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One-Dimensional Elastic-Plastic
Calculations for Aluminum

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**One-Dimensional Elastic-Plastic
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by

Charles L. Mader

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ONE-DIMENSIONAL ELASTIC-PLASTIC

CALCULATIONS FOR ALUMINUM

by

Charles L. Mader

ABSTRACT

Taylor's experimental measurements of the free-surface velocity of aluminum targets being shocked by aluminum flying plates moving at 0.10 and 0.041 cm/ μ sec are used to calibrate the Hooke's law-Von Mises yield model of elastic-plastic flow for aluminum at 33 and 84 kbar.

I. Introduction. This report describes a numerical study of the elastic-plastic behavior of aluminum. The object of the study was to determine the usefulness of the Hooke's law-Von Mises yield model, with a realistic equation of state, in reproducing the detailed experimental observations of elastic-plastic behavior of aluminum recently reported by Taylor¹ of GMX-6.

Taylor's experimental observations²⁻⁵ for many materials show that the elastic precursor of a shock is time-dependent and that the plastic shock profiles are not sharp and are also time-dependent. McQueen,⁵ Taylor and Rice,³ and Johnson⁶ have attempted to develop a numerical model to describe the time-dependent elastic behavior of the iron precursor. The area of primary interest in the study described here was not the elastic precursor, but the elastic component of a rarefaction. Although there is no experimental evidence to show that the elastic rarefaction is time-dependent, it very probably is. Therefore, the usefulness of a time-independent elastic-plastic model in describing even the gross features of elastic-plastic flow is not obvious.

This study will show that one can obtain a reasonable description of the experimental observations of the elastic-plastic behavior of aluminum in the 30- to 110-kbar range using the Hooke's law-

Von Mises yield model if the yield is permitted to vary with initial shock pressure. It also will show that spalling at these low pressures is a function of more than the tension rate; that partial spalling occurs and is, therefore, a three-dimensional time-dependent phenomenon.

II. The Calculational Model. The Hooke's law-Von Mises yield model has been elegantly described and studied by Hermann.^{7,8,9}

Since the experimentally observed single-shock Hugoniot is a measure of the elastic-plastic components, one must calculate a hydrostat (pure plastic component) from the experimental data. Various models were tried to describe the low pressure hydrostat, and the one found satisfactory is:

$$P_{\text{hydrostat}} = P_{\text{Hugoniot}} - \left(\frac{2}{3} Y_0 \right) \left[\frac{P_{\text{Hugoniot}}}{P(\text{for } P_{\text{Hug}} - P_{\text{hyd}} = \frac{2}{3} Y_0)} \right]$$
$$\frac{P_{\text{Hugoniot}}}{0.050} \leq 1.0$$

In this report we used 0.050 Mbar for the pressure where the Hugoniot stress minus the hydrostat pressure was equal to $2/3 Y_0$. The computed results were insensitive to the value used between 0.05 and 0.02 Mbar.

The results described in this report were calculated using the JIN code^{10,11} to compute the one-dimensional hydrodynamic flow of slabs of aluminum.

The stress, σ_x , was calculated as follows.

First, for each cell and at each time step, the elastic stress deviator, S_j , was calculated from

$$S_j^{n+1} = S_j^n + 2\mu \left[- \left(\frac{U_{j+1} - U_j}{R_{j+1} - R_j} \right) (\Delta t) + \frac{1}{3} \left(\frac{v_j^{n+1} - v_j^n}{0.5(v_j^{n+1} + v_j^n)} \right) \right]$$

If $|S_j^{n+1}| > |2/3 Y_0|$, $\sigma_x = P_{\text{hydrostat}} \pm (2/3 Y_0)$ where the sign of $(2/3 Y_0)$ is identical to the sign of S_j^{n+1} .

If $|S_j^{n+1}| < |2/3 Y_0|$, $\sigma_x = P_{\text{hydrostat}} + S_j^{n+1}$.

