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COO-3235-55

NUCLEAR EXCITATIONS AND REACTION MECHANISMS
Report of Progress Feb. 1, 1974 to 31 October 1974

Department of Physics
Brown University
Providence, Rhode Island

Contract AT(11-1)-3235

November 1, 1974

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ABSTRACT

- A. (Fallieros). Electron scattering, gauge transformations, nuclear currents, correlations of radiative (M1) widths and neutron-emission widths, giant collective excitations, dispersion relations for the nuclear photoeffect.
- B. (Levin). Soluble model for identical particle scattering; inhomogeneous equation method for e^-+H collisions; K-matrix methods; unitarity in many-body scattering; new coupled channel T and K operator approach to many-body scattering; general theory, time-dependent formulation, application to e^-+H scattering, connection with the method of Hahn, analysis of breakup, unitarity modifications of the theory; zero-range approximation in DWBA using a three-body model.
- C. (Friar). Methods for determining nuclear charge distributions from elastic electron scattering and muonic atom energy level data; relativistic corrections to charge form factors; dispersion corrections to elastic electron scattering; relativistic effects in photoabsorption sum rules; analyticity of the Compton Amplitude; pion-nucleon absorption operator.
- D. (Hirata). Pion-nucleus scattering, optical potentials, nucleon resonances in nuclei.

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SCOPE

The Nuclear Physics Program at Brown University currently includes five faculty members, two in experimental physics and three theorists, plus a research associate in theory. The activities of the latter four persons are described in this report. As noted in past progress reports, a collaboration between the experimentalists and the theorists exists, although there is no current AEC support for the former.

The research work (completed or in progress) is presented separately in the next section. Parts A, B, C, and D describe the work of Professors Fallieros, Levin, and Friar, and of Dr. Hirata. This work is carried on in the Barus and Holley Physics - Engineering Building, and at Orsay, France, and Surrey, England, in the cases of Professors Fallieros and Levin who are on sabbatical leave from Brown until June, 1975. Numerical analyses have been done at Brown on the IBM System 360/67, at Waltham, Massachusetts, on the CDC 6600 computer, and at MIT.

CURRENT RESEARCH

2A. (Fallieros)

Electron scattering and nuclear currents

Several results that we obtained in the past on sum rules, transition densities and form factors for inelastic electron scattering related directly to the fundamental law of current conservation. Since this law is in turn related to the principle of invariance under gauge transformations, a study was made aiming at the direct derivation of these results from gauge invariance. There are two (partially overlapping) approaches one can follow which can be referred to as the direct and the indirect route, respectively. The first proceeds through the shortest possible path from the general statement of gauge invariance to the final expressions. This approach often improves our insight of the physical significance of the results and has led to the rederivation of several sum rules previously obtained by other methods as well as to some new ones. The second approach considers the consequences of two (or more) different choices of gauge. Since the physically-meaningful quantities must be independent of the choice of a particular gauge one tries to establish the equivalence of the two results and in doing so, one finds it necessary to introduce the sum rules that were already obtained by the direct method. This clearly can also be considered as an indirect proof of the validity of the sum rule. An example of this was worked out in some detail and will be published in collaboration with Prof. Friar. It concerns the calculation of the diamagnetic susceptibility of an atom. When this is calculated in the standard Coulomb gauge ($\vec{A} = \frac{1}{2} \vec{B} \times \vec{r}$; $\vec{A}$: vector potential, $\vec{B}$: magnetic field) it is found to be directly related to the familiar A^2 term in the interaction Hamiltonian. We have calculated the same quantity in the so-called Landau

gauge (introduced by Landau in the study of the motion of free particles in a uniform magnetic field). It was found that the quadratic (A^2) term in the Hamiltonian leads now to a susceptibility different by a factor 2 relative to the previous calculation. One notices, however, that now there exists a non-vanishing second-order contribution coming from the part of the Hamiltonian that is linear in $\vec{A}$ (this type of contribution is what in solid state physics is usually called Van Vleck paramagnetism). It is easy to relate this result to the diamagnetic susceptibility and recover the factor 2 mentioned above, provided one uses a sum rule familiar from the study of isoscalar electric quadrupole transitions in nuclei and which is also applicable to atoms.

No new attempts have been made during the past year to fit and analyse more experimental data. With the expectation of experimental results at momentum transfers higher than those presently provided, we have been more interested in systematizing methods for analysing and interpreting data on the basis of what we have previously referred to as a two doorway situation.

