LA-AC-6232

FEB 18 1965

83768

THE (He^3, d) REACTION
ON THE $N = 28$ NUCLEI

MASTER

By

Dale D. Armstrong

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission, is in any way responsible for the results or for any use of the information contained in this report, or for any damages resulting from the use of the information, or for any infringement of any patent rights, or for any other legal liability. The Commission, however, does not warrant the accuracy or the completeness of the information, or the process or procedure by which it was obtained, or the results or the use of the information, or for any damages resulting from the use of the information, or for any infringement of any patent rights, or for any other legal liability. The Commission, however, does not warrant the accuracy or the completeness of the information, or the process or procedure by which it was obtained, or the results or the use of the information, or for any damages resulting from the use of the information, or for any infringement of any patent rights, or for any other legal liability.

A Thesis

Submitted in Partial Fulfillment of the
Requirements for the Degree of
Doctor of Philosophy in Physics

The University of New Mexico

1965

PATENT CLEARANCE OBTAINED. RELEASE TO
THE PUBLIC IS APPROVED. PROCEDURES
ARE ON FILE IN THE RECEIVING SECTION.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

ACKNOWLEDGMENTS

This work was performed under the auspices of the U. S. Atomic Energy Commission.

I wish to express my thanks to Dr. R. F. Taschek, Physics Division Leader, Dr. R. B. Leachman, Group Leader, and the administration of the Laboratory for granting me permission to conduct this program of research.

I am particularly indebted to Dr. A. G. Blair for his guidance and encouragement during the course of this study. His willingness to discuss all facets of the study and his participation in the experimental phase of the program are gratefully acknowledged.

I want to express my appreciation to Dr. C. P. Leavitt of the University of New Mexico for reading the manuscript prior to its completion.

I wish, also, to express my gratitude to Larry Allen and Judith Gursky who devoted much of their time and efforts to the preparation or assistance in the preparation of the target foils that were used in the present study. To W. S. Hall goes the credit for the development of the new computer codes and the modifications of the existing computer codes that were necessary for the analysis of the data.

Any excellence in the form of this manuscript is due to efforts of Marjorie R. Rector, who typed and proofread the manuscript.

Dale D. Armstrong
December 1, 1964

ABSTRACT

A beam of 22-MeV He^3 ions from the Los Alamos variable energy cyclotron was used to investigate the (He^3, d) reaction on Ca^{48} , V^{51} , Cr^{52} , and Fe^{54} . A solid-state E- ΔE mass-identification detector system was used. The use of a summing preamplifier for performing the coherent addition of the E and ΔE pulses allowed the energy resolution to be reduced to the inherent energy spread of the cyclotron beam, which is 100 to 120 keV. Levels or groups of levels were resolvable up to an excitation of 5 to 6 MeV for all the nuclei that were studied, and angular distributions were obtained for the deuterons corresponding to these levels. A modified version of the Gibbs-Tobocman distorted-wave calculation was used to extract spectroscopic information concerning the levels. The formalism developed by Macfarlane and French for the j-j coupling version of the nuclear shell model was used to assign the levels to the various single-particle states. The isobaric-spin splitting of the single-particle strength was observed for the $2p_{3/2}$ proton single-particle state in Co^{55} . However, only the lower isobaric-spin components of the SP states were observed for all the other nuclei studied. The seniority mixing of the two 4^+ states in Cr^{52} belonging to the $(f_{7/2})^4$ configuration was determined and compares favorably with the predictions made by Komoda. Elastic-scattering data for 22-MeV He^3 ions elastically scattered from Ca^{40} , Ca^{48} , V^{51} , Cr^{52} , Fe^{54} , and Fe^{56} were obtained. Optical-model fits to the elastic-scattering data provided

the optical-model parameters for He^3 ions that were required for the distorted-wave calculation.

TABLE OF CONTENTS

	Page
ACKNOWLEDGMENTS	ii
ABSTRACT	iii
LIST OF ILLUSTRATIONS	vii
LIST OF TABLES	ix
INTRODUCTION	1
I. EXPERIMENTAL PROCEDURE	4
A. Scattering Chamber and Detectors	4
B. Electronic Apparatus	5
C. Targets	7
D. Beam Monitoring	12
E. Data Reduction	13
1. (He^3, d) Reaction Data	13
2. Elastic-Scattering Data	20
3. Energy Calibrations	21
II. GENERAL DISCUSSION OF THE RESULTS	22
A. Deuteron Angular Distributions	22
B. Distorted-Wave Calculations	28
C. Optical-Model Parameters	38
D. Spectroscopic Factors	46
III. DISCUSSION OF SPECIFIC REACTIONS	54
A. $\text{Ca}^{48}(\text{He}^3, d)\text{Sc}^{49}$ Reaction	54

B.	$V^{51}(He^3,d)Cr^{52}$ Reaction	59
C.	$Cr^{52}(He^3,d)Mn^{53}$ Reaction	67
D.	$Fe^{54}(He^3,d)Co^{55}$ Reaction	69
IV.	SUMMARY	77
	REFERENCES	82

LIST OF ILLUSTRATIONS

Figure	Page
1. Block Diagram of the Electronics	6
2. Pulse-height Spectrum of the Deuterons from the Ca ⁴⁸ (He ³ ,d)Sc ⁴⁹ Reaction	14
3. Pulse-height Spectrum of the Deuterons from the V ⁵¹ (He ³ ,d)Cr ⁵² Reaction	15
4. Pulse-height Spectrum of the Deuterons from the Cr ⁵² (He ³ ,d)Mn ⁵³ Reaction	16
5. Pulse-height Spectrum of the Deuterons from the Fe ⁵⁴ (He ³ ,d)Co ⁵⁵ Reaction	17
6. Pulse-height Spectrum of the Deuterons from the Fe ⁵⁶ (He ³ ,d)Co ⁵⁷ Reaction	18
7. Angular Distributions for the Deuteron Groups from the Ca ⁴⁸ (He ³ ,d)Sc ⁴⁹ Reaction	23
8. Angular Distributions for the Deuteron Groups from the V ⁵¹ (He ³ ,d)Cr ⁵² Reaction	24
9. Angular Distributions for the Deuteron Groups from the Cr ⁵² (He ³ ,d)Mn ⁵³ Reaction	25
10. Angular Distributions for the Deuteron Groups from the Fe ⁵⁴ (He ³ ,d)Co ⁵⁵ Reaction	26
11. Decomposition of the Angular Distributions for Groups from the Fe ⁵⁴ (He ³ ,d)Co ⁵⁵ Reaction and V ⁵¹ (He ³ ,d)Cr ⁵² Reaction into their $\ell = 1$ and $\ell = 3$ Components	29

Figure	Page
12. Comparison of the Predictions of the DW Calculation to the Angular Distributions for Group 1 and Group 2 of Figure 7 .	34
13. Comparison of the Predictions of the DW Calculation to the Angular Distributions for Group 5 and Group 10 of Figure 8 .	35
14. Comparison of the Predictions of the DW Calculation to the Angular Distributions for Group 1 and Group 4 of Figure 9 .	36
15. Comparison of the Predictions of the DW Calculation to the Angular Distributions for Group 1 and Group 2 of Figure 10 .	37
16. Comparisons of the Optical-Model Predictions to the He ³ Elastic-Scattering Data	42
17. Comparisons of the Elastic-Scattering Angular Distributions Predicted by the DW Calculation to the Data	45
18. Q Dependence of the Cross Sections Predicted by the DW Calculation for the Cr ⁵² (He ³ ,d)Mn ⁵³ Reaction	53
19. Comparison of the Prediction of the DW Calculation for an $\ell = 2$ Transfer to the Angular Distribution for the Deuterons from the 2.28-MeV Level of Sc ⁴⁹	58

LIST OF TABLES

Table	Page
I. Description of the Targets Used in the Present Study . . .	8
II. Tabulation of the Magnitudes and Relative Errors of the Differential Cross Sections for 22-MeV He^3 Ions, Elastically Scattered from Ca^{48} , Ca^{40} , V^{51} , Cr^{52} , Fe^{54} , and Fe^{56}	39
III. Optical-Model Parameters Obtained for the He^3 Ions Elastically Scattered from the Listed Nuclei	41
IV. Results from the Present Study of the $\text{Ca}^{48}(\text{He}^3, \text{d})\text{Sc}^{49}$ Reaction	56
V. Results for the $1f_{7/2}$ SP State from the Present Study of the $\text{V}^{51}(\text{He}^3, \text{d})\text{Cr}^{52}$ Reaction	61
VI. Results for the $1f_{5/2}$, $2p_{3/2}$, and $2p_{1/2}$ SP States from the Present Study of the $\text{V}^{51}(\text{He}^3, \text{d})\text{Cr}^{52}$ Reaction	63
VII. Results from the Present Study of the $\text{Cr}^{52}(\text{He}^3, \text{d})\text{Mn}^{53}$ Reaction	68
VIII. Results from the Present Study of the $\text{Fe}^{54}(\text{He}^3, \text{d})\text{Co}^{55}$ Reaction	71
IX. The Properties of Some of the Fe^{55} Excited States	73
X. Location of the Centroid for Each Proton SP State that was Determined in the Present Study	77

INTRODUCTION

Studies of direct reactions have proved to be fruitful in obtaining spectroscopic information about nuclear energy levels and nuclear structure. The (d,p) stripping reaction and (d,t) pickup reaction have been utilized to amass a large amount of information concerning the single-particle (SP) neutron levels of nuclei (see, e.g., the summaries in Ref. 1). Much less is known about the SP proton states of nuclei. Proton levels have been studied by the (d,n) stripping reaction, but until recently the instrumental energy resolution of a few hundred keV limited the usefulness of this reaction to the region of light nuclei for which the energy levels are, in general, more widely spaced. Similarly, the use of the (He^3 ,d) stripping reaction has been limited to light elements because of the low He^3 beam energies that could be obtained with existing Van de Graaff accelerators. The Los Alamos variable energy cyclotron has been capable for several years of accelerating He^3 ions² to energies from 10.5 to 25 MeV, but the instrumental energy resolution of 300 to 400 keV has limited its usefulness. Recently, however, the development of high-resolution semiconductor detectors has allowed the energy resolution to be reduced to 100-120 keV, the inherent energy spread of the cyclotron beam. This development has made it possible to extend the study of the (He^3 ,d) stripping reaction into the region of intermediate-weight nuclei.

The results of a study of the (He^3, d) reaction on Ca^{48} , V^{51} , Cr^{52} , and Fe^{54} , reported in this paper, were obtained by using a beam of 22-MeV He^3 ions from the cyclotron. Levels or groups of levels were resolvable up to an excitation energy of 5 to 6 MeV for all these nuclei, and a partial angular distribution for each deuteron group was obtained. Some gross-structure analysis was done for the deuterons from unresolved levels above 6 MeV of excitation for the $\text{Ca}^{48}(\text{He}^3, d)\text{Sc}^{49}$ and $\text{V}^{51}(\text{He}^3, d)\text{Cr}^{52}$ reactions.

Comparisons of the results of a modified version of the Gibbs-Tobocman distorted-wave (DW) calculation³ to the data were used to extract spectroscopic factors. The optical-model parameters used in the DW calculation were obtained from optical-model fits to elastic-scattering data.^{4,5} The orbital angular momentum of the captured proton was unambiguously determined by a comparison of the shape of the calculated angular distribution to the shape of the experimentally obtained angular distribution. Except for a group at 2.28 MeV of excitation in Sc^{49} which may be an $\ell = 2$ transfer, only $\ell = 1$ and $\ell = 3$ angular-momentum transfers were observed.

Since all of the nuclei studied have a closed shell of 28 neutrons and differ from having a magic number of protons by no more than four protons, they are to good approximation spherical nuclei. Thus, the j-j coupling version of the nuclear shell model was used as the basis for interpreting the results. The sum rules for spectroscopic factors developed by Macfarlane and French for the j-j coupling scheme were

used to identify the components of the $1f_{7/2}$, $1f_{5/2}$, $2p_{3/2}$, and $2p_{1/2}$ SP states.

When a proton is added to a neutron-rich nucleus, it is possible to form states which are the isobaric analogs of the states formed by adding a neutron to the same initial nucleus.^{6,7} Such states were observed for the $\text{Fe}^{54}(\text{He}^3, d)\text{Co}^{55}$ reaction. Lee et al.⁸ have recently reported a study of those Cu^{65} states which are the isobaric analogs of the low-lying Ni^{65} states. They found that the spin sequence for these levels was the same for both isotopes and that the energy spacings of the levels were very nearly the same for both nuclei. No such one-to-one correspondence of the Co^{55} states to their isobaric analogs, the Fe^{55} states, could be definitely established in the present study.

I. EXPERIMENTAL PROCEDURE

A. Scattering Chamber and Detectors

The beam geometry and the basic construction of the scattering chamber have been reported earlier.^{9,10}

The detector assembly was of the type suitable for use in our electronic E- ΔE particle identification system and consisted of a transmission semiconductor ΔE detector, followed by a thick semiconductor E detector. Data for the elastic scattering of He^3 ions from the various nuclei were obtained using gold surface-barrier detectors¹¹ with depletion depths of 100 μ and 500 μ for the ΔE and E detectors, respectively. Data for the (He^3, d) reaction were obtained using as the ΔE detector a gold surface-barrier detector with a depletion depth of 500 μ , and as the E detector a gold surface-barrier lithium-drifted detector^{12,13} with a depletion depth of 3 mm.

Previously, the detector assembly was composed of a transmission gas ion chamber followed by a NaI scintillation system. This bulky package had to be mounted external to the vacuum shell, but the compact semiconductor detector assembly could be mounted inside the scattering chamber. The internal mounting eliminates the spread in the energy of the detected particles due to the effect of the chamber window and the air path which exists between the target and the detector for the external detector mount. An internal detector-support boom was installed and attached to the rotary table. The angle between the detector and the

beam direction could be set and read remotely. A thermoelectric cooler¹⁴ was mounted on the boom, and onto the cooler was mounted the detector assembly. (Cooling the semiconductor detectors reduces their contribution to the electronic noise width and enhances their energy-resolving capability.) The boom and hot junction of the thermoelectric cooler were water-cooled to approximately 14°C. With a current of 25 to 30 amperes through the cooler, the detector assembly was cooled to approximately -17°C.