For pressures greater than initial pressure (1 bar),

$$P_{\text{hyd}} = P_{\text{HOM}} \text{ equation of state} - \left(\frac{2}{3} Y_0 \right) \left(\frac{P_{\text{HOM}}}{0.05} \right)$$

$$\frac{P_{\text{HOM}}}{0.05} \leq 1.0.$$

For pressures less than initial pressure (1 bar),

$$P_{\text{hydrostat}} = P_{\text{HOM}} \text{ equation of state.}$$

The nomenclature is identical with that used in references 10 and 11.

S	stress deviator	U	particle velocity
μ	shear modulus	R	position
Y_0	yield strength	V	specific volume
j	space index	Δt	time increment
n	time index		

The HOM equation of state is described in Appendix A, and equation of state constants are given. Spalling was treated as described in reference 12.

III. The Experimental Data. John Taylor¹ of GMX-6 measured the free-surface velocity as a function of time for drivers of various thicknesses moving at various velocities hitting 0.5-in. aluminum plates. Taylor used the capacitor technique to measure the free-surface velocities continuously and accelerated the drivers with a propellant-driven gun. The techniques used have been described by Taylor and Rice³ and by McQueen.⁵ The Stanford Research Institute (SRI) group has published several papers and reports describing their experimental studies of elastic-plastic flow.^{13,14} While their experimental data are not precise enough to be useful in calibrating the

theoretical model, they are used for an additional comparison with the calibrated model. The particular SRI data used were described by Erkman.¹⁴

IV. The Results. The comparison of the experimental and calculational results is shown in Figs. 1 through 7. The yield strengths found necessary to reproduce the experimental data are given in Table I. The yield strength increases from a 2.4 kbar at 33 kbar plastic shock pressure to 5.5 kbar at 84 kbar plastic shock pressure. Also shown in Table I are the spalling constants required to match the experimentally deduced spall thicknesses. It is clear that spalling at these low pressures is a function of more than the tension rate. The experiments exhibiting thin spalls (Figs. 1 and 3) apparently result in partial spallation. Partial spallation was approximated by using a fixed negative spall interface pressure rather than the usual 1-atm interface pressure characteristic of sharp, clean spall interfaces. The negative spall interface pressure was used to approximate the actual case of an interface pressure that is initially under tension and slowly relieves to 1 atm. Various time-dependent spall interface pressure techniques were attempted, and one can obtain close agreement at later times than those shown in Fig. 1.

The effect of changing the parameters is discussed in Appendix B.

V. Conclusions. These results indicate that a properly calibrated Hooke's law-Von Mises yield model can yield a reasonable description of elastic-plastic behavior if one does not require great detail. The time-dependent features of elastic-plastic flow are not described by this model. Properly calibrated, the elastic-plastic model is certainly better than the assumption that the flow is pure plastic.

As more experimental data become available, one could determine how the yield strength changes with pressure, temperature, and physical properties of the material.

Table I

SUMMARY OF CALCULATED RESULTS

Exp. No.	Driver Velocity (cm/ μ sec)	Driver Thickness (cm)	Target Thickness (cm)	Y_0 (kbar)	Elastic Precursor Pressure (kbar)	Plastic Pressure (kbar)	Elastic Rarefaction Pressure (kbar)	ΔP Elastic Rarefaction (kbar)	Spall Thickness (cm)	Spall A (Mbar)
1	0.0412	0.1016	1.27	2.4	5.4	32.6	20.9	12.7	0.138	0.035
	0.0412	0.1016	1.27	0	0	32.3	0	0	0.112	0.035
2	0.0412	0.3175	1.27	2.4	5.4	32.6	20.9	12.7	0.310	0.045
	0.0412	0.3175	1.27	0	0	32.3	0	0	0.281	0.045
3	0.10	0.3175	1.27	5.5	12.0	84.0	52.6	31.4	0.231	0.050
	0.10	0.3175	1.27	0	0	83.9	0	0	0.246	0.050
4	0.10	0.635	1.27	5.5	12.0	84.0	52.6	31.4	0.474	0.070
	0.10	0.635	1.27	0	0	83.9	0	0	0.534	0.070
5	0.034	0.635 Fe 0.3175 Al	1.27	2.4	5.4	26.8	-	-	-	-
6	0.125	0.15875	2.54	5.5	12.0	108.0	74.0	34.0	-	-
	0.125	0.15875	2.54	0	0	108.0	0	0	-	-
7	0.330	0.25+	2.54	5.5	-	348.0	297.0	51.0	-	-
	0.330	0.254	2.54	-	-	348.0	0	0	-	-

Fig. 1a. The Taylor experimental and the computed free-surface velocities of a 1.27-cm aluminum plate driven by a 0.1016-cm aluminum plate initially traveling at 0.0412 cm/μsec.

