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DESIGN OF PROTECTIVE STRUCTURES
(A New Concept of Structural Behavior)

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DESIGN OF PROTECTIVE STRUCTURES

(A New Concept of Structural Behavior)

SYNOPSIS

The design of protective structures is no longer confined to the exclusive domain of military construction. As a result of atomic warfare and with the elimination of previous boundaries of military objective, protective design is now a common problem for both the military installations as well as for civil and industrial buildings. Presently there is but little information available to serve as a guide for the design of such structures. With the objective of providing some aid to the structural engineer in his new and difficult task, this paper presents certain data and design procedure. The presentation is made in two parts: The first part includes declassified experimental data and a procedure used by the Bureau of Yards and Docks of the Navy Department in designing structures to resist conventional weapons of the last war, such as bombs and projectiles. The second part is devoted to a discussion of atomic bomb blast and to a new concept of structural resistance. Based on this concept, an analysis is presented, together with a simplified procedure, for the design of structures to resist atomic blast.

INTRODUCTION

Statements regarding proof construction often convey wrong impressions of strength and structural adequacy. This is due mainly to improper or unqualified use of such stock phrases as "bomb-proof," "atomic-proof," "blast-proof," "splinter-proof," and other expressions of similar generality.

The designation of a building or a structure as proof against a weapon, without complete qualifications regarding the physical and ballistic properties, as well as the range and the orientation of the weapon, is meaningless. Even with such qualifications, no true appraisal can be made of the obtainable protection without a change in our conventional concept and understanding of strength and resistance.

Unlike structural behavior anticipated and obtained in conventional design, the extent of damage due to weapons' loading in a protective construction cannot be fully or clearly determined. In conventional work, a structure is designed to sustain a given condition of loading within limits of elastic strain. Except for rare instances, the intensity of a loading sustained by a building during its service life is not appreciably surpassed by that contemplated in the original design. In contrast, there is no control on the loading which may be imposed on a protective structure. There is no assurance, except that given by the laws of probability, that the condition and the severity of loading as assumed will prevail.

Consequently, the extent of damage may be far beyond the range conceived in design. As a practical corollary, we may have to assume that a building will suffer maximum damage which, in turn, may be taken as a condition short of collapse of local or individual members of framing.

The analysis of the strength of a structure under atomic blast presented here is predicated on this concept of structural behavior. It may be considered as a limit design where locally use is made of the full dynamic resistance of members under relatively large plastic deformations. However, such local overstreses, or even some failures, may not necessarily mean a serious impairment of overall structural adequacy of a building, since in most cases, by proper arrangement of framing, it will still be possible to retain the needed strength and the stability of the structure as a whole.

The two methods of analysis given in the paper--one for impact penetration and the other for blast resistance--are adaptable for practical use. No claim is advanced regarding the exactness of either method. However, owing to the many indeterminate factors affecting the conditions of loading and assumptions of behavior, such exactness is neither deemed feasible of attainment nor essential for a practical or adequate solution of the problem.

For the needed brevity, the scope of the paper is limited primarily to design problems of surface structures built of reinforced concrete.

CATEGORIES OF WEAPONS

For purposes of structural analysis, weapons of modern warfare may be classed in two general groups:

1. Those which cause greatest damage by the impact of their penetrating mass, and
2. Those which cause greatest damage by the blast of their explosive mass.

The first category, which may be referred to as weapons of impact, includes projectiles fired from guns, conventional bombs having a charge-to-weight ratio smaller than 20%, rocket-assisted bombs and guided missiles. The other class, which may be referred to as weapons of blast, includes atomic bombsand high explosive or conventional bombs having a charge-to-weight ratio higher than 20%. In general, weapons of impact cause severe damage locally, while the effect of weapons of blast is characterized by overall damage of relatively less severity.

PART I. RESISTANCE TO IMPACT

<u>Concept of Protection</u>.- The primary objective of a bomb or projectile, as a weapon of offense, is to breach a barrier of defense. The barrier may consist of a barricade, a shield, an outer shell of a structure, or some combination of the three. When the breaching or the penetration is complete, the weapon is said to have attained its optimum offensive efficiency. This efficiency

is usually ascertained by actual tests at arms proving grounds. Hence, the obtained depth of the penetration is called the proof or limit thickness of the weapon. Accordingly, for protection from a weapon of this category, it will be necessary to provide a structural shell where the minimum thickness exceeds that of the proof with a margin of safety yet to be determined.