A vertical slit aperture whose dimensions were 0.040" x 0.250" was used before the detectors. The resulting angular resolution was approximately 1° and the solid angle approximately 7×10^{-4} steradians for all runs, the exact values depending on the measured distance between the aperture and the center of the target.

Preliminary runs evidenced an appreciable rate of deposition of carbon onto the targets. The source of contamination was untrapped vapors from the oil vacuum pumps and vapors from other sources of hydrocarbon such as O-rings and their lubricants. A liquid-nitrogen-cooled baffle was installed in the scattering chamber. The rate of carbon deposition thereafter was negligible.

B. Electronic Apparatus

A block diagram of the electronic apparatus is shown in Fig. 1. Basically, the diagram describes the electronic E-ΔE particle-identification system which has been discussed very thoroughly in the literature (see, e.g., Refs. 2, 10, 15 and 16). However, prior to the present

Fig. 1. Block diagram of the electronic apparatus. PA is used as an abbreviation for preamplifier, A for linear pulse amplifier, M for analog computer multiplier, D for discriminator, C for coincidence unit, and S for pulse adder.

study, a summing circuit (S in Fig. 1), which coherently adds the E and ΔE pulses, was included in the basic system. Coherent addition of the pulses has the important advantage of improving the energy-resolving capability of the system by eliminating the effect of the nonuniformity of the semiconductor ΔE detector and by eliminating the Landau energy spread introduced into the detected particles during their passage through the ΔE detector. Although the summing circuit was designed to allow the addition of the pulses in any desired proportion, a ratio of 1:1 was always found to be satisfactory. Without the summing circuit, the energy resolution was 300 to 400 keV (fwhm); with it, the energy resolution was 100 to 120 keV (fwhm), the inherent energy spread of the cyclotron beam.

The particle-identification system was adjusted to generate gating pulses for only those E + ΔE pulses corresponding to the type of particle of interest.^{10,16} A gated 400-channel pulse-height analyzer was then used to display the pulse-height spectrum of the E + ΔE pulses. Pulse-height spectra of the deuterons from the (He^3, d) reactions and of the scattered He^3 ions were obtained for a suitable range of scattering angles. Each pulse-height spectrum was read out as information on punched paper tape and printed paper tape and was simultaneously plotted. The information on the punched paper tape was later transferred to punched cards suitable for use on the Los Alamos digital computers.

C. Targets

Table I lists the targets used in the present study and their

TABLE I

Description of the targets used in the present study

Target	Isotopic composition	Areal density (mg/cm ²)	Method of Preparation	Backing Material
Ca	Natural abundance	1.59	Evaporation	None
Ca ⁴⁸	90.33% Ca ⁴⁸ , 9.38% Ca ⁴⁰	0.06	Evaporation	~ 60-μg/cm ² C
Ca ⁴⁸	90.33% Ca ⁴⁸ , 9.38% Ca ⁴⁰	0.32	Evaporation	740-μg/cm ² Mg ²⁴
V ⁵¹	99.9% V ⁵¹	0.17	Evaporation	None
V ⁵¹	99.9% V ⁵¹	1.33	Rolling	None
Cr ⁵²	99.9% Cr ⁵²	0.55	Evaporation	None
Fe ⁵⁴	93% Fe ⁵⁴ , 7% Fe ⁵⁶	0.35	Evaporation	~ 80-μg/cm ² Au
Fe ⁵⁴	93% Fe ⁵⁴ , 7% Fe ⁵⁶	1.17	Rolling	None
Fe ⁵⁶	99.9% Fe ⁵⁶	1.34	Electroplating	None
Fe	Natural abundance	0.29	Evaporation	~ 80-μg/cm ² Au

isotopic composition, areal density, method of preparation, and backing material, if any. The thicker V^{51} , Fe^{54} , and Fe^{56} targets had been used in a previous experiment. All the targets were prepared from isotopically enriched material.¹⁷ All the evaporated targets except Cr^{52} were prepared locally.^{17,a}

The areal densities of the thick foils were obtained by weighing the foils and measuring their areas. The areal densities of the thin V^{51} , Fe^{54} , and Fe foils were obtained from comparisons of the elastic scattering of He^3 ions from them to that from the thicker foils. The areal densities of the Ca^{48} foils were obtained from comparisons of the elastic scattering of He^3 ions from the 9.38% Ca^{40} contained in them to the elastic scattering from the natural Ca target. The areal density of the Cr^{52} target was supplied by the fabricator.¹⁷ The estimated error of the areal densities is $\pm 15\%$.

Many attempts were made to obtain self-supporting foils of Fe^{54} because all backing materials have some disadvantage. The (He^3, d) reaction on light-element backings competes seriously with the (He^3, d) reaction on Fe^{54} because of the low ground-state Q value ($Q = -0.436$ MeV) for the $Fe^{54}(He^3, d)Co^{55}$ reaction. Heavy element backings, such as Au , add appreciably to the elastic scattering of He^3 ions; thus, in order to avoid pileup of pulses in the electronics, one must reduce the beam

^aLarry Allen of this Laboratory prepared the calcium targets. Judith Gursky, also of this Laboratory, prepared the thin V^{51} foils and assisted in the preparation of the Au -backed Fe^{54} and natural iron targets.

intensity and, therefore, the rate at which data are taken. However, all attempts to obtain an unbacked target were unsuccessful. Gold was chosen as the backing since the disadvantage for the heavy element backing seemed least serious and since the (He^3, d) reaction on Au for 22-MeV He^3 ions is completely negligible compared to the same reaction on Fe^{54} .

Carbon was initially tried as the backing for the Ca^{48} foil since the difference in the ground-state Q values for the $\text{C}^{12}(\text{He}^3, \text{d})\text{N}^{13}$ and $\text{Ca}^{48}(\text{He}^3, \text{d})\text{Sc}^{49}$ reactions is such as to allow energy levels up to 7.7 MeV of excitation in Sc^{49} to be studied without interference of the deuterons from the $\text{C}^{12}(\text{He}^3, \text{d})$ reaction. However, the natural carbon backings contained 1% of C^{13} for which the deuterons from the $\text{C}^{13}(\text{He}^3, \text{d})$ reaction would interfere with those from the $\text{Ca}^{48}(\text{He}^3, \text{d})$ reaction. All the carbon-backed Ca^{48} foils for which the ratio of C^{13} to Ca^{48} was negligible consistently broke during the cooling period immediately following the evaporation. Thus, carbon proved to be unsatisfactory as a backing material for the Ca^{48} foil. Preliminary data taken with a carbon-backed foil as a target showed that the deuteron groups from levels above 6 MeV of excitation in Sc^{49} could not be resolved with the detection system, so no real loss of information would result if Mg^{24} were used as the backing material. [The Q for the $\text{Mg}^{24}(\text{He}^3, \text{d})$ reaction is -3.22 MeV and for the $\text{C}^{12}(\text{He}^3, \text{d})$ reaction -3.55 MeV.]

O^{18} would have been a troublesome contaminant in the Ca^{48} foil for the same reasons that C^{13} was. Therefore, to minimize the amount of O^{18} present in the foil and to prevent the deterioration of the foil from

oxidation, the backing material was mounted in the target frame, the Ca^{48} was evaporated onto the backing, and all subsequent storage, transfer, and use of the target occurred with the target maintained in a vacuum.

The Ca^{48} was evaporated from a tantalum boat loaded with $\text{Ca}^{48}\text{CO}_3$. As the temperature of the boat was raised, first the carbonate decomposed into the oxide, then the oxide was reduced by the tantalum, and finally the liberated Ca^{48} evaporated. Unfortunately, the reduction and subsequent evaporation occurs at a temperature only slightly below the decomposition temperature for the Ta_2O_5 which was always present on the surface of the boat. During the preparation of the Mg^{24} -backed Ca^{48} targets, the Ta_2O_5 decomposition temperature was exceeded and as a result $\sim 80 \mu\text{g}/\text{cm}^2$ of Ta was deposited on the target. Although this introduced the disadvantage discussed above for the Au-backed Fe^{54} target, the target was, nevertheless, suitable for use in studying the $\text{Ca}^{48}(\text{He}^3, \text{d})$ reaction.

Except for Ca^{48} , the thinnest target listed in Table I for each isotope was used to obtain the (He^3, d) reaction data. For no isotope studied was the energy spread due to the target thickness the limiting factor in determining the energy resolution of the system.

The elastic-scattering data were obtained with no loss of quality by using the thicker targets. In all cases the elastically-scattered He^3 group was well separated from the inelastic group from the first excited state. No thick target of Ca^{48} and only one thickness of Cr^{52}

foil was available. However, the carbon-backed Ca^{48} foil proved to be satisfactory.

D. Beam Monitoring

The He^3 beam passing through the target was collected in a Faraday cup, and the total charge for each experimental run was integrated by means of a vibrating-reed electrometer current integrator.¹⁸

The beam energy was determined and monitored by means of a slowing-down technique similar to that described by Northrop and Stokes.¹⁹ Briefly, the technique consists of observing beam particles scattered at 90° by a thick Au foil mounted in the Faraday cup. The scattered particles are detected in a diffused-junction semiconductor detector,²⁰ and the amplified pulses displayed by a multichannel analyzer. One or more members of an arrangement of uniform, accurately-weighed Al foils may be inserted between the Au scattering foil and the detector to slow down the scattered particles until the pulse height equals that due to a standard alpha-particle source which can be moved in front of the detector. The beam energy is then determined by reference to Al range-energy curves.²¹ The system has been checked by comparing results to energy measurements made with nuclear emulsions. In the present experiment, the error limit in the determination of the mean energy is estimated to be ± 200 keV. The variation of the beam energy at the center of a target was less than ± 50 keV over an entire angular distribution.

E. Data Reduction

1. (He^3, d) Reaction Data

Typical pulse-height spectra of the deuterons from the (He^3, d) reaction on Ca^{48} , V^{51} , Cr^{52} , Fe^{54} , and Fe^{56} are shown in Figs. 2-6. Except for the Fe^{56} spectrum, the solid line drawn through the data points is the result of a least squares computer routine which fits a skewed Gaussian distribution with an exponential tail to each peak in the pulse-height spectrum and computes the area under each peak.²² Except for Fe^{56} , each pulse-height spectrum was so analyzed and the differential cross section was computed by standard methods. The Fe^{56} target was a natural iron target intended and satisfactory only for use in making a background subtraction for the Fe^{54} data. Each deuteron group has been assigned the number indicated in the figures for ease of identification in the discussion that follows.

The shapes and widths of the resolved deuteron groups indicate that, predominantly, each one corresponds to the excitation of a single level in the residual nucleus. However, evidence will be presented later (see the discussion in III B) that a few of the well-defined groups correspond to at least two closely spaced levels. Groups 18 through 26 for the $\text{V}^{51}(\text{He}^3, \text{d})\text{Cr}^{52}$ reaction (Fig. 3) represent summation over approximately 300 keV of unresolved levels. These latter groups were included because they were free from the interference of deuterons from reactions on the contaminants in the target and because the information they provided was useful.

Fig. 2. Pulse-height spectrum of the deuterons from the $Ca^{48}(He^3,d)Sc^{49}$ reaction. The solid line represents the results of a least squares computer routine which fit a skewed Gaussian distribution with an exponential tail to each resolved peak in the pulse-height spectrum. The number indicated in the figure has been assigned to each deuteron group for ease of identification in the discussion in the text.

Fig. 3. Pulse-height spectrum of the deuterons from the $V^{51}(He^3,d)Cr^{52}$ reaction. See Fig. 2 or the text for the explanation of the solid lines and numbers displayed in the figure.

Fig. 4. Pulse-height spectrum of the deuterons from the $Cr^{52}(He^3,d)Mn^{53}$ reaction. See Fig. 2 or the text for an explanation of the solid lines and numbers displayed in the figure.

Fig. 5. Pulse-height spectrum of the deuterons from the $\text{Fe}^{54}(\text{He}^3, d)\text{Co}^{55}$ reaction. See Fig. 2 or the text for an explanation of the solid lines and numbers displayed in the figure.

Fig. 6. Pulse-height spectrum of the deuterons from the $\text{Fe}^{56}(\text{He}^3, \text{d})\text{Co}^{57}$ reaction. The solid line drawn through the data points is intended as a visual guide.

The (He^3, d) reaction on the Fe^{56} impurity (see Table I) in the Fe^{54} target was a serious source of background. Therefore, pulse-height spectra for the (He^3, d) reaction on the Au-backed natural iron target were obtained. (Such a spectrum has been labeled as $\text{Fe}^{56}(\text{He}^3, \text{d})\text{Co}^{57}$ in Fig. 6 since natural iron is 91.66% Fe^{56} .) A channel-by-channel subtraction of the Fe^{56} data from the Fe^{54} data was satisfactory and was done prior to the area analysis of the peaks in the $\text{Fe}^{54}(\text{He}^3, \text{d})\text{Co}^{55}$ pulse-height spectrum. A comparison of the intensity of the $\text{Fe}^{56}(\text{He}^3, \text{d})$ reaction for the Fe^{54} target to the intensity of the same reaction for the natural iron target determined the fraction of the Fe^{56} spectrum to be subtracted from the Fe^{54} spectrum. (In fact, the isotopic composition of the Fe^{54} targets was obtained from this procedure.) Natural iron does contain Fe^{54} , Fe^{57} , and Fe^{58} , for which no allowance was made in the background subtraction. However, the errors introduced by the presence of these isotopes in the natural iron target are negligible compared to the statistical errors of the $\text{Fe}^{54}(\text{He}^3, \text{d})\text{Co}^{55}$ reaction data.

