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System Dynamics Study for the
Ultra High Temperature Reactor Experiment

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System Dynamics Study for the
Ultra High Temperature Reactor Experiment

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ABSTRACT

This report describes a study of the Ultra High Temperature Reactor Experiment (UHTREX) system dynamics. A mathematical model of the system is developed and investigated with various digital computer programs. The results show that the UHTREX system can be operated safely at design conditions and over a wide range of off-design conditions. The system is stable and relatively insensitive to inaccuracies and changes in heat transfer coefficients.

A startup route and a hot restart procedure for the UHTREX reactor are studied and found to be satisfactory. A variety of hypothetical system failures are investigated. It is shown that the overall system is safe from damage even in the event of serious equipment failures.

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1. INTRODUCTION

This report describes a study of the Ultra High Temperature Reactor Experiment (UHTREX) system dynamics. In what follows, it is assumed that the reader is familiar with the general characteristics of UHTREX.¹

1.1 Definition of System

The major components of the system being investigated are shown in Fig. 1-1. As noted there, the blower speeds and pressures in the primary and secondary coolant loops, the rate of air flow through the heat dump, the fraction of primary flow bypassed around the recuperator, and the reactivities of the control rods and fuel are independent and controllable variables. All desired reactor operating conditions must be achieved by manipulation of these eight variables.

The system shown in Fig. 1-1 is complex. Its behavior, as treated in this report, is described by about 200 basic equations, and a hand solution of these equations is out of the question. To understand and predict how the system as a whole will behave, it is necessary to solve the equations on an analog or digital computer. The particular digital computer simulation approach that has been taken is described here.

1.2 Objective of Study

The objective of this study is to obtain an understanding of the general behavior of the UHTREX system prior to its construction and

Fig. 1-1. Major UHTREX System Components and Nominal Design Operating Conditions

operation. This understanding will be used in designing and checking various system components, in writing operating procedures, and in checking the effects of hypothetical equipment failures.

Specifically, the information obtained has been, and will be, used to aid in the design of:

1. Control rod drives, safety systems, valve characteristics, etc.
2. Control systems, including digital computer programs
3. Routine operating procedures
4. Emergency procedures
5. Special early experiments and general experiments to determine system parameters.

It is expected that this study will also lead to an analog model of the UHTREX system that will be useful in training operators and in getting a feel for the behavior of the system prior to real operation.

1.3 Summary of Progress

1.3.1 Mathematical Model of System

Equations describing the basic components and phenomena in the UHTREX system have been derived. These equations form the mathematical model of the system.

Sufficient detail has been included in the model to attain the objectives of the study. In particular, the model has enough detail to determine system stability and to obtain the time response of reactor power, gas flow rates, inlet and outlet gas temperatures, gross average solid temperatures, etc., to changes in blower speed, reactivity, and bypass setting.

It might be noted that the mathematical model described here, although adequate, is not final. Additions and refinements, some of which are discussed in Section 6, will be made from time to time until the UHTREX reactor has been run and is fully understood.

1.3.2 Computer Programs

An IBM 7094 digital computer program for solving the system equations has been written and debugged. With this program, UHXCOM, one can simulate UHTREX behavior in the time domain. One can obtain system transfer functions by performing calculations on the transient response of the system to small step changes. Also, the system equations have been linearized, Laplace transformed, and solved with digital computer frequency domain programs to obtain the system transfer functions directly.² The transfer functions obtained from the frequency domain programs have been checked with transfer functions obtained from the time domain simulation program. These checks ensure that there are no basic errors in any of the programs and that the numerical methods used to solve the system differential equations in the time domain program are accurate.

The UHXCOM simulation program and the frequency domain programs are flexible. They allow one to run a variety of problems by varying parameters such as heat transfer coefficients, and temperature coefficients of reactivity. In addition, they are relatively easy to modify to incorporate new features or refinements to the mathematical model. Automatic film plot output routines have been used in all programs.

1.3.3 Simulation Runs

A variety of simulation runs have been made to see how the UHXCOM simulation program works and to see how the UHTREX system behaves. The UHXCOM program has proved accurate and easy to use. The simulation runs require relatively little 7094 machine time. The results of the simulation runs are encouraging. They indicate that the UHTREX system can be run safely and conservatively. The results of the simulation runs have been useful in determining overall system temperatures and flow rates at nominal design conditions, in determining appropriate actions to be taken in hypothetical emergency situations, and in gaining a general understanding of how the system can be operated.

1.4 Outline of Report

Section 2 describes the mathematical model of the UHTREX system. Section 3 describes the UHXCOM simulation program and gives a brief description of the frequency domain transfer function programs. Sections 4, 5, and 6 discuss system operation, give selected simulation results, and describe plans for future analysis work.

Appendices A and B contain instructions for using the UHXCOM program and a discussion of basic UHTREX system constants. Appendix C is an introductory discussion of Laplace transforms, linear systems analysis, and transfer functions. It is suggested that readers who are not familiar with these concepts read Appendix C before starting Section 2. Appendix D discusses the heat transfer convergence criterion. Appendix E explains the numerical methods used to solve the neutron kinetics equations for the reactor. The method presented in Appendix E has proved particularly useful in that it allows the use of a fairly large time increment in the UHXCOM simulation program. Appendix F presents an example of a derivation used in the frequency domain programs.

2. MATHEMATICAL MODEL OF THE SYSTEM

A variety of simple and complicated mathematical models of the system has been investigated. The model described here is preferred because it seems to represent the system accurately. In addition, it is simple enough to be analyzed or at least simulated and understood in some gross sense.

In general, the models for the various system components are quite crude, containing few details of the temperatures within a given component. However, they are relatively good input-output models. That is, given a set of flow rates and inlet gas temperatures as functions of time, the outlet gas temperatures as functions of time are reasonably accurate. Thus, the individual models and the overall system model are good enough to determine system stability and to obtain usable curves of power, gas flow rates, inlet and outlet gas temperatures, gross average solid temperatures, etc., as functions of time. Individual internal solid temperatures available from the model might be used occasionally to estimate rates of change of temperature or changes in temperature in the real system, but such applications of the results of this study should be examined carefully and qualified in an appropriate manner.

Table II-1 gives a list of abbreviations used frequently in this report.

TABLE II-1

SOME ABBREVIATIONS USED IN UHTREX SYSTEM DYNAMICS REPORT

BS ₁	Primary loop blower speed
BS ₂	Secondary loop blower speed
°F	Degrees Fahrenheit
FB	Fraction of primary flow bypassed around recuperator
HP ₁	Primary blower horsepower
HP ₂	Secondary blower horsepower
P ₁	Primary loop coolant pressure
P ₂	Secondary loop coolant pressure
Q _d	Heat dump thermal power
Q _n	Reactor neutronic power
Q _t	Reactor thermal power
°R	Degrees Rankin
T _{air}	Heat dump inlet air temperature
T _{avg}	Average reactor solid temperature
T _{dci}	Heat dump cold inlet air temperature
T _{dco}	Heat dump cold outlet air temperature
T _{dhi}	Heat dump hot inlet coolant temperature
T _{dho}	Heat dump hot outlet coolant temperature
T _i	Reactor inlet coolant temperature
T _o	Reactor outlet coolant temperature
T _{pli}	Pipe No. 1 inlet coolant temperature
T _{plo}	Pipe No. 1 outlet coolant temperature
T _{p2i}	Pipe No. 2 inlet coolant temperature
T _{p2o}	Pipe No. 2 outlet coolant temperature
T _{rci}	Recuperator cold inlet coolant temperature
T _{rco}	Recuperator cold outlet coolant temperature
T _{rhi}	Recuperator hot inlet coolant temperature
T _{rho}	Recuperator hot outlet coolant temperature

TABLE II-1 (continued)

T_{xci}	Heat exchanger cold inlet coolant temperature
T_{xco}	Heat exchanger cold outlet coolant temperature
T_{xhi}	Heat exchanger hot inlet coolant temperature
T_{xho}	Heat exchanger hot outlet coolant temperature
$\dot{W}_1$	Primary loop coolant flow rate
$\dot{W}_2$	Secondary loop coolant flow rate
$\dot{W}_3$	Heat dump air flow rate
X_b	Recuperator bypass stem position
ΔP_1	Primary loop pressure drop
ΔP_2	Secondary loop pressure drop
ΔP_{21}	Pressure difference between primary and secondary loops ($P_2 - P_1$)
ρ	Net reactivity
ρ_r	Rod reactivity
ρ_t	Temperature reactivity
ρ_s	Shutdown reactivity
ϕ_1	Primary blower flow coefficient
ϕ_2	Secondary blower flow coefficient

2.1 General Features

Some general features of the model and some approximations used in the model are described below. Selected results from computer programs have been included where they seem helpful.

2.1.1 Lumped Parameter Models

The reactor heat exchanger, the recuperator, the main heat exchanger, the heat dump, and the system pipes are distributed parameter devices in which gas and solid temperatures are functions of four independent variables in time and space. Such devices can be described by partial differential equations which may be linear in temperature but are nonlinear in flow rate. With such equations, there is no analytic solution available for, say, the exit gas temperature as a function of time when the inlet gas temperature and coolant flow rates vary with time.

To study such devices prior to their full-scale construction, one might construct physical analogs or scaled models which would allow direct observation of experimental results; a variety of such studies have been made on components in the UHTREX system.³ As an alternative, one might solve the partial differential equations describing the system on a digital computer. Again, studies of this sort have been made on various UHTREX components.³ These digital programs contain considerable detail, require a considerable amount of computation time, and, in most cases, yield only a steady-state solution. They give excellent results on the problems for which they were intended.

However, the particular interest here is not so much in the detailed internal conditions of a particular component, but rather in the interactions of system components and in the behavior of the connected system as a whole. The simultaneous numerical solution of the partial differential equations for all system components would require

far too much computing time. Consequently, in this study we have approximated the distributed parameter systems with lumped parameter mathematical models. That is, the real system, in which solid temperatures vary with time and distance, has been approximated by a lumped mathematical system, in which distances as such do not appear explicitly, and in which the solid temperature in any one "lump" is uniform. The acceptance of such compromise lumped mathematical models has led to a digital simulation program that requires relatively little computing time.

The diagram and equations for a simple two-lump heat exchanger are shown in Fig. 2-1. The symbols in Eqs. 2.1-1 through 2.1-8 have the following meanings.

C_1, C_2	= specific heats of solids (Btu/lb $^{\circ}$ R)
W_1, W_2	= weights of lumps (lb)
T_1, T_2	= temperatures of lumps ($^{\circ}$ R)
c_p	= specific heat of coolant gas (Btu/lb $^{\circ}$ R)
$\dot{W}$	= gas flow rate (lb/sec)
T_{i1}, T_{i2}	= inlet gas temperatures ($^{\circ}$ R)
T_{o1}, T_{o2}	= Outlet gas temperatures ($^{\circ}$ R)
T_{a1}, T_{a2}	= average gas temperatures ($^{\circ}$ R)
H_1, H_2	= heat transfer coefficients (Btu/ $^{\circ}$ R sec)
h_1, h_2	= heat transfer constants (Btu/ $^{\circ}$ R sec (lb/sec) $^{0.8}$)
k	= average gas temperature weighting factor (-)

In Fig. 2-1, the gas passing through lump 1 sees only the solid temperature T_1 , and the gas in lump 2 sees only T_2 . Intuitively, it would seem that as the number of lumps becomes large the behavior of the lumped system would approach that of the distributed parameter system, and in some problems one can show analytically that this is indeed the case.

Having chosen to use lumped parameter mathematical models, the next question is, "How many lumps are necessary?". The object, of

$$C_1 W_1 \frac{dT_1}{dt} = H_1 (T_{a_1} - T_1) \quad (2.1-1)$$

$$T_{a_1} = \frac{T_{i_1} + kT_{o_1}}{k + 1} \quad (2.1-2)$$

$$c_p \dot{W} (T_{i_1} - T_{o_1}) = H_1 (T_{a_1} - T_1) \quad (2.1-3)$$

$$H_1 = h_1 W^{0.8} \quad (2.1-4)$$

$$C_2 W_2 \frac{dT_2}{dt} = H_2 (T_{a_2} - T_2) \quad (2.1-5)$$

$$T_{a_2} = \frac{T_{i_2} + kT_{o_2}}{k + 1} \quad (2.1-6)$$

$$c_p \dot{W} (T_{i_2} - T_{o_2}) = H_2 (T_{a_2} - T_2) \quad (2.1-7)$$

$$H_2 = H_2 W^{0.8} \quad (2.1-8)$$

Fig. 2-1 Simple Two-Lump Heat Exchanger Model

course, is to use enough lumps to obtain accurate results; but at the same time, one should not use more lumps than are necessary, for the use of unnecessary lumps will waste digital computer time.

Two factors were considered in determining the number of lumps to be used in the models. First, it was required that each system component be represented by a model having a sufficient number of lumps to satisfy an appropriate convergence criterion.⁴ The criterion usually has the form

$$H \leq (k + 1) c_p \dot{W} \quad (2.1-9)$$

where

H = heat transfer coefficient (Btu/°R sec)

$\dot{W}$ = gas flow rate (lb/sec)

c_p = specific heat of coolant gas (Btu/lb°R)

k = average gas temperature weighting factor (—).

In Appendix D it is shown that each lump of the model shown in Fig. 2-1 must satisfy the criterion given in Eq. 2.1-9. If this criterion is not satisfied, a step increase in the inlet gas temperature T_{in} will produce an immediate decrease in T_{o1} , the outlet gas temperature of the first lump. This disturbing phenomenon can be avoided by choosing a large enough number of lumps so that the heat transfer coefficient for any one lump satisfies Eq. 2.1-9. In the models discussed in detail later in this chapter, sufficient lumps were taken so that Eq. 2.1-9 is satisfied over a wide range of gas flow rates, generally from "design" flow rates down to about one-half of the design values. For extremely low flow rates at which the convergence criterion ordinarily would not be satisfied, the heat transfer coefficient is set equal to $(k + 1) c_p \dot{W}$. Thus, the convergence criterion is still satisfied and H is now proportional to $\dot{W}$ rather than, say, $W^{0.8}$ as is shown in Eq. 2.1-4. This approximation allows the extension of the useful flow rate change of the existing model to lower flow rates than would otherwise be available. The approximation is a reasonable one, especially considering the overall indeterminacies in the heat transfer coefficients of the system.

Having satisfied the convergence criterion by using say, ℓ lumps, one is assured that the behavior of any one lump will be at least reasonable. The next question is, "Is the behavior of the ℓ -lump model sufficiently like that of the real system?". If the answer is "yes," ℓ lumps are enough. If the answer is "no," more than ℓ lumps are required.

To answer the question one must determine the behavior of the real system, and one must have a convenient way of comparing the behavior of the real system with that of the ℓ -lump model. In this work, the behavior of the real system was determined, approximately, by the study of an ideal model. The ideal model was either the distributed parameter equations or a lumped model having more lumps than the ℓ -lump trial model. The behavior of the trial model and the ideal model were compared by examining their transfer functions⁵ in the frequency region of interest.* A good match of the transfer functions over an appropriate frequency range assures one of the similarity of the linear characteristics of the two models at the operating point under consideration. Comparison of transfer functions at more than one operating point and comparisons of the time response of the two models to a few large system disturbances help verify the nonlinear characteristics of the trial model.

The heat dump can serve as an example to illustrate the use of the transfer function in selecting the number of lumps required. Figures 2-2 and 2-3 show the gain and phase of the heat dump transfer function $T_{dho}(s)/T_{dhi}(s)$ for 1-, 2-, 3-, and 4-lump models and for two distributed parameter models. The gain functions for the 3- and 4-lump models are close to that of the distributed parameter model out to 0.05 rad/sec. The corresponding phase functions begin to differ significantly at about 0.02 rad/sec. However, at this frequency, the gain is -32 db; as the frequency increases, the gain goes down to -60 db. Thus, the phase

* The frequency range of interest is that range in which the expected driving function has an appreciable energy content.

Fig. 2-2 Gain of the Heat Dump Transfer Function
 T_{dho}/T_{dhi} for Various Models

Fig. 2-3 Phase of the Heat Dump Transfer Function T_{dho}/T_{dhi} for Various Models

differences between the 3- and 4-lump models and the distributed parameter model occur in a frequency range where the gain is very low, and either the 3- or 4-lump model would suffice. The data given in Figs. 2-2 and 2-3, combined with the convergence criterion, led us to choose the 4-lump model for use in this study. The selection of the 4-lump model is consistent with the general philosophy of picking models which are conservatively good.

The investigation of lumped models led us to choose a reactor heat exchanger that has four axial lumps, or lumps along the axis of gas flow, and two radial lumps, at right angles to the direction of gas flow. The recuperator and pipes are represented by a model with three axial lumps and one radial lump. The heat dump has four axial lumps. The heat exchanger has seven lumps along each major gas flow axis, but there are three major paths for the gas flow, so that it has a total of 21 lumps. It is described in detail in Section 2.5.

2.1.2 Average Gas Temperature Weighting

The model presented in Fig. 2-1 and its accompanying equations are typical of those found elsewhere in this report. Note, in particular, Eqs. 2.1-2 and 2.1-6 in which average gas temperatures are computed. The accuracy of these average temperatures is dependent on the actual variation of gas temperature with distance in the real system and on the choice of the average gas temperature weighting factor k .

A set of "best" gas weighting factors have been determined for all system elements by comparing transfer functions of trial models having different weighting factors with transfer functions of an ideal model. For instance, an inspection of the transfer functions shown in Figs. 2-4 and 2-5 leads one to choose a heat dump hot gas weighting factor of $k = 1.25$ for a good match of gain and phase at low frequencies. That is, for the gas on the hot side of the heat dump,

$$T_{\text{avg}} = T_{\text{in}} + \frac{1.25 T_{\text{hot}}}{2.25} \quad (2.1-10)$$

Fig. 2-4 Gain of the Heat Dump Transfer Function T_{dho}/T_{dhi} for Various Average Gas Temperature Weighting Factors

Fig. 2-5 Phase of the Heat Dump Transfer Function T_{dho}/T_{dhi} for Various Average Gas Temperature Weighting Factors

Various combinations of weighting factors for the hot-side and cold-side gas were tried for all system components, and the best combinations have been incorporated in the model. For the number of lumps chosen, weighting factors of 1 give the best results, except in the hot side of the heat dump where the hot gas was weighted by a factor of 1.25 as shown above, and in the hot side of the recuperator where the hot gas was weighted by a factor of 1.5.

No attempt has been made to select different weighting factors for each lump of a many-lump model. Satisfactory transfer functions are obtained with the same weighting used in all lumps.

2.1.3 Coolant Gas Transit Times

Transit times for the primary and secondary coolant gas have been neglected. At design conditions, these times are of the order of seconds, while pertinent system time constants are of the order of hundreds of seconds. At off-design conditions, the ratio of the time constants to the transit times is not so favorable, but it is still large enough to justify neglecting the transit times.

Results from various distributed parameter transfer function programs described in Chapter 3 also justify neglect of the transit times. For example, Figs. 2-2 and 2-3 show the transfer function gain and phase of two distributed parameter models of the heat dump. In one model, the effect of finite gas velocity has been included. In the other model, the gas velocity has been taken as infinite; in other words, the gas transit time has been neglected. The gain of these two models is identical over the range $10^{-4} < \omega < 10$. The phases differ significantly only for $\omega > 0.5$. Thus, the neglect of the gas transit time affects the results only at frequencies outside the range of interest.

2.1.4 Heat Transfer Coefficients

As illustrated in Eqs. 2.1-4 and 2.1-8, the heat transfer coefficients for the model are of the form*

$$H = hW^\alpha, \quad (2.1-11)$$

where h is not a function of temperature. This approximation holds very well at and near design conditions.

Of all the numbers in the model, the heat transfer coefficients are probably the least well known. Parameter studies to determine the effect of heat transfer coefficients have been made. These studies, which are discussed in Section 5.3, show that the system is quite insensitive to changes in heat transfer coefficients. Thus, the results given by the model should be good in spite of the fact that the heat transfer coefficients are poorly known or poorly approximated.

2.1.5 Comments on Other Approximations in Model

A variety of additional approximations, such as neglecting $c_p \frac{dT_{\text{gas}}}{dt}$ terms in all heat exchangers, have been made in establishing the system model. Some of these approximations will be eliminated at a later date. For instance, gas pressure changes, which are neglected now, will be incorporated in the model. Other approximations, including those discussed above, will be reviewed as our knowledge of the system and the way that it will be operated increases.

2.2 Reactor Heat Exchanger

The reactor core is a hollow vertical graphite cylinder 39-inches high, with an outside diameter of 70 inches, and an inside diameter of 23 inches. Radial holes 1.1-inches in diameter have been bored through

* As noted in Section 2.1.1, H is made equal to $(k + 1)\dot{W}_p$ for convergence at low flow rates.

the cylinder wall at 15° intervals in 13 horizontal planes. These 312 holes hold the reactor fuel elements, which are hollow cylinders of graphite that have been highly enriched with uranium. The fuel element cylinders have an outside diameter of 1 inch, an inside diameter of 0.5 inch, and a length of 5.5 inches. Four fuel elements rest end-to-end in each of the 312 radial holes to make a total of 1248 elements in the reactor when the core is fully loaded. Helium coolant flows from the center of the core to the outside via the fuel element channels and the holes in the fuel elements.

The mathematical model for the reactor heat exchanger is one which represents an average fuel channel and its associated graphite moderator. The reactor heat exchanger model is shown in Fig. 2-6.

Equations for the third lump of the reactor heat exchanger are given below.

$$\begin{aligned}
 C_{m\text{ma}}(3) \frac{dT_m(3)}{dt} = & Q_m(3) + \frac{H_a}{4}(T_a(3) - T_m(3)) + \frac{H_{\text{rad}}}{4}(T_f^4(3) - T_m^4(3)) \\
 & + X_{kl}(3)(T_m(2) - T_m(3)) \\
 & + X_{kr}(3)(T_m(4) - T_m(3)) \quad (2.2-1)
 \end{aligned}$$

$$\begin{aligned}
 C_{f\text{fa}}(3) \frac{dT_f(3)}{dt} = & Q_f(3) + \frac{H_a}{4}(T_a(3) - T_f(3)) + \frac{H_{\text{rad}}}{4}(T_m^4(3) \\
 & - T_f^4(3)) + \frac{H_e}{4}(T_e(3) - T_f(3)) \quad (2.2-2)
 \end{aligned}$$

$$Q_m(3) = F_{m_3}(0.05 \text{ } q) \quad (2.2-3)$$

$$Q_f(3) = F_{f_3}(0.95 \text{ } q) \quad (2.2-4)$$

$$T_a(3) = \frac{T_{a_1}(4) + T_{a_1}(3)}{2} \quad (2.2-5)$$

Note: $Y_{(i)}$ is notation used in the UHXCOM FORTRAN program for temperature of i th solid lump.

Fig. 2-6 Reactor Heat Exchanger Model

$$T_{e(3)} = \frac{T_{e1(4)} + T_{e1(3)}}{2} \quad (2.2-6)$$

$$C_p \dot{W}_{ad} (T_{a1(4)} - T_{a1(3)}) = \frac{H_a}{4} (T_{a(3)} - T_{m(3)}) + \frac{H_a}{4} (T_{a(3)} - T_{f(3)}) \quad (2.2-7)$$

$$C_p \dot{W}_{ed} (T_{e(4)} - T_{e(3)}) = \frac{H_e}{4} (T_{e(3)} - T_{f(3)}) \quad (2.2-8)$$

$$\dot{W}_{ad} = F_a \dot{W} \quad (2.2-9)$$

$$\dot{W}_{ed} = F_e \dot{W} \quad (2.2-10)$$

$$H_e = h_{ek} \dot{W}_e^{0.8} \quad (2.2-11)$$

where

- q = total power generated in reactor (Btu/sec)
- $Q_{m(3)}$ = power generated in third lump of moderator (Btu/sec)
- $Q_{f(3)}$ = power generated in third lump of fuel (Btu/sec)
- F_{m3} = fraction of total moderator power generated in the third lump (—)
- F_{f3} = fraction of total fuel power generated in the third lump (—)
- C_m = specific heat of graphite moderator (Btu/lb°R)
- C_f = specific heat of fuel elements (Btu/lb°R)
- C_p = specific heat of helium gas (Btu/lb°R)
- $W_{ma(3)}$ = weight of moderator in third lump (lb)
- $W_{fa(3)}$ = weight of fuel in third lump (lb)
- $T_{m(3)}$ = temperature of third lump of moderator (°R)
- $T_{f(3)}$ = temperature of third lump of fuel (°R)
- $T_{a1(4)}$ = temperature of annulus inlet gas to third lump (°R)

- $T_{a1}(3)$ = temperature of annulus outlet gas of third lump ($^{\circ}\text{R}$)
 $T_a(3)$ = average temperature of annulus gas in third lump ($^{\circ}\text{R}$)
 $T_{e1}(4)$ = temperature of gas entering third lump through fuel element ($^{\circ}\text{R}$)
 $T_{e1}(3)$ = temperature of gas leaving third lump from fuel element ($^{\circ}\text{R}$)
 $T_e(3)$ = average temperature of gas in third lump of fuel element ($^{\circ}\text{R}$)
 H_e = heat transfer coefficient between fuel element and helium gas ($\text{Btu}/^{\circ}\text{R sec}$)
 h_{ek} = heat transfer constant ($\text{Btu}/^{\circ}\text{R sec (lb/sec)}^{0.8}$)
 H_a = heat transfer coefficient between gas in annulus and the adjacent fuel and moderator ($\text{Btu}/^{\circ}\text{R sec}$)
 H_{rad} = radiative heat transfer coefficient between moderator and fuel ($\text{Btu/sec } ^{\circ}\text{R}^4$)
 $\dot{W}_1$ = primary loop helium gas flow rate (lb/sec)
 F_e = fraction of primary flow going through fuel elements (—)
 F_a = fraction of primary flow going around fuel elements through annulus (—)
 $\dot{W}_{ed}$ = gas flow rate through elements (lb/sec)
 $\dot{W}_{ad}$ = gas flow rate through annulus (lb/sec)
 $X_{kl}(3)$ = left direction heat transfer constant ($\text{Btu}/^{\circ}\text{R sec}$)
 $X_{kr}(3)$ = right direction heat transfer constant ($\text{Btu}/^{\circ}\text{R sec}$)

Equations 2.2-1 and 2.2-2 are conventional heat rate balance equations. Equations 2.2-3 and 2.2-4 are used to calculate the power levels in the graphite moderator and in the fuel elements. As shown in these equations, 95% of the reactor power is generated in the fuel and 5% in the moderator.⁶ F_{m3} , F_{f3} , and similar constants for the other lumps allow the adjustment of the power profile in the reactor along the axis of gas flow through the fuel elements. The weightings used in computing the average gas temperatures in Eqs. 2.2-5 and 2.2-6 were determined by inspection of various reactor transfer functions as described

in Section 2.1. In the model a fixed 85% of the primary flow is through the fuel element holes, so F_a and F_e in Eqs. 2.2-9 and 2.2-10 are fixed at 0.15 and 0.85, respectively. The minor functional dependence of H_a and H_{rad} on flow rate and temperature has been neglected, and these two heat transfer coefficients are treated as constants. Expressions similar to Eqs. 2.2-1 through 2.2-8 can be written for each of the other three axial reactor lumps. These additional equations, combined with an expression to calculate the reactor outlet gas temperature,

$$T_o = \frac{T_{ao}\dot{W}_{ad} + T_{eo}\dot{W}_{ed}}{\dot{W}_1}, \quad (2.2-12)$$

describe the reactor heat exchanger model completely.

2.3 Reactor Neutron Kinetics

The basic reactor neutron kinetic equations used in this study are shown below.

$$\frac{dq}{dt} = \left(\frac{\rho - \beta}{\ell^*} \right) q + \sum_{i=1}^6 \lambda_i c_i \quad (2.3-1)$$

$$\frac{dc_i}{dt} = \frac{\beta_i}{\ell^*} q - \lambda_i c_i \quad (i = 1, \dots, 6) \quad (2.3-2, \dots, 7)$$

$$\beta = \sum_{i=1}^6 \beta_i \quad (2.3-8)$$

$$\rho_t = \gamma \left(\frac{W_m \bar{T}_m + W_f \bar{T}_f}{W_m + W_f} - T_{amb} \right) \quad (2.3-9)$$

$$\rho = \rho_t + \rho_{sd} + \rho_r \quad (2.3-10)$$

$$\bar{T}_m = \frac{1}{W_m} \sum_{i=1}^4 T_{m(i)} W_{ma(i)} \quad (2.3-11)$$

$$\bar{T}_f = \frac{1}{W_f} \sum_{i=1}^4 T_{f(i)} W_{fa(i)} \quad (2.3-12)$$

where

q	= reactor neutronic power (Btu/sec)
ρ	= total absolute reactivity (—)
β	= fraction of neutrons which are delayed (—)
ℓ^*	= mean effective neutron lifetime (sec)
λ_i	= precursor decay constants (sec^{-1})
c_i	= power derived from release of neutrons from the i th precursor group (Btu/sec)
β_i	= fraction of delayed neutrons held by i th precursor group (—)
ρ_t	= absolute reactivity due to change in core temperature from ambient temperature (—)
γ	= reactor temperature coefficient of reactivity ($^{\circ}\text{R}^{-1}$)
W_m	= total weight of moderator (lb)
W_f	= total weight of fuel elements (lb)
$\bar{T}_m$	= average temperature of moderator ($^{\circ}\text{R}$)
$\bar{T}_f$	= average temperature of fuel ($^{\circ}\text{R}$)
$W_{ma(i)}$	= weight of i th stage of moderator
$W_{fa(i)}$	= weight of i th stage of fuel
$T_{m(i)}$	= temperature of i th stage of moderator
$T_{f(i)}$	= temperature of i th stage of fuel
T_{amb}	= ambient (room) temperature ($^{\circ}\text{R}$)
ρ_{sd}	= absolute shutdown reactivity, or reactivity by which reactor is subcritical with all rods inserted at room temperature (—)
ρ_r	= absolute rod reactivity, which is zero with absorber rods fully inserted and increases as the rods are withdrawn (—)

The one-point neutron kinetic equations, Eqs. 2.3-1 through 2.3-8, are the ones ordinarily used in reactor engineering system studies. In Eq. 2.3-9 the temperature dependence of γ has been neglected, and γ as listed in Appendix B is the value at design conditions.

The simulation program described in Section 3 does not use Eqs. 2.3-1 through 2.3-8 directly in the form that they are given here. These equations are Laplace transformed and rearranged to yield an alternate set of equations whose solution requires less computing time. The transformed equations and the methods used to solve them are outlined in Appendix E.

2.4 Recuperator

The recuperator is a graphite regenerative heat exchanger which reduces the temperature of the helium gas coming out of the reactor from about 2860°R to 1860°R. The lower temperature of the recuperator outlet gas can be tolerated by the stainless steel in the main heat exchanger.

Heat exchange in the recuperator is accomplished in a solid graphite block 12 feet long and 3 feet in diameter. This block has 437 one-half-inch-diameter holes bored parallel to the long axis. The two helium streams flow countercurrently through the graphite block. At design conditions, the recuperator exchanges a maximum of 1.3×10^7 Btu/h between the hot helium from the reactor and the cooler helium returning to the reactor. The cold-side flow rate can be adjusted by means of a bypass flow valve.

A sketch of the model of the recuperator heat exchanger is shown in Fig. 2-7. As can be seen, the model has three axial lumps and one radial lump. Equations for the second lump of the recuperator model are given below.

$$C_r \frac{W_r}{3} \frac{dT_r(2)}{dt} = \frac{H_{rh}}{3} (T_{rh(2)} - T_r(2)) + \frac{H_{rc}}{3} (T_{rc(2)} - T_r(2)) \quad (2.4-1)$$

$$T_{rh(2)} = \frac{T_{rh1(1)} + 1.5 T_{rh1(2)}}{2.5} \quad (2.4-2)$$

- Notes: 1. $T_{rh(i)}$ and $T_{rc(i)}$ are average gas temperatures.
 2. $Y_{(i)}$ is FORTRAN notation for solid temperatures.

Fig. 2-7 Recuperator Model

$$T_{rc}(2) = \frac{T_{rc1}(3) + T_{rc1}(2)}{2} \quad (2.4-3)$$

$$C_p \dot{W}_1 (T_{rh1}(1) - T_{rh1}(2)) = \frac{H_{rh}}{3} (T_{rh}(2) - T_r(2)) \quad (2.4-4)$$

$$C_p \dot{W}_1 (1 - FB) (T_{rc1}(3) - T_{rc1}(2)) = \frac{H_{rc}}{3} (T_{rc}(2) - T_r(2)) \quad (2.4-5)$$

$$H_{rh} = h_{rhk} W_1^{0.8} \quad (2.4-6)$$

$$H_{rc} = h_{rck} [\dot{W}_1 (1 - FB)]^{0.8} \quad (2.4-7)$$

where

- C_r = specific heat of graphite (Btu/lb °R)
- C_p = specific heat of helium gas (Btu/lb °R)
- W_r = weight of graphite in recuperator (lb)
- $\dot{W}_1$ = primary loop helium gas flow rate (lb/sec)
- FB = fraction of primary flow bypassed around recuperator (—)
- $T_r(2)$ = temperature of graphite in second lump (°R)
- $T_{rh1}(1)$ = temperature of hot gas entering second lump (°R)
- $T_{rh1}(2)$ = temperature of hot gas leaving second lump (°R)
- $T_{rh}(2)$ = average temperature of hot gas in second lump (°R)
- $T_{rc1}(3)$ = temperature of cold gas entering second lump (°R)
- $T_{rc1}(2)$ = temperature of cold gas leaving second lump (°R)
- $T_{rc}(2)$ = average temperature of cold gas in second lump (°R)
- H_{rh} = heat transfer coefficient between graphite and hot gas (Btu/°R sec)
- h_{rhk} = heat transfer constant (Btu/°R sec (lb sec)^{0.8})
- H_{rc} = heat transfer coefficient between graphite and cold gas (Btu/°R sec)
- h_{rck} = heat transfer constant (Btu/°R sec (lb/sec)^{0.8})

These equations are similar to those for the reactor heat exchanger. As noted in Section 2.1, however, the average gas temperature on the hot side of the recuperator is weighted slightly in favor of the outlet gas temperature.

Expressions similar to Eqs. 2.4-1 through 2.4-5 can be written for the first and third lumps of the recuperator. These additional equations, combined with an expression to calculate the temperature of the gas delivered to the reactor

$$T_i = (1 - FB)T_{rco} + (FB)T_{xho} , \quad (2.4-8)$$

describe the recuperator model completely.

2.5 Primary-Secondary Coolant Loop Heat Exchanger

A countercurrent-flow shell-and-tube heat exchanger made of austenitic steel transfers heat from the coolant gas in the primary loop to that in the secondary loop. On the tube side of the heat exchanger, the temperature of the primary coolant gas drops from 1860°R to 1060°R, a temperature compatible with the operation of the electric blowers. A schematic drawing of the UHTREX heat exchanger is shown in Fig. 2-8.

The mathematical model for the heat exchanger contains seven axial lumps as shown in Fig. 2-9. Equations for the first lump of the model of Fig. 2-9 are shown below.

$$C_x \frac{M_1}{7} \frac{dT_{x_1}(1)}{dt} = \frac{H_1}{7} (T_{xh_1}(1) - T_{x_1}(1)) + \frac{X_k}{7} (T_{x_2}(1) - T_{x_1}(1)) \quad (2.5-1)$$

$$C_x \frac{M_2}{7} \frac{dT_{x_2}(1)}{dt} = \frac{H_2}{7} (T_{xh_2}(1) - T_{x_2}(1)) + \frac{X_k}{7} (T_{x_1}(1) - T_{x_2}(1)) \quad (2.5-2)$$

Fig. 2-8 UHTREX Primary to Secondary Heat Exchanger

Note: $Y_{(10)}$ through $Y_{(22)}$ are FORTRAN program notation for temperatures of solid

Fig. 2-9 Heat Exchanger Model

$$C_x \frac{M_3}{7} \frac{d}{dt} T_{x_3(1)} = \frac{H_3}{7} (T_{xh_2(1)} - T_{x_3(1)}) + \frac{H_4}{7} (T_{xc(1)} - T_{x_3(1)}) \quad (2.5-3)$$

$$C_x \frac{M_4}{7} \frac{d}{dt} T_{x_4(1)} = \frac{H_5}{7} (T_{xc(1)} - T_{x_4(1)}) \quad (2.5-4)$$

$$c_p \dot{W}_1 (T_{xhi} - T_{x_1(1)}) = \frac{H_1}{7} (T_{xh_1(1)} - T_{x_1(1)}) \quad (2.5-5)$$

$$c_p \dot{W}_1 (T_{x_2(2)} - T_{xho}) = \frac{H_2}{7} (T_{xh_2(1)} - T_{x_2(1)}) + \frac{H_3}{7} (T_{xh_2(1)} - T_{x_3(1)}) \quad (2.5-6)$$

$$c_p \dot{W}_2 (T_{dci} - T_{xc_1(1)}) = \frac{H_4}{7} (T_{xc(1)} - T_{x_3(1)}) + \frac{H_5}{7} (T_{xc(1)} - T_{x_4(1)}) \quad (2.5-7)$$

$$T_{xh_1(1)} = \frac{T_{xhi} + T_{x_1(1)}}{2} \quad (2.5-8)$$

$$T_{xh_2(1)} = \frac{T_{x_2(2)} + T_{xho}}{2} \quad (2.5-9)$$

$$T_{xc(1)} = \frac{T_{dci} + T_{xc_1(1)}}{2} \quad (2.5-10)$$

$$H_1 = H_{1k} \dot{W}_1^{0.8} \quad (2.5-11)$$

$$H_2 = H_{2k} \dot{W}_1^{0.8} \quad (2.5-12)$$

$$H_3 = H_{3k} \dot{W}_1^{0.8} \quad (2.5-13)$$

$$H_4 = H_{4k} \dot{W}_2^{0.55} \quad (2.5-14)$$

$$H_5 = H_{5k} \dot{W}_2^{0.55} \quad (2.5-15)$$

where

C_x	= specific heat of stainless steel (Btu/°R lb)
C_p	= specific heat of helium (Btu/°R lb)
M_1	= weight of first pass hot flow tube (lb)
M_2	= weight of insulating tube (lb)
M_3	= weight of flow divider (lb)
M_4	= shell weight (lb)
$\dot{W}_1$	= hot side helium flow rate (lb/sec)
$\dot{W}_2$	= cold side helium flow rate (lb/sec)
X_k	= conductivity across (helium gap) insulating tube (Btu/°R sec)
H_1	= heat transfer coefficient to first pass hot flow tube (Btu/°R sec)
H_2	= heat transfer coefficient return flow to insulating tube (Btu/°R sec)
H_3	= heat transfer coefficient return flow to flow divider (Btu/°R sec)
H_4	= heat transfer coefficient cold flow to flow divider (Btu/°R sec)
H_5	= heat transfer coefficient cold flow to shell (Btu/°R sec)
H_{1k}	= constant used in calculating heat transfer coefficients (Btu/°R sec (lb/sec) ^{0.8})
H_{2k}	= constant used in calculating heat transfer coefficients (Btu/°R sec (lb/sec) ^{0.8})
H_{3k}	= constant used in calculating heat transfer coefficients (Btu/°R sec (lb/sec) ^{0.8})
H_{4k}	= constant used in calculating heat transfer coefficients (Btu/°R sec (lb/sec) ^{0.55})
H_{5k}	= constant used in calculating heat transfer coefficients (Btu/°R sec (lb/sec) ^{0.55})

$$\begin{aligned}
T_{x_1(i)} &= \text{solid temperature of } i\text{th lump of first pass hot flow tube } (^{\circ}\text{R}) \\
T_{x_2(i)} &= \text{solid temperature of } i\text{th lump of insulating tube } (^{\circ}\text{R}) \\
T_{x_3(i)} &= \text{solid temperature of } i\text{th lump of flow divider } (^{\circ}\text{R}) \\
T_{x_4(i)} &= \text{solid temperature of } i\text{th lump of shell } (^{\circ}\text{R}) \\
T_{x_{h1}(i)} &= \text{average gas temperatures in first pass hot flow tube } (^{\circ}\text{R}) \\
T_{x_1(i)} &= \text{exit gas temperatures in first pass hot flow tube } (^{\circ}\text{R}) \\
T_{x_{h2}(i)} &= \text{average gas temperatures in hot side return flow } (^{\circ}\text{R}) \\
T_{x_2(i)} &= \text{exit gas temperatures in hot side return flow } (^{\circ}\text{R}) \\
T_{x_c(i)} &= \text{average gas temperatures in cold side flow } (^{\circ}\text{R}) \\
T_{x_{c1}(i)} &= \text{exit gas temperatures in cold side flow } (^{\circ}\text{R})
\end{aligned}$$

Relationships similar to those in Eqs. 2.5-1 through 2.5-10 can be written for each of the other two lumps of the heat exchanger.