<u>Penetration</u>.- There are three main factors governing the penetration of the missile into a resisting mass. These are:

1. The velocity of the missile at impact,
2. The physical properties of the missile, and
3. The material's characteristic of the resisting mass.

These factors have been combined in various forms to obtain an empirical expression for penetration by a number of authorities in ballistics. One of these relations, the modified Petry formula, expresses proof thickness as follows:

$$D = k A_p V' \qquad \cdots\cdots (1)$$

in which D is the depth (in feet) of penetration, k is an experimentally obtained material's coefficient for penetration (see Table 1*), A_p is the sectional pressure, obtained by dividing the weight of the missile by its maximum cross-sectional area (expressed as pounds per square foot), and V' is a velocity factor which, in turn, is given by the expression

$$V' = \log_{10}\left(1 + \frac{V^2}{215{,}000}\right) \qquad \cdots\cdots (2)$$

* from "Civil Protection," by F. J. Samuely & C. W. Hamann, the Architectural Press, London, 1939.

where V represents the terminal or striking velocity (in feet per second) of the missile. (Values of V may be obtained from the curve in Fig. 1.)

The striking velocity, V, of a bomb dropped from a plane is computed from its vertical and horizontal components. The vertical component is due to gravity. Neglecting the effect of air resistance or the aerodynamic characteristics of the bomb, it may be expressed as

$$V = \sqrt{2gH} \qquad \cdots\cdots (3)$$

where g is the acceleration due to gravity (in feet per second per second) and H is the height of fall (in feet). The horizontal component is given by the velocity of the plane at the time of release of the bomb. The striking velocities of bombs released at various altitudes from planes flying with various speeds are given in Fig. 2. It is to be noted that the actual velocities are somewhat less than that indicated, due to the compressive and frictional resistances of the air, both of which depend upon the shape and weight of the bomb. These factors have the effect of limiting the velocity to a terminal or maximum possible value, no matter at what height the bomb may be released. This effect would vary according to the type of the bomb, being as a rule greater for heavy bombs. For example, a 200-lb incendiary bomb may have a terminal velocity of only 400 ft per sec, whereas a 500-lb bomb may reach 1600 ft per sec, a 1600-lb bomb, 1100 or 1200 ft per sec, and a 2000-lb bomb, 1300 ft per sec. It is estimated that a 2000-lb A.P. bomb, released from an elevation 16,000 ft, would attain a

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striking velocity of approximately 1000 ft per sec.

Angle of Impact.- Owing to its two components, a bomb will land on its target at an angle with respect to a normal, when released from a plane. The angle of incidence is dependent on the height and the speed of the plane. In addition, the angle of incidence is somewhat dependent on the ballistic properties of the bomb. Approximate angles of impact of bombs released from level-flying planes at various altitudes and air speeds are given in Fig. 2. In these curves, air resistance and the aerodynamics of the bombs have been neglected.

It is estimated that a 2000-lb A.P. bomb released from a plane flying horizontally at 250 miles per hour, at approximately 16,000 ft elevation, attains a striking velocity of 1000 ft per sec, and lands at an angle of impact approximately 70° to the horizontal or with an obliquity of 20°.

From the foregoing, it is evident that bombs may strike both the roof as well as the walls or the sides of a building, and that a pitched roof having a slope of 20° may sustain normal hits from bombs.

The Effect of Relative Thickness on Depth of Penetration.- The penetration formula given in Eq. (1) is applicable only to slabs where the thickness of the slab is many times larger than the depth of penetration. Experiments indicate that for a given condition of impact loading, there will be a minimum depth of penetration when the thickness of the resisting slab attains a minimum value. Expressed in terms of the depth of minimum penetration, D, this minimum thickness of the resisting slab is about 3D. That is to say, in slabs having a thickness 3D or more, the depth of penetration under a given projectile of a stated terminal velocity will remain about constant. However, if the thickness of the resisting slab is less than 3D, the depth of penetration, D' will be larger than D. With the decrease in the thickness of the slab, the depth of penetration will increase until perforation is obtained. This is the condition where the resisting slab has a thickness of only 2D. According to the Navy* experiments, the relation between slab thickness and depth of penetration may be expressed by

$$D' = D\left[1 + e^{-4(a'-2)}\right] \qquad \cdots\cdots (4)$$

This relation is shown diagramatically in Fig. 3 and also by a curve in Fig. 4.

