.000178
1	14345-36183	12	0.7753	8412					0.001636	0.000136
1	40454-85276	9	0.4977	9789					0.016186	0.001798
2	2-38000	23(b)	0.3473	3769	0.6688	8848			0.003923	0.000171
2	2-85276	32	0.5444	4955	0.4607	9916			0.023910	0.000747
3	2-85276	32	0.5857	5191	0.4157	10182	-0.000059	-149269	0.023316	0.000729

(a) T_n = half-time in minutes of nth exponential term.

(b) 21542-minute value omitted.

at 92,156 and 99,360 minutes, which fall about equal distances on opposite sides of this last segment, were omitted from the analyses since they introduce a large component into the error sum and unduly influence the final values of the parameters obtained with the VMM program.

The 23 measurements between 2 and 36,183 minutes can be fitted by the sum of two exponential terms (maximum error of 2.3 percent) but a better fit is obtained if these measurements are separated into two groups and each group fitted with a separate term. The 32 measurements between 2 and 85,276 minutes were fitted with both the sum of two and three exponential terms. A slightly better fit to the data was obtained with three terms than with two terms, but the VMM program always set the coefficient and exponent of the third term negative. Consequently, the magnitude of this negative third term increases with time. For this reason, this series of three terms cannot be used to predict the amount of Cs^{137} present in mice at times considerably longer than that covered by these measurements since it is impossible for the mouse to have an increasing body content of Cs^{137} .

32-Month-Old Mice

When plotted on semilog paper (Figure 21), the retention data for the 32-month-old mice fell along straight-line segments from 2 through 19,947 minutes, from 22,982 through 64,659 minutes, and from 64,659 through 84,760 minutes. The measurements within each segment could be fitted very closely with a single exponential term. The coefficients and half-times of these three terms are given in Table 35 and are very similar to those obtained with 22-month-old mice (Table 34). These terms are discontinuous in that each one fit only a section of the data.

The data from 2 through 64,659 minutes were fitted with the sum of two exponentials. The computed values exceed the observed values by several percent at each end of the straight-line segments and are lower by several percent in the middle of each straight-line segment, while the appropriate single term fits the data over the entire straight-line segment. This trend demonstrates that, while retention of cesium does not actually follow the sum of two exponentials, the sum of the two terms provides an approximate fit to the data.

The entire set of data from 2 through 84,769 minutes was fitted with the sum of both two and three exponential terms. As demonstrated by the error sums, a considerably better fit to the data was obtained with three terms than with two terms. However, an even better fit was obtained with the three single discontinuous terms. Significantly, the coefficient of the third term was positive while the exponent was negative.

194-89

Fig. 21. Cesium-137 retention by mice after administration by intravenous injection at age of 32 months. Data have been fitted with three discontinuous functions.

TABLE 35
Parameters of exponential terms to fit retention of Cs^{137} in 32-month-old mice

No. of exponentials	Time span of measurements, minutes	No. of measurements	a_1	$T_1^{(a)}$	a_2	$T_2^{(a)}$	a_3	$T_3^{(a)}$	Total error sum	Mean error sum
1	2-19947	17	0.9868	6908					0.007111	0.000418
1	22982-64659	13	0.6212	8937					0.005732	0.000441
1	64659-84760	4	0.2478	10883					0.000364	0.000091
2	2-64659	31	0.4873	4450	0.5257	9270			0.230460	0.000743
2	2-84760	34	0.6064	5169	0.3957	9914			0.362310	0.001066
3	2-84760	34	0.4637	4352	0.5494	9131	0.000072	-48033	0.025590	0.000753

(a) T_n = half-time in minutes of nth exponential term.

Discussion

The retention in very young mice (21-day-old at time of injection) can be expressed as the sum of three exponential terms for the first 54,719 minutes or until the Cs^{137} content has decreased to 0.1 percent of the amount injected. The retention for the 30-day-old mice follows the sum of three exponential terms for the first 44,674 minutes or until the Cs^{137} content has decreased to 0.4 percent of the amount injected. After

this time, the retention seems to follow a slightly longer single exponential. The retention in both the 3-month and 11-month-old mice from 2 until 20,181 and 17,275 minutes, respectively, after injection could be considered to follow the sum of two exponential terms. The retention after these times follows a series of two straight-line sections for the 3-month-old mice and three straight-line segments for the 11-month-old mice. The retention curve for the 11-month-old mice demonstrates the largest number of distinct changes of slope. The retention data for the 22-month and 32-month-old mice do not demonstrate an early curvilinear section but both follow three distinct, discontinuous, single exponential terms. The half-times of the three terms found for the 22-month-old mice are not greatly different from those found for the 32-month-old mice, but the coefficients are considerably different. The older (32-month-old) mice follow the shorter half-time term for a longer time span and thus effectively lose the Cs^{137} faster.

The observation that the retention does not follow the sums of a series of exponentials but rather a series of single terms whose biological half-times vary with time and age is not incomprehensible. The sum of a series of terms would be associated with a multicompartmental model, whereas Cs^{137} is deposited mainly in the soft tissue that could be represented by a one-compartmental model in which the fraction of a single dose deposited in the skeleton is evidently very small.

Conclusions and Summary

Analyses of the retention data of mice of various ages demonstrate that after intravenous injection of the chloride, the retention of Cs^{137} varies as a function both of the age of the animal at the time of injection and of the time since injection. The effective retention of cesium was lowest in the youngest mice, progressively higher for the older mice and then somewhat lower again for the oldest group. Furthermore, these analyses demonstrate that the retention does not change in a uniform manner that can be described by the sum of a series of exponential terms. If the retention is to be described accurately, it is necessary to use a series of discontinuous terms whereby each term is employed to describe the retention over a limited time span.

These analyses also demonstrate the danger inherent in the use of a function determined from one time span to predict the amount of Cs^{137} present at much later times.

Acknowledgement

On the basis of these results with mice, it can be predicted that studies on very young humans will yield a much shorter biological half-time for cesium retention than would be shown by young or old adults.

