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CF- 56-9-132

External Distribution

COPY NO. 171

DATE: September 30, 1956

SUBJECT: Heat Transfer to Bingham Plastics, ThO_2 Slurries Flowing
Turbulently in Tubes: An Exploratory Study

TO: Distribution

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Heat Transfer to Bingham Plastic:

ThO₂ Slurries Flowing Turbulently in Tubes: An Exploratory Study

C. G. Lawson

Summary

An expression is derived relating the heat transfer and friction for Bingham plastic slurries. The expression is

$$\frac{hD}{k} = \frac{f}{2} \left(\frac{DV\rho}{\eta} \right) \left(\frac{C\eta}{k} \right)^{1/3} \left(1 - \frac{2g_c T_y}{f\rho V^2} \right)^{5/3}$$

The expression is modified after consideration of heat transfer rates measured with ThO₂ slurries and of data reported in the literature to the form

$$\frac{hD}{k} = 0.023 \left(\frac{DV\rho}{\eta} \right)^{0.8} \left(\frac{C\eta}{k} \right)^{1/3} \left(1 - \frac{2g_c T_y}{\left(\frac{0.023}{\left(\frac{DV\rho}{\eta} \right)^{0.2}} \right) \rho V^2} \right)^3$$

More data are required on other plastic slurries to confirm the validity of the above relation and to extend its usefulness. In particular, heat transfer measurements are required in the region of the critical velocity ranges where the flow changes from laminar to turbulent.

*The above work reported here was presented at a Homogeneous Reactor Project Monthly Information meeting in August, 1956.

The author acknowledges the help of R. Behrend III in helping with the data taking and computing and the help of D. G. Thomas in data taking.

Introduction

The purpose of this paper is to accomplish the following:

1. To report a relation derived for the rate of heat transfer between a circular tube wall and a Bingham plastic flowing turbulently within the tube by considering the analogy between fluid friction and heat transfer.
2. To make a preliminary evaluation of the derived expression by comparison with experiment.
3. To point out further experimentation required to solve the problems relating to heat transfer with Bingham plastics, primarily Bingham plastic slurries.

Reynolds in 1874 assumed that in a turbulently flowing fluid, heat and momentums are transferred in the same way, and as a consequence, the heat transferred is proportional to the friction in similar systems.⁽¹⁾ This idea, called the analogy between fluid friction and heat transfer, has been extended successfully to explain heat transfer to virtually all ordinary fluids ranging in properties from liquid metals through gases, water, and oils.^{(2) (3) (4) (5)} There is every reason to believe that the analogy should apply to Bingham plastics and to any other fluid having a definable relation between shear stress and velocity gradient. In general, the relation for round tubes is

$$\frac{du}{dr} = \frac{-\tau_c T}{\mu} \quad (1)$$

For ordinary fluids μ is a constant and is called the viscosity of the fluid. For Bingham plastics, on the other hand, the "viscosity" is a function of the shear stress.

$$\frac{1}{\mu} = \frac{\left(1 - \frac{\tau_y}{\tau}\right)}{\eta} \quad \text{for } \tau > \tau_y \quad (2)$$

$$\frac{1}{\mu} = 0 \quad \text{for } \tau \leq \tau_y \quad (3)$$

For all fluids in round tubes

$$\tau = \frac{r}{2} \frac{dp}{dL} \quad (4)$$

η is called the coefficient of rigidity and τ_y is called the yield stress⁽⁶⁾ or the stress required to shear the fluid. Both η and τ_y are properties of the fluid alone.

The belief is expressed here that if the above relation is combined with the current knowledge of Newtonian fluid heat transfer, quantitative relations between heat transfer and fluid physical properties may be obtained. Further, these relations may be adequate for many engineering purposes.

Application of the Analogy Concept to Bingham Plastics

We know that for a turbulently flowing fluid, the velocity structure may be considered as having three regions⁽⁷⁾ Region I is the turbulent core, wherein at each radial location the time average velocity is essentially constant in the axial direction of flow, and there is no net radial flow. Region II

is the sublayer where the flow may be described as laminar or unidirectional parallel to the wall. These two regions are separated by a buffer region, Region III, wherein a transition occurs from laminar to fully turbulent flow. For our purposes, the flow will first be considered turbulent throughout, and the results will be applied to the Colburn⁽⁹⁾ expression of the analogy to take into account the effects of Regions II and III.

