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**UNIQUE DETERMINATION OF EDDY CURRENT FLOW PATTERNS
FROM MEASUREMENTS OF EXTERNAL FIELDS**

M. J. CHURCHILL

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February 1967

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UNIQUE DETERMINATION OF EDDY CURRENT FLOW PATTERNS FROM MEASUREMENTS OF EXTERNAL FIELDS

M. J. Churchill

INTRODUCTION

The unique determination of eddy current flow patterns in a material by surface field measurements is important in the further development of electromagnetic nondestructive tests. This study establishes the theoretical basis for the transformation of field information to current pattern information.

Well-known eddy current testing techniques now make it possible to detect the presence of an inhomogenous area or defect in a piece of conducting material.⁽¹⁾ This defect is generally sensed by detecting the variation of the impedance of a test coil placed at the surface of the material.

As an interesting parallel, similar methods⁽²⁾ are being employed in determining the nature of the earth's crust. Surface impedances are calculated from measurements of the electromagnetic fields existing at the surface of the earth due to currents flowing within the earth. Plots of these impedances, as a function of position, aid geophysists in the detection of faults and other regions having differing physical properties. Similar techniques may further the development of nondestructive eddy current tests.

SUMMARY AND CONCLUSIONS

This study shows that the eddy current flow patterns within a bounded material are uniquely determined by the electromagnetic fields at the surface. Thus, in theory, it is possible to predict current flow patterns from surface field measurements.

A study of published material in related areas reinforced the theoretical basis for the development of improved electromagnetic nondestructive test techniques.

Two possible methods of calculating field-current pattern distributions were investigated; one involves integration of the field equations, and the other uses a series approximation method. Both are difficult and require the use of a computer for useful geometries.

The complexity of the mathematics seems to rule out applying these ideas to the general area of electromagnetic nondestructive testing at this time. However, investigation of specific problems could greatly increase the basic understanding of the testing problem. From an experimental study of the field patterns from specific types of interesting defects, it may be possible to develop definite relationships between these field and current patterns without extensive theoretical treatment. Then with this information, more general relationships could be studied.

BASIC THEORY

The problem of determining the electromagnetic field within a bounded region has been treated extensively, as has the uniqueness condition. Stratton⁽³⁾ gives a treatment of the problem including the conditions for a unique solution. By applying Poyntings Theorem (See Appendix 1),

$$-\frac{\partial}{\partial t} \int_V \left(\frac{\epsilon E^2}{2} + \frac{\mu H^2}{2} \right) dV = \int_V \frac{1}{\sigma} J^2 dV + \int_S (\vec{E} \times \vec{H}) \cdot \vec{n} dA \quad ,$$

to a bounded region, he shows that the fields within a region are completely determined by the initial fields and the tangential components of E or H at the surface. This holds for regions of arbitrary permeability (μ), permittivity (ϵ), and conductivity (σ).

Now, since by definition the current density $\vec{J}$ is given by $\vec{J} = \sigma \vec{E}$, the current patterns associated with the fields should be uniquely determined also. The conductivity (σ) can generally be assumed to be a constant or scalar function of position.

Thus, in theory at least, it should be possible to accomplish our objective. Upon impressing an electromagnetic field on a sample, measuring the resultant fields at the surface, and making some calculations, one could predict the current patterns within the sample. Generally however, the computations are extremely difficult.

CALCULATION OF CURRENT PATTERNS

Two promising methods of relating the field outside a region to the currents within that region were found in the literature. One involves the use of a Green's dyadic; the other involves a series expansion of the fields.

One method of relating the field outside a region to the current within that region involves the use of a Green's dyadic or tensor Green's function.^(4,5) The basic equation represents the electric field at a point $\vec{r}_0$ as

$$\vec{E}(\vec{r}_0) = \iiint_V j_{\omega\mu} \vec{J}(\vec{r}) \cdot G(\vec{r}|\vec{r}_0) dV$$

where

$$G(\vec{r}|\vec{r}_0) = -\frac{1}{4\pi} \left(I + \frac{1}{k^2} \text{grad grad} \right) \frac{e^{-jk|\vec{r} - \vec{r}_0|}}{|\vec{r} - \vec{r}_0|}.$$

Here, $\vec{J}(\vec{r})$ is the current density, I is the unit dyadic, $\vec{r}$ and $\vec{r}_0$ are position vectors, and $e^{j\omega t}$ time dependence has been assumed.