Equally serious complications of the study of the $\text{Fe}^{54}(\text{He}^3, \text{d})$ reaction were the $\text{C}^{12}(\text{He}^3, \text{d})\text{N}^{13}$ and $\text{O}^{16}(\text{He}^3, \text{d})\text{F}^{17}$ reactions on the carbon and oxygen contaminants in the target. The deuteron groups corresponding to the ground state of N^{13} and the ground state and first excited state of F^{17} were the troublesome ones. Since the kinematic shift in the deuteron energy as a function of scattering angle is larger for lighter elements than for heavier elements, the positions of the contaminant groups relative to the groups from the $\text{Fe}^{54}(\text{He}^3, \text{d})$ reaction were

different for each scattering angle. At nearly every scattering angle one or more of the contaminant groups were not resolved from a group corresponding to an excited state of Co^{55} . For use in such instances, angular distributions for the deuteron groups from the (He^3, d) reaction on carbon and oxygen were obtained by using a Mylar film as a target. The angular distributions were then normalized to the Fe^{54} target data by comparisons of the intensity of the contaminant groups at those scattering angles for which they were resolved to the intensity determined for them by using the Mylar film. Thus, satisfactory subtractions of the contaminant groups were possible when they were not resolved. The normalization was done to an accuracy of $\pm 10\%$; however, this uncertainty in the contaminant group strength was sometimes reflected as a much larger error in the determination of the strength of weak groups.

2. Elastic Scattering Data

Pulse-height spectra were also obtained for the elastic scattering of 22-MeV He^3 ions from Ca^{40} , Ca^{48} , V^{51} , Cr^{52} , Fe^{54} , and Fe^{56} . In general, the number of counts in the elastic peak was easily determined by a simple summation of the counts in all channels comprising the peak. The number of counts was increased by 1 to 2% in order to include the contribution from the low energy tail of the peak. However, in the case of Ca^{48} , heavy element contaminants in the target necessitated a correction to the data for $\theta_{\text{c.m.}} < 30^\circ$. Also, groups elastically scattered from the carbon and oxygen contaminants began to interfere with the group from Ca^{48} at $\theta_{\text{c.m.}} \approx 24^\circ$. Between 24° and 15° it was possible to separate

the Ca^{48} peak from the contaminant peaks with the aid of the least squares computer routine mentioned in section II E.1. At still smaller angles no such separation was possible, and no data for $\theta_{\text{c.m.}} < 15^\circ$ are included in the angular distribution.

The angular distribution from Ca^{48} was corrected for the isotopic impurity of the target material (see Table I) by using the Ca^{40} data and making the appropriate subtractions from the Ca^{48} data. Similarly, the Fe^{54} distribution was corrected by using the Fe^{56} data. The amount of contaminants in the other targets was small enough to be neglected.

3. Energy Calibration

Energy calibration of the deuteron spectra was accomplished by comparison with the well-known spectra from the (He^3, d) reaction on the isotopes $\text{Ni}^{58,60,62,64}$ in a natural nickel target, and on C^{12} which was present in all the targets. From the knowledge of the energies of the deuterons, the laboratory scattering angle, and the masses of the particles involved in the reaction, Q values were calculated for each deuteron group. Excitation energies based on the ground-state Q values given in the Nuclear Data Tables²³ were calculated for the Cr^{52} , Mn^{53} , and Co^{55} excited states. The ground-state Q value reported by Erskine et al.²⁴ was used to calculate the excitation energies for the Sc^{48} states. For all possible comparisons, the excitation energies obtained in the present study agreed to within ± 25 keV with the excitation energies reported by other investigators.

II. GENERAL DISCUSSION OF THE RESULTS

A. Deuteron Angular Distributions

The angular distributions obtained for the deuterons from the (He^3, d) reaction on Ca^{48} , V^{51} , Cr^{52} , and Fe^{54} are shown in Figs. 7-10. Each angular distribution is associated with the cross-section scale adjacent to its forward angle points. The error bars indicate the relative cross-section errors; points without error bars are in error by less than the size of the symbol used to plot them. The numbers correspond to the numbers assigned to the deuteron groups in the associated pulse-height spectra (see Figs. 2-5).

The most distinctive features of the assembled distributions are the strong forward peaking and the predominance of only two types of structure. One type of distribution has a maximum near 20° , and the other has a minimum near 20° , but rises rapidly toward smaller angles. The strong forward peaking of both types of distributions suggests that the reaction proceeds by the direct-interaction stripping mechanism.²⁵

The differential cross section for the (He^3, d) reaction on V^{51} at a laboratory scattering angle of 150° was found to be three to four orders of magnitude less than the differential cross section at 30° , which is typical for the (He^3, d) reaction on all the isotopes that were studied. This large asymmetry about 90° indicates that the reaction does not proceed significantly by compound-nucleus formation and reinforces the argument that stripping must be the dominant reaction mechanism.

Fig. 7. Angular distributions of all but the weakest deuteron groups from the $\text{Ca}^{48}(\text{He}^3, d)\text{Sc}^{49}$ reaction, up to 5.9 MeV of excitation. The number assigned to each angular distribution corresponds to the group number of Fig. 2. The lines drawn through the data points are intended only as visual guides.

Fig. 8. Angular distributions of all but the weakest deuteron groups from the $V^{51}(\text{He}^3, d)\text{Cr}^{52}$ reaction, up to 8.7 MeV of excitation. The number assigned to each angular distribution corresponds to the group number of Fig. 3.

Fig. 9. Angular distributions of all but the weakest deuteron groups from the $\text{Cr}^{52}(\text{He}^3, d)\text{Mn}^{53}$ reaction, up to 5.1 MeV of excitation. The number assigned to each angular distribution corresponds to the group number of Fig. 4.

Fig. 10. Angular distributions of all but the weakest deuteron groups from the $\text{Fe}^{54}(\text{He}^3, \text{d})\text{Co}^{55}$ reaction, up to 6.1 MeV of excitation. The number assigned to each angular distribution corresponds to the group number of Fig. 5.

Similar asymmetries have been observed in a concurrent study of the (He^3, d) reaction on the nickel isotopes.²⁶ Because the (He^3, d) reaction does proceed by the stripping mechanism, the shape of the angular distribution can be used to determine the orbital angular momentum of the captured proton.²⁵

Since all the nuclei that were studied contained 28 neutrons, on the basis of the j-j coupling version of the nuclear shell model the neutrons completely fill all the neutron SP states up to and including the $1f_{7/2}$ SP state, thus forming a closed-shell structure characterized by zero angular momentum.²⁷ Therefore, the angular momenta of the states of the residual nuclei excited by the (He^3, d) reaction are determined by the proton structure of these nuclei. Since all the nuclei studied contained between 20 and 28 protons, the ground states of the residual nuclei are formed by adding $1f_{7/2}$ ($l = 3$) protons to the target nuclei. The angular distributions for all the ground-state groups exhibit a maximum near 20° ; therefore, this type of distribution must correspond to an orbital angular momentum of $l = 3$ for the captured proton. The lower excited states are expected to be formed by populating the $2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$ SP states.²⁷ Residual interactions result in a fragmentation of the SP strengths so one expects to observe more than one level corresponding to each of these SP states. Thus, for Sc^{49} , Mn^{53} , and Co^{55} , those excited-state groups which are also characterized by an $l = 3$ distribution must be components of the $1f_{5/2}$ SP state. The other groups are then the components of the p states and their angular distributions must be characteristic of $l = 1$ for the captured proton.

A few of the angular distributions [e.g., group 8 for the $\text{Ca}^{48}(\text{He}^3, d)\text{Sc}^{49}$ reaction] appear to be characteristic of neither an $\ell = 1$ nor an $\ell = 3$ orbital angular momentum transfer. However, they were quite easily shown to be sums of $\ell = 1$ and $\ell = 3$ angular distributions, indicating that at least two closely spaced levels had not been resolved. The strength of each component was determined by employing two computer routines. The first routine calculated polynomial fits to those experimentally determined $\ell = 1$ and $\ell = 3$ angular distributions which were chosen as standards. The second routine used the polynomials to compute a least squares fit to the composite angular distribution. The results of two such analyses are shown in Fig. 11. The standard deviation of the computed strengths of the components varied between $\pm 10\%$ and $\pm 15\%$. The values for the strengths were not very sensitive to the choice of standards and agreed to within the standard deviations of the calculations for all choices of the standards.

B. Distorted-Wave Calculations

The extraction of information about nuclear structure from the stripping-reaction data by a discussion such as that given in the preceding section is somewhat unsatisfactory since only plausibility arguments based on the assumption of a specific model of the nucleus were used to establish ℓ -transfer values. The verification of these ℓ values and the extraction of more detailed spectroscopic information are necessarily based on comparisons of the data to reliable theoretical calculations of the stripping-reaction cross sections. The calculation

Fig. 11. Decomposition of the angular distributions for groups from the $\text{Fe}^{54}(\text{He}^3, d)\text{Co}^{55}$ reaction and the $\text{V}^{51}(\text{He}^3, d)\text{Cr}^{52}$ reaction into their $l = 1$ and $l = 3$ components. The method used to extract the strengths of the $l = 1$ and $l = 3$ components is described in the text.

used in the present study was a modified version of the Gibbs-Tobocman DW calculation³ for stripping reactions. The mathematical details of this calculation are discussed in the literature and will not be repeated here. However, an outline of the theoretical foundations of the calculation is pertinent.

Although the discussion of stripping reactions that follows is in terms of the (He^3, d) stripping reaction, only a change in notation would be necessary to apply it to other one-particle stripping reactions.

The (He^3, d) stripping reaction is described as follows: An incident He^3 ion approaches a target nucleus I and, while it is in the vicinity of I , loses a proton p to I forming a residual nucleus $R = I + p$. The deuteron remnant of the He^3 nucleus then proceeds out from the residual nucleus. Early calculations neglected the interactions of the deuteron and He^3 ion with R and I , respectively. The DW calculation derives its name from the fact that it includes these interactions.

The matrix element for this process is given by

$$M_{fi} = \int_0^\infty dr \psi_f^* V_{dp} \psi_i \quad (1)$$

where ψ_f is the final-state wave function for the system, ψ_i is the initial-state wave function, and V_{dp} is the potential which describes the interaction of the deuteron and proton in the He^3 nucleus. The cross section is proportional to the square of the absolute value of M_{fi} .

In a DW calculation of M_{fi} the ψ_f and ψ_i are written as the product of two wave functions:

$$\psi_i = \varphi_{\text{He}^3}(\underline{r}_d - \underline{r}_p) \chi_{\text{He}^3}(\underline{r}_{\text{He}^3} - \underline{r}_I) \quad (2)$$

$$\psi_f = \xi_p(\underline{r}_p - \underline{r}_I) \chi_d(\underline{r}_d - \underline{r}_R) \quad (3)$$

where φ_{He^3} is the wave function which describes the relative motion of the proton and deuteron in the He^3 nucleus, χ_{He^3} is the optical-model wave function of the He^3 nucleus in the entrance channel, ξ_p is the bound-state wave function of the captured proton, and χ_a is the optical-model wave function of the deuteron in the exit channel.

Measuring all coordinates relative to I and substituting (2) and (3) into (1) gives

$$M_{fi} = \int_0^\infty \left[d\underline{r}_p d\underline{r}_d \xi_p^*(\underline{r}_p) \chi_d^*\left(\underline{r}_p - \frac{M_p}{M_R} \underline{r}_p\right) V_{dp}(\underline{r}_d - \underline{r}_p) \right. \\ \left. \times \varphi_{\text{He}^3}(\underline{r}_d - \underline{r}_p) \chi_{\text{He}^3}\left(\frac{\underline{r}_d - \underline{r}_p}{2}\right) \right] . \quad (4)$$

Most DW calculations, including the present one, simplify the evaluation of this overlap integral by assuming a zero range for the interaction of the proton and deuteron; that is,

$$V_{dp} = V_0 \delta(\underline{r}_d - \underline{r}_p) . \quad (5)$$

With this assumption, (4) becomes

$$M_{fi} = V_0 \varphi_{\text{He}^3}(0) \int_0^\infty d\underline{r} \xi_p(\underline{r}) \chi_d^*\left(\frac{M_I}{M_R} \underline{r}\right) \chi_{\text{He}^3}(\underline{r}) , \quad (6)$$

where the subscript on $\underline{r}$ has been dropped since only one radius parameter remains in the calculation of the matrix element. The quantity $V_0 \varphi_{\text{He}^3}(0)$

is expressed in the present calculation in terms of the parameters of the Hulthén wave function, which is assumed for φ_{He^3} , and the binding energy of the proton and deuteron in the He^3 nucleus.

Finally, the proton is assumed to be captured into a state of the residual nucleus having a well-defined orbital angular momentum.

The calculation used in the present study employed a harmonic oscillator function for the bound-state wave function, ξ_p . The wave functions χ_{He^3} and χ_d were calculated considering both the nuclear and Coulomb interactions of the He^3 ion with the initial nucleus and the deuteron with the residual nucleus. Complex potential wells (optical model) were used to describe the nuclear interactions. The parameters for the potential wells were obtained from optical-model fits to elastic-scattering data and are discussed in detail in section III C.

Recently a DW calculation which employed a finite range for the n-p interaction in the deuteron, corresponding to a finite range for V_{dp} in the present study, has been used to calculate cross sections for the (d,p) stripping reaction.²⁸ The predictions of the calculation corresponded more closely to experimental results than those of a zero-range calculation. However, it was found that a zero-range calculation, in which the integration over the interior of the nucleus is deleted, can simulate the predictions of the finite-range calculation. The effect of a finite range of interaction is to severely attenuate the contribution to the stripping process from the interior of the nucleus.²⁸ Therefore, an extensive search was made to determine a suitable value for the cutoff

radius in the calculation of the (He^3, d) stripping cross sections in the present study. The calculations employing a lower limit in (6) of 4.2 fermis, which give the best prediction of the structure of the experimentally obtained angular distributions, were used in the present study.

The comparisons of the predictions of the DW calculation to typical experimental angular distributions are shown in Figs. 12-15. The $\ell = 3$ predictions were arbitrarily normalized to the data at the maximum near 20° and $\ell = 1$ calculations at the maximum near 25° . One immediately notes that the structures of the distributions are predicted well for $\ell = 1$ and $\ell = 3$ orbital angular momentum transfers, thus confirming the assignments in section III A. The fits could probably have been improved in detail by varying the optical model parameters. However, such a variation of parameters is most often arbitrary and would have introduced uncertainties in the use of the calculation to extract spectroscopic information from the data.