Fig. 1b. The computed pressure-distance profiles. The driver plate is shown as a light line, and the target plate as a heavy line.

Fig. 2a. The Taylor experimental and the computed free-surface velocities of a 1.27-cm aluminum plate driven by a 0.3175-cm aluminum plate initially traveling at 0.0412 cm/μsec.

Fig. 2b. The computed pressure-distance profiles. The driver plate is shown as a light line, and the target plate as a heavy line.

Fig. 5a. The Taylor experimental and the computed free-surface velocities of a 1.27-cm aluminum plate driven by a 0.3175-cm aluminum plate initially traveling at 0.10 cm/μsec.

Fig. 3b. The computed pressure-distance profiles. The driver plate is shown as a light line, and the target plate as a heavy line.

Fig. 4a. The Taylor experimental and the computed free-surface velocities of a 1.27-cm aluminum plate driven by a 0.635-cm aluminum plate initially traveling at 0.10 cm/μsec.

Fig. 4b. The computed pressure-distance profiles. The driver plate is shown as a light line, and the target plate as a heavy line.

Fig. 5a. The Taylor experimental and the computed free-surface velocities of a 1.27-cm aluminum plate driven by a 0.3175-cm aluminum plate backed by a 0.635-cm iron plate initially traveling at 0.034 cm/μsec.

Fig. 5b. The computed pressure-distance profiles. The iron driver plate and the target plate are shown as heavy lines, and the aluminum driver plate as a light line.

Fig. 6a. The SRI experimental and the computed initial particle velocities as a function of target thickness divided by driver thickness. The driver velocity was 0.125 cm/μsec.

Fig. 6b. The computed pressure-distance profiles. The driver plate is shown as a light line, and the target plate as a heavy line.

Fig. 7a. The SRI experimental and the computed initial particle velocities as a function of target thickness divided by driver thickness. The driver velocity was 0.330 cm/μsec.

Fig. 7b. The computed pressure-distance profiles. The driver plate is shown as a light line, and the target plate as a heavy line.

APPENDIX A

THE HOM EQUATION OF STATE FOR A CONDENSED COMPONENT

I. The Nomenclature.

C, S	coefficients to a linear fit of U_s and U_p
C_v	heat capacity of condensed component in cal/gram/deg
I	total internal energy in Mbar-cc/gram
P	pressure in Mbar
T	temperature in $^{\circ}K$
U_p	particle velocity
U_s	shock velocity
V	total volume in cc/gram
V_o	initial volume of condensed component in cc/gram
α	linear coefficient of expansion
γ	Grüneisen constant
μ	shear modulus
A	Spall A
Subscripts	
H	Hugoniot
s	condensed component
o	initial condition

II. Equation of State Parameters.

	Aluminum		Iron	
C	+5.35	-001	+4.63	-001
S	+1.35	+000	+1.275	
F_s	+7.96115866874	+001		
G_s	-3.17533561633	+002		
H_s	-4.38525371533	+002		
I_s	-2.64248248960	+002		
J_s	-5.79734965732	+001		
γ_s	+1.7	+000	+1.6811	+000
C_v	+2.2	-001	+1.07	-001
α	+2.4	-005	+1.17	-005
V_o	+3.59066427289	-001	+1.27291242	-001
μ	+2.5	-001		

III. The Equation of State.

Knowing I and V , we wish to calculate P and T . For volumes smaller than V_o , the experimental Hugoniot data are expressed as a linear fit of the

shock and particle velocities. The Hugoniot temperatures are computed using the technique described in reference 10.