The integral relationship is derived from Maxwell's Equations. An inhomogeneous wave equation is obtained and a Green's

function solution is assumed. The differential equation is then integrated, giving a relationship for electric field in terms of current density. A detailed discussion of this derivation is beyond the scope of this paper, but is covered by Stratton.⁽³⁾

Although the above equation is rather formidable, practical problems can be solved when the geometry is appropriate to allow direct integration or approximations to the integral. In this method, however, it would be necessary to start with a current distribution rather than a field distribution

A second method of relating fields and currents involves expanding the fields inside and outside the material in powers of $-jk$, where $k = 2\pi/\lambda$ and λ is the wavelength of the incident radiation.^(6,7) In this method, an incident wave is assumed; the scattered and internal fields are calculated, all as power series. Starting with

$$\vec{E} = \vec{E}_0 - jk\vec{E}_1 - k^2\vec{E}_2 + \dots$$

and

$$\vec{H} = \vec{H}_0 - jk\vec{H}_1 - k^2\vec{H}_2 + \dots$$

and using Maxwell's Equations, conditions may be established on each term by equating like powers of jk . In a similar manner, the boundary conditions are applied to the series giving additional conditions. Scattered fields and internal fields may then be calculated term by term using potential theory. Thus, any number of terms may be calculated if Laplace's Equation can be solved in the appropriate coordinate system. Van Bladel⁽⁶⁾ shows that he solved Laplace's Equation when separation of variables was impossible by using numerical approximations. When the differential equations and continuous curves were replaced by difference equations and nets of points, curves were obtained showing first order eddy currents and scattered fields for a cross section of an infinite cylinder with a gap in the wall giving an end view like the letter "C".

OTHER PROBLEMS OF CURRENT INTEREST

Other problems and solutions related to the eddy current testing field were found; including calculations of the impedance of a solenoid enclosing a cylindrical conductor,⁽⁸⁾ the impedance of a single turn coil placed near a conducting sheet,⁽⁹⁾ and problems related to the magnetotelluric investigations of various geophysical problems. Most of these papers treat idealized situations without trying to predict the result of defects in the material being examined.

FUTURE STUDY

Extensive mathematical study does not appear feasible at this time. Instead, specific defect types will be studied experimentally to get a better idea of the relationship between the fields and current patterns. Then, it may be possible to develop definite useful relationships that will lead to the transformation of field information to current patterns.

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APPENDIX

The basis for determining the uniqueness of the field-current relationship is Poynting's Theorem,

$$-\frac{\partial}{\partial t} \int_V \left(\frac{\epsilon E^2}{2} + \frac{\mu H^2}{2} \right) dV = \int_V \frac{1}{\sigma} J^2 dV + \int_S (\vec{E} \times \vec{H}) \cdot \vec{n} dA$$

This is a very basic theorem of field theory, derived from Maxwell's Equations. The term

$$-\frac{\partial}{\partial t} \int_V \left(\frac{\epsilon E^2}{2} + \frac{\mu H^2}{2} \right) dV$$

represents the time rate of change of electric and magnetic energies in the total volume. The surface integral

$$\int_S (\vec{E} \times \vec{H}) \cdot \vec{n} dA$$

represents the flow of energy per unit time out of the volume, through the boundary S, and

$$\int_V \frac{1}{\sigma} J^2 dV$$

represents energy dissipated per unit time in heating. Thus, a relation is established for the rate of change of energy in a volume (V) bounded by a surface (S). This relation is then used to establish the conditions for uniquely determining the fields within a region. It is assumed that two fields will satisfy simultaneously the boundary conditions and Maxwell's Equations. If the tangential components of the electric field or the magnetic field are equal along the surface of the region, the two fields will be identical. Thus, only a single field is determined within a region by a given field distribution on the surface.

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