DW calculations do not predict the correct magnitudes of the cross sections for stripping reactions involving incident particles more complex than the deuteron. The present calculation underestimated them by a factor of the order of 20. However, the utility of DW calculations is based on the correct prediction of the structure of the angular distributions and the Q-value dependence of the magnitudes of the cross sections. The success of DW calculations in the interpretation of a large amount of experimental data is justification of these essential features of the calculation.

Fig. 12. Comparison of the predictions of the DW calculation to the angular distributions for group 1 ($Q = 4.16 \text{ MeV}$) and group 2 ($Q = 1.06 \text{ MeV}$) of Fig. 7.

Fig. 13. Comparison of the predictions of the DW calculation to the angular distributions for group 5 ($Q = 1.910 \text{ MeV}$) and group 10 ($Q = 0.35 \text{ MeV}$) of Fig. 8.

Fig. 14. Comparison of the predictions of the DW calculation to the angular distributions for group 1 ($Q = 1.070$ MeV) and group 4 ($Q = -1.33$ MeV) of Fig. 9.

Fig. 15. Comparison of the predictions of the DW calculation to the angular distributions for group 1 ($Q = -0.436$ MeV) and group 2 ($Q = -2.63$ MeV) of Fig. 10.

Yntema and Satchler²⁹ have reported a dependence of the cross sections on the spin-orbit coupling of the captured proton, which is not normally included in a DW calculation. Fulmer and McCarthy have found it necessary to assume different normalization factors for different l values.³⁰ Both of these considerations were used to modify the predictions of the calculation used in the present study.

C. Optical-Model Parameters

Fortunately, during the progress of the present study a comprehensive optical-model analysis of deuteron elastic-scattering data was reported by Perey and Perey.⁴ The results of this analysis were quite easily extrapolated to furnish the optical-model parameters required for the DW calculations. However, relatively little experimental or theoretical information concerning the elastic scattering of He^3 ions from nuclei exists in the literature. Thus, a study of the elastic scattering of 22-MeV He^3 ions from Ca^{48} , V^{51} , Cr^{52} , and Fe^{54} was initiated.

The experimental results are presented in Table II. The relative cross-section errors include the estimated uncertainties in target thickness as seen by the He^3 beam, as well as the statistical errors. The data obtained for Ca^{40} and Fe^{56} were included since they were used to make the contaminant subtractions discussed in section II E.

The computer program used for the optical-model analysis of the data was furnished by Oak Ridge National Laboratory and has been reported in the literature by Perey.³¹ The form of the potential that was used in the analysis of the He^3 data was

TABLE II

Tabulation of the magnitudes and relative errors of the differential cross sections for 22-MeV He^3 ions elastically scattered from Ca^{40} , Ca^{48} , V^{51} , Cr^{52} , Fe^{54} , and Fe^{56}

$\text{Ca}^{40}(\text{He}^3, \text{He}^3)\text{Ca}^{40}$			$\text{Ca}^{48}(\text{He}^3, \text{He}^3)\text{Ca}^{48}$			$\text{V}^{51}(\text{He}^3, \text{He}^3)\text{V}^{51}$			$\text{Cr}^{52}(\text{He}^3, \text{He}^3)\text{Cr}^{52}$			$\text{Fe}^{54}(\text{He}^3, \text{He}^3)\text{Fe}^{54}$			$\text{Fe}^{56}(\text{He}^3, \text{He}^3)\text{Fe}^{56}$		
$\theta_{\text{c.m.}}$	$\sigma(\theta)$	Rel. error	$\theta_{\text{c.m.}}$	$\sigma(\theta)$	Rel. error	$\theta_{\text{c.m.}}$	$\sigma(\theta)$	Rel. error	$\theta_{\text{c.m.}}$	$\sigma(\theta)$	Rel. error	$\theta_{\text{c.m.}}$	$\sigma(\theta)$	Rel. error	$\theta_{\text{c.m.}}$	$\sigma(\theta)$	Rel. error
(deg)	(mb/sr)	(%)	(deg)	(mb/sr)	(%)	(deg)	(mb/sr)	(%)	(deg)	(mb/sr)	(%)	(deg)	(mb/sr)	(%)	(deg)	(mb/sr)	(%)
16.12	1.16×10^4	± 4	15.94	1.1×10^4	± 5	13.77	2.88×10^4	± 4	13.75	3.02×10^4	± 4	15.83	2.19×10^4	± 4	13.70	3.82×10^4	± 4
19.34	4.27×10^3		19.12	3.94×10^3		15.88	1.57		16.92	1.34		18.99	9.57×10^3		15.80	2.08	
22.56	1.72		22.30	1.45		19.05	6.40×10^3		19.03	7.62×10^3		22.15	4.25		18.96	9.49×10^3	
25.77	8.73×10^2		25.47	8.66×10^2		22.22	2.76		22.20	3.18		25.31	2.14		22.11	4.17	
28.97	5.48		28.64	5.30		25.39	1.43		25.36	1.52		28.46	1.21		25.26	2.16	
32.17	3.52		31.81	3.01		28.55	8.59×10^2		28.52	9.34×10^2		31.61	7.42×10^2		28.41	1.23	
35.37	1.84		34.97	1.70		31.70	5.05		31.67	5.53		34.75	4.23		31.55	7.54×10^2	
38.55	9.88×10^1		38.13	8.69×10^1		34.86	2.97		34.82	3.16		37.89	2.32		34.69	4.17	
41.73	6.28		41.28	6.20		38.00	1.63		37.96	1.70		41.02	1.42		37.82	2.54	
44.91	4.54		44.42	4.33		41.14	1.04		41.10	1.04		44.15	9.83×10^1		40.95	1.53	
48.07	3.33		47.56	3.01		44.28	7.13×10^1		44.24	7.30×10^1		47.27	7.05		44.08	1.04	
51.23	1.99		50.69	1.68		47.41	5.22		47.36	5.16		50.39	4.56		47.19	7.24×10^1	
54.38	1.02		53.81	1.10		50.53	3.46		50.48	3.78		53.50	3.08		50.31	4.77	
57.51	5.91×10^0		56.93	8.93×10^0		53.64	2.27		53.60	2.31		56.60	2.26		53.41	3.23	
60.64	5.25		60.04	8.35		56.76	1.60		56.70	1.68		59.70	1.66		56.51	2.26	
63.76	6.19		63.14	6.58		59.86	1.24		59.80	1.26		62.79	1.24		59.60	1.71	
66.87	5.63		66.23	4.45		62.95	1.00		62.89	9.79×10^0		65.87	8.86×10^0		62.69	1.34	
69.97	4.67		69.31	2.49		66.04	7.47×10^0		65.98	7.43		68.94	6.06		65.76	1.01	
73.06	2.82		72.38	1.33	± 7	69.11	5.03		69.05	4.96		71.90	3.97		68.83	6.94×10^0	
76.13	1.29		75.44	9.25×10^{-1}		72.18	3.17		72.12	3.33		75.06	3.25		71.90	4.67	
79.20	4.93×10^{-1}		78.50	9.16		75.24	2.07		75.18	2.33		78.11	2.56		74.95	3.42	
82.25	2.80		81.54	1.09×10^0		78.29	1.78		78.23	1.87		81.15	2.14		78.00	2.75	
85.29	4.18		84.58	1.10		81.33	1.62		81.27	1.57		84.18	1.79		81.03	2.34	
88.32	5.76		87.60	8.26×10^{-1}		84.37	1.39		84.30	1.29		87.20	1.43		84.06	1.97	
91.34	6.34		90.62	5.46		87.39	1.21		87.32	1.13		90.21	9.98×10^{-1}		87.09	1.59	
94.35	6.02					90.40	9.34×10^{-1}		90.34	8.08×10^{-1}		93.22	8.09		90.10	1.22	
97.34	4.53					93.41	6.36	± 5	93.34	5.57		96.21	6.04		93.10	8.72×10^{-1}	
100.32	3.19					96.40	4.33		96.34	4.23		99.20	5.28	± 6	96.10	6.56	
103.29	1.93					99.39	3.07		99.32	3.37		102.18	4.58		99.09	5.14	
106.25	1.16					102.36	2.57		102.30	2.70		105.15	3.85		102.06	4.36	
109.20	7.82×10^{-2}					105.33	2.26		105.27	2.33		108.11	3.09		105.03	3.73	
112.13	7.43					111.24	2.33	± 7	108.23	2.11	± 5	111.06	2.51		108.00	3.21	
115.06	7.37					114.18	1.93		111.18	1.88		114.00	2.04		110.95	2.79	
117.97	7.09	± 5				117.11	1.68		114.12	1.57		116.94	1.81		113.90	2.41	± 6
120.87	6.33					120.04	1.17		117.05	1.44		119.87	1.52		116.83	2.00	
123.76	6.02								119.98	1.10		122.79	1.47		119.76	1.51	
126.64	5.03								122.89	8.79×10^{-2}		125.70	1.21		122.69	1.31	
129.51	5.39								125.80	6.8	± 7	128.60	9.82×10^{-2}		125.60	1.15	
132.37	5.58								128.70	6.77		131.50	8.04		128.51	1.00	
135.23	4.86								131.60	5.34		134.39	6.76		131.41	8.32×10^{-2}	
138.07	4.41								134.48	5.27					134.30	7.27	
140.90	3.71								137.36	4.57					137.19	6.89	
															140.07	5.93	

$$V = -V_0 f(r, r_{OR}, a_R) - 1 W f(r, r_{OI}, a_I) + V_c(r_{oc}) ; \quad (7)$$

where V_0 specifies the depth of the potential well used to describe the target nucleus, W simulates the absorption of the He^3 ions by the target nucleus, and

$$f(r, r_0, a) = \left[1 + \exp \left(\frac{r - r_0 A^{\frac{1}{3}}}{a} \right) \right]^{-1} \quad (8)$$

is the Woods-Saxon form factor for the potential well. The subscripts R and I in (7) index, respectively, the potential parameters for the real and imaginary parts of the potential. V_c is the Coulomb potential for a point-charge He^3 ion and a uniformly-charged spherical target nucleus:

$$V_c = \begin{cases} \frac{Ze^2}{R_c} \left(3 - \frac{r^2}{R_c^2} \right), & \text{for } r \leq R_c; \\ \frac{Ze^2}{r}, & \text{for } r > R_c. \end{cases} \quad (9)$$

$R_c = r_{oc} A^{\frac{1}{3}}$, Z is the atomic number of the target nucleus, e is the electronic charge, and A is the mass number of the target nucleus.

The program executed an automatic search for those values of the parameters which minimized the quantity

$$\chi^2 = \sum_i \left[\frac{(\frac{d\sigma}{d\Omega})_{th} - (\frac{d\sigma}{d\Omega})_{exp}}{\Delta(\frac{d\sigma}{d\Omega})_{exp}} \right]^2_i \quad (10)$$

where the subscripts th and exp are abbreviations of theoretical and experimental, respectively, and $\Delta(\frac{d\sigma}{d\Omega})_{exp}$ is the relative cross-section error.

The comparisons of the calculated distributions to the data are shown in Fig. 16, and the associated potential parameters are listed in Table III. Previous analysis of the same data by R. H. Bassel⁵ had established preliminary values for the parameters; the values in the tables are not markedly different from those he had obtained.

TABLE III

Optical-model parameters obtained for He^3 ions elastically scattered from the listed nuclei. These parameters were used in the DW calculations.

Isotope	V_0 (MeV)	r_{OR} (fermis)	a_R (fermis)	W (MeV)	r_{OI} (fermis)	a_I (fermis)
Ca^{40}	100.5	1.076	0.822	7.57	1.945	0.704
Ca^{48}	100.2	1.061	0.830	9.15	1.856	0.705
V^{51}	99.6	1.069	0.795	10.8	1.775	0.711
Cr^{52}	98.8	1.069	0.814	13.5	1.705	0.726
Fe^{54}	96.8	1.069	0.873	16.5	1.705	0.726
Fe^{56}	95.6	1.069	0.821	13.6	1.705	0.736

Except for Cr^{52} , all the angular distributions are normalized to the calculated angular distributions in the forward-angle region. The magnitudes and shapes of the calculated distributions in this region were not sensitive to the values of the potential parameters so the observed differences of the magnitudes of the theoretical and experimental values were assumed to indicate errors in the absolute magnitudes of the

Fig. 16. Comparisons of the optical-model predictions to the He^3 elastic-scattering data. The optical-model parameters obtained from the calculations are listed in Table III.

experimental cross sections. The errors did not exceed the estimated uncertainties of the target areal densities (see section II C) except for V^{51} which was in error by 20%. The thick V^{51} target was the poorest quality target used in the study and the uncertainty in its areal density could easily have been underestimated. The cross sections listed in Table II and the cross sections for the (He^3, d) reactions are the normalized cross sections.

The fits to the data are quite good except for the back-angle data for Ca^{40} . No parameters were found for the potentials employed in the present calculation which would fit this data. R. H. Bassel has included a spin-orbit potential in his calculations in an attempt to fit the data and has found the fit is actually poorer.³² This inability to fit the data for Ca^{40} could be explained by the fact that Ca^{40} is a doubly-magic nucleus which implies a lower density of states in the compound system; thus, the optical-model assumption that only the average characteristics of the nucleus are important³¹ is less valid. Similar anomalies have been observed for optical-model analyses of the elastic scattering of particles from light nuclei which are also characterized by a low density of states in the compound system.

The DW program could be instructed to calculate the elastic-scattering cross sections for the particles in the entrance and exit channels by using the optical-model wave functions that it determined for each of these particles. These cross sections should be exactly the same as those determined by the optical-model analysis of the

elastic-scattering data, except that the DW calculation employed form factors for the complex potential wells which were approximations to the Woods-Saxon form factor and a different charge radius. The comparisons of the elastic-scattering cross sections predicted by the DW calculation to the data are shown in Fig. 17. The comparable quality of these fits to those obtained from the optical-model analysis of the data (see Fig. 16) is verification of the equivalence of the approximation.