2.6 System Piping

As noted in Fig. 1-1, the pipes between the heat exchanger and the heat dump are included in the mathematical model. The gas transit times in these pipes are negligible, but the effects of their heat capacity are not. The three-lump models used to represent the pipes are shown in Fig. 2-10.

Equations for the first lump of pipe No. 1 are shown below.

$$C_{p1} \frac{W_{p1}}{3} \frac{dT_{pm1}(1)}{dt} = \frac{H_{p1}}{3} (T_{p1g(1)} - T_{pm1}(1)) \quad (2.6-1)$$

$$T_{p1g(1)} = \frac{T_{p1i} + T_{p1(1)}}{2} \quad (2.6-2)$$

- Notes: 1. $T_{pg(i)}$ and $T_{pg(i)}$ are average gas temperatures.
 2. $Y_{(i)}$ are FORTRAN notation for solid temperatures.

Fig. 2-10 Pipe Models

$$C_{p1} \dot{W}_2 (T_{p1i} - T_{p1(1)}) = \frac{H_{p1}}{3} (T_{p1g(1)} - T_{pm1(1)}) \quad (2.6-3)$$

$$H_{p1} = h_{p1k} \dot{W}_2^{0.8} \quad (2.6-4)$$

where

- C_{p1} = specific heat of pipe No. 1 (Btu/lb °R)
- W_{p1} = weight of pipe No. 1 (lb)
- $T_{pm1(1)}$ = temperature of first lump of pipe No. 1 (°R)
- T_{p1i} = temperature of gas entering first lump of pipe No. 1 (°R)
- $T_{p1(1)}$ = temperature of gas leaving first lump of pipe No. 1 (°R)
- $T_{p1g(1)}$ = average temperature of gas in first lump of pipe No. 1 (°R)
- H_{p1} = heat transfer coefficient of pipe No. 1 (Btu/°R sec)
- h_{p1k} = heat transfer constant (Btu/lb °R (lb/sec)^{0.8})
- $\dot{W}_2$ = secondary loop helium gas flow rate (lb/sec)

Similar equations can be written for the two remaining lumps of pipe No. 1 and the three lumps of pipe No. 2.

2.7 Heat Dump

The heat dump is an air-cooled heat exchanger designed to transfer heat to the atmosphere at a maximum rate of 3 MW. It is composed of stainless steel finned tubes arranged in four horizontal banks inside a 13-foot cubical shroud. Two 5-foot-diameter fans below the tube banks force as much as 42.8 lb/sec of air past the tubes at maximum power. The rate of heat transfer to the atmosphere can be varied by adjusting the speed of the fans. A sketch of the heat dump model is shown in Fig. 2-11. The gas temperatures shown there are nominal ones associated with design conditions.

- Notes: 1. $T_{dh(i)}$ and $T_{dc(i)}$ are average gas temperatures.
 2. $Y_{(i)}$ are FORTRAN notation for solid temperatures.

Fig. 2-11 Heat Dump Model

Equations for the first lump of the heat dump are shown below.

$$C_d \frac{W_d}{4} \frac{dT_{d(1)}}{dt} = \frac{H_{dh}}{4} (T_{dh(1)} - T_{d(1)}) + \frac{H_{dc}}{4} (T_{dc(1)} - T_{d(1)}) \quad (2.7-1)$$

$$T_{dh(1)} = \frac{T_{dhi} + 1.25T_{dh_1(1)}}{2.25} \quad (2.7-2)$$

$$T_{dc(1)} = \frac{T_{dc_1(1)} + T_{dc_1(2)}}{2} \quad (2.7-3)$$

$$C_p \dot{W}_2 (T_{dhi} - T_{dh_1(1)}) = \frac{H_{dh}}{4} (T_{dh(1)} - T_{d(1)}) \quad (2.7-4)$$

$$C_a \dot{W}_3 (T_{dc_1(2)} - T_{dc_1(1)}) = \frac{H_{dc}}{4} (T_{dc(1)} - T_{d(1)}) \quad (2.7-5)$$

$$H_{dh} = h_{dhk} \dot{W}_2^{0.8} \quad (2.7-6)$$

$$H_{dc} = h_{dck} \dot{W}_3^{0.725} \quad (2.7-7)$$

where

- C_d = specific heat of the heat dump solid (Btu/lb °R)
- C_p = specific heat of helium gas (Btu/lb °R)
- C_a = specific heat of air (Btu/lb °R)
- $T_{d(1)}$ = temperature of first lump of heat dump (°R)
- T_{dhi} = temperature of hot helium gas entering first lump of heat dump (°R)
- W_d = weight of stainless steel in heat dump (lb)
- $T_{dh_1(1)}$ = temperature of hot helium gas leaving first lump of heat dump (°R)
- $T_{dh(1)}$ = average temperature of hot helium gas in first lump of heat dump (°R)

- $T_{dc1}(2)$ = temperature of cold air entering first lump of heat dump ($^{\circ}\text{R}$)
 $T_{dc1}(1)$ = temperature of cold air leaving first lump of heat dump ($^{\circ}\text{R}$)
 $T_{dc}(1)$ = average temperature of air in first lump of heat dump ($^{\circ}\text{R}$)
 H_{dh} = heat transfer coefficient between hot gas and heat dump ($\text{Btu}/^{\circ}\text{R sec}$)
 H_{dc} = heat transfer coefficient between cold gas and heat dump ($\text{Btu}/^{\circ}\text{R sec}$)
 h_{dhk} = heat transfer constant ($\text{Btu}/^{\circ}\text{R sec (lb/sec)}^{0.8}$)
 h_{dck} = heat transfer constant ($\text{Btu}/^{\circ}\text{R sec (lb/sec)}^{0.725}$)
 $\dot{W}_2$ = secondary loop helium gas flow rate (lb/sec)
 $\dot{W}_3$ = heat dump air flow rate (lb/sec)

These equations are similar to ones shown in previous sections. Note, however, that the average temperature of the hot side gas is weighted toward the outlet temperature, as discussed in Section 2.1.2.

Equations like those shown above can be written for the other lumps of the heat dump.

2.8 Connective Equations

The nine equations shown below relate the system components to one another. Specifically, these equations define inlet gas temperatures of system components in terms of outlet gas temperatures of other components.

$$T_i = T_{ai} = T_{ei} = T_{rco}(1 - \text{FB}) + T_{rci}(1 - \text{FB}) \quad (2.8-1)$$

$$T_{rhi} = T_o \quad (2.8-2)$$

$$T_{rci} = T_{xho} \quad (2.8-3)$$

$$T_{xhi} = T_{rho} \quad (2.8-4)$$

$$T_{xci} = T_{p2o} \quad (2.8-5)$$

$$T_{pli} = T_{xco} \quad (2.8-6)$$

$$T_{p2i} = T_{dho} \quad (2.8-7)$$

$$T_{dhi} = T_{plo} \quad (2.8-8)$$

$$T_{dci} = T_{air} \quad (2.8-9)$$

All terms shown above are defined in previous figures or equations and most of them are also shown in Fig. 1-1.

As noted in Section 3, one can use UHXCOM to study the components separately, in a "disconnected" mode. In this mode the connective equations shown above are omitted, and the inlet gas temperatures to the components are treated as independent variables. Whereas the system itself cannot really be run in a disconnected mode, the model can; operation of the model in the disconnected mode allows one to make valuable checks on the simulation of individual system components.

2.9 System Pressure Drops

The pressure drop in each system component is assumed to have the following general form.*

$$\Delta P = k \frac{T}{P} \dot{W}^2 \Delta P' \quad (2.9-1)$$

ΔP = pressure drop at gas temperature T , flow rate $\dot{W}$, and pressure P (psia)

$\Delta P'$ = pressure drop at gas temperature flow rate and pressures given in Fig. 1-1 (psia)

T = weighted average gas temperature in component ($^{\circ}R$)

P = coolant gas pressure in component (psia)

$\dot{W}$ = coolant flow rate (lb/sec)

k = constant (-).

* The change in friction factor is sufficiently small to justify its being considered a constant.

Significant pressure drops in the primary loop occur in the reactor, the hot side of the recuperator, the hot side of the heat exchanger, the outlet pipe on the hot side of the heat exchanger,* and the cold side of the recuperator. These pressure drops are calculated as shown below.

$$\Delta P_p = \frac{\dot{W}_1^2}{P_p} \left[0.015 \left(\frac{T_{ai} + T_{rhi}}{2} \right) + 0.063 \left(\frac{T_{rhi} + 1.5 T_{rh1(3)}}{2.5} \right) + 0.09 \left(\frac{T_{rh1(3)} + T_{x2(1)}}{2} \right) + 0.0261 T_{x2(1)} + 0.063(1 - FB)^2 \left(\frac{T_{x2(1)} + T_{rc1(1)}}{2} \right) \right] \quad (2.9-2)$$

ΔP_p = primary loop pressure drop (psia)

$\dot{W}_1$ = primary loop coolant flow rate (lb/sec)

P_p = primary loop coolant pressure (psia)

T_{ai} = reactor inlet gas temperature ($^{\circ}R$)

$T_{rh1(3)}$ = recuperator hot inlet gas temperature, which is equal to reactor outlet gas temperature ($^{\circ}R$)

$T_{x2(1)}$ = heat exchanger hot side outlet gas temperature ($^{\circ}R$)

$T_{rc1(1)}$ = recuperator cold side outlet gas temperature ($^{\circ}R$).

Significant pressure drops in the secondary loop occur in the shell side of the heat exchanger, pipe No. 1, the hot side of the heat dump, and pipe No. 2. These drops are calculated as shown in Eq. 2.9-3.

$$P_s = \frac{\dot{W}_2^2}{P_s} \left[0.141 \left(\frac{T_{p2(1)} + T_{xc1(7)}}{2} \right) + 0.034 \left(\frac{T_{xc1(7)} + T_{p1(3)}}{2} \right) + 0.103 \left(\frac{T_{p1(3)} + 2T_{dh1(4)}}{3} \right) + 0.0717 \left(\frac{T_{dh1(4)} + T_{p2(1)}}{2} \right) \right] \quad (2.9-3)$$

* This component has a pressure drop which is barely large enough to justify its inclusion. It has little effect on gas temperature and for that reason is not shown in the model of Fig. 1-1.

ΔP_s = secondary loop pressure drop (psia)
 $\dot{W}_2$ = secondary loop coolant flow rate (lb/sec)
 P_1 = primary loop coolant pressure (psia)
 $T_{p2}(1)$ = pipe No. 2 outlet gas temperature ($^{\circ}R$)
 $T_{xc1}(7)$ = heat exchanger shell side outlet gas temperature ($^{\circ}R$)
 $T_{p1}(3)$ = pipe No. 1 outlet gas temperature ($^{\circ}R$)
 $T_{dh1}(4)$ = heat dump hot side outlet gas temperature ($^{\circ}R$)
 $T_{p2}(1)$ = pipe No. 2 outlet gas temperature ($^{\circ}R$)

The constants in Eqs. 2.9-2 and 2.9-3 are chosen so that the pressure drops in each component are those calculated in Appendix B when the gas temperatures, coolant flow rates, and gas pressures are those shown in Fig. 1-1.

It should be noted that these expressions for pressure drop are only approximate. The pressure drop functional variation with flow rate, gas temperature, and pressure is close to that given by Eqs. 2.9-1, 2.9-2, and 2.9-3, and it is anticipated that the dynamic behavior of the system will be quite similar to that predicted by the simulation. The pressure drop calculations and the expressions given in Eqs. 2.9-2 and 2.9-3 will be checked experimentally and adjusted, if necessary, prior to critical operation of the reactor.

2.10 Blowers

Helium in the primary and secondary coolant loops is circulated by two identical centrifugal impeller blowers. Each blower shaft is supported on two self-acting hydrodynamic journal bearings and a hydrodynamic thrust bearing and is driven by a squirrel cage induction motor. Power for the motor is furnished by a 200-cps ac generator whose shaft is coupled to a 60-cps ac motor by means of an eddy current clutch. Blower speed can be varied by controlling the slippage in the magnetic clutch, and thereby changing the speed of the 200-cps generator and the frequency of the voltage supplied to the blower motor. This system

for producing desired blower speeds has an extremely small response time compared to the main system time constants, and the main system has little feedback effect on the blower speed system. Consequently, the blower speed system is not included in the model, and both blower speeds are considered independent variables.

In actual operation the blower speed control equipment may be slaved to an automatic flow control system. A crude flow control system, described in Section 3.2.3.2, has been included in the UHXCOM program as a calculational convenience.

From consideration of similarity requirements and experimental observation, the performance characteristics of the blowers in the primary and secondary loops may be described by a single-valued function of the form

$$\psi = f(\phi) \quad (2.10-1)$$

where

$$\psi = \text{head coefficient } (-)$$

$$\phi = \text{flow coefficient } (-)$$

and ψ and ϕ are as given below.

$$\psi = \frac{H \cdot 2g}{U_t^2} \quad (2.10-2)$$

$$\phi = \frac{4Q_m}{U_t D_t^2} \quad (2.10-3)$$

$$H = \text{head in feet} \approx \frac{144 \Delta P}{\rho_g} \text{ (ft)}$$

$$Q_m = \text{inlet volume flow} = \frac{\dot{W}}{3600 \rho_g} \text{ (ft}^3\text{/sec)}$$

$$g = \text{gravitational constant} = 32.2 \text{ (ft/sec}^2\text{)}$$

$$U_t = \text{tip speed (ft/sec)}$$

D_t = tip diameter (ft)

$\dot{W}$ = flow rate (lb/sec)

ΔP = pressure drop (lb)

ρ_g = density (lb/ft³)

A curve relating ψ and ϕ for the UHTREX blowers was found experimentally. It is shown in Fig. 2-12. For use in the UHXCOM program, the experimental data are approximated by six straight lines.

A curve of blower efficiency versus flow coefficient, also obtained experimentally, is shown in Fig. 2-13. Blower horsepower is obtained from the expression

$$H_p = \frac{1}{\eta} \frac{H Q_m \rho_g}{60 \times 550} \quad (2.10-4)$$

where H , Q_m , and ρ_g are defined above and η is blower efficiency. At steady-state operation the primary and secondary blowers are limited to 60 horsepower.

Fig. 2-12 Performance of the 60-hp UHTREX Prototype Blower

Fig. 2-13 Efficiency of the 60-hp UHTREX Prototype Blower

3. COMPUTER PROGRAMS

The mathematical model of the UHTREX system described in the previous section has been investigated with a variety of digital computer programs. Some of these programs are discussed below. UHXCOM, the overall system simulation program, is discussed in considerable detail.

3.1 Frequency Domain Equations and Programs

There are a number of steps that must be completed before a "frequency domain" program can be written. A mathematical model must be defined, the system equations must be linearized, and the Laplace transforms of these equations must be obtained; then a computer program can be written to solve the linearized transformed equations for the system transfer functions.

The mathematical model used for the frequency domain programs is the one presented in Section 2, except that for simplicity the blowers and pressure drops have been omitted. Thus, the coolant flow rates are treated as independent variables. It is possible that the real system will be operated in just this fashion; that is, with flow rate demands as independent variables and blower speeds adjusted to produce zero coolant flow rate errors. In any case, the blowers do not make a big difference in system behavior, and since the UHXCOM simulation program can be run without blowers if desired, transfer functions from the frequency domain program still can be checked with transfer functions obtained from UHXCOM.

The linearization and Laplace transformation of all the lumped parameter equations in the mathematical model is a lengthy process. A short example is given below in Section 3.1.1 where the equations for a one-lump heat exchanger are linearized and transformed into a set of equations that can be handled by a frequency domain program. These programs are discussed in Section 3.1.3.

Additional frequency domain programs, based on equations for distributed parameter mathematical models, have also been written. The distributed parameter equations for the heat dump and the Laplace transformed equations that can be derived* from them are presented in Section 3.1.2. In general, the material presented here is included only to illustrate how the required frequency domain equations were developed, what the programs calculate, and how their results have been used.

3.1.1 Transfer Function Derivation

A heat dump derivation has been chosen to illustrate the methods used in obtaining transfer functions for the UHTREX components. The objective of the derivation presented below is to obtain equations that allow the calculation of the eight transfer functions relating any one of the two dependent variables

1. Temperature of the heat dump hot outlet gas
2. Temperature of the heat dump cold outlet gas

to any one of the four independent variables

1. Temperature of the heat dump hot inlet gas
2. Temperature of the heat dump cold inlet gas
3. Flow rate of helium gas in the secondary loop
4. Air flow rate through heat dump.

* A sample derivation is given in Appendix F.

The model for the nth lump of an ℓ -lump counterflow heat dump is shown in Fig. 3-1. The basic equations are given below.

$$C_d \frac{W_d}{\ell} \frac{dT_{d(n)}}{dt} = \frac{H_{dh}}{\ell} (T_{dh(n)} - T_{d(n)}) + \frac{H_{dc}}{\ell} (T_{dc(n)} - T_{d(n)}) \quad (3.1-1)$$

$$T_{dh(n)} = \frac{T_{dh1(n-1)} + k_{dhot} T_{dh1(n)}}{k_{dhot} + 1} \quad (3.1-2)$$

$$T_{dc(n)} = \frac{T_{dc1(n+1)} + k_{dcold} T_{dc1(n)}}{k_{dcold} + 1} \quad (3.1-3)$$

$$C_p \dot{W}_2 (T_{dh1(n-1)} - T_{dh1(n)}) = \frac{H_{dh}}{\ell} (T_{dh(n)} - T_{d(n)}) \quad (3.1-4)$$

$$C_a \dot{W}_3 (T_{dc1(n+1)} - T_{dc1(n)}) = \frac{H_{dc}}{\ell} (T_{dc(n)} - T_{d(n)}) \quad (3.1-5)$$

$$H_{dh} = h_{dhk} \dot{W}_2^{0.8} \quad (3.1-6)$$

$$H_{dc} = h_{dck} \dot{W}_3^{0.725} \quad (3.1-7)$$

Fig. 3-1 Nth Lump of Heat Dump

where

ℓ	= number of lumps in model (—)
C_d	= specific heat of the heat dump solid (Btu/lb °R)
C_p	= specific heat of helium gas (Btu/lb °R)
C_a	= specific heat of air (Btu/lb °R)
$T_{d(n)}$	= temperature of nth lump of heat dump (°R)
$T_{dh1(n-1)}$	= temperature of hot helium gas entering the nth lump of heat dump (°R)
$T_{dh1(n)}$	= temperature of hot helium gas leaving nth lump of heat dump (°R)
$T_{dh(n)}$	= average temperature of hot helium gas in nth lump of heat dump (°R)
$T_{dcl(n+1)}$	= temperature of cold air entering nth lump of heat dump (°R)
$T_{dcl(n)}$	= temperature of cold air leaving nth lump of heat dump (°R)
$T_{dc(n)}$	= average temperature of cold air in nth lump of heat dump (°R)
H_{dh}	= heat transfer coefficient between hot gas and heat dump (Btu/°R sec)
H_{dc}	= heat transfer coefficient between cold air and heat dump (Btu/°R sec)
h_{dhk}	= heat transfer constant (Btu/°R sec (lb/sec) ^{0.8})
h_{dck}	= heat transfer constant (Btu/°R sec (lb/sec) ^{0.725})
$\dot{W}_2$	= secondary loop helium gas flow rate (lb/sec)
$\dot{W}_3$	= heat dump air flow rate (lb/sec)
k_{dhot}	= heat dump average gas temperature weighting factor, hot helium gas side (—)
k_{dcold}	= heat dump average air temperature weighting factor, cold air side (—)
W_d	= weight of heat dump stainless steel (lb)

These equations are nonlinear in flow rate because the flow rate appears in the heat transfer coefficients. The equations are made linear and Laplace transformed below.

To linearize the basic equations, assume a small perturbation in each variable. Let

$$\dot{W} = \dot{W}_o + \Delta\dot{W} \quad (3.1-8)$$

$$T = T_o + \Delta T \quad (3.1-9)$$

$$H = H_o + \Delta H \quad (3.1-10)$$

where the subscript "o" indicates the steady-state value of the variable and the Δ 's denote small perturbations about the steady-state values.

In Eq. 3.1-10, ΔH will depend on ΔW , for H is a function of W_o . Equations 3.1-1, 3.1-2, and 3.1-3 can be combined to give Eq. 3.1-11.

Equation 3.1-11 is on the following page.

$$\begin{aligned}
C_d W_d \frac{dT_{do}(n)}{dt} + C_d W_d \frac{d\Delta T_d(n)}{dt} = & H_{dho} \left[\frac{T_{dh1o}(n-1) + k_{dhot} T_{dh1o}(n)}{k_{dhot} + 1} - T_{do}(n) \right] \\
& + H_{dco} \left[\frac{T_{dc1o}(n+1) + k_{dcold} T_{dc1o}(n)}{k_{dcold} + 1} - T_{do}(n) \right] \\
& + H_{dho} \left[\frac{\Delta T_{dh1}(n-1) + k_{dhot} \Delta T_{dh1}(n)}{k_{dhot} + 1} - \Delta T_d(n) \right] \\
& + H_{dco} \left[\frac{\Delta T_{dc1}(n+1) + k_{dcold} \Delta T_{dc1}(n)}{k_{dcold} + 1} - \Delta T_d(n) \right] \\
& + \Delta H_{dh} \left[\frac{T_{dh1o}(n-1) + k_{dhot} T_{dh1o}(n)}{k_{dhot} + 1} - T_{do}(n) \right] \\
& + \Delta H_{dc} \left[\frac{T_{dc1o}(n+1) + k_{dcold} T_{dc1o}(n)}{k_{dcold} + 1} - T_{do}(n) \right] \\
& + \Delta H_{dh} \left[\frac{\Delta T_{dh1}(n-1) + k_{dhot} \Delta T_{dh1}(n)}{k_{dhot} + 1} - \Delta T_d(n) \right] \\
& + \Delta H_{dc} \left[\frac{\Delta T_{dc1}(n+1) + k_{dcold} \Delta T_{dc1}(n)}{k_{dcold} + 1} - \Delta T_d(n) \right]
\end{aligned}
\tag{3.1-11}$$

This equation can be simplified by:

1. Dropping all steady-state terms which, when taken together, sum to zero.
2. Dropping all second order differentials.

The simplified equation is shown below.

$$\begin{aligned}
& C_d W_d \frac{d\Delta T_d(n)}{dt} + (H_{dho} + H_{dco}) \Delta T_d(n) \\
&= \frac{H_{dho}}{k_{dhot} + 1} \left[\Delta T_{dh1(n-1)} + k_{dhot} \Delta T_{dh1(n)} \right] \\
&+ \frac{H_{dco}}{k_{dcold} + 1} \left[\Delta T_{dc1(n+1)} + k_{dcold} \Delta T_{dc1(n)} \right] \\
&+ \Delta H_{dh} \left[\frac{T_{dh1o(n-1)} + k_{dhot} T_{dh1o(n)}}{k_{dhot} + 1} - T_{do(n)} \right] \\
&+ \Delta H_{dc} \left[\frac{T_{dc1o(n+1)} + k_{dcold} T_{dc1o(n)}}{k_{dcold} + 1} - T_{do(n)} \right] \quad (3.1-12)
\end{aligned}$$

Equations 3.1-4 and 3.1-5 can be treated in a similar manner to yield

$$\begin{aligned}
& \ell C_p \left[T_{dh1o(n-1)} - T_{dh1o(n)} \right] \Delta \dot{W}_2 \\
&= \left[\frac{H_{dho}}{k_{dhot} + 1} - \ell C_p \dot{W}_{20} \right] \Delta T_{dh1(n-1)} + \left[\frac{k_{dhot} H_{dho}}{k_{dhot} + 1} + \ell C_p \dot{W}_{20} \right] \Delta T_{dh1(n)} \\
&+ \Delta H_{dh} \left[\frac{T_{dh1o(n-1)} + k_{dhot} T_{dh1o(n)}}{k_{dhot} + 1} - T_{do(n)} \right] - H_{dho} \Delta T_d(n) \quad (3.1-13)
\end{aligned}$$

and

$$\begin{aligned}
& \ell C_a \left[T_{dc1o(n+1)} - T_{dc1o(n)} \right] \Delta \dot{W}_3 \\
&= \left[\frac{H_{dco}}{k_{dcold} + 1} - \ell C_a \dot{W}_{30} \right] \Delta T_{dc1(n+1)} + \left[\frac{k_{dcold} H_{dco}}{k_{dcold} + 1} + \ell C_a \dot{W}_{30} \right] \Delta T_{dc1(n)} \\
&+ \Delta H_{dc} \left[\frac{T_{dc1o(n+1)} + k_{dcold} T_{dc1o(n)}}{k_{dcold} + 1} - T_{do(n)} \right] - H_{dco} \Delta T_d(n) \quad (3.1-14)
\end{aligned}$$

Equation 3.1-6 becomes

$$H_{dho} + \Delta H_{dho} = h_{dhk} \left[\dot{W}_{2o}^{0.8} + 0.8 \dot{W}_{2o}^{-0.2} \Delta \dot{W}_2 + \frac{(0.8)(-0.2)}{2!} \dot{W}_{2o}^{-1.2} \Delta \dot{W}_2^2 + \dots \right] \quad (3.1-15)$$

Dropping appropriate terms, one obtains

$$\Delta H_{dh} = \frac{0.8 h_{dhk}}{\dot{W}_{2o}^{0.2}} \Delta \dot{W}_2. \quad (3.1-16)$$

Similarly, Eq. 3.1-7 yields

$$\Delta H_{dc} = \frac{0.725 h_{dck}}{\dot{W}_{3o}^{0.275}} \Delta \dot{W}_3. \quad (3.1-17)$$

The substitution of Eqs. 3.1-16 and 3.1-17 into Eqs. 3.1-12, 3.1-13 and 3.1-14 gives the following results.

$$\begin{aligned} C_d W_d \frac{d\Delta T_d(n)}{dt} + (H_{dho} + H_{dco}) \Delta T_d(n) \\ = \frac{H_{dho}}{k_{dhot} + 1} \left[\Delta T_{dh1}(n-1) + k_{dhot} \Delta T_{dh1}(n) \right] \\ + \frac{H_{dco}}{k_{dcold} + 1} \left[\Delta T_{dc1}(n+1) + k_{dcold} \Delta T_{dc1}(n) \right] + D_{1(n)} \Delta \dot{W}_2 + D_{2(n)} \Delta \dot{W}_3 \end{aligned} \quad (3.1-18)$$

$$D_{3(n)} \Delta \dot{W}_2 = D_5 \Delta \bar{T}_{dh1}(n-1) + D_6 \Delta T_{dh1}(n) - H_{dho} T_d(n) \quad (3.1-19)$$

and

$$D_{4(n)} \Delta \dot{W}_3 = D_7 \Delta T_{dc1}(n+1) + D_8 \Delta T_{dc1}(n) - H_{dco} \Delta T_d(n) \quad (3.1-20)$$

where

$$D_1(n) = \left[\frac{0.8 h_{dhk}}{\dot{W}_{2o}^{0.2}} \right] \left[\frac{T_{dh1o(n-1)} + k_{dhot} T_{dh1o(n)}}{k_{dhot} + 1} - T_{do(n)} \right] \quad (3.1-21)$$

$$D_2(n) = \left[\frac{0.725 h_{dck}}{\dot{W}_{3o}^{0.275}} \right] \left[\frac{T_{dc1o(n+1)} + k_{dcold} T_{dc1o(n)}}{k_{dcold} + 1} - T_{do(n)} \right] \quad (3.1-22)$$

$$D_3(n) = \ell C_p \left[T_{dh1o(n-1)} - T_{dh1o(n)} \right] - D_1(n) \quad (3.1-23)$$

$$D_4(n) = \ell C_a \left[T_{dc1o(n+1)} - T_{dc1o(n)} \right] - D_2(n) \quad (3.1-24)$$

$$D_5 = \frac{H_{dho}}{k_{dhot} + 1} - \ell C_p \dot{W}_{2o} \quad (3.1-25)$$

$$D_6 = \frac{k_{dhot} H_{dho}}{k_{dhot} + 1} + \ell C_p \dot{W}_{2o} \quad (3.1-26)$$

$$D_7 = \frac{H_{dco}}{k_{dcold} + 1} - \ell C_a \dot{W}_{3o} \quad (3.1-27)$$

$$D_8 = \frac{k_{dcold} H_{dco}}{k_{dcold} + 1} + \ell C_a \dot{W}_{3o} \quad (3.1-28)$$

Equations 3.1-18, 3.1-19 and 3.1-20 are the linear differential equations for the nth lump of an ℓ -lump heat dump. The next step toward obtaining the transfer functions is to take the Laplace transform of these equations. When this transformation is carried out and the variable $\Delta T_{d(n)}(s)$ is eliminated, the heat dump equations* become

* For finite perturbations of the independent variables of this model ($\dot{W}_2, \dot{W}_3, T_{dh1(n-1)}$ and $T_{dc1(n+1)}$) the value of $\Delta T_{d(n)}$ at $t = 0$ is zero. Thus, the constant $\Delta T_{d(n)}(0^+)$ does not appear in Eq. (3.1-29).

$$\begin{aligned}
& \left[-D_1(n) \right] \Delta \dot{W}_2 + \left[-\frac{U_4(n)}{H_{dco}} (C_d W_{ds} + H_{dho} + H_{dco}) - D_2(n) \right] \Delta \dot{W}_3 \\
& + \left[-\frac{H_{dho}}{k_{dhot} + 1} \right] \Delta T_{dh1(n-1)} \\
& + \left[\frac{D_7}{H_{dco}} (C_d W_{ds} + H_{dho} + H_{dco}) - \frac{H_{dco}}{k_{dcold} + 1} \right] \Delta T_{dc1(n+1)} \\
& = \left[\frac{k_{dhot} H_{dho}}{k_{dhot} + 1} \right] \Delta T_{dh1(n)} \\
& + \left[-\frac{D_8}{H_{dco}} (C_d W_{ds} + H_{dho} + H_{dco}) + \frac{k_{dcold} H_{dco}}{k_{dcold} + 1} \right] \Delta T_{dc1(n)} \quad (3.1-29)
\end{aligned}$$

and

$$\begin{aligned}
& \left[D_3(n) \right] \Delta \dot{W}_2 + \left[-\frac{U_4(n) H_{dho}}{H_{dco}} \right] \Delta \dot{W}_3 + \left[-D_5 \right] \Delta T_{dh1(n-1)} \\
& + \left[\frac{D_7 H_{dho}}{H_{dco}} \right] \Delta T_{dc1(n+1)} = \left[D_6 \right] \Delta T_{dh1(n)} + \left[-\frac{D_8 H_{dho}}{H_{dco}} \right] \Delta T_{dc1(n)} \quad (3.1-30)
\end{aligned}$$

One should note that in these equations, $\Delta \dot{W}_2$, $\Delta \dot{W}_3$, $\Delta T_{dh1(n)}$, etc. are the transformed functions $\Delta \dot{W}_2(s)$, $\Delta \dot{W}_3(s)$, $\Delta T_{dh1(n)}(s)$, etc. The argument has been omitted for brevity.

Let

$$X1R_{(n)} = - D_3(n) \quad (3.1-31)$$

$$X2R = D_6 \quad (3.1-32)$$

$$X3R_{(n)} = D_1(n) \quad (3.1-33)$$

$$X4R_{(n)} = - \frac{D_4(n)(H_{dho} + H_{dco})}{H_{dco}} - D_2(n) \quad (3.1-34)$$

$$X4I_{(n)} = - \frac{D_4(n)C_d W_d}{H_{dco}} \quad (3.1-35)$$

$$X5R = \frac{D_8(H_{dho} + H_{dco})}{H_{dco}} - \frac{k_{dcold}H_{dco}}{k_{dcold} + 1} \quad (3.1-36)$$

$$X5I = \frac{D_8 C_d W_d}{H_{dco}} \quad (3.1-37)$$

$$X6R = \frac{D_8 H_{dho}}{H_{dco}} \quad (3.1-38)$$

$$X7R_{(n)} = - \frac{D_4(n)H_{dho}}{H_{dco}} \quad (3.1-39)$$

$$X8R = \frac{k_{dhot}H_{dho}}{k_{dhot} + 1} \quad (3.1-40)$$

$$X9R = \frac{H_{dho}}{k_{dhot} + 1} \quad (3.1-41)$$

$$X10R = \frac{D_7(H_{dho} + H_{dco})}{H_{dco}} - \frac{H_{dco}}{k_{dcold} + 1} \quad (3.1-42)$$

$$X_{10I} = \frac{D_7 C_d W_d}{H_{dco}} \quad (3.1-43)$$

$$X_{11R} = D_5 \quad (3.1-44)$$

$$X_{12R} = \frac{D_7 H_{dho}}{H_{dco}} \quad (3.1-45)$$

When these variables are substituted into Eqs. 3.1-29 and 3.1-30, the transformed equations for the nth lump of the heat dump become

$$\begin{aligned} & (-X_{3R}(n))\Delta\dot{W}_2 + (X_{4I}(n)s + X_{4R}(n))\Delta\dot{W}_3 + (-X_{9R})\Delta T_{dh1(n-1)} \\ & + (X_{10Is} + X_{10R})\Delta T_{dc1(n+1)} = (X_{8R})\Delta T_{dh1(n)} + (-X_{5Is} - X_{5R})\Delta T_{dc1(n)} \end{aligned} \quad (3.1-46)$$

$$\begin{aligned} & (-X_{1R}(n))\Delta\dot{W}_2 + (X_{7R}(n))\Delta\dot{W}_3 + (-X_{11R})\Delta T_{dh1(n-1)} \\ & + (X_{12R})\Delta T_{dc1(n+1)} = (X_{2R})\Delta T_{dh1(n)} + (-X_{6R})\Delta T_{dc1(n)} \end{aligned} \quad (3.1-47)$$

At this point there are two equations, each with the Laplace transforms of the perturbations of the four independent variables on the left-hand side and the transforms of the perturbations of the two dependent variables on the right side. The transfer function between a dependent variable and any of the independent variables can be obtained from the simultaneous solution of these equations. For instance, by eliminating $\Delta T_{dc1(n)}$ from Eqs. 3.1-46 and 3.1-47 one obtains

$$\begin{aligned}
& \left[\frac{-X3R(n)}{-X5Is - X5R} + \frac{X1R}{-X6R} \right] \Delta \dot{W}_2 + \left[\frac{X4I(n)s + X4R(n)}{-X5Is - X5R} + \frac{-X7R(n)}{-X6R} \right] \Delta \dot{W}_3 \\
& + \left[\frac{-X9R}{-X5Is - X5R} + \frac{X11R}{-X6R} \right] \Delta T_{dh1(n-1)} + \left[\frac{X10Is + X10R}{-X5Is - X5R} + \frac{-X12R}{-X6R} \right] \Delta T_{dc1(n+1)} \\
& = \left[\frac{X8R}{-X5Is - X5R} + \frac{-X2R}{-X6R} \right] \Delta T_{dh1(n)} \quad (3.1-48)
\end{aligned}$$

To obtain the transfer function between $\dot{W}_2$ and $T_{dh1(n)}$, let

$$\Delta \dot{W}_3 = \Delta T_{dh1(n-1)} = \Delta T_{dc1(n+1)} = 0 \quad (3.1-49)$$

and solve for the quotient $\Delta T_{dh1(n)} / \Delta \dot{W}_2$. Thus,

$$\frac{\Delta T_{dh1(n)}(s)}{\Delta \dot{W}_2(s)} = \frac{\frac{X3R(n)}{X5Is + X5R} - \frac{X1R}{X6R}}{-\frac{X8R}{X5Is + X5R} + \frac{X2R}{X6R}} \quad (3.1-50)$$

The transfer function between gas flow rate in the secondary loop and the temperature of the hot outlet gas is given by Eq. 3.1-50. For a two-lump model there would be four equations similar to Eqs. 3.1-46 and 3.1-47, the process of solving the larger matrix would be more complicated, and the resulting transfer function would be more complicated.

A complex matrix solution was used to calculate the transfer functions for ℓ -lump models using ℓ sets of basic equations plus appropriate connecting equations. The connecting equations tie the lumps together and are of the form $T_{in}(\text{ith lump}) = T_{out}(\text{previous lump})$. After substitution of the connecting equations, a 4-lump model of the heat dump gives an 8 x 8 matrix, in which some of the elements are complex numbers. This complex matrix is discussed more fully in Section 3.1.3.

Similar sets of transfer functions have been developed for the reactor heat exchanger, reactor neutron kinetics, recuperator, primary-secondary heat exchanger, and the pipes. The simultaneous solution of all these equations is also discussed in Section 3.1.3.

3.1.2 Distributed Parameter Equations

Temperature transfer functions can also be obtained from the distributed parameter equations for various systems. These transfer functions can be compared with those from the lumped models to check the validity of the lumped model approach. This section presents the distributed parameter equations for the heat dump and shows the form of the transfer functions that can be derived from them. The derivation itself is given in Appendix F.

In the distributed parameter model of the heat dump, the solid and gas temperatures vary throughout the system as continuous functions of time and space. Thus, the physical length of the heat dump and the gas flow velocity are included. These terms, in the form length/velocity, represent a pure time delay equivalent to the transit time required for the gas to flow through the model.