REFERENCES

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APPENDIX

TABLE 36

Retention of Cs^{137} chloride injected into 21-day-old mice

Measurement No.	Time after injection, minutes	Relative amount of Cs^{137} retained
1	2	1.00000
2	500	0.86227
3	1408	0.61610
4	1838	0.56950
5	2826	0.43257
6	3150	0.40467
7	4230	0.32527
8	5680	0.24542
9	7134	0.18707
10	8616	0.14164
11	9961	0.11257
12	11408	0.08860
13	12856	0.07070
14	14291	0.05656
15	16085	0.04622
16	17230	0.03700
17	18698	0.02981
18	20083	0.02494
19	21544	0.02060
20	22976	0.01708
21	24376	0.01408
22	28760	0.00895
23	31610	0.00684
24	33377	0.00578
25	34543	0.00501
26	38843	0.00324
27	46150	0.00173
28	48896	0.00122
29	54719	0.00070

TABLE 37
Retention of Cs¹³⁷ chloride injected into 30-day-old mice

Measurement No.	Time after injection, minutes	Relative amount of Cs ¹³⁷ retained
1	2	1.00000
2	189	0.95578
3	445	0.93860
4	1460	0.74458
5	2925	0.55826
6	4380	0.39449
7	5786	0.33109
8	7727	0.23021
9	8765	0.20163
10	10322	0.15517
11	11765	0.13232
12	12970	0.11233
13	14478	0.09321
14	15872	0.08070
15	17297	0.06910
16	18738	0.05651
17	20162	0.04781
18	21612	0.04009
19	24487	0.02993
20	28930	0.01926
21	30192	0.01679
22	33156	0.01245
23	39086	0.00730
24	43163	0.00499
25	44674	0.00421
26	49021	0.00303
27	59209	0.00140
28	64971	0.00082

TABLE 38

Retention of Cs¹³⁷ chloride injected into 3-month-old mice

Measurement No.	Time after injection, minutes	Relative amount of Cs ¹³⁷ retained
1	2	1.00000
2	168	0.96531
3	400	0.95882
4	1350	0.80724
5	2716	0.63678
6	4285	0.50925
7	5790	0.41084
8	7129	0.34227
9	9205	0.26474
10	10025	0.24531
11	11680	0.20791
12	13230	0.18200
13	14550	0.15742
14	15940	0.13712
15	17387	0.11829
16	18746	0.10494
17	20181	0.08756
18	23051	0.06917
19	25929	0.05507
20	30291	0.03833
21	31620	0.03426
22	34600	0.02677
23	40562	0.01675
24	44654	0.01218
25	46361	0.01060
26	50646	0.00745
27	60745	0.00364
28	66452	0.00226
29	70842	0.00163
30	74986	0.00131
31	82322	0.00098
32	86649	0.00061

TABLE 39

Retention of Cs¹³⁷ chloride injected into 11-month-old mice

Measurement No.	Time after injection, minutes	Relative amount of Cs ¹³⁷ retained
1	2	1.00000
2	68	0.99166
3	180	0.96752
4	280	0.95887
5	377	0.95109
6	630	0.91987
7	1445	0.77783
8	1695	0.75242
9	1895	0.73929
10	2800	0.61897
11	4235	0.50426
12	5670	0.41117
13	7110	0.34536
14	8590	0.29769
15	10060	0.25502
16	11475	0.22644
17	12900	0.19597
18	15790	0.15133
19	17275	0.13113
20	19100	0.11350
21	20415	0.10182
22	22075	0.09052
23	23170	0.08239
24	24380	0.07557
25	25827	0.06627
26	27428	0.05878
27	30274	0.04917
28	31799	0.04358
29	35969	0.03306
30	37650	0.02848
31	40451	0.02251
32	46436	0.01494
33	50518	0.01068
34	52230	0.00945
35	60864	0.00547
36	66613	0.00354
37	72354	0.00252
38	80886	0.00153
39	88057	0.00131
40	92381	0.00106

TABLE 40

Retention of Cs¹³⁷ chloride injected into 22-month-old mice

Measurement No.	Time after injection, minutes	Relative amount of Cs ¹³⁷ retained.
1	2	1.00000
2	421	0.98849
3	1495	0.84465
4	2873	0.73690
5	4289	0.64834
6	5686	0.55289
7	7145	0.48140
8	8597	0.41605
9	10002	0.35647
10	11461	0.31501
11	12908	0.27051
12	14345	0.24201
13	15785	0.20853
14	17571	0.18387
15	18682	0.16526
16	20182	0.14483
17	21542	0.12003
18	22980	0.11754
19	23170	0.11558
20	24430	0.10373
21	25829	0.09220
22	30254	0.06301
23	34918	0.04330
24	36183	0.04011
25	40454	0.02850
26	44863	0.02048
27	50407	0.01363
28	57721	0.00835
29	64826	0.00541
30	66555	0.00482
31	71956	0.00284
32	76273	0.00224
33	85276	0.00118
34	92156	0.00087
35	99360	0.00035

TABLE 41

Retention of Cs¹³⁷ chloride injected into 32-month-old mice

Measurement No.	Time after injection, minutes	Relative amount of Cs ¹³⁷ retained
1	2	1.00000
2	165	0.97521
3	1266	0.85300
4	1561	0.84221
5	2728	0.73445
6	4178	0.67724
7	5576	0.58109
8	7537	0.46949
9	8560	0.41124
10	9950	0.35620
11	11349	0.30887
12	12743	0.26743
13	14438	0.23326
14	15600	0.20217
15	16959	0.18036
16	18628	0.15576
17	19947	0.13577
18	22982	0.10314
19	24130	0.09491
20	25954	0.08389
21	28594	0.06821
22	30016	0.05953
23	32882	0.05019
24	38708	0.03159
25	42336	0.02217
26	44625	0.01942
27	45821	0.01820
28	48929	0.01385
29	53358	0.01002
30	59048	0.00635
31	64659	0.00407
32	73180	0.00231
33	80801	0.00144
34	84760	0.00113

MOVE OF THE WHOLE-BODY COUNTER

Charles E. Miller

Introduction

The whole-body counter, usually designated either as the Argonne National Laboratory Human Gamma-ray Spectrometer No. 1 or the portable Iron Room, was developed in 1954 in order to reduce the background count detected by NaI scintillation crystals.^{1,2} It was disassembled, moved, and reassembled in a new location during January, 1965. Since its construction in 1954, the counter had been located at one end of a large unfinished area, 30 feet wide by 60 feet long, of the service floor (basement) of the Physics building. During recent years for a number of reasons, it had become progressively more difficult, if not impossible, to make accurate measurements of the radioactive content of normal, healthy humans with the room in this location.

As additional equipment for upstairs laboratories was installed on the service floor, the room temperature varied greatly. In the vicinity of the whole-body counter, the temperature varied seasonally from 70°F to over 100°F and diurnally during the summer from perhaps 80°F to 100°F. While the changes in ambient room temperature may not have significantly affected the older vacuum-tube type of pulse height analyzer, the temperature changes caused the transistorized equipment in use since 1963 to drift widely and, in some instances, to malfunction completely.

The gamma-ray backgrounds within the whole-body counter, which originally had been very reproducible, were no longer stable. During the first few years the presence of Ar⁴¹ from the stack effluent of the research reactor CP-5, which was located about three-quarters of a mile to the south, might be detected for a short period of time on a few occasions per year when the wind was directly out of the south. More recently, larger concentrations of Ar⁴¹ have been detected more frequently as the power of the reactor was increased. In addition, short-lived airborne radioisotopes produced by the beam of either a cyclotron or a Van de Graaff also became a problem as more particle accelerators were built in various directions not far from the location of the whole-body counter. Before the whole-body counter was moved, a study was made to evaluate these sources of increased background.