Penetration Due to Explosive Charge.- As discussed above, the efficiency of an impact bomb is gaged by its power of penetration which, in turn, is reflected by the cross-sectional weight or density. For a given weight of bomb, the heavier the casing the larger is the penetration. Since the amount of explosive charge contained in a bomb is obtained by a corresponding reduction in the thickness of the casing, the weight of the explosives in A.P. (armour-piercing) and S.A.P. (semi-armour-piercing)

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* All references to Navy pertain to the Bureau of Yards and Docks of the Navy Department.

types of bombs is kept to a minimum. The total penetration of a bomb with a charge may be assumed to be the sum of two separate penetrations; one due to impact and the other to explosion. In the Navy experiments, the effect of such bombs was simulated by first firing an inert projectile into the resisting model, then placing a charge in the impact crater and exploding it.

An approximate value for depth of penetration due to explosion of a TNT charge in a concrete slab of great thickness (about 3 to 4 times the depth of the resulting penetration) may be obtained from the relation

$$D_e = c'\sqrt[3]{C} \qquad \cdots\cdots (5)$$

where D_e is the depth of penetration (in ft) C, is the weight of the charge (in pounds) and c' is a penetration constant. In "Civil Protection" the following values are given for c': mass concrete, 0.26; ordinary reinforced concrete, 0.22; specially reinforced concrete, 0.18. The value obtained in the Navy experiments, where use was made of special reinforcing and a concrete having an actual 28-day strength of about 4000 psi, was about 0.2.

Extent of Damage in Concrete Slabs Due to Impact and Explosion.- When a bomb or projectile strikes a concrete slab, there results a crater of rather irregular shape. In addition to the cavity in the face of impact, there may be considerable cracking in the opposite face of the slab. The severity of such cracking increases with decreased thickness, becoming critical in the form of scabbing and spalling when limit or perforation thickness is neared. Figs.

5 to 8 illustrate typical cases of damage obtained in the Navy tests From the deformation pattern of the reinforcing it can be seen that concrete in both faces of the slab tends to break and displace outwardly. This is due to the inherent weakness of concrete in tension to resist the scouring action of the reflected shock wave in the impact face and the scabbing effect of the propagated wave in the opposite face. The extent and tendency of failures of this type may be minimized by the use of special systems of reinforcing to provide the needed tensile strength. Fig. 9 shows the details of such a system of reinforcing utilized by the Navy in its bomb-resistant structures built during the last war. The arrangement, consisting of welded bar trusses, is obtained by joining the main reinforcing of two faces of a slab and the zigzag webbing to form a Warren truss. The limited extent of scabbing and the pattern of failure observable in Fig. 7(b) are traceable to the use of this type of reinforcing in that particular test slab.

<u>The Principle of Divided Thickness of Protection.</u>- Since it is not feasible to eliminate scabbing entirely, a roof slab possessing sufficient thickness for minimum penetration will still fail to provide full protection against the possibility or hazard of falling debris from the ceiling. For the needed safety, in some cases use is made of a so-called anti-scabbing plate, consisting of a steel plate attached to the ceiling by means of anchors cast in the slab. Another method for securing the desired protection is to use a double-slab construction. For this purpose, the design

thickness is divided into two parts; one thickness for an outer or roof slab and another for an inner or ceiling slab. The outer slab is designed for impact perforation, that is, a thickness of about 2D, while the inner slab is designed for minimum penetration due to the explosive charge or a thickness of about $3D_e$.

The basic advantage of this arrangement is that no appreciable strains due to the impact shock wave are imparted to the inner slab. Should explosion occur on contact or during penetration of the bomb into the outer slab, the inner slab will be protected also against the strains from this source. On the other hand, should explosion occur after perforation, that is, the charge exploding in the gap provided between the two slabs, its scabbing effect will again be minimized to the extent of difference of severity of shock of an open vs. confined explosion. As an additional advantage, by localizing the main damage in the outer slab, the work necessary for subsequent repairs is simplified, and the continued use of the affected building is assured during such repair operations.