The distributed parameter heat dump model is shown in Fig. 3-2. The equations used in the derivation of the distributed parameter transfer function for the heat dump are given below.

$$C_d W_d \frac{dT_d(x,t)}{dt} = H_{dh} \left[T_{dh}(x,t) - T_d(x,t) \right] - H_{dc} \left[T_d(x,t) - T_{dc}(x,t) \right] \quad (3.1-51)$$

$$q_h(x,t) = H_{dh} \left[T_{dh}(x,t) - T_d(x,t) \right] \quad (3.1-52)$$

$$q_c(x,t) = H_{dc} \left[T_d(x,t) - T_{dc}(x,t) \right] \quad (3.1-53)$$

Fig. 3-2 Distributed Parameter Heat Dump

$$H_h(x,t) = \rho_h C_p A_1 T_{dh}(x,t) = \frac{\dot{W}_2 C_p}{V_h} T_{dh}(x,t) \quad (3.1-54)$$

$$H_c(x,t) = \rho_c C_a A_2 T_{dc}(x,t) = \frac{\dot{W}_3 C_a}{V_c} T_{dc}(x,t) \quad (3.1-55)$$

$$V_h H_h(o,t) - V_h H_h(l, t-l/V_h) = \int_l^0 q_h(x, t-x/V_h) dx \quad (3.1-56)$$

$$V_c H_c(l,t) - V_c H_c(o, t-l/V_c) = \int_l^0 q_c(x, t-l/V_c + x/V_c) dx \quad (3.1-57)$$

where

- C_d = specific heat of the heat dump solid (Btu/lb °R)
- C_p = specific heat of helium gas (Btu/lb °R)
- C_a = specific heat of air (Btu/lb °R)
- $T_d(x,t)$ = temperature of heat dump solid at point x, time t (°R)
- $T_{dh}(x,t)$ = temperature of hot helium gas at point x, time t (°R)
- $T_{dc}(x,t)$ = temperature of cold air at point x, time t (°R)
- H_{dh} = heat transfer coefficient between hot gas and heat dump per foot of length (Btu/°R sec ft)
- H_{dc} = heat transfer coefficient between cold air and heat dump per foot of length (Btu/°R sec ft)
- $\dot{W}_2$ = secondary loop helium gas flow rate (lb/sec)
- $\dot{W}_3$ = heat dump air flow rate (lb/sec)
- V_h = secondary loop helium gas velocity (ft/sec)
- V_c = heat dump air velocity (ft/sec)
- A_1 = cross sectional area of helium gas passage (ft²)
- A_2 = cross sectional area of air passage (ft²)
- ρ_h = density of helium gas (lb/ft³)
- ρ_c = density of air (lb/ft³)
- q_h = heat given up by hot helium gas per foot of length (Btu/sec ft)

q_c	= heat taken in by cold air per foot of length (Btu/sec ft)
H_h	= heat content of hot helium per foot of length (Btu/ft)
H_c	= heat content of cold air per foot of length (Btu/ft)
W_d	= weight of heat dump solid per foot of length (lb/ft)
t	= time (sec)
x	= axial distance along heat dump (ft)
ℓ	= specific value of x (ft)
L	= length of heat dump (ft)

These equations can be transformed and juggled as shown in Appendix F to obtain the transfer functions given below in Eqs. 3.1-58 to 3.1-61.

$$\frac{T_{\text{hot out}}(s)}{T_{\text{hot in}}(s)} = \frac{T_{dh}(o,s)}{T_{dh}(L,s)} = \frac{Y_1(s) - Y_2(s)}{\left[Y_1(s) + C_3(s) \right] e^{Y_1(s)L} - \left[Y_2(s) + C_3(s) \right] e^{Y_2(s)L}} \quad (3.1-58)$$

$$\frac{T_{\text{hot out}}(s)}{T_{\text{cold in}}(s)} = \frac{T_{dh}(o,s)}{T_{dc}(L,s)} = \frac{-C_2(s) \left[e^{Y_1(s)L} - e^{Y_2(s)L} \right]}{\left[Y_1(s) + C_3(s) \right] e^{Y_1(s)L} - \left[Y_2(s) + C_3(s) \right] e^{Y_2(s)L}} \quad (3.1-59)$$

$$\frac{T_{\text{cold out}}(s)}{T_{\text{cold in}}(s)} = \frac{T_{dc}(o,s)}{T_{dc}(L,s)} = \frac{\left[Y_1(s) + C_1(s) \right] e^{Y_1(s)L} - \left[Y_2(s) + C_1(s) \right] e^{Y_2(s)L}}{Y_1(s) - Y_2(s)} - \frac{C_4(s)C_2(s) \left[e^{Y_1(s)L} - e^{Y_2(s)L} \right]}{\left[Y_1(s) - Y_2(s) \right] \left\{ \left[Y_1(s) + C_3(s) \right] e^{Y_1(s)L} - \left[Y_2(s) + C_3(s) \right] e^{Y_2(s)L} \right\}} \quad (3.1-60)$$

$$\frac{T_{\text{cold out}}(s)}{T_{\text{hot in}}(s)} = \frac{T_{\text{dc}}(o,s)}{T_{\text{dh}}(L,s)} = \frac{C_4(s) \left[e^{Y_1(s)L} - e^{Y_2(s)L} \right]}{\left[Y_1(s) + C_3(s) \right] e^{Y_1(s)L} - \left[Y_2(s) + C_3(s) \right] e^{Y_2(s)L}} \quad (3.1-61)$$

The length/velocity terms are contained in the "C" variables. For example,

$$C_1(s) = -\frac{1}{V_h} s - \frac{H_{\text{dh}} \left(\tau s + 1 - \frac{H_{\text{dh}}}{H_{\text{dh}} + H_{\text{dc}}} \right)}{\dot{W}_2 C_p (\tau s + 1)} \quad (3.1-62)$$

where

$$\tau = \frac{C_d W_d}{H_{\text{dh}} + H_{\text{dc}}} \quad (3.1-63)$$

The terms $Y_1(s)$ and $Y_2(s)$ are the roots of the characteristic equation of the system and are functions of the C variables. The $\frac{1}{V_h} s$ term on the right side of Eq. 3.1-62 results from the finite gas velocity. As shown in Figs. 2-4 and 2-5 of Section 2.1.2, the effect of the finite gas velocity in the heat dump is negligible in the frequency range of interest.

A FORTRAN program has been written to calculate the distributed parameter transfer functions shown above. Some of these transfer functions are discussed in Section 2.1, where they are compared with various lumped parameter transfer functions.

3.1.3 Transfer Function Programs

The previous two subsections discuss the derivation of transfer functions for lumped and distributed parameter heat dump models. Similar derivations have been completed for other system components. This subsection describes some features of FORTRAN programs that have been written to calculate the magnitude and phase of the transfer functions from the Laplace transformed equations.

The computer programs for calculating the lumped parameter transfer functions must solve a set of linear equations similar to Eqs. 3.1-46 and 3.1-47. The programs are complicated because the matrix includes many complex numbers when the substitution $s = j\omega$ is made. Fortunately, a subroutine "CMPMAT" which solves matrices of this type is available. The matrix for the four-lump model of the heat dump discussed in Section 3.1.1 takes the form shown in Fig. 3-3.

The subroutine CMPMAT is set up so that the matrix elements of the independent variables must be either 1 or 0. Thus, the matrix shown in Fig. 3-3 must be normalized. To solve for the transfer function $\Delta T_{dho}(j\omega)/\Delta \dot{W}_2(j\omega)$, the matrix takes the form shown in Fig. 3-4.

To make use of the subroutine it is necessary only to specify the real and imaginary part of each element and then call the subroutine. The subroutine calculates and prints the gain and phase of the transfer functions $\frac{\Delta T_{dho}(s)}{\Delta \dot{W}_2(s)}$ and $\frac{\Delta T_{dco}(s)}{\Delta \dot{W}_2(s)}$ at various values of $s = j\omega$. The transfer functions $\frac{\Delta T_{dcl(4)}(s)}{\Delta \dot{W}_2(s)}$, $\frac{T_{dcl(3)}(s)}{\Delta \dot{W}_2(s)}$, $\frac{T_{dcl(2)}(s)}{\Delta \dot{W}_2(s)}$, $\frac{T_{dhl(3)}(s)}{\Delta \dot{W}_2(s)}$, $\frac{\Delta T_{dhl(2)}(s)}{\Delta \dot{W}_2(s)}$ and $\frac{\Delta T_{dhl(1)}(s)}{\Delta \dot{W}_2(s)}$ are calculated and are available for print-out if desired.

It is possible to calculate the transfer functions of part or all of the connected UHTREX system by combining the equations of each component in the system, along with the appropriate equations connecting the components, into one large complex matrix. The digital computer programs can produce numerical data printouts and film plots of gain and phase versus frequency. The film plot routines were used to produce the curves shown in Figs. 2-2 through 2-5 of Section 2. These film plots are a great labor saver; and they present the results in a form that facilitates comparison of results from parameter studies.

The frequency domain digital computer programs are of two types. One type of program deals with the disconnected system, where each

$\Delta \dot{W}_2$	$\Delta \dot{W}_3$	$\Delta T_{dh1(o)} \equiv \Delta T_{dhi}$	$\Delta T_{dc1(5)} \equiv \Delta T_{dci}$	
X3R ₍₁₎	- X4R ₍₁₎ - j ω X4I ₍₁₎	X9R	0	
X1R ₍₁₎	- X7R ₍₁₎	X11R	0	
X3R ₍₂₎	- X4R ₍₂₎ - j ω X4I ₍₂₎	0	0	
X1R ₍₂₎	- X7R ₍₂₎	0	0	
X3R ₍₃₎	- X4R ₍₃₎ - j ω X4I ₍₃₎	0	0	
X1R ₍₃₎	- X7R ₍₃₎	0	0	
X3R ₍₄₎	- X4R ₍₄₎ - j ω X4I ₍₄₎	0	- X10R - j ω X10I	
X1R ₍₄₎	- X7R ₍₄₎	0	- X12R	=

$\Delta T_{dc1(4)}$	$\Delta T_{dc1(3)}$	$\Delta T_{dc1(2)}$	$\Delta T_{dc1(1)} \equiv \Delta T_{dco}$	$\Delta T_{dh1(4)} \equiv \Delta T_{dho}$	$\Delta T_{dh1(3)}$	$\Delta T_{dh1(2)}$	$\Delta T_{dh1(1)}$
0	0	X10R+j ω X10I	X5R+j ω X5I	0	0	0	- X8R
0	0	X12R	X6R	0	0	0	- X2R
0	X10R+j ω X10I	X5R+j ω X5I	0	0	0	- X8R	- X9R
0	X12R	X6R	0	0	0	- X2R	- X11R
X10R+j ω X10I	X5R+j ω X5I	0	0	0	- X8R	- X9R	0
X12R	X6R	0	0	0	- X2R	- X11R	0
X5R+j ω X5I	0	0	0	- X8R	- X9R	0	0
X6R	0	0	0	- X2R	- X11R	0	0

Fig. 3-3 Matrix for Four-Lump Model of Heat Dump

$\Delta \dot{W}_2$		$\Delta T_{dc1(4)}$	$\Delta T_{dc1(3)}$	$\Delta T_{dc1(2)}$	ΔT_{dco}	ΔT_{dho}	$\Delta T_{dh1(3)}$	$\Delta T_{dh1(2)}$	$\Delta T_{dh1(1)}$
1.0	=	0	0	$\frac{X10R+j\omega X10I}{X3R_{(1)}}$	$\frac{X5R+j\omega X5I}{X3R_{(1)}}$	0	0	0	$\frac{-X8R}{X3R_{(1)}}$
1.0		0	0	$\frac{X12R}{X1R_{(1)}}$	$\frac{X6R}{X1R_{(1)}}$	0	0	0	$\frac{-X2R}{X1R_{(1)}}$
1.0		0	$\frac{X10R+j\omega X10I}{X3R_{(2)}}$	$\frac{X5R+j\omega X5I}{X3R_{(2)}}$	0	0	0	$\frac{-X8R}{X3R_{(2)}}$	$\frac{-X9R}{X3R_{(2)}}$
1.0		0	$\frac{X12R}{X1R_{(2)}}$	$\frac{X6R}{X1R_{(2)}}$	0	0	0	$\frac{-X2R}{X1R_{(2)}}$	$\frac{-X11R}{X1R_{(2)}}$
1.0		$\frac{X10R+j\omega X10I}{X3R_{(3)}}$	$\frac{X5R+j\omega X5I}{X3R_{(3)}}$	0	0	0	$\frac{-X8R}{X3R_{(3)}}$	$\frac{-X9R}{X3R_{(3)}}$	0
1.0		$\frac{X12R}{X1R_{(3)}}$	$\frac{X6R}{X1R_{(3)}}$	0	0	0	$\frac{-X2R}{X1R_{(3)}}$	$\frac{-X11R}{X1R_{(3)}}$	0
1.0		$\frac{X5R+j\omega X5I}{X3R_{(4)}}$	0	0	0	$\frac{-X8R}{X3R_{(4)}}$	$\frac{-X9R}{X3R_{(4)}}$	0	0
1.0		$\frac{X6R}{X1R_{(4)}}$	0	0	0	$\frac{-X2R}{X1R_{(4)}}$	$\frac{-X11R}{X1R_{(4)}}$	0	0

Fig. 3-4 Normalized Heat Dump Matrix

component is taken as an individual unit independent of any other component. The second type of program deals with the entire system, with all components connected.

A complete list of the transfer function programs for the UHTREX system is shown in Tables III-1 and III-2, with the transfer functions that the programs calculate and the IBM 7094 machine time required to assemble and execute each program. The problem time shown is the time required to compute the gain and phase of one transfer function at approximately sixty different frequencies.

3.1.4 Application of Frequency Domain Program Results

Some applications of the results of the frequency domain programs were discussed previously in Section 2.1. These applications and a few additional ones are summarized below.

3.1.4.1 Determination of Number of Component Lumps - The transfer function programs were used in conjunction with time response programs to determine the number of lumps required in each component model. This choice was made by comparing the transfer functions of n-lump models, where $n = 1, 2, 3$, and 4, etc., with the distributed parameter transfer functions. The authors chose that model which had the minimum number of lumps whose transfer functions approximated the distributed parameter transfer functions to a frequency considered suitable for system operating conditions.

3.1.4.2 Determination of Best Average Gas Temperature Weighting - The actual gas temperatures through each lump do not vary linearly. Therefore, a weighting factor was introduced in the calculation of average gas temperature to bias the calculated value toward the actual average value. The k_{dhot} and k_{dcold} constants appearing in the heat dump equations given in Section 3.1.1 are such weighting factors. The transfer

TABLE III-1

DISCONNECTED MODE TRANSFER FUNCTION PROGRAMS

Component	Program Name, Assembly, and Problem Running Times (minutes)	Independent Variables	Dependent Variables		
Heat Dump	Lumped Par. "NLHDUM"	T_{dhi}	T_{dho}	T_{dco}	
	Assembly Time 5.2	T_{dci}	T_{dho}	T_{dco}	
	Problem Time 0.07	$\dot{W}_2$	T_{dho}	T_{dco}	
		$\dot{W}_3$	T_{dho}	T_{dco}	
	Dist. Par. "TRFDUM"	T_{dhi}	T_{dho}	T_{dco}	
	Assembly Time 2.5	T_{dci}	T_{dho}	T_{dco}	
Recuperator	Lumped Par. "NLRECP"	T_{rhi}	T_{rho}	T_{rco}	
	Assembly Time 5.3	T_{rci}	T_{rho}	T_{rco}	
	Problem Time 0.07	$\dot{W}_1$	T_{rho}	T_{rco}	
		FB	T_{rho}	T_{rco}	
	Dist. Par. "TRFREC"	T_{rhi}	T_{rho}	T_{rco}	
	Assembly Time 2.5	T_{rci}	T_{rho}	T_{rco}	
Heat Exchanger	Lumped Par. "NLHETX"	T_{xhi}	T_{xho}	T_{xco}	
	Assembly Time 6.5	T_{xci}	T_{xho}	T_{xco}	
	Problem Time 0.25	$\dot{W}_1$	T_{xho}	T_{xco}	
		$\dot{W}_2$	T_{xho}	T_{xco}	
	Dist. Par. "TRFHEX"	T_{xhi}	T_{xho}	T_{xco}	
	Assembly Time 4.0	T_{xci}	T_{xho}	T_{xco}	
Reactor with Neutronics with or with- out Tempera- ture Reactiv- ity Feedback	Lumped Par. "NLNCOR"	T_i	T_{ao}	T_{eo}	T_o
	Assembly Time 1.9	$\dot{W}_1$	T_{ao}	T_{eo}	T_o
	Problem Time 0.31	ρ	T_{ao}	T_{eo}	T_o
		Q	T_{ao}	T_{eo}	T_o
Reactor Core Heat Exchanger	Approx. Dist. Par. "TRFCOR"	T_i	T_{ao}	T_{eo}	T_o
	Assembly Time 6.7				
	Problem Time 0.06				
Pipes	Lumped Par. "PIPE12"	T_{pi}	T_{po}		
	Assembly Time 3.0				
	Problem Time 0.08				
	Dist. Par. "TRFPIP"	T_{pi}	T_{po}		
	Assembly Time 1.3				
	Problem Time 0.04				

TABLE III-2

CONNECTED MODE TRANSFER FUNCTION PROGRAMS

<u>Description</u>	<u>Program Name</u>	<u>Independent Variables</u>	<u>Dependent Variables</u>
Connected Partial System Looking Out From Reactor	"UHXTRF" Version 1 Assembly Time 8.6 Problem Time 1.0	T_o $\dot{W}_1$	T_i T_i
Reactor Plus Simple Version of System Looking Out From Reactor	"TRFCNS" Assembly Time 4.0 Problem Time 0.23	ρ $\dot{W}_1$	Q, ρ_t, T_o Q, ρ_t, T_o
Complete Connected System	"UHXTRF" Version 2 Assembly Time 8.6 Problem Time 1.5	ρ T_{dci} $\dot{W}_1$ $\dot{W}_2$ $\dot{W}_3$ FB	Any System Gas Temp., Plus Reactor Power and Reactor Core Avg. Solid Temperature

functions for different values of the weighting factors were compared with the distributed parameter transfer functions to obtain the best match.

3.1.4.3 Determination of Effects of Transit Time - As mentioned previously, the effect of the transit time is included in the distributed parameter transfer functions. Appropriate programs have been written to calculate component transfer functions either with or without transit time so that this effect can be evaluated.

3.1.4.4 Time Domain Simulation and Frequency Domain Program Checking - The time domain simulation program is capable of calculating transfer functions from step input responses. These transfer functions were compared with the frequency domain transfer functions to check the numerical methods and programming in both problems. These cross checks are an invaluable aid in eliminating errors. The checks have been quite good with the maximum errors of 5-db gain and 10° phase in the significant frequency range. Generally errors can be made less than 2-db gain and 5° phase through a large frequency range by using a small ΔT and a large running time.

The comparisons of the transfer functions made at design operating conditions serve as a check on the simulation from the linear standpoint, but they do not check the nonlinearities of the system. Comparisons made at different operating conditions check at least some of the system nonlinearities.

A considerable amount of checking of individual component and overall system transfer functions has been performed. Consequently the authors feel that the results produced by frequency and time domain programs are accurate for the mathematical model given in Section 2.

3.2 Time Domain Simulation Program, UHXCOM

The UHXCOM digital computer program obtains a numerical solution to the UHTREX mathematical model described in Section 2. Thus, the program can simulate the reactor system in the time domain, and can obtain the response of the model to changes in independent variables such as reactivity and blower speed.

The following material describes the general features of the UHXCOM program, its uses, and a few of its special features. Specific instructions for using the program are given in Appendix A.

3.2.1 Ways to Run Problems

3.2.1.1 Connected Mode - The UHXCOM program is ordinarily run with all system components connected as they will be in the actual system. In this connected mode the outlet gas from the reactor is the inlet gas to the recuperator, etc., as described by the connective equations of Section 2.8.

In the connected mode one may include or exclude the blowers as desired by specifying an input variable, NOBLWR, as 0 or 1. A surge condition in the blowers may be ignored in special problems by setting another input variable JSPEC = 1.

In the standard problem the program uses a large steady-state matrix to determine the system initial conditions of temperature, power, and flow rates corresponding to a user's choice of blower speeds (or flow rates if the blowers are not included), air flow rate in the heat dump, bypass valve setting, system pressures, and reactivity. Thus, the steady-state matrix provides a starting point for transient calculations. However, the initial system conditions of temperature, power, and flow rate may be read in from data cards, if desired, by specifying KMX = 2. In this case the steady-state matrix is not used at the beginning of the problem; rather, it is used at the end of the problem to determine the final

steady-state conditions that would prevail if the problem were run until it reached steady-state.

3.2.1.2 Disconnected Mode - In the disconnected mode the six-system heat exchange elements are decoupled and the inlet gas temperatures are treated as independent variables. In this circumstance the connective equations of Section 2.8 are not used, and there is no gas temperature feedback between components. The disconnected mode allows the study of individual system components. In particular, it allows one to obtain transfer functions for each component which can be checked in detail with transfer functions for each component obtained from the frequency domain programs. A good check on individual components almost assures one that the connected system checks will be obtained on the first try, for the connected system differs from the disconnected system only in that it contains the connective equations.

System hot (or cold) inlet gas temperatures may be perturbed at one time by specifying an input variable KVAR2 = 2 (or 3). Coolant gas flow rates, bypass valve settings, etc., may be perturbed by specifying KVAR2 = 4. The blowers are not used in the disconnected mode.

As in the connected case, initial conditions can be calculated from the steady-state matrix or read in from data cards. If the problem is run in the disconnected mode, and if the steady-state matrix is used, the system is connected at the beginning of the problem to obtain the initial conditions. Once these conditions are obtained the solution proceeds in the disconnected mode.

3.2.2 Discussion of Program

3.2.2.1 Flow Diagram - A simplified flow diagram of the main UHXCOM program is given in Fig. 3-5. It is supplemented by other flow diagrams and discussion in this section and in Appendix A.

Fig. 3-5 UHXCOM Flow Diagram

Fig. 3-5 UHXCOM Flow Diagram (continued)

3.2.2.2 Steady-State Solution - At the beginning and the end of most problems one would like to obtain a steady-state solution; that is, a consistent solution for system power, solid and gas temperatures, and flow rates, corresponding to given values of bypass valve setting, reactivity, blower speeds, etc. Some mathematical difficulties arise because of nonlinearities in the heat transfer coefficients and in the expression for radiative heat transfer between fuel and moderator. These nonlinearities prevent one from obtaining a direct solution to the whole system by inverting a matrix of linear equations. The problem of finding a steady-state solution in spite of the nonlinearities has been solved by breaking the problem into two parts, each of which is solvable, and iterating between the two.

The first part, described in Sections 2.2 through 2.8, includes all heat exchange processes and the reactor power generation. Given a set of system flow rates, a bypass valve setting, and the reactor reactivity, one would like to calculate steady-state gas and solid temperatures and reactor power. That is, one would like to find a simultaneous solution to a large set of relationships like Eqs. 2.2-1 through 2.8-9, after modifying them by setting all derivatives equal to zero. The only difficulties here are the nonlinear radiation terms like that in the right side of Eq. 2.2-1. A typical term is treated below to obtain a linear equation suitable for use in a matrix. By inverting the matrix one can obtain the steady-state solution for power and solid and gas temperatures.

The expression for radiative heat transfer from moderator to fuel in the i th reactor lump is

$$Q_{r(i)} = \frac{H_{\text{rad}}}{4} (T_{m(i)}^4 - T_{f(i)}^4) \quad (3.2-1)$$

where

$$\begin{aligned} Q_{r(i)} &= \text{radiative heat transfer rate in } i\text{th lump (Btu/sec)} \\ H_{\text{rad}} &= \text{radiative heat transfer coefficient between moderator} \\ &\quad \text{and fuel (Btu/sec } ^\circ\text{R}^4) \\ T_{m(i)} &= \text{temperature of } i\text{th moderator lump (} ^\circ\text{R)} \\ T_{f(i)} &= \text{temperature of } i\text{th fuel lump (} ^\circ\text{R)} \end{aligned}$$

Expanding Eq. 3.2-1,

$$Q_{r(i)} = \frac{H_{\text{rad}}}{4} \left[(T_{m(i)}^2 + T_{f(i)}^2) (T_{m(i)} + T_{f(i)}) \right] \left[T_{m(i)} - T_{f(i)} \right] \quad (3.2-2)$$

or

$$Q_{r(i)} = \frac{H_{\text{rad}}}{4} \left[TA_{(i)} \right] \left[T_{m(i)} - T_{f(i)} \right] \quad (3.2-3)$$

With $TA_{(i)}$ considered constant, Q_r depends on the linear difference between moderator and fuel temperatures. Now, given an initial set of fuel and moderator temperatures one can calculate the $TA_{(i)}$ terms, enter them as coefficients in a matrix, and invert the matrix to obtain a new set of values for reactor power and gas and solid temperatures. From this new set, one can calculate the new $TA_{(i)}$ terms. In the program this iterative process is repeated until the change in the $TA_{(i)}$'s from one cycle to the next is very small. When the cycle has converged, the program proceeds to the second part.

The second part includes the system pressure drops and blowers. Given gas temperatures from the first part, and a set of blower speeds and system pressures, equations of Sections 2.9 and 2.10 are used to calculate a new set of system flow rates. If these new flow rates differ by more than a prescribed amount from the flow rates used in the calculations of part one, the program returns to part one with the new flow rates, and the whole cycle, including recalculation of heat transfer coefficients, is repeated. If the new flow rates differ from the old ones used in part one by less than a prescribed amount, the consistent set of steady-state system temperature, flow rates, and power is

printed with the heading "Initial Profiles," and the program proceeds to the transient portion of the calculation.

Usually both the $TA_{(i)}$ s and the flow rates are required to change less than 0.01% of their old value before the system values are accepted. If the steady-state matrix calculation of part one does not converge in 10 cycles, the program will print "Not Converged," give the values of $TA(i)$, and proceed. If more than 20 cycles through the second part are required, the old and new values of flow are printed and the program proceeds. By reading a control variable $NRM = 1$, a printout of the number of cycles required for the flow to converge can be obtained. It has been found that one cycle is usually sufficient, with two or three required immediately after a system disturbance.

The same sort of steady-state calculation is carried out at the end of each connected type problem with the results printed under the heading "Final SS Results."

3.2.2.3 Transient Solution - Figure 3-6 is an outline of the sequence of transient calculations performed after an initial set of system conditions is obtained. This outline is a summary of portions of the flow diagram given in Fig. 3.5.

At the beginning of each problem, values of power, solid and gas temperatures, and flow rates are either read from data cards or calculated as described in the preceding subsection. Then the program enters a major calculational cycle, shown in Fig. 3-6. The cycle is repeated once each time step until the end of the problem. Here, as in the steady-state calculation, an iterative subcycle is required to obtain a consistent set of gas temperatures and flow rates. Once a consistent set is obtained, the program calculates all derivatives of solid temperatures and uses Euler's method to solve for solid temperatures at the next time interval. Following Euler, if

Fig. 3-6 Sequence of Transient Calculations

$$\frac{dy(t)}{dt} = f(y,t) \quad (3.2-4)$$

then

$$y(t_0 + \Delta t) \approx y(t_0) + \left[\frac{dy(t)}{dt} \right]_{t=t_0} \Delta t \quad (3.2-5)$$

Basic time constants in the UHTREX system range from 10 to 1000 seconds, and good simulation results can be obtained using a time increment of $\Delta t = 1.0$ second.

Originally Euler's method was also used to solve the reactor kinetic equations. However, it was found that the time step had to be very short if the answers were to be accurate. Consequently, an alternate solution to the reactor kinetic equations was developed. This method, described in Appendix E, solves an integral equation⁷ for the neutron density $n(t)$ by using the method of collocation.⁸ In applying the method of collocation it is assumed that the time variation of reactivity $\rho(t)$ and neutron density $n(t)$ can be matched by quadratic functions in a short time interval. The quadratic coefficients for $\rho(t)$ are determined directly, for $\rho(t)$ is known and the unknown coefficients of the quadratic fit to $n(t)$ can be determined by use of the integral equation. This method of solving the reactor neutron kinetic equations allows the use of time steps as large as 2 seconds for many problems.* The use of such large time steps has led to an overall simulation program that requires relatively little 7094 computing time.

3.2.2.4 Blower Calculations - The pressure drops in the system are functions of temperature, flow, and pressure.

$$\Delta P_s = f \left[(\dot{W})^2, T_g, P_s \right] \quad (3.2-6)$$

* A study of the heat transfer convergence criteria has shown that at design conditions the maximum time step that can be used is set at 2.5 seconds.

A similar expression for the pressure rise in the primary and secondary loop blowers can be obtained from the blower characteristic shown in Fig. 2-12. For use in the UHXCOM program, the head coefficient versus flow coefficient curve of this figure is approximated by six straight lines of the form

$$\psi = a_i + m_i \phi \quad (3.2-7)$$

where a_i and m_i are the intercept and slope for the i th approximation interval. Solving for the blower ΔP from Eqs. 2.10-1 through 2.10-3, replacing density by P/RT , and substituting equivalent mass flow for volume flow, one obtains

$$\Delta P_b = a_i 1.04 \times 10^{-7} \frac{N^2 P}{T} + m_i (2.33 \times 10^{-5} N) \dot{W} \quad (3.2-8)$$

where

- ΔP_b = blower pressure rise (psia)
- N = blower speed (rpm)
- P = blower inlet pressure (psia)
- T = coolant temperature ($^{\circ}R$)
- $\dot{W}$ = coolant flow rate.

The total system pressure drop, ΔP_s , is set equal to blower pressure rise to yield a quadratic equation in mass flow rate. This equation is solved for the flow rate corresponding to given values of blower speed, coolant temperature, etc. The resulting solution for flow is checked against upper and lower flow limits of the i th interval, and the range is changed to the $i + 1$ st or $i - 1$ st if necessary to get a flow and pressure drop consistent with a given range. The results are saved for plotting on film and for calculating the horsepower requirement of the blower.

A surge indication in one of the blowers precludes further system operation. Thus, in case of surge, the temperatures, flows, and power are printed and the problem is terminated unless it is a special problem involving blower coastdown.

3.2.3 Special Features

3.2.3.1 Time Function Subroutine - The time function subroutine is a flexible, general purpose function generator. It enables one to specify that a given system independent variable, such as bypass valve setting FB, has the form

$$FB = FBIC(1 + TFN) \quad (3.2-9)$$

where

$$TFN = a + b[\cos \omega(\tau - \tau_{\text{delay}})] + c(\tau - \tau_{\text{delay}}). \quad (3.2-10)$$

Thus, any independent variable can be made to be a step plus a cosine plus a ramp by choosing the appropriate input variables. Sections A.2 and A.3 explain how to specify input data to obtain the proper time functions.

It is a considerable advantage to have the time function portion of the program written as a subroutine. If one wishes a function other than one of the type shown in Eq. 3.2-9 he can simply rewrite the appropriate portions of the time function subroutine, assemble, and compute.

3.2.3.2 Closed Loop Routines - A large number of trial and error simulation runs would be required to find what particular setting of the system independent variables yielded a set of desired system operating conditions, such as 3-MW power and 2860°R reactor exit gas temperature. Thus, for calculational convenience alone, a number of closed loops have been included in the UHXCUM program. These closed loops automatically adjust a variable such as primary blower speed to obtain, say, a primary loop flow rate of 2.85 lb/sec. The simultaneous adjustment of more than one variable allows one to obtain more than one desired operating condition. At present the closed loop routine is written so that by adjusting recuperator bypass flow, reactivity, and the primary and secondary loop blower speeds, one can obtain any reasonable set of desired values for power, reactor outlet gas temperature, and primary and secondary loop coolant flow rates.

This routine has proved useful from a calculational standpoint, and has given some insight into possible closed loop operation of the reactor. This subject will be pursued with interest in the future. Additional information on the closed loop routine and its use in the UHXCOM program is given in Section A.4 of Appendix A.

3.2.3.3 Internal Mapping - By specifying control variable NRM = 1, one can obtain a print of the number of passes through the blower calculations (ICONVP, ICONVS) and the number of passes through the steady-state matrix (ICONVM). This information is useful in determining the effect of a change in convergence criteria.

3.2.3.4 Film Output - During the transient portion of the calculation, the values of system variables are periodically stored for plotting on 35 mm film. The subroutine "PLOT1" draws the curves on film using the SC 4020 plotter at the end of each problem. Examples of the film output are shown in Section 5.

The film output routine has proved to be one of the most valuable accessories in the UHXCOM program. It enables one to obtain the full benefit of the calculations in a minimum of off-computer time.

3.2.3.5 Transfer Function Calculation from Time Domain Response - As shown in Section C.4 of Appendix C, the transfer function of a linear system can be calculated from the system's response to a step input. The expressions for the real and imaginary parts of the transfer function are shown below.

$$\text{Re } H(s) \Big|_{s=j\omega} = \frac{1}{A} g(0^+) + \frac{1}{A} \int_0^{\infty} \cos \omega t \frac{d}{dt} g(t) dt \quad (3.2-11)$$

$$\text{Im } H(s) \Big|_{s=j\omega} = - \frac{1}{A} \int_0^{\infty} \sin \omega t \frac{d}{dt} g(t) dt \quad (3.2-12)$$

where

$H(s)$ = system transfer function
 s = complex frequency variable
 ω = frequency at which $H(s)$ is evaluated
 A = magnitude of input step
 $g(t)$ = change in output due to input step.

The UHXCOM program calculates transfer functions by solving these equations with subroutines outlined in Figs. 3-7 and 3-8. The TRFN1 subroutine is entered each time cycle in the transient calculation, and calculates quantities proportional to the real and imaginary part of the transfer function. The TRFN2 subroutine is entered at the end of each problem and finishes the transfer function calculation. The user can specify the independent variable to be perturbed, the set of transfer functions to be calculated, and the frequencies at which the transfer function is to be calculated. Section A.5 of Appendix A explains how one can use UHXCOM to obtain transfer functions.

As noted previously, the transfer functions obtained from the UHXCOM program have been checked extensively with those obtained from the frequency domain program. The availability of such checks has allowed us to learn a lot about the practical aspects of the use of UHXCOM to obtain transfer functions. A few general statements concerning the accuracy of the kind of transfer calculation done in UHXCOM and some quantitative information pertaining especially to the UHXCOM system are given below.

a. Good precision (of the transfer functions) at high frequencies requires one to use a short time step. To obtain precisions of ± 2 db, and $\pm 5^\circ$ at 10 rad/sec, a time increment of 0.2 sec is required. There are not many UHTREX system problems of interest at this relatively high frequency, and runs with $\Delta t = 0.2$ sec are ordinarily made only for checking purposes.

This routine has proved useful from a calculational standpoint, and has given some insight into possible closed loop operation of the reactor. This subject will be pursued with interest in the future. Additional information on the closed loop routine and its use in the UHXCOM program is given in Section A.4 of Appendix A.

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where

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s = complex frequency variable

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A = magnitude of input step

$g(t)$ = change in output due to input step.

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Fig. 3-7 Subroutine TRFN1

Fig. 3-8. Subroutine TRFN2

b. Good precision at low frequencies requires a long simulation run. For accuracies of ± 2 db and $\pm 5^\circ$ at 0.001 radians, a 3000-second run is adequate.

c. Good transfer function precision from a nonlinear system or one whose differential equations have time varying coefficients requires that a small step disturbance be used. Use of a small step does not drive the system to the point where its nonlinear characteristics are significant. In the UHXCOM system, steps of 2% to 5% of the independent variables nominal operating value give good results, as does a 5¢ reactivity step for the reactor. In the latter case, of course, the reactor heat exchanger and temperature feedback effects must be included. Without them the reactor has a pole at the origin, and its response to a step of reactivity is not bounded.

d. Good precision at points on the transfer function where the gain is low requires strict computational convergence criteria. Otherwise the system output response to the step input is lost in the calculational noise. Convergence criteria of 0.00001 are required to obtain good accuracy when the input step is 2% and the gain is down 40 db. This tight convergence criterion requires more passes through the iterative cycles in the steady-state and transient calculations. The number of passes is not excessive, however, and the 7094 computing time is not lengthened severely. As a practical matter one ordinarily has little interest in the actual gain and phase when the gain is so low, so the tendency toward poor accuracy at low gains is not serious.

The authors feel that the ability to obtain system transfer functions from two independent sources, the UHXCOM program and the frequency domain programs, is one of the most useful features of this analysis. It has allowed a check of the coding of both kinds of programs, and it checks the numerical methods used in UHXCOM and the many practical aspects of the use of UHXCOM to obtain transfer functions.

3.2.4 Miscellaneous Comments

The UHXCOM simulation program, including subroutines, consists of about 2000 FORTRAN statements. It takes seven minutes to assemble, and in assembled form requires some 30,000 words of storage.

With $\Delta t = 1.0$ sec the UHXCOM simulation program runs in approximately 1/10 real time. That is, it takes one minute on the 7094 to simulate a 10-minute reactor run. The authors are pleased with this favorable ratio of machine time to simulation time, especially in view of the size of the program.

As noted in the previous subsection, to make most transfer functions accurate to ± 2 db in gain and $\pm 5^\circ$ in phase from 0.001 to 10 rad/sec, Δt must be about 0.2 sec and the simulated run must last about 3000 sec. Such a run requires about 20 minutes of 7094 time, so the data are fairly costly. These runs are made only for the purpose of checking with the frequency domain program results, and on that account they are not required frequently. When such runs are needed, the results are well worth the machine time.

4. SYSTEM OPERATION

This section discusses a set of nominal system operational aims and presents a summary of known limitations on operation. Both the aims and limitations pertain to the equipment shown in the model of Fig. 1-1. Various system operation results obtained from the UHXCOM simulation program are given in Section 5.

4.1 Operational Aims and Requirements

4.1.1 Full Power Design Conditions

It is desired to operate the UHTREX reactor for extended periods at full power design conditions which are:

3 MW of heat to the helium coolant

2860°R (2400°F) reactor exit coolant temperature.

These conditions will probably, but not necessarily, be achieved at a primary helium coolant flow rate of 2.85 lb/sec and a primary system pressure of approximately 500 psia. At this time there are no accompanying specific requirements placed on other system variables except that they fall within limits described in Section 4.2.

4.1.2 Off-Design Operation

The system will be operated at "cold critical" conditions of low power and no coolant flow during an initial check-out period. Conversely, the coolant blowers will be tested to their full capacity with the reactor subcritical prior to high power operation. Additional off-design operating points will be encountered as the reactor is loaded and brought to full power.

4.1.3 Stability, Regulation, and Response Times

It is required that the system be stable at full power design conditions and at a variety of off-design conditions. At present there is no requirement as to the ability of the overall system to regulate power, but at least one mode of operation which yields an approximate constant power is planned.

In view of the very large system time constants, most reactor operation can be accomplished without extensive use of automatic control loops. If automatic loops are used, they will be designed to give a smooth well-damped response to demand changes. There are no minimum rise time control requirements to be met. Special circumstances that do require prompt action are discussed in Section 5.

4.1.4 Safety and Emergency Procedures

It is required that the reactor system be operated in a safe manner at all times. It should not be a hazard to operating personnel, to reactor site personnel, or to communities in Los Alamos County. In addition, the system should be designed so that over-all damage from an individual component failure is minimized.

These safety and emergency condition requirements lead to a variety of necessary operational capabilities. The control rod system must have a fast scram, it must be possible to shut off the blowers quickly, etc. To initiate these actions effectively, the system must be able to diagnose power excursions, or other system emergencies, in a short time. As noted in Section 5, a variety of special safety systems to detect emergency failures and initiate the proper shutdown are being built. It is anticipated that data from the on-line electronic process computer installed at the UHTREX facility will be of considerable help in making an accurate diagnosis of unusual system conditions. As shown in Section 5, the UHXCOM simulation program is quite helpful in determining the adequacy of various operations initiated to control emergency situations.

4.2 System Limitations

Table IV-1 contains a variety of known limitations that pertain to system components considered in this study. It should be noted that the gas temperature limitations are bounds on steady-state operation and could be exceeded for short periods of time, of the order of minutes.

4.3 Possible Modes of Operation

For calculational purposes only, a variety of optional closed loops have been incorporated in the UHXCOM program.* Using these loops, the following operating variables were adjusted to obtain given values of demand variables.

<u>Demand Variable</u>	<u>Operating Variable</u>
1. Reactor power, Q	Reactor bypass, FB
2. Reactor outlet gas temperature, T_o	Reactivity, ρ
3. Primary loop gas flow rate, $\dot{W}$	Primary loop blower speed, N

These loops, which contain only integral compensation, worked very well for their intended purpose, which was to achieve a steady-state set of operating conditions for the model at desired values of Q , T_o , and $\dot{W}$. However, the rods are to be fully withdrawn in normal full power design conditions, so one can adjust reactivity only through fuel loading and unloading, which is a time-consuming process. Thus, unlimited adjustment of the variable ρ is available for calculational purposes, but not for real reactor operation. For real operation, ρ can be changed

* Appendix A contains a description of these loops and how to use them.

TABLE IV-1
KNOWN SYSTEM LIMITATIONS

System Variable	Abbreviation	Limits	Comments
Primary loop blower speed (rpm)	N_1	$4000 < N_1 < 12,000$	Gas bearings not operable at lower speeds without gas jacking. Upper speed limitation is fixed by limit on frequency output of alternator.
Secondary loop blower speed (rpm)	N_2	$4000 < N_2 < 12,000$	
Primary blower motor power (hp)	HP_1	$HP_1 < 65$	Motor powers limited by balance of losses, heat transmitted to motor via drive shaft, and cooling.
Secondary blower motor power (hp)	HP_2	$HP_2 < 65$	
Heat dump air flow rate (lb/sec)	$\dot{W}_3$	$\dot{W}_3 < 50$	Approximate maximum design value for air blowers.
Reactor inlet gas temperature ($^{\circ}R$)	T_i	$T_i < 2060$	Design limit for connecting piping at reactor inlet.
Recuperator hot outlet gas temperature ($^{\circ}R$)	T_{rho}	$T_{rho} < 1860$	Maximum design temperature for heat exchanger inlet.
Heat exchanger cold outlet gas temperature ($^{\circ}R$)	T_{xco}	$T_{xco} < 1560$	Temperature rating of shell of heat exchanger.
Heat exchanger tube temperature ($^{\circ}R$)	T_{xt}	$T_{xt} < 1660$	Temperature rating of tube.
Heat exchanger hot outlet gas temperature ($^{\circ}R$)	T_{xho}	$T_{xho} < 1060$	Maximum design temperatures for steady-state operation of primary blowers.
Heat dump hot outlet gas temperature ($^{\circ}R$)	T_{dho}	$T_{dho} < 1060$	
Primary loop helium pressure (psia)	P_1	$P_1 < 550$	Design limit of pressure vessels.
Secondary loop helium pressure (psia)	P_2	$P_2 < 550$	

TABLE IV-1 - continued

System Variable	Abbreviation	Limits	Comments
Secondary pressure-primary pressure (psia)	$P_2 - P_1$	$0 < (P_2 - P_1) < 50$	} Design limit of heat exchanger.
Fraction of primary flow bypassed around recuperator (-)	FB	$0.01 < FB < 0.8$	} Both limits are established by bypass valve and recuperator pressure drop characteristics.
Recuperator bypass valve 90° travel time from full closed to full open (sec)	T_b	$T_b \geq 72$	} Lower time limit set by bypass valve motor.