Sealed Room Experiment

Small Cave

The contribution of airborne radioactivity to the background within an iron shield was first studied with a small cave that had 8-in.-thick walls

of laminated steel. A sodium iodide, thallium activated crystal, 9 inches in diameter by 4 inches thick, was placed in the cave and an aluminum box was built around the cave. The base of the aluminum box was sealed to the single floor plate, which projected several inches beyond each side of the cave. The cable from the preamplifier and photomultiplier tube attached to the crystal was brought out through a gasketed hole so that the box was practically airtight. The gamma-ray background of the crystal contained enough well-defined photopeaks from the radium in the photomultiplier tubes to permit us to determine the energy calibration of the over-all system at any time.

The output pulses of the crystal were sorted and stored with a Technical Measurements Corporation Model 404 pulse height analyzer. The analyzer was operated in the automatic mode so that pulses were accumulated for a preset time and the data were punched onto tape, after which the memory was cleared to zero and the cycle repeated. A series of 40-minute and 100-minute backgrounds was accumulated in this way over a several-week period. The energy calibration drifted downward slightly during the first few days and then became stabilized. The backgrounds in the various energy bands (after the particular groups of channels were adjusted to correct for drift) fell within the range of values expected by probability statistics and demonstrated 1) that earlier variations in background were due to airborne contamination and 2) that stable, reproducible backgrounds could be obtained under the proper conditions.

Whole-body Counter

The large whole-body counter was sealed as well as possible without disassembling the room in order to evaluate the influence of airborne contamination and to determine if stable backgrounds could be obtained by some air lock system. The floor of the room could not be sealed satisfactorily.

The base of the Iron Room consisted of three sheets of steel plates, 3 feet wide by 10 feet long by $1/4$ inch thick, laid side by side. The internal frame of the room had been bolted to the floor plate and 32 layers of $1/4$ -inch-thick floor plates were stacked on this base. The three steel base plates projected about 4 inches beyond the 8-inch-thick walls, which were attached to the upright frame. It was not possible to seal or weld the two cracks between the three base plates without removing the entire 8-inch-thick floor as well as two of the walls. Aluminum plates, 6 inches wide by $1/4$ inch thick, were placed around the four sides of the base plates and sealed to the steel base plates. A welded aluminum box was built around the entire room and welded to the aluminum plates attached to the steel base plate. A gasketed aluminum door that could be screwed tightly to the aluminum box around all edges of the door was provided. All electrical leads and service lines into the room were gasketed and sealed where they passed through the aluminum wall. The same 9-inch-diameter by 4-inch-thick NaI(Tl) crystal was employed.

A series of 40-minute and 100-minute backgrounds was recorded during the next several weeks. The gamma-ray background within the sealed room increased to about twice the background of the unsealed room and then fluctuated systematically around this high value. Two 40-minute backgrounds would yield about the same count rate, the next three would increase equal amounts, the next two were about equal, and then the next three backgrounds would decrease to the minimum. This cycle repeated itself again and again. Examination of the difference between the spectrum of the unsealed room and that of the sealed room suggested that the increase in background was due to the buildup of radon daughters within the room and not to the presence of any other airborne contaminant.

It was not possible to locate the source of the radium producing the radon without dismantling the room to seal the cracks in the floor. The radium might have been present as a contaminant on the surface of the 32 layers of 1/4-inch-thick steel plates in each wall, floor and ceiling. However, it was felt that a more likely source was the radon that escaped from the concrete floor beneath the room and seeped up through the cracks in the floor and into the interior of the room. This hypothesis was substantiated by Mr. Henry F. Lucas of the Radiological Physics Division, who found that when a small hole was drilled through the aluminum wall and a tube inserted into the room, samples of room air showed very high humidity, as well as high radon concentration. These results indicated that water vapor was definitely entering the room from the outside.

The cyclic changes in the background were found to be due to systematic changes in the energy calibration. The count rates in fixed groups of channels had been used to compare the backgrounds of one run to the next. A careful analysis of the energy calibration disclosed that the gain of the detector (crystal, phototube and cathode follower preamplifier) was increasing and decreasing about 1 percent. The apparent change in background count rate actually reflected the fact that the fixed group of channels spanned a different energy band from one run to the next. When the number of channels in each group were adjusted to span the same energy bands for each measurement, reproducible stable backgrounds were obtained. Further investigation disclosed that the heat exchanger inside the whole-body counter ran for about 20 minutes commencing with start of the two lowest 40-minute backgrounds. Consequently, it was concluded that the temperature changes within the room accounted for the change in energy calibration and the apparent variation in backgrounds.

Temperature-sensing thermocouples were attached to the glass envelope of the photomultiplier, and the gain changes were correlated with temperature changes of the photomultiplier tube. The temperature of the glass envelope of the photomultiplier did not begin to drop until 60 to 80 minutes after the heat exchanger had cycled and cooled the interior of the room. Consequently, the energy calibration was the same for these two

runs and these two backgrounds were equal. The temperature of the photomultiplier then decreased during the next three 40-minute measurements, remained relatively constant for the next two 40-minute measurements, then increased during the next three 40-minute measurements. The gain varied inversely as did the temperature. This series of measurements demonstrated the necessity of maintaining the detector at a constant temperature if accurate and reproducible results are to be obtained.

Move of the Iron Room

It was necessary either to move the counter into an air-conditioned laboratory or to air-condition the area in which the counter was located. In the latter event it would still be desirable to place a vapor barrier under the room to eliminate the radon seepage into the room.

An air-conditioned, humidity-controlled space, formerly occupied by the Applied Mathematics Division, became available. This space is ideally suitable since it is centrally located near the building elevator and has well-regulated temperature and humidity, tiled floor, and finished walls and ceiling. A further advantage is that the air of this laboratory is recirculated, a fact which makes it practically a closed system. Although the air conditioning equipment is located outside the room, the air is taken from the room, cooled, humidified, and distributed back into the room through a plenum. A small amount of makeup air is obtained from the service floor outside the room. Consequently, the whole-body counter could be disassembled, moved to the new location and reassembled on a large metal plate at a small cost compared with the installation of a gas-tight, metal barrier beneath the counter in its old location and refurbishing and air-conditioning the area.

The physical dimensions of the whole-body counter were not altered for the following reasons. This whole-body counter had been used since 1957 to measure the radium content of humans who had ingested radium or had received it by intravenous injection 30 to 40 years ago. These patients, who have been measured two or three times during the past eight years, will be remeasured every two or three years, if possible, in order to delineate the long-term retention of radium in man. Since the radium content of humans decreases very slowly at long times after intake of the radium, each measurement must either be very accurate or, if incorrect on an absolute basis, at least contain the same systematic error.

Consequently, in order to avoid recalibration and the inherent errors therein, the whole-body counter was reassembled almost exactly as it was before except for two details. The Iron Room was placed on a 1/8-inch-thick sheet of aluminum, which projected several inches beyond all sides of the

Iron Room, instead of directly on the concrete floor, in order to provide a vapor barrier between the floor and the Iron Room. The aluminum box was placed around the room and welded to the base sheet of aluminum under the room. As the room was originally constructed, the steel plates that formed the floor of the room did not touch two of the exterior walls. This space, which was originally filled with scrap bars of iron, was now filled instead with iron shot. Neither of these changes altered the dimensions, geometrical arrangement or scatter within the room. Thus the calibration constants previously determined for any isotope with any counting geometry should be valid in the room at the new location.