Studies in two additional directions have been made in connection with sum rules, transition densities, etc. that will not be presented here since they involve work still in progress which will be described in the proposal for next year. They concern first an investigation of the cancellations between various multipolarities (both isoscalar and isovector) in a fixed momentum transfer sum rule for form factors and second, calculations of equal-time commutators between density operators and their second (or higher) time derivatives.

Magnetic excitations and width correlations

The work described in last year's proposal is essentially completed and will be prepared for publication. On the question of correlations between particle-

emission and radiative widths the situation appears to be the following: There is a state (with quantum numbers 1^+) rather well removed from the dominant M1 giant resonance, which has quite strong matrix elements for neutron emission but very small radiative width. Relatively weak components of this state are expected to be found in the region of the giant M1 mode due to fragmentation (coupling with more complex configurations). The weakness of these fragments is, however, compensated by the rather large particle-emission probability and the overall effect turns out to be significant. The neutron ejection is thus dominated by two states which interfere in the overall process whereas the γ emission (or absorption) uses only one of these doorway states (the giant M1). No correlation is, therefore, expected between the radiative and particle widths.

The essential point of this argument is clearly the existence of the state mentioned above whose γ matrix element is small, whereas that for decay by neutron emission is large. It is not difficult to understand this behavior: in this state the protons and the neutrons appear with the same phase (that is why its characteristic energy is lower than that of the giant mode). Since the magnetic moments of the proton and the neutron have opposite signs the two contributions cancel and the result is a small M1 matrix element. In the nuclear interaction, however, which is responsible for the neutron emission there is no such difference between protons and neutrons. The two contributions, therefore, add and the resulting matrix element is enhanced.

Some attention has been directed toward a physical problem that has some similarity with the transitions mentioned above. This is neutron emission induced by muonic transitions, in a sense the inverse of an internal conversion process. It is not clear yet how important M1 transitions will be in this case, If they turn out to be a small fraction of the overall effect, work in this direction will not be continued in the near future.

Nuclear force fields and densities

The work described in last year's proposal under the same heading has been completed, but its publication has been withheld because of the recent appearance of a similar article in the Physical Review by Japanese physicists. Some new work planned in this direction will be mentioned in the proposal for next year's program. The work already completed (as well as that of the Japanese group) starts from results we have previously published on the consequences of the doorway dominance approximation as applied to a nuclear-density sum rule, introduces simple assumptions relating a transition potential to a transition density, derives a relation between the transition potential and the real part of the nuclear optical potential, determines the strength of the former in terms of the strength of the latter and obtains results concerning, e.g., the excitation energies of collective excitations. The result for the energy of the isoscalar quadrupole resonance is essentially identical to that obtained by Bohr and Mottelson, namely, $E = 58 \text{ MeV } A^{-1/3}$ to be compared with $E_{\text{exp}} = (63 \pm 1.5) A^{-1/3} \text{ MeV}$ recently reported by the experimental group at Orsay.

CURRENT RESEARCH 2B. (Levin)

During the 12 months since submission of the 1973 Progress Report, considerable progress has been made in developing the coupled channel T and K operator approach to many-body scattering of Kouri and Levin (K-L). A number of collaborations in connection with this work have begun and joint articles submitted for publication. In addition a number of the articles that had been submitted for publication at the time of writing last year's progress report have been accepted for publication.

Further work is continuing at a number of different centers and is expected to go forward for some time. The investigation of low energy, e^- -H scattering parameters using the inhomogeneous equations method has been completed, and further calculations on this system (and on e^- -He) are now being undertaken using the K-L method.

Eigenstate expansions in a soluble three-body model

A study with D. J. Kouri of the soluble three-body problem consisting of two non-interacting identical particles, one incident on and one bound to an infinitely heavy structureless core was undertaken, with the aim of comparing two approaches to the identical particle scattering problem. In the first approach, a Mott and Massey type of expansion for the total wave function was used. This involves a termwise antisymmetric expansion based on the complete set of target states. In the second approach, the state function obeying the inhomogeneous equation for identical particle scattering is expanded in the complete set of states; antisymmetry is not a feature of the expansion, since it is built into the structure of the inhomogeneous equation. Both exact and

approximate solutions to the problem were examined and it was shown in detail how the solution to the inhomogeneous equation produces the correct scattering amplitude, even though the antisymmetry of the problem is not built into the solution itself. A paper on this topic has been published recently (Nucl. Phys. A225 (1974) 521).