Rod System Limitations:

- Articulated rods cannot be inserted when core temperature in the vicinity of their receiving holes exceeds 1660°R.
- Individual rods can be run in and/or out at a maximum speed of 18 inches/minute. Scram time is approximately 0.6 second.
- Total rod worth of the whole rod system is approximately \$40. Scram rod worth is approximately \$10.
- All rods are to be fully withdrawn, if possible, at and around full power design conditions.

Reactor Temperature Cycling:

Reactor temperature cycles from room temperature to design temperature should be minimized.

occasionally, system pressures P_1 and P_2 can be adjusted, and BS_1 , BS_2 , FB , and $\dot{W}_3$ can be changed almost at will.

One way of using the independent variables to achieve desired system operation is discussed in Section 5.1.3.

5. RESULTS

This chapter contains a description of various hypothetical reactor experiments and the results of these experiments as predicted by the UHXCOM simulation program.

5.1 Steady-State Operation

The following three sections deal with steady-state operation at and near full power design conditions.

5.1.1 Possible Full Power Steady-State Conditions

An UHXCOM simulation run was made to determine overall system conditions when the reactor power is 3 MW, the reactor outlet gas temperature is 2400°F, and the primary flow rate is 2.85 lb/sec. In this run, calculational closed loops were used to obtain the 3 MW, 2400°F, and 2.85 lb/sec values. Specifically, the following variables were adjusted to obtain these desired values.

<u>Adjust</u>	<u>To Obtain → Desired Values</u>
1. Reactor Bypass FB	Reactor Power $Q = 2845 \text{ Btu/sec}$
2. Reactivity TAMB	Reactor Outlet Gas Temperature $T_o = 2400^\circ\text{F}$
3. Primary Loop Blower Speed BS ₁	Primary Flow Rate $\dot{W}_1 = 2.85 \text{ lb/sec}$

Secondary loop blower speed was set at 9000 rpm, and the heat dump flow rate was set at 42.8 lb/sec.

The results of this run, as shown in Fig. 5-1, show that there is at least one way of operating the system at full power so that the system limitations of Section 4 are satisfied. The conclusion is by no means obvious, and can be based on nothing less than a complete steady-state solution of the equations presented in this report.

It should be emphasized that the conditions given in Fig. 5-1 represent only one possible way of operating the system, and are not the final or design conditions for the system. For convenience, the conditions of Fig. 5-1 will be referred to here as the "standard operating conditions." These standard conditions are used as the initial conditions in all simulation runs reported here except where noted otherwise.

5.1.2 Off-Design Operation

Steady-state runs were made in operating regions close to the standard conditions to determine the effects of changes in such things as bypass setting and blower speed. The results are shown in Figs. 5-2a through 5-2f. These figures contain a considerable amount of information in condensed form, and have proved to be of value to the authors. However, they can be misinterpreted, and the reader is advised to read the following explanation of how to use the figures before looking at them in detail.

Each figure contains five curves. Each curve shows how the dependent ordinate variable changes when only one of the abscissa variables is changed. The other four abscissa variables are held constant. Thus, curve FB in Fig. 5-2a shows that if BS_1 , BS_2 , and $\dot{W}_3$ are held constant at their standard condition points and FB is changed from 0.2 to 0.1, the reactor power goes down to about 2.8 MW. Again, each figure contains five completely independent curves which show what will happen if only one independent variable is changed.

Fig. 5-1 Possible Full Power Steady-State Conditions

A short discussion of each figure is given below.

Fig. 5-2a Reactor Power

Reactor power increases when the recuperator bypass flow is increased, when system reactivity is increased, or when the flow in any of the three coolant loops is increased. ($\dot{W}_1$ and $\dot{W}_2$ are increasing functions of BS_1 and BS_2 as shown in Figs. 5-2e and 5-2f.) BS_1 and FB have considerably more effect than $\dot{W}_3$ or BS_2 . Indeed, $\dot{W}_3$, the heat dump air flow, has little effect on power in the range shown.

Fig. 5-2b Reactor Gas Temperatures

The reactor inlet and outlet gas temperatures are relatively insensitive to blower speeds and flow rates. They are more sensitive to changes in bypass flow and reactivity, with both temperatures decreasing as the flow bypassed around the recuperator is increased, or as the reactivity is decreased. The reactor inlet gas temperature is particularly sensitive to changes in bypass and reactivity.

Fig. 5-2c Heat Exchanger Primary Loop Gas Temperatures

The heat exchanger hot inlet gas temperature is quite sensitive to changes in bypass flow, with an increased bypass leading to higher gas temperatures. An increase in ρ or BS_1 also leads to higher primary loop heat exchanger gas temperatures. It should be noted that the increase in FB, ρ , or BS_1 is due mainly to the fact that increases in these variables also give an increase in power.

Fig. 5-2d Heat Dump Secondary Loop Gas Temperatures

An increase in bypass flow, reactivity, or BS_1 yields an increase in the heat dump secondary loop inlet gas temperature. The heat dump secondary outlet gas temperature is relatively insensitive to changes in operating variables.

Figs. 5-2e, 5-2f Primary and Secondary Loop Flow Rates

Primary and secondary loop flow rates increase with the speed of their respective blowers but are relatively insensitive to changes in other independent variables.

Fig. 5-2a Reactor Power as a Function of Blower Speeds, Air Flow and Recuperator Bypass

Fig. 5-2b Reactor Inlet and Outlet Gas Temperatures

Fig. 5-2c Heat Exchanger Primary Loop Inlet and Outlet Gas Temperatures

Fig. 5-2d Heat Dump Secondary Loop Inlet and Outlet Gas Temperatures

Fig. 5-2e Primary Flow Rate

Fig. 5-2f Secondary Flow Rate

The numbers in Figs. 5-2 also can be used to obtain approximate results for cases in which two variables change. For instance, suppose one would like to know the effect of BS_1 on the primary loop heat exchanger hot inlet temperature, T_{xhi} , when the bypass value is used to maintain constant power. Let BS_1 increase to 1.2 times its standard value. From Fig. 5-2a, it can be seen that FB would have to be decreased to about 0.02 to keep the power constant. From Fig. 5-2c, a change in BS_1 from 1 to 1.2 times its standard value increase T_{xhi} by about 35°F, while decreasing FB to 0.02 decreases T_{xhi} by 155°F. The two changes would result in a decrease in T_{xhi} of about 120°F. These results are approximate because the system is not linear and the results of independent changes in FB and BS_1 cannot be superimposed with perfect accuracy. However, such superposition is good enough to give useful answers in a region near the standard conditions. The conclusion obtained above, namely, that T_{xhi} decreases with an increase in BS_1 if the power is held constant, is in agreement with intuitive results.

5.1.3 Comments

Overall, Figs. 5-2 show that reactor power and most system gas temperatures are relatively insensitive to changes in secondary coolant and heat dump air flow rates. However, bypass flow, reactivity, and primary blower speed have a considerable effect on power and, consequently, on gas temperatures. It is also noted that critical system temperatures decrease with increasing secondary and heat dump flow rates. As explained previously, system temperatures also decrease with increasing primary flow rate as long as the reactor power is held constant.

Figs. 5-2 lead one to suggest that the five system independent variables be used at and near full power to obtain desired values of other system variables as shown in Table V-1. Thus, from Table V-1, BS_1 could be adjusted to obtain a particular value of BS_1 , or a desired $\dot{W}_1$, or a desired Q_t . Such adjustment could be made by an operator or by appropriate closed loop control equipment.

TABLE V-1

SUGGESTED USE OF SYSTEM INDEPENDENT VARIABLES

<u>Independent Variable Adjusted</u>	<u>Desired System Values</u>
ρ	T_o
X_b	X_b or T_i
BS_1	$BS_1, \dot{W}_1$ or Q_t
BS_2	$BS_2, \dot{W}_2$ or T_{xco}
FS	FS or T_{xho}

The first three rows of Table V-1 were chosen so that the three independent variables that have the most effect on system behavior could be used to adjust reactor gas temperatures and thermal power. The temperatures T_{xco} and T_{xho} on the right-hand side of the last two rows are somewhat arbitrary, but seem like good choices at this time. Regulation of these two temperatures, or almost any other two mid-system temperatures, will help isolate the reactor from changes in air temperature.

The variables on the right side of each row in Table V-1 are, in general, quite strongly affected by changes in the corresponding independent variable on the left side. Indeed, the strength of coupling is an important consideration in the construction of Table V-1.

The observation that critical system temperatures decrease with increasing secondary and heat dump flow rates suggests an alternative adjustment of these two independent variables: BS_2 and $\dot{W}_3$ might well be set to the highest values at which they can be run permanently and left there. Such high settings will lead to somewhat lower system temperatures and temperature gradients than might otherwise be obtained.

5.2 Open Loop Dynamic Response to Modest Changes in Independent Variables

The figures in this section show the results of modest changes in the UHTREX system independent variables. The results display the "open loop" dynamics in that an independent variable is changed and then the system is allowed to respond as it will, with no corrective action being taken by an operator or by closed loop control external to the basic system itself.

The size and rate of changes in independent variables were selected to correspond roughly to physical changes that might be introduced through normal operator actions, or at most, unusual weather changes. Thus, when the blower speeds are varied, they are changed on a 5-minute ramp rather than an abrupt step, because this would be the usual procedure in changing them. Later sections will discuss system response to more spectacular changes in independent variables.

All the curves in this and later sections are taken from digital computer plots. Because of computer storage problems a limited number of points, usually 50 or 60, can be plotted on each curve during any one run. Thus, some minor detail may be missing in the output curves when a step input is made. For instance, a step change in an independent variable will appear in the computer plot as a ramp change from one plot point to the next. In the solution, however, a step is indeed a step. This limitation should be kept in mind when looking at step response data close to the time of the step.

A similar limitation arises in computing when the time interval is large compared to, say, changes in reactor period. In the 5¢ step problem, when a computing interval of 0.5 second is used, the prompt step behavior in the first 0.5 second after the step is missed. The results at 0.5 second are correct, but what happened in the interim is not known. Thus the computer minimum period in this case is 10 seconds, whereas the prompt period β/ℓ^* is 3 seconds. This is not a real

limitation in this problem, because the reactivity rise that goes with the addition of a fuel element is not in fact a step, but is probably a 1-second ramp up with a small step down at the end if a spent fuel element drops out. In problems involving truly fast changes such as occur in a scram, one can obtain the details of what happens in the first few seconds by running the problems with a small Δt .

The system response to the 5¢ step and other system changes will be discussed below. In general, only the highlights of the responses will be mentioned.

5.2.1 Reactivity Step

Fig. 5-3 shows the response to a +5¢ reactivity step. This step corresponds roughly to loading a fuel element which does not displace a fuel element from the center of the reactor. The reactivity difference between a newly loaded element and the central one that is discharged would ordinarily be about 2¢ or 3¢.

In response to the step of reactivity, the neutronic power rises about 20% with a peak some 36 seconds after the step. It displays two oscillations having a period of 276 seconds, and more or less reaches steady-state in ten minutes. The reactor thermal power shown in the same figure exhibits a minor oscillation, and the heat dump thermal power does not change at all. The average core temperature rises a few degrees to negate the +5¢ step.

The net system reactivity also oscillates, but is reduced to less than 2¢ in three minutes. Nearly all of the temperature reactivity change in this run is due to the moderator temperature change rather than to fuel element changes, although the fuel element temperatures change considerably more than the moderator temperatures, as one might expect. The inverse period is less than 10^{-3} in 3 minutes. The reactor exit gas temperature increases less than 50°R during the

Fig. 5-3 Response to a +5¢ Reactivity Step

power excursion. The recuperator outlet temperature and gas temperatures in the heat exchanger, heat dump, and other components show no appreciable changes.

The system's behavior toward this reactivity perturbation is typical in that a disturbance at the reactor end of the UHTREX system is delayed and attenuated almost completely as it travels toward the heat dump. Similarly, heat dump disturbances are delayed and attenuated as they travel toward the reactor.

One concludes from the 5¢-step case that, outside of a modest power oscillation, nothing exciting happens. To avoid unnecessary shut-downs, the power oscillation of Fig. 5-3 should be kept in mind when setting power level scrams during fuel loading.

5.2.2 Bypass

The fraction of primary loop coolant flow that is bypassed around the recuperator was decreased from 0.2 to 0.1 on a 5-minute ramp. The reactor inlet and outlet gas temperatures rise about 140° and 40°, respectively, during the ramp and then settle to new equilibrium values. The reactor solid temperatures rise enough to lower the reactivity by about 1¢, and the reactor power changes as shown in Fig. 5-4. The thermal power and neutronic power are fairly close in this case, because it is the gas temperature change that causes the neutronic power change. The heat dump thermal power change lags considerably however. Recuperator and heat exchanger gas temperatures change by less than 100°R.

5.2.3 Primary Loop Blower

The primary loop blower speed was decreased by 20% on a 5-minute ramp. The primary flow rate decreased by about the same percentage, while the secondary flow rate rose slightly. System pressure drops and

Fig. 5-4 Response to a Slow Decrease in Bypass Flow

blower powers behaved similarly. Reactor neutronic and thermal power responses, as shown in Fig. 5-5, are similar to those produced by a bypass decrease, but the power changes are somewhat larger. Also, the heat dump thermal power change does not lag quite so much as it did in the bypass problem.

5.2.4 Secondary Loop Blower

The secondary loop blower speed was reduced 20% in 5 minutes. The secondary loop flow rate, pressure drop, and blower power went down by a similar percentage, while the corresponding quantities in the primary loop decreased only slightly. The heat exchanger hot outlet temperature rose by about 140°F. As shown in Fig. 5-6, heat dump thermal power fell rapidly during the term of the blower ramp, and then recovered to a value about 4% below full power in about 15 minutes. Reactor thermal and neutronic powers drop slowly and finally reach the heat dump thermal power.

5.2.5 Heat Dump Air Flow

If the heat dump air flow rate is decreased 20%, the three dependent heat dump gas temperatures rise less than 100°R in 20 minutes. System pressure drops and blower powers decrease by less than 5%, and the reactor powers go down about 4% in 25 minutes, as shown in Fig. 5-7.

5.2.6 Heat Dump Air Inlet Temperature

To simulate what would happen during an extreme air temperature change, the heat dump air inlet temperature was decreased 40° in 5 minutes. As a result, system flow rates and pressure drops rose by approximately 5%, the reactor gas temperatures decreased by about 25°R, and the reactor powers increased slowly to about 4% above their initial values. As shown in Fig. 5-8, it took about 2½ minutes for the reactor to start changing. The minimum period was about 1000 seconds. In this run

Fig. 5-5 Response to a Slow 20% Decrease in Primary Loop Blower Speed

Fig. 5-6 Response to a Slow 20% Decrease in Secondary Loop Blower Speed

Fig. 5-7 Response to a Slow 20% Decrease in Heat Dump Air Flow Rate

Fig. 5-8 Response to a Slow 40°F Decrease in Heat Dump Inlet Air Temperature

the initial conditions were the standard conditions except for the heat dump air inlet temperature.

5.2.7 Comments

In general, the entire system behaves in an unexciting manner when stimulated by inputs of the kind shown in this section. Reactor power is almost the only physical variable of interest that responds quickly or with appreciable overshoot, and the effects of its changing are hardly felt elsewhere in the system.

5.3 Parameter Sensitivity Studies

Many physical parameters in the UHXCOM mathematical model are not known accurately. This is particularly true of heat transfer coefficients. Also, some of these parameters may change during the lifetime of the reactor. For instance, the heat transfer characteristics of the heat exchanger may change if material deposits on the tube walls during operation. Thus, some of the numbers used in the UHXCOM model may be inaccurate now or later, and one would like to know the effects of such inaccuracies on system behavior. These effects can be determined by changing a poorly known parameter in the UHXCOM model, repeating a previous problem, and comparing the results. The comparison shows the sensitivity of the system to change, and allows one to judge the seriousness of the situation in which a physical constant is poorly known or changes with time. This section reports the results of UHXCOM problems in which poorly known parameters were changed to determine their effects on system behavior.

5.3.1 Reactor Temperature Coefficient of Reactivity

Three 5¢-step problems with different values of reactor temperature coefficient of reactivity (ρ_t) were run. The results are summarized in runs 1, 2 and 3 of Table V-2. In run 1, ρ_t is halved; in run 2 it is left as in the standard problem; and in run 3 it is doubled. These runs show that as the temperature coefficient of reactivity ρ_t decreases, the size of the power overshoot in response to a small step of reactivity increases slightly, and the period of the oscillation increases considerably. As one might expect, the dynamic behavior of the reactor is strongly dependent on ρ_t . However, even for variations of -50% to +100% in ρ_t , the system is still stable and responds in a reasonable way.

5.3.2 Heat Transfer Coefficients

Runs 4, 5, and 6 of Table V-2 were made to provide some measure of the sensitivity of the system to the prediction of heat transfer coefficients. Each run was started with a +5¢ reactivity step to determine the changes in dynamic reactor behavior caused by changes in heat transfer coefficients. Table V-2 shows that the reactor power overshoot and period of power oscillation in response to a +5¢ step are not affected significantly by heat transfer coefficients that were changed. The thermal power, gas temperatures, etc. in the right-hand columns of Table V-2 are the steady-state values reached after the transients due to the +5¢ step have subsided. These steady-state values should be compared to the standard condition values in the first row of the table. It can be seen that an arbitrary increase in the recuperator heat transfer coefficient by 50% is similar to a decrease in bypass flow. It decreases the reactor inlet and outlet temperature, decreases the heat exchanger inlet and outlet temperature, and reduces power. A 33% reduction in the shell side coefficient in the heat exchanger reduced power about 9% and increased temperatures in the primary and secondary loops as much as 80°R.

UHTREX SYSTEM BEHAVIOR WITH VARIOUS PARAMETER CHANGES
(All runs include an initial +5¢ reactor reactivity step)

Run	Parameter Varied	Change	Power Overshoot Max Q_n	Period of Power Oscillation (sec)	Q_t (Btu/sec)	Reactor Gas Temp. ($^{\circ}$ R)		Heat Exchanger Gas Temp. ($^{\circ}$ R)				Maximum Heat Exchanger Tube Temp. ($^{\circ}$ R)
			Initial Q_n			T_i	T_o	T_{xhi}	T_{xho}	T_{xci}	T_{xco}	
Standard Conditions					2844	2062	2860	1653	854	619	1177	1241
Design Limits						2060	2860	1860	1060	1060	1560	1660
Run 1	Reactor temp. coef- ficient of reactivity	- 50%	1.26	400	2851	2068	2870	1657	855	619	1179	1244
Run 2	Reactor temp. coef- ficient of reactivity	0%	1.18	276	2847	2065	2865	1655	855	619	1178	1243
Run 3	Reactor temp. coef- ficient of reactivity	+100%	1.14	172	2846	2063	2862	1654	855	619	1177	1242
Run 4	Recuperator heat transfer coefficient	+ 50%	1.14	280	2713	2129	2878	1586	837	615	1141	1202
Run 5	Heat exchanger heat transfer coefficient on shell side	- 33%	1.14	280	2735	2079	2867	1673	885	616	1147	1262
Run 6	Heat dump heat trans- fer coefficient on tube side	- 33%	1.08	284	2589	2104	2877	1702	929	691	1250	1308

Abbreviations:

T_i	- Reactor inlet gas temperature
T_o	- Reactor outlet gas temperature
T_{xhi}	- Heat exchanger hot side inlet gas temperature
T_{xho}	- Heat exchanger hot side outlet gas temperature
T_{xci}	- Heat exchanger cold side inlet gas temperature
T_{xco}	- Heat exchanger cold side outlet gas temperature
Q^n	- Reactor neutronic power
Q_t	- Reactor thermal power

In general, even large changes in the heat transfer coefficient of individual components do not seem to cause serious problems. There is no evidence that it will be difficult to find operating conditions within design limitations of equipment that will allow required power and temperature conditions in the reactor to be achieved.

5.3.3 Comments

The results presented in this section show that the UHTREX system is encouragingly insensitive to parameter changes. Thus, parameter inaccuracies should not seriously affect the results presented here.

5.4 Special Simulation Runs

This section contains a variety of simulation runs, some of which are associated with hypothetical power excursions and system failures. The runs given here do not exhaust the possibilities of the program, but are ones of current interest for which there is adequate input information.

5.4.1 Startup Route

A problem was run to determine the feasibility of a particular reactor startup route. The word route is used to emphasize that the startup described here is not a procedure but merely a path that might be followed at one's own pace with stops where appropriate. This path takes one from low temperature conditions to design conditions without violating any of the system limitations described in Section 4.

The startup route initial conditions, adjustments enroute, and final conditions are shown in Table V-3. In brief, all three flow rates, the bypass flow fraction, and system pressures are kept at the standard conditions. Reactivity is added slowly to bring the reactor power and temperature to their standard values. In this problem,

TABLE V-3
POSSIBLE UHTREX STARTUP ROUTE

System Variable	Initial Conditions	Adjustments Enroute	Final conditions after 24-hour reactivity removal followed by 1 hour to achieve steady-state
Blowers, Fans			
BS ₁	4000 rpm	Adjust BS ₁ to give $\dot{W}_1 = 2.85^* \text{ lb/sec}$	8186 rpm
$\dot{W}_1$	2.9 lb/sec		2.84 lb/sec
ΔP_1	2.1 psi		6.0 psi
HP ₁	6.8 hp		49.1 hp
σ_1	0.17		0.13
BS ₂	6500 rpm	Adjust BS ₂ to give $\dot{W}_2 = 4.08^* \text{ lb/sec}$	8946 rpm
$\dot{W}_2$	4.1 lb/sec		4.07 lb/sec
ΔP_2	6.1 psi		9.9 psi
HP ₂	25.7 hp		49.1 hp
σ_2	0.14		0.13
$\dot{W}_3$	42.8 lb/sec	Keep $\dot{W}_3 = 42.8^* \text{ lb/sec}$	42.8 lb/sec
Pressures			
P ₁	500	Keep P ₁ = 500 psi	500 psi
P ₂	500	Keep P ₂ = 500 psi	500 psi
Bypass			
FB	0.20	Adjust bypass valve to keep FB = 0.20*	0.20
Reactor			
Q _t	0.6 kW	Increase ρ at an average rate of about 2¢ per minute for 24 hours by removing rods or loading fuel.	2.97 MW
T _{avg}	100°F		2720°F
ρ	0		0
T _i	81°F		1599°F
T _o	97°F		2399°F

* Standard condition values.

reactivity was added at a rate of about 2¢ per minute, which brings the system to standard conditions in 24 hours. The reactivity addition rate is somewhat arbitrary. The rate chosen here is low enough to avoid large initial transients but high enough to allow a reasonable simulation time on the digital computer. The computer time for the 25 hours of simulated reactor operation in this problem is less than 35 minutes.

The reactor behavior during the first 5000 seconds of operation is shown in Fig. 5-9a. The reactivity increase leads to mild oscillations in the beginning. These oscillations could be damped out by use of control rods, or other independent system variables if desired. After the first 5000 seconds, the system powers and temperatures increase monotonically and nearly linearly to the standard conditions. Figs. 5-9b and 5-9c show profiles typical of system behavior along the startup route.

It should be noted that at no point along the startup route do system conditions approach or violate known limitations. In this regard, blower operation along the startup route is particularly conservative. It should also be noted that the startup route is simple mechanically and conceptually, and leads to conservative uniform temperature increases everywhere.

As noted above, the rate and method of addition of reactivity are somewhat arbitrary. If reactivity is added in frequent steps of say 5¢, then multiples of the typical 5¢ transient response will be superimposed on any startup route considered. The bypass valve and/or primary blower might be used to damp such transients if desired. It would seem that large reactivity steps or high rates of reactivity increase should not be used if one wishes to avoid large power overshoots, extreme fuel temperatures, and out-of-limit operation in general.

Fig. 5-9a System Power Responses During First Portion of Startup Route

Fig. 5-9b System Power Responses Along Startup Route

Fig. 5-9c Fuel Element Temperatures Along Startup Route

5.4.2 Restart

This section deals with the problem of restarting the reactor soon after a shutdown has occurred, while the reactor is still quite hot. The problem is somewhat complex in that there are many possible courses of restart action and, at the same time, there are many system limitations to be considered. The 4000-rpm lower limit on blower speed is one of the most interesting constraints. Unless one goes into a time consuming system depressurization, the minimum blower speed establishes a minimum flow rate. This minimum flow rate is accompanied by a minimum cooling rate in the hot reactor, and it sets a minimum rate of recovery of reactivity due to cooling. Any restart solution must accommodate the reactivity recovery rate given by the minimum blower speed.

The reactivity recovery rate is a maximum, and the restart is most difficult, if one tries a restart immediately after shutdown, when the reactor is at its hottest.* In the problems discussed here, the initial conditions for the reactor and other solid temperatures are specified to be their standard values which are their normal maximums. Thus, the restart considered here is made under the most difficult conditions. If the reactor has been allowed to cool after shutdown, the subsequent restart is even easier than described below.

The restart considered is outlined in Table V-4. In the shutdown condition at the left, the reactor control rods are full in. The bypass valve is at its standard position, and all blowers and fans are off. The following steps are taken in restart.

Phase A -- approximately 15 minutes

- 1) Move the bypass valve to that position which will give 25% bypass flow. This step is taken to ensure that the

* It is recognized that there will be some modest amount of cooling prior to any restart. Immediate restart without any cooling would be impossible, for full removal of all rods would only bring the reactor critical, and there would be no rod worth available to increase the power.

TABLE V-4
RESTART SCHEDULE

Independent Variable	Shutdown Condition	Restart Phases				Standard Conditions
		A	B	C	D	
BS ₁ (rpm)	0	0	Turn on to minimum speed of 4000 rpm.	4000	Increase slowly to standard values over 10-minute interval.	8255
BS ₂ (rpm)	0	0		4000		9000
$\dot{W}_3$ (lb/sec)	0	0	42.8	42.8	42.8	42.8
FB (—)	0.20	0.25	0.25	0.25	Change slowly to standard value of 0.20 in 10 minutes.	0.20
Control Rods	In	Withdraw to establish neutronic power at approximately 30 kW.	Use rods to moderate power rise. Remove entirely or almost entirely at end.	Out or almost out.	Move out slowly if not out already.	Out
Time Required (min)	--	~15	15	--	15	--
Remarks	Starting Condition			Can pause here if desired.		End Condition

reactor inlet gas temperature will not be excessive when a low flow, corresponding to minimum blower speed, is established.

- 2) Withdraw and manipulate rods to establish a reactor power of approximately 30 kW. The reactor power need not be established precisely at 30 kW, nor is it necessary to try to level out the power. As soon as the power is about 30 kW and the period is say, greater than 60 seconds, proceed to Phase B.*

Phase B -- 15 minutes

- 3) Turn on primary and secondary blowers to their minimum speeds, turn on the heat dump fans to their standard speeds.
- 4) As the reactor cools, its reactivity will increase and the power will tend to rise. The operator should insert and manipulate a control rod to moderate any potential power overshoot, and should keep the positive period greater than 30 seconds. Toward the end of the 10-minute period, the rods should be removed entirely or almost so. It might be helpful to leave 50¢ of rod in the reactor, until Phase D is started, to lower the reactor outlet gas temperature about 50°F and keep it at or below 2400°F.

Phase C -- arbitrary length

At this point, the reactor power primary and secondary flow rates are very close to one-half their standard values. System gas and solid temperatures are at or near their standard values except near the heat dump where they are a little lower. So far as is known now,

* At a power of 30 kW and with no coolant flow, the reactor fuel element temperatures increase about 1°R in 4 seconds. One should not delay unnecessarily under these conditions.

one could operate at this point for many minutes if it were desirable. From the standpoint of startup, however, there is no requirement for a delay at this point.

Phase D -- 15 minutes

In this phase, the primary and secondary blower speeds are slowly increased to their standard values in a 10-minute operating period. At the same time, the bypass valve is slowly changed to its standard position. If a control rod was left part way in the reactor at the end of Phase B, it should be removed slowly and entirely in this same 10 minutes. An additional 5 minutes is allowed to let the system more or less come to steady-state at the standard operating conditions. This completes the restart. The power profiles in Phase D of restart are shown in Fig. 5-10a. This portion of the restart is quite smooth without operator manipulation of the control rods.

Phase B of the restart is particularly interesting and deserves additional comment. During the first part of Phase B, the hot reactor operating at a low power is cooled by the primary flow. Consequently, the system reactivity increases at a rate of about 0.3¢ per second. If the operator does not move the control rods to moderate the impending power rise, the system power will respond as shown in Fig. 5-10b. The neutronic power rises to 5.3 MW about 160 seconds after the blowers are turned on, then decreases, oscillates, and finally comes to equilibrium in 15 or 20 minutes. The peak reactivity is 31¢ at $t = 120$ seconds. The minimum period of 17 seconds occurs at $t = 120$ also. The maximum fuel element temperature in the UHXCUM model is 3680°R, which is 220°R above its usual value. This high fuel temperature, which exists for only a few minutes, does not present mechanical or chemical problems to the fuel elements. There is no significant fuel migration in the elements and little gas release from them in such a transient. The reactor inlet gas temperature stays at or below its limiting value, as do heat exchanger temperatures. As noted previously, the reactor

Fig. 5-10a Power Profiles in Phase D of Restart

Fig. 5-10b Power Response in Phase B of Restart Without Operator Rod Correction

outlet gas temperature is about 50°R high and can be lowered by leaving a portion of a rod in the system if so desired.

It is recommended that the operator, or possibly the computer, manipulate the reactor control rods during Phase B of restart to moderate the power peak shown in Fig. 5-10b. Typically, one might try to keep the reactor period greater than 30 seconds and the reactor power less than 3 MW. This does not appear to be a difficult job. The reactivity and power rise slowly, and the reactor rod system is capable of reducing reactivity at a rate considerably in excess of 0.3¢/second. A good check of the operator or computer ability to moderate the power rise on Phase B of restart will be made with a reactor analog model prior to operation of the real system.

A variety of problems based on alternate Phase B restarts have been run. In one problem, the bypass was left fully closed in Phase B. This bypass position sends fairly hot gas to the reactor, and the reactor cooling rate is lower than in the restart of Table V-4. Consequently, the reactivity recovery rate drops to about 0.22¢/second and the uncorrected power transient is less severe than that shown in Fig. 5-10b. However, closing the bypass to moderate the power transient raises the reactor inlet gas temperature about 300°R above its limiting value, and this condition is unacceptable. In fact, in the proposed restart the bypass valve is opened slightly from its standard position to lower the reactor inlet gas temperature.

The power level established in Phase A of restart also affects the results in Phase B. If the power level set in A is higher than 1% of standard power, it takes less time in Phase B to reach a power at which the reactor is heating rather than cooling, and the uncorrected power peak is lower. Conversely, starting at a power lower than 1% leads to an even larger uncorrected power peak. The results of three

problems run to determine the effects of the Phase A power level on the Phase B restart are shown in Table V-5. As expected, this table indicates that it is easier to do the Phase B restart from higher Phase A powers. However, one cannot raise the power in Phase A too high, or the fuel elements will overheat before Phase B is started. At 30 kW, the fuel elements rise 1°R in about 4 seconds. A somewhat higher power level might be chosen for Phase A if it could be established easily and quickly and one could proceed to Phase B promptly. The data in Table V-4b can be interpolated to determine probable power peaks associated with different initial powers.

In conclusion, the UHTREX reactor can be easily and safely restarted when hot as described above.

TABLE V-5

EFFECTS OF INITIAL POWER ON PHASE B OF RESTART

Phase A Initial Power (kW)	Characteristics of Reactor Response in Phase B Without Operator Rod Correction				
	Power Peak Standard Power	Time of Power Peak (Sec)	Maximum Reactivity Recovery Rate (¢/Sec)	Maximum Reactivity (¢)	Minimum Period (Sec)
3	2.48	180	0.32	39	11
30	1.76	160	0.3	31	17
300	1.02	~160	0.25	17	42

5.4.3 Abrupt Changes

Four problems were run to determine the effects of accidental abrupt changes in operation of the primary and secondary loop blowers and the bypass valve. In the first two problems, the blower speeds were increased in a step by 20% of their standard values. In problem three, the bypass flow fraction was increased on a ramp from its standard 0.20 to 0.36 in 16 seconds, and in problem four the bypass flow was decreased in a similar way. The bypass ramp corresponds roughly to the speed limited operation of the bypass valve motor. The results of these four problems are summarized in Table V-6. The reactor power rose temporarily by 24 and 32% in response to primary blower and bypass increases, but only 2% in response to the secondary blower change. The power fell by 16% in response to the bypass flow decrease. These transient power changes are substantial but not damaging. The changes had little effect outside the reactor.

It was noted that the reactor inlet gas temperature, T_i , is strongly and immediately affected by changes in bypass flow. In particular, in the fourth problem, in which the bypass is closed, T_i rises from 2060°R to 2280°R in the 16 seconds during which the bypass valve is in motion. Such an increased T_i , if maintained for many minutes, would lead to system damage. The bypass valve must be opened or the primary blower must be shut down to avoid damage. An automatic primary blower shutdown based on excessive T_i is being designed. The blower coastdown itself takes place in a short time.

In conclusion, abrupt increases in flow rate and bypass flow should be avoided, but if they do occur the consequences are not severe. There is plenty of time to take corrective action before the long term effects of such changes propagate to components that will be operating near their design limit temperatures. However, an abrupt decrease in bypass gives an immediate increase in T_i and corrective action should be taken quickly.

TABLE V-6

UHTREX SYSTEM RESPONSE TO ABRUPT CHANGES

Variable Changed	Initial Value	Final Value	Type of Change	Resultant Neutronic Peak Power (MW)	Ratio, Peak Power Initial Power	Time of Neutronic Power Peak (Sec)	Minimum Power Period (Sec)	Comments
BS ₁	8255 rpm	9906 rpm	Step	3.72	1.24	105	312	$\dot{W}_1$ changed from 2.85 to 3.32 lb/sec
BS ₂	9000 rpm	10,800 rpm	Step	3.08	1.03	140	3180	$\dot{W}_2$ changed from 4.08 to 4.84 lb/sec
FB	0.20	0.36	16-sec ramp	3.96	1.32	110	206	$\dot{W}_1$ changed from 2.85 to 2.87 lb/sec
FB	0.20	0.04	-2% of the full range of FB. 16- sec ramp	$Q_{\min}$ ≈ 2.4 MW	--	--	--	

5.4.4 Power Excursion Problem

A run was made to determine what would happen to the UHTREX system if a power excursion deposited a substantial amount of energy in the reactor fuel elements. The primary point of interest was whether the reactor, recuperator, or heat exchanger would be damaged in such a way that the primary containment would be ruptured.

To simulate the power excursion, it was assumed that there was an instantaneous energy release in the fuel elements sufficient to raise their temperatures 1000°C above their full power design values.* The run was started with the system at design conditions except for the fuel element temperatures, and the neutronic power which was arbitrarily set at 0.3 MW. The system was then allowed to relax.

Some results of this run are shown in Figs. 5-11a and 5-11b. The exit gas temperature from the reactor falls to its nominal design value in about 22 seconds, and the recuperator muffles the effects of the power excursion as far as the rest of the system is concerned. From discussions with other UHTREX personnel, it appears that the short-term excessive reactor exit gas temperatures would not damage the recuperator. Also, the temperature of the gas flowing to the recuperator would be lower than shown, for there would be considerable heat transfer to the stationary reflector, a part of the reactor not included in the model. The stationary reflector is a large and relatively strong piece of graphite which would not be damaged by short but high gas temperature exposures.

Thus, a power excursion which elevates the fuel elements 1000°C would not produce damage sufficient to lead to rupture of the primary containment.

* It takes about 7.5×10^{18} fissions, or a 0.075-second excursion at 3000 MW to produce the energy required to raise the fuel elements 1000°C.

Fig. 5-11a Fuel Element Temperatures After Hypothetical Power Excursion

Fig. 5-11b Recuperator Gas Temperatures After Hypothetical Power Excursion

5.4.5 Bypass to Full Open Position

In a trial problem, the bypass flow was stepped from its nominal value of $0.20 \dot{W}_1$, to its maximum value of $0.80 \dot{W}_1$. As shown in Fig. 5-12, the reactor thermal power rises as the bypass valve is opened, and reactor neutronic power follows afterwards, reaching a peak of 6.9 MW in about 75 seconds. The heat dump power rises slowly, and at steady state all three powers settle to 4.38 MW, almost 50% above the initial power level. During the power transient, the maximum net reactivity was 12¢ and the minimum period was 57 seconds. Fuel element temperatures oscillate and spread, as do the moderator temperatures. The recuperator hot outlet gas temperature rises at about 2°R/second and exceeds the allowable heat exchanger hot inlet temperature in 100 seconds. The heat exchanger tube temperature rises more slowly, at a maximum rate of 1°R/second. It comes to within 30°R of its design limit in 1000 seconds, and eventually exceeds the limit by 6°R.

A second problem including reactor scram and primary blower shutdown was run. Both these corrective actions were initiated when the reactor neutronic power reached 150% of its standard value. The temperature of the hot gas to the heat exchanger rises more rapidly than it did in the uncorrected case, due to the reduction in primary flow. However, this lower flow is able to transmit less heat to the heat exchanger, and the heat exchanger tube temperature rises less than 50°R, reaching a maximum of 1275°R at about 32 seconds. Thus, the corrective actions taken here are more than adequate to protect the heat exchanger.

5.4.6 Coastdown of Primary Loop Blower

The coastdown curve of the primary loop blower was obtained experimentally and incorporated in the UHXCUM program. A problem including coastdown of the blower was run. Ten seconds after the start of coastdown, $\dot{W}_1 < 0.5$ lb/sec, and the reactor thermal power is about 0.6 MW. The inlet gas temperature to the reactor rises 50°R in the first 5 seconds

Fig. 5-12 Response to Bypass Failure to Full Open Position
Without Corrective Action

and another 25°R in the following 20 seconds. The reactor outlet gas temperature, too, rises 400°R in 50 seconds. However, $\dot{W}_1$ is so low that little heat can be delivered to the reactor inlet or outlet pipe, so the rising reactor gas temperatures are of no particular concern. The temperature of the hottest UHXCUM fuel element lump rises 150°R in 25 seconds to about 3610°R. Moderator temperatures increase less than 25°R. The fuel and moderator temperature changes decrease the system reactivity, and the neutronic power falls linearly to 1454 Btu/sec 50 seconds after the blower coastdown is initiated. The decrease of thermal, neutronic, and heat dump power is shown in Fig. 5-13. Most heat exchanger and heat dump gas temperatures decrease. The highest heat exchanger tube temperature decreases over 300°R in 50 seconds.

The conclusion is that inadvertent coastdown of the primary blower will not lead to system damage because the decreasing primary flow cannot transmit a damaging amount of heat to potentially vulnerable parts. It might be noted that in the event of loss of $\dot{W}_1$ for any reason the reactor control rods will be run in. Such corrective action will ensure that the reactor power will continue to decrease to a very low value without oscillation.

5.4.7 Primary Loop Rupture

In this problem the primary loop pressure was decreased from 500 to 15 psi on a 10-second ramp corresponding to a hypothetical rupture of the primary loop. The primary flow rate decreases with the pressure drop to approximately 0.1 lb/sec. Because of the low primary flow, the results are quite similar to those found in the primary blower coastdown problem.

Reactor thermal power falls to 110 Btu/sec in 10 seconds. Neutronic power is down by 50% in 50 seconds. The fuel element lump that is at the highest temperature in the UHXCUM model rises about 170°R above its usual 3457°R in 30 seconds and then starts to decrease. The

Fig. 5-13 System Power Responses to Primary Blower Coastdown

moderator temperatures show little change. The reactor outlet gas temperature rises abruptly by 400°R in 15 seconds, but the hot gas should have little effect on the reactor outlet piping because the flow rate is so low. Heat exchanger tube temperatures decrease because the tubes continue to be cooled by the secondary flow which is only slightly affected by the primary rupture.