The lead liner for the interior surfaces of the room and the first few interior layers of steel plates were washed before reassembly. Otherwise, the plates were not cleaned unless a worker had obviously stepped on a plate and left a footprint.

Gamma-Ray Background at New Location

The gamma-ray background in the energy band from 25 keV to 1.575 MeV is about 800 counts per minute with the whole-body counter in the new location versus 910 counts per minute at the old location. This reduction in background might result from 1) increased shielding of cosmic-ray secondary particles provided by the presence of two concrete floors overhead instead of the one floor at the old location, 2) a lower radon concentration in the room since radon cannot seep in through the bottom, 3) the fact that some radioactivity might have been cleared from the interior lining of lead and the first few plates, or 4) the use of iron shot instead of scrap at the floor and wall seams.

Radon Buildup

The gamma-ray spectrometer was calibrated on a Friday afternoon and the aluminum box sealed with rubber gaskets around the door. A series of 100-minute background measurements was made for the next 5300 minutes, or until the following Tuesday morning. The background from 25 keV to 1.575 MeV was 802 counts per minute for the first 100-minute measurement and was 63.8 counts per minute higher after the following 5300-minute period. The doors were then opened, and after the interior of the Iron Room was flushed with air from the new laboratory, a series of 40-minute backgrounds was measured. The gamma-ray background immediately after flushing was 810 counts per minute as compared to 802 counts per minute immediately before the room was sealed.

The rate of increase of background while the room was sealed was consistent with the pattern expected for the buildup of radon from an emanating radium source. Since the background increased 64 counts in

5300 minutes during the time required for radon to reach one-half of the equilibrium value, the background would have increased 130 counts per minute if the room had remained sealed for 30 days. Previous calibrations demonstrated that $1.0 \mu\mu\text{Ci}$ of radon per liter of air in the room yielded about 100 counts per minute. The above 130 counts per minute indicates the presence of about $1.3 \mu\mu\text{Ci}$ of radon per liter, equivalent to about $0.01 \mu\text{Ci}$ in the 8000 liters volume of the room, or the presence of $0.01 \mu\text{Ci}$ of emanating Ra^{226} within the aluminum box. This relatively small amount of radium, which might be distributed over the surfaces of the 32 layers of plates in each wall, will not affect the background significantly if the Iron Room is ventilated with air from the surrounding laboratory.

Stability of Background

As a matter of standard procedure, series of 40-minute or 100-minute gamma-ray backgrounds are recorded from approximately 5:00 p.m. until 8:30 a.m. every night and from Friday evening until Monday morning each weekend unless the room is otherwise in use. The presence of a trace of Ar^{41} has been observed on a few occasions. The concentration of Ar^{41} , while high enough to influence the measurement of normal K in humans on a few occasions, does not build up to the high levels observed in the previous location. The presence of Ar^{41} can probably be avoided completely if the makeup air intake is closed any time the wind blows from the south. The integral background in the energy band from 25 keV to 1.575 MeV has changed about 20 counts per minute during some series of measurements. This variation undoubtedly reflects changes in the radon concentration in the atmosphere.³

Summary

The translocation of the original 46-ton Iron Room to an air-conditioned room was accomplished in January, 1965. In its new location the Iron Room rests on a single sheet of aluminum and has a controlled air supply. Background counts in the energy range from 25 keV to 1.575 MeV are quite stable and have been reduced from 910 to 800 counts per minute.

Acknowledgement

Mr. John B. Corcoran and Mr. Carter A. Priess made many of the technical measurements before and after the whole-body counter was moved.

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CALIBRATION OF CESIUM-137 ROD STANDARDS

Charles E. Miller and Carter A. Priess

In September, 1962 a special meeting was held at the Second Symposium on Radioactivity in Man¹ to discuss the fluctuations in Cs¹³⁷ measurements made by whole-body counter groups located in various parts of the United States. These investigators had made weekly measurements of normal persons who did not work with radioactive materials in order to acquire information on the amount of Cs¹³⁷ in the civilian population living in various areas of the United States. The rate of change of whole-body Cs¹³⁷ values also provided at least a rough indication of the fallout rate in these sections of the country.

The average Cs¹³⁷ values reported by some of the laboratories varied up and down by a factor of 2 or 3 from week to week. Since the half-time of Cs¹³⁷ in the adult human lies between 60 and 150 days,^{2,3} the average in vivo Cs¹³⁷ content of a group of humans could not decrease more than about 8% during any one week if the half-time in every individual in the study was the minimum 60-day value. The Cs¹³⁷ content could increase significantly during any week if the intake of cesium were high during that week, but the Cs¹³⁷ value for the following weeks would also remain high. Moreover, the average Cs¹³⁷ values reported by some groups were higher by a factor of 2 or 3 than the results reported by other groups. This variability might reflect different fallout patterns or it could indicate that the absolute calibrations of the whole-body counters were incorrect. The average weekly Cs¹³⁷ levels reported by some laboratories possibly reflected either instrumentation, positioning, or calibration errors, or the presence of surface contamination on the subject.

An ad hoc committee was organized to review the problems encountered in Cs¹³⁷ measurement and to recommend procedures for calibration of whole-body counters. This committee proposed a three-step program. 1) Each laboratory would receive a calibrated National Bureau of Standards ring-type massless point source that could be used to check absolute calibration of its whole-body counter. 2) Each group would receive a set of three rod-type secondary Cs¹³⁷ standards of 0.01, 0.1 and 1.0 μ Ci. The activity was to be deposited at a specified depth at a point on the axis within the rod so that each rod could be counted from the sides or from one end. The ratios among these sources were to be known to within 1.0%. Each group was to measure these sources accurately in order to determine if its equipment corrected for dead time at high count rates, did not drift at high count rates, or otherwise performed accurately. 3) After each group had measured both the primary point source and the secondary standards and had eliminated any electronic difficulties, a normal person who contained the average amount of Cs¹³⁷ found in the civilian population plus a tagged subject who contained a considerably larger, well-documented amount

would visit each laboratory. Each laboratory would then measure these two persons for a final in vivo calibration. The Division of Biology and Medicine of the U. S. Atomic Energy Commission agreed to supply the point sources and the rod-type secondary standards and to support the travel of the persons who would be the in vivo standards.

This three-phase intercalibration program has progressed very slowly and has now required about three years. The National Bureau of Standards primary point standard sources of Cs^{137} were obtained, distributed and measured by each laboratory quite quickly (phase 1 of the program). The main problem has been that of acquiring accurate rod-type point standards (phase 2 of the program). In the first 20 sets of rods that were received from a commercial supplier, the activity was not located at a point source but was distributed nonuniformly throughout a large plastic plug in the rod. As a result, the ratio between sources in a set depended both upon the distance between the source and the crystal and upon the rotation of the source with respect to the crystal if counted from the side. In other sets, the absolute magnitudes and ratios were incorrect or the external surfaces were contaminated. Finally, early in 1965 20 sets of decade rod sources were acquired in which the Cs^{137} was deposited in beads located a specified distance within the rods.