Consider a fluid having a yield stress flowing in a circular pipe. The following restrictions are imposed:

1. The flow is turbulent throughout the fluid and is fully developed.
2. The velocity profile is square with the velocity equal to 0 at the wall and V elsewhere. Also V is constant in time.
3. There is a temperature difference ($\theta = t_f - t_s$) between the fluid and the wall. The fluid temperature is constant radially at any distance down the length of the tube and the heat flux, q , is constant.

We assume that heat and momentum are both transferred between the tube and the fluid by the velocity fluctuations perpendicular to the direction of flow. Consider an element of area, ΔA on the wall of the tube and observe the number of fluid particles, n , impinging on ΔA during time ΔT . Call the average mass of these particles m . The temperature of these particles is changed by an amount, θ , then the heat transferred between the wall and the fluid through ΔA during ΔT is the heat carried away by all of these particles. To compute this amount of heat, we let the total mass of particles impinging on ΔA during time ΔT be

$$nm = \Delta M$$

and

$$\rho \Delta A \Delta T = \Delta M C \theta = h \theta \Delta A \Delta T \quad (5)$$

or

$$\frac{h}{C} = \frac{\Delta M}{\Delta A \Delta T} \quad (6)$$

For no acceleration in the pipe, the rate of change of momentum associated with the particles impinging on ΔA of the wall and changing their velocities from V to 0 exerts a stress which must be balanced by the difference between the shear force acting on ΔA and the force required to overcome the yield stress. This must be so since no momentum is generated at stresses on the fluid that are less than the yield stress. Hence

$$g_c (\tau_w - \tau_y) \Delta A = \frac{V \Delta M}{\Delta T} \quad (7)$$

or

$$\frac{g_c (\tau_w - \tau_y)}{V} = \frac{\Delta M}{\Delta A \Delta T} \quad (8)$$

By equating (6) and (8), we obtain the relation

$$\frac{h}{C} = \frac{g_c (\tau_w - \tau_y)}{V} \quad (9)$$

Multiply equation (9) by $\frac{1}{\rho V}$ and obtain

$$\frac{h}{C \rho V} = \frac{\tau_w g_c}{\rho V^2} \left(1 - \frac{\tau_y}{\tau_w} \right) \quad (10)$$

The Fanning friction factor is defined as⁽⁷⁾

$$f = \frac{2g_c \tau_w}{\rho V^2} \quad (11)$$

and for τ_w

$$\tau_w = \frac{f}{2} \frac{\rho V^2}{g_c} \quad (12)$$

By substituting equations (11) and (12) into equation (10), we may write

$$\frac{h}{C_p V} = \frac{f}{2} \left(1 - \frac{2g_c \tau_y}{f \rho V^2} \right) \quad (13)$$

Equation (13) is identical to the statement of Reynolds original analogy excepting for the term $\left(1 - \frac{2g_c \tau_y}{f \rho V^2} \right)$. For an ordinary fluid, τ_y is zero and equation (5) reduces to Reynolds statement.⁽⁸⁾

The above analysis considers a fluid flowing with only a turbulent region, but even with this, we see that the equations developed for Newtonian fluids would be expected to apply to plastics only when the ratio $\frac{\tau_y}{\tau_w}$ is small compared to unity.

We need now consider the effect of laminar and buffer regions upon the heat transfer.

Colburn⁽⁹⁾ has developed an expression for the analogy wherein he accounts for the wall effects (laminar and buffer regions) with the Prandtl

Modulus $\frac{C\mu_f}{k}$ evaluated at the arithmetic average temperature between the fluid bulk and the wall temperatures. The fluid having this temperature is located near the tube wall. Colburn's heat transfer expression is

$$\frac{h}{Cv\rho} \left(\frac{C\mu_f}{k} \right)^{2/3} = \frac{f}{2} \quad (14)$$

From the arguments presented above, it appears reasonable that for a Bingham plastic equation (7) may be modified directly to

$$\frac{h}{Cv\rho} \left(\frac{C\mu_f}{k} \right)^{2/3} = \frac{f}{2} \left(1 - \frac{\tau_y}{\tau_w} \right) \quad (15)$$

The Prandtl Modulus is a ratio of the molecular diffusivity of momentum to the molecular diffusivity of heat. The molecular diffusivity of momentum is defined as the ratio of shear stress to velocity gradients divided by the density of the fluid⁽⁷⁾ or the kinematic viscosity, μ/ρ . Since the flow is laminar near the wall, this may be evaluated from the laminar shear diagram. For a Bingham plastic, we may use equation (2) evaluated at the wall shear stress to evaluate μ_f , and hence the "viscosity" of the fluid near the wall of the tube is

$$\mu_f = \frac{g_c \tau_w}{\left(\frac{du}{dr} \right)_w} = \frac{\eta_f}{1 - \frac{\tau_y}{\tau_w}} \quad (16)$$