Thus, a primary rupture does not present a heating problem. As it is now planned, an automatic system will detect primary rupture and initiate venting of the secondary loop. This system will also run in the reactor control rods and shut down the primary and secondary blower.

5.4.8 Accidental Isolation Valve Closure

In this problem the secondary flow was shut off to simulate the effect of accidental closure of the secondary isolation valves. During the first 50 seconds of the problem, there is little change in reactor power or system temperatures except at the heat exchanger, where the highest tube temperature rises 210°R and the hot outlet gas temperature, T_{xho} , rises 300°R in 50 seconds. The average rate of these two rises is about 4°R/second and 6°R/second, respectively. The heat exchanger hot outlet gas temperature reaches its limit of 1060°R 32 seconds after the isolation valves are closed, and other limiting system temperatures are exceeded soon thereafter.

As noted in Section 4, gas temperature limits can be exceeded for at least a few minutes before transmitting enough heat to damage solid parts of the system. Thus, the blowers can stand a high T_{xho} for 3 or 4 minutes, and the fact that T_{xho} does exceed its limit temporarily is of no particular concern. What is needed is a system that will detect the accidental isolation valve closure and take appropriate corrective action promptly.

The safety system planned now provides for automatic control rod run in and primary and secondary blower shutdown on detection of loss

of secondary flow. The accidental isolation valve closure problem was rerun with the corrective primary blower coastdown. The results, which are similar to those of the primary blower coastdown problem, show that the corrective actions planned are sufficient to protect the system from damage.

5.4.9 Secondary Loop Rupture

The secondary loop pressure was decreased from 500 to 50 psi in one second to simulate rupture of the secondary loop. The heat exchanger tube temperature rises 180°R in the first 50 seconds, and T_{xho} rises about 200°R to exceed its limit in 43 seconds. Other heat exchanger solid and gas temperature limits are reached in 200 to 300 seconds.

These results indicate that the primary blower should be shut down soon after secondary loop rupture to avoid excessive system temperatures. A system that will run in the reactor rods, isolate the secondary loop, and shut down the blowers upon detection of secondary rupture is being planned. These corrective actions will protect the system against blower damage and high temperatures if the secondary loop should rupture.

5.4.10 Slow Secondary Loop Pressure Drop

There is equipment in the UHTREX system to detect a sudden drop in secondary loop pressure such as might accompany a secondary loop rupture. If a sudden drop is detected, the isolation valves are closed, the blowers are turned off, and the reactor is shut down. Thus, the sudden secondary loop pressure drop situation is in good hands. The question as to how to detect a slow pressure drop arose, and an UHXCOM problem including a "slow drop" was run. In this problem the secondary loop blower speed was kept constant. The run shows that system variables that changed at all changed linearly as the secondary loop pressure fell from 515 to 400 psi in 60 seconds. Some of the results are summarized in Table V-7.

Table V- 7 shows what might be expected. The secondary flow rate, secondary loop pressure drop, secondary blower horsepower, and heat dump power all fall with the pressure drop, and T_{xco} rises slightly. Thus, if desired, a drop in secondary loop blower horsepower and a rising T_{xco} could be used to confirm a loss of secondary loop pressure.

TABLE V- 7

RESULTS OF SLOW SECONDARY LOOP PRESSURE DROP

<u>Variable</u>	<u>Initial Value</u>	<u>Value at End of 60 Sec</u>	<u>Change</u>	<u>Final Value at Steady State</u>
Time, t (sec)	0	60	60	∞
P_2 (psi)	515	400	115	400
Q_t (MW)	3.01	2.97	0.04	2.89
T_{xho} ($^{\circ}$ R)	852	871	19	884
T_{xco} ($^{\circ}$ R)	1167	1204	37	1260
Q_{dump} (MW)	3.01	2.43	0.58	2.89
$\dot{W}_2$ (lb/sec)	4.19	3.26	0.83	3.29
Secondary Loop System Pressure Drop	10.2	8.0	2.4	8.4
Secondary Blower Horsepower	51.2	39.8	11.4	40.3

5.4.11 Loss of Heat Dump Air Flow

In an UHXCOM problem, the air flow in the heat dump was decreased in a step from 100% to 1% of its usual value. System solid and gas temperature increase, and the reactor powers and the coolant flow rates decrease. $\dot{W}_1$ drops 20% and $\dot{W}_2$ drops about 35% in 500 seconds. Some 350 seconds after the problem is started, T_{xho} exceeds its limit of 1060° R, and the primary blower will begin to overheat. Other system gas temperatures also exceed their limits in the following 200 seconds.

The reactor rods should be run in and the blowers should be shut down within 2 or 3 minutes after loss of heat dump air flow. A problem in which the blowers were shut down 200 seconds after loss of $\dot{W}_3$ was run, and it confirms that such corrective action is adequate. Type-writer out-of-limit messages and annunciator alarms will be generated in time to allow the operator to shut down in case of loss of heat dump air flow.

6. CONCLUSIONS

6.1 UHTREX Operation

The results in this report show that the UHTREX system can be operated at design conditions with heat exchanger temperatures conservatively low. The independent variables have considerable influence on system behavior, and they can be adjusted to achieve design conditions and a wide range of off-design conditions.

The UHTREX system response to anticipated operator actions is slow and mild. These dynamic responses and steady-state conditions are relatively insensitive to inaccuracies and changes in heat transfer coefficients and the reactor temperature coefficient of reactivity.

A startup route for the UHTREX reactor has been investigated and found to be satisfactory; and a simple hot restart procedure has been determined. A variety of hypothetical system failures have been investigated. It has been shown that the overall system is safe from damage even in the event of serious equipment failures.

At this time, there are no known and unsolved operational or safety problems in the UHTREX equipment included in the scope of this report.

6.2 Analysis Program

6.2.1 Experience Thus Far

The general approach that has been taken to obtain a better understanding of the UHTREX system has been satisfying and successful. It would appear that the system chosen for study does contain the most significant components and that the mathematical model for these components is adequate for the purposes of the investigation. The UHXCOM program to solve the model equations is neither so large nor so complicated as to prohibit its practical use with existing computers at the Laboratory.

The feature of being able to check the UHXCOM program with transfer functions obtained directly from frequency domain programs has been invaluable in debugging UHXCOM. Film output from both programs has saved considerable time in evaluating results.

In general, the digital computer has proved to be a satisfactory simulation tool, but its associated turn-around-time* of one-half to one day is a considerable obstacle to smooth and efficient progress on such a problem.

6.2.2 Plans

Recent interest in the UHTREX pressure control systems has led to a request that pressures in both primary and secondary loops be calculated as a function of gas temperatures in UHXCOM rather than be treated as independent variables. From previous work, it appears that such a change should have little effect on system behavior as predicted by UHXCOM. However, inclusion of the pressure calculation will allow one to determine what pressure changes would occur during various transients, and

* Delay from time problem is submitted to time problem results are available.

this information will help in designing the pressure regulation system. Consequently, plans to include the pressure calculation are being made.

A variety of possible closed loop controls are discussed in Section 5.1.3. Such closed loop options will be included in UHXCOM so that the loops can be designed and so that their effects and interactions can be studied.

There is increasing interest in operational problems in which heat leak, reactivity poisoning, and fuel burnup play an important role. These three phenomena have very long time constants, and their inclusion in UHXCOM should have little effect on problems studied thus far. However, they are interesting effects in themselves because of their influence on day to day operation. Thus, they will be considered for inclusion in UHXCOM.

REFERENCES

1. Descriptions of the UHTREX system are given in two Los Alamos Scientific Laboratory reports.
 - a. "Ultra High Temperature Reactor Experiment (UHTREX) Hazard Report," LA-2689, March 1962.
 - b. "Ultra High Temperature Reactor Experiment Facility Description and Safety Analysis Report," LA-3556, 1966.
2. The present report contains examples of the linearizations and transformations used to obtain frequency domain equations appropriate for direct machine solution. It also contains a brief discussion of the frequency domain programs and an explanation of how the results of these programs have been used in model studies, in checking the simulation program, etc. A complete description of the derivation of the frequency domain equations, the frequency domain programs themselves, and the results from these programs is given in an unpublished report by C. E. Stiles, Los Alamos Scientific Laboratory.
3. Information on various experimental model studies, analytical studies, and digital computer analysis programs is given in the following reports.

Quarterly Status Reports on Ultra High Temperature Reactor Experiment (UHTREX)

LAMS-2623, August 20, 1961	LAMS-2933, June 20, 1963
LAMS-2650, November 20, 1961	LAMS-2984, September 20, 1963
LAMS-2678, February 20, 1962	LAMS-3036, December 20, 1963
LAMS-2722, May 20, 1962	LAMS-3069, March 20, 1964
LAMS-2782, September 20, 1962	LAMS-3112, June 20, 1964
LAMS-2835, December 20, 1962	LAMS-3190, September 20, 1964
LAMS-2894, March 20, 1963	

Quarterly Status Reports on Advanced Reactor Technology (ART)

LA-3244-MS, January 31, 1965	LA-3482-MS, January 31, 1966
LA-3370-MS, July 30, 1965	LA-3524-MS, April 30, 1966
LA-3431-MS, October 31, 1965	

4. A discussion of various convergence criteria is given in "Calculation of Transient Temperatures in Pipes and Heat Exchangers by Numerical Methods" by G. M. Dusenberre, Transactions of the ASME, Vol. 72, No. 3, April 1954, p. 421. Also see Appendix D, "Convergence Criteria" of present report.
5. A good discussion of the use of transfer functions in finding appropriate lumped parameter approximations to distributed parameter heat transfer systems is given in "Approximate Models for Distributed-Parameter Heat-Transfer System," S. J. Ball, Proceedings of 1963 Joint Automatic Control Conference, pp. 131-139. This paper also contains a good bibliography on the subject.
6. Personal communication from B. M. Carmichael, Los Alamos Scientific Laboratory.
7. "General Solution of the Reactor Kinetic Equations," G. R. Keepin and C. W. Cox, Nuclear Science and Engineering, Vol. 8, No. 6, December 1960, p. 670.
8. The method of collocation used in this report was supplied by K. Lathrop of the Los Alamos Scientific Laboratory in a personal communication.

References for each appendix are given at the end of the appendix.

APPENDIX A

INSTRUCTIONS FOR USING UHXCOM

This appendix is divided into nine parts:

- A.1 Standard Problem
- A.2 Control Variables
- A.3 Time Dependent Functions
- A.4 Closed Loop Variables
- A.5 Transfer Function Calculations (see heading)
- A.6 Sample Problem Input
- A.7 Sample Problem Output List
- A.8 Sample Computer Plot
- A.9 Problem Running Time on IBM 7094

A.1 Standard Problem

Most problems of interest in the UHTREX System Dynamics Study involve transients about the standard operating conditions. A "standard problem" which reflects these operating conditions is stored in the program to minimize input to the computer code. A portion of the standard problem is listed below for convenience. The variables may be modified (read in) at the beginning of the problem.

A.1.1 Control Variables

<u>Symbol</u>	<u>Value</u>	<u>Description</u>
KMX	1	Indicates start from SS matrix (see A.2)
NRM	0	No internal mapping (see A.2)
KVAR2	1	Run in the connected mode (see A.2)
KVAR1	1	(see A.2)
KV(i)	11	i=1,11 (see A.2)
JN	0	Indicates <u>no</u> transfer function calculation (see A.5)
JM	1	(see A.5)
JØ	1	(see A.5)
JV(i)	13	i=1,30 (see A.5)
ØM(i)	0.0 rad/sec	i=1,21 (see A.5)
EPIWD	0.00001	Convergence criterion in blower calculations
EPISS	0.00001	Convergence criterion for steady-state matrix calculation
JSPEC	0	If JSPEC=1, does not terminate problem if blower surge condition occurs (for special problems)
NØBLWR	0	If NØBLWR=1, ignores blower calculations after initial SS matrix calculation
KCLPW	0	If KCLPW=1, closes control loops (see A.4)
QDEM	2845 Btu/sec	Desired power level (see A.4)
TØW	2860°R	Desired reactor outlet gas temperature (see A.4)
W1DW	2.85 lb/sec	Desired primary loop flow rate (see A.4)
W2DW	4.078 lb/sec	Desired secondary loop flow rate (see A.4)

<u>Symbol</u>	<u>Value</u>	<u>Description</u>
TØGAIN	0.01	Closed loop gain constant (see A.4)
QGAIN	1×10^{-6}	Closed loop gain constant (see A.4)
WGAIN	1.	Closed loop gain constant (see A.4)
TAU	0 sec	Time at start of problem
DT	1 sec	Time increment for transients
TØT	100 sec	Time at end of problem
CT4(i)	1×10^6 sec	Time to start perturbation of ith independent variable(s) = $n \times DT$
CT6(i)	1×10^6 sec	Time to stop perturbation of ith independent variable(s) = $n \times DT$
PLTT1	10^6 sec	End point of first plot range
PT	2 sec	Time increment of first plot range (TAU < PLTT1)
PT2	0 sec	Time increment of final plot range
DKL	-100 cents	Lower bound for reactivity film plotting
DKU	+100 cents	Upper bound for reactivity film plotting
YT	3500.°R	Normal upper range for temperature plots
YB	500.°R	Normal lower range for temperature plots
XFMT	4000.°R	Normal upper range for fuel, moderator temperatures
XFMB	1000.°R	Normal lower range for fuel, moderator temperatures
XQT	6000.°R Btu/sec	Normal upper range for power
XQB	0. Btu/sec	Normal lower range for power

A.1.2 Neutronics Constants

GAMMA	-5.83×10^{-5} reactivity/°R	Temperature coefficient of reactivity
TAMB	3183°R	Reference level for reactivity due to temperature

<u>Symbol</u>	<u>Value</u>	<u>Description</u>
DLKRIC	+0.0065	1 \$ initial rod worth (initial condition)
DKSDIC	-0.0065	-1 \$ initial shut down worth (initial condition)
A ₁ (1)	1.1919 x 10 ¹	Reactor kinetics constants for ²³⁵ U reactor with $\ell^* = 0.001$, $\beta = 0.0065$. See Appendix E.
A ₁ (2)	1.3167	
A ₁ (3)	8.5608	
A ₁ (4)	1.1446 x 10 ¹	
A ₁ (5)	2.8952 x 10 ¹	
A ₁ (6)	4.6785 x 10 ¹	
A ₁ (7)	8.8451 x 10 ²	
S(1)	0.0	
S(2)	-1.5032 x 10 ⁻²	
S(3)	-7.0394 x 10 ⁻²	
S(4)	-1.9626 x 10 ⁻¹	
S(5)	-1.2099	
S(6)	-3.6891	
S(7)	-7.0596	

A.1.3 Flow Parameters

W1DIC	2.85 lb/sec	Primary loop flow rate (initial condition)
W2DIC	4.078 lb/sec	Secondary loop flow rate (initial condition)
W3DIC	42.78 lb/sec	Heat dump air flow rate (initial condition)
FBIC	0.202	Fraction of primary flow which bypasses cold side of the recuperator (initial condition)
FA	0.15	Fraction of primary flow through the annulus between the fuel element and channel

<u>Symbol</u>	<u>Value</u>	<u>Description</u>
FE	0.85	Fraction of primary flow through the fuel element hole
CP	1.25 Btu/°R lb	Helium heat capacity
CA	0.25 Btu/°R lb	Air heat capacity
SP1IC	500 psi	Primary loop system pressure (initial condition)
SP2IC	500 psi	Secondary loop system pressure (initial condition)
BS1IC	8110 rpm	Primary loop main blower speed (initial condition)
BS2IC	9000 rpm	Secondary loop main blower speed (initial condition)

A.1.4 Reactor Heat Exchanger*

CM	0.48 Btu/°R lb	Moderator heat capacity
CF	0.48 Btu/°R lb	Fuel element heat capacity
WM	8000 lb	Weight of moderator
WF	241 lb	Weight of fuel
HA	1.9 Btu/°R sec	Heat transfer coefficient in annulus = hA
HEK	$1.4 \frac{\text{Btu/°R sec}}{(\text{lb/sec})^{0.8}}$	Heat transfer coefficient in fuel element hole = $hA/(\text{WID} \times \text{FE})^{0.8}$
HRAD	$5.75 \times 10^{-11} \text{ Btu/°R}^4 \text{ sec}$	Radiation coefficient between fuel and moderator
FF1	0.255	Fraction of fuel element power generated in fuel element No. 1 (starting at center of core)

* Values given are for the whole component (not per lump values).

<u>Symbol</u>	<u>Value</u>	<u>Description</u>
FF2	0.256	Element No. 2
FF3	0.269	Element No. 3
FF4	0.220	Element No. 4
FM1	FF1	Fraction of moderator power generated in lump No. 1 (starting from center of core)
FM2	FF2	Lump No. 2
FM3	FF3	Lump No. 3
FM4	FF4	Lump No. 4

A.1.5 Recuperator*

HRHK	$8.2 \frac{\text{Btu}/^{\circ}\text{R sec}}{(\text{lb/sec})^{0.8}}$	Hot side heat transfer coefficient = $hA/(W1D)^{0.8}$
HRCK	$6.32 \frac{\text{Btu}/^{\circ}\text{R sec}}{(\text{lb/sec})^{0.8}}$	Cold side heat transfer coefficient = $hA/(W1D(1.0 - FB))^{0.8}$
CR	0.44 Btu/ $^{\circ}\text{R lb}$	Heat capacity of recuperator solid
WR	3116 lb	Weight of recuperator solid
WTHREC	1.5	Hot side mean gas temperature weight- ing factor. $T_{\text{ave}} = (T_{\text{in}} + \text{WTHREC} \times T_{\text{out}})/(1. + \text{WTHREC})$

A.1.6 Heat Exchanger*

CX	0.18 Btu/ $^{\circ}\text{R lb}$	Heat capacity of heat exchanger solids
WX1	285 lb	Weight of either solid in bayonet tube
WX2	1663 lb	Weight of flow divider

* Values given are for the whole component (not per lump values).

<u>Symbol</u>	<u>Value</u>	<u>Description</u>
WX3	1694 lb	Weight of shell
H1K	$5.6 \frac{\text{Btu}/^{\circ}\text{R sec}}{(\text{lb/sec})^{0.8}}$	Heat transfer coefficient to bayonet tube (inlet side) = $hA/(W1D)^{0.8}$
H2K	$7.78 \frac{\text{Btu}/^{\circ}\text{R sec}}{(\text{lb/sec})^{0.8}}$	Heat transfer coefficient to bayonet tube (outlet side) = $hA/(W1D)^{0.8}$
H3K	$9.92 \frac{\text{Btu}/^{\circ}\text{R sec}}{(\text{lb/sec})^{0.8}}$	Heat transfer coefficient to flow divider (hot side) = $hA/(W1D)^{0.8}$
H4K	$12.8 \frac{\text{Btu}/^{\circ}\text{R sec}}{(\text{lb/sec})^{0.55}}$	Heat transfer coefficient to flow divider (cold side) = $hA/(W2D)^{0.55}$
H5K	$1.84 \frac{\text{Btu}/^{\circ}\text{R sec}}{(\text{lb/sec})^{0.55}}$	Heat transfer coefficient to shell = $hA/(W2D)^{0.55}$
XK	2.33 Btu/ $^{\circ}$ R sec	Conductivity between solids in bayonet tube = kA/L .

A.1.7 Cold Outlet Heat Exchanger Pipe (Pipe No. 1)*

CP1	0.18 Btu/ $^{\circ}$ R lb	Heat capacity of pipe
WP1	2153 lb	Weight of pipe
HP1K	$2.13 \frac{\text{Btu}/^{\circ}\text{R sec}}{(\text{lb/sec})^{0.8}}$	Heat transfer coefficient to pipe = $hA/(W2D)^{0.8}$

A.1.8 Cold Inlet Heat Exchanger Pipe (Pipe No. 2)*

CP2	0.18 Btu/ $^{\circ}$ R lb	Heat capacity of pipe
WP2	1941 lb	Weight of pipe
HP2K	$1.23 \frac{\text{Btu}/^{\circ}\text{R sec}}{(\text{lb/sec})^{0.8}}$	Heat transfer coefficient to pipe = $hA/(W2D)^{0.8}$

* Values given are for the whole component (not per lump values).

A.1.9 Heat Dump*

<u>Symbol</u>	<u>Value</u>	<u>Description</u>
CD	0.18 Btu/°R lb	Heat capacity of heat dump solid
WD	10,600 lb	Weight of heat dump solid
HDHK	$11.7 \frac{\text{Btu}/^{\circ}\text{R sec}}{(\text{lb}/\text{sec})^{0.8}}$	Heat transfer coefficient (hot side) = $hA/(W2D)^{0.8}$
HDCK	$1.34 \frac{\text{Btu}/^{\circ}\text{R sec}}{(\text{lb}/\text{sec})^{0.725}}$	Heat transfer coefficient (air side) = $hA/(W3D)^{0.725}$
TAIRIC	510°R	Inlet air temperature (initial condition)
WTHDP	1.25	Hot side mean gas temperature weighting factor. $T_{ave} = (T_{in} + WTHDP \times T_{out}) / (1. + WTHDP)$

A.2 Control Variables

The variable KMX denotes whether initial temperatures and power are to be read in or solved from the steady matrix.

KMX = 1** Steady-state matrix solution as starting point
 = 2 Read in temperatures, power and flow rates.

If temperatures, flows, etc., are read in (KMX = 2) and the initial flows calculated in the blower calculations do not match the flows read in, the following will be printed out:

POSSIBLE INPUT ERROR WØLD1 = xxx (value) W1D = xxx (value) WØLD2 =
 xxx (value) W2D = xxx (value).

NRM is a variable which permits one to print out the number of passes required for convergence of flow rates in the transient and steady-state blower calculations. It also supplies machine computing times.

* Values given are for the whole component (not per lump values).

** Denotes standard problem value.

NRM = 0* No, do not want feature described above
 = 1 Yes, want to use feature described above.

KVAR2 is the variable one uses to specify whether the components are to be connected or disconnected. Three options can be exercised in the disconnected mode of operation.

KVAR2 = 1* Connected system
 = 2 Disconnected (for perturbing all cold inlet temperatures and the reactor inlet temperature)
 = 3 Disconnected (for perturbing all hot inlet temperatures)
 = 4 Disconnected (for perturbing flow rates, recuperator bypass, etc., except hot or cold inlet temperatures).

KVAR1 (≤ 11) tells how many variables are to be perturbed; (exception if disconnected with KVAR2 = 2 or 3 and with KVAR1 = 1, a number of hot or cold inlet gas temperatures will be perturbed.) KV(i), ($i \leq$ KVAR1) specifies the variable to be perturbed.

KVAR1 = 1*

Example: If KVAR1 was specified = 2 one could choose 2 variables to perturb. KV(1) = N_{i1} , KV(2) = N_{i2} where the N_i 's are chosen from the following list.

$N_i = 1$	Perturb reactivity ρ (or TAMB in closed loop operation. See A.4)**
= 2	Perturb heat dump inlet air temperature
= 3	Perturb primary loop flow rate
= 4	Perturb secondary loop flow rate
= 5	Perturb air flow rate
= 6	Perturb fraction of primary flow which bypasses the cold side of the recuperator
= 7	Perturb primary loop blower speed
= 8	Perturb secondary loop blower speed

* Denotes standard problem value.

** CT4(1) should be $\geq$ DT.

- = 9 Perturb primary loop system pressure
- = 10 Perturb secondary loop system pressure
- = 11 Blowers in/out vs time (sets NOBLWR = 1)
- = 12 Perturb (disconnected hot or cold inlet temperatures)
or nothing

KV(i) = 12*

The value of JN determines whether transfer functions are to be calculated or not.

- JN = 0* Don't calculate transfer functions
- = 1 Calculate transfer functions.

JM (≤ 30), specifies the number of dependent variables chosen for which transfer functions are desired.

JM = 1*

$J\emptyset$ (≤ 21), specifies the number of frequencies at which transfer functions are evaluated.

$J\emptyset = 1*$

JV(i) specifies the dependent variable for which a transfer function is calculated. (See list in Section A.5.)

JV(i) = 11*

$\emptyset M(i)$ gives the values of frequencies used above.

$\emptyset M(i) = 0.0*$

JSPEC	0*	Terminates problem if blower surge occurs
	1	Does not terminate problem for blower surge
N $\emptyset$ BLWR	0*	Normal value
	1	Ignores blower calculations after initial steady-state calculation (may be function of time)
KCLPW	0*	Do not close loops
	1	Close loops on power, outlet reactor gas temperature, etc., (depends on time functions chosen, see A.4).

See
A.4

* Denotes standard problem value.

A.3 Time Dependent Functions

Section A.2 showed that many variables may be perturbed. The variables are perturbed using a generalized time function:

$$f(\tau) = a + b\sin[\omega(\tau - \tau_{\text{delay}})] + c(\tau - \tau_{\text{delay}})$$

where:

$$a = CT1(i)$$

$$b = CT2(i)$$

$$\omega = CT3(i)$$

$$\tau_{\text{delay}} = CT4(i) \quad \text{Time to start perturbation}$$

$$c = CT5(i)$$

These arbitrary constants produce $f_i(\tau)$. The i 's used here are the N_i 's chosen in Section A.2.

Example: Let X be the variable perturbed

$$\text{then } (X + \delta X) = (1.0 + f_i(\tau)) \times (X_{\text{initial}})$$

Note that if $X_{\text{initial}} = 0.0$ it follows that $(X + \delta X) = 0.$, always. Also note that $f_{11}(\tau)$ is used for all perturbations of disconnected cold or hot inlet temperatures. Thus, when the system is disconnected and the hot (or cold) inlet gas temperatures are perturbed, they are all perturbed by the same per cent.

A.4 Closed Loop Variables

The closed loop function is used in conjunction with the time dependent function routine. Figure A-1 illustrates the closed loop used.

KCLPW 0* If KCLPW = 1, one can close loops.

Table A-1 gives the available choices for closing loops with the proper KV(i) value to specify.

* Denotes standard problem value.

Fig. A-1 Feedback Control Loop for Calculational Purposes Only

An example of how the loops are closed numerically is shown below. The particular case chosen is that in which recuperator bypass is adjusted to obtain a desired reactor power level.

$$FB(new) = FB(old) + (QDEM - QT) \times QGAIN \times \Delta\tau$$

This calculation is done at every time step in the simulation.

It should be noted that the integral compensation chosen for these loops was selected arbitrarily for calculational convenience only. If at a later date it is determined that specific loops will be closed in the real system, these loops and their compensation will be designed carefully to achieve good system stability and response characteristics.

TABLE A-1
CALCULATIONAL CLOSED LOOPS

<u>Demand Variable</u>	<u>Demand Value*</u>	<u>Operating Variable</u>	<u>Gain Term</u>	<u>Value*</u>	<u>KV(i) Value</u>
System power QT	2845. Btu/ sec (3 MW) QDEM	Fraction of recuperator bypass flow, FB	QGAIN	1×10^{-6}	6
Reactor outlet gas temperature T $\emptyset$	2860°R T $\emptyset$ W	Average reactor solid temperature, TAMB	T $\emptyset$ GAIN	0.01	1
Primary loop flow rate W1D	2.85 lb/ sec W1DW	Primary loop blower speed, BS ₁	WGAIN	1.0	7
Secondary loop helium flow rate W2D	2.85 lb/ sec W2DW	Secondary loop blower speed, BS ₂	WGAIN	1.0	8

* Standard problem values.

A.5 Transfer Function Calculations

The transfer functions are calculated from the time response of the dependent variable(s) caused by a 2% to 5% step of the independent variable. A value of 1 for JN tells the computer to calculate the transfer functions. KV(1) = N_1 specifies the independent variable which is perturbed. CT1(N_1) is the only time function constant which should have a value different from 0.

JN = 0*

JØ(≤ 21) specifies the number of frequencies at which the transfer functions are to be evaluated.

ØM(i) $i \leq JØ$, gives the values of the frequencies in rad/sec.

JM(≤ 11), specifies the number of dependent variables for which transfer functions are desired.

JV(i) = K, $i \leq JM$, specifies which dependent variable is selected. The selection of the dependent variable is made from the following list.

K = 1	Reactor outlet temperature
= 2	Recuperator hot outlet temperature
= 3	Recuperator cold outlet temperature
= 4	Heat exchanger hot outlet temperature
= 5	Heat exchanger cold outlet temperature
= 6	Heat exchanger cold outlet pipe (No. 1) outlet temperature
= 7	Heat exchanger cold inlet pipe (No. 2) outlet temperature
= 8	Heat dump hot outlet temperature
= 9	Heat dump cold outlet temperature
= 10	Power
= 11	None

* Denotes standard problem value.

A.6 Sample Problem Input

Suppose one wishes to get the transfer functions from hot inlet temperatures to outlet temperatures for the recuperator, the heat exchanger, and the heat dump.

For this problem, a time increment of $DT = 0.5$ second, a total time $TOT = 1000$ seconds, and a 5% step of hot inlet temperatures might be typical. A value of $PT = 20.$ will give one a transient response curve which has 50 plotted values.

One need not specify $KVAR1 = 1$ or $KV(1) = 12$ since these values are given in the standard problem. $KVAR2$ here must = 3, $CT1(12) = +0.05$. Suppose that transfer functions for two frequencies, 0.01 and 0.05 rad/sec, are desired. Then $J\emptyset = 2$, $\emptyset M(1) = 0.01$, $\emptyset M(2) = +0.05$. Since transfer functions are desired, $JN = 1$. From the list in Section A.5, six dependent variables are all that apply since the reactor is not perturbed. From the foregoing, $JM = 6$, $JV(1) = 2$, $JV(2) = 3$, $JV(3) = 4$, $JV(4) = 5$, $JV(5) = 8$, $JV(6) = 9$.

UHXCOM uses the input routine "SYMBOL," an adaptation of the SHARE subroutine ARSYM1. The input deck for the preceding problem would look as follows. The symbol ' indicates a blank space on the data card.

Col

1

(1) H'TRANSFER FUNCTIONS FOR HOT INLET TEMP PERT.

A "P" column in 1 of the data cards will cause the input cards to be listed, see Section A.7.

Data cards (cards 2 through 4)

Col

1

(2) P CASE'060266.01'DT'.5'TOT'1000.'CT4(12)'1.0'PT'20.

- (3) P KVAR2'3"CT1(12)'+.05"JØ'2"ØM(1)'.01"ØM(2)'.05
- (4) P JN'1"JM'6"JV(1)'2"3"4"5"8"9

Symbol control cards (cards 5 and 6)

Col

1

- (5) = (rest of card blank)
- (6) E (rest of card blank)

A.7 Sample Problem Output List

The computer output listing from a sample problem in which a variety of transfer functions were obtained is shown in Figs. A-2a through A-2f.

A.8 Sample Computer Plot

The raw computer plot of reactor power in the event of bypass failure to full open is shown in Fig. A-3. This plot corresponds to Fig. 5-12 of this report.

Each computer problem produces 19 pages of such plots. In all, a total of 48 system variables are shown in the computer output.

A.9 Problem Running Time on 7094

The ratio of problem time "TOT" to 7094 computing time is approximately 13 to 1. Short problems of 3 or less 7094 minutes have a slightly poorer time ratio of about 10 to 1. This is due to fixed times required for the steady-state matrix calculation, monitor setup time, etc. which are required in nearly every problem. An equation for determining 7094 computing time required follows.

$$C.T. = 0.6 + 0.0013 \times \frac{T\emptyset T}{\Delta t}$$

where

C.T. = 7094 computing time (minutes)

T $\emptyset$ T = Problem simulation time (seconds)

Δt = Time step (seconds).

```

      B * 05 JOB,KOODUERRE K HTX K00          *050 KOODUERR 125181 6/02/66 MACH 6
      * XEQ

LIBRARY ENTRY POINTS,
(FPT) (STHM) (FIL) EXP13 SQRT EXP12 EXP LOG10 COS SIN
LOG ATAN LINCNT FLUSH EXIT (IOS) (WRS) (RCH) (TES) (WTC)
(WER)

125216 EXECUTION

CLOCK TIME = 772.180
CASE 060266.01 DT .5 TOT 1000. CT4(12) 1. PT 20.
KVAR2 3 CT1(12) +.05 JD 2 OM(1) .01 OM(2) .05
JN 1 JM 6 JV(1) 2 3 4 5 8 9

```

Start time of problem

Print out of data cards (with P in column 1)

Fig. A-2a First Page of Output List

```

TRANSFER FUNCTIONS FOR HOT INLET TEMP PERT.

INPUT RETURN

PROB 60266.01 TOTAL TIME 1.000E 03 DELTA T 5.000E-01 PLOT CYCLE 2.000E 01

NUMBER OF VAR TIME FUNCTION DATA
CHANGED 1 VAR 12 CT3 CT4 CT5 CT6
TIME FUNCTION CONSTANTS (Start Perturbation) (End Perturbation)
1 0. 0. 0. 1000000.000000 0. 1000000.000000
2 0. 0. 0. 1000000.000000 0. 1000000.000000
3 0. 0. 0. 1000000.000000 0. 1000000.000000
4 0. 0. 0. 1000000.000000 0. 1000000.000000
5 0. 0. 0. 1000000.000000 0. 1000000.000000
6 0. 0. 0. 1000000.000000 0. 1000000.000000
7 0. 0. 0. 1000000.000000 0. 1000000.000000
8 0. 0. 0. 1000000.000000 0. 1000000.000000
9 0. 0. 0. 1000000.000000 0. 1000000.000000
10 0. 0. 0. 1000000.000000 0. 1000000.000000
11 0. 0. 0. 1000000.000000 0. 1000000.000000
12 0.050000 0. 0. 1000000.000000 0. 1000000.000000
      (KVAR2) 1.000000

SYSTEM CONNECT OPTION 3 TEMP INPUT OPTION 1 ← KMX
TRANSFER FN OPTION 1 NO OF G'S. 6 NO OF OMEGAS 2
% NCS 2 3 4 5 8
FREQS 0.01000 0.05000

OPTIONS
SPECIAL PROBLEM 0 BLOWERS IN 0 DEBUG 0 CLOSED LOOP 0 RESTART 0

```

*Dependent variable indices, see section A.5

Fig. A-2b Second Page of Output List

INITIAL PROFILES

HEAT DUMP									
HOT SIDE									
1.177E 03	9.638E 02	8.101E 02	6.991E 02	6.189E 02					
SOLID									
9.379E 02	7.914E 02	6.856E 02	6.092E 02						
COLD SIDE									
7.759E 02	6.744E 02	6.011E 02	5.482E 02	5.100E 02					
PIPE ONE									
HOT SIDE									
1.177E 03	1.177E 03	1.177E 03	1.177E 03						
SOLID									
1.177E 03	1.177E 03	1.177E 03							
PIPE TWO									
HOT SIDE									
6.189E 02	6.189E 02	6.189E 02	6.189E 02						
SOLID									
6.189E 02	6.189E 02	6.189E 02							
HEAT EXCHANGER									
HOT SIDE ONE									
1.653E 03	1.600E 03	1.555E 03	1.518E 03	1.489E 03	1.467E 03	1.454E 03	1.449E 03		
SOLID ONE									
1.525E 03	1.492E 03	1.466E 03	1.447E 03	1.436E 03	1.434E 03	1.442E 03			
SOLID TWO									
9.601E 02	1.014E 03	1.071E 03	1.133E 03	1.204E 03	1.288E 03	1.392E 03			
HOT SIDE TWO									
8.544E 02	9.194E 02	9.851E 02	1.054E 03	1.130E 03	1.218E 03	1.321E 03	1.449E 03		
SOLID THREE									
7.588E 02	8.323E 02	9.043E 02	9.777E 02	1.056E 03	1.142E 03	1.241E 03			
COLD SIDE									
6.189E 02	7.012E 02	7.783E 02	8.525E 02	9.262E 02	1.002E 03	1.084E 03	1.177E 03		
SHELL									
6.601E 02	7.398E 02	8.154E 02	8.893E 02	9.642E 02	1.043E 03	1.131E 03			
RECUPERATOR									
HOT SIDE									
2.860E 03	2.520E 03	2.121E 03	1.653E 03						
SOLID									
2.452E 03	2.042E 03	1.559E 03							
COLD SIDE									
2.368E 03	1.942E 03	1.442E 03	8.544E 02						
REACTOR									
MODERATOR									
3.424E 03	3.275E 03	3.039E 03	2.692E 03						

See Chapter 2 figures for locations of temperatures.

Fig. A-2c Third Page of Output List

3.431E 03	3.270E 03	ANNULUS	3.034E 03	2.663E 03	2.062E 03
		FUEL ELEMENT			
3.457E 03	3.295E 03		3.076E 03	2.709E 03	
		HOLE			
2.759E 03	2.574E 03		2.382E 03	2.198E 03	2.062E 03
		POWER			
2.844E 03					
		PRIMARY LOOP			
HPOWER=	29.095	DELTA P=	6.027	BLOWER SPEED=	8255.2
				SYST PRESS=	500.00
				FLOW RATE=	2.850
		SECONDARY LOOP			
HPOWER=	49.791	DELTA P=	9.987	BLOWER SPEED=	9000.0
				SYST PRESS=	500.00
				FLOW RATE=	4.078
TIME=	0.	FB=	0.20222	TAMB=	3.183E 03
				QMAX=	0.
				INVPERMAX=	0.

A-20

Fig. A-2d Fourth Page of Output List

```

FINAL TEMPS
HEAT DUMP
HOT SIDE
1.236E 03    1.004E 03    8.365E 02    7.157E 02    6.285E 02
SOLID
9.757E 02    8.162E 02    7.010E 02    6.179E 02
COLD SIDE
7.993E 02    6.889E 02    6.091E 02    5.516E 02    5.100E 02
PIPE ONE
HOT SIDE
1.177E 03    1.177E 03    1.177E 03    1.177E 03
SOLID
1.177E 03    1.177E 03    1.177E 03
PIPE TWO
HOT SIDE
6.189E 02    6.189E 02    6.189E 02    6.189E 02
SOLID
6.189E 02    6.189E 02    6.189E 02
HEAT EXCHANGER
HOT SIDE ONE
1.735E 03    1.678E 03    1.630E 03    1.590E 03    1.559E 03    1.535E 03    1.520E 03    1.515E 03
SOLID ONE
1.597E 03    1.561E 03    1.533E 03    1.513E 03    1.502E 03    1.499E 03    1.508E 03
SOLID TWO
9.873E 02    1.046E 03    1.107E 03    1.174E 03    1.251E 03    1.342E 03    1.454E 03
HOT SIDE TWO
8.732E 02    9.434E 02    1.014E 03    1.089E 03    1.171E 03    1.266E 03    1.378E 03    1.515E 03
SOLID THREE
7.700E 02    8.493E 02    9.272E 02    1.006E 03    1.090E 03    1.184E 03    1.291E 03
COLD SIDE
6.189E 02    7.078E 02    7.911E 02    8.712E 02    9.507E 02    1.033E 03    1.122E 03    1.221E 03
SHELL
6.634E 02    7.494E 02    8.311E 02    9.109E 02    9.918E 02    1.077E 03    1.171E 03
RECUPERATOR
HOT SIDE
3.003E 03    2.638E 03    2.211E 03    1.709E 03
SOLID
2.566E 03    2.125E 03    1.608E 03
COLD SIDE
2.475E 03    2.018E 03    1.483E 03    8.544E 02
REACTOR
MODERATOR
3.424E 03    3.275E 03    3.039E 03    2.692E 03

```

Fig. A-2e Fifth Page of Output List

3.431E 03 3.270E 03 ANNULUS 3.034E 03 2.663E 03 2.062E 03

3.457E 03 3.295E 03 FUEL ELEMNT 3.076E 03 2.709E 03

2.759E 03 2.574E 03 HOLE 2.382E 03 2.198E 03 2.062E 03

2.844E 03 POWER

PRIMARY LOOP
HPOWER= 29.095 DELTA P= 6.027 BLOWER SPEED= 8255.2 SYST PRESS= 500.00 FLOW RATE= 2.850

SECONDARY LOOP
HPOWER= 49.791 DELTA P= 9.987 BLOWER SPEED= 9000.0 SYST PRESS= 500.00 FLOW RATE= 4.078

TIME= 1000.0000 FB= 0.20222 TAMB= 3.183E 03 QMAX= 2.844E 03 INVPERMAX= 1.095E-06

TRANSFER FUNCTION DATA

FREQUENCY= 0.010000
OP VAR=TRHO THETA=-9.5541E 01 MAG.= 1.2169E-01 MAGDB=-1.8295E 01
OP VAR=TRCO THETA=-5.2016E 01 MAG.= 3.9072E-01 MAGDB=-8.1627E 00
OP VAR=TXHO THETA=-2.5535E 01 MAG.= 1.9064E-01 MAGDB=-1.4396E 01
OP VAR=TXCO THETA=-3.2292E 01 MAG.= 4.2552E-01 MAGDB=-7.4216E 00
OP VAR=TDHO THETA=-1.0512E 02 MAG.= 7.2991E-02 MAGDB=-2.2735E 01
OP VAR=TDCC THETA=-4.6083E 01 MAG.= 2.5544E-01 MAGDB=-1.1854E 01

FREQUENCY= 0.050000
OP VAR=TRHO THETA=-8.6687E 01 MAG.= 9.3476E-03 MAGDB=-4.0586E 01
OP VAR=TRCO THETA=-7.8898E 01 MAG.= 1.0801E-01 MAGDB=-1.9330E 01
OP VAR=TXHO THETA=-5.6438E 01 MAG.= 9.0195E-02 MAGDB=-2.0896E 01
OP VAR=TXCO THETA=-8.1638E 01 MAG.= 2.0790E-01 MAGDB=-1.3643E 01
OP VAR=TDHO THETA=-1.7388E 02 MAG.= 2.3514E-03 MAGDB=-5.2574E 01
OP VAR=TDCC THETA=-7.6580E 01 MAG.= 7.5776E-02 MAGDB=-2.2409E 01

CLOCK TIME = 773.320

223 LINES OUTPUT.

125338

*The values of maximum power and inverse period are printed in case the peak values are missed on the film plot.

