KEYPUNCHED

REON TECHNICAL MEMORANDUM
CALCULATION OF EQUATION FOR
SPECIFIC HEAT AND SPECIFIC HEAT RATIO
AS A FUNCTION OF TEMPERATURE
RANGE LIMIT 600°R to 1800°R

MASTER

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Curves:

1.2.3-65-C-046

1.2.3-65-C-047

I. INTRODUCTION

In order to update the Job 341 computer program to be acceptable for hot turbine gas drive turbopump tests, Reference (D.2.3-64-1-021), the equations for specific heat as a function of temperature, ratio of specific heats as a function of temperature, were required in the temperature limits of 600°R to 1800°R.

II. RESULTS

When the hydrogen gas is in the temperature range of 600°R to 1800°R, the equation to be used is:

$$C_p = c$$

$$(1.1168874 + 4.4505024 \times 10^{-4} T - 5.4378556 \times 10^{-7} T^2$$

$$+ 2.8803207 \times 10^{-10} T^3 - 5.2819706 \times 10^{-14} T^4)$$

where: C_p = Specific heat - BTU/lb

T = Temperature - °R

$$\gamma = (5.5901210 \times 10^{-1} - 1.1851554 \times 10^{-3} T + 8.6693075 \times 10^{-6} T^2$$

$$- 3.2880152 \times 10^{-9} T^3 + 2.5590210 \times 10^{-12} T^4$$

$$- 1.0332078 \times 10^{-15} T^5 + 2.4555808 \times 10^{-19} T^6$$

$$- 2.4339549 \times 10^{-23} T^7)$$

γ = ratio of specific heats

III. DISCUSSION

Specific heat constants and temperatures were input in AUC computer program number 409 (Seventh Order Polynomial Curve Fit Program) and the best fit occurred in a fourth order polynomial, as shown by the table of variance, (Reference Appendix I).

The equation obtained was:

$$1.1166874 + 4.4505024 \times 10^{-4} T - 5.4378556 \times 10^{-7} T^2$$

$$C_p = c$$

$$+ 2.8803207 \times 10^{-10} T^3 - 5.2819706 \times 10^{-14} T^4)$$

The temperatures and specific heat constants were chosen from "The Specific Heats of Certain Gases Over Wide Ranges of Pressures and Temperatures" by Ellenwood, Kulik and Gay.

The variation of pressure, within the limits of the Mark III Mod 4 turbopump, is not considered in this equation.

The need for this equation can be seen in Curve 1.2.3-65-C-003. The characteristic equation $\ln C_p = f(T)$ was used due to the fit obtained in Curve 1.2.3-65-C-034.

In order to check this equation, a computer program was written to compute specific heat and temperature and the table is presented in Appendix II.

The same procedure was used to calculate the equation for the ratio of specific heats. The values for the ratio of specific heats as a function of temperature were taken from the "Gas Tables" by the Keenan and Kaye. The equation obtained was:

$$C_p = e^{(5.5901210 \times 10^{-1} - 1.1381557 \times 10^{-3}T + 2.6691075 \times 10^{-6}T^2 - 3.2880152 \times 10^{-9}T^3 + 2.3990119 \times 10^{-12}T^4 - 1.0582078 \times 10^{-15}T^5 + 2.4566868 \times 10^{-19}T^6 - 2.4389143 \times 10^{-23}T^7)}$$

Appendix III presents the printout for the seventh order polynomial curve fit program.

Tabulation of these two parameters are presented in Appendix IV.

The curves for these two parameters, plotted on regular and semi-logarithmic graph paper, are presented in 1.2.3-65-C-046 and 1.2.3-65-C-047 respectively.

