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ALGORITHMS FOR REDUCING YTTERBIUM

STRESS-GAGE DATA

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ALGORITHMS FOR REDUCING YTTERBIUM STRESS-GAGE DATA

ABSTRACT

Computer-compatible algorithms for obtaining stress-time histories from the voltage-time histories of two-lead ytterbium gages have been developed from the data of M. J. Ginsberg of the Stanford Research Institute. The technique divides both the loading and unloading behavior of the ytterbium into several regions and fits curves suggested by Ginsberg's work to each region.

INTRODUCTION

Ytterbium has proved to be a satisfactory material for the construction of piezo-resistive stress gages for the range below the phase transformation (i.e., ~30 kbars).¹⁻⁴ However, calibration, particularly for dynamic unloading, has up to now been inaccurate. Recently Ginsberg⁵ produced an excellent report on the detailed calibration of ytterbium stress gages up to 33 kbar, including both loading and unloading. Unfortunately the equations he developed are not satisfactory for the computerized conversion of ytterbium gage data to stress-time histories.

Our practice is to use a digitized microscope to read voltage-time histories from two-lead ytterbium stress gages and to use these digital data to compute stress-time histories. The process requires two steps: conversion of voltage to resistance change, and then, conversion of resistance change to stress. This report discusses the algorithms used to perform these conversions.

RESISTANCE

The most convenient measure of stress in a piezoresistive gage is $\Delta R/R_0$, where ΔR is the stress-induced change in resistance and R_0 the initial resistance of the gage. The circuit used to obtain ΔR is shown in Fig. 1. S. Spataro³ of LLL has solved the circuit. Its behavior is

$$V = \frac{5000 \Delta R}{(50 + R_0)(50 + R_0 + \Delta R)} \cdot \quad (1)$$

All oscilloscopes, of course, must be calibrated for voltage and time vs position on the film. Spataro simplifies the calibration by calibrating for resistance instead of voltage. The oscilloscope is baselined with R_0 in the circuit, and then a series of known resistances is inserted in the circuit to provide steps of known ΔR . Figure 2

Fig. 1. Bridge circuit (after Spataro) for the yttterbium gages.

Fig. 2. Voltage or resistance calibration picture.

shows a resistance calibration picture, and it is clearly nonlinear, as would be expected from Eq. (1). It is now necessary to interpolate between calibration levels, and simple linear interpolation is clearly not justified. We develop the interpolation algorithm as follows:

Let two calibration levels be called level 1 and level 2, where the ΔR represented by level 2 is greater than that for level 1. Then, from Eq. (1)

$$V_1 = V_2 - \frac{5000}{(50 + R_0)} \left(\frac{\Delta R_2}{50 + R_0 + \Delta R_2} - \frac{\Delta R_1}{50 + R_0 + \Delta R_1} \right) \quad (2)$$

and for any voltage V_i between levels 1 and 2

$$V_i = V_1 + \frac{5000}{(50 + R_0)} \left(\frac{\Delta R_i}{50 + R_0 + \Delta R_i} - \frac{\Delta R_1}{50 + R_0 + \Delta R_1} \right). \quad (3)$$

Combining Eq. (2) and Eq. (3)

$$\frac{V_i - V_1}{V_2 - V_1} = \frac{\frac{\Delta R_i}{50 + R_0 + \Delta R_i} - c_1}{c_2 - c_1}, \quad (4)$$

where

$$c_1 = \frac{\Delta R_1}{50 + R_0 + \Delta R_1} \quad (5)$$

and

$$c_2 = \frac{\Delta R_2}{50 + R_0 + \Delta R_2}. \quad (6)$$

We then assume that voltage vs height on the film, Y , is linear between levels 1 and 2, so that

$$\frac{V_i - V_1}{V_2 - V_1} = \frac{Y_i - Y_1}{Y_2 - Y_1}. \quad (7)$$

Then, with a little algebra,

$$\Delta R_i = \frac{(50 + R_o) Q}{1 - Q}, \quad (8)$$

where

$$Q = \frac{Y_i - Y_1}{Y_2 - Y_1} (c_2 - c_1) + c_1. \quad (9)$$

Eq. (8) can be used directly for interpolation between levels.

STRESS-LOADING

Ginsberg developed two calibration curves for loading. For simplicity in writing equations let R represent $\Delta R/R_o$. Ginsberg fits

$$\sigma = 1.082 [1 - \text{EXP}(-20.8 R)] + 9.168 R \quad (10)$$

(where σ is stress in kbars) to the range of 0 to 10 kbar, and

$$\sigma = 13.65 R - 4.358 R^2 + 0.9227 R^3 - 0.0684 R^4 \quad (11)$$

to the range of 0 to 33.4 kbar. Figure 3 shows these curves and Ginsberg's loading data, and it is clear that Eq. (10) is unsatisfactory above 10 kbar and more satisfactory than Eq. (11) below about 4 kbar. We therefore decided to use Eq. (10) in its best region, and Eq. (11) in its best region, and interpolate in the region where it is not clear which is best. Since we start out knowing R , not σ , the regions are based on R . Then where $R < 0.355$ we use Eq. (10), where $R > 0.9351$ we use Eq. (11), and in between, we use the interpolating expression

$$\sigma = 0.541 + 11.409 R - 2.179 R^2 + 0.4613 R^3 - 0.0342 R^4. \quad (12)$$

This expression is satisfactory since by $R = 0.355$ the exponential in Eq. (10) is very small. See Figs. 4 to 6.

STRESS-UNLOADING

Ginsberg was unable to develop a simple scheme to describe unloading. He found that below a peak loading stress σ_p of 10 kbar he could fit his data well with straight line unloading paths, while above 10 kbar he required quadratic or higher order

Fig. 3. The two loading curves of Ginsberg (Eqs. 10 and 11) compared to his loading data.