Electron-hydrogen scattering using the inhomogeneous equations method

This work, the main portion of the Ph.D. study of P. W. Dahlgren has finally been concluded. We are satisfied that the computer program is free of error and that the negative resonance widths found for P waves at energies just below the $n=2$ threshold are an indication of the basic inability of the inhomogeneous equations method to produce unitary results when approximations are introduced. The difficulty is that the solution to the inhomogeneous equation is a sum of two terms, the complementary function and the particular integral. For the case of interest, i.e. the incident energy less than that needed to excite the atom, the scattering is described by a phase shift δ_ℓ in each partial wave. A K-matrix form of solution was developed to ensure a unitary final result. This procedure cannot guarantee that $\delta_\ell (= \tan^{-1} K_\ell)$ has the proper sign, but, as is well known, a determination of the sign is unimportant. However, when resonances are present, causality requires that as the energy increases through the resonance energy, K_ℓ must jump from $+\infty$ to $-\infty$. Stated in an alternate way, near resonance δ_ℓ may be written as $\tan^{-1} K_\ell = \delta_\ell = \Gamma_\ell / (E_\ell - E) + \phi_\ell$, where Γ_ℓ and E_ℓ , the resonance width and energy, must be positive, and where ϕ_ℓ is small and slowly varying with E for $E \approx E_\ell$. The inhomogeneous equation gives K_ℓ as a sum, $K_\ell^\pm = K_\ell(d) \pm K_\ell(e)$, where $+$ ($-$) refers

to singlet (triplet) spin states, and $d(e)$ refers to the complementary function (particular integral), or equivalently to direct (exchange) effects. In order that the quantity Γ_l be non-negative, we must have $K_l(d) \geq K_l(e)$ in the resonance region. This inequality was found to hold for S waves but not for P waves in the $1s - 2s - 2p$, or close coupling, approximation. There is no way, within approximate solutions to the inhomogeneous equations, to guarantee that $K_l(d) \geq K_l(e)$ for $E = E_l$, and hence to provide for unitary, causal answers. For this reason, the appearance of $T_{l=1} < 0$, should not be regarded as a surprise. It does however mean that calculations based on the inhomogeneous equation method, at least in the region of resonances, cannot be relied on. (An article describing these calculations is in preparation). It is interesting that a similar case of non-unitary results has occurred, also for S-wave, $e^- + H$ scattering, but here using integral equations known in advance not to lead to unitary approximations. These are discussed below. Considerable effort has been spent trying to connect the inhomogeneous equations with these particular integral equations, but without success. Mr. Dehlgren is now altering his computer program so as to perform similar $e^- + H$ calculations using the differential form of the Kouri-Levin equations. These will be his final Ph.D. thesis calculations.

Electron-helium scattering using the inhomogeneous equations method

The failure of this method to ensure $T_{l=1} \geq 0$ at resonance in the $e^- + H$ case has led to the abandoning of the $e^- + He$ calculations began with Professor Bergeron of Laval University. The calculations are now being programmed to be done using the differential form of the K-L equations.

K-matrix methods for scattering theory

Three articles with D. J. Kouri dealing with this topic were mentioned in last year's progress report, two to be submitted to Annals of Physics and one to Physics Letters. These have been published: Ann. Phys. (NY) 83 (1974) 316, 326; Physics Letters 48B (1974) 203. In addition, new work using K matrix methods has been completed, and an article accepted for publication in Physical Review C. In this work, a half-off-shell K and T matrix relationship has been used to establish a constraint on the half-off-shell T matrix elements. In the notation used in last year's progress report, this constraint is: $\langle k | \text{Im } T_{\ell}(k^2) | q \rangle = \tan^2 \delta_{\ell}(k) \times \langle k | \text{Re } T_{\ell}(k^2) | q \rangle$ for all values of q . This relation is automatically satisfied for potentials; it should be a useful check on phenomenological values for $\langle k | T_{\ell}(k^2) | q \rangle$.

Coupled channel T and K operator approach to many-body scattering

In last year's progress report it was noted that the solution to the problems connected with defining a K operator for identical particle scattering could be extended to the three-body problem. The article noted there as in rough draft form has been completed, as has a second on the treatment of the problem of N two-body channels in an n-body scattering system. These articles have been submitted to Nuclear Physics. A third article outlining this new method for many-body scattering and establishing the connectedness properties of the iterated kernels of the coupled integral equations for two-body transition operators has been established in Physics Letters (Vol. 50B (1974) 421). In addition, the article on identical particles and the Heitler damping equation has been accepted for publication in Physical Review A.