The Perey and Perey optical-model analysis⁴ of deuteron elastic-scattering data used surface absorption instead of the volume absorption that proved satisfactory for the description of He³ elastic scattering. The complex potential they used differed from the one in (7) by only the form factor for W which in their case was

$$g(r, r_{oI}, a_I) = -4a_I (d/dr) \left[f(r, r_{oI}, a_I) \right], \quad (11)$$

where the symbols have the same meaning as before. Set B of their parameters was chosen arbitrarily for use in the present study, and for convenience is listed below:

$$\begin{aligned} r_{oR} &= r_{oc} = 1.15 \text{ fermis,} \\ a_R &= 0.81 \text{ fermis,} \\ r_{oI} &= 1.34 \text{ fermis,} \\ a_I &= 0.68 \text{ fermis,} \\ V_o &= 81 + 2 \left(Z/A^{1/3} \right) - 0.22E, \\ W &= 14.24 + 0.24E. \end{aligned}$$

The formulas for V and W are given in the text of the article. The value

Fig. 17. Comparison of the elastic-scattering angular distributions calculated by the DW calculation to the data for the elastic scattering of He^3 ions from Ca^{48} , V^{51} , Cr^{52} , and Fe^{54} . The quality of the fits displayed in this figure are to be compared to the optical-model fits displayed in Fig. 16.

of 14.24 used in the calculation of W was determined by using the value for W given in Appendix III of the article for 21.5-MeV deuterons elastically scattered from Ti^{48} . E is the deuteron energy.

D. Spectroscopic Factors

Although the l values for the captured protons are unambiguously determined by comparisons of the predictions of the DW calculation to the experimental data, the partial angular distributions obtained for the present study cannot be used to determine their total angular momentum, j . Thus, a proton captured with an orbital angular momentum of l may be associated with either the SP state having $j = l + \frac{1}{2}$ or the one having $j = l - \frac{1}{2}$. The fact that the SP state with the largest j lies lowest in excitation can be very useful in making the proper associations; but when there is a multiplicity of levels associated with each SP state, additional criteria are needed. Studies of neutron SP states indicate that there is a grouping of the levels associated with each SP state and the half-width of the distribution of the levels is approximately one-third of the excitation energy for the centroid of the levels.³³ Such groups are not always apparent from a qualitative examination of the data, especially for p states which have a small spin-orbit splitting. However, the combination of these criteria with the quantitative methods developed by Macfarlane and French¹ provides a reasonable, but not infallible, method of assigning the levels.

Many applications of the formalism are discussed in Ref. 1 and all assume that the predictions of the plane-wave Born-approximation

(PWBA) calculation of the stripping-reaction cross sections are valid. The application of the formalism using the predictions of a DW calculation is an obvious modification of the methods of Ref. 1.

The measured differential cross section is related to the DW cross section by

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}} = N \frac{2J_f + 1}{2J_o + 1} \left[C \left(T_o, \frac{1}{2}, T_f; T_{oz}, T_{fz} - T_{oz} \right) \right]^2 S \sigma(l, Q, \theta), \quad (12)$$

where o and f index the ground state of the target nucleus and the final state of the residual nucleus, respectively, N is the normalization factor, C the isobaric-spin Clebsch-Gordan coupling coefficient, S the spectroscopic (nuclear overlap) factor, and σ the DW-calculation prediction of the cross section. The total angular momentum and isobaric spin are denoted by J and T, respectively.

The isobaric-spin Clebsch-Gordan coefficient was included in Eq. (12) since a proton added to a neutron-rich nucleus in an orbit empty of neutrons forms one of two final states which differ in their isobaric spin.^{6,7} The C^2 factors simply determine the way in which the total cross section divides between the states of different T. The normalization factor N was included since the DW calculation does not predict the correct magnitude for the cross section. (See section II B.)

The spectroscopic factor S is the quantity of interest because it contains the information concerning the structure of the target and residual nuclei. It is a measure of the overlap of the ground-state

wave function of the target nucleus and the final-state wave function of the residual nucleus. The theoretical value for S depends on the particular model of nuclear structure that is assumed in its calculation, since different nuclear models imply that different basic sets of wave functions provide an adequate description of nuclear states. Comparisons of the theoretical and experimental values are useful in establishing the validity of any nuclear model; or conversely, assuming the validity of the model, such comparisons form the basis for interpreting experimental results.

The method of evaluating the overlap integrals is considered in Ref. 1, and will not be developed here in detail. Basically, the problem is to construct antisymmetrized wave functions for the initial and final states from a set of basic wave functions. The final-state wave function is expressed as a vector coupling of the wave function of the initial state and the wave function of the captured nucleon. After the wave functions are determined, the evaluation of the overlap integral is straightforward. The expressions that have been developed for the spectroscopic factors in Ref. 1 were used to predict the strengths used for the interpretation of the experimental results. For the purposes of the present study, C^2S is defined as the strength of the transition to a level of the residual nucleus. The ΣC^2S , where the sum includes the strengths for all levels associated with a given SP state, was taken to be the strength of the SP state. This definition of strength differs from the conventional one by the omission in the present case of the

statistical factor $(2J_f + 1)/(2J_o + 1)$. The interpretation of the data in terms of these strengths has the advantage of indicating the splitting of the total SP strength between the two isobaric-spin states that are formed when protons inequivalent (different n , ℓ , or j) to all nucleons in the target nucleus are added to the target nucleus.

Two categories of expressions for the strengths were used. One type was more easily applied to the prediction of strengths for the states formed from the target nucleus by adding a proton which was equivalent (same n , ℓ , j) to some of the nucleons in the target nucleus. The other type was most useful for predicting the strengths for states formed by adding an inequivalent proton. Both categories were based on the j - j coupling version of the nuclear shell model.

For equivalent protons the strength is determined by the overlap of the initial state $x_o J_o T_o$ with the final state $x_f J_f T_f$ and, for the case for which the corresponding neutron shell is filled, is given by

$$\left[C \left(T_o, \frac{1}{2}, T_o - \frac{1}{2}; T_{oz}, -\frac{1}{2} \right) \right]^2 S \left(n, x_f J_f T_o - \frac{1}{2}; n - 1, x_o J_o T_o \right) =$$

$$n < j^n : x_f J_f | j^{n-1} : x_o J_o >^2, \quad (13)$$

where x_f and x_o represent those quantum numbers other than angular momentum quantum numbers which may be needed to specify the state, n is the number of equivalent protons in the j shell, and $< >$ is the coefficient of fractional parentage that connects the initial and final states.^{35,36} The tables of the coefficients of fractional parentage published by Flowers and Edmonds for the j - j representation were used

in the present study.^{37,38} No isobaric-spin quantum numbers appear on the right side of Eq. (13) since only the $T_f = |T_0 - \frac{1}{2}| = T_{fz}$ state is allowed by the exclusion principle for the coupling of equivalent protons if the corresponding neutron shell is filled. The C^2 coefficient and the isobaric spin could be omitted from the left-hand side of Eq. (13) in this case, but could not be omitted from the expressions to be given later that are relevant for the transfer of an inequivalent proton. To avoid confusion, S has been consistently expressed in the isobaric-spin formalism in this paper.

The sum rules for spectroscopic factors that were developed by Macfarlane and French from the weak-coupling formalism of Lane, Thomas, and Wigner³⁹ are more appropriate for the transfer of an inequivalent proton. The weak-coupling formalism predicts a multiplicity of levels associated with each SP state in agreement with experimental observations. The multiplicity of levels is predicted by considering the coupling of the added nucleon not only to the ground state, but also to the low excited states of the parent nucleus. The possibility that other final states with the same spin and parity as the SP state exist is greatly enhanced. If one or more such levels are very close in excitation to the SP state, final-state interactions will then mix the configurations resulting in a fragmentation of the SP strength. This formalism modifies, but does not invalidate, the conclusions based on the j - j coupling model; i.e., the SP description of nuclear structure is still meaningful.

The sum rule that results from this formalism is $\sum_i S^i = 1$, where the sum extends over only those components of the SP state that are

characterized by unique values of T_f and J_f . Since the sum does not extend over different T_f values, the expression for the strength is obtained from this relation by multiplying both sides by C^2 which gives

$$\sum_i C^2 S_{J_f=J}^i = C^2, \quad (14)$$

$T_f=T$

where the isobaric-spin dependence of C and S and the xJ dependence of S is understood to be the same as for Eq. (13). Since $T_f = T_0 \pm \frac{1}{2}$ in the present study and since C^2 is a function of T_f , there were two values for the strength for each J_f value. For all the nuclei that were studied, $T_0 = (\frac{1}{2})(N - Z)$, where N and Z are the number of neutrons and protons, respectively, in the target nucleus.

The relation in (14) is directly applicable for the (He^3, d) reaction on even-even target nuclei for which $J_0 = 0$ and thus $J_f = j$. However, an inequivalent proton added to an odd-even target nucleus in a j SP state has available final states with a range of J values given by $|J_0 - j| \leq J_f \leq J_0 + j$. It was not possible to determine J_f for the states formed by the (He^3, d) reaction on V^{51} , the only odd-even target nucleus used in the present study. Therefore, the relation in Eq. (14) was not useful for this reaction since Eq. (14) assumes the knowledge of J_f . If one multiplies Eq. (14) by $(2J_f + 1)/(2J_0 + 1)$ and further sums the resulting expression over all possible values for J_f , the following expression results:

$$\sum_{J_f, i} \frac{2J_f + 1}{2J_0 + 1} C^2 S_{J_f}^i = (2j + 1)C^2. \quad (15)$$

The left-hand side of Eq. (15) is just the sum of all the values of $(d\sigma/d\Omega)_{\text{exp}}/\sigma(\ell, Q, \theta)$ [see Eq. (12)] corresponding to the capture of an inequivalent proton with angular momentum j .

The sum rules [Eqs. (13), (14), and (15)] formed the basis for interpreting the data. The experimental values for the sums were determined from Eq. (12) by using the predictions of the DW calculation and making tentative assignments of the levels to the various SP states. Reassignments were made when necessary if it was reasonable to do so.

For each isotope, the predicted values of $\sigma(\ell, Q, \theta)$ were determined at 1-MeV intervals for an appropriate range of Q values. The values for σ at a center-of-mass scattering angle of approximately 20° for $\ell = 3$ angular momentum transfers and at approximately 25° for $\ell = 1$ transfers were used to establish the Q dependence of the predicted cross sections. A typical Q -dependence plot is shown in Fig. 18. The curve drawn through the predicted points was visually determined to be the best representation of the Q dependence. All subsequent comparisons of the theoretical predictions to the data employed cross-section values read from this graph for Cr^{52} or from similar ones for the other isotopes. The discussion of the normalization factors that were required for the predictions of the DW calculation (see section II B) are included in the next section.

Fig. 18. Q dependence of the cross sections predicted by the DW calculation for the $\text{Cr}^{52}(\text{He}^3, d)\text{Mn}^{53}$ reaction.

III. DISCUSSION OF SPECIFIC REACTIONS

A. Ca⁴⁸(He³,d)Sc⁴⁹ Reaction

The nucleus Sc⁴⁹ has one proton more than the nucleus Ca⁴⁸, whose ground-state configuration has been known for some time to be doubly magic. Very little has been known about the level structure of Sc⁴⁹, however. From the study of the β decay of Ca⁴⁹, low-lying Sc⁴⁹ levels were observed at 3.1, 4.1, and 4.7 MeV of excitation, and it was suggested at that time that the ground state and the first two of these excited states might be interpreted rather simply as the $1f_{7/2}$, $2p_{3/2}$, and $1f_{5/2}$ SP states, respectively.^{40,41} More recently, however, the results of an experiment on the radiative capture of protons in Ca⁴⁸, in which several new Sc⁴⁹ levels were observed, has been reported.⁴² Two of these levels appear at 2.22 and 2.34 MeV, and it was suggested that the 2.34-MeV level was probably the $2p_{3/2}$ SP state.⁴² The results of the present study are consistent with the $1f_{7/2}$ SP assignment to the ground state, but show that the $2p_{3/2}$, $1f_{5/2}$, and $2p_{1/2}$ SP states are split over several energy levels and that no significant component of these states appears below 3 MeV of excitation.

Only those levels corresponding to the lower T value ($T_f = T_{fz} = 7/2$, $C^2 = 0.89$) components of the SP state were observed for the Ca⁴⁸(He³,d) reaction. The $T_f = 9/2$, $T_{fz} = 7/2$ components are the isobaric analogs of the levels formed by adding a neutron to Ca⁴⁸ and are of considerable interest. However, the lowest of these levels is

expected to occur at ~ 12 MeV of excitation, a region of excitation that was obscured by reactions on the Ca^{48} target backing material and contaminants in the target. Moreover, since only $1/9$ of the (He^3, d) strength for an SP state is contained in the $T_f = 9/2$ component, the weak isobaric state would undoubtedly have been obscured by the large number of $T_f = 7/2$ components of other SP states that one would expect at 12 MeV of excitation. For convenience in the following discussion, "single-particle state" is to be understood to mean the $T_f = T_{fz} = 7/2$ component of a SP state.

Since the $\text{Ca}^{48}(\text{He}^3, d)$ reaction does not appreciably excite any other state up to 3.10 MeV, the ground state must be a nearly pure $1f_{7/2}$ SP state. From Eq. (13) one has $C^2S = 1$ for this transition; thus, the normalization factor N in Eq. (12) was established. All other $\ell = 3$ transitions have been assigned as $1f_{5/2}$. Table IV lists these groups, their excitation energies, and their strengths based on this normalization. The sum of the $1f_{5/2}$ strengths, $\sum C^2S = 1.04$, compares favorably with the predicted strength of 0.89. In obtaining these experimental strengths, account has been taken of recent work which indicates that the cross section should depend upon the value of $\underline{\ell} \cdot \underline{s}$ of the captured proton; a dependence of $+22\%$ and -22% for $j = 7/2$ and $5/2$, respectively, was used.²⁹ Table IV shows that the $1f_{5/2}$ strength is distributed over many levels.