These sources, which were also measured by Los Alamos Scientific Laboratory, have been measured extensively with the whole-body counter of the Health Division, Argonne National Laboratory, in order to determine the ratios between the three sources in each set. Since each of the participating whole-body counter groups may spend considerable time in verifying the ratios found in its set of sources, the results of these measurements are given in laborious detail in this report so that they will be available in ensuing years.

Equipment Employed

A sodium iodide, thallium-activated crystal, 8 inches in diameter by 4 inches thick, canned in 1956, was employed as the detector. This crystal, which has relatively poor resolution, was used in preference to crystals of better resolution so that the mechanical positioning fixtures built for this detector some years ago could be used. The physical arrangement of the detector and source position are shown in Figure 22. Two steel rods, 5/8 inch (1.59 cm) in diameter by 50 cm long, were screwed into the steel yoke that supports the NaI(Tl) crystal. Two movable steel collars attached to a Lucite tray were slipped over the two rods. The steel collars (guides) maintained the surface of the tray parallel to the

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Fig. 22. Diagram of detector, beta shield, and source holder

face of the crystal, and locking screws located in each collar permitted the tray to be clamped at any distance up to 46 cm from the face of the crystal. Fine indexing grooves, 1 cm apart, had been machined around the rods, and a 10 millimeter scale was built into each collar. With reasonable care it is thus possible, by aligning the marks on the rod and the lines on the scale, to position or reposition the tray at any millimeter distance from the crystal face. It was not possible to reposition the tray accurately to a fraction of a millimeter since a vernier scale was not built into the fixture. A Lucite "candlestick" holder was placed on the center of the tray and the source was placed handle-end down in this holder. The bead source was thus positioned along the axis of the crystal and the distance of the bead from the crystal face was determined by the distance from the handle end of the rod to the face of the crystal.

The pulses from the single 5-inch diameter photomultiplier that is optically coupled to the 8- by 4-inch crystal were routed to the first 200 channels of a Technical Measurements Corporation Model 404 pulse height analyzer via input 1. The gain of this amplifier was adjusted on May 26, 1965 so that the Cs^{137} photopeak was centered at channel 100. Neither the gain nor the high voltage changed during the next six weeks, and the Cs^{137} photopeaks all fell in channel 100 during the hundreds of measurements that were made during this six-week period.

The output of a mercury-wetted relay type pulse generator driven from the 60-cycle line voltage was fed into the second 200 channels of the analyzer through a second amplifier via input 3 in the analyzer. The gain of this amplifier and the pulse generator output were adjusted so that the pulse generator pulses all fell in channel 300.

The recorded count rate from the pulse generator was employed to evaluate several systems in the Technical Measurements Corporation analyzer. The number of pulses recorded per minute, which should number 3600 counts per minute, provides a check on the live timer built with the analyzer. In addition, any base level shifts introduced into the analog-to-digital converter (ADC) section of the analyzer by high count rates produced by the Cs^{137} source or chance coincidences between the pulse generator pulse and the Cs^{137} pulse would shift the pulse generator pulses from channel 300 into an adjacent channel, since the output pulses from both amplifiers are sorted by the same ADC.

Tray Position Employed

The rod sources were measured with the tray located at both 16 and 44.5 cm from the face of the crystal. When the rod sources were placed handle end down in the candlestick holder with the tray at these two distances, the center of the bead source was located either 3.3 or 31.8 cm from

the crystal face. Consequently, the flat National Bureau of Standards point source, which was employed to obtain an absolute calibration of the system, was measured with the tray located at 3.3 and 31.8 cm.

Energy Band Employed to Determine Count Rate under the Photopeak

The count rate within a relatively wide energy band around the photopeak was employed in order to minimize the effect of any potential change in energy calibration. A Cs^{137} spectrum was displayed on the scope of the pulse height analyzer and a group of channels selected visually with the aid of the "spectrum resolver." Channel 78 was chosen as the lower limit of the band since this channel fell in the trough between the Compton region and the photopeak. Channel 119 was chosen as the upper limit since the net count rate from Cs^{137} was practically zero in channels around 119.

The count rate in channels 130 through 199 was employed to monitor the room background in order to provide assurance that Ar^{41} or any other airborne contamination was not present in the room, particularly while counting the $0.01 \mu\text{Ci}$ sources at 3.3 cm or the $0.1 \mu\text{Ci}$ and $0.01 \mu\text{Ci}$ sources at 31.8 cm. Any increase of the count rate in this energy band, which averaged 65% of that in the photopeak band, indicated either an increased gamma-ray background within the room, or a count rate high enough that pulses were being missorted because of either pileup or chance coincidences.

Data Reduction

The data accumulated during each measurement were punched on paper tape. The tapes were analyzed with the aid of an existing computer program and the Argonne-built computer called "Gus." As used for these analyses, the computer was programmed to divide each measurement by the length of time of the run, which is punched in the first data block, subtract the net background measurement from the net source measurement, and then add the net count rates in any desired groups of channels--in this case, channels 78 through 119 and 130 through 199.

The net counts under the photopeak of the several determinations made on each source were then punched onto cards, and means, variances, standard deviations and standard errors were computed for each group of measurements with the aid of a Control Data Corporation 160-A computer.

Dead Time, Pileup, and Coincidence Correction

Because of pileup and chance coincidences between pulses, the count rate per microcurie of activity accumulated under the photopeak will still vary, at least to a small degree, as a function of count rate even though the

pulse height analyzer accumulates pulses for corrected live time. Pulses that are produced by Cs^{137} gamma rays, which are totally absorbed in the crystal and thus should be sorted into the photopeak channels, may be sorted into higher channels if this pulse occurs either in coincidence with another pulse or soon enough after the other pulse to ride on its tail. The count rate in the photopeak recorded with some types of pulse height analyzers, such as the one employed for these measurements, will not be affected by pileup. These analyzers sort pulses according to the rate of change of the pulse height rather than by the total voltage difference above some arbitrary base line. Since the pulse tail usually has a relatively flat slope, the first pulse does not produce a large rate of change. The count rate per microcurie obtained with some analyzers, particularly older types, may change significantly at high count rates because of pileup.

The count rate per microcurie of activity under the photopeak will decrease at very high count rates because of chance coincidence losses, regardless of the type of analyzer employed. No pulse height analyzer can differentiate into two pulses a single pulse that results from two gamma rays interacting in the crystal simultaneously. The local representative of the manufacturer of our equipment states that any two pulses that occur within 3 microseconds of each other will be summed and sorted as one pulse. Although the pulse height analyzer that we used runs for the correct live time and correctly sorts pulses that ride on the tails of other pulses, it is still necessary to correct for chance coincidences if the correct ratios of activities among the three sources are to be obtained.