HYDRO1 - 1

TABLE OF RESIDUALS

FIRST THROUGH SIXTH ORDER RESIDUALS

Y	X	R1	R2	R3	R4	R5	R6
3.4800000+00	672	-1.3655573-03	1.4660970-03	1.9370704-04	0	0	0
3.4900000+00	762	-1.5802421-03	-8.6847693-04	-3.5973638-04	0	0	0
3.5000000+00	942	8.6804479-04	-1.5923120-03	2.0145222-04	-3.7252003-09	0	0
3.5300000+00	1392	5.6069270-03	1.4547743-03	-4.3168664-05	0	0	0
3.6100000+00	1842	-3.5291836-03	-4.6098977-04	7.7448785-06	-3.7252903-09	0	0

SEVENTH ORDER POLYNOMIAL CURVE FIT PROGRAM AGC 429

TYPE

LN Y=F(X)

POLYNOMIAL COEFFICIENTS

	A0	A1	A2	A3	A4	A5	A6	A7
FIRST ORDER	1.2258445+00	2.9497310+05						
SECOND ORDER	1.2557804+00	-2.5622600+05	2.2005296+08					
THIRD ORDER	1.2011428+00	1.2339281+04	-1.0392656+07	3.3265211+11				
FOURTH ORDER	1.1168874+00	4.4505024+04	-5.4378566+07	2.8803207+10	-5.2819706+14			

TABLE OF VARIANCES

FIRST ORDER	SECOND ORDER	THIRD ORDER	FOURTH ORDER	FIFTH ORDER	SIXTH ORDER	SEVENTH ORDER
9.8016360+06	1.5541339+06	4.1887910+08	5.5514151+18	-8.6847693+04	7.7448785+06	0

APPENDIX C

HYDROGEN TABLE OF GAMMA VS TEMPERATURE

GAMMA T(R)

1.4019 600.

1.4015 610.

1.4011 620.

1.4008 630.

1.4005 640.

1.4002 650.

1.4000 660.

1.3997 670.

1.3995 680.

1.3993 690.

1.3992 700.

1.3990 710.

1.3989 720.

1.3987 730.

1.3986 740.

1.3985 750.

1.3984 760.

1.3983 770.

1.3982 780.

1.3982 790.

1.3981 800.

1.3980 810.

1.3980 820.

1.3979 830.

1.3978 840.

1.3978 850.

1.3977 860.

1.3977 870.

1.3976 880.

1.3976 890.

1.3975 900.

1.3974 910.

1.3974 920.

1.3973 930.

1.3973 940.

1.3972 950.

1.3972 960.

1.3971 970.

1.3970 980.

1.3970 990.

1.3969 1000.

1.3968 1010.

1.3967 1020.

1.3966 1030.

1.3966 1040.

1.3965 1050.

1.3964 1060.

1.3963 1070.

1.3962 1080.

1.3961 1090.

1.3960 1100.

1.3959 1110.

1.3958 1120.

1.3957 1130.

1.3955 1140.

1.3954 1150.

1.3953 1160.

1.3952 1170.

1.3950 1180.

1.3949 1190.

1.3948	1200.
1.3946	1210.
1.3945	1220.
1.3943	1230.
1.3942	1240.
1.3940	1250.
1.3939	1260.
1.3937	1270.
1.3935	1280.
1.3933	1290.
1.3932	1300.
1.3930	1310.
1.3928	1320.
1.3926	1330.
1.3924	1340.
1.3922	1350.
1.3920	1360.
1.3918	1370.
1.3916	1380.
1.3914	1390.
1.3912	1400.
1.3910	1410.
1.3907	1420.
1.3905	1430.
1.3903	1440.
1.3900	1450.
1.3898	1460.
1.3895	1470.
1.3893	1480.
1.3890	1490.
1.3888	1500.
1.3885	1510.
1.3883	1520.
1.3880	1530.
1.3877	1540.
1.3875	1550.
1.3872	1560.
1.3869	1570.
1.3866	1580.
1.3863	1590.
1.3860	1600.
1.3857	1610.
1.3854	1620.
1.3852	1630.
1.3849	1640.
1.3846	1650.
1.3843	1660.
1.3840	1670.
1.3836	1680.
1.3833	1690.
1.3830	1700.
1.3827	1710.
1.3824	1720.
1.3821	1730.
1.3818	1740.
1.3815	1750.
1.3812	1760.
1.3809	1770.
1.3806	1780.
1.3803	1790.
1.3800	1800.