For a system where the yield stress and coefficient of rigidity are relatively independent of temperature, the Colburn equation becomes

$$\frac{h}{C_p V} \left(\frac{C\eta}{k} \right)^{2/3} = \frac{f}{2} \left(1 - \frac{\tau_y}{\tau_w} \right)^{5/3} \quad (17)$$

$$\frac{f}{2} \left(1 - \frac{2\tau_y g_c}{\rho V^2} \right) \quad (18)$$

Application of the Colburn equation implies the velocity structure near the tube wall is similar for a Bingham plastic and an ordinary fluid. The similarity implied is that in the generalized velocity profiles, μ would be replaced by $\frac{\eta}{1 - \frac{\tau_y}{\tau_w}}$, particularly near the wall. Whether this im-

plication is justified or not can only be resolved by an experiment measuring the velocity profile. Consequently, it may be more judicious to express equation (17) as

$$\frac{h}{C_p V} \left(\frac{C\eta}{k} \right)^{2/3} = \frac{f}{2} \left(1 - \frac{\tau_y}{\tau_w} \right)^p \quad (19)$$

where p is an arbitrary constant. The above equation requires a method of evaluating the friction factor in the turbulent flow region.

A literature survey has been made and previously reported wherein it was concluded that the friction factor could be evaluated using the conventional smooth pipe curve for turbulent flow providing the density of the slurry and the viscosity of the dispersion medium were used in the Reynolds

number⁽¹⁰⁾⁽¹¹⁾. This procedure appears to be accurate within about 20% for a range extending through to four times the critical Reynolds number. A generalized friction factor Reynolds number plot has been prepared and reported previously.⁽¹⁰⁾ This graph is presented in Figure 6 and may be used for determining the critical Reynolds number and for calculating the wall shear stress from the equation

$$\tau_w = \frac{f}{2} \frac{\rho V^2}{g_c} \quad (20)$$

The value of f may also be estimated from the Drew, Koo, and McAdams relation⁽¹⁸⁾

$$f = 0.046 \left(\frac{DV\rho}{\mu} \right)^{-0.2} = \frac{0.046}{\left(\frac{DV\rho}{\eta} \right)^{0.2} \left(\frac{\eta}{\mu} \right)^{0.2}} \quad (21)$$

and

$$\tau_w = \frac{.023}{\left(\frac{DV\rho}{\mu_1} \right)^{.2}} \frac{\rho V^2}{g_c} \quad (22)$$

by substituting equations (21) and (22) into equation (17) and multiplying both sides of the equation by the products $\left(\frac{DV\rho}{\eta} \right) \left(\frac{C\eta}{k} \right)$, we obtain the relation

$$\frac{hD}{k} = \frac{f}{2} \frac{DV\rho}{\eta} \left(\frac{C\eta}{k} \right)^{1/3} \left(1 - \frac{\tau_y}{\tau_w} \right)^{5/3} \quad (23)$$

(24)

or

$$\frac{hD}{k} = 0.023 \left(\frac{DV_p}{\eta} \right)^{0.8} \left(\frac{\mu}{\eta} \right)^{0.2} \left(\frac{C_p}{k} \right)^{1/3} \left(1 - \frac{2g_c T_y}{0.023 \rho V^2} \left(\frac{DV_p}{\eta} \right)^{0.2} \right)^{5/3}$$

Literature-Survey

Forced convection turbulent heat transfer measurements with slurries have been made by several investigators. (12)(13)(14)(15)(16)

Bonilla et al investigated heat transfer to chalk-water slurries of ~1.5 micron mean particle size and concentrations up to ~10% by volume and found that at higher velocities the heat transfer coefficients were predictable from the Colburn equation, but as the velocity was decreased, the coefficient dropped below the predicted value. As the slurry concentration was increased, the discrepancy increased. The data appear to be similar to those that would be obtained in the transition region for a Newtonian fluid.