The basic idea of this new approach is to take the exact forms for the "post" or "prior" transition operators (The "Low equations") and rewrite them as two sets of coupled equations for modified transition operators describing transitions to or from various channel states. These exact coupled equations are written with the aid of a set of real parameters W_{jk} , elements of the channel coupling array W . The W_{jk} at first serve the purpose of expressing the outgoing wave boundary condition in all arrangement channels rather than in only one channel, as in the standard Lippman-Schwinger equation. The most important aspect of the W_{jk} is that they may be chosen so as to guarantee that the iterated kernel of the coupled equations is connected, thus avoiding the divergences associated with disconnected diagrams. Another important aspect of the W_{jk} is that they give rise to T operator equations for which analogous K operator equations can be defined; these lead to Heitler damping equations. The reason that this occurs is that the new equations are written as a matrix form of Lippmann-Schwinger equation. Not all manipulations lead to such results, implying that the concomitant matrix elements will not be unitary, as discussed below. An advantage of working with the K - L type of equations is that all the results of one-body scattering theory such as the introduction of Moller operators, variational principles, and a time-dependent approach (in terms of single "super" operators) can be easily carried over to the many-body case. Articles on the latter topics have been submitted for publication, and are discussed below. An invited and a contributed paper on aspects of this new approach were presented at the Conference on Few-Body Problems, held at Laval University, Quebec, Canada, August 1974, and will be presented at the Institute of Physics meeting at Birmingham

University, England, in December 1974.

Relation between solutions of different sets of equations derived with W_{jk}

It was noted above that results not leading to a damping equation could be derived using the W_{jk} (see also last year's progress report). One such was derived by Baer and Kouri (B-K), and calculations on the e^-+H system were performed by Kouri, Craigie and Secrest for low-energy, S-wave scattering using both the B-K and K-L equations. The latter gave unitary results of the form $\exp(i\delta) \sin\delta$, while the former gave non-unitary results of the form $A \exp(i\delta)$, where A is real, but $A \neq \sin \delta$. An article explaining these results in terms of K operators and the damping equation, by Kouri, Craigie and Secrest, has been published: J. Chem Phys. 61 (1974) 17.

Effect on e^-+H scattering parameters of varying the W_{jk}

Only certain values of the W_{jk} are allowed in order to ensure connectivity of the K-L coupled equations. However, if the approximation of neglecting continuum states in all channels is made then connectivity is ensured independent of the values of the W_{jk} . Such an approximation was made for the e^-+H system (only the $n=1$, $1s$, hydrogenic ground state was retained) in order to study the effect of varying W_{11} . This is a two-channel problem, and the two values of W_{jk} , W_{11} and W_{12} (or W_{21} and W_{22}), are reduced to one by the general constraint $\sum_k W_{1k} = 1$. Both singlet and triplet scattering lengths and $E = 0.1$ Ryd phase shifts were studied as a function of W_{11} . Best results were found for the value $W_{11} = 0$, which is the value guaranteeing connectivity. Comparison with the many adjustable-parameter variational calculations of C. Schwartz gave surprisingly good agreement, given the very crude approximation used:

the triplet results differed by about 4%, while the singlet differed by about 17%. A paper on these results by Kouri, Levin, Craigie and Secrest has been submitted to Physical Review. Further calculations are being carried out by Craigie and Secrest and by Dahlgren and Levin.

Connection with the reduction method of Hahn

A method for deriving connected equations in the n-body problem, in which the Schrödinger equation is written in matrix form and certain interactions are removed from the rows and columns of the matrix, has recently been introduced by Y. Hahn. A derivation of Hahn's results for the cases $n=2$ and $n=3$ using the channel coupling array has been given, and a general derivation for arbitrary n has been suggested. A paper by Hahn, Kouri and Levin on this topic has been accepted for publication in Physical Review C.

Lack of Federbush disease in the K-L equations

An article by Hahn, Kouri and Levin, on the non-existence of spurious solutions (or lack of "Federbush disease") in the coupled T and K operator equations derived using the channel coupling array W, has been accepted for publication in Physical Review C.

Unitarity in many-body scattering using the post-prior formalism

A new study of the unitarity (discontinuity) relations using the standard post-prior form of the transition operators has been made. Attention has focussed mainly on enumerating the states of the total Hamiltonian H that occur in these relations. Using these states, a generalization of the results of Lovelace for the 3-body problem was obtained in two different ways. This

form for the unitarity relation establishes that only by coincidence will approximate calculations of post and prior transition operator matrix elements be unitary. As such, this provides an alternate and more fundamental explanation for the non-unitary matrix elements found by Kouri, Craigie and Secrest; the first explanation, noted above, was based on the K operator and damping equation. An article written with Kouri on this topic has been submitted to Nuclear Physics.