PROCEDURE OF DESIGN

Design Data.- Complete information is needed regarding the weapons and the conditions of impact against which protection is to be provided. This information, which may be called the basic design data, should include the weight, charge and sectional properties of the missile, and the velocities and angles of anticipated impact. In most cases, the critical loading for the roof and the walls of a

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structure will differ. For example, if the building is to be located near the shore, within the range of naval ship guns, then the critical loading for the exposed walls would be that due to an A.P. type projectile fired from a distance just beyond the range of the defending shore battery. On the other hand, the design of the roof will be governed by the loading of an A.P. type bomb dropped from an altitude deemed as safe for the carrier plane.

The following criteria were used by the Bureau of Yards and Docks in providing the heaviest type of protection to the Navy's vital shore structures built during World War II.

(a) A.P. type aerial bomb.

Weight = 2,000 lb; charge = 300 lb; sectional pressure = 1,500 lb per sq ft; terminal velocity = 1,000 ft per sec; angle of impact = 20° (from vertical);

(b) A.P. type projectile.

Weight = 200 lb (8-inch projectile); sectional pressure = 745 lb per sq ft; terminal velocity = 1,300 ft per sec; angle of impact, normal hit.

Computation of Thicknesses.- The total thickness to be provided for each part of a framing will depend on the minimum depth of penetration and the desired degree of protection.

The minimum depth of penetration under impact and explosion are given by Eqs. (1) and (5), respectively. Assuming that use will be made of special reinforcing and class E concrete, having a nominal 28-day compressive strength of 3,000 psi and an actual

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strength of 4000 psi*, the factor in Eq. (1) may be taken as $k = 0.0028$. For other strengths of concrete, an approximate value for k may be obtained from Fig. 10. For the bomb and the conditions of impact specified in (a) above, we will have:

$$D = 0.0028 \times 1500 \times 0.75 = 3.15 \text{ ft.},$$

$$D_e = 0.2 \sqrt[3]{300} = 1.34 \text{ ft.}$$

The axis of the crater in a slab will about coincide with the direction of the force of impact. Since the bomb is assumed to land at an angle of 20^o, the depth of penetration measured normal to the face of a slab in a given position will then be

1. Slab in a horizontal plan:

$$D_h = D \cos 20^o = 2.96 \text{ ft.},$$

$$D_{eh} = D_e \cos 20^o = 1.26 \text{ ft.}$$

2. Slab in a vertical position:

$$D_v = D \sin 20^o = 1.08 \text{ ft.},$$

$$D_{ev} = D_e \sin 20^o = 0.46 \text{ ft.}$$

The condition of impact specified in (b) is that of a horizontal hit. The depth of penetration in a vertically placed slab is then

$$D = 0.0028 \times 745 \times 0.95 = 1.98 \text{ ft.}$$

To obtain the design thicknesses, we must now apply a thickness factor against the minimum penetration corresponding to each condition of impact.

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* In the Navy tests the actual 28-day compressive strength of this class of conrete varied from 3500 to 4500 psi, and the obtained average k value was about 0.0028.

If a double-slab roof framing is to be used, the factor of thickness for the outer slab will be 2, which would result in perforation due to impact penetration of the bomb. The required thickness is then

$$T_t = 2 \times 2.96, \text{ or about } 6 \text{ ft.}$$

For the inner slab, a factor of 3 will be ample for protection against the explosive charge of the bomb. Accordingly,

$$T_b = 3 \times 1.26, \text{ or about } 4 \text{ ft.}$$

In the case of a roof composed of a single slab, the factor of thickness, to be applied against the total penetration of the bomb, may vary from 2.5 to 3, depending whether an anti-scabbing plate is to be used or not. The required design thickness will correspondingly be

$$T_1 = 2.5(2.96 + 1.26), \text{ or about } 11 \text{ ft.},$$

and $$T_2 = 3(2.96 + 1.26), \text{ or about } 13 \text{ ft.}$$

For the walls of a building a thickness factor of 2.5 will suffice. Then, if the wall is to be protected against shell fire, the required design thickness is

$$T_1 = 2.5 \times 1.98, \text{ or about } 5 \text{ ft.}$$

If the wall is not oriented towards the sea, or located within critical range of naval gun fire, then the required design thickness for protection against oblique hits from bombs will be

$$T_2 = 2.5(1.08 + 0.46), \text{ or about } 4 \text{ ft.}$$

The design thicknesses for protection against other impact weapons may similarly be computed. The procedure may also be used for determining protective thickness against bomb fragments and missiles of various type, provided reasonably satisfactory assumptions can be made regarding their weights, shapes and the velocities at impact.