Table IV also shows that there are significant $\ell = 1$ transitions to many levels of Sc^{49} . Because one cannot distinguish $p_{3/2}$ from $p_{1/2}$ on the basis of the $\ell = 1$ angular distributions obtained for the present

TABLE IV

Results from present study of the $\text{Ca}^{48}(\text{He}^3, \text{d})\text{Sc}^{49}$ reaction

Single-particle state	Group number	Excitation energy (MeV)	C^2S (experimental)	$\Sigma\text{C}^2\text{S}$ (experimental)	$\Sigma\text{C}^2\text{S}$ (predicted)
$1f_{7/2}$	1	g.s.	1.00	1.00	1.00
	3	3.86	0.14		
	4	4.11	0.20		
	5	4.34	0.08		
$1f_{5/2}$	7	4.76	0.13	1.04	0.89
	8	5.09	0.34		
	9	5.39	0.15		
$2p_{3/2}$	2	3.10	0.61		
	6	4.51	0.28	0.89	0.89
	7	4.76	0.02		
	8	5.09	0.07		
$2p_{1/2}$ ^a	10	5.69	0.24		
	11	5.83	0.07		
	b	5.9	0.03	0.67	0.89
	b	6.3	0.05		
	b	6.6	0.09		
	b	6.9	0.10		

^aSome of the groups assigned to this state may actually be $2p_{3/2}$ transitions.

^bThis group represents a summation over approximately 300 keV of unresolved states. The major component of its angular distribution appears to be $\ell = 1$.

study, and since the splitting of the $2p_{3/2}$ and $2p_{1/2}$ SP states is expected to be only 2 MeV or so,⁴³ the assignment of each of these groups is less certain. The application of the sum rule Eq. (14) indicates that the $2p_{3/2}$ SP strength must be split between at least two levels. If one assumes that most of the important $2p_{3/2}$ and $2p_{1/2}$ transitions have been observed, then, guided by the sum rule, the criteria of paragraph one in section II D and the concurrent studies of the states of Cr^{52} , Mn^{53} , and Co^{55} , one obtains the assignments shown in Table IV. The normalization factor required is 70% of that used for the $l = 3$ transitions,^a and the $l \cdot s$ cross-section dependence has been taken as +10% and -10% for $j = 3/2$ and $j = 1/2$, respectively. If the assumption is incorrect, and substantial amounts of the $2p_{1/2}$ strength actually lie above 7 MeV, some of the groups now assigned to the $2p_{1/2}$ state in Table IV must be reassigned to the $2p_{3/2}$ state.

A very weak group was observed at 2.28 MeV. The angular distribution for this group is shown in Fig. 19 and as can be seen from the figure resembles the prediction of the DW calculation for an $l = 2$ transfer. This indicates, perhaps, a very small admixture of the $(1d_{3/2})^{-2}$ configuration in the Ca^{48} ground state. This group was poorly defined statistically but, nevertheless, appeared to be composed of at least two levels. Since the excitation energy for this group equals the mean energy of the levels at 2.22 and 2.34 MeV reported by Dubois and

^aSeparate normalizations for different l values have been found necessary in other applications of DW calculations when rather precise determinations of transition strengths are sought. See e.g., Ref. 30.

Fig. 19. Comparison of the prediction of the DW calculation for an $\ell = 2$ transfer to the angular distribution for the deuterons from the 2.28-MeV level of Sc^{49} .

Maripuu,⁴² it is interesting to conjecture that the $l = 2$ group does indeed correspond to these levels. Such a correspondence would not conflict with their spin assignment of $3/2$ to the 2.34-MeV level and $3/2$ or $5/2$ to the 2.22-MeV level.

The assignment of the levels to the $1f_{7/2}$ and $1f_{5/2}$ SP states is unambiguous since the spin-orbit splitting is large. The assignments to the $2p_{3/2}$ and $2p_{1/2}$ states are less certain and must be considered as being tentative. The assignment of the 3.1-MeV level to the $2p_{3/2}$ SP state and the 4.1-MeV level to the $1f_{5/2}$ SP state confirms the assignments of these levels from the results of the beta-decay studies,^{40,41} but with the modification that these levels represent only a portion of the total SP strength of their respective SP state.

B. $v^{51}(\text{He}^3, d)\text{Cr}^{52}$ Reaction

The nucleus Cr^{52} has four protons outside closed shells of 20 protons and 28 neutrons. The low excited states are expected to be simply described in terms of the various $(f_{7/2})^4$ configurations. From such configurations one would expect to observe the following states: a 0^+ ground state with seniority $v = 0$ (no unpaired protons), 2^+ , 4^+ , and 6^+ excited states with $v = 2$ (2 unpaired protons), and 2^+ , 4^+ , 5^+ , and 8^+ excited states with $v = 4$ (4 unpaired protons). These levels have been studied extensively both theoretically^{44,45} and experimentally.^{46,47,48} $\text{Cr}^{52}(n, n'\gamma)$ reaction studies⁴⁶ and studies⁴⁷ of the γ decay following the beta decay of Mn^{52} have been used to assign spins and parities of 2^+ , 4^+ , 4^+ , and 6^+ to the levels at 1.434, 2.370, 2.766,

and 3.112 MeV, respectively. The results of the present study confirm these assignments and indicate that there is a considerable amount of seniority mixing for the 4^+ states. The multiplicity of levels associated with each SP state is larger for Cr^{52} than Sc^{48} because there is an additional J splitting of the SP strength. Also, the $2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$ SP states lie higher in excitation where the states of other configurations but with the spins and parities of these SP states are more numerous. Final-state interactions mix the SP configuration with the other configurations, resulting in a fragmentation of the SP strength. No positive assignment of the $l = 1$ groups to the $2p_{3/2}$ and $2p_{1/2}$ SP states was possible. The components of the $1f_{5/2}$ SP state are above 6.5 MeV of excitation and were not resolved by the techniques of the present study; however, some gross-structure analysis of the data was done which gives an indication of the excitation energy for the major portion of the $1f_{5/2}$ SP strength.

Only the stronger $T(T_f = T_{fz} = 2, C^2 = 0.83)$ component of each SP state was observed. The $T_f = 3, T_{fz} = 2 (C^2 = 0.17)$ components of the $2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$ SP states in Cr^{52} lie above 11 MeV of excitation and were obscured by the near continuum of the $T_f = T_{fz} = 2$ components of other SP states.

Since the DW calculation did not predict the correct magnitude for the cross section (see section II B), the most unambiguous analysis of the data resulted when the normalization factor for the DW calculation was determined for each isotope from the assumption that the ground state

was a pure $1f_{7/2}$ configuration. On this basis, Eq. (13) gives $C^2S = 4$ for the ground state transition of the $V^{51}(\text{He}^3, d)$ reaction; thus, the proper normalization factor for Eq. (12) was determined. The strengths for the $1f_{7/2}$ transitions listed in Table V are based on this normalization.

TABLE V

Results for the $1f_{7/2}$ SP state from the present study of the
 $V^{51}(\text{He}^3, d)\text{Cr}^{52}$ reaction

Single- particle state	Group number	Excitation energy (MeV)	C^2S (experimental)	ΣC^2S (experimental)	ΣC^2S (predicted)
	1	g.s.	4.00	4.00	4.00
	2	1.434	1.06	1.06	1.33
$1f_{7/2}$	3	2.370	0.49	1.27	1.33
	4	2.766	0.78		
	5	3.112	1.26	1.26	1.33

The correspondence of the excitation energies for groups 2, 3, 4, and 5 to those reported for the 2^+ , 4^+ , 4^+ , and 6^+ levels, respectively, was used to assign the spins and parities of these groups. The experimental values for the strengths of these groups--1.06 for the 2^+ level, 1.27 for the summed strength for the 4^+ levels, and 1.26 for the 6^+ level--are in reasonable agreement with the value of 1.33 predicted by Eq. (13), thus confirming the spin assignments. However, the strength for the 2^+ level is 20% smaller than predicted indicating that like the 4^+ component the

strength of the 2^+ component could be divided between two or more levels. The only candidates for another 2^+ level are groups 6, 7, and 9. Unfortunately, the interference of deuterons from the (He^3, d) reaction on an unknown contaminant in the target made it impossible to obtain unambiguous angular distributions for these deuteron groups.

The $1f_{5/2}$ SP strength was distributed over a large number of levels and was located in a region of excitation in which single levels or groups of 2 or 3 levels could not be resolved. In addition, some of the levels in this region were components of the $2p_{3/2}$ or $2p_{1/2}$ SP states. Groups 19 through 26 represent sums over approximately 300 keV of these unresolved levels. The angular distributions for these deuteron groups were composites of $\ell = 1$ and $\ell = 3$ angular distributions. The method of section II A was used to decompose each angular distribution into its components. The strengths for the $1f_{5/2}$ components are listed in Table VI and were determined using the same $\ell \cdot s$ dependence for the cross section that was used for the $\text{Ca}^{48}(\text{He}^3, d)$ reaction. The excitation energies listed for these groups of unresolved levels are the mean energies. The comparison of the experimental and theoretical strengths shows that not all of the $1f_{5/2}$ strength was observed. The procedure of summing groups of levels could not be extended to higher excitation because of the interference of deuterons from the (He^3, d) reaction on the C^{12} contaminant in the target.

To illustrate the difficulty encountered in attempts to assign the $\ell = 1$ groups to SP states, a very crude assignment of these groups

TABLE VI

Results for the $1f_{5/2}$, $2p_{3/2}$, and $2p_{1/2}$ SP states from the present study of the $V^{51}(\text{He}^3, d)\text{Cr}^{52}$ reaction. The assignments of the groups to the $2p_{3/2}$ and $2p_{1/2}$ SP states are for illustrative purposes only.

Single-particle state	Group number	Excitation energy (MeV)	$\frac{2J_f + 1}{2J_o + 1} C^2S$ (experimental)	$\sum \frac{2J_f + 1}{2J_o + 1}$ (experimental)	$\sum \frac{2J_f + 1}{2J_o + 1} C^2S$ (predicted)
$1f_{5/2}^a$	19	6.8	0.06	3.37	5.00
	20	7.1	0.38		
	21	7.4	0.74		
	22	7.6	0.36		
	23	7.9	0.44		
	24	8.2	0.50		
	25	8.4	0.47		
$2p_{3/2}$	26	8.7	0.42	3.05	3.33
	8	3.78	0.13		
	10	4.67	0.54		
	11	5.09	0.37		
	12	5.42	0.40		
	13	5.57	0.21		
	14	5.77	0.15		
	15	5.96	0.16		
	16	6.18	0.27		
	17	6.34	0.21		
	18	6.60	0.09		
$2p_{1/2}^a$	19 ^a	6.8	0.25	1.68	1.67
	20 ^a	7.1	0.27		
	21	7.4	0.50		
	22	7.6	0.27		
	23	7.9	0.25		
	24	8.2	0.30		
	25	8.4	0.21		
	26	8.7	0.15		

^aThese groups represent summations over approximately 300 keV of unresolved levels.

to the $2p_{3/2}$ and $2p_{1/2}$ SP states is given in Table VI. The same l and $l \cdot s$ dependence of the normalization factor used for the $\text{Ca}^{48}(\text{He}^3, d)$ reaction was used for this reaction. The agreement of the experimental and theoretical strengths is very good. However, the centroids for the $2p_{3/2}$ and $2p_{1/2}$ SP strengths are approximately 5.5 and 7.5 MeV, respectively, and the rule that the half-width of the distribution of the levels is one-third the excitation energy for the centroid indicates that in the region of 5 to 7.3 MeV of excitation the components of the $2p_{3/2}$ and $2p_{1/2}$ SP states are intermingled. The methods of the present study cannot be used to separate them. To make meaningful assignments of the $l = 1$ levels to these SP states, an experiment that is capable of determining the j of the captured proton will be necessary.

A further interpretation of the strengths of the $1f_{7/2}$ transitions is possible in terms of the seniorities of the 2^+ and 4^+ states. As was indicated in the introduction to this section, states of the $(f_{7/2})^4$ configuration with $\nu = 2$ and $\nu = 4$ are allowed for both the 2^+ and 4^+ states of Cr^{52} . Talmi⁴⁴ predicts that the $\nu = 4, 4^+$ state lies lower in excitation than the $\nu = 2, 4^+$ state, but concludes that the experimental information^{46,47} indicates that the seniorities of these two states may be mixed. The uncertainties in his method of estimating the magnitude of the mixing and the fact that no $4^+ \rightarrow 4^+ \gamma$ transition was observed in the work of Wilson et al.⁴⁷ prevented him from drawing any definite conclusions concerning the magnitude of the seniority mixing. Komoda⁴⁵ has pointed out the fact that two states with different seniority do not

interact directly but mix through intermediate states of other configurations. Considering all possible configurations of 4 protons in $1f_{7/2}$, $1f_{5/2}$, $2p_{3/2}$, and $2p_{1/2}$ SP states, Komoda calculated the expected mixing on the basis that the only important interactions were two-body interactions of either the Serber type or Rosenfeld type. The lower 4^+ state is predicted to be primarily $\nu = 2$ for a Rosenfeld interaction and primarily $\nu = 4$ for a Serber interaction. The $V^{51}(\text{He}^3, d)\text{Cr}^{52}$ reaction can be used to determine the mixing since the ground state of V^{51} is a unique $\nu = 1$ state and the seniority selection rule for this reaction is $\Delta\nu = 1$. Thus, one can excite only those $(1f_{7/2})^4$ states of Cr^{52} with either $\nu = 0$ or $\nu = 2$. Since both 4^+ states are observed in this study, it is clear that the strength for each of these states is a measure of its $\nu = 2$ component. The experimental ratio of the strength of the $\nu = 2$ component of the lower 4^+ state to that for the higher state is 0.62 and agrees well with Komoda's value of 0.63 for a range parameter of 1.1 for the Serber interaction. However, Komoda predicts that as a result of the configuration interaction approximately 20% of the $\nu = 2$ strength should be found in other 4^+ states. A comparison of the experimental and theoretical values of the summed strengths (C^2S values in Table V) does not indicate such a reduction in the strength of the two 4^+ states. The summed strength could be in error as much as $\pm 10\%$ which could account for some disagreement. Also, to the extent that the configuration responsible for the mixing is $[(1f_{7/2})^3_{J=7/2}(1f_{5/2})]_{J=4}$, the strength would be in error by an amount corresponding to the amplitude

of this component in the wave functions of the 4^+ states, since the transfer of a $1f_{7/2}$ proton cannot be distinguished from the transfer of a $1f_{5/2}$ proton by the methods of the present study. Komoda has not, as of the present time, published the amplitudes for each component of the wave functions of the 4^+ states, so no definite conclusions can be reached concerning the magnitude of this error. Neither of these considerations would affect the ratio obtained for the $\nu = 2$ components seriously; therefore, the agreement with Komoda's predictions can be considered as being good.