Unitarity modifications of the K-L equations for n-body scattering

The methods used in the preceding analysis have been applied to a study of unitarity in the K-L coupled equation formalism. Using the set of states of H noted above to enumerate the cuts in the complex energy plane, and also Cauchy's theorem, an interesting modification of the coupled equations, when the choice of W so permits, has been derived. This occurs when W couples together the transition operators for all the channels, including the n-body break-up channel (choices of W exist for which the break-up operator does not occur, as in the Faddeev, Lovelace, and Alt-Grassberger - Sandhas formulations for $n=3$). The modification consists of multiplying each channel Green's operator (resolvent) by the projection operator onto the set of bound states in that channel (for the n-body break-up channel, this projection operator is unity). As a result, only bound states occur in every channel (apart from plane wave states of relative motion between the clusters or fragments forming the channel). Although this increases the number of equations, the restriction to bound intermediate states provides a great simplification. This is especially important when working with unitary approximations.

Calculations using these modified equations are planned for the nuclear 3-body problem. Two articles on this topic have been written with D. J. Kouri, one submitted to Physics Letters and one to Nuclear Physics. The latter article includes a treatment of Pauli principle effects for a model identical-particle scattering problem.

Variational principles for multiparticle scattering

An article applying the K-L equations to deriving variational principles for multiparticle scattering has been published recently by Tobochnik. In this work only matrix elements for a single transition operators are coupled together in the exact set of equations. An article extending the analysis to the entire set of coupled transition operators as well as introducing additional variational functionals has been written by Kouri and Levin and submitted to Physical Review.

Time dependent formulation of the K-L equations

A general, time-dependent approach to many-body scattering based on the K-L coupled channel formalism has been recently developed in collaboration with H. Krüger and D. J. Kouri. The similarity of the matrix form of the K-L equation with the usual one-body scattering formalism facilitates the analysis considerably. Because non-orthogonality of different arrangement channel eigenstates is not a problem in our method, it is relatively straightforward to cast all the operators of a particular type into such a form that they are the matrix elements of "super" operators, i.e., elements of new operators that are matrices in channel space. Thus, new super S, T and K matrices

can be defined, whose elements are the more usual operators describing transitions from one channel state to another. This work generalizes that of other workers, eg., Eckstein. An article on this topic has been submitted for publication to Physical Review.

Analysis of break-up channels in the K-L formalism

The procedure used to modify the post or prior forms of coupled equations to the K-L form, valid when the initial state has at least one bound fragment, does not hold when the initial state describes n free particles. The analysis of this situation, within the framework noted in preceding sections has recently been carried out. Identification has been made of disconnected terms, and the unitary (discontinuity) relations have been derived. An article on this topic is in preparation.

Applications of the K-L formalism

Several applications of the K-L coupled equations method to 3-body and many-body systems are now being made. These include calculations of n -d scattering using unitary approximations, construction and analysis of the complex potential well and theoretical optical potential for scattering of a particle by a bound system of particles all identical to it, and investigation of a multichannel effective range formalism taking into account resonances or bound states of strongly interacting particles.

Zero-range DWBA approximation in a 3-body model of stripping

Calculations of $l=2$ and 3 partial wave amplitudes for this model problem, discussed in last year's progress report and renewal proposal, were

resumed late in summer, 1974. Final results were not available prior to my coming to University of Surrey in early September, 1974, and no further calculations can be carried out until my return to Brown next summer. An assessment of the desirability of continuing such calculations versus writing up the results so far obtained and then terminating this project will be made in the next few months. However, the possibility of using the 3-body model as a means of investigating stripping theories other than DWBA, projects noted in previous renewal proposals, is still open.

CURRENT RESEARCH 2C. (Friar)

The determination of the nuclear charge distribution from elastic electron scattering

A method for determining the charge distribution of nuclei from electron scattering and muonic data, previously developed and described in the previous year's progress report, has been extended slightly and applied to ^{12}C . This is significant because of the wide spread use of ^{12}C as an experimental standard and because the electron scattering data for this nucleus extend to large momentum transfers and thus probe distances in ^{12}C on a fairly small scale. The method uses a Fourier expansion of the density with the coefficients being determined from the data by minimizing χ^2 . Errors and error correlations in the coefficients are also determined so that the resulting density, statistical errors in the density, and error correlations in the density are all determined subject to the assumptions made.