THERMODYNAMIC PROPERTIES OF HYDROGEN

1.2.3-65-C-033

THERMODYNAMIC PROPERTIES
OF
HYDROGEN

SPECIFIC HEAT AT CONSTANT PRESSURE

BTU
LB-MOLE °RBTU
LB °R500 600 700 800 900 1000 1100 1200 1300 1400 1500 1600 1700
TEMPERATURE °R

APPENDIX III

SEVENTH ORDER POLYNOMIAL CURVE FIT PROGRAM AGC 429

TYPE

LN Y=F(X)

POLYNOMIAL COEFFICIENTS

	A0	A1	A2	A3	A4	A5	A6	A7
FIRST ORDER	3.4480979-01	-1.1372398-05						
SECOND ORDER	3.3482995-01	7.5867198-06	-8.0342882-09					
THIRD ORDER	3.4986097-01	-3.5394794-05	3.0210745-08	-1.0688053-11				
FOURTH ORDER	3.8621192-01	-1.7557193-04	2.2319064-07	-1.2335535-10	2.3647861-14			
FIFTH ORDER	4.2695702-01	-3.7231000-04	5.8874915-07	-4.5050031-10	1.6499565-13	-2.3658459-17		
SIXTH ORDER	5.0810408-01	-8.4262943-04	1.6898673-06	-1.7844074-09	1.0479675-12	-3.2697089-16	4.2319930-20	
SEVENTH ORDER	5.5901210-01	-1.1881554-03	2.6693075-06	-3.2880152-09	2.3990219-12	-1.0382078-15	2.4566868-19	-2.4389549-23

TABLE OF VARIANCES

FIRST ORDER	SECOND ORDER	THIRD ORDER	FOURTH ORDER	FIFTH ORDER	SIXTH ORDER	SEVENTH ORDER
1.1905689-06	2.8545345-07	1.1648932-07	3.5452696-08	2.8251204-08	2.6151575-08	2.6093708-08

TABLE OF RESIDUALS

FIRST THROUGH SIXTH ORDER RESIDUALS

Y	X	R1	R2	R3	R4	R5	R6
1.4020000+00	600	8.6566433-05	-1.4101462-03	-7.0844870-04	-2.6303902-04	-1.4338456-04	-8.7316148-05
1.4000000+00	650	9.4549824-04	-1.0540150-04	2.1094829-04	2.5997590-04	2.1884032-04	1.7112214-04
1.3990000+00	700	1.0914179-03	4.4615939-04	4.6417769-04	2.8322078-04	1.8698163-04	1.3092998-04
1.3990000+00	750	5.2279793-04	2.4301000-04	4.1696243-05	-2.3946445-04	-3.2520294-04	-3.4841150-04
1.3980000+00	800	6.6923164-04	7.1474165-04	3.6508217-04	8.0429018-05	3.9493665-05	5.5972487-05
1.3980000+00	850	1.0061171-04	4.3124892-04	-3.7876889-06	-2.2474397-04	-2.0998251-04	-1.6582850-04
1.3970000+00	900	2.4755392-04	8.2314759-04	3.5768468-04	2.4164282-04	3.0659419-04	3.5902858-04
1.3970000+00	1000	-8.8968594-04	5.5304728-05	-3.3821538-04	-2.0694174-04	-9.4020739-05	-7.5972639-05
1.3960000+00	1100	-1.3108477-03	-1.5714578-04	-3.5510585-04	-3.4663826-05	4.3534674-05	5.5730343-06
1.3950000+00	1200	-1.7315000-03	-5.2977260-04	-4.7268346-04	-1.0131486-04	-1.0997616-04	-1.6887020-04
1.3930000+00	1300	-1.4340198-03	-3.4495257-04	-3.7453137-05	2.2325758-04	1.3309997-04	1.0899268-04
1.3910000+00	1400	-1.1344766-03	-3.1875540-04	1.7038640-04	1.9227061-04	8.2033686-05	1.1944957-04
1.3890000+00	1500	-8.3286781-04	-4.5117829-04	8.6709857-05	-1.6823318-04	-2.1091755-04	-1.4694035-04
1.3860000+00	1600	1.9205455-04	-2.0973384-05	3.6863796-04	-5.4209493-05	2.6277266-05	3.4810975-05
1.3830000+00	1700	1.2216642-03	2.5323313-04	2.3341458-04	-4.4735148-05	9.5913187-05	1.2455508-05
1.3800000+00	1800	2.2559771-03	3.7145615-04	-3.8307253-04	5.6511723-05	-3.9325096-05	-5.0477683-06