Orr and DallaValle studied several Newtonian slurry systems and found the heat transfer data could be correlated to within ~50% with the Sieder-Tate modification of the Dittus-Boelter equation

$$\frac{hD}{k} = .027 \left(\frac{DV_p}{\mu} \right)^{.8} \left(\frac{C_p}{k} \right)^{.4} \left(\frac{\mu}{\mu_w} \right)^{.14}$$

where the physical properties are evaluated at the average bulk temperatures of the slurry and

h = heat transfer coefficient log mean based on ΔT .

k = thermal conductivity of the slurry calculated from Maxwell's equation

$$k = k_1 \frac{2k_1 + k_s - 2P(k_1 - k_s)}{2k_1 + k_s + P(k_1 - k_s)}$$

k_1 = thermal conductivity of the liquid

k_s = thermal conductivity of the solid

P = volume fraction of solids

$$\mu_s = \mu_1 \left[\frac{1}{1 - \frac{P}{P_b}} \right]^{1.8}, \text{ "Viscosity" of slurry.}$$

P_b = volume fraction of solids in settled bed.

μ_1 = viscosity of suspending liquid.

White and DallaValle continued the investigation started by Orr and DallaValle and found a better correlation could be obtained if the viscosity were calculated as

$$\mu = \mu_1 \left[\frac{1}{1 - \frac{P}{P_b}} \right]$$

The suspensions used by Orr, DallaValle and White were: water with chalk, graphite, glass beads, aluminum powder, and copper as well as ethylene glycol with aluminum powder and graphite.

Miller has studied turbulent heat transfer to suspensions of water-graphite and kerosene-graphite of concentration up to about 10% by volume of solids. He found his data could be correlated by the Colburn equation

$$\frac{hD}{k} = .029 \left(\frac{D\nu\rho}{\mu} \right)^{0.8} \left(\frac{C_p \mu_s}{k} \right)^{0.4}$$

where the physical properties are evaluated at the mean temperatures. Thermal conductivity, specific heat, and density are linearly weighted averages and the viscosity was taken as the coefficient of rigidity. The slurries were practically Newtonian in all cases.

Salamone and Newman studied heat transfer to several slurries of low concentration which were overlapped in the above studies. They attempted to correlate their data using the solid and liquid properties separately along with the "turbulent viscosity" measured from friction factor-Reynolds number plots. Their correlation is

$$\frac{hD}{k_f} = 0.131 \left(\frac{D\nu\rho}{\mu_b} \right)^{.62} \left(\frac{\mu_b C_f}{k_f} \right)^{.72} \left(\frac{k_s}{k_f} \right)^{.05} \left(\frac{D}{D_p} \right)^{.05} \left(\frac{C_f}{C_s} \right)^{.35}$$

k_s, k_f = thermal conductivity of liquid and solids

C_s, C_f = specific heat of liquid and solid

μ_b = turbulent viscosity obtained from friction factor-Reynolds number plot

D_p = particle diameter.

The correlation of Salamone and Newman has the Nusselt modulus, $\frac{hD}{k}$, proportional to viscosity to the +0.1 power. Normally the Nusselt modulus is proportional to viscosity to the -0.4 or -0.467 power. For this reason their correlation should be used cautiously.

No data were found in the literature where heat transfer studies were made with slurries having yield stresses in the order of 1-lb/sq. ft. Also no analysis for heat transfer to turbulently flowing Bingham plastic was found. Insufficient data were presented in the above studies to permit analysis according to the procedure herein presented.

Discussion of Data

Heat transfer measurements were made with water and 4 different thorium-water slurries.

The heat transfer test section consisted of a steam heated 1" black iron pipe 107 diameters in length. The test section was preceded by a 50 diameter calming section. The exit to the test section was 45 diameters in length. A partially closed plug valve was used as a mixing chamber in taking the slurry exit temperature. In all cases the system was allowed to reach equilibrium and repeat readings were taken about 15 minutes apart. Two sets of data were taken with each slurry and the data were analyzed by the Wilson line plot technique to extract the slurry side heat transfer coefficients. The water data were used to obtain the resistance of the tube and steam condensate to heat transfer. It is emphasized that the data were exploratory and preliminary in nature.

The physical properties of the slurries evaluated at the mean slurry temperature (about 75°C) were as in Table I.

Table I

Slurry	ρ gms/ml	τ_y	η Centipoise	k (17)	C_p (17)
		$\frac{\text{lb}_f}{\text{sq.ft.}}$		$\frac{\text{BTU}}{\text{hr ft}^2 \text{ } ^\circ\text{F}}$	$\frac{\text{BTU}}{\text{lb}_m \text{ } ^\circ\text{F}}$
I	1.58	0.155	1.83	0.459	0.590
II	1.58	0.155	1.83	0.459	0.590
III	1.82	0.338	2.84	0.488	0.511
IV	1.82	0.00	1.86	0.488	0.511

Slurries I and II had essentially the same physical properties. The shear diagram of these slurries is shown in Figure 1.