Where steel plates are to be used in combination with concrete, in protective value 1-inch thick ordinary structural grade steel may be taken as equal to 12-inch thick concrete of about 3000 psi strength, and for specially-treated steel the equivalence is about 18 inches of concrete.

Arrangement of Framing.- A typical arrangement for a bomb-resistant structure is shown in Fig. 11. The building, the Navy's bomb-resistant personnel shelter, was designed in accordance with protection criteria outlined above. The main features of framing include the following.

The roof consists of two slabs: an outer slab of 6-ft depth and a ceiling slab of 4-ft depth. The outer slab, supported on a series of piers at the walls, projects 5 feet beyond the wall line to form a protective canopy over the ceiling slab against a direct hit.

All walls, including the barricades sheltering the entrance passage ways, are 4 ft thick.

A 4-ft slab is also used in the floor to resist underground explosions. In addition, a 20-ft wide burster slab provides protection against under-floor penetration of light-case, high-explosive bombs.

Typical details of reinforcing are shown in Fig. 9. The arrangement marked Type A, with sizes as indicated in Detail 1, may be considered as minimum reinforcing. Should the elastic analysis of the framing under dead, snow, wind and other live loading indicate the need of additional reinforcement, Type B arrangement, with proper size of reinforcement, may be used.

PART II. RESISTANCE TO BLAST

A. DESIGN DATA

Atomic Blast.- With the development and use of the atomic bomb, a new criteria is introduced in the design of protective structures. The efficiency and destructive power of this new weapon is more vividly illustrated by equivalence or comparison of its energy of fission to that resulting from conventional explosives. For an atomic bomb of the type dropped in Japan, the equivalence is 20,000 tons of TNT. Hence, this bomb, which serves as a basis for establishing a design loading for blast, is referred to as a 20 kilo-ton bomb.

A great deal of information is now available regarding the nature and effects of atomic explosion*. The destructive effects of the fission are caused by the shock wave and the thermal and

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* "The Effects of Atomic Weapons," by Los Alamos Scientific Laboratory; Govt. Print. Office, Aug. 1950.

nuclear radiations associated with the phenomenon. From the point of view of structural adequacy, resistance to the shock wave constitutes the primary problem in protective construction.

A shock wave is essentially a pressure wave initiated by the energy released during an explosion. The general pattern of this pressure wave is shown in Fig. 12. As will be noted, it is characterized by a shock front, where the intensity of pressure sharply rises from atmospheric to a maximum or peak value, followed first by a positive then a negative phase. The intensities of the pressures and their durations decrease with the propagation of the shock wave. Similarly, the velocity of the shock front decreases from an indeterminate maximum near the center of the explosion to the velocity of sound at a distance of about 10,000 ft. For design purposes, in the distance range of 2000 ft to 10,000 ft from the center of explosion, the velocity of the shock wave may be considered as constant and equal to 1400 ft per sec. The variations in peak pressures in this range, and corresponding to a 20 kilo-ton bomb, are shown in Fig. 13*; and the durations of the positive phase are given in Fig. 14*.

In the following analysis, the investigation will be confined to the positive phase of the shock wave. Since the pressures in the negative phase are relatively small, no investigation for this period is required.

Blast Pressures on a Structure.- When a shock wave encounters

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* From "The Effects of Atomic Weapons" pp. 52 and 54.

an obstruction in its path, such as a structure, the pressure pattern is disturbed. The extent of the disturbances will vary at the various faces of the structure in accordance with the angularity and orientation of the element or the created interference. Owing to the many variables involved, the disturbance is in the form of a complex phenomenon for which complete information is as yet not available. Presently the most convenient and practical method of studying the disturbances caused by obstacles in the path of a blast wave is by means of "shock tubes." A number of investigations in this field have been made, of which those at Princeton University* constitute a valuable source of information. The pressure distribution curves shown in Figs. 15, 16, and 17 are based primarily on these data. In this connection, the following are to be noted. All pressures represent an average condition over each face of the structure. In Fig. 15, a factor of 3 was applied to peak pressure, p_0, to obtain the maximum reflected pressure on the front wall. This corresponds to an angle of incidence of about 70°**. The stagnation pressure at the end of the period t_1 is taken as equal p_0. To simplify the analysis, the variations in pressure in both time intervals t_1 and t_2 are assumed to be linear. This assumption was also made for the pressures shown in Fig. 16 and 17. In

* "The Diffraction of Shock Waves Around Obstacles and the Transient Loading of Structures," by Walker Bleakney. Princeton University, Dept. of Physics. March 16, 1950.