$$(a) \quad U_s = C + SU_p$$

$$P_H = \frac{C^2(V_o - V_s)}{[V_o - S(V_o - V_s)]^2}$$

$$(b) \quad \ln T_H = F_s + G_s \ln V_s + H_s (\ln V_s)^2 + I_s (\ln V_s)^3 + J_s (\ln V_s)^4$$

$$(c) \quad I_H = \frac{1}{2} P_H (V_o - V_s)$$

$$(d) \quad P_s = \frac{\gamma_s}{V_s} (I_s - I_H) + P_H \quad \text{where } \gamma_s = V \left(\frac{\partial P}{\partial I} \right)_V$$

$$(e) \quad T_s = T_H + \frac{(I_s - I_H)(23,890)}{C_v}$$

For volumes larger than V_o we use the $P = 0$ line as the standard curve and the Grüneisen equation of state for state points off the standard curve.

$$P_s = \left[I_s - \frac{C_v}{(3)(23890)(\alpha)} \left(\frac{V_s}{V_o} - 1 \right) \right] \frac{\gamma_s}{V_s}$$

$$T_s = \frac{(I_s)(23890)}{C_v} + T_o \quad \text{where } T_o = 300^{\circ}K,$$

$$P_{\text{spall}} = -A \sqrt{\frac{\Delta P}{\Delta X}} \quad \text{If } P < P_{\text{spall}}, \text{ spall occurs.}$$

APPENDIX B

PARAMETER STUDY

The results of calibrating the Hooke's law-Von Mises yield model have been described in the body of this report. This Appendix will discuss the effect of the parameters Y_o , γ , C_v , α , and μ .

The yield strength, Y_o , determines the magnitude of the elastic precursor and the elastic rarefaction. Figures 1 through 7 show its effect for $Y_o = 0$ and $Y_o = 2.4$ or 5.5 .

The velocity of the elastic precursor or the elastic rarefaction is determined by μ , the shear modulus. Walsh¹⁵ shows that

$$C_R^2 = C_B^2 + \frac{4}{3} \mu V$$

where C_R is sound speed in an elastic-plastic model and C_B is sound speed in a pure plastic

model. Aluminum μ is a constant of 0.250 Mbar up to at least 300 kbar. Figure B1 shows the calculated results for the same system as that shown in Fig. 2 with a μ of 0.40. The magnitude of the elastic precursor and the elastic rarefaction decreases, and the velocity increases with increasing μ .

The parameters C_V and α enter the calculation only in the negative pressure equation of state as C_V/α . Figure B2 shows the calculated results for the same system as that shown in Fig. 1 ($\alpha = 2.4 \times 10^{-5}$, $C_V = 0.22$) with an α of 5×10^{-5} and 1.25×10^{-5} or C_V of 0.44 or 0.11. Decreasing C_V or increasing α results in a less steep tension wave.

The Grüneisen γ enters the calculation both in the calculation of pressures off the Hugoniot and those off the $P = 0$ line. Decreasing γ results in a less steep rarefaction wave and a less steep tension wave. Figure B3 shows the calculated results for the same system as that shown in Fig. 1 ($\gamma = 1.70$) with a γ of 0.85.

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Fig. B1. The computed free-surface velocity of a 1.27-cm aluminum plate driven by a 0.3175-cm aluminum plate initially traveling at 0.0412 cm/μsec using a μ of 0.25 and 0.40.

Fig. B2. The computed free-surface velocity of a 1.27-cm aluminum plate driven by a 0.1016-cm aluminum plate initially traveling at 0.0412 cm/μsec using an α of 5×10^{-5} , 2.5×10^{-5} , and 1.25×10^{-5} or a C_V of 0.44, 0.22, and 0.11. Spall A was 0.025, 0.035, and 0.049.

Fig. B5. The computed free-surface velocity of a 1.27-cm aluminum plate driven by a 0.1016-cm aluminum plate initially traveling at 0.0412 cm/ μ sec using a γ of 1.7 and 0.85. Spall A was 0.035, 0.035, and 0.025.