Slurry III was a higher concentration material with a yield stress of about twice that of I and II. Slurry IV was the identical slurry used in III with sodium silicate added to the amount of 1/2% by wt. of ThO_2 present. This was done to show the effect of the shear diagram upon the heat transfer. The shear diagram of slurry III is shown in Figure 2. The silicated slurry was Newtonian with a viscosity of 1.86 centipoise.

The heat transfer data are shown in Figure 3 as a plot of the log mean heat transfer coefficient versus the average fluid velocity.

The graph shows that the water heat transfer and the Newtonian slurry heat transfer both increase with $(\text{velocity})^{0.8}$ and the slurries having a yield stress show a much greater effect of velocity on the heat transfer. The slurries with identical physical properties give essentially the same results.

Figure 4 is a plot comparing the data with the equation

$$\frac{hD}{k} = .023 \left(\frac{Dv\rho}{\eta} \right)^{0.8} \left(\frac{\mu}{\eta} \right)^{.2} \left(\frac{Cp}{k} \right)^{1/3} \left(1 - \frac{\tau_y}{\tau_w} \right)^{5/3}$$

It is seen that the experimental data lay somewhat below the predicted values at the lower end of the curve. This may be because that the flow is not fully developed in this region, or it may be because a larger exponent is required on the group $\left(1 - \frac{\tau_y}{\tau_w}\right)$. Both reasons suggest that in this region, the effect of the yield stress is to suppress the formation of eddies.

For slurry III, the critical velocity between laminar and turbulent flow is about 9 ft/sec. Also, the values of $\frac{\tau_y}{\tau_w}$ are between about .2 and 0.5. This means that theoretically, there is a region between 0.2 and 0.5 of the pipe diameter which is not being sheared. However, the radial velocity fluctuations prevalent in turbulent flow would tend to break up this plug. It seems plausible then that for velocities not much greater than 9 ft/sec remnants of this plug flow would exist. Where this type of flow is present, the analysis would not apply. Further, one would expect the flow to be sinuous and there could be uncommonly long entrance lengths with a not too large effect upon the heat transfer. Another way of looking at it is that the yield stress tends to promote stability of the laminar flow structure and would result in an uncommonly high number of larger eddies which are not broken down unless subjected to very high shear stresses. This effect may be observed when agitating plastic fluids in drums.

Similarly, at the higher Reynolds numbers, the data are higher than the predicted values. Examination of the data of DallaValle and Orr, DallaValle and White, Miller and Bonilla suggests that the asymptotic equation for plastics at high Reynolds number would be the same as for Newtonian materials. Therefore the term $\left(\frac{\mu}{n}\right)^{0.2}$ was dropped and the data were fitted

to obtain an exponent for the group $\left(1 - \frac{\tau_y}{\tau_w}\right)$. The resultant equation is

$$\frac{hD}{k} = .023 \left(\frac{D\bar{V}\rho}{\eta}\right)^{.8} \left(\frac{c\eta}{k}\right)^{1/3} \left(1 - \frac{\tau_y}{\tau_w}\right)^3$$

This equation is compared with the experimental data in Figure 5. The agreement is considered to be within the total experimental error for this preliminary investigation. The method used for computing τ_w is from equation 23.

$$\tau_w = \frac{.023}{\left(\frac{D\bar{V}\rho}{\mu_1}\right)^{.2}} \frac{\rho v^2}{\epsilon_0}$$

This is done because much of the turbulent flow pressure drop data suggests this gives a more realistic value than any other method of calculation⁽¹⁰⁾⁽¹¹⁾.

It is clear that additional, more accurate experimental data are required to determine the adequacy of the above equation. The region of the unstable transitional flow must be determined as well as the effect of the thermal entrance length. However, it is also clear that the analogy between fluid friction and heat transfer may be applied profitably to non-Newtonian fluids and therefore, this approach should be extended.

Conclusions

1. A heat transfer equation has been derived relating the fluid friction to the heat transfer coefficient for Bingham plastic slurries. The equation is

$$\frac{hD}{k} = \frac{f}{2} \left(\frac{DV\rho}{\eta} \right) \left(1 - \frac{2g_c \tau_y}{f\rho V^2} \right)^{5/3} \left(\frac{c\eta}{k} \right)^{1/3}$$

2. This equation has been modified empirically to

$$\frac{hD}{k} = .023 \left(\frac{DV\rho}{\eta} \right)^{.8} \left(1 - \frac{\tau_y}{\tau_w} \right)^3 \frac{c\eta}{k}^{1/3}; \tau_w = \frac{.023 \rho V^2}{g_c \left(\frac{DV\rho}{\mu_1} \right)^{.2}}$$

The modified equation describes the heat transfer data within $\pm 50\%$ through an apparent transition region for slurries having yield stress values up to 0.355 lb/sq ft.