** "The Effects of Atomic Weapons," p. 123.

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Fig. 16, the initial pressure at any transverse section of the roof is taken as p_0, dropping to a value of $\frac{1}{3}p_0$ in the time required for the vortex to reach that section. The vortex velocity is assumed to be constant and equal to $\frac{1}{7}$ of the shock velocity. In Fig. 17, an initial pressure of $\frac{2}{3}p_0$ is used.

B. ANALYSIS

General Phases of Investigation.- The analysis of a structure under blast involves two main investigations: (1) Strength, and (2) Stability.

The strength computation, in turn, may be divided in two parts: (a) Local strength or the resistance of individual members, and (b) Overall strength of the framing or assembly.

The investigation of stability will include: (1) a study for stability against sliding, and (2) a study for stability against rotation or overturning.

LOCAL STRENGTH

Concept of Deformation.- To ascertain the strength of a framing arrangement, we must first consider the strength of each individual member. Assuming that the member will have adequate supports for transfer of its load, then the problem becomes a study of anticipated deformations of the member relative to its supports. Under an impulse loading, the strength behavior of the member, or its capacity to absorb an impulse loading, will be governed in a large measure by its ductility. That is to say, the larger the de-

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formation or its axial elongation and the corresponding sag or deflection, the greater its capacity to absorb the imposed loading. The ductility of a reinforced concrete member to produce desired deformations for resistance is governed by the ductility of its reinforcement.

When a reinforced concrete member is subjected to an impulse loading many times greater than its elastic resistance, the member will pass through three stages of deformations: (a) an initial stage corresponding to elastic behavior; (b) an intermediate stage of partly elastic and partly plastic behavior; and (c) a final stage of wholly plastic behavior. In the first stage, the member will behave as a fully restrained element, developing maximum resisting moments at the supports. This period of elastic behavior terminates when the end moments reach the ultimate elastic capacity of the section. As the deformations increase, the resisting moments at the supports will increase to the plastic capacity of the section and the reinforcing will deform locally to form plastic hinges at these locations. As a result, the ends will rotate and the member will deflect as in the case of a simply supported beam. This is the second stage of deformation. When the plastic moment capacity of the section at the center of the span is reached, the member enters into its final stage of deformation, characterized by extensive cracking and by overall elongation of the reinforcement. The passage from the intermediate to the final stage of deformation is accompanied by a loss in moment capacity and an increase in the axial tension. With continued deformations, the moment resistance rapidly diminishes to a negligible value and the member axially elongates in the form of a simple parabolic curve. Failure will generally occur by rupture of the reinforcement similar to a bar in tension. If a factor is applied against the maximum deflection to prevent such failure, then the resulting deformation may be considered as a limiting condition in obtaining a practical or working resistance in design. This deformation may be stated in terms of axial elongation, or by unit strain. Assuming that the reinforcement will elongate uniformly, then the unit strain could be obtained by dividing the total elongation of the reinforcement by the overall length of the member. A maximum deflection of one-tenth of the span length is suggested as a limiting value. This corresponds to an average elongation of about 2.8 percent.

Modulus of Resistance.- Fig. 18 illustrates the foregoing concept of deformation of a member. The deflection curve shown in Fig. 18 (a), corresponding to the first stage of deformation, may be considered as that due to a uniformly distributed load q_a. The deflection at the center of the span, y_a, will then equal

$$y_a = \frac{1}{384} \cdot \frac{q l^4}{E_c I_c}$$

in which E_c is the modulus of elasticity and I_c the moment of inertia per unit width of member. From which

$$q_a = \frac{384 \, E_c I_c}{l^4} \cdot y_a$$

In Fig. 18 (b), the deformation indicated by the dashed line represents the beginning of plastic deformation where the moments at the supports have reached their ultimate plastic or yield value,