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Fig. 23

Diagram of multiholder
used to position from 1 to
19 sources under the
 NaI(Tl) crystal detector

Since we were fortunate to have all twenty sets of sources available, the following procedure was employed to determine the correction at high count rate for our system. A Lucite multiholder was made that had 19 holes arranged in two concentric rings (Figure 23). The first ring of 6 holes was $7/8$ inch (2.22 cm) from the center and the second ring of 12 holes had a radius of 1.75 inches (4.44 cm). In order to obtain square corners at the bottom so that the rods would not bind in the holder and thus seat at different depths, the 19 holes were first drilled through a block of Lucite and then a flat plate was carefully glued to the bottom. The thickness of the bottom plate was the same as that of the candlestick holder so that the bead source was the same distance from the crystal when either holder was placed in the tray. A set of 19 dummy Lucite rods were made that were the same diameter and length as the decade rod sources. The holes and Lucite rods were numbered as shown in Figure 23.

The multiholder was centered on the tray so that the center rod was on the axis of the crystal. One at a time, the first ten $0.01\text{-}\mu\text{Ci}$ sources were

each placed successively in holes 1 through 10 and counted for at least five 10-minute counts. Sources 1-2, 1-2-3, 1-2-3-4, 2-3-4, as well as sources 2 through 7, and 2 through 10, were counted as groups. In order to obtain the identical absorption and scatter for all measurements, dummy Lucite rods were placed in holes not occupied by rod sources so that each hole was occupied by either the same numbered source or a Lucite rod.

The sums of the count rates obtained when the sources were counted individually were compared to those obtained when the same sources were counted as a group. The mean, variance, standard deviation, and standard error of each set of measurements of the individual sources and of the combinations were computed. The mean net counts and associated standard deviation of the various combinations are given in Table 42. Substantially the same count rate was obtained by counting any combination of up to nine sources together as was obtained by counting the sources individually and then summing the individual counts. Consequently, it would seem that the ratio between the 0.1- and 0.01- μ Ci sources could be calculated directly from the count rates obtained from the sources without correcting the count rate for coincidence losses. Actually, the standard deviation of about 0.25% present in each measurement might obscure very small coincidence losses at these count rates.

TABLE 42
Comparison of counts when selected Cs^{137} sources
were counted individually or in groups

Sources	Sources counted in groups		Sum of sources counted individually	
	c/m	S.D.	c/m	S.D.
1-2	5782.0	15.40	5778.9	11.06
1-2-3	8379.8	35.14	8372.7	29.70
1-2-3-4	10,993.8	34.99	10,993.7	29.80
2-3-4	8093.6	25.25	8113.3	9.50
5-6-7	8490.0	19.87	8484.5	27.00
8-9-10	7561.9	20.90	7548.9	26.20
2 through 7	16,561.2	39.92	16,597.8	17.85
2 through 7	16,564.6	47.87	16,597.8	17.85
2 through 10	24,153.2	57.76	24,146.7	47.70
2 through 10	24,125.4	81.23	24,146.7	47.70

The same procedure employed to count the 0.01- μ Ci sources was also used to count each of the 0.1- μ Ci sources in order to determine the coincidence losses present when the 1.0- μ Ci sources and the National Bureau of Standards sources were measured. Sources 1 through 13 were

counted individually and, in addition to the groups listed above, group counts were made of sources 10 through 13, 14 through 16, 18 through 20, 2 through 13, and 2 through 16 plus 18 through 20.

Again the sums of the count rates obtained when the sources were counted individually were compared to those obtained when the sources were counted as groups. Since sources 14 through 20 were not measured individually, the sum count expected for groups 2 through 16 and 2 through 20 (17 omitted) if they had been counted individually were obtained as follows. The group count rates obtained for sources 2-3-4, 5-6-7, 8-9-10, and 11-12-13 were compared to the sum of the count rates obtained when the three sources of each group were counted separately and found to average 0.33% less than the sums of the individual counts. The group count rates of groups 14 through 16 and 18 through 20 were accordingly increased by 0.33% and added to the sum obtained for sources 2 through 13 when counted individually. These corrected count rates were then compared to those obtained from the group of 15 and 18 sources.

The mean and variance were calculated for each set of measurements on the individual sources and on the groups. The standard deviation in counts per minute associated with the sum of a group of individual counts or with the group count was about 0.1% of the mean count rate. The percentage differences between the true count rate and the observed count rate for groups of 3, 6, 9, 12, 15, and 18 sources were calculated and indicated by dots in Figure 24. The observed

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Fig. 24. Dead time correction in percent plotted against count rate

count rate is given on the abscissa and the percent difference or coincidence loss on the ordinate. A smooth curve has been drawn through these points with the lower end arbitrarily passing through the origin. It will be observed that a coincidence loss of 0.08% would be predicted for the count rate of 24,000 counts per minute obtained when nine 0.01- μ Ci sources were counted as a group. This minor coincidence loss was, in fact, obscured by the variations present in the counts on the 0.01- μ Ci sources.

Check of Accuracy of Time Interval Set by Dial on Analyzer

The count rate recorded in channel 300 from the pulse generator provided a check on the accuracy of the length of time of each measurement.

A count rate of 3614 counts per minute was recorded in channel 300 for all runs of from 1 to 100 minutes, while very low count rates from the Cs^{137} were being accumulated in the first 200 channels. The fact that this analyzer accumulates pulses for about 0.4% longer (3614 counts per minute instead of the expected 3600 counts per minute) than the actual time arbitrarily selected by a dial is not important as long as the duration of any time measurement is always long or short by the same percentage. The internal timer would introduce a significant error, particularly for a short measurement, if each measurement were a constant number of seconds longer or shorter than observed.

The recorded count rate from the pulse generator decreased when a high count rate from the Cs^{137} source was sorted via input 1 because of coincidence losses in the ADC between the Cs^{137} pulses and the pulse generator pulse. The percentage decrease in the pulse generator count rate agreed with the percentage coincidence losses derived experimentally for these count rates by counting sources individually and as groups. Nevertheless, another test of the timing system was made.

Since counting times of 1, 4, and 10 minutes were used when the 1.0-, 0.1-, and 0.01- μCi sources, respectively, were counted at 3.3 cm, series of runs were made at various count rates to determine if the 4- and 10-minute counts were actually four and ten times as long as the 1-minute count. The NBS source was located at both 31.8 cm and 3.3 cm from the face of the crystal, and numerous repetitive 1-, 4-, and 10-minute measurements recorded. The mean and standard deviation of each series of runs was computed and found to agree with the other measurement made at that distance. Consequently, the 4- and 10-minute measurements are four and ten times as long as the 1-minute measurement for the count rates present when the 1.0- μCi source was counted at 31.8 or 3.3 cm or the 0.1 μCi source at 3.3 cm.