In order to check the validity of the error analysis part of the calculation, 25 sets of pseudo-data were generated using an assumed charge distribution, and calculating the cross-sections for the energies and scattering angles of the Stanford electron scattering data set. These "data" sets were randomized using a random number generator according to the percentage errors assigned to the (real) Stanford data. The charge distribution was then determined using our procedure for each pseudo-data set and compared to the original (assumed) charge distribution. Plots of these distributions indicate that our error analysis procedure is indeed an accurate estimate of the statistical errors.

Previous experience has shown us that systematic effects such as the ef-

fect of finite spectrometer acceptance angles on the measured cross-sections can have an important effect on the unfolded charge distribution. Taking account of the effect of the vertical and horizontal spectrometer acceptance angles was found to be important in the case of ^{12}C , reducing the χ^2 by a factor of 2 and changing the density more than one standard deviation. The effect of non-static contributions to the electron-nucleus scattering, namely dispersion corrections (see previous year's progress report for a description of this calculation), was also included in the analysis and it was found that at the present level of precision, these effects are unimportant in ^{12}C . A comparison was made between the density deduced from the data and deformed Hartree-Fock calculations, and it is tentatively concluded that there is little evidence for ^{12}C being a deformed rotator. A detailed analysis of the tail of the charge distribution, which should be approximately exponential, was also made.

Because this technique, and several others which are similar, offer an order of magnitude more flexibility and sensitivity in determining the charge distribution of a nucleus from data, a review of the results of this and similar techniques is being prepared. Because new electron linacs are capable of higher precision experiments than before, it is necessary to have theoretical tools of similar precision to obtain useful information. Such tools are now available and a wide dissemination of these tools is hoped for. Submitted to Nucl. Phys. (with J. W. Negele, MIT).

Relativistic and retardation effects in the Thomas-Reiche-Kuhn sum rule for a bound particle

Lowest order relativistic corrections to the current operator and the effect of retardation (multipoles other than $E1$ and retarded contributions to $E1$) are included in a calculation of the Thomas-Reiche-Kuhn (TRK) sum rule for a particle bound in a potential. It is found that the retardation corrections partially cancel and the remainder is cancelled by part of the relativistic corrections. The latter corrections depend on the Lorentz-transformation properties of the potential which binds the particle (e.g. scalar or vector). The complete integrated photoabsorption cross-section, including the first-order relativistic and retardation corrections, is found to be equal to the high energy limit of the Compton Amplitude calculated by Goldberger and Low, a result which is the dispersion relation form of the TRK sum rule. A heuristic derivation of the latter results is also presented. The calculation shows that relativistic effects may be considerably more important than the effect of higher multipoles. Submitted to Phys. Rev. C. (with S. Fallieros).

The non-relativistic Compton amplitude

On the assumption that the Compton amplitude for scattering (of a photon) from a non-relativistic system has "good" analytic properties as a function of the energy, ω , of the incoming photon, a dispersion relation can be written. Gerasimov used this dispersion relation to prove that the retardation corrections to the TRK (dipole) sum rule must vanish. In the previous paragraph we noted that these corrections do not vanish. A simple heuristic argument is presented which demonstrates that the non-relativistic Compton amplitude is

not analytic in the upper half of the ω -plane and therefore no dispersion relation can be written. There are additional cuts which are due to the structure of the bound-state wave function and to the use of the non-relativistic relationship between energy and momentum. The relativistic problem, where relativistic kinematics is used, is free of these anomalous singularities. This result is also interpreted in terms of virtual particle-anti-particle states of the bound system. Submitted to Phys. Rev. C. (with S. Fallieros).

Pion-nucleon absorption operator ambiguity

A non-relativistic reduction is made of the equations of motion of a particle bound in a potential and interacting with an external pion field. This procedure yields an effective Hamiltonian, which can be used to calculate pion absorption by a nucleus. Keeping all terms which are linear in the pion-nucleon coupling constant and which are also first order in $(v/c)^2$, relativistic corrections to the usual non-relativistic result, it is found that the effective Hamiltonian depends on the Lorentz-transformation properties of the potential binding the particle (e.g. scalar or vector). A simple physical interpretation is made of this result. The Foldy-Wouthuysen technique for performing the reduction in powers of $(v/c)^2$ is not unique, since a simple unitary equivalence exists. This is shown to have no effect on physical results if care is used in treating the potential-dependent terms in the pion-nucleon absorption operator. The question of Galilean invariance of this operator is also discussed. The relationship of the unitary equivalence to the two common forms of πNN vertex, γ_5 and $\gamma_5 \gamma_\mu \partial_\mu$, is demonstrated. (Phys. Rev. C10, 955 (1974)).