Fig. A-2f Sixth Page of Output List

A-22

Fig. A-3 Sample Computer Plot

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APPENDIX B

SYSTEMS CONSTANTS

In this appendix, the calculations that were made to determine the heat transfer coefficients, coolant pressure losses, flow and heat transfer areas, and weights for the reactor, heat exchangers, and piping are presented. All calculations are for the operation at 3-MW heat generation, 2860°R reactor outlet, and 2.85 lb/sec coolant flow in both primary and secondary loops.

REACTOR

Dimensions

<u>Core:</u>	Outside diameter	70.0 in.
	Inside diameter	23.0 in.
	Height (fueled region only)	39.0 in.
<u>Fuel:</u>	Element diameter	1.0 in.
	Element length	5.5 in.
	Diameter of hole in element	0.5 in. 0.0417 ft
	Number of elements/channel	4
	Number of channels	312
	Total elements	1248

Flow Area: Inside element = $\frac{\pi}{4}(0.5)^2 \times 312/144 = 0.425 \text{ ft}^2$

Eccentric annulus = $\frac{\pi}{4}(1.1^2 - 1.0^2) \times 312/144$

= 0.357 ft^2

where fuel channel hole is 1.1 in. diam for major portion of length

Heat Transfer Area: Inside element = $\pi(0.5) \times 5.5 \times 0.248/144$
= 75 ft^2

Eccentric annulus = $\pi(1.0) \times 5.5 \times 1248/144$
= 150 ft^2

Weights: Fuel element = $1248 \frac{\pi}{4}(1.0^2 - 0.5^2)5.5 \frac{103}{1728}$
= 241 lb

Core = $\frac{\pi}{4}(70^2 - 23^2)(39) \frac{103}{1728} = 8000 \text{ lb}$

Material and Coolant Properties

Graphite

<u>Temp.</u> <u>(°F)</u>	<u>Density</u> <u>(lb/ft³)</u>	<u>Specific Heat</u> <u>(Btu/lb °F)</u>	<u>Conductivity</u> <u>(Btu/h ft °F)</u>
77	103	0.172	94.0
482		0.298	70.2
932		0.376	52.0
1292		0.432	43.5
1832		0.458	31.4

Data for type AGOT graphite as given in reference 1 was assumed typical and agrees reasonably well with current data.

Helium

Specific Heat = 1.25 Btu/lb °F

<u>Temp.</u> <u>(°F)</u>	<u>Density</u> <u>(lb/ft³ @ 500 psia)</u>	<u>Conductivity</u> <u>(Btu/h ft °F)</u>	<u>Viscosity</u> <u>(lb/ft h)</u>
500	0.198	0.124	0.072
1000	0.128	0.165	0.090
1500	0.095	0.201	0.114
2000	0.076	0.236	0.133
2500	0.062	0.270	0.151

This data corresponds to data used in reference 2.

Coolant Flow

Total Flow Rate - for 3.0 MW with 800°R coolant temperature rise from inlet to outlet of core.

$$Q = C_p \dot{W} \Delta T$$

$$\dot{W} = (3.0 \times 3.413 \times 10^6 \frac{\text{Btu/h}}{\text{MW}}) / 1.25 \times 800 = 10,250 \text{ lb/h or } 2.85 \text{ lb/sec}$$

The flow balance between the fuel element hole and annulus is assumed to be 15% in the eccentric annulus and 85% in the hole. This division of flow is based on unpublished calculations and on experimental observation.

Heat Transfer Coefficients

Fuel Element Hole - for full flow and maximum temperature

$$\text{Correlation: } N_{Nu} = 0.036(N_{Re})^{0.8}(N_{Pr})^{0.4} \left[\frac{T_s}{T_g} \right]^{-0.5} \left[\frac{X}{D} \right]^{-0.1} \quad (\text{Ref. 2})$$

$$\text{Average gas temperature } T_g = 2460^\circ\text{R}$$

$$\text{Average wall temperature } T_s = 3240^\circ\text{R}$$

$$\text{Diameter } D = 0.0417 \text{ ft}$$

$$\text{Thermal conductivity } k = 0.235 \text{ Btu/h ft } ^\circ\text{R}$$

$$\begin{aligned} \text{Specific mass flow } G &= \dot{W}/A = 10,250 \times 0.85/0.425 \\ &= 20,500 \text{ lb/h ft}^2 \end{aligned}$$

$$\text{Viscosity } \mu = 0.133 \text{ lb/h ft}$$

$$\text{Nusselt number } N_{nu} = \frac{hD}{k}$$

$$\text{Reynold number } N_{Re} = \frac{DG}{\mu}$$

$$\text{Prandtl number } N_{Pr} = \frac{C_p \mu}{k}$$

$\frac{X}{D}$ was set at 11 which is equal to $\frac{L}{D}$ for a single element.

Substituting in the above, the heat transfer coefficient in the fuel element hole

$$\begin{aligned} h &= \frac{0.235}{0.0417} 0.036(6420)^{0.8}(0.71)^{0.4}(1.32)^{-0.5}(11)^{-0.1} \\ &= 135 \text{ Btu/h ft}^2 \text{ } ^\circ\text{R} \end{aligned}$$

Annulus

Flow in the eccentric annulus was assumed to be laminar. However, the channel has so many flow disturbances that considerable turbulence may be introduced. At a distance from the entrance where Nusselt number may be considered a constant, and for a fully eccentric annulus

$$N_{Nu} = \frac{h D_e}{k} = 0.44 \times 3.65 \quad (\text{Ref. 3})$$

$$h = 0.44 \times 3.65 \frac{0.235}{0.00833} = 45 \text{ Btu/h ft}^2\text{°R}$$

The constant, 0.44, is a correction factor for an eccentric annulus that was calculated by V. J. Skoglund, Consultant, in an unpublished LASL memorandum.

Pressure Loss

The pressure loss in the coolant channel is a small part of the total loss in the reactor. The pressure loss in a one-half scale flow model, scaled to the prototype condition, was found to be 0.6 psi between inlet and outlet nozzles of the reactor.

$$\Delta P_{\text{reactor}} = 0.6 \text{ lb/in.}^2 \text{ at } 10,250 \text{ lb/h coolant flow}$$

2060°R inlet
3.0 MW power

Miscellaneous

Average Velocities

$$V_{\text{element}} = \frac{0.85 \dot{W}}{A_x \rho_g} \times \frac{1}{3600} = \frac{0.85 \times 10,250}{0.425 \times 3600 \times 0.075} = 100 \text{ ft/sec}$$

$$V_{\text{annulus}} = \frac{0.15 \dot{W}}{A_x \rho_g} \times \frac{1}{3600} = \frac{0.15 \times 10,250}{0.357 \times 0.075 \times 3600} = 18.5 \text{ ft/sec}$$

Sonic Velocity at 2000°R

$$V = \sqrt{\gamma g R T} = \sqrt{1.66 \times 32.2 \times \frac{1545}{4} \times 2000} = 7120 \text{ ft/sec}$$

Neutronics Constants

Temperature coefficient of reactivity at full power design conditions = -5.83×10^{-5} reactivity/°R

Mean neutron lifetime $\ell^* = 10^{-3}$ sec

Reactor kinetic constants are given in Section A.1.2 of Appendix A.

RECUPERATOR

Dimensions

Heat exchanger graphite block	Width	23 inches
	Height	20 inches
	Length	137 inches
Number of holes	Hot side	225
	Cold side	225

Spacing of holes: 24 columns on 15/16-inch centers 19 holes (typical) per column; holes on 15/16-inch centers with centers staggered from one column to the other.

Flow pattern: Hot and cold pass coolant flows in alternate columns.

Flow Area $225 \frac{\pi}{4} (0.5)^2 / 144 = 0.307 \text{ ft}^2$

Heat Transfer Area $225\pi(0.5)(137)/144 = 337 \text{ ft}^2$

Weight of Recuperator Core $[23 \times 20 \times 137 - 450 \frac{\pi}{4} (0.5)^2 137] \frac{103}{1728}$
 $= 3116 \text{ lb}$

Heat Transfer Coefficients

$$N_{Nu} = 0.023(N_{Re})^{0.8}(N_{Pr})^{0.4} \left[\frac{T_s}{T_g} \right]^{-0.5}$$

		<u>Cold Side</u>	<u>Hot Side</u>	<u>Units</u>
Average coolant temperature	T_g	= 1560	2360	°R
Average wall temperature	T_s	= 1960	1960	°R
Thermal conductivity	k	= 0.172	0.229	Btu/h ft°R
Viscosity	μ	= 0.097	0.129	lb/ft h
Density	ρ_g	= 0.120	0.078	lb/ft ³
Specific heat	C_p	= 1.25	1.25	Btu/lb°R
Diameter	D	= 0.0417	0.0417	ft

		Cold Side	Hot Side	Units
Coolant flow rate	$\dot{W}$	10,250	10,250	lb/h
Mass flow $\dot{W}$ /area flow	G	= 33,400	33,400	lb/h ft ²
Nusselt number	$N_{Nu} = \frac{hD}{K}$			
Reynolds number	$N_{Re} = \frac{DG}{\mu}$			
Prandtl number	$N_{Pr} = \frac{C_p \mu}{k}$			

$$h_{cold} = \frac{0.172}{0.0417} 0.023 \left(\frac{0.0417 \times 33400}{0.097} \right)^{0.8} \left(\frac{1.25 \times 0.097}{0.172} \right)^{0.4} \left(\frac{1960}{1560} \right)^{-0.5}$$

$$= \frac{0.172}{0.0417} 0.023 (14380)^{0.8} (0.71)^{0.4} (1.25)^{-0.5} = 156 \text{ Btu/h ft}^2 \text{ } ^\circ\text{R}$$

$$h_{hot} = \frac{0.229}{0.0417} 0.023 \left(\frac{0.0417 \times 33400}{0.129} \right)^{0.8} \left(\frac{1.25 \times 0.129}{0.229} \right)^{0.4} \left(\frac{1960}{2360} \right)^{-0.5}$$

$$= \frac{0.229}{0.0417} 0.023 (10800)^{0.8} (0.71)^{0.4} (0.83)^{-0.5} = 202 \text{ Btu/h ft}^2 \text{ } ^\circ\text{R}$$

Pressure Loss

LASL specification allows 4.0 psi total for sum of hot and cold side. Due to the complexity of the flow path, it is assumed that the actual pressure loss will be 4.0 psi and will be divided between the hot and cold sides proportional to the reciprocal of the coolant density.

$$\Delta P_h + \Delta P_c = 4.0$$

$$\frac{\Delta P_h}{\Delta P_c} = \frac{2360}{1560}$$

$$\Delta P_c = 1.6 \text{ psi}$$

$$\Delta P_h = 2.4 \text{ psi}$$

Average Velocity

<u>Hot Side</u>	$v = \frac{\dot{W}}{3600 \rho_g A} = \frac{10250}{3600 \times 0.078 \times 0.307} = 115 \text{ ft/sec}$
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<u>Cold Side</u>	$v = \frac{\dot{W}}{3600 \rho_g A} = \frac{10250}{3600 \times 0.120 \times 0.307} = 77 \text{ ft/sec}$
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HEAT EXCHANGER

<u>Bayonet Tubes:</u>	Number	96
	Effective length	171 inches
	Core tube i.d.	0.710 inches
	o.d.	0.750 inches
	Insulating tube i.d.	1.0225 inches
	o.d.	1.0625 inches
	Outer tube i.d.	1.356 inches
	o.d.	1.500 inches

Flow Areas: Core tube $= \frac{\pi}{4} (0.710)^2 \frac{96}{144} = 0.264 \text{ ft}^2$

Annulus $= \frac{\pi}{4} (1.356^2 - 1.0625^2) \frac{96}{144} = 0.371 \text{ ft}^2$

Heat Transfer Areas: Core tube i.d. $= \pi(0.710) 171 \times 96/144 = 254.3$

Insulating tube o.d. $= \pi(1.0625) 171 \times 96/144 = 380.5$

Outer tube o.d. $= \pi(1.500) 171 \times 96/144 = 537.2$

Heat Transfer Coefficients:

Core Tube

$$N_{Nu} = 0.034 (N_{Re})^{0.8} (N_{Pr})^{0.4} \left(\frac{T_s}{T_g} \right)^{-0.5} \left(\frac{L}{D} \right)^{-0.1}$$

Average gas temperature $T_s = 1760^\circ\text{R}$

Average wall temperature $T_g = 1760^\circ\text{R}$

Diameter $D = 0.710 \text{ in.}; 0.0592 \text{ ft}$

Thermal conductivity $k = 0.186 \text{ Btu/h ft}^\circ\text{R}$

Specific mass flow $G = 10,250/0.264 = 38,825 \text{ lb/h ft}^2$

Viscosity $\mu = 0.106$

Density $\rho_g = 0.106$

Length of tube $L = 14.25 \text{ ft}$

Nusselt number $N_{Nu} = \frac{hD}{k} = \frac{h \times 0.0592}{0.186}$

Reynold number $N_{Re} = \frac{DG}{\mu} = 21,700$

Prandtl number $N_{Pr} = \frac{C_p \mu}{k} = 0.71$

$$h = \frac{0.186}{0.0592} 0.034(21,700)^{0.8}(0.71)^{0.4}(1)^{-0.5}(60)^{-0.1}$$

$$= 183 \text{ Btu/h ft}^2\text{°R}$$

Hot Side Annulus:

Insulating tube o.d. 1.0625 in.

Outer tube i.d. 1.356 in.

$$N_{Nu} = 0.023(N_{Re})^{0.8}(N_{Pr})^{1/3} \quad (\text{Ref. 4})$$

$$\text{Average coolant temperature} = \frac{1660 + 1060}{2} = 1360^\circ\text{R}$$

$$\text{Equivalent diameter } D_e = \frac{1.356 - 1.0625}{12} = 0.0243 \text{ ft}$$

$$\text{Thermal conductivity } k = 0.158 \text{ Btu/h ft}^\circ\text{R}$$

$$\text{Density } \rho_g = 0.137 \text{ lb/ft}^3$$

$$\text{Viscosity } \mu = 0.09 \text{ lb/ft h}$$

$$\text{Specific mass flow } G = 10,250/0.371 = 27,600 \text{ lb/h ft}^2$$

$$N_{Re} = \frac{D_e G}{\mu} = \frac{0.0243 \times 27,600}{0.09} = 7450$$

$$h = \frac{0.158}{0.0243} 0.023(7450)^{0.8}(0.71)^{1/3} = 170 \text{ Btu/h ft}^2\text{°R}$$

Shell Side Coefficient

$$N_{Nu} = \frac{hd}{k} = 1.9 \left(\frac{P-d}{P} \frac{d}{D_i} \right)^{0.4} \left(\frac{S_w}{S_w + S_e} \right)^2 \left(\frac{G_{av} d}{\mu} \right)^{0.6} \left(\frac{C_p \mu}{k} \right)^{0.3} \left(\frac{\mu}{\mu_w} \right)^{0.14} F$$

(Ref. 5)

$$P = \text{tube pitch} = 1\text{-}13/16 \text{ in.}; 0.151 \text{ ft}$$

$$d = \text{tube o.d.} = 1.50 \text{ in.}; 0.125 \text{ ft}$$

$$D_1 = \text{shell i.d.} = 21.0 \text{ in.}; 1.750 \text{ ft}$$

$$S_w = \text{free area in baffle window} = 0.309 \text{ ft}^2$$

$$S_e = \text{leakage area between baffle and shell} = 0.029 \text{ ft}^2$$

$$S_M = \text{minimum free area for cross flow} = 0.224 \text{ ft}^2$$

$$S_P = \text{maximum free area for cross flow} = 1.180 \text{ ft}^2$$

$$G_{av} = \text{average mass velocity} = \frac{1}{3} \left(\frac{Q}{S_w} + \frac{Q}{S_M} + \frac{Q}{S_P} \right) = 29,202 \text{ lb/h ft}^2$$

Q = mass flow rate = 10,250 lb/h

μ = viscosity = 0.076 lb/h ft

k = thermal conductivity = 0.133 Btu/h ft²R

$$F = \text{Tinker's end space factor} = \frac{L_2 + (L_1 - L_2) \left(\frac{2L_3}{L_1 - L_2} \right)^{0.6}}{L_1} = 1.00$$

L_1 = length between tube sheets = 171 in.

L_2 = length between end baffles = 154-3/4 in.

L_3 = length between adjacent baffles = 8-1/16 in.

$$\left(\frac{P - d}{P} \frac{d}{D_1} \right)^{0.4} = \left(\frac{1.813 - 1.500}{1.813} \times \frac{1.500}{21.0} \right)^{0.4} = 0.1725$$

$$\left(\frac{S_W}{S_W + S_E} \right)^2 = \left(\frac{0.309}{0.309 + 0.029} \right)^2 = 0.837$$

$$\left(\frac{G_{av} d}{\mu} \right)^{0.6} = \left(\frac{29,202 \times 0.125}{0.076} \right)^{0.6} = (48,029)^{0.6} = 644$$

$$\left(\frac{C_p \mu}{k} \right)^{0.3} = \left(\frac{1.25 \times 0.076}{0.133} \right)^{0.3} = (0.71)^{0.3} = 0.9$$

$$h = \frac{0.133}{0.125} 1.9(0.1725)(0.837)(644)(0.9)(1.00) = 169.2$$

Effective Conduction from Recuperator Core to Insulating Tube

The stagnant gap between the recuperator core tube and the insulating tube serves to insulate the coolant in the core tube from the hot side annulus. Heat transfer across the gap is by radiation and conduction in the coolant.

$$Q = \sigma A_r (T_c^4 - T_I^4) + k A_c (T_c - T_3) / \Delta r = K (T_c - T_I)$$

$$\sigma = 0.173 \times 10^{-8} \text{ Btu/h ft}^2 \text{R}^4$$

$$A_r = \text{surface area on recuperator core tube} = \pi(0.750)(171)(96)/144 \\ = 268 \text{ ft}^2$$

$$A_c = \text{mean area in gap} = \pi(0.8863)(171)(96)/144 = 317 \text{ ft}^2$$

$$F = \text{radiation interchange factor} = \frac{1}{\frac{1}{\epsilon_c} + \frac{A_c}{A_I} \left(\frac{1}{\epsilon_I} - 1 \right)}$$

$$\epsilon_c = \epsilon_I = 0.6$$

$$k = \text{average thermal conductivity} = 0.165 \text{ Btu/h ft}^\circ\text{R @ } 1460^\circ\text{R}$$

$$\Delta r = \text{width of gap} = (1.0225 - 0.750)/2 \times 12 = 0.01135 \text{ ft}$$

$$F = \frac{1}{\frac{1}{0.6} + \frac{0.750}{1.0225} \left(\frac{1}{0.6} - 1 \right)} = 0.464$$

$$\text{Assume } T_c = 1860^\circ\text{R}, T_I = 1060^\circ\text{R}[\text{condition at tube sheet}]$$

$$Q = 0.173 \times 10^{-8} \times 268 \times 0.464(1860^4 - 1060^4) + \frac{0.165 \times 317}{0.01135}(1860 - 1060) \\ = 215.1(1.86^2 + 1.06^2)(1.86 + 1.06)(1860 - 1060) + 4608(1860 - 1060) \\ = (2879 + 4608)(1860 - 1060)$$

$$K = 7487$$

$$\text{Assume } T_c = 1760^\circ\text{R}, T_2 = 1760^\circ\text{R}[\text{condition at tube end}]$$

$$Q = 215.1(1.76^2 + 1.76^2)(1.76 + 1.76)(\Delta T) + 4608(\Delta T) \\ = (4690 + 4608)\Delta T$$

$$K = 9298$$

$$\text{Average coefficient} = \frac{7487 + 9298}{2} = 8392 \text{ Btu/h }^\circ\text{R}$$

$$\text{or} = 589 \text{ Btu/h ft }^\circ\text{R}$$

Pressure Loss

Tube Side

$$\Delta P_{\text{friction loss in core tube}} = \frac{fG^2L}{g\rho_g R}$$

$$f = \frac{0.079}{N_{\text{Re}}^{1/4}}$$

$$N_{Re} = 21,700 \text{ from previous section}$$

$$G = 38,825 \text{ lb/h ft}^2$$

$$L = 14.25 \text{ ft}$$

$$\rho_g = 0.106 \text{ lb/ft}^3$$

$$D = 0.0592 \text{ ft} = 2R$$

$$g = 32.2 \times (3600)^2 \text{ ft/h}^2$$

$$f = \frac{0.079}{(21,700)^{1/4}} = \frac{0.079}{12.14} = 0.0065$$

$$\Delta P_{\text{core tube}} = \frac{0.0065(3.8825 \times 10^4)^2 \times 14.25}{32.2(3.6 \times 10^3)^2 \times 0.106 \times 0.0296 \times 144} = 0.74 \text{ lb/in.}^2$$

$$\Delta P_{\text{friction loss in annulus}} = \frac{f_2 G^2 L}{g \rho_g R (1 - \lambda^2)}$$

for concentric annulus. This calculation is similar to that given in Ref. 3.

$$\lambda^2 = \frac{1}{2} \frac{1 - K^2}{\ln(1/K)}$$

$$K = \frac{r}{R}$$

$$r = \text{inner diam} = 0.0885/2 = 0.4425 \text{ ft}$$

$$R = \text{outer radius} = 0.1130/2 = 0.0565 \text{ ft}$$

$$f_2 = \frac{0.079}{N_{Re}^{1/4}}$$

$$N_{Re} = \frac{2RG}{\mu} (1 - \lambda^2)$$

$$L = 14.25 \text{ ft}$$

$$\rho_g = 0.128 \text{ lb/ft}^3 \text{ @ the average coolant temperature of } 1460^\circ\text{R}$$

$$G = 27,600 \text{ lb/h ft}^2$$

$$K = \frac{0.04425}{0.0565} = 0.785$$

$$\lambda^2 = \frac{1}{2} \frac{1 - (0.785)^2}{\ln \left(\frac{1}{0.785} \right)} = 0.794$$

$$N_{re} = \frac{2 \times 0.0565 \times 27600}{0.09} (1 - 0.794) = 7130$$

$$f_2 = \frac{0.079}{(7130)^{1/4}} = \frac{0.079}{9.19} = 0.00860$$

$$\Delta P = \frac{0.00860 (27600)^2 (14.25)}{32.2 \times (3600)^2 (0.128) (0.0565) (1 - 0.794) (144)} = 1.03 \text{ lb/in.}^2$$

Summary of Pressure Losses on Tube Side

Losses in entrances, exits, etc. are taken from the Baldwin-Lima-Hamilton (B-L-H) design report.

<u>Source</u>	<u>B-L-H</u>	<u>LASL</u>
1. Inlet nozzle expansion	0.051	
2. Contraction into core tube	0.049	
3. Turning loss - core tube to annulus	0.080	
4. Expansion - core tube to annulus	0.011	
5. Core tube support clips	0.119	
6. Expansion loss from annulus	0.023	
7. Cross flow between tube sheets	0.006	
8. Contraction around core tube sheet	0.016	
9. Contraction in outlet nozzle	<u>0.023</u>	
	0.378 psi	0.378
10. Friction loss through core tube	0.774	0.740
11. Friction loss through annulus	<u>0.850</u>	<u>1.030</u>
	2.002 psi	2.148 psi

Shell Side Pressure Loss

Correlation of Bell. The nomenclature is the same as used in Ref. 6.

$$\Delta P_s = \left[2\Delta P_c \left(1 + \frac{N_w}{N_c} \right) + (N_B - 1)\Delta P_c \left(\frac{\Delta P_L}{\Delta P_{NL}} \right)_{\text{exch}} \right] + \left[N_B \Delta P_6 \left(\frac{\Delta P_L}{\Delta P_{NL}} \right)_{\text{exch}} \right]$$

$$\Delta P_c' = \frac{2f_c (\epsilon \Delta P) (G_c)^2 N_c}{g_c \rho_s \phi} = \text{cross flow loss}$$

$$S_c = \frac{[d_s - N_L d_t]_{BP}}{144} = \frac{[20.5 - 11(1.5)] 12.375}{144}$$

= 0.224 ft² min. cross flow area, one tube row from center

$$G_c = \frac{10,250}{0.224} = 45,760 \text{ lb/h ft}^2$$

$$\phi = \left(\frac{\mu}{\mu_w} \right)^{0.14} = 1.0$$

$$N_{re} = \frac{D_e G_c}{\mu} = \frac{0.125 \times 45760}{0.076} = 75,263$$

$$f_c = 0.67 (N_{re})^{-0.182} = 0.67 (75263)^{-0.182} = 0.086$$

$$F_{BP} = \frac{\left\{ ds - \frac{[(N_d - 1)P + dt]}{144 S_c} \right\}_{BP}}{144 S_c} = \frac{\{20.75 - [(11 - 1)1.813 + 1.5]\} 8.08}{144(0.224)}$$

$$= \frac{1.12 \times 8.08}{144 \times 0.224} = 0.28$$

$$m = 3.8(0.28) = 1.06$$

$$\epsilon_{\Delta P} = e^{-m} = e^{-1.06} = 0.345$$

$$\Delta P_c' = \frac{2 \times 0.094 \times 0.345 \left(\frac{45760}{3600} \right)^2 \times 7}{32.2 \times 0.18 \times 1.0} = 12.658 \text{ lb/ft}^2$$

$$\Delta P_c = \frac{12.656}{144} = 0.0878 \text{ lb/in.}^2$$

$$\Delta P_b' = [2 + 0.6 N_w] \left[\frac{\rho V_z^2}{2 g_c} \right] = \text{baffle loss}$$

$$S_b = \frac{44.5}{144} = 0.309 \text{ ft}^2 \text{ (from B-L-H design report)}$$

$$G_b = \frac{10,250}{0.309} = 33171 \text{ lb/h ft}^2$$

$$N_{re} = \frac{D_e G_b}{\mu} = \frac{0.125 \times 33171}{0.076} = 54,500$$

$$V_c = \frac{10250/3600}{0.224 \times 0.18} = 70.6 \text{ ft/sec}$$

$$V_b = \frac{10250/3600}{0.309 \times 0.18} = 51.3 \text{ ft/sec}$$

$$V_z = \sqrt{70.6 \times 51.3} = 60.2 \text{ ft/sec}$$

$$\Delta P_b = [2 + 0.6 \times 2] \left[\frac{0.18(60.2)^2}{2 \times 32.2} \right] = 3.2 \times 10.13 = 3241 \text{ lb/ft}^2$$

$$\left[1 - \frac{\Delta P_L}{\Delta P_{NL}} \right]_{\text{exch}} = \left(1 - \frac{\Delta P_L}{\Delta P_{NL}} \right)_0 \left(\frac{S_{TB} + 2 S_{SB}}{S_L} \right) \text{ leakage correction factor}$$

$$S_{TB} = N_{BT} \frac{\pi}{4} \left[\frac{(d_{BT})^2 - (d)^2}{144} \right] = 85 \frac{\pi}{4} \left(\frac{1.531^2 - 1.500^2}{144} \right)$$

$$S_{SB} = \left(\frac{360 - A^\circ}{360} \right) \frac{\pi}{4} \left[\frac{d_S^2 - d_G^2}{144} \right] = \frac{360 - 114}{360} \times \frac{\pi}{4} \left(\frac{21^2 - 20.875^2}{144} \right)$$

$$= 0.0195 \text{ ft}^2$$

$$S_L = S_{TB} + S_{SB} = 0.0434 + 0.0195 = 0.0629$$

$$\frac{S_L}{S_c} = \frac{0.0629}{0.344} = 0.1828$$

$$\left(1 - \frac{\Delta P_L}{\Delta P_{NL}} \right)_0 = 0.375 \quad (\text{Fig. 4 in reference 6})$$

$$\left[1 - \frac{\Delta P_L}{\Delta P_{NL}} \right]_{\text{exch}} = 0.375 \times \left(\frac{0.0434 + 2 \times 0.0195}{0.0629} \right) = 0.491$$

$$\left(\frac{\Delta P_L}{\Delta P_{NL}} \right)_{\text{exch}} = 1.0 - 0.491 = 0.509$$

$$\Delta P_s' = \left[2 \times 12.656 \left(1 + \frac{2}{7} \right) + (12 - 1)(12.656)(0.509) \right] + [12(32.41)(0.509)]$$

$$= 32.54 + 70.86 + 197.9 = 301.36 \text{ lb/ft}^2$$

$$\Delta P_s = \Delta P_s' / 144 = 301.36 / 144 = 2.09 \text{ lb/ft}^2$$

Total Pressure Loss on Shell Side

<u>Source of Loss</u>	<u>B-L-H</u>	<u>LASL</u>
Heat exchanger shell		2.093
Inlet nozzle	0.082	0.082*
Outlet nozzle	<u>0.254</u>	<u>0.254*</u>
		2.429

Weights

$$\text{Density} = \rho = \frac{62.4 \times 7.8}{1728} = 0.282 \text{ lb/in.}^3$$

$$\begin{aligned} \text{Weight core tubes} &= 96 \left[\frac{\pi}{4} (0.750^2 - 0.710^2) \right] 171 \times 0.282 = 212 \text{ lb} \\ \text{Insulating tubes} &= 96 \left[\frac{\pi}{4} (1.0625^2 - 1.0225^2) \right] 171 \times 0.282 = 302 \text{ lb} \\ \text{Outer tubes} &= 96 \left[\frac{\pi}{4} (1.500^2 - 1.356^2) \right] 171 \times 0.282 = 1495 \text{ lb} \\ \text{Shell (180 in. length)} &= \frac{\pi}{4} \left[(21.75^2 - 20.75^2) \right] 180 \times 0.282 = 1694 \text{ lb} \\ \text{Main tube sheet} &= \frac{\pi}{4} \left[20.75^2 - 96(1.356)^2 \right] 3 \times 0.282 = 168 \text{ lb} \\ \text{Hot tube sheet} &= \frac{\pi}{4} \left[20.75^2 - 96(0.710)^2 \right] \frac{5}{8} \times 0.282 = \underline{53 \text{ lb}} \\ \text{Total of above**} &= 3924 \text{ lb} \end{aligned}$$

Average Velocities

$$\begin{aligned} \text{Core tube} \quad v &= \frac{\dot{W}}{3600 \times \rho_g A} = \frac{10,250}{3600 \times 0.106 \times 0.264} = 102 \text{ ft/sec} \\ \text{Annulus} \quad v &= \frac{\dot{W}}{3600 \times \rho_g A} = \frac{10,250}{3600 \times 0.137 \times 0.371} = 156 \text{ ft/sec} \\ \text{Shell} \quad v &= 60.2 \text{ ft/sec (see calculated shell side pressure loss)} \end{aligned}$$

* From B-L-H design report.

** Does not include baffles, tie rods, nozzles, flanges, etc., the total of which may weigh as much as 3000 lb.

HEAT DUMP

Dimensions

<u>Tubes:</u>	Outside diameter	1.00 inch
	Inside diameter	0.87 inch
	Number	56
	Length/pass	12.0 ft
	Number of passes/tube	4
<u>Fins:</u>	Outside diameter	2.00 inch
	Thickness	0.03 inch
	Number/inch	7

Areas

$$\begin{aligned}\text{Flow area in tubes} &= 56 \times \frac{\pi}{4} (0.87/12)^2 = 0.231 \text{ ft}^2 \\ \text{Heat transfer surface in tubes} &= 56 \times \pi (0.87/12) (48) = 611 \text{ ft}^2 \\ \text{External heat transfer surface} &= \text{fin area and tube area} \\ &= 56 \times 48 [2 \times 7 \times 12 \frac{\pi}{4} (2.0^2 - 1.0^2) + \pi (1.0) (12 - 7 \times 12 \times 0.03)] / 144 \\ &= 56 \times 48 [2.75 + 0.207] = 8000 \text{ ft}^2\end{aligned}$$

Weights (from B-L-H)

Fins	5200 lb
Tubes	1980 lb
Manifold	1720 lb
Other	<u>1700 lb</u>
Total	10600 lb

Air Flow Rate = 154,000 lb/h @ 560°R inlet temperature = 42.8 lb/sec

Tube Side Heat Transfer Coefficient

$$N_{Nu} = 0.023 (N_{Re})^{0.8} (N_{Pr})^{0.4} \left[\frac{T_s}{T_g} \right]^{-0.5} \quad (\text{Ref. 2})$$

Average coolant temperature	$T_g = 1060^\circ\text{R}$
Average wall temperature	$T_s = 1360^\circ\text{R}$
Diameter	$D = 0.0725 \text{ ft}$
Thermal conductivity	$k = 0.133 \text{ Btu/h ft}^\circ\text{R}$
Viscosity	$\mu = 0.076 \text{ lb/ft h}$

Coolant flow rate

$$\dot{W} = 10,250 \text{ lb/h}$$

Mass flow rate

$$G = 10,250/0.231 = 44300 \text{ lb/h ft}^2$$

Nusselt Number

$$N_{Nu} = hD/k$$

Reynold Number

$$N_{Re} = DG/\mu$$

Prandtl Number

$$N_{Pr} = C_p \mu/k$$

$$h = \frac{0.133}{0.0725} 0.023 \left(\frac{0.0725 \times 44300}{0.076} \right)^{0.8} \left(\frac{1.25 \times 0.076}{0.133} \right)^{0.4} \left(\frac{1360}{1060} \right)^{-0.5}$$

$$= \frac{0.133}{0.0725} \times 0.023(41300)^{0.8}(0.715)^{0.4} \left(\frac{1360}{1060} \right)^{-0.5}$$

$$= 160 \text{ Btu/h ft}^2\text{°R}$$

Air Side Heat Transfer Coefficient

(Ref. 7)

Equivalent diameter of finned tubes = D_e

$$= \frac{2(\text{fin area} + \text{bare tube area})}{\pi(\text{projected perimeter})}$$

$$\text{Projected perimeter} = \left[2 \times 2 \times \frac{1}{2} \times 7 \times 12 + 2(12 - 7 \times 12 \times 0.03) \right] \left(\frac{56 \times 48}{12} \right) = 41800 \text{ ft}$$

$$D_e = \frac{2 \times 8000}{\pi 41800} = 0.121 \text{ ft}$$

$$\text{Frontal area} = 5.25 \times 12 = 64 \text{ ft}^2$$

Flow area = frontal area - projected area of tubes and fins

$$= 64.0 - 28 \times 0.0833 \times 12 - 28 \times 2 \left(\frac{0.03}{12} \right) \times \left(\frac{1}{12} \right) \times (7 \times 12) \times 12 = 64.0 - 28 - 11.75 = 24.25 \text{ ft}^2$$

$$\dot{W}_{\text{air}} = 154,000 \text{ lb/h @ } 100^\circ\text{F inlet}$$

$$G = 154,000/24.25 = 6350$$

$$\mu = 0.049 \text{ lb/ft h}$$

$$N_{Re} = \frac{D_e G}{\mu} = \frac{0.121 \times 6350}{0.049} = 15700$$

$$J = \frac{h_f D_e}{k} (N_{Pr})^{1/3} = 100 \quad (\text{Ref. 7})$$

$$k = 0.018 \text{ Btu/h ft}^\circ\text{F}$$

$$h_f = 100 \frac{0.018}{0.121} (0.74)^{1/3} = 13.3 \text{ Btu/h ft } ^\circ\text{R}$$

Fin Efficiency

(Ref. 7)

$$h_f = 13.3 \text{ Btu/h ft}^2{}^\circ\text{R}$$

$$k_f = 16.6 \text{ Btu/h ft}^\circ\text{R}$$

$$y = (0.03/2)/12 = 0.00125 \text{ ft}$$

$$r_e = 1.0/12 = 0.0833 \text{ ft}$$

$$r_b = 0.5/12 = 0.0417 \text{ ft}$$

$$(r_e - r_b) \sqrt{\frac{h_f}{k_y}} = (0.0833 - 0.0417) \sqrt{\frac{13.3}{16.6 \times 0.00125}} = 1.05 \text{ ft}$$

$$\frac{r_e}{r_b} = 2.0$$

$$\Omega = 0.67$$

$$\text{Effective } h_f = (0.67 \times 2.75 + 0.207) \frac{13.3}{2.957} = 9.2 \text{ Btu/h ft}^2{}^\circ\text{R}$$

$$\begin{aligned} \text{Overall coefficient} = U &= \frac{1}{\frac{1}{h_o} + \frac{A_o}{A_i} \frac{1}{h_i}} = \frac{1}{\frac{1}{9.2} + \frac{2.957}{\pi \times 0.0725} \times \frac{1}{160}} \\ &= 5.27 \text{ Btu/h ft}^2{}^\circ\text{R} \end{aligned}$$

Tube Side Pressure Loss

$$\Delta P_t = \frac{f G^2 L}{g_c \rho_g R} \times \frac{1}{144} = \text{friction loss in tube}$$

$$f = \frac{0.079}{N_{Re}^{0.25}}$$

$$G = 44300 \text{ lb/h ft}^2$$

$$\rho_g = 0.18 \text{ lb/ft}^3 \text{ @ average temperature} = 1060^\circ\text{R}$$

$$R = 0.725/2 = 0.362 \text{ ft radius}$$

$$g = 32.2 (3600)^2 \text{ ft/h}^2$$

$$L = 48 \text{ ft/tube}$$

$$\mu = 0.076 \text{ lb/ft h @ } 1060^\circ\text{R}$$

$$N_{Re} = \frac{DG}{\mu} = \frac{0.0725 \times 44300}{0.076} = 42300$$

$$f = \frac{0.079}{(42300)^{0.25}} = \frac{0.079}{14.35} = 0.0055$$

$$\Delta P_t = \frac{0.0055(44300)^2 \times 48}{32.2(3600)^2(0.18) \times 0.0363} \times \frac{1}{144} = 1.32 \text{ psi}$$

Entrance, exit, and 180° bend losses - assume one velocity head per each.

$$\Delta P_E = \frac{\rho_g V^2}{2g_c} = \frac{G^2}{2g_c \rho_g} = \frac{(44800)^2}{2 \times 32.2(3600)^2 \rho_g} \times \frac{1}{144} = \frac{0.0159}{\rho_g}$$

$$\text{Entrance : } \rho_g @ 1460^\circ\text{R} = 0.128 \text{ lb/ft}^3 \quad \Delta P = 0.124 \text{ psi}$$

$$\text{Exit : } \rho_g @ 660^\circ\text{R} = 0.280 \quad \Delta P = 0.057 \text{ psi}$$

$$180^\circ \text{ bends: ave } \rho_g @ 1060^\circ\text{R} = 0.180 \quad \Delta P = \underline{0.265} \text{ psi (3 bends per tube)}$$

$$0.446 \text{ psi}$$

$$\text{Tube side pressure loss} = 1.320 + 0.446 = 1.77 \text{ psi}$$

SECONDARY LOOP PIPING

Size 8-inch Schedule 40

Inside diameter 0.665 ft

Flow area 0.3474 ft²

Inside surface area 2.089 ft²/ft

Weight/foot 28.55 lb/ft²

Length 660°R 61.3 ft + 9-90° elbows

1460°R 96.5 ft + 12-90° elbows

Coolant Properties 660°R 1460°R

ρ 0.280 0.128 lb/ft³

k 0.098 0.165 Btu/h ft °R

μ 0.056 0.095 lb/ft h

N_{Pr} 0.71 0.71

Average Velocity 29 64 ft/sec

Heat Transfer Coefficients

$$N_{Nu} = 0.023(N_{Re})^{0.8}(N_{Pr})^{0.4}$$

$$G = \frac{10250}{0.3474} = 29500 \text{ lb/h ft}^2$$

660°R Coolant Pipe

$$N_{Re} = \frac{DG}{\mu} = \frac{0.3474 \times 29500}{0.056} = 350,000$$

$$\frac{h \times 0.3474}{0.098} = 0.023(350,000)^{0.8}(0.71)^{0.4}$$

$$h = \frac{0.098}{0.3474} 0.023(27,200)(0.87) = 78.3 \text{ Btu/h ft}^2\text{°R}$$

1460°R Coolant Pipe

$$N_{Re} = \frac{DG}{\mu} = \frac{0.3474 \times 29500}{0.095} = 206,500$$

$$h = \frac{0.165}{0.3474} 0.023(206,500)^{0.8}(0.71)^{0.4} = 87.6 \text{ Btu/h ft}^2\text{°R}$$

HEAT TRANSFER COEFFICIENTS

For the fuel element hole, recuperator, heat exchanger tubes, and heat dump, the correlation for heat transfer coefficient has the form

$$N_{Nu} = 0.034(N_{Re})^{0.8}(N_{Pr})^{0.4}\left(\frac{L}{D}\right)^{-0.1}\left(\frac{T_s}{T_g}\right)^{-0.5}$$

or

$$\frac{hD}{k} = 0.34\left(\frac{D \times \dot{W}}{\mu \times A_x}\right)^{0.8}\left(\frac{C_p \mu}{k}\right)^{0.4}\left(\frac{L}{D}\right)^{-0.1}\left(\frac{T_s}{T_g}\right)^{-0.5}$$

$$h = \left[\frac{0.034}{D} \left(\frac{D}{A_x} \right)^{-0.8} \left(\frac{C_p \mu}{k} \right)^{0.4} \left(\frac{L}{D} \right)^{-0.1} \right] \frac{k}{\mu^{0.8}} \dot{W}^{0.8} \left(\frac{T_s}{T_g} \right)^{-0.5}$$

where

$$\left(\frac{C_p \mu}{k} \right) = \text{constant for range of temperatures and pressures of interest.}$$

$$\therefore h \propto \mu^{0.2} \left(\frac{k}{\mu} \right) \dot{W}^{0.8} \left(\frac{T_s}{T_g} \right)^{-0.5}$$

$\mu^{0.2}$ varies about 8% for 1000°R change in temperature.