Seniority mixing of the 2^+ states is also possible. Wilson et al.⁴⁷ have reported a level at 2.965 MeV which they tentatively identify as the $\nu = 4, 2^+$ state. This level is not observed for the $V^{51}(\text{He}^3, d)$ reaction which is in agreement with Komoda's prediction that less than 0.1% of the strength of the $\nu = 2, 2^+$ state is mixed with the $\nu = 4, 2^+$ state; this prediction is based on the assumption of a Serber type interaction with a range parameter of 1.1. Komoda also predicts that approximately 23% of the $\nu = 2, J^\pi = 2^+$ strength is contained in other 2^+ states which are not predominantly $(f_{7/2})^4$ configurations, in good agreement with the difference in the experimental and theoretical values of C^2S in Table V of 20%. Van Patter et al.⁴⁸ have argued that the 2.965-MeV level is a collective state and that the $\nu = 4, 2^+$ level lies higher in excitation. The results of the present study cannot answer this question.

A level at 3.80 MeV has been tentatively identified as a 5^+ or 6^+ level.^{46, 47} If the 3.78-MeV level of the present study corresponds to

this 3.80-MeV level, the $l = 1$ angular distribution for the deuterons from this level would prohibit the 6^+ assignment and would be compatible with the 5^+ assignment.

C. $\text{Cr}^{52}(\text{He}^3, d)\text{Mn}^{53}$ Reaction

Very little is known about the levels of Mn^{53} from previous work. The ground state spin and parity have been established as $7/2^-$, the spin and parity of the 0.383-MeV level are in doubt, and no spin assignments have been made for the 1.298- and 2.275-MeV levels.⁵¹ Of these levels, that at 2.275 MeV was not observed in the present study. The 0.383-MeV level was very weakly excited and no new information about its properties results from the present study; however, the probable spins and parities of the 1.298-MeV level and of several additional levels were determined. For the same reasons as for the $\text{Cr}^{52}(\text{He}^3, d)$ reaction, only the $T_f = T_{fz} = 3/2$ components of the SP states were observed.

The assignments of the observed levels to SP states, their excitation energies, and their strengths are listed in Table VII. The strengths are based on normalization factors for the DW calculation determined by the methods of the two preceding sections. The agreement of the theoretical and experimental values of the strengths for the $1f_{5/2}$ and $2p_{3/2}$ SP states is good. The comparison of the strengths for the $2p_{1/2}$ SP state indicates that not all of the components of this state were observed. The centroid for the $2p_{3/2}$ strength is at 2.49 MeV and for the $2p_{1/2}$ at approximately 5 MeV. The rule for determining the half-width of the distribution of the levels (see section II D) indicates

TABLE VII

Results from the present study of the $\text{Cr}^{52}(\text{He}^3, \text{d})\text{Mn}^{53}$ reaction

Single-particle state	Group number	Excitation energy (MeV)	C^2S (experimental)	$\Sigma\text{C}^2\text{S}$ (experimental)	$\Sigma\text{C}^2\text{S}$ (predicted)
$1f_{7/2}$	1	g.s.	0.500	0.50	0.50
	7	3.10	0.04		
	9	3.67	0.38		
	11	4.06	0.16	0.82	0.80
	12	4.29	0.06		
$1f_{5/2}$	14	4.57	0.04		
	16	4.96	0.14		
	3	1.29	0.07		
	4	2.40	0.46		
	5	2.66	0.10		
	6	2.88	0.02	0.75	0.80
	7	3.10	1.01		
	8	3.47	0.07		
$2p_{3/2}$	10	3.90	0.02		
	11	4.06	0.02		
	13	4.43	0.14		
	14	4.57	0.04	0.51	0.80
	15	4.74	0.18		
	16	4.96	0.06		
	17	5.08	0.07		

that there is very little overlapping of the $2p_{3/2}$ and $2p_{1/2}$ levels in the Mn^{53} nucleus. The assignments of group 8 and 10 might be questionable, but these groups are weak and would not appreciably affect the conclusions as to the locations of the centroids of the $2p_{3/2}$ and $2p_{1/2}$ SP states.

D. $Fe^{54}(He^3, d)Co^{55}$ Reaction

At the time the present study was initiated, no information concerning the excited levels of Co^{55} had been reported in the literature. However, during the progress of this study, three levels at 1.8, 2.2, and 2.45 MeV (all ± 0.2 MeV) were reported by Kumabe *et al.*⁴⁹ from a study of the $Ni^{58}(p, \alpha)Co^{55}$ reaction. Also, preliminary results of the study of the $Fe^{54}(d, n)Co^{55}$ reaction were reported indicating levels at 2.15, 2.55, 2.92, 3.55, and 4.15 MeV (all ± 0.05 MeV).⁵⁰ On the basis of the results of the study of the $Fe^{54}(d, n)$ reaction, the captured proton was definitely assigned a value of $l = 1$ for both the 2.15- and 2.55-MeV levels and tentative l values of 1, 3, and 1 for the 2.92-, 3.55-, and 4.15-MeV levels, respectively.⁵⁰ The level at 1.8 MeV was not excited by either the (d, n) reaction or by the (He^3, d) reaction of the present study. The results of the present study confirm the l assignments for the 2.15-, 2.55-, and 2.92-MeV states and indicate that the 4.15-MeV state is an unresolved group of levels. Several other levels were observed and were assigned probable spins and parities.

Once again the normalization for the DW calculation was established by using the procedures discussed in sections III A and III B. The

excitation energies, the assignments to the various SP states, and the strengths for the observed levels are listed in Table VIII. Since both the $T_f = 1/2$ and $T_f = 3/2$ components of the $2p_{3/2}$ SP state were observed, the SP states are indexed as to the total isotopic spin associated with each component; T_{fz} is $1/2$ for both components. Comparisons of the theoretical and experimental strengths show that the agreement is good. Further, the distribution of the levels within each SP state is in accord with the rule for estimating the width of the distribution (see section II D). Except for the 3.55-MeV level, the excitation energies obtained for the levels observed in the study of the $Fe^{54}(d,n)$ reaction are in agreement to within experimental error with those obtained in the present study. The 3.55-MeV level may correspond to the 3.67-MeV level (group 6 in Table VIII), but the correspondence could not be definitely established, especially since the 3.55-MeV level has been, tentatively, reported to be an $\ell = 3$ level and in this study the 3.67 level was found to be an $\ell = 1$ level.

Of the several other levels that were excited by the $Fe^{54}(He^3,d)$ reaction, the most interesting are those that are the isobaric analogs of the low-lying Fe^{55} levels. The difficulty experienced in the study of these levels was their proper identification. The excitation energies for the isobaric states can be calculated theoretically. However, the simpler and more reliable method of determining the excitation energy for the isobaric analog of the Fe^{55} ground state was from the difference in the Q value for the $Fe^{55}(p,n)Co^{55}$ reaction leading to this state and

TABLE VIII

Results from the present study of the $\text{Fe}^{54}(\text{He}^3, \text{d})\text{Co}^{55}$ reaction

Single-particle state	Group number	Excitation energy (MeV)	C^2S (experimental)	$\Sigma\text{C}^2\text{S}$ (experimental)	$\Sigma\text{C}^2\text{S}$ (predicted)
$(1f_{7/2})_{T_f=1/2}$	1	g.s.	0.25	0.25	0.25
$(1f_{5/2})_{T_f=1/2}$	5	3.34	0.53		
	8	4.18	0.21	0.74	0.67
$(2p_{3/2})_{T_f=1/2}$	2	2.19	0.37		
	3	2.59	0.22	0.68	0.67
	4	2.98	0.09		
$(2p_{1/2})_{T_f=1/2}$	5	3.34	0.27		
	6	3.67	0.10		
	7	3.92	0.05	0.72	0.67
	8	4.18	0.21		
	11	5.53	0.09		
$(2p_{3/2})_{T_f=3/2}$	9	4.71	0.15		
	10	5.14	0.09	0.23	0.33
a	9	4.71			
a	12	5.72			
a	13	6.01			

^aThe proper assignment of this state may be to either the $1f_{5/2}$ or $1g_{9/2}$ SP state. See the discussion in the text.

the Q value for the same reaction leading to the ground state of Co^{55} . The (p,n) reaction Q value for the transition to the isobaric analog of the Fe^{55} ground state was calculated by using the data reported by Anderson, Wong, and McClure.⁵¹ The (p,n) reaction Q value for the transition to the Co^{55} ground state was taken from the Nuclear Data Tables.²³ From the difference of these two Q values, the excitation energy for the isobaric analog of the Fe^{55} ground state in Co^{55} was estimated to be 4.66 MeV. Such estimates are usually within ± 100 keV of the correct value; therefore, group 9 at 4.71 MeV of excitation undoubtedly corresponds to the isobaric ground state. With this assumption, one would then expect to correlate by spin, strength, and energy spacing some of the Co^{55} states higher in excitation than 4.71 MeV with the low-lying states of Fe^{55} . For the purposes of such correlations, the pertinent properties of all the low-lying excited states of Fe^{55} are tabulated in Table IX. Table IX is based on the data obtained by Fulmer and McCarthy from the study of the $\text{Fe}^{54}(\text{d},\text{p})\text{Fe}^{55}$ reaction.³⁰

Unfortunately, in the present experiment group 9 exhibited a composite distribution typical for either $\ell = 1$ and $\ell = 3$ or $\ell = 1$ and $\ell = 4$ angular-momentum transfers. The strength of the $\ell = 1$ component was extracted using both types of mixtures establishing to within an error of $\pm 20\%$ the strength listed in Table VIII. Since no other $\ell = 4$ angular distribution was available for use in the analysis, such a distribution was obtained from the study of the $\text{Ni}^{58}(\text{He},\text{d})\text{Cu}^{59}$ reaction.³² Since $\ell = 3$ and $\ell = 4$ angular distributions for the deuterons

corresponding to the excitation of a state of the residual nucleus for which the proton is only weakly bound are very similar, the analysis was equally good for both types of mixtures and no choice of one over the other was possible.

TABLE IX

The properties of some of the Fe^{55} excited states. This summary is based on a study of the $\text{Fe}^{54}(\text{d},\text{p})\text{Fe}^{55}$ reaction by Fulmer and McCarthy.³⁰

Excitation energy (MeV)	Probable SP state	S	(1/3)S
g.s.	$2p_{3/2}$	0.81	0.27
0.417	$2p_{3/2}$	0.30	0.10
0.935	$1f_{5/2}$	0.60	0.20
1.327	$1f_{5/2}$	0.06	0.02
1.413	$(1f_{5/2})$	0.02	0.01
1.926	$2p_{1/2}$	0.13	0.04
2.063	$2p_{1/2}$	0.19	0.06

Since $C^2 = 1/3$ for the transitions to the isobaric levels, it is clear that the strengths obtained for the isobaric states from the present study of the $\text{Fe}^{54}(\text{He}^3, \text{d})$ reaction should be equal to one-third of the S value listed in Table IX. The value of $(1/3)S = 0.27$ for the ground state of Fe^{55} is probably in satisfactory agreement with the value of $C^2S = 0.15$ obtained for the $l = 1$ component of group 9. Similarly,

group 10 can be established as the isobaric analog of the 0.417-MeV Fe^{55} level: both levels are characterized by $\ell = 1$, the 0.43-MeV energy spacing of groups 9 and 10 compares favorably with the spacing between the ground state and 0.417-MeV level of Fe^{55} , and the value of $(1/3)S = 0.10$ for the 0.417-MeV level agrees well with the value of $C^2S = 0.09$ for group 10.

The uncertainty in the ℓ assignment for the second component of group 9 complicates the identification of other isobaric levels in Co^{55} . Furthermore, groups 12 and 13 are unbound, and there is no DW calculation that is capable of predicting the angular distribution for the deuterons corresponding to the excitation of unbound levels. However, from comparison to bound-state distributions, one is inclined to believe that the major component of groups 12 and 13 is $\ell = 3$ or 4. Thus, from the correlation in energy spacing and probable correlation in spin, group 12 could be identified as the isobaric analog of the 0.935-MeV level of Fe^{55} . An extrapolation of the predictions of the DW calculation then gives a value of $C^2S = 0.27$ which is to be compared to the value of $(1/3)S = 0.20$ for the 0.935-MeV level of Fe^{55} . Considering the uncertainties in the method of determining the C^2S value for group 12, the agreement is reasonable. Assuming this assignment of group 12 to be correct, one would then assign group 13 and the second component of group 9 to the $T_f = 1/2$ component of the $1g_{9/2}$ SP state. Since no other $\ell = 4$ levels were observed in the present study, the proper normalization factor for the DW calculation for such levels could not be determined and no realistic estimate of the strengths for groups 9 and

13 was possible. However, on the basis of the shell model, $1g_{9/2}$ levels are expected in the region of excitation for groups 9 and 13.

Additionally, Fulmer et al.⁵² have pointed out that any splitting of the $1g_{9/2}$ SP strength in this region of the periodic table must arise through the configuration interaction of particles in odd-parity SP states coupled to the 3^- level of the parent nucleus. Such splitting will be observed if the 3^- level occurs at an excitation less than the excitation observed for the $1g_{9/2}$ levels, and such splitting is expected to result in a rather small number of fragments. A level at 4.85 MeV in Fe^{54} has been identified by Bellicard and Barreau⁵² as a 3^- level. Thus, a splitting of the $1g_{9/2}$ strength is expected, and groups 9 and 13 could contain the major portion of the $1g_{9/2}$ strength since both these groups are strongly excited.

The alternative to the assignments just discussed for groups 12 and 13 and the second component of group 9 is to assign this portion of group 9 as a $T_f = 3/2$ component of the $1f_{5/2}$ SP state and groups 12 and 13 as the $T_f = 1/2$ component of the $1g_{9/2}$ SP state. On such a basis, the C^2S value for the $l = 3$ component of group 9 is 0.33, which would imply that nearly all the strength of the $T_f = 3/2$ component of the $1f_{5/2}$ SP state was contained in this level. Further, the correspondence of the Fe^{55} levels to their isobaric analogs in Co^{55} would be destroyed except for group 10 and the $l = 1$ component of group 9. That there should be a correspondence of these states appears to have been established by Lee et al.⁸ in their study of those Cu^{65} levels which are the

isobaric analogs of the low-lying Ni^{65} levels. Therefore, the first assignments of groups 9, 12, and 13 are probably correct, although an anomalous behavior of the isobaric levels in Co^{55} cannot be ruled out.