Relativistic corrections to inelastic electron scattering

Using the techniques and results of previous work (see previous year's progress report) on relativistic corrections to the Coulomb part of elastic electron scattering, the corrections to the Coulomb part of inelastic scattering have been examined. It is found that the formula for the inelastic cross-section is slightly modified from the elastic case. The effective form factors, furthermore, have the correct threshold behavior when relativistic corrections are introduced and vanish as $\vec{q} \rightarrow 0$ (the 3-momentum transfer). The special case of the quasi-elastic breakup of the deuteron is examined in detail and several errors in the literature are corrected. (to be published).

Dispersion corrections to elastic electron scattering

Work on a variety of topics connected with dispersion corrections has begun. This topic has assumed new importance due to new linacs which have recently been constructed. In particular, an approximation method which corrects the usual closure approximation technique is being tested (see previous year's report) against an exact (model) calculation. If the approximation method is accurate, the accurate calculation of Coulomb dispersion corrections will be considerably facilitated. (with T. deForest, IKO, Amsterdam).

CURRENT RESEARCH 2D. (Hirata)

Dr. Hirata joined the nuclear physics group at Brown University in August, 1974. During the three months he has participated in the contract he has been investigating off-shell effects in pion-nucleus optical potentials and in pion production from nuclei. Rescattering mechanisms not included in the usual DWBA treatments have been examined and the possibility of performing scattering calculations in momentum space has been considered.

3. OTHER ACTIVITIES

The following invited lectures were given by members of the group:

"Giant Resonances"

(S. Fallieros, Brookhaven National Laboratory, July 1974)

"Nuclear Electroexcitation"

(S. Fallieros, Bartol Research Foundation, May 1974)

"A New Theory of Many-Body Scattering"

(P. S. Levin, Laval University, February 1974)

"Many-Body Scattering Theory: Diseases and Cures"

(F. S. Levin, Physics Dept., Brown University, February 1974)

"A New Theory of Reactive Scattering"

(F. S. Levin, Chemistry Department, Brown University, March 1974)

"A New Theory of Rearrangement Collisions"

(F. S. Levin, University of Connecticut, March 1974)

"A New Theory of Rearrangement Collisions"

(F. S. Levin, New York University, May 1974)

"A New Method in the Theory of Many-Body Scattering"

(F. S. Levin, National University of Mexico, July 1974)

"A New Method in the Theory of Many-Body Scattering"

(F. S. Levin, National Institute of Nuclear Energy, Mexico, August 1974)

"An Optimal Expansion Basis for Many-Body Scattering"

(F. S. Levin, Mini-Symposium on Nuclear Physics, National University of Mexico, August 1974)

"A New Method in the Theory of Many-Body Scattering"

(F. S. Levin, Invited paper, International Conference on Few Body Problems in Nuclear and Particle Physics, Laval University, August 1974)

"A New Method in the Theory of Many-Body Scattering"

(F. S. Levin, Liverpool University, Liverpool, England, October 1974)

"Relativistic Effects in Electron-Nucleus Scattering"

(J. L. Friar, Brookhaven National Laboratory, November 20, 1973)

"High Energy Scattering of Protons from ^{12}C "

(J. L. Friar, University of Montreal, December 6, 1973)

"Relativistic Effects in Electron-Nucleus Scattering"

(J. L. Friar, SUNY, Stony Brook, January 31, 1974)

Other Activities (continued)

"Determination of the Charge Density of ^{208}Pb from Elastic Electron Scattering and Muonic Atom X-Ray Data"
(J. L. Friar, University of Massachusetts, Amherst, February 28, 1974)

"Determination of the Charge Density of ^{208}Pb from Elastic Electron Scattering and Muonic Atom X-Ray Data"
(J. L. Friar, Naval Research Laboratory, Washington, D. C., May 31, 1974)