Measurements of Rod Standards

31.8-cm Position

The tray was first positioned so that the center of the radioactive bead of Cs^{137} was 31.8 cm from the face of the 8- by 4-inch crystal. Each of the 20 "nominal 1.0" μCi cesium sources was counted in turn for three 4-minute measurements, and the entire series was repeated. The six measurements of each source were examined to determine if the individual measurements fell within the expected Poisson statistical limits.* In one instance, one set of three measurements was consistently above the other

$$*\text{S.D.} = \sqrt{\frac{\text{counts/minute}}{\text{minutes}}}$$

three measurements. Investigation disclosed that this source rod was oversized and tended to bind just before being seated on the base of the candlestick holder. When it was counted a third time while properly seated in the holder, a count rate in agreement with the lower of the first two sets of counts was obtained. A "nominal 1.0" μCi source at this distance yielded about 29,000 counts per minute under the photopeak. Based on the coincidence loss curve of Figure 24, this count rate is approximately 0.1% low.

Each 0.1- μCi source was then counted at the same distance for at least three 10-minute counts. The 0.1- μCi sources yielded on the average about 2330 counts per minute under the photopeak.

The 0.01- μCi sources numbered 1, 7, and 15 were also counted at this distance. Seven replicate 100-minute measurements were made of background, source 7, background, source 13, background, source 1, background. The 0.01- μCi source yielded about 220 counts per minute under the photopeak. It was obviously impractical to measure all the 0.01- μCi sources at this distance.

3.3-cm Position

In order to obtain a higher and more accurate count rate from the 0.01- μCi sources, the tray was raised so that the center of the bead source was only 3.3 cm from the face of the crystal. The distance of 3.3 cm was not selected as an optimum distance but rather from the fact that the tray was positioned so that two engraved lines 1 cm apart on the rod matched exactly the engraved 0 and 10 mm lines on the tray. With the tray at this height, the 1.0- μCi sources were each counted for at least four 1-minute counts, the 0.1- μCi sources for five or more 4-minute counts and the 0.01- μCi sources for three or more 10-minute counts. These sources yielded count rates under the photopeak of about 376,000, 30,400, and 2,911 counts per minute with sustained coincidence losses of about 2.22, 0.11, and 0.01%, respectively. The appropriate correction for each 1.0- μCi source was obtained from the curve, while a correction of 0.1% was arbitrarily used for all measurements of the 0.1- μCi sources at 3.3 cm.

Ratios between the 1.0- μCi and 0.1- μCi Sources in Each Set

The net count under the photopeak of each measurement was punched onto a conventional computer card. All of the measurements on each source at each distance were analyzed with a computer to determine the mean, variance, standard deviation, and standard error of each set of measurements. The mean count rates, corrected for coincidence losses, of each 1.0- μCi source in the set when counted at 3.3 and 31.8 cm were divided by the mean count rates of the 0.1- μCi sources (corrected for losses if necessary) from that set when counted at the same distance. The two ratios

so obtained for each set were tabulated and examined. In those instances in which the ratios differed by more than 0.5%, the sources were recounted at both distances. The mean count rate obtained when the sources were remeasured, while slightly different from the first determination, fell within the expected standard error. Consequently, the individual measurements of both sets of measurements were pooled and new means, variances, standard deviations, and standard errors were obtained.

The ratios between the 1.0- and 0.1- μ Ci sources obtained at both distances and the standard error of these ratios⁴ are given in Table 43. The standard error of the ratio was calculated with the equation:

$$\text{S.E. of the ratio} = \sqrt{\left(\frac{M_1}{M_2}\right)^2 \left[\left(\frac{\text{S.E.}_1}{M_1}\right)^2 + \left(\frac{\text{S.E.}_2}{M_2}\right)^2 \right]}$$

where

M_1 = mean of count rate of 1.0- μ Ci source,

S.E._1 = standard error of M_1 ,

M_2 = mean of count rate of 0.1- μ Ci source,

S.E._2 = standard error of M_2 .

TABLE 43

Ratio of 1.0- to 0.1- μ Ci sources at 31.8 and 3.3 cm from face of crystal.
All measurements corrected for coincidence losses if significant.

Source No.	3.3 cm		31.8 cm		Mean ratio	% Deviation from mean
	Ratio	S.E.	Ratio	S.E.		
DC-1	13.084 $\pm$ 0.032		13.090 $\pm$ 0.017		13.087	0.02
DC-2	12.360 $\pm$ 0.045		12.348 $\pm$ 0.040		12.354	0.05
DC-3	13.120 $\pm$ 0.045		13.161 $\pm$ 0.021		13.140	0.16
DC-4	12.881 $\pm$ 0.025		12.805 $\pm$ 0.051		12.835	0.30
DC-5	13.005 $\pm$ 0.041		12.987 $\pm$ 0.064		12.996	0.07
DC-6	11.703 $\pm$ 0.024		11.721 $\pm$ 0.029		11.712	0.08
DC-7	12.684 $\pm$ 0.036		12.722 $\pm$ 0.038		12.703	0.15
DC-8	12.687 $\pm$ 0.017		12.744 $\pm$ 0.020		12.715	0.22
DC-9	10.701 $\pm$ 0.034		10.659 $\pm$ 0.015		10.680	0.20
DC-10	12.551 $\pm$ 0.023		12.617 $\pm$ 0.068		12.584	0.26
DC-11	13.016 $\pm$ 0.014		13.103 $\pm$ 0.038		13.060	0.33
DC-12	13.177 $\pm$ 0.025		13.224 $\pm$ 0.060		13.200	0.18
DC-13	12.002 $\pm$ 0.012		12.039 $\pm$ 0.034		12.021	0.15
DC-14	10.984 $\pm$ 0.024		11.050 $\pm$ 0.024		11.017	0.30
DC-15	11.366 $\pm$ 0.015		11.445 $\pm$ 0.021		11.405	0.34
DC-16	12.972 $\pm$ 0.030		13.063 $\pm$ 0.049		13.017	0.35
DC-17	0.1- μ Ci source contaminated with Mn ⁵⁴					
DC-18	11.785 $\pm$ 0.021		11.827 $\pm$ 0.035		11.806	0.18
DC-19	12.411 $\pm$ 0.017		12.460 $\pm$ 0.045		12.436	0.20
DC-20	13.106 $\pm$ 0.013		13.179 $\pm$ 0.044		13.142	0.28

The average of the two ratios obtained on each set of sources as well as the difference between the average value and the two measurements, expressed as the percentage of the average value, are given in the table.

The original specification of the Committee stated that the ratio between sources should be known within 1.0%. Since the average ratio falls within 0.35% of the actual ratios found at both 3.3 and 31.8 cm, this average ratio can be employed when the sources are measured at any intermediate distance.

Ratios between the Sources in the Set When Counted at 3.3 cm

The individual measurements on each 0.01- μ Ci source made at 3.3 cm from the crystal were analyzed and the means, variances, standard deviations, and standard errors were calculated. The ratios (and their standard errors) between the 0.1- and the 0.01- μ Ci sources and between the 1.0- and 0.1- μ Ci sources were calculated in the same manner described above, and are given in Table 44 along with the ratio between the 1.0- and 0.1- μ Ci sources also listed in Table 43.