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M_r. The full line represents the beginning of the transitional period. At this point the deflection is y_{b1}, which again may be considered as produced by the load q_{b1}. The relation between q_b and y_b is then given by

$$q_{b1} = \frac{192\, E_c I_c}{l^4} \cdot y_{b1} \qquad (6)$$

Fig. 18(c) shows the transitional stage in which it is assumed that the member continues to deflect under a constant load q_c. The value of this load may be expressed either in terms of M_r obtained at the beginning of the period or in terms of a reduced moment, M'_r, and an axial tension, S'. It is known that the total stress in the reinforcement resulting from both sources of strain will remain constant and equal to the yield value strength of the steel in this period. However, since the relation between M'_r and S' cannot be clearly defined and conveniently expressed, the value of y_c may be obtained from M_r alone. Thus

$$q_c = \frac{16\, M_r}{l^2} \qquad (7)$$

Fig. 18(d) shows the final stage of deformation where, owing to extensive cracking, no appreciable resisting moments exist. The relation between deflection and load will then become

$$q_d = \frac{8\, S}{l^2} \cdot y_d \qquad (8)$$

in which S is the ultimate yield strength of the reinforcement per unit width.

In the relations given above, the load q is a measure of strength of the member and accordingly may called the "modulus of resistance" of the member, or simply the "resistance."

The relation between resistance and deflection, throughout the full range of deformations is shown diagrammatically in Fig. 18 A. Here the line O A indicates the first or elastic stage; A B, the partly elastic and partly plastic stage; B C, the transitional period; and C D the final or fully plastic stage of deformation. At point C, q_c is equal to q_d. Hence, from Eqs. (7) and (8)

$$\frac{16 M_r}{l^2} = \frac{8 S}{l^2} \cdot y_{c1}$$

from which

$$y_{c1} = \frac{2 M_r}{S} \tag{9}$$

If the effect of resisting moments is neglected, then the deflection-load relation will be represented by the straight line O C D. This is the case for a relatively thin element supported on two ends.

In the case of a thin framing element supported on four sides, Fig. 19, such as a reinforced concrete slab of small depth, or a steel plate, the moments M_r will have only a negligible effect on the resistance of the member, and, hence, may be omitted in the resistance equation. Under large plastic deformations, such an element will deflect as a membrane. The deflection y' at any point (x, z) is given by

$$y' = \frac{4 q l_1^2}{\pi^3 S} \sum_{n=1,3,5}^{n=\infty} \frac{(-1)^{\frac{n-1}{2}}}{n^3} \cdot \left(1 - \frac{\cosh \frac{n \pi z}{l_1}}{\cosh \frac{n \pi l_2}{2 l_1}} \right) \cos \frac{n \pi x}{l_1} \, ,$$

and at the center

$$y = \frac{4 q l_1^2}{\pi^3 S} \sum_{n=1,3,5}^{n=\infty} \frac{(-1)^{\frac{n-1}{2}}}{n^3} \cdot \left(1 - \frac{1}{\cosh \frac{n \pi l_2}{2 l_1}} \right)$$

From which the modulus of resistance, q, is obtained as

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$$q = \frac{\pi^3 S y}{4 l_1^2 \sum_{n=1,3,5}^{n=\infty} \frac{(-1)^{\frac{n-1}{2}}}{n^3} \cdot \left(1 - \frac{1}{\cosh \frac{n\pi l_2}{2 l_1}}\right)} ,$$

or,

$$q = \frac{\gamma S y}{l_1^2} \tag{10}$$

Values of the resistance factor, γ, are given in Fig. 20.

Fundamental Relation of Motion.- The effect of q is that of a negative force, acting in an opposite direction to the applied load p. The net force f, equalling p - q, will then represent the accelerating force of a unit mass, w/g. Thus

$$f = p - q = \frac{w}{g} \alpha \;;$$

which is the fundamental relation of motion.

For a unit area located at a distance x from the supports, the displacement, y', will diminish from y to 0. Assuming that the ratio of y' to y during the period of motion remains constant, it is then possible to express the motion of any point in terms of the motion at the center line by means of a reduction factor applied to the unit area and the unit mass at that point. That is to say, the equation of motion for a unit area having a displacement y' may be written as that of an equivalent reduced area having the same displacement y at the center and an equivalent reduced mass having the the corresponding acceleration α. Thus

$$f' = \beta (p - q) = \beta_1 \frac{w}{g} \alpha \;;$$

where β is the reduction factor for area, and β_1 for mass. Then the expression of motion for the entire member becomes

$$F = \beta A (p - q) = \beta_1 \frac{W}{g} \alpha \tag{11}$$

in which A is the total surface of the member, W its total weight,

Y-200-25

$$\beta = \frac{\int y' \, dA}{y A} \, , \quad \text{and} \quad \beta_1 = \frac{\int (y')^2 \, dW}{y^2 W} \, .$$

For one-way slabs or beams $\beta = \frac{2}{3}$ and $\beta_1 = \frac{8}{15}$. For two-way slabs values of β and β_1, for various length-to-width ratios, are given in Fig. 20.