4. PERSONNEL

Professor S. Palleros

Associate Professor F. S. Levin

Assistant Professor J. L. Friar

Research Associate M. Hirata

YEAR'S PUBLICATIONS

1. D. J. Kouri and F. S. Levin, On Standing Wave Solutions to the Schrödinger Equation, Ann. Phys. 83, 316 (1974).
2. D. J. Kouri and F. S. Levin, On Standing Wave Solutions to the Inhomogeneous Equation for Identical Particle Scattering, Ann. Phys. 83, 326 (1974).
3. D. J. Kouri and F. S. Levin, A New Method for Determining K-Matrix Elements, Phys. Letts. 48B, 203 (1974).
4. D. J. Kouri and F. S. Levin, Eigenstate Expansions in a Soluble Model of Identical Particle Scattering, Nucl. Phys. A225, 521 (1974).
5. D. J. Kouri, F. S. Levin, M. Craigie and D. Secrest, On a General Relationship Between Solutions of Alternate Sets of Equations for Rearrangement Channel T Operators, J. Chem. Phys. 61, 17 (1974).
6. D. J. Kouri and F. S. Levin, A New Method in the Theory of Many-Body Scattering, Phys. Letts. 50B, 421 (1974).
7. D. J. Kouri and F. S. Levin, A New Method in the Theory of Many-Body Scattering, invited paper to appear in the Proceedings of the International Conference on Few Body Problems in Nuclear and Particle Physics.
8. D. J. Kouri and F. S. Levin, Unitarity Modifications of the Coupled Channel T and K Operator Equations for Many-Body Scattering, to appear in the Proceedings of the International Conference on Few Body Problems in Nuclear and Particle Physics.
9. D. J. Kouri and F. S. Levin, Channel T and K Operators and the Heitler Damping Equation for Identical Particle Scattering, Phys. Rev. A. (in press).
10. Y. Hahn, D. J. Kouri, and F. S. Levin, Channel Coupling Arrays and the Reduction Method in Many-Body Scattering, Phys. Rev. C. (in press).
11. Y. Hahn, D. J. Kouri, and F. S. Levin, Coupled Channel T Operator Formalism and Federbush Disease, Phys. Rev. C (in press).
12. D. J. Kouri and F. S. Levin, The K Matrix and Unitarity Constraints on Off-Shell T-Matrix Elements, Phys. Rev. C. (in press).
13. D. J. Kouri and F. S. Levin, New Coupled Equations for Channel T Operators, Proceedings of the March 1974, meeting of the German Physical Society at Stuttgart, p. 375.
14. D. J. Kouri and F. S. Levin, Coupled Equations for the N Channel Scattering Problem, Bull. Am. Phys. Soc. 19, 489 (1974).
15. D. J. Kouri and F. S. Levin, Unitarity Relations for the N Channel Scattering Problem, Bull. Am. Phys. Soc. 19, 489 (1974).

16. D. J. Kouri and F. S. Levin, Coupled Channel T and K Operator Equations with Connected Kernels. I. Three-Body Problem with Pairwise Interactions; II. N. Coupled, Two-Body Channels (submitted to Nuclear Physics).
17. D. J. Kouri, F. S. Levin, M. Craigie and D. Secrest, Computational Study of Channel Coupling Arrays: Low Energy $e^- + H$ Scattering Parameters in the 1s Approximation (submitted to Physical Review).
18. D. J. Kouri and F. S. Levin, Unitarity in Non-Relativistic, Many-Body Scattering. I. Post-Prior Formalism; II. Modification of the Coupled Channel T and K Operator Equations (submitted to Nuclear Physics).
19. D. J. Kouri and F. S. Levin, A. Resolution of Some Orthogonality and Expansion Set Problems in Many-Body Scattering Theory (submitted to Physics Letters).
20. D. J. Kouri and F. S. Levin, General Many-Channel Variational Principles for Multiparticle Scattering (submitted to Physical Review).
21. D. J. Kouri, H. Krüger, and F. S. Levin, Arrangement Channel Quantum Mechanics. I. A. General Time-Dependent Formalism for Multiparticle Systems (submitted to Physical Review).
22. B. Goulard, T. J. Deal, and S. Fallieros, Isoscalar Transitions Densities for Isovector Excitations, Can. Journal Phys. 52, 84 (1974).
23. J. L. Friar and M. Rosen, Dispersion Corrections to Elastic Electron Scattering by ^{12}C and ^{16}O , Ann. Phys. (N.Y.) 87, (1974).
24. J. L. Friar, Pion-Nucleon Absorption Operator Ambiguity, Phys. Rev. C10, 955 (1974).
25. J. L. Friar and S. Fallieros, Relativistic and Retardation Effects in the Thomas-Reiche-Kuhn Sum Rule for a Bound Particle, Phys. Rev. C (in press).
26. J. L. Friar and S. Fallieros, A Comment on the Non-Relativistic Compton Amplitude, Phys. Rev. C (in press).
27. J. L. Friar and J. W. Negele, The Determination of the Nuclear Charge Distribution of ^{12}C from Elastic Electron Scattering, submitted to Nucl. Phys.