$\frac{k}{\mu}$ is constant.

$\left(\frac{T_s}{T_g} \right)^{-0.5}$ is typically about 1.1 and will not change significantly during most transients of interest.

Thus

$h \propto \dot{W}^{0.8}$ is assumed to represent the heat transfer coefficient for the above

and

$$h = \frac{h_o}{(\dot{W}_o)^{0.8}} \times \dot{W}^{0.8} \text{ for reasonable changes in flow and temperature}$$

where subscript o denotes calculated reference value.

Likewise, for the shell side of the heat exchanger

$$h = \frac{h_o}{\dot{W}_o^{0.6}} \times \dot{W}^{0.6}$$

The heat transfer coefficient in the fuel channel annulus is assumed to be a constant. For laminar flow the Nusselt Number is essentially constant; i.e.,

$$N_{Nu} = \frac{hD}{k} = \text{constant}$$

and, therefore the heat transfer coefficient should be proportional to the conductivity, k. Conductivity varies by about 30% for a change of 1000°R in the region of normal operation. This small change in heat transfer coefficient with temperature is not considered.

The (H')'s which appear in UHXCOM are calculated below. The h_o 's (Btu/h ft °R) are transformed to include the area and converted to Btu/sec, thus

$$H' = \frac{h_o \times A}{3600 \text{ sec/h}} \times \frac{1}{\dot{W}_o^n} = \frac{\text{Btu}}{\text{sec } ^\circ\text{R}(\text{lb/sec})^n}$$

Reactor

$$HA = \frac{45 \times 150}{3600} = 1.9 \text{ Btu/sec } ^\circ\text{R (independent of flow)}$$

$$HEK = \frac{135 \times 75}{3600(2.85)^{0.8}} = 1.4$$

Recuperator

$$HRHK = \frac{202 \times 337}{3600(2.85)^{0.8}} = 8.2$$

$$HRCK = \frac{156 \times 337}{3600(2.85)^{0.8}} = 6.32$$

Heat Exchanger

$$H1K = \frac{183 \times (17.84 \times 14.25)}{3600(2.85)^{0.8}} = 5.6$$

$$H2K = \frac{170 \times (26.7 \times 14.25)}{3600(2.85)^{0.8}} = 7.78$$

$$H3K = \frac{170 \times (34.08 \times 14.25)}{3600(2.85)^{0.8}} = 9.92$$

$$H4K = \frac{169.2 \times (37.7 \times 14.25)}{3600(2.85)^{0.6}} = 12.8$$

$$H5K = \frac{169.2}{3600} \times \frac{(5.44 \times 14.25)}{(2.85)^{0.6}} = 1.84$$

The conductivity across the bayonet tube insulation is converted to Btu/sec $^\circ\text{R}$.

$$XK = 8392 \text{ Btu/h } ^\circ\text{R} = 2.33 \text{ Btu/sec } ^\circ\text{R}$$

Heat Dump

$$HDHK = \frac{160 \times 611}{3600(2.85)^{0.8}} = 11.7$$

$$HDCK = \frac{9.2 \times 8000}{3600(42.78)^{0.725}} = 1.34$$

Secondary Loop Piping

660 $^\circ\text{R}$ Coolant Pipe

$$HP2K = \frac{78.3 \times (2.089 \times 61.3)}{3600(2.85)^{0.8}} = 1.21$$

1460 $^\circ\text{R}$ Coolant Pipe

$$HP1K = \frac{87.6 \times (2.089 \times 96.5)}{3600(2.85)^{0.8}} = 2.13$$

PRESSURE LOSSES

Pressure loss in the system is calculated from equations of the form

$$\Delta P = \frac{fG^2L}{2g_c\rho_g R} = \frac{f\dot{W}^2L}{2g_cA^2\rho_g R}$$

If it is assumed that $f = \text{constant}$ and $\rho_g \propto \frac{P}{T}$ $P = \text{pressure}$ then

$$\Delta P \propto \frac{\dot{W}^2T}{P}$$

or

$$\Delta P = \frac{\Delta P_o}{\dot{W}_o^2} \left(\frac{P}{T} \right)_o \times \frac{\dot{W}^2T}{P}$$

The change in friction factor, $\propto \sqrt[4]{\dot{W}}$, is sufficiently small to justify its being considered a constant.

The $(\Delta P')$'s which appear indirectly in the blower quadratic equations are calculated below.

$$\Delta P' = \frac{\Delta P_o}{\dot{W}_o^2} \times \frac{P_o}{T_o} = \frac{\text{sec}^2}{\text{in.}^4 \text{ } ^\circ\text{R}}$$

Reactor

$$\Delta P' = \frac{0.6 \times 500}{2.85^2 \times 2060} = 0.015$$

1060°R Piping (Primary Loop)

$$\Delta P' = \frac{0.45 \times 500}{2.85^2 \times 1060} = 0.0261$$

Recuperator

$$\Delta P' (\text{cold side}) = \frac{1.6 \times 500}{2.85^2 \times 1560} = 0.063$$

$$\Delta P' (\text{hot side}) = \frac{2.4 \times 500}{2.85^2 \times 2360} = 0.063$$

Heat Exchanger

$$\Delta P' (\text{tube side}) = \frac{2.148 \times 500}{2.85^2 \times 1460} = 0.09$$

$$\Delta P' (\text{shell side}) = \frac{2.429 \times 500}{2.85^2 \times 1060} = 0.141$$

Secondary Loop Piping

$$\Delta P' (660^\circ\text{R pipe}) = \frac{0.745 \times 500}{2.85^2 \times 640} = 0.0717$$

$$\Delta P' (1460^\circ\text{R pipe}) = \frac{0.803 \times 500}{2.85^2 \times 1460} = 0.034$$

Heat Dump

$$\Delta P' (\text{helium side}) = \frac{1.77 \times 500}{2.85^2 \times 1060} = 0.103$$

BLOWERS

From consideration of similarity requirements and experimental observation, the performance characteristics of the UHTREX blowers may be described by a single value function of the form

$$\psi = f(\phi)$$

where

$$\psi = \text{head coefficient} = \frac{H \times 2g}{U_t^2}$$

$$\phi = \text{flow coefficient} = \frac{4 Q_m}{U_t D_t^2}$$

$$H = \text{head in feet} \approx \frac{144 \Delta P}{\rho_g}$$

$$Q_m = \text{inlet flow in ft}^3/\text{sec} = \frac{\dot{W}}{3600 P_i}$$

$$g = 32.2 \text{ ft/sec}^2$$

$$U_t = \text{tip speed} = 5.079 \times 10^{-2} N$$

$$D_t^2 = (\text{tip diameter})^2 = 0.9409 \text{ ft}^2$$

A curve of this type was found experimentally for the UHTREX blowers. For use in the computer program it was divided into six linear sections.

Blower surge is estimated to occur when the flow coefficient has a value of 0.103.

The blower horsepower is calculated by

$$HP = \frac{1}{\eta} \frac{H \times Q \times \rho_g}{60 \times 550}$$

where

η = efficiency is also found from experimental data.

At steady-state operation, the blower is limited to 60 horsepower, maximum.

The curves are fitted with straight lines over several ranges. The ψ versus ϕ curve was fitted for six ranges; the η versus ϕ curve was fitted for four ranges.

The tables below give the coefficients of the straight line equations. The η or ψ axis was not shifted for each range.

TABLE B-1

ψ VS ϕ CURVE

	<u>Range of ϕ</u>	<u>Intercept</u>	<u>Slope</u>
1.	0.000-0.114	1.445	+0.333
2.	0.114-0.123	1.567	-0.736
3.	0.123-0.132	1.750	-2.222
4.	0.132-0.147	1.895	-3.310
5.	0.147-0.164	2.147	-5.028
6.	0.164-0.190	2.372	-6.372

TABLE B-2

η VS ϕ CURVE

	<u>Range of ϕ</u>	<u>Intercept</u>	<u>Slope</u>
1.	0.000-0.120	0.669	+3.666
2.	0.120-0.133	0.762	-0.407
3.	0.133-0.155	0.860	-1.140
4.	0.155-0.180	1.092	-2.640

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APPENDIX C

LINEAR SYSTEMS ANALYSIS

This report contains frequent reference to "transfer functions," "frequency domain," and other terms commonly used in systems analysis work. Some readers may not be familiar with these terms; and this appendix has been included to provide them with a background such that they can understand all of the material in the report.

The phrase "linear system" as used here, and as it is ordinarily used in the engineering profession, denotes a system described by linear differential equations with constant coefficients. This definition is different from that used by mathematicians who define a linear system as "a system described by linear differential equations," without restricting the coefficients of these equations to be constant. Thus, in this appendix the authors may speak of "linearizing" a set of equations which, in the strict mathematical sense, are already linear.

C.1 Laplace Transform Theory¹

The Laplace transform of a function $f(t)$ is defined as

$$F(s) = \int_0^{\infty} f(t)e^{-st}dt.$$

By using this equation one can establish a set or table of transforms similar to a Fourier transform table or a logarithm table. Thus, given

a particular time function $f(t)$, one can find the corresponding transform, $F(s)$, and vice versa. A short set of Laplace function-transform pairs, operation-transform pairs, and pertinent theorems is given in Table C-1.

To illustrate the application of the Laplace transform, consider the following problem. A mechanical system is shown in Fig. C-1. The object is to determine the displacement, $x(t)$, given a driving force $f(t)$.

Fig. C-1 Simple Mechanical System

The differential equation describing the system is

$$B \frac{dx}{dt} + kx = f(t) \quad (C.1-1)$$

One can take the Laplace transform of this equation to give

$$B[s X(s) - x(0^+)] + kX(s) = F(s) \quad (C.1-2)$$

$$\therefore X(s) = \frac{F(s)}{Bs + k} + \frac{BX(0^+)}{Bs + k} \quad (C.1-3)$$

or

$$X(s) = \frac{1/B F(s)}{s + k/B} + \frac{x(0^+)}{s + k/B} \quad (C.1-4)$$

TABLE C-1

FUNCTION-TRANSFORM PAIRS

<u>Name of Function of t</u>	<u>f(t)</u>	<u>F(s)</u>
Unit impulse (or delta function)	$\delta(t)$	1
Step function	$\mu(t)$	$\frac{1}{s}$
Damped exponential	e^{-at}	$\frac{1}{s + a}$
Sine	$\sin bt$	$\frac{b}{s^2 + b^2}$
Cosine	$\cos bt$	$\frac{s}{s^2 + b^2}$
Damped sinusoid	$e^{-at} \sin bt$	$\frac{b}{(s + a)^2 + b^2}$

OPERATION-TRANSFORM PAIRS

<u>Name of Operation</u>	<u>f(t)</u>	<u>F(s)</u>
Real differentiation	$\frac{d}{dt} f(t)$	$s F(s) - f(0^+)$
Real integration	$\int_0^t f(\tau) d\tau$	$\frac{F(s)}{s} + \frac{f^{(-1)}(0^+)}{s}$
Complex multiplication	$\int_0^t f_1(t-\tau) f_2(\tau) d\tau$	$F_1(s) F_2(s)$
Where $f(0^+) = \lim_{t \rightarrow 0} f(t)$, and $f^{(-1)}(0^+) = \lim_{t \rightarrow 0} \int_{-\infty}^t f(\tau) d\tau$		

Both limits above are taken as $t \rightarrow 0$ from the positive side.

THEOREMS

Final value theorem	$\lim_{t \rightarrow \infty} f(t)$	=	$\lim_{s \rightarrow 0} sF(s)$
Initial value theorem	$\lim_{t \rightarrow 0} f(t)$	=	$\lim_{s \rightarrow \infty} sF(s)$

Now if, for instance,

$$\left. \begin{aligned} k &= 4 \text{ lb/in.}, \\ B &= 2 \text{ lb/in./sec} \\ x(0^+) &= 3 \text{ in.} \end{aligned} \right\} \quad (\text{C.1-5})$$

and

$$f(t) = \sin 4t, \quad (\text{C.1-6})$$

we have

$$F(s) = \frac{4}{s^2 + 16} \quad (\text{C.1-7})$$

and

$$X(s) = \frac{2}{(s + 2)(s^2 + 16)} + \frac{3}{s + 2} \quad (\text{C.1-8})$$

A little algebraic juggling leads to

$$X(s) = \frac{1}{20} \left(\frac{62}{s + 2} - \frac{2s}{s^2 + 16} + \frac{4}{s^2 + 16} \right) \quad (\text{C.1-9})$$

By applying the table of transforms given above, one finds that

$$x(t) = \frac{1}{20} \left(62 e^{-2t} - 2 \cos 4t + \sin 4t \right) \quad (\text{C.1-10})$$

or

$$x(t) = 3.1 e^{-2t} + 0.1117 \sin (4t - \theta) \quad (\text{C.1-11})$$

where

$$\theta = 1.109 \text{ radians} \equiv 63.5^\circ \quad (\text{C.1-12})$$

As a partial check, one can verify that this solution contains the initial condition.

Thus,

$$\lim_{t \rightarrow 0} x(t) = \frac{1}{20} (62 - 2) = 3 \equiv x(0^+) \quad (\text{C.1-13})$$

A similar result can be obtained by applying the initial value theorem to Eq. C.1-9.

$$\lim_{t \rightarrow 0} x(t) = \lim_{s \rightarrow \infty} sX(s)$$

$$\therefore \lim_{t \rightarrow 0} x(t) = \lim_{s \rightarrow \infty} \frac{s}{20} \left(\frac{62}{s+2} - \frac{2s}{s^2+16} + \frac{4}{s^2+16} \right) = 3 \equiv x(0^+) \quad (\text{C.1-14})$$

The Laplace transform solution of more complicated problems involving many simultaneous differential equations proceeds in a similar fashion. The original differential equations are transformed, and the transformed equations are solved for $x(s)$, the transform of the unknown of interest. Finally, the inverse transform of both sides of the equation is taken to obtain $x(t)$. Thus, the Laplace transform method reduces the solution of a set of differential equations in the variable t to the solution of a set of algebraic equations in the variable s . The method has the advantage that it takes initial conditions into account automatically, and the steps in the solution proceed in a well defined manner.

In the rest of this report, t is a real variable associated with time, whereas the Laplace transform variable s is a complex variable

$$s = \sigma + j\omega$$

associated with frequency. Thus, system equations written as functions of t have to do with the "time domain" and transformed equations with the argument s are associated with the "frequency domain. One can get from one domain to the other by use of the Laplace transform table.

C.2 Linear Systems

The linear system shown in Fig. C-2 can be characterized by $h(t)$, the response of the system to a unit impulse; or it can be characterized by $H(s)$, the system transfer function, which is the Laplace transform of $h(t)$. In Fig. C-2, the output $g(t)$ is that part of the total output which is due to the input $f(t)$; and $g(t)$ is given by the convolution of $f(t)$ and $h(t)$ as shown in Eq. C.2-1.

Fig. C-2 Linear System

$$g(t) = \int_0^t f(\tau) h(t - \tau) d\tau \quad (C.2-1)$$

By taking the Laplace transform of this expression one obtains

$$G(s) = F(s)H(s) \quad (C.2-2)$$

Thus, the system transfer function $H(s)$ is given by

$$H(s) = \frac{G(s)}{F(s)} \quad (C.2-3)$$

In particular, if the transfer function is evaluated at a realizable frequency $j\omega$ where

$$\omega = 2\pi f \quad (C.2-4)$$

then

$$H(s) \Big|_{s=j\omega} = \frac{G(j\omega)}{F(j\omega)} = R(\omega)e^{j\theta(\omega)} \quad (C.2-5)$$

Here R is the magnitude and θ is the phase of the transfer function $H(s)$. Both R and θ are functions of ω . As a practical matter, one ordinarily plots $20 \log_{10} R$ vs ω and θ vs ω .

To illustrate the concept of a transfer function, consider the example of Section C.1. The input output transfer function of the system shown in Fig. C-1 is available directly from Eq. C.1-4. $H(s)$ is defined as the ratio

$$H(s) = \frac{X(s)}{F(s)} = \frac{1/B}{s + k/B} \quad (C.2-6)$$

Thus,

$$\begin{aligned} H(j\omega) &= \frac{1/B}{k/B + j\omega} \\ &= \frac{1/2}{2 + j\omega} \\ &= \frac{1}{2} \frac{1}{\sqrt{4 + \omega^2}} e^{j\theta} \end{aligned} \quad (C.2-7)$$

$$R(\omega) = \frac{1}{2\sqrt{4 + \omega^2}} \quad (C.2-8)$$

and

$$\theta = \tan^{-1} - \frac{\omega}{2} \text{ radians} \quad (C.2-9)$$

Sketches of $20 \log R$ and θ vs ω are shown in Figs. C-3 and C-4. One can determine the attenuation and phase shift of the mechanical network from these graphs directly. For instance, suppose one supplies

$$f(t) = \sin 4t \quad (C.2-10)$$

as a driving force to the network. From Figs. C-3 and C-4, at 4 radians the output sine wave is "down 19 db" and shifted in phase by -63.5° .

The -19 db corresponds to magnitude ratio of 0.1117. After the initial transient has died away, the output $x(t)$ will be

$$x(t) = 0.1117 \sin(4t - \theta) \quad (t \gg 0) \quad (C.2-11)$$

where

$$\theta = 63.5^\circ \quad (C.2-12)$$

Note that Eq. C.2-11 corresponds exactly to the "steady-state" term on the right-hand side of Eq. C.1-11, as it should.

Thus, the transfer function tells one how much the sine wave output is attenuated, and how many degrees it is shifted, with respect to the input. Transfer functions as such are probably not new to the average reader. They are used by manufacturers of hi-fidelity sound equipment in specifying the characteristics of an amplifier. For instance,

Fig. C-3 Gain, $20 \log_{10} R$ Vs Frequency ω

Fig. C-4 Phase $\theta(\omega)$ Vs Frequency ω

one will read that an audio amplifier is "flat to within ± 1.5 db from 20 cycles to 18 kc." Thus, the amplifier input-output transfer function probably looks something like that shown in Fig. C-5.

Fig. C-5 Hypothetical Audio Amplifier Transfer Function Gain

Of course, an amplifier should have a "flat" transfer function in the frequency range of interest to reproduce its input without distortion. If, for instance, the amplifier gain was down 20 db from its flat portion at say 2 kc, the 2-kc energy in the input signal would be attenuated severely and the output signal would be distorted. On the other hand, it does little good for an audio amplifier to be "flat to 100 kc," for there is little audio energy in the frequency spectrum from 20 kc to 100 kc. For zero distortion, an amplifier also should have a phase shift that is a linear function of frequency. With such a phase characteristic, energies at different frequencies are delayed the same amount of time in the amplifier. They leave the amplifier with the same time relation as they had when they entered it, and there is no distortion due to phase shift.

C.3 Poles and Zeros

The zeros of a transfer function are those values of s at which the magnitude of the transfer function is zero. The poles are those values of s at which the magnitude of the transfer function is infinite. The transfer function $H(s)$ is written in factored form in Eq. C.3-1 so as to show its m zeros $z_1, z_2, \dots, z_m$ and its n poles $p_1, p_2, \dots, p_n$.

$$H(s) = k \frac{(s - z_1)(s - z_2) \dots (s - z_m)}{(s - p_1)(s - p_2) \dots (s - p_n)} . \quad (C.3-1)$$

The transfer function of Eq. C.2-6,

$$H(s) = \frac{1/B}{s + k/B}$$

has a single pole at $s = -k/B$ and a single zero at $s = \infty$. For physically realizable systems, the poles and zeros always appear in conjugate pairs when they are complex. All poles of stable systems have negative real parts and lie in the left-hand side of the s plane. Zeros of stable and unstable systems may lie anywhere in the s plane.

The poles and zeros of $H(s)$, along with the gain constant k , describe $H(s)$ completely. As may be seen in the following paragraphs, they may be interpreted and applied in a variety of ways.

C.3.1 Geometrical Interpretation

Consider Fig. C-6 in which the finite poles and zeros of the following transfer function have been plotted.

$$\begin{aligned} H(s) &= \frac{(s - z_1)(s - z_2)}{(s - p_1)(s - p_2)(s - p_3)} \\ &= \frac{(s + 2)(s - 6)}{(s + 1 - j5)(s + 1 + j5)(s + 4)} \end{aligned} \quad (C.3-2)$$

Fig. C-6 Plot in s Plane of Poles and Zeros of

$$H(s) = \frac{(s + 2)(s - 6)}{(s + 1 - j5)(s + 1 + j5)(s + 4)}$$

Now suppose that one wishes to evaluate the magnitude and phase of $H(s)$ at some point in the s plane, say $s_1 = 3 + j4$. Thus

$$\begin{aligned} H(s) \Big|_{s=s_1} &= \frac{(3 + j4 + 2)(3 + j4 - 6)}{(3 + j4 + 1 - j5)(3 + j4 + 1 + j5)(3 + j4 + 4)} \\ &= \frac{(5 + j4)(-3 + j4)}{(4 - j1)(4 + j9)(7 + j4)} \end{aligned} \quad (C.3 - 3)$$

Equation C.3 - 3 can be rewritten as

$$\begin{aligned} H(s) \Big|_{s=s_1} &= \frac{R_1 e^{j\theta_1} R_2 e^{j\theta_2}}{R_3 e^{j\theta_3} R_4 e^{j\theta_4} R_5 e^{j\theta_5}} \\ &= \frac{R_1 R_2}{R_3 R_4 R_5} e^{j(\theta_1 + \theta_2 - \theta_3 - \theta_4 - \theta_5)} \end{aligned} \quad (C.3 - 4)$$

where

$$\begin{aligned} R_1 &= \sqrt{5^2 + 4^2} & \theta_1 &= \tan^{-1} \frac{4}{5} \\ R_2 &= \sqrt{(-3)^2 + 4^2} & \theta_2 &= \tan^{-1} \frac{4}{3} \\ R_3 &= \sqrt{4^2 + (-1)^2} & \theta_3 &= \tan^{-1} \frac{-1}{4} \\ R_4 &= \sqrt{4^2 + 9^2} & \theta_4 &= \tan^{-1} \frac{9}{4} \\ R_5 &= \sqrt{7^2 + 4^2} & \theta_5 &= \tan^{-1} \frac{4}{7} \end{aligned}$$

It can be seen that R_1 is the magnitude of the vector from z_1 to s_1 in Fig. C-6. Also, the angle θ_1 is the angle measured from the σ axis to the vector from z_1 to s_1 . Thus, the values of R_1 and θ_1 , and all other R 's and θ 's can be obtained directly from the s plane by making measurements with a scale and a protractor. Using either method, one obtains

$$\begin{aligned} R_1 &= 6.4 & \theta_1 &= 39^\circ \\ R_2 &= 5.0 & \theta_2 &= 127^\circ \\ R_3 &= 4.1 & \theta_3 &= 346^\circ \\ R_4 &= 9.8 & \theta_4 &= 66^\circ \\ R_5 &= 8.1 & \theta_5 &= 3^\circ \end{aligned}$$

Then the magnitude and phase of $H(s) \Big|_{s=s_1}$ can be found by taking the quotient of the magnitudes and the differences of the phases as shown in Eq. C.3 - 4 to obtain $H(s) \Big|_{s=s_1} \approx 0.098 e^{j4.81}$ where the angle θ is 4.81 radians.

A special tool, called a Spirule, has been developed as an aid in finding the quotients of the magnitudes and the differences of the phases from a pole zero plot such as is shown in Fig. C-6. Using it, one can obtain the magnitude and phase of a transfer function with specified poles and zeros in a short time.

C.3.2 Role of Poles in Determining System Response

Suppose a linear system with a transfer function $H(s)$ is disturbed by a unit impulse

$$f(t) = \delta(t).$$

The transform of the system output is then

$$G(s) = F(s)H(s) = H(s)$$

for $F(s) = 1$. In the time domain,

$$\begin{aligned} f(t) &= \mathcal{L}^{-1}[H(s)] \\ &= \mathcal{L}^{-1}\left(\frac{R_1}{s - p_1} + \frac{R_2}{s - p_2} + \cdots + \frac{R_n}{s - p_n}\right) \end{aligned}$$

Thus, $f(t)$ will be made up of n terms, each due to one pole. As noted previously, poles with nonzero imaginary parts will occur in pairs, and the time response due to such a pair will be of an oscillatory nature. As may be seen from Table C-1, poles with zero imaginary parts will yield time functions that decrease or increase exponentially. A rough sketch of pole locations and the kind of time response they yield is given in Fig. C-7. One should note that as the poles move away from the origin on the σ axis their associated exponential response has a smaller and smaller time constant. Similarly, as poles move away from the origin along the $j\omega$ axis their associated sinusoidal frequency increases.

Fig. C-7 Time Response Associated With Various Pole Locations

Thus, the locations of the poles of a transfer function are important in determining the system characteristic response, and it is quite helpful in linear system analysis to keep a mental image of the relations shown in Fig. C-7.

C.4 Transfer Functions From Time Domain Response

To determine the transfer function or some related property of a linear physical system, one ordinarily applies a specific input to the system and records the resulting output. The output itself or some function of the output yields the desired information. Inputs consisting of steps, ramps, sinusoids, and noiselike signals are commonly used.

In the case at hand, one would like to determine the transfer functions of a computer simulation of a physical system. A step input to the simulation was chosen because it is easy to generate and because the magnitude and phase of the transfer functions can be obtained at many frequencies from the results of just one such simulated step input experiment. The material given below explains how the transfer function of a linear system can be computed from its time domain response to a step input.

Consider the linear system shown in Fig. C-8.

Fig. C-8 Linear System

The input-output relations for this system are

$$g(t) = \int_0^t f(\tau) h(t-\tau) d\tau \quad (C.4-1)$$

$$G(s) = F(s) H(s) \quad (C.4-2)$$

Let the input $f(t)$ be a step of magnitude A starting at $t = 0$.

$$f(t) = A u(t) \quad (C.4-3)$$

The transform of $f(t)$ is

$$F(s) = \frac{A}{s} \quad (C.4-4)$$

and thus

$$G(s) = \frac{A}{s} H(s) \quad (C.4-5)$$

or

$$H(s) = \frac{1}{A} s G(s) \quad (C.4-6)$$

Now from Table C-1.

$$\mathcal{L}\left[\frac{d}{dt} g(t)\right] = sG(s) - g(0^+) \quad (C.4-7)$$

Therefore

$$\frac{1}{A} sG(s) = \frac{1}{A} g(0^+) + \frac{1}{A} \mathcal{L}\left[\frac{d}{dt} g(t)\right] \quad (C.4-8)$$

and

$$H(s) = \frac{1}{A} g(0^+) + \frac{1}{A} \int_0^\infty e^{-st} \frac{d}{dt} g(t) dt \quad (C.4-9)$$

We are interested in evaluating $H(s)$ at physically realizable frequencies on the $j\omega$ axis, so let $s = j\omega$ in Eq. C.4-9.

$$H(j\omega) = \frac{1}{A} g(0^+) + \frac{1}{A} \int_0^{\infty} e^{-j\omega t} \frac{d}{dt} g(t) dt \quad (C.4-10)$$

$$H(j\omega) = \frac{1}{A} \left[g(0^+) + \int_0^{\infty} \cos \omega t \frac{d}{dt} g(t) dt \right] - \frac{j}{A} \int_0^{\infty} \sin \omega t \frac{d}{dt} g(t) dt \quad (C.4-11)$$

The real and imaginary parts of $H(j\omega)$ are

$$\text{Re } H(j\omega) = \left[\frac{1}{A} g(0^+) + \int_0^{\infty} \cos \omega t \frac{d}{dt} g(t) dt \right] \quad (C.4-12)$$

and

$$\text{Im } H(j\omega) = - \frac{1}{A} \int_0^{\infty} \sin \omega t \frac{d}{dt} g(t) dt \quad (C.4-13)$$

The UHXCOM program uses these two relations to compute the real and imaginary parts of $H(j\omega)$ and the related magnitude and phase as described in Section 3.2.3.5. The computation is carried out for a number of values of ω to give the entire transfer function from the results of only one step input simulation run.

REFERENCES

1. Two practical references on Laplace transforms are given below.
M. F. Gardner and J. L. Barnes, "Transients in Linear Systems,"
John Wiley and Sons, New York (1951).
Earl D. Rainville, "The Laplace Transform: An Introduction,"
Macmillan Co., New York, (1963).

APPENDIX D

CONVERGENCE CRITERIA

Solutions of system model equations are achieved through finite difference techniques which require that certain criteria¹ be satisfied or the model will yield results that depart from reality. With a fixed lump size for a given component, some gas temperature equations violate one criterion at low flows and some solid temperature equations violate another criterion at high flows.

D.1 Gas Temperature Equations

The UHXCOM model assumes that the gas temperatures change essentially instantaneously which yields heat balance equations of the form:

$$C_p \dot{W}(T_{gin} - T_{gout}) = hA(\bar{T}_g - T_s) \quad (D.1-1)$$

where

$$\bar{T}_g = \frac{T_{gin} + n \cdot T_{gout}}{n + 1} \quad (D.1-2)$$

Solving for the outlet gas temperature in terms of the solid temperature and inlet gas temperature we find:

$$T_{gout} = \left[\left(1 - \frac{hA}{C_p \dot{W}(1 + n)} \right) \cdot \frac{C_p \dot{W}(1 + n)}{C_p \dot{W}(1 + n) + hAn} \right] T_{gin} + \left[\frac{hA}{C_p \dot{W}} \cdot \frac{C_p \dot{W}(1 + n)}{C_p \dot{W}(1 + n) + hAn} \right] T_s \quad (D.1-3)$$

The coefficients of D.1-3 must be positive so that the second law of thermodynamics is obeyed. Since the coefficient of T_s is always positive, we have to satisfy the condition that:

$$hA \leq C_p \dot{W}(n + 1) \quad (D.1-4)$$

where

$$h = h(h_0 \dot{W}^c) \text{ and } c < 1.$$

The left-hand side of the inequality decreases more slowly than the right-hand side. When D.1-4 is violated, the affected heat transfer coefficient is arbitrarily adjusted to satisfy D.1-4. This causes some of the fine structure of the transient response of the affected temperatures to be lost; however, the effect on the overall system response is negligible.

D.2 Solid Temperature Equations

A generalized solid temperature equation compatible with D.1-1 is:

$$C_s M_s \frac{dT_{s1}}{d\tau} = H_{g1}(\bar{T}_{g1} - T_{s1}) + H_{g2}(\bar{T}_{g2} - T_{s1}) + k(T_{s2} - T_{s1}) \quad (D.2-1)$$

$$T_{s1}(\tau + \Delta\tau) = T_{s1}(\tau) + \Delta T_{s1} = T_{s1}' \quad (D.2-2)$$

Juggling these equations, one obtains T_{s1}' as shown below.

$$\begin{aligned} T_{s1}' = & \left(1 - \frac{\Sigma H_g + \Sigma k}{C_s M_s} \cdot \Delta\tau\right) T_{s1}(\tau) + \frac{H_{g1} \Delta\tau}{C_s M_s} \cdot \bar{T}_{g1} \\ & + \frac{H_{g2} \Delta\tau}{C_s M_s} \bar{T}_{g2} + \frac{k \Delta\tau}{C_s M_s} T_{s2} \end{aligned} \quad (D.2-3)$$

Again the second law of thermodynamics insists that the coefficients must be positive, and we note that the coefficient of $T_{s1}(\tau)$ is the only

one which might become negative with increasing flow. The criterion which must be satisfied is:

$$\Delta\tau \leq \frac{C_s M_s}{\Sigma H_g + \Sigma k} \quad (D.2-4)$$

An approximate analysis revealed that the primary loop portion of the heat exchanger was the sensitive component in establishing the maximum time step. Fig. D-1 shows the maximum permissible time step as a function of primary loop flow.

The symbols used in the equations are:

C_p	= specific heat of gas in lump (Btu/lb-°R)
C_s	= specific heat of solid lump (Btu/lb-°R)
$\dot{W}$	= mass flow of gas (lb/sec)
M_s	= mass of solid lump (lb)
τ	= time (sec)
h	= heat transfer coefficient (Btu/ft-sec-°R)
A	= heat transfer area for lump (ft ²)
H_{g1}, H_{g2}	= lump total heat transfer coefficients (Btu/sec-°R)
k	= conductivity between adjacent solids (Btu/sec-°R)
T_s, T_{s1}, T_{s2}	= mean solid temperatures for lump (°R)
T_{gin}, T_{gout}	= inlet, outlet gas temperatures for lump (°R)
$\bar{T}_g, \bar{T}_{g1}, \bar{T}_{g2}$	= average gas temperatures for lump (°R)
n	= weighting factor for average gas temperatures

REFERENCE

1. G. M. Dusinberre, "Calculation of Transient Temperatures in Pipes and Heat Exchangers by Numerical Methods," Transactions of the ASME, Vol. 76, No. 3, April 1954, p. 421.

Fig. D-1 Maximum Time Steps for UHXCOM

APPENDIX E

NUMERICAL SOLUTION OF THE REACTOR NEUTRON KINETICS EQUATIONS

E.1 Comments

When the UHXCOM program was written originally, Euler's method was used to solve the reactor neutron kinetic equations. However, it was found that the time step had to be extremely small if the solution was to converge.

An alternate solution to the reactor kinetic equations has been developed. Briefly, this alternate method involves solution of an integral equation¹ for the neutron density by the method of collocation. In applying the method of collocation it is assumed that the time variation of reactivity, $\rho(t)$, and the neutron density $n(t)$ can be matched by quadratic functions in a short time interval. The quadratic coefficients for $\rho(t)$ are determined directly, for $\rho(t)$ is known, and the unknown coefficients of the quadratic fit to $n(t)$ can be determined by use of the integral equation.

This method of solving the reactor neutron kinetic equations allows us to use time steps as large as 2 seconds for most UHXCOM problems. It is the use of such large time steps that has led to such a favorable ratio of computing time to simulation time.

The details of the method of solution discussed above are presented in the remainder of this appendix.

E.2 Development of Integral Equation

The development of the integral equation for $n(t)$ is essentially that given by Keepin and Cox.¹

The basic reactor kinetic equations used are:

$$\frac{dn(t)}{dt} = \frac{k(t)(1 - \bar{\gamma}\beta) - 1}{\Lambda} n(t) + \sum \lambda_i \gamma_i C_i(t) + \gamma_s s(t) \quad (\text{E.2-1a})$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\Lambda} k(t) n(t) - \lambda_i C_i(t) \quad (i = 1, 2, \dots, 6) \quad (\text{E.2-1b})$$

where

- $n(t)$ = neutron density at time t (neutrons/in.³)
- $k(t)$ = neutron reproduction number for finite geometry at time t (-)
- Λ = prompt neutron generation time for finite geometry (sec)
- λ_i = decay constant of the i th delayed neutron group (sec⁻¹)
- $C_i(t)$ = density of i th delayed neutron group precursor at time t (neutrons/in.³)
- β_i = i th group fraction of total neutrons from fission (-)
- $\beta = \sum_i \beta_i$ = total delayed neutron fraction (-)
- γ_i = effectiveness (in producing fission) of i th group delayed neutrons compared to prompt neutrons (-)
- $\bar{\gamma} = \beta^{-1} \sum_i \gamma_i \beta_i$ = average delayed neutron effectiveness (-)
- $\bar{\gamma}_s s(t)$ = effective contribution from extraneous sources at time t (neutrons/in.³ sec)

One might note that these equations are not the same as the reactor neutron kinetic equations given in Section 2. However, the differences are small and may be neglected for the purposes of this investigation. The effectiveness terms γ_i and $\bar{\gamma}$ are perhaps within ½% of unity for the UHTREX reactor, and may be taken as unity. Also, for operation at substantial powers the source term $s(t)$ can be neglected. Thus,

$$\frac{dn(t)}{dt} = \frac{k(t)(1 - \beta) - 1}{\Lambda} n(t) + \sum \lambda_i C_i(t) \quad (\text{E.2-1c})$$

Now make the change of variables

$$\ell^* = \frac{\Lambda}{k(t)} \quad (\text{E.2-1d})$$

and

$$\rho = 1 - \frac{1}{k(t)} \quad (\text{E.2-1e})$$

to give

$$\frac{dn(t)}{dt} = \frac{(\rho - \beta)}{\ell^*} n(t) + \sum \lambda_i C_i(t) \quad (\text{E.2-1f})$$

and

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\ell^*} n(t) - \lambda_i C_i(t) \quad (\text{E.2-1g})$$

These equations are now identical in form to Eqs. 2.3-1 to 2.3-7 of Section 2.3. The error made in taking ℓ^* as a constant is negligible for reactivities less than a few dollars. For instance, suppose $k(t) = 1.04$. Then

$$\rho = 1 - \frac{1}{1.04} = 1 - 0.961 = 0.039 \text{ absolute reactivity}$$

and $\frac{\rho}{\beta} = \frac{0.039}{0.0065} = \6 reactivity. But this \$6 reactivity gives an error of less than 4% in taking ℓ^* as a constant rather than recalculating it as $\frac{\Lambda}{k(t)}$; and for most reactor control system studies $\frac{\rho}{\beta}$ is considerably less than \$1.