3. Additional experimental data are required to determine the adequacy of the above equation, to extend its range of application, and to develop a better equation.

4. Analytical and experimental work should be directed toward developing the fluid friction-heat transfer analogy for slurries of a non-Newtonian nature.

5. The fact that h is not proportional to (velocity)^{0.8} is insufficient evidence of unstable flow for plastic fluids. Therefore, effort should be made to determine the existence of an unstable flow regime in the region of critical Reynolds number and to define the boundaries of this region.

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Table of Nomenclature

A = Area on Heat transfer surface	$\frac{\text{Dimensions}}{\text{sq. ft.}}$
C = Specific heat of the slurry	$\text{Btu/lb}_m \text{ } ^\circ\text{F}$
D = Diameter	feet
f = Fanning friction factor = $\frac{g_c D \Delta P}{2 \rho V^2 L}$ for round tubes = $\frac{\lambda}{4}$	none
g_c = Conversion constant	$\frac{(\text{ft})(\text{lb})_m}{(\text{sec})(\text{lb})_f}$
h = Heat transfer coefficient, based on log mean temperature difference for turbulent flow and arithmetic mean temperature difference for laminar flow.	$\frac{\text{Btu}}{(\text{ft})^2(\text{hr})(^\circ\text{F})}$
k = Thermal conductivity of the slurry	$\frac{(\text{Btu})(\text{ft})}{(\text{hr})(\text{ft})^2(^\circ\text{F})}$
L = Length of pipe across which ΔP is measured	feet
m = Average mass of fluid particle	lb_m
ΔM = Total mass of fluid particles impinging on a given area in a given time increment.	lb_m
$n = \Delta M/m$ = Number of fluid particles impinging on a given area in a given time interval.	none
p = A constant	
ΔP = Pressure difference	$(\text{lb})_f/(\text{sq})(\text{ft})$
q = Heat flux	$\frac{\text{Btu}}{(\text{hr})(\text{sq})(\text{ft})}$
r = Radial distance from the pipe axis	ft
ΔT = Increment of time	hours
t_f, s = Fluid and wall temperature respectively	$^\circ\text{F}$

V = Fluid velocity	<u>Dimensions</u>
$\frac{du}{dr}$ = Radial velocity gradient measured from the axis outward	hours ⁻¹
μ = Viscosity	Centipoise or (lb) mass/(ft)(hr)
μ_f = Viscosity measured at the arithmetic average between the wall temperature and the bulk fluid temperature	same
μ_1 = Viscosity of the dispersion medium	same
η = Coefficient of rigidity of a Bingham plastic (slope of the shear diagram)	same
τ = Shear stress acting on the fluid, = $\frac{V}{2} \frac{\Delta P}{L}$ for all fluids in round tubes	$\frac{(\text{lb})_f}{(\text{sq})(\text{ft})}$
τ_y = The yield stress of a Bingham plastic or the stress re- quired to initiate flow in a Bingham plastic	$\frac{(\text{lb})_f}{(\text{sq})(\text{ft})}$
τ_w = Shear stress at the wall of the pipe = $\frac{D \Delta P}{4L}$ for round tubes	$\frac{(\text{lb})_f}{(\text{sq})(\text{ft})}$
ρ = Density of the slurry	$\frac{(\text{lb})_m}{(\text{cu})(\text{ft})}$
θ = Difference between the wall temperature and the fluid temperature = $t_f - t_s$	$^{\circ}F$
λ = Moody friction coefficient = $\frac{2g_c D \Delta P}{\rho V^2 L} = 4f$

Fig 1 Pseudo Shear Diagram for Slurries I and II used in Heat Transfer Study

Fig. 2. Pseudo Shear Diagram for Slurry III used in Heat Transfer Study

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Fig. 3. The Effect of Velocity on the Heat Transfer Coefficient.

Fig. 4. Comparison of Experimental Data with the Derived Heat Transfer Expression

Fig. 5. Empirical Modification of Heat Transfer Equation Compared with Data.

Fig. 6. Friction Factor - Reynolds Number Diagram for Bingham Plastic Slurries in Smooth Pipes.

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