Fig. 4. Part of the composite loading curve and some computed straight line unloading paths (Eq. 13) compared to Ginsberg's data.

Fig. 5. The composite loading curve, the quadratic through $R_p = 1.1$ (Eq. 22) (this is the lowest R_p through which we fitted a quadratic), and the quadratic through $R_p = 2.0$ (Eq. 21) down which all curves with $R_p > 2.0$ unload are shown. Data from $\sigma_p = 24$ and 29.97 kbar are compared to the unloading quadratics.

Fig. 6. The interpolation scheme. Shown are the straight line through $R_p = 0.5$, the quadratic through $R_p = 1.1$ (and Ginsberg's data from that point), and the interpolated curve through $R_p = 0.77$. They are compared to Ginsberg's data for that region.

expressions to represent his data. He developed an expression for formulating the straight lines from any σ_p up to 10 kbar, but only showed higher order curves from specific values of σ_p . Also, neither his lines nor his higher order curves intersect the loading curve at σ_p , but rather are fitted through data points that scatter about the loading curve.

For computer reduction of the data it is imperative that unloading be continuous with loading — that there be no jump in stress going from one to the other. The unloading curve and loading curve must be the same at σ_p . We therefore had to modify Ginsberg's unloading expressions for our use. Again, we changed methods at the R corresponding to σ_p , which is denoted by R_p .

For $R_p < 0.1$ we follow Ginsberg and unload down the loading path. To develop his straight lines Ginsberg fitted lines to each of several sets of unloading data, fitted another straight line to the slopes of these lines, and then developed a general expression from these slopes and the zero-stress residual resistance. Our procedure was slightly different. We fitted a quadratic (rather than a straight line) to the slopes of Ginsberg's lines, but used lines in $(\sigma - \sigma_p)$ and $(R - R_p)$, and obtained the slopes of the overall fitted lines from this quadratic:

$$\text{slope} = 29.543 - 4.426 \sigma_p + 0.2871 \sigma_p^2$$

$$\sigma = \sigma_p - \text{slope} (R_p - R). \quad (13)$$

The line was then constrained to pass through σ_p, R_p with its slope determined from the quadratic (Fig. 4). This procedure was used between $R_p = 0.1$ and $R_p = 0.5$, which was about the point corresponding to the minimum of the fitted quadratic (σ_p was 5.756 there).

Above $R_p = 1.1$ ($\sigma_p = 11.07$ kb), we followed Ginsberg's polynomial ideas. He uses a quartic fit for $\sigma_p \approx 24, 30$, and 33 kbar. This fit seems to follow the loading curve down to about 15 kbar. We therefore decided that above $R_p = 2.0$ ($\sigma_p = 16.15$ kbar) unloading should follow the loading curve, and then follow the quadratic through the $R_p = 2.0$ point.

So between $R_p = 1.1$ and $R_p = 2.0$ quadratics had to be fitted. It seemed necessary that the quadratic should pass through R_p, σ_p , and from the nature of Ginsberg's data it also seemed desirable that the slope of the unloading curve at that point should be the same as the slope of the loading curve. The third constraint on the quadratic was obtained by fitting a straight line

$$R_0 = 0.0998 + 0.002658 \sigma_p \quad (14)$$

to the zero-stress unloading values R_0 from Ginsberg's data above $\sigma_p = 8$ kbar, and requiring that the quadratic pass through that zero point.

The quadratic is obtained by finding the slope of the unloading curve DP at R_p

$$DP = 11.409 - 4.358 R_p + 1.3839 R_p^2 - 0.1368 R_p^3 \quad (15)$$

for R_p less than 0.9551 (the loading interpolation region) and

$$DP = 13.65 - 8.761 R_p + 2.768 R_p^2 - 0.2736 R_p^3 \quad (16)$$

for R_p greater than 0.9551. Then

$$\sigma = A + BR + CR^2, \quad (17)$$

where

$$C = \frac{DP(R_p - R_0) - \sigma_p}{(R_p - R_0)^2} \quad (18)$$

$$B = DP - 2 CR_p \quad (19)$$

$$A = -BR_0 - CR_0^2. \quad (20)$$

The curve at $R_p = 2.0$, followed by all unloading curves with $R_p > 2.0$ after they come down the loading curve to that point, is

$$\sigma = -1.794 + 12.8492 R - 1.937 R^2. \quad (21)$$

We are still left with the problem of a smooth transition between the straight lines at $R_p < 0.5$ and the quadratics with $R_p > 1.1$. For this we resort to interpolation. For any R whose R_p is in this range we compute two values of stress, σ_1 and σ_2 . σ_2 is obtained from the quadratic through $R_p = 1.1$

$$\sigma_2 = -2.111 + 16.9195 R - 4.487 R^2. \quad (22)$$

If $R < 0.5$ (not R_p , which is always > 0.5), we use the unloading line from 0.5

$$\sigma_1 = 5.756 - 13.579 (0.5 - R) \quad (23)$$

to obtain σ_1 ; otherwise we use the loading curve. Finally, we interpolate by

$$\sigma = \sigma_1 - (\sigma_1 - \sigma_2) \frac{\sigma_p - 5.756}{11.07 - 5.756} . \quad (24)$$

Figures 5 and 6 show these unloading schemes.

STRESS-RELOADING

In the absence of information to the contrary, we simply reload up the unloading curve until $\sigma = 1.01 \sigma_p$. The multiplier is needed to avoid looping because of inaccuracies in the interpolation schemes. Then we proceed up the loading curve.

CONCLUSION

These algorithms have been introduced into GANDALF, our interactive version of the SRI Lagrange-Gage Code.⁶ So far, they have proved satisfactory.

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