Proceeding with the main derivation, Keepin integrates Eq. E.2-1b to obtain

$$C_i(t) = \frac{\beta_i}{\Lambda} \int_{-\infty}^t e^{-\lambda_i(t-t')} k(t') n(t') dt' \quad (\text{E.2-1h})$$

The validity of this integration can be checked by differentiating Eq. E.2-1h. Following Courant,² if one wishes to differentiate

$$F(x) = \int_{\psi_1(x)}^{\psi_2(x)} f(x,y) dy, \quad (\text{E.2-1i})$$

start with the expression

$$F(s) = \int_{\mu}^v f(x,y) dy = \phi(\mu, v, x). \quad (\text{E.2-1j})$$

Now use the chain rule to obtain

$$F'(s) = \frac{\partial \Phi}{\partial x} + \frac{\partial \Phi}{\partial \mu} \frac{d\mu}{dx} + \frac{\partial \Phi}{\partial v} \frac{dv}{dx} \quad (\text{E.2-1k})$$

From the fundamental Theorem of Calculus, the indefinite integral

$$\Phi(x) = \int_a^x f(\mu) d\mu \quad (\text{E.2-11})$$

of a continuous function $f(x)$ always possesses a derivative $\Phi'(x)$ given

$$\text{by} \quad \Phi'(x) = f(x) \quad (\text{E.2-1m})$$

Application of this theorem to Eqs. E.2-1j and E.2-1k yields

$$F'(x) = \int_{\Psi_1(x)}^{\Psi_2(x)} f_x(x, y) dy - \Psi_1'(x) f(x, \Psi_1(x)) + \Psi_2'(x) f(x, \Psi_2(x)) \quad (\text{E.2-1n})$$

Now Eq. E.2-1n can be used to obtain the derivative of $C_i(t)$ in Eq.

E.2-1h.

$$\begin{aligned} \frac{d}{dt} C_i(t) = \frac{\beta_i}{\Lambda} \left[\int_{-\infty}^t -\lambda_i e^{-\lambda_i(t-t')} k(t') n(t') dt' \right. \\ \left. - 0 + (1) e^{-\lambda_i(t-t)} k(t) n(t) \right] \end{aligned} \quad (\text{E.2-1o})$$

$$\frac{d}{dt} C_i(t) = \frac{\beta_i}{\Lambda} k(t) n(t) - \lambda_i C_i(t) \quad (\text{E.2-1p})$$

Equations E.2-1p and E.2-1b are identical, so the integration performed to obtain Eq. E.2-1h is indeed correct.

Substitution of Eq. E.2-1h into Eq. E.2-1a gives

$$\begin{aligned} \frac{dn(t)}{dt} = \frac{k(t) (1 - \bar{\gamma}\beta) - 1}{\Lambda} n(t) \\ + \int_{-\infty}^t \sum_{i=1}^6 \frac{\lambda_i \gamma_i \beta_i}{\Lambda} e^{-\lambda_i(t-t')} k(t') n(t') dt + \gamma_s S(t) \end{aligned} \quad (\text{E.2-2})$$

In principle this last equation could be solved numerically for $n(t)$ if $k(t)$ and the initial conditions on n , k , S , and C_i were given. However, the appearance of Λ in Eq. E.2-2 limits the maximum time incre-

ment that can be used in the computation, especially for fast systems with a Λ as small as 10^{-8} . In what follows Keepin and Cox manipulate

Eq. E.2-2 to obtain an expression in which Λ no longer appears explicitly, First, define

$$\delta k(t) = k(t) - 1 \quad (\text{E.2-2a})$$

$$n(o) = n(t) \big|_{t=0} \quad (\text{E.2-2b})$$

$$C_i(o) = C_i(t) \big|_{t=0} \quad (\text{E.2-2c})$$

These definitions can be combined with Eq. E.2-2 to give

$$\begin{aligned} \frac{1 - \bar{\gamma}\beta}{\Lambda} \delta k(t) n(t) &= \frac{dn(t)}{dt} + \frac{\bar{\gamma}\beta}{\Lambda} n(t) \\ &- \sum_{i=1}^6 \frac{\gamma_i \beta_i \lambda_i}{\Lambda} \int_0^t e^{-\lambda_i(t-t')} n(t') dt' \\ &- \sum_{i=1}^6 \frac{\gamma_i \beta_i \lambda_i}{\Lambda} \int_0^t e^{-\lambda_i(t-t')} \delta k(t') n(t') dt' \\ &- \sum \lambda_i \gamma_i C_i(o) e^{-\lambda_i t} - \gamma_s S(t) \quad (\text{E.2-2d}) \end{aligned}$$

The fifth term, T5, on the right side of Eq. E.2-2d can be obtained as follows

$$T5 = - \int_{-\infty}^t \sum_{i=1}^6 \frac{\lambda_i \gamma_i \beta_i}{\Lambda} e^{-\lambda_i(t-t')} k(t') n(t') dt' \big|_{t_o=0} \quad (\text{E.2-2e})$$

$$T5 = - \sum_{i=1}^6 \lambda_i \gamma_i e^{-\lambda_i t} \left[\frac{\beta_i}{\Lambda} \int_{-\infty}^0 e^{-\lambda_i(o-t')} k(t') n(t') dt' \right] \quad (\text{E.2-2f})$$

$$T5 = - \sum_{i=1}^6 \lambda_i \gamma_i e^{-\lambda_i t} C_i(o) \quad (\text{E.2-2g})$$

Next carry out the integration shown in the third term T3 on the right side of Eq. E.2-2d to give

$$T_3 = - \sum_{i=1}^6 \frac{\lambda_i \beta_i \lambda_i}{\Lambda} \left[n(t') \frac{e^{-\lambda_i(t-t')}}{\lambda_i} \Big|_{t'=0}^{t'=t} - \int_0^t \frac{e^{-\lambda_i(t-t')}}{\lambda_i} \frac{dn(t')}{dt'} dt' \right] \quad (E.2-2h)$$

$$T_3 = - \sum_{i=1}^6 \frac{\lambda_i \beta_i \lambda_i}{\Lambda} \left[\frac{n(t)}{\lambda_i} - \frac{n(o)}{\lambda_i} e^{-\lambda_i t} - \frac{1}{\lambda_i} \int_0^t e^{-\lambda_i(t-t')} \frac{dn(t')}{dt'} dt' \right] \quad (E.2-2i)$$

$$T_3 = - \frac{\bar{\lambda}\beta}{\Lambda} n(t) + \sum_{i=1}^6 \frac{\lambda_i \beta_i}{\Lambda} \left[n(o) e^{-\lambda_i t} + \int_0^t e^{-\lambda_i(t-t')} \frac{dn(t')}{dt'} dt' \right] \quad (E.2-2j)$$

Substitution of this expression into Eq. E.2-2d and combination of terms yields

$$\begin{aligned} \frac{1 - \bar{\gamma}\beta}{\Lambda} \delta k(t) n(t) &= \frac{dn(t)}{dt} + \sum_{i=1}^6 \frac{\gamma_i \beta_i}{\Lambda} \int_0^t e^{-\lambda_i(t-t')} \frac{dn(t')}{dt'} dt \\ &\quad - \frac{\gamma_i \beta_i \lambda_i}{\Lambda} \int_0^t e^{-\lambda_i(t-t')} \delta k(t') n(t') dt' \\ &\quad + \sum_{i=1}^6 \gamma_i e^{-\lambda_i t} \left[\frac{\beta_i}{\Lambda} n(o) - \lambda_i C_i(o) \right] - \lambda_s S(t) \end{aligned} \quad (E.2-3)$$

The next step is to take the Laplace transform of Eq. E.2-3. One can make use of the identity

$$\mathcal{L} \left[\int_0^t f(t-\tau) g(\tau) d\tau \right] = \mathcal{L}[f(t)] \mathcal{L}[g(t)] \quad (E.2-3a)$$

to find the transform of the integral terms in Eq. E.2-3. The first integral term becomes

$$\begin{aligned}
& \mathcal{L} \left\{ \sum_{i=1}^6 \frac{\gamma_i \beta_i}{\Lambda} \int_0^t \left[e^{-\lambda_i(t-t')} \right] \left[\frac{dn(t')}{dt'} \right] dt \right\} \\
&= \mathcal{L} \left[\sum_{i=1}^6 \frac{\gamma_i \beta_i}{\Lambda} e^{-\lambda_i t} \right] \mathcal{L} \left[\frac{dn(t)}{dt} \right] \quad (E.2-3b)
\end{aligned}$$

$$= \sum_{i=1}^6 \frac{\lambda_i \beta_i}{\Lambda(s + \lambda_i)} \left\{ s \mathcal{L} [n(t)] - n(o) \right\} \quad (E.2-3c)$$

Similarly,

$$\begin{aligned}
& \mathcal{L} \left[- \sum_{i=1}^6 \frac{\gamma_i \beta_i \lambda_i}{\Lambda} \int_0^t e^{-\lambda_i(t-t')} \delta k(t') n(t') dt' \right] \\
&= \mathcal{L} \left[- \sum_{i=1}^6 \frac{\gamma_i \beta_i \lambda_i}{\Lambda} e^{-\lambda_i(t)} \right] \mathcal{L} [\delta k(t) n(t)] \quad (E.2-3d)
\end{aligned}$$

$$= \left[- \sum \frac{\gamma_i \beta_i \lambda_i}{\Lambda} \frac{1}{s + \lambda_i} \right] \mathcal{L} [\delta k(t) n(t)] \quad (E.2-3e)$$

Thus, Eq. E.2-3 can be rewritten as

$$\begin{aligned}
& \left(1 + \sum \frac{\gamma_i \beta_i}{\Lambda} \frac{1}{s + \lambda_i} \right) \left\{ s \mathcal{L} [n(t)] - n(o) \right\} \\
&= \left(\frac{1 - \bar{\gamma} \beta}{\Lambda} + \sum \frac{\gamma_i \beta_i \lambda_i}{\Lambda} \frac{1}{s + \lambda_i} \right) \mathcal{L} [\delta k(t) n(t)] \\
&\quad - \sum \gamma_i \left(\frac{\beta_i}{\Lambda} n(o) - \lambda_i C_2(o) \right) \frac{1}{s + \lambda_i} + \mathcal{L} [\gamma_s S(t)] \quad (E.2-4)
\end{aligned}$$

One can now take the inverse transform of Eq. E.2-4 and solve for $n(t)$ to obtain

$$\begin{aligned}
n(t) = n(o) + \frac{1 - \bar{\gamma}\beta}{\Lambda} \int_0^t \left\{ \mathcal{L}^{-1} [G(s)] \right\}_{t=t-t'} \delta k(t') n(t') dt' \\
+ \int_0^t \left\{ \mathcal{L}^{-1} [H(s)] \right\}_{t=t-t'} \delta k(t') n(t') dt' \\
+ \int_0^t \left\{ \mathcal{L}^{-1} [G(s)] \right\}_{t=t-t'} \left\{ \gamma_s S(t') - \sum \gamma_i e^{-\lambda_i t} \left[\frac{\beta_i}{\Lambda} n(o) - \lambda_i C_i(o) \right] \right\} dt'
\end{aligned} \tag{E.2-5}$$

where

$$G(s) = \frac{1}{s \left(1 + \sum_{i=1}^6 \frac{\gamma_i \beta_i}{\Lambda} \frac{1}{s + \lambda_i} \right)} = \sum_{j=0}^6 \frac{B_j}{s - S_j} \tag{E.2-6}$$

and

$$H(s) = G(s) \sum_{i=1}^6 \frac{\gamma_i \beta_i \lambda_i}{\Lambda} \frac{1}{s + \lambda_i} = \sum_{j=0}^6 \frac{R_j}{s - S_j} \tag{E.2-6a}$$

The constants B_j , S_j , and R_j are calculated using appropriate values of γ_i , β_i , λ_i , and Λ . Evaluation of these constants allows one to express the inverse transforms of $G(s)$ and $H(s)$.

$$\mathcal{L}^{-1} [G(s)]_{t=t-t'} = \sum_{j=0}^6 B_j e^{S_j(t-t')} \tag{E.2-6b}$$

$$\mathcal{L}^{-1} [H(s)]_{t=t-t'} = \sum_{j=0}^6 R_j e^{S_j(t-t')} \tag{E.2-6c}$$

Now define A_j as shown below.

$$A_j = \left(\frac{1 - \bar{\gamma}\beta}{\Lambda} \right) B_j + R_j \tag{E.2-6d}$$

Equation E.2-5 can now be rewritten as

$$n(t) = n(o) + \sum_{j=0}^6 A_j \int_0^t e^{S_j(t-t')} \delta k(t') n(t') dt' + \Omega_o(t) \tag{E.2-7}$$

where $\Omega_o(t)$ is defined as

$$\Omega_0(t) = \int_0^t \left[\sum_{j=0}^6 B_j e^{S_j(t-t')} \right] \left\{ \gamma_s S(t') + \sum_{i=1}^6 \left[\gamma_i C_i(o) - \frac{\beta_i}{\Lambda} n(o) \right] \gamma_i e^{-\gamma_i t'} \right\} dt' \quad (E.2-7a)$$

Keepin and Cox obtain a numerical solution for the neutron density, n_m , in the m th time interval by use of the following expression

$$n_m \approx \frac{n(o) + \sum_{j=0}^6 A_j e^{S_j} \sum_{\ell=0}^{m-1} e^{-S_j \ell h} \delta k_{\ell} n_{\ell} h + \Omega_{om}}{1 - \sum_{j=0}^6 A_j \delta k_m h} \quad (E.2-7b)$$

Here h , the integration time interval is restricted as shown in Eq. E.2-7c.

$$h < \frac{1}{\sum_{j=0}^6 A_j \delta k_m} \approx \frac{\Lambda}{\delta k_m} \quad (E.2-7c)$$

Their 704 program takes about 10 minutes to compute $n(t)$ for a 6-second run with $\Lambda = 10^{-4} \text{ sec}^{-1}$. Thus the ratio of 704 time to simulation time is approximately 100 to 1 for this case, in which they apparently used an h of

$$h = \frac{\Lambda}{\beta} \approx 0.02 \text{ seconds.}$$

A similar 7094 program would require considerably less computing time.

E.3 Application of Collocation Method³

For the purposes of this study it is satisfactory to assume that the reactor kinetics are at steady-state initially and that the source term is zero. Thus, Ω_0 of Eq. E.2-7 is zero. Eq. E.2-7 can be rewritten as

$$n(t_o) = n(o) + \sum_{j=0}^6 A_j \int_0^{t_o} e^{S_j(t_o-t')} \delta k(t') n(t') dt' \quad (E.3-1)$$

Similarly,

$$n(t_0 + \tau) = n(t_0) + \sum_{j=0}^6 A_j \int_0^{t_0 + \tau} e^{S_j(t_0 + \tau - t')} \delta k(t') n(t') dt' \quad (E.3-2)$$

These two equations can be combined to give

$$\begin{aligned} n(t_0 + \tau) = n(t_0) &+ \sum_{j=0}^6 A_j (e^{S_j \tau} - 1) \int_0^{t_0} e^{S_j(t_0 - t')} \delta k(t') n(t') dt' \\ &+ \sum_{j=0}^6 A_j \int_{t_0}^{t_0 + \tau} e^{S_j(t_0 + \tau - t')} \delta k(t') n(t') dt' \quad (E.3-3) \end{aligned}$$

or

$$\begin{aligned} n(t_0 + \tau) = n(t_0) &+ F(t_0, \tau) \\ &+ \sum_{j=0}^6 A_j \int_0^{\tau} e^{S_j(\tau - s)} \delta k(s + t_0) n(s + t_0) ds \quad (E.3-4) \end{aligned}$$

where $F(t_0, \tau)$ is a function of n for $t < t_0$ and is known.

It is assumed that the time variation of δk and n could be matched by a quadratic function in the time interval h . Thus,

$$\delta k(t_0 + \tau) = k_1 + k_2(\tau/h) + k_3(\tau/h)^2 \quad 0 < \tau \leq h \quad (E.3-5)$$

$$n(t_0 + t) = n_1 + n_2(\tau/h) + n_3(\tau/h)^2 \quad 0 < \tau \leq h \quad (E.3-6)$$

The function δk is known over the interval h , and the coefficients k_2 and k_3 can be evaluated so that the fit is accurate at, say, $h/2$ and h . The problem is to evaluate n_2 and n_3 . The two equations above may be combined with Eq. E.3-4 to give

$$n_2(\tau/h) + n_3(\tau/h)^2 = F(t_0, \tau)$$

$$\begin{aligned} &+ \sum_{j=0}^6 A_j e^{S_j \tau} \int_0^{\tau} e^{-S_j s} [k_1 + k_2(s/h) + k_3(s/h)^2] [n_1 + n_2(s/h) + n_3(s/h)^2] ds \\ &\quad (E.3-7) \end{aligned}$$

Eq. E.3-7 can be rewritten as

$$n_2(\tau/h) + n_3(\tau/h)^2 = F(t_0, \tau) + \sum_{i=1}^3 n_i \sum_{\ell=1}^3 k_{\ell} \sum_{j=0}^6 A_j e^{S_j \tau} \int_0^{\tau} e^{-S_j \xi} (\xi/h)^{i+\ell-2} d\xi \quad (E.3-8)$$

The coefficients n_2 and n_3 can be evaluated from this equation by choosing $\tau = h/2$ and $\tau = h$. The integrals on the right side can be evaluated by recursion formulae. The integrals and the summation on j need be calculated only once in any problem.

When n_2 and n_3 are determined $F(t_0, \tau)$ can be computed for the next time step. It is possible to do the integrations in $F(t_0, \tau)$ in the same manner as in Eq. E.3-8, accumulating the integral in steps of h rather than integrating to t_0 each time.

A variety of problems have been run to verify the accuracy of the collocation method. Excellent results have been obtained for $\delta k < \$1$, $\rho^* = 10^{-4}$ and time steps as large as 2 seconds. Positive reactivities of over $\$1$ require fairly small time steps and a double precision routine for good accuracy. Such a routine was not required in the problems studied thus far, for the positive reactivities involved are considerably less than $\$1$.

The collocation program requires relatively little calculation time at each cycle, and allows the use of large time steps. In all, the collocation method has proved to be an excellent way of handling the reactor kinetics equations for the purposes of this report.

REFERENCES

1. G. R. Keepin and C. W. Cox, "General Solution of the Reactor Kinetic Equations," Nuclear Science and Engineering, Vol. 8, No. 6, December 1960, p. 670. (See integral equation for $n(t)$, Eq. 7 of this article.)
2. R. Courant, "Differential and Integral Calculus," Vol. II, Interscience, New York, (1950), p. 220.
3. This work was done by K. Lathrop of the Los Alamos Scientific Laboratory.

APPENDIX F

DERIVATION OF THE TRANSFER FUNCTIONS OF A DISTRIBUTED PARAMETER HEAT EXCHANGER

The four transfer functions between inlet and outlet gas temperatures of a counterflow heat exchanger are derived in this appendix.

The effect of the transit time delay is included.

The model considered and the associated assumptions are shown in Fig. F-1.

The basic equations for the model are given below.

$$\text{MC } \frac{dT_w(x,t)}{dt} = A_h [T_h(x,t) - T_w(x,t)] - A_c [T_w(x,t) - T_c(x,t)] \quad (\text{F-1})$$

$$q_h(x,t) = A_h [T_h(x,t) - T_w(x,t)] \quad (\text{F-2})$$

$$q_c(x,t) = A_c [T_w(x,t) - T_c(x,t)] \quad (\text{F-3})$$

$$V_h H_h(o,t) - V_h H_h\left(\ell, t - \frac{\ell}{V_h}\right) = \int_{\ell}^0 q_h\left(x, t - \frac{x}{V_h}\right) dx \quad (\text{F-4})$$

$$V_c H_c(\ell, t) - V_c H_c\left(o, t - \frac{\ell}{V_c}\right) = \int_0^{\ell} q_c\left(x, t - \frac{\ell}{V_c} + \frac{x}{V_c}\right) dx \quad (\text{F-5})$$

$$H_h(x,t) = \rho_h C_{ph} A_1 T_h(x,t) = \frac{\dot{W}_h C_{ph}}{V_h} T_h(x,t) \quad (\text{F-6})$$

$$H_c(x,t) = \rho_c C_{pc} A_2 T_c(x,t) = \frac{\dot{W}_c C_{pc}}{V_c} T_c(x,t) \quad (\text{F-7})$$

ASSUMPTIONS

1. Temperatures of gas and solid constant with respect to y coordinate
2. Effects of conduction in solid negligible
3. Gas flow rates constant
4. Constant specific heats
5. Constant gas densities
6. Heat transfer coefficients constant

Fig. F-1 Model and Assumptions

In these equations,

M = weight of solid per foot of length, (lb/ft)

$\dot{W}_h$ = gas flow rate, hot side, (lb/sec)

$\dot{W}_c$ = gas flow rate, cold side, (lb/sec)

C = specific heat of solid, (Btu/lb °R)

C_{ph} = specific heat of hot gas, (Btu/lb °R)

C_{pc} = specific heat of cold gas, (Btu/lb °R)

T_w = solid temperature

T_h = hot gas temperature

T_c = cold gas temperature

A_h = hot side heat transfer coefficient per foot of length,
(Btu/sec ft °R)

A_c = cold side heat transfer coefficient per foot of length,
(Btu/sec ft °R)

V_h = velocity of hot gas, (ft/sec)

V_c = velocity of cold gas, (ft/sec)

A_1 = cross sectional area of hot gas passage, (ft²)

A_2 = cross sectional area of cold gas passage, (ft²)

ρ_h = density of hot gas, (lb/ft³)

ρ_c = density of cold gas, (lb/ft³)

q_h = heat given up by hot gas per foot of length, (Btu/sec ft)

q_c = heat taken up by cold gas per foot of length, (Btu/sec ft)

H_h = heat content of hot gas per foot of length, (Btu/ft)

H_c = heat content of cold gas per foot of length, (Btu/ft)

The derivation of the transfer functions between inlet and outlet gas temperatures proceeds as follows: First take the Laplace transform of Eqs. F-1 through F-7, dropping the initial conditions. From Eq. F-1

$$MC S T_w(x,s) = A_h T_h(x,s) + A_c T_c(x,s) - (A_h + A_c) T_w(x,s) \quad (F-8a)$$

Let

$$\tau = \frac{MC}{A_h + A_c} \quad (F-8b)$$

Then

$$T_w(x,s) = \frac{\frac{A_h}{A_h + A_c}}{\tau s + 1} T_h(x,s) + \frac{\frac{A_c}{A_h + A_c}}{\tau s + 1} T_c(x,s) \quad (F-9)$$

From Eq. F-2

$$q_h(x,s) = A_h [T_h(x,s) - T_w(x,s)] \quad (F-10)$$

From Eq. F-3

$$q_c(x,s) = A_c [T_w(x,s) - T_c(x,s)] \quad (F-11)$$

Combine Eqs. F-4 and F-6 to give

$$\begin{aligned} \dot{W}_h C_{ph} T_h(0,s) - \dot{W}_h C_{ph} T_h(\ell,s) e^{-\ell/V_h s} \\ = \int_0^\ell q_h(x,s) e^{-xs/V_h} dx = - \int_0^\ell q_h(x,s) e^{-xs/V_h} dx \end{aligned} \quad (F-12)$$

Combine Eqs. F-5 and F-7 to give

$$\begin{aligned} \dot{W}_c C_{pc} T_c(\ell,s) - \dot{W}_c C_{cp} T_c(0,s) e^{-\ell/V_c s} \\ = \int_0^\ell q_c(x,s) e^{-\ell s/V_h} e^{x/V_c s} dx = e^{-\ell/V_c s} \int_0^\ell q_c(x,s) e^{xs/V_h} dx \end{aligned} \quad (F-13)$$

Substitute Eq. F-9 into Eq. F-10.

$$q_h(x,s) = A_h T_h(x,s) - \frac{\frac{A_h^2}{A_h + A_c}}{\tau s + 1} T_h(x,s) - \frac{\frac{A_h A_c}{A_h + A_c}}{\tau s + 1} T_c(x,s) \quad (F-14)$$

or

$$q_h(x,s) = \frac{A_h}{\tau s + 1} \left[\left(\tau s + 1 - \frac{A_h}{A_c + A_h} \right) T_h(x,s) - \frac{A_c}{A_h + A_c} T_c(x,s) \right] \quad (F-15)$$

Substitute Eq. F-9 into Eq. F-11.

$$q_c(x,s) = A_c \left[\frac{\frac{A_h}{A_h + A_c}}{\tau s + 1} T_h(x,s) + \frac{\frac{A_c}{A_h + A_c}}{\tau s + 1} T_c(x,s) - T_c(x,s) \right] \quad (F-16)$$

$$\text{or } q_c(x,s) = \frac{A_c}{\tau s + 1} \left[\frac{A_h}{A_h + A_c} T_h(x,s) + \left(\frac{A_c}{A_h + A_c} - \tau s - 1 \right) T_c(x,s) \right] \quad (\text{F-17})$$

$$\text{or } q_c(x,s) = \frac{A_c}{\tau s + 1} \left[\frac{A_h}{A_h + A_c} T_h(x,s) - \left(\tau s + 1 - \frac{A_c}{A_h + A_c} \right) T_c(x,s) \right] \quad (\text{F-18})$$

Now substitute Eq. F-15 into Eq. F-12.

$$\begin{aligned} \dot{W}_h C_{ph} T_h(o,s) - \dot{W}_h C_{ph} T_h(\ell,s) e^{-\ell s/V_h} \\ = - \int_0^\ell \frac{A_h}{\tau s + 1} \left[\left(\tau s + 1 - \frac{A_h}{A_h + A_c} \right) T_h(x,s) - \frac{A_c}{A_h + A_c} T_c(x,s) \right] e^{-x/V_h} dx \end{aligned} \quad (\text{F-19})$$

Substitute Eq. F-18 into Eq. F-13.

$$\begin{aligned} \dot{W}_c C_{pc} T_c(\ell,s) - \dot{W}_c C_{pc} T_c(o,s) e^{-\ell s/V_c} \\ = e^{-\ell s/V_c} \int_0^\ell \frac{A_c}{\tau s + 1} \left[\frac{A_h}{A_h + A_c} T_h(x,s) - \left(\tau s + 1 - \frac{A_c}{A_h + A_c} \right) T_c(x,s) \right] e^{xs/V_c} dx \end{aligned} \quad (\text{F-20})$$

Divide Eq. F-19 by $\dot{W}_h C_{ph}$.

$$\begin{aligned} T_h(o,s) - T_h(\ell,s) e^{-\ell s/V_h} \\ = - \frac{\frac{A_h}{\dot{W}_h C_{ph}}}{\tau s + 1} \int_0^\ell \left[\left(\tau s + 1 - \frac{A_h}{A_h + A_c} \right) T_h(x,s) - \frac{A_c}{A_h + A_c} T_c(x,s) \right] e^{-xs/V_h} dx \end{aligned} \quad (\text{F-21})$$

Divide Eq. F-20 by $\dot{W}_c C_{pc} e^{-\ell s/V_c}$

$$\begin{aligned} T_c(\ell,s) e^{\ell s/V_c} - T_c(o,s) \\ = \frac{\frac{A_c}{\dot{W}_c C_{pc}}}{\tau s + 1} \int_0^\ell \left[\frac{A_h}{A_h + A_c} T_h(x,s) - \left(\tau s + 1 - \frac{A_c}{A_h + A_c} \right) T_c(x,s) \right] e^{xs/V_c} dx \end{aligned} \quad (\text{F-22})$$

Differentiate Eq. F-21 with respect to ℓ .

$$\begin{aligned}
 & \frac{s}{V_h} T_h(\ell, s) e^{-\ell s/V_h} - \frac{d T_h(\ell, s)}{d\ell} e^{-\ell s/V_h} \\
 &= - \frac{\frac{A_h}{\dot{W}_h C_{ph}}}{\tau s + 1} \left[\left(\tau s + 1 - \frac{A_h}{A_h + A_c} \right) T_h(\ell, s) - \frac{A_c}{A_h + A_c} T_c(\ell, s) \right] e^{-\ell s/V_h}
 \end{aligned} \tag{F-23}$$

Differentiate Eq. F-22 with respect to ℓ

$$\begin{aligned}
 & \frac{s}{V_c} T_c(\ell, s) e^{\ell s/V_c} + \frac{d T_c(\ell, s)}{d\ell} e^{\ell s/V_c} \\
 &= \frac{\frac{A_c}{\dot{W}_c C_{pc}}}{\tau s + 1} \left[\frac{A_h}{A_h + A_c} T_h(\ell, s) - \left(\tau s + 1 - \frac{A_c}{A_h + A_c} \right) T_c(\ell, s) \right] e^{\ell s/V_c}
 \end{aligned} \tag{F-24}$$

From Eq. F-23

$$\begin{aligned}
 & \frac{d T_h(\ell, s)}{d\ell} - \left[\frac{s}{V_h} + \frac{A_h \left(\tau s + 1 - \frac{A_h}{A_h + A_c} \right)}{\dot{W}_h C_{ph} (\tau s + 1)} \right] T_h(\ell, s) \\
 &= - \frac{A_h A_c}{\dot{W}_h C_{ph} (A_h + A_c) (\tau s + 1)} T_c(\ell, s)
 \end{aligned} \tag{F-25}$$

From Eq. F-24

$$\begin{aligned}
 & \frac{d T_c(\ell, s)}{d\ell} + \left[\frac{s}{V_c} + \frac{A_c \left(\tau s + 1 - \frac{A_c}{A_h + A_c} \right)}{\dot{W}_c C_{pc} (\tau s + 1)} \right] T_c(\ell, s) \\
 &= \frac{A_c A_h}{\dot{W}_c C_{pc} (A_h + A_c) (\tau s + 1)} T_h(\ell, s)
 \end{aligned} \tag{F-26a}$$

Let

$$C_1(s) = -\frac{s}{V_h} - \frac{A_h \left(\tau s + 1 - \frac{A_h}{A_h + A_c} \right)}{\dot{W}_h C_p (\tau s + 1)} \quad (\text{F-26b})$$

$$C_2(s) = -\frac{A_h A_c}{\dot{W}_h C_{ph} (\tau s + 1) (A_h + A_c)} \quad (\text{F-26c})$$

$$C_3(s) = \frac{s}{V_c} + \frac{A_c \left(\tau s + 1 - \frac{A_c}{A_h + A_c} \right)}{\dot{W}_c C_{pc} (\tau s + 1)} \quad (\text{F-26d})$$

$$C_4(s) = \frac{A_h A_c}{\dot{W}_c C_{pc} (A_h + A_c) (\tau s + 1)} \quad (\text{F-26e})$$

Then Eq. F-25 becomes

$$\frac{dT_h(\ell, s)}{d\ell} + C_1(s) T_h(\ell, s) = C_2(s) T_c(\ell, s) \quad (\text{F-27})$$

and Eq. F-26 becomes

$$\frac{dT_c(\ell, s)}{d\ell} + C_3(s) T_c(\ell, s) = C_4(s) T_h(\ell, s) \quad (\text{F-28})$$

Now solve for $T_c(\ell, s) = f(T_{ci}, T_{co})$

and $T_h(\ell, s) = f(t_{hi}, T_{ho})$

where

T_{ci} = cold gas inlet temperature
 T_{co} = cold gas outlet temperature
 T_{hi} = hot gas inlet temperature
 T_{ho} = hot gas outlet temperature

Define $F(p, s) = \int_0^\infty f(\ell, s) e^{-\ell p} d\ell$ and transform Eqs. F-27 and F-28.

$$p T_h(p,s) - T_h(o^+,s) + C_1(s) T_h(p,s) = C_2(s) T_c(p,s) \quad (F-29)$$

$$p T_c(p,s) - T_c(o^+,s) + C_3(s) T_c(p,s) = C_4(s) T_h(p,s) \quad (F-30)$$

Collecting terms, one obtains

$$(p + C_1(s)) T_h(p,s) - C_2(s) T_c(p,s) = T_h(o^+,s) \quad (F-31)$$

and

$$-C_4(s) T_h(p,s) + (p + C_3(s)) T_c(p,s) = T_c(o^+,s) \quad (F-32)$$

It follows that

$$T_h(p,s) = \frac{\begin{vmatrix} T_h(o^+,s) & -C_2(s) \\ T_c(o^+,s) & p + C_3(s) \end{vmatrix}}{\begin{vmatrix} p + C_1(s) & -C_2(s) \\ -C_4(s) & p + C_3(s) \end{vmatrix}} \quad (F-33)$$

and

$$T_c(p,s) = \frac{\begin{vmatrix} p + C_1(s) & T_h(o^+,s) \\ -C_4(s) & T_c(o^+,s) \end{vmatrix}}{\begin{vmatrix} p + C_1(s) & -C_2(s) \\ -C_4(s) & p + C_3(s) \end{vmatrix}} \quad (F-34)$$

$$\begin{aligned} \text{Let } D &= \begin{vmatrix} p + C_1(s) & -C_2(s) \\ -C_4(s) & p + C_3(s) \end{vmatrix} = p^2 + (C_1(s) + C_3(s))p \\ &\quad + C_1(s) C_3(s) - C_2(s) C_4(s) \\ &= (p - \gamma_1(s))(p - \gamma_2(s)) \end{aligned} \quad (F-35)$$

Expanding Eqs. F-33 and F-34 and using Eq. F-35,

$$T_h(p,s) = \frac{(p + C_3(s)) T_h(o^+,s)}{(p - y_1(s))(p - y_2(s))} + \frac{C_2(s) T_c(o^+,s)}{(p - y_1(s))(p - y_2(s))} \quad (F-36)$$

and

$$T_c(p,s) = \frac{(p + C_1(s)) T_c(o^+,s)}{(p - y_1(s))(p - y_2(s))} + \frac{C_4(s) T_h(o^+,s)}{(p - y_1(s))(p - y_2(s))} \quad (F-37)$$

By use of partial fractions, Eqs. F-36 and F-37 become

$$T_h(p,s) = \left[\frac{y_1(s) + C_3(s)}{p - y_1(s)} - \frac{y_2(s) + C_3(s)}{p - y_2(s)} \right] \frac{T_{ho}(o^+,s)}{y_1(s) - y_2(s)} + \left[\frac{C_2(s)}{p - y_1(s)} - \frac{C_2(s)}{p - y_2(s)} \right] \frac{T_{co}(o^+,s)}{y_1(s) - y_2(s)} \quad (F-38)$$

and

$$T_c(p,s) = \left[\frac{y_1(s) + C_1(s)}{p - y_1(s)} - \frac{y_2(s) + C_1(s)}{p - y_2(s)} \right] \frac{T_{co}(o^+,s)}{y_1(s) - y_2(s)} + \left[\frac{C_4(s)}{p - y_1(s)} - \frac{C_4(s)}{p - y_2(s)} \right] \frac{T_h(o^+,s)}{y_1(s) - y_2(s)} \quad (F-39)$$

Taking the inverse transform of Eqs. F-38 and F-39 and noting that $T_h(o^+,s) = T_{ho}(s)$ and $T_c(o^+,s) = T_{ci}(s)$,

$$T_h(\ell,s) = \frac{(y_1(s) + C_3(s))e^{y_1(s)\ell} - (y_2(s) + C_3(s))e^{y_2(s)\ell}}{y_1(s) - y_2(s)} T_{ho}(s) + \frac{C_2(s)(e^{y_1(s)\ell} - e^{y_2(s)\ell})}{y_1(s) - y_2(s)} T_{ci}(s) \quad (F-40)$$

and

$$T_c(\ell,s) = \frac{(y_1(s) + C_1(s))e^{y_1(s)\ell} - (y_2(s) + C_1(s))e^{y_2(s)\ell}}{y_1(s) - y_2(s)} T_{ci}(s) + \frac{C_4(s)(e^{y_1(s)\ell} - e^{y_2(s)\ell})}{y_1(s) - y_2(s)} T_{ho}(s) \quad (F-41)$$

Let $\ell \Rightarrow L$. Then $T_h(\ell,s) \Rightarrow T_h(L,s) = T_{hi}(s)$
 $T_c(\ell,s) \Rightarrow T_c(L,s) = T_{co}(s)$

Then Eqs. F-40 and F-41 become

$$T_{hi}(s) = \frac{(y_1(s) + C_3(s))e^{y_1(s)L} - (y_2(s) + C_3(s))e^{y_2(s)L}}{y_1(s) - y_2(s)} T_{ho}(s) + \frac{C_2(s)(e^{y_1(s)L} - e^{y_2(s)L})}{y_1(s) - y_2(s)} T_{ci}(s) \quad (F-42)$$

and

$$T_{c2}(s) = \frac{(y_1(s) + C_1(s))e^{y_1(s)L} - (y_2(s) + C_1(s))e^{y_2(s)L}}{y_1(s) - y_2(s)} T_{ci}(s) + \frac{C_4(s)(e^{y_1(s)L} - e^{y_2(s)L})}{y_1(s) - y_2(s)} T_{ho}(s) \quad (F-43)$$

Solving for $T_{ho}(s)$ from Eq. F-42 gives

$$T_{ho}(s) = \frac{y_1(s) - y_2(s)}{(y_1(s) + C_3(s))e^{y_1(s)L} - (y_2(s) + C_3(s))e^{y_2(s)L}} T_{hi}(s) - \frac{C_2(s)(e^{y_1(s)L} - e^{y_2(s)L})}{(y_1(s) + C_3(s))e^{y_1(s)L} - (y_2(s) + C_3(s))e^{y_2(s)L}} T_{ci}(s) \quad (F-44)$$

Substituting Eq. F-44 into Eq. F-43 gives

$$T_{co}(s) = \left\{ \frac{(y_1(s) + C_1(s))e^{y_1(s)L} - (y_2(s) + C_1(s))e^{y_2(s)L}}{y_1(s) - y_2(s)} - \frac{C_2(s)(e^{y_1(s)L} - e^{y_2(s)L})C_4(s)(e^{y_1(s)L} - e^{y_2(s)L})}{(y_1(s) - y_2(s)) \left[(y_1(s) + C_3(s))e^{y_1(s)L} - (y_2(s) + C_3(s))e^{y_2(s)L} \right]} \right\} T_{ci}(s) + \frac{C_4(s)(e^{y_1(s)L} - e^{y_2(s)L})}{(y_1(s) + C_3(s))e^{y_1(s)L} - (y_2(s) + C_3(s))e^{y_2(s)L}} T_{hi}(s) \quad (F-45)$$

From Eq. F-44

$$\left. \frac{T_{ho}(s)}{T_{hi}(s)} \right|_{T_{ci} = \text{const}} = \frac{y_1(s) - y_2(s)}{(y_1(s) + C_3(s))e^{y_1(s)L} - (y_2(s) + C_3(s))e^{y_2(s)L}} \quad (\text{F-46a})$$

$$\left. \frac{T_{ho}(s)}{T_{ci}(s)} \right|_{T_{hi} = \text{const}} = - \frac{C_2(s)(e^{y_1(s)L} - e^{y_2(s)L})}{(y_1(s) + C_3(s))e^{y_1(s)L} - (y_2(s) + C_3(s))e^{y_2(s)L}} \quad (\text{F-46b})$$

From Eq. F-45

$$\left. \frac{T_{co}(s)}{T_{ci}(s)} \right|_{T_{hi} = \text{const}} = \frac{(y_1(s) + C_1(s))e^{y_1(s)L} - (y_2(s) + C_1(s))e^{y_2(s)L}}{y_1(s) - y_2(s)} - \frac{C_2(s)(e^{y_1(s)L} - e^{y_2(s)L}) C_4(s)(e^{y_1(s)L} - e^{y_2(s)L})}{(y_1(s) - y_2(s)) \left[(y_1(s) + C_3(s))e^{y_1(s)L} - (y_2(s) + C_3(s))e^{y_2(s)L} \right]} \quad (\text{F-47a})$$

$$\left. \frac{T_{co}(s)}{T_{hi}(s)} \right|_{T_{ci} = \text{const}} = \frac{C_4(s)(e^{y_1(s)L} - e^{y_2(s)L})}{(y_1(s) + C_3(s))e^{y_1(s)L} - (y_2(s) + C_3(s))e^{y_2(s)L}} \quad (\text{F-47b})$$

Equations F-46 and F-47 give the desired transfer functions. The heat dump transfer function between hot outlet and hot inlet gas temperatures, corresponding to Eq. F-46a, is plotted in Figs. 2-2 and 2-3 of Sec. 2. These figures show the gain and phase for both the finite and the infinite gas velocity cases.