SEVENTH ORDER RESIDUALS

1.4020000+00	600	-8.0123544-05
1.4000000+00	650	1.6067550-04
1.3990000+00	700	1.2477487-04
1.3990000+00	750	-3.4597982-04
1.3980000+00	800	6.3337386-05
1.3980000+00	850	-1.5906058-04
1.3970000+00	900	3.6109984-04
1.3970000+00	1000	-8.4807165-05
1.3960000+00	1100	-3.3117831-06
1.3950000+00	1200	-1.6705133-04
1.3930000+00	1300	1.1926237-04
1.3910000+00	1400	1.2485497-04
1.3890000+00	1500	-1.5508290-04
1.3860000+00	1600	2.5138259-05
1.3830000+00	1700	2.4716370-05
1.3800000+00	1800	-8.4824860-06

3.51	1190.
3.5107	1200.
3.5103	1210.
3.5131	1220.
3.5138	1230.
3.5146	1240.
3.5154	1250.
3.5162	1260.
3.5170	1270.
3.5179	1280.
3.5188	1290.
3.5198	1300.
3.5208	1310.
3.5218	1320.
3.5228	1330.
3.5239	1340.
3.5250	1350.
3.5261	1360.
3.5273	1370.
3.5285	1380.
3.5297	1390.
3.5310	1400.
3.5323	1410.
3.5336	1420.
3.5350	1430.
3.5364	1440.
3.5378	1450.
3.5393	1460.
3.5408	1470.
3.5423	1480.
3.5438	1490.
3.5454	1500.
3.5470	1510.
3.5486	1520.
3.5503	1530.
3.5520	1540.
3.5537	1550.
3.5554	1560.
3.5572	1570.
3.5590	1580.
3.5608	1590.
3.5626	1600.
3.5645	1610.
3.5663	1620.
3.5682	1630.
3.5701	1640.
3.5721	1650.
3.5740	1660.
3.5759	1670.
3.5779	1680.
3.5799	1690.
3.5818	1700.
3.5838	1710.
3.5858	1720.
3.5878	1730.
3.5898	1740.
3.5918	1750.
3.5938	1760.
3.5958	1770.
3.5978	1780.
3.5998	1790.
3.6018	1800.

HYDROGEN TABLE OF SPECIFIC HEAT VS TEMPERATURE

CP	T (R)				
0.111688740E+01	0.445050240E-03	-0.543765660E-06	0.288032070E-09	-0.528197061E-13	
3.4678	600.				
3.4698	610.				
3.4716	620.				
3.4734	630.				
3.4751	640.				
3.4767	650.				
3.4783	660.				
3.4797	670.				
3.4811	680.				
3.4824	690.				
3.4836	700.				
3.4848	710.				
3.4859	720.				
3.4870	730.				
3.4880	740.				
3.4889	750.				
3.4898	760.				
3.4907	770.				
3.4915	780.				
3.4922	790.				
3.4929	800.				
3.4936	810.				
3.4942	820.				
3.4949	830.				
3.4954	840.				
3.4958	850.				
3.4965	860.				
3.4970	870.				
3.4974	880.				
3.4979	890.				
3.4983	900.				
3.4987	910.				
3.4992	920.				
3.4995	930.				
3.4999	940.				
3.5003	950.				
3.5007	960.				
3.5010	970.				
3.5014	980.				
3.5017	990.				
3.5021	1000.				
3.5025	1010.				
3.5028	1020.				
3.5032	1030.				
3.5036	1040.				
3.5040	1050.				
3.5044	1060.				
3.5048	1070.				
3.5052	1080.				
3.5056	1090.				
3.5061	1100.				
3.5065	1110.				
3.5070	1120.				
3.5075	1130.				
3.5081	1140.				
3.5086	1150.				
3.5092	1160.				
3.5098	1170.				

THERMODYNAMIC PROPERTIES OF HYDROGEN

SPECIFIC HEAT RATIO, γ

1.40
1.38
1.36
1.34
1.32
1.30

400 600 800 1000 1200 1400 1600 1800 2000 2200

TEMPERATURE °R

1.2.3-65-C-046

THERMODYNAMIC PROPERTIES
OF
HYDROGEN

RATIO OF SPECIFIC HEATS

TEMPERATURE °R

K-E SEMI-LOGARITHMIC 359-51G
KEUFFEL & ESSER CO. MADE IN U.S.A.
1 CYCLE X 70 DIVISIONS