IV. SUMMARY

The strengths and SP assignments of the previous section were used to calculate the centroid for the levels associated with the $T_f = T_z$ component of each SP state and the $T_f = T_z + 1$ component of the $2p_{3/2}$ SP state in Fe^{54} . The excitation energies for the centroids of these components of the SP states are summarized in Table X. Since a

TABLE X

Location of the centroid for each proton SP state that was determined in the present study. The location of the centroids for the similar neutron SP states in Ca^{41} are included for the comparisons that are discussed in the text.

Single-particle state	Excitation energy for the centroids of the SP states (MeV)				
	Sc ⁴⁹	Cr ⁵²	Mn ⁵³	Co ⁵⁵	Ca ⁴¹
$(1f_{7/2})_{T_f=T_z}$	0	1.15	0	0	0 ^a
$(2p_{3/2})_{T_f=T_z}$	3.55	5.6	2.49	2.43	2.08 ^a
$(2p_{1/2})_{T_f=T_z}$	5.9	7.8	4.7	3.96	4.12 ^a
$(1f_{5/2})_{T_f=T_z}$	4.70	7.9	4.01	3.58	5.50 ^a
$(2p_{3/2})_{T_f=T_z+1}$	-	-	-	4.87	-

^aThis is the excitation energy for the centroid for this neutron SP state in Ca^{41} . This value was reported by Belote et al.⁵⁴ from their study of the $Ca^{40}(d,p)Ca^{41}$ reaction.

comparison of the proton SP states for Sc^{49} to the neutron SP states for Ca^{41} is of interest, the results of the study of the $\text{Ca}^{40}(\text{d},\text{p})\text{Ca}^{41}$ reaction by Belote et al.⁵⁴ are also included in Table X.

The comparison of the results for Sc^{49} to those for Ca^{41} is complicated by the fact that only the $T_f = 7/2$ component of the $1f_{5/2}$, $2p_{3/2}$, and $2p_{1/2}$ proton SP states were observed for Sc^{49} . Thus, the true centroids for the unperturbed SP states which are desired for use in the comparisons are not those given in Table X, but rather are the centroids determined by including the $T_f = 9/2$ components of the SP states. Since the $T_f = 9/2$ levels in Sc^{49} are the isobaric analogs of the Ca^{49} levels, an estimate of the excitation energies for these components of the SP states in Sc^{49} was obtained by using the methods discussed in section III D and the results reported for the study of the $\text{Ca}^{48}(\text{d},\text{p})\text{Ca}^{49}$ reaction.⁵⁴ From such an analysis the $T_f = 9/2$ components of the $2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$ SP states are at excitations of approximately 11, 13, and 15 MeV, respectively, in Sc^{49} . If these values for the excitation energies together with the predicted strengths for these components are included in the calculation of the centroids, the result is that the centroids of the unperturbed $2p_{3/2}$, $1f_{5/2}$, and $2p_{1/2}$ SP states are at excitations of approximately 4.4, 5.7, and 6.9 MeV, respectively.

The spin-orbit splitting of 5.7 MeV for the $1f_{7/2}$ and $1f_{5/2}$ proton SP states in Sc^{49} compares closely to the value of 5.50 MeV for the similar neutron SP states in Ca^{41} . Similarly, the spin-orbit splitting

of the $2p_{3/2}$ and $2p_{1/2}$ proton (2.5 MeV) and neutron (2.04 MeV) SP states is very nearly the same for these nuclei. In this respect, there appears to be little difference between the proton and neutron SP structure. However, one also notes that the sequence of the $2p_{1/2}$ and $1f_{5/2}$ is interchanged for the proton SP states. Such an interchange can be understood on the basis of the self-binding effect which has been discussed in the literature for neutron SP states. The self-binding effect arises from the extra-strong attraction between particles of the same orbital angular momentum because their radial wave functions overlap perfectly.⁴³ This perfect overlap is independent of the value for j . Therefore, the $1f_{7/2}$ and $1f_{5/2}$ proton SP states in Sc^{49} would be expected to be lowered in excitation with respect to the $2p_{3/2}$ and $2p_{1/2}$ proton SP states since the $1f_{7/2}$ neutron SP state is filled for Sc^{49} . Comparison of the relative positions of the centroids of the $1f$ and $2p$ proton SP states in Sc^{49} to the neutron SP states in Ca^{41} indicates a lowering of the $1f$ states of approximately 2.5 MeV due to the self-binding effect. The more pertinent comparison would be of these states in Sc^{49} to the same states in Sc^{41} . However, the information that exists in the literature for the Sc^{41} levels is not sufficiently detailed to be very useful. An $\ell = 1$ group of levels at 1.69 MeV in Sc^{41} has been reported from a study of the $Ca^{40}(He^3, d)Sc^{41}$ reaction⁹ which is probably the major fragment of the $2p_{3/2}$ proton SP state. From such an assumption, the comparison of the separation of the $1f_{5/2}$ and $2p_{3/2}$ SP states in Sc^{49} and Sc^{41} then gives a rough estimate of 1.8 MeV for the

magnitude of the self-binding effect, roughly in agreement with the previous value. The similar effect for the $1f_{5/2}$ neutron SP state as the $1f_{7/2}$ proton SP state fills has been discussed by Cohen.⁴³

Cohen found that the self-binding effect is evidenced but is much smaller for the $1f_{5/2}$ neutron SP state when the $1f_{7/2}$ neutron (rather than proton) SP state is filling. If there were a perfect correspondence of isobaric states of Co^{55} and the Fe^{55} states, the centroids for the unperturbed $2p_{3/2}$, $1f_{5/2}$, and $2p_{1/2}$ proton SP states in Co^{55} would be at 2.85, 4.15, and 4.87 MeV, respectively. Comparisons of the centroids for the $1f$ and $2p$ proton states in both Sc^{49} and Co^{55} indicate that for both isotopes the $1f_{5/2}$ state is approximately 0.6 MeV above the $2p$ centroid. Thus, the results of the present study indicate that the self-binding effect is negligible for proton-proton interactions.

The similarities of the neutron and proton SP structures are, in general, good. The methods that have been used to extract information concerning the neutron SP structure of nuclei from studies of the (d,p) reaction also proved to be satisfactory in the present study of the proton SP structure of nuclei using the (He^3,d) reaction. The extension of studies of the (He^3,d) reaction on nuclei in other regions of the periodic table should insure that in a few years fully as much will be known about the proton SP structure of nuclei as is presently known about the neutron SP structure.

The study of the isobaric states of nuclei is possible through use of the (He^3,d) reaction but will probably be restricted to nuclei

with states of small T , since only for these nuclei will the isobaric levels be strong enough to be observed. The present analysis of the isobaric levels of Co^{55} indicates the need for not only better resolution experiments but also better DW calculations for levels which are unbound or nearly unbound. If, as has been demonstrated for (d,p) studies,⁵⁵ the angular distributions for the deuterons from the (He^3,d) reaction can be demonstrated to be dependent on the j for the captured proton, the identification of the isobaric levels (and all other levels!) will be considerably easier.

On the basis that the ground states are pure $1f_{7/2}$ configurations, the normalization factors for the DW calculation are 24.0, 21.0, 20.0, and 17.3 for Sc^{49} , Cr^{52} , Mn^{53} , and Co^{55} , respectively. The average value of these normalization factors is 20.6 with the largest deviation being 16%. The uncertainties in the DW calculation are of this magnitude so one would say that observed strengths for the ground states agree well with the predicted values. However, this result would be more certain if a proton pickup reaction had been possible for those $N = 28$ nuclei which are stable, i.e., if the purity of the ground states could have been tested. This fact points out the need to augment the (He^3,d) studies with either (t,α) or (d,He^3) studies in the same way the (d,t) reaction has been used in conjunction with the (d,p) reaction in studies of neutron SP states. Such studies will be possible at this Laboratory in the near future when the installation of a tandem Van de Graaff accelerator is completed.

LIST OF REFERENCES

1. M. H. Macfarlane and J. B. French, Revs. Modern Phys. 32, 567 (1960).
2. H. E. Wegner and W. S. Hall, Rev. Sci. Instr. 29, 1100 (1958).
3. W. Tobocman, Phys. Rev. 115, 98 (1959); W. R. Gibbs, Ph.D. thesis, Rice University (1961); W. S. Hall and W. R. Gibbs, private communication.
4. C. M. Perey and F. G. Perey, Phys. Rev. 132, 755 (1963).
5. R. H. Bassel, D. D. Armstrong, and A. G. Blair, to be published.
6. J. B. French, Nuclear Phys. 26, 161 (1960).
7. J. B. French and M. H. Macfarlane, Nuclear Phys. 26, 168 (1960).
8. L. L. Lee, Jr., A. Marinov, and J. P. Schiffer, Physics Letters 8, 352 (1964).
9. H. E. Wegner and W. S. Hall, Phys. Rev. 119, 1654 (1960).
10. D. D. Armstrong, M.S. thesis, University of New Mexico (1961).
11. Models RMEJ150-100 and RMEJ150-500, Oak Ridge Technical Enterprises Corporation, Oak Ridge, Tennessee.
12. Model W-60-3SB, Technical Measurement Corporation, San Mateo, California.
13. Catalog No. 100-LSBG-3000-1D, Solid State Radiations, Los Angeles, California.
14. Type 814-J, Westinghouse, Semiconductor Division, Youngwood, Pennsylvania.
15. R. H. Stokes, J. A. Northrop, and K. Boyer, Rev. Sci. Instr. 29, 61 (1958).

16. R. H. Stokes, Rev. Sci. Instr. 31, 768 (1960).
17. Oak Ridge National Laboratory, Isotope Division, Oak Ridge, Tennessee.
18. R. J. Helmer and A. H. Hemmendinger, Rev. Sci. Instr. 28, 649 (1957).
19. J. A. Northrop and R. H. Stokes, Rev. Sci. Instr. 29, 287 (1958).
20. Model 4PL42, Solid State Radiations, Los Angeles, California.
21. D. R. F. Cochran, private communication.
22. P. T. McWilliams, W. S. Hall, and H. E. Wegner, Rev. Sci. Instr. 33, 70 (1962); W. S. Hall, private communication.
23. Nuclear Data Tables, Part I (National Academy of Sciences - National Research Council, Washington, D. C., 1961).
24. J. R. Erskine, J. P. Schiffer, and A. Marinov, Bull. Am. Phys. Soc. 11, 80 HC15 (1964).
25. S. T. Butler, Nuclear Stripping Reactions (John Wiley and Sons, Inc., New York, 1957).
26. A. G. Blair, private communication.
27. M. G. Mayer and J. H. D. Jensen, Theory of Nuclear Shell Structure (John Wiley and Sons, Inc., New York, 1955).
28. N. Austern, R. M. Drisko, E. C. Halbert, and G. R. Satchler, Phys. Rev. 133, B3 (1964).
29. J. L. Yntema and G. R. Satchler, to be published.
30. R. H. Fulmer and A. L. McCarthy, Phys. Rev. 131, 2133 (1963).
31. C. M. Perey, Phys. Rev. 131, 745 (1963).
32. R. H. Bassel, private communication.

33. B. L. Cohen, R. H. Fulmer, and A. L. McCarthy, Phys. Rev. 126, 698 (1962).
34. R. F. Bacher and S. Goudsmit, Phys. Rev. 46, 948 (1934).
35. G. Racah, Phys. Rev. 63, 367 (1943).
36. G. Racah, Phys. Rev. 76, 1352 (1949).
37. B. H. Flowers, Proc. Roy. Soc. (London) A212, 248 (1952).
38. A. R. Edmonds and B. H. Flowers, Proc. Roy. Soc. (London) A214, 515 (1952).
39. A. M. Lane, R. G. Thomas, and E. P. Wigner, Phys. Rev. 98, 693 (1955).
40. G. D. O'Kelley, N. H. Lazar, and E. Eichler, Phys. Rev. 101, 1059 (1956).
41. D. W. Martin, J. M. Cork, and S. B. Burson, Phys. Rev. 102, 457 (1956).
42. J. Dubois and S. Maripuu, Physics Letters 8, 349 (1964).
43. B. L. Cohen, Phys. Rev. 130, 227 (1963).
44. I. Talmi, Phys. Rev. 126, 1096 (1962).
45. T. Komoda, Nuclear Phys. 51, 234 (1964).
46. D. M. Van Patter, N. Nath, S. M. Shafroth, S. S. Malik, and M. A. Rothman, Phys. Rev. 128, 1246 (1962).
47. R. R. Wilson, A. A. Bartlett, J. J. Kraushaar, J. D. McCullen, and R. A. Ristinen, Phys. Rev. 125, 1655 (1962).
48. T. Kato, M. Nozawa, Y. Yoshizawa, and Y. Koh, J. Phys. Soc. Japan 15, 2140 (1960).
49. S. Kumabe, H. Ogata, T. Kamatuzaki, N. Inone, S. Tonita, Y. Yamada, and S. Matsumoto, Nuclear Phys. 46, 437 (1963).

50. D. S. Gemmell, L. L. Lee, Jr., J. P. Schiffer, and A. B. Smith,
Nuclear Spectroscopy with Direct Reactions I. Contributed Papers
(Argonne National Laboratory, Argonne, Illinois).
51. J. D. Anderson, C. Wong, and J. W. McClure, Phys. Rev. 126, 2170
(1962).
52. R. H. Fulmer, A. L. McCarthy, B. L. Cohen, and R. Middleton, Phys.
Rev. 133, B955 (1964).
53. J. Bellicard and M. Barreau, Nuclear Phys. 36, 476 (1962).
54. T. A. Belote, E. Kashy, A. Sperduto, H. A. Enge, and W. W. Buechner,
Nuclear Spectroscopy with Direct Reactions I. Contributed Papers
(Argonne National Laboratory, Argonne, Illinois).
55. L. L. Lee, Jr., and J. P. Schiffer, Phys. Rev. Letters 12, 108 (1964).