TABLE 44

Ratios between Cs¹³⁷ sources when counted 3.3 cm from the face of the crystal

Source No.	Microcurie content "1.0- μ Ci source"	Ratios and associated standard errors		
		1:0.1	0.1:0.01	1:0.01
DC-1	1.296	13.084 $\pm$ 0.032	9.730 $\pm$ 0.028	127.31 $\pm$ 0.41
DC-2	1.288	12.360 $\pm$ 0.045	9.962 $\pm$ 0.037	123.13 $\pm$ 0.51
DC-3	1.303	13.120 $\pm$ 0.045	10.661 $\pm$ 0.039	139.88 $\pm$ 0.52
DC-4	1.334	12.881 $\pm$ 0.025	11.037 $\pm$ 0.024	142.18 $\pm$ 0.38
DC-5	1.364	13.005 $\pm$ 0.041	10.217 $\pm$ 0.026	132.87 $\pm$ 0.44
DC-6	1.238	11.703 $\pm$ 0.024	10.248 $\pm$ 0.026	119.93 $\pm$ 0.36
DC-7	1.333	12.684 $\pm$ 0.036	10.457 $\pm$ 0.029	132.63 $\pm$ 0.30
DC-8	1.362	12.687 $\pm$ 0.017	10.678 $\pm$ 0.027	135.48 $\pm$ 0.33
DC-9	1.260	10.701 $\pm$ 0.034	11.796 $\pm$ 0.042	126.23 $\pm$ 0.51
DC-10	1.315	12.551 $\pm$ 0.023	11.197 $\pm$ 0.029	140.54 $\pm$ 0.32
DC-11	1.344	13.016 $\pm$ 0.014	10.840 $\pm$ 0.027	141.10 $\pm$ 0.32
DC-12	1.296	13.177 $\pm$ 0.025	10.197 $\pm$ 0.040	134.37 $\pm$ 0.53
DC-13	1.183	12.002 $\pm$ 0.012	10.438 $\pm$ 0.025	125.27 $\pm$ 0.29
DC-14	1.534	10.984 $\pm$ 0.024	10.261 $\pm$ 0.029	112.71 $\pm$ 0.38
DC-15	1.208	11.366 $\pm$ 0.015	10.406 $\pm$ 0.034	118.28 $\pm$ 0.40
DC-16	1.373	12.972 $\pm$ 0.030	10.030 $\pm$ 0.039	130.11 $\pm$ 0.57
DC-17	0.1- μ Ci source contaminated with Mn ⁵⁴			
DC-18	1.211	11.785 $\pm$ 0.021	10.221 $\pm$ 0.036	120.51 $\pm$ 0.44
DC-19	1.306	12.411 $\pm$ 0.017	10.203 $\pm$ 0.037	126.63 $\pm$ 0.47
DC-20	1.354	13.106 $\pm$ 0.013	9.525 $\pm$ 0.028	124.82 $\pm$ 0.35

The ratios (not given) between the 0.1- and 0.01- μ Ci sources of sets 1, 7, and 13 when located at 31.8 cm agreed within the standard errors of the ratios found when these sources were located at 3.3 cm. This fact also supports the contention that the appropriate correction for coincidence losses has been made on the 0.1- μ Ci measurement at 3.3 cm.

Absolute Activity

Although the rod sources were originally intended to be used only as secondary standards, we have attempted to determine fairly precisely the amount of Cs^{137} present in the "nominal 1.0" μ Ci rods. To do this, the NBS massless point source was placed on the tray and the height of the tray was adjusted so that the point source coincided with the position occupied by the center of the bead in the rod source (31.8 cm from the crystal face). After several measurements were made, a right cylinder block of Lucite, 5/8 inch (1.59 cm) in diameter by 0.310 inch (0.79 cm) thick,* was placed over the point source to introduce the same absorption and scatter present in the rod sources and several more measurements were made. The tray was raised so that the point source was 3.3 cm from the crystal and the same procedure was repeated. The count rate in the photopeak decreased 5.5% at both distances when the Lucite block was added.

The calibration factor used to calculate the absolute Cs^{137} content of the rod sources was based on National Bureau of Standards Cs^{137} point source 189. This Cs^{137} source was certified to emit 50.1×10^3 gamma rays per second on August 1, 1963. A ratio of 85 gamma rays per 100 disintegrations was employed to calculate the microcurie content and a half-time of 29.78 years was used to correct for physical decay from August 1, 1963 until July 1, 1965.

The count rates under the photopeaks obtained when the NBS source was covered with a 0.310-inch thick block of Lucite and located at both 31.8 and 3.3 cm were divided by the calculated microcurie content to obtain the calibration factors. These count rates were also corrected for coincidence losses.

The count rates obtained from the 1.0- μ Ci sources at 31.8 and 3.3 cm were divided by the count rates per microcurie calculated from the NBS source at these two positions. As would be expected from the fact that the ratios obtained for some sets at 31.8 and 3.3 cm differed by as much as 0.7%, the two values calculated for some sources also differed by as much as 0.7%. The values calculated for the two distances were averaged and are listed in Table 44. These values should be well within 1.0% of the correct value.

*A shadowgraph was used to measure the distance from the center of the bead to the end of the rod, and thereby to determine the 0.310-inch thickness of the block.

The absolute activities of the 0.1- and 0.01- μ Ci sources were not given. If the value for the "nominal 1.0" μ Ci source in the set is divided by the appropriate ratio, the values for the other sources in the set can be calculated to within 1.0% of the true value.

Ratio from the Side

The 1.0- and 0.1- μ Ci sources of set 1 were also counted from the side to determine if the ratio between the sources differed significantly from the ratio found when the sources were counted from the end. Each source was placed on its side on the tray with the label placed first up toward the crystal and then with the label placed away from the crystal. The bead source was on the axis of the crystal about 15.25 cm from the face of the crystal. The axis of the handle of the rod source was at right angles to the crystal axis.

The ratios between the 1.0- and 0.1- μ Ci sources of set 1 were 13.16 when the label was placed toward and 13.20 when the label was placed away from the crystal face. These ratios are in fair agreement with the ratio of 13.09 found from the end of the source and verify the fact that the same ratio should be obtained by counting the source either from the side or the end.

Conclusions

The ratios of activities between 19 sets of these decade sources and the absolute activities of the "nominal 1.0" μ Ci sources of each set have been measured and set forth in detail. Since the standard errors of the ratios are all within 0.5%, substantially all of the ratios should be within 1.0% of the true value if the sources are counted from the end. The absolute activities of the "nominal 1.0" μ Ci sources should also be within 1.0% of the true value. When compared to a point massless source, these rod standards would appear to contain 5.5% less Cs^{137} than stated because of absorption and scatter in the Lucite rods in which they are imbedded.

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