APPLICATIONS

Having established the fundamental relation of motion pertaining to a local element, the next step of the analysis consists of the application of the relation to various conditions of blast loading. For this purpose, use will be made of a simple rectangular building having vertical walls and a flat roof, as shown in Fig. 15. The critical condition of each wall would be an orientation or exposure to frontal attack of the blast wave.

Case I. Front Wall.

Condition 1. Moment Resistance Neglected.- The relation between load and deformation is given by the straight line O D of the resistance diagram shown in Fig. 18 A. The pressure-time curve for this case is shown in Fig. 15. The pressure p_1, in period t_1, is given by

$$p_1 = p_0 \left(3 - 2 \frac{t}{t_1} \right) \tag{12}$$

and that in the period t_2 is

$$p_2 = p_0 \left(\frac{\tau - t}{t_2} \right) \tag{13}$$

Y-200-26

Assuming that the wall framing consists of a slab reinforced in one direction (in the direction of h), the equation of motion corresponding to the two time periods is then obtained by substitution of the values of p, given by Eqs. (12) and (13) and q, given by Eq. (8), in Eq. (11),

$$\beta A \left[p_o \left(3 - 2\frac{t}{t_1} \right) - \frac{8S}{h^2} y \right] = \beta_1 \frac{W}{g} \alpha \; ; \tag{14}$$

$$\beta A \left[p_o \left(\frac{\tau - t}{t_2} \right) - \frac{8S}{h^2} y \right] = \beta_1 \frac{W}{g} \alpha \quad , \tag{14a}$$

or in the general form

$$\frac{d^2y}{dt^2} + r^2 y = a - bt \; , \tag{15}$$

in which $\dfrac{d^2y}{dt^2}$ is the acceleration α, $r^2 = \dfrac{8\beta g A S}{\beta_1 h^2 W}$, and a and b, for the periods t_1 and t_2, are:

$$\left.\begin{aligned} a_1 &= \frac{\beta g A}{\beta_1 W} \cdot 3 p_o \quad , & b_1 &= \frac{2\beta g A}{\beta_1 W t_1} \cdot p_o \; , \\ a_2 &= \frac{\beta g A}{\beta_1 W} \cdot \frac{\tau}{t_2} p_o \quad , & b_2 &= \frac{\beta g A}{\beta_1 W t_2} \cdot p_o \; . \end{aligned}\right\} \tag{16}$$

The general solution of this ordinary differential equation is in the form

$$y = c_1 \sin rt + c_2 \cos rt + \frac{a - bt}{r^2} \tag{17}$$

and

$$\frac{dy}{dt} = r (c_1 \cos rt - c_2 \sin rt) - \frac{b}{r^2} \tag{18}$$

The values of the two constants, c_1 and c_2, are obtained from the consideration

Y-200-27

$$t = 0, \quad y = 0 ;$$

$$t = 0, \quad \frac{dy}{dt} = 0 ;$$

which gives

$$c_1 = \frac{b_1}{r^3} , \qquad c_2 = - \frac{a_1}{r^2} .$$

Substituting these values in Eqs. (17) and (18), the displacement and velocity equations,

$$y = \frac{1}{r^2} \left[b_1 \left(\frac{\sin rt}{r} - t \right) + a_1 (1 - \cos rt) \right] ; \qquad (19)$$

$$\frac{dy}{dt} = \frac{1}{r^2} \left[b_1 \left(\cos rt - 1 \right) + a_1 r \sin rt \right] . \qquad (20)$$

Similarly, in the period, t_2,

$$y = c_3 \sin rt + c_4 \cos rt + \frac{a_2 - b_2 t}{r^2} ; \qquad (21)$$

$$\frac{dy}{dt} = r(c_3 \cos rt - c_4 \sin rt) - \frac{b_2}{r^2} . \qquad (22)$$

The constants c_3 and c_4 are determined from the consideration that at the beginning of this period the displacement and velocity will be the same as the respective values at the end of the first period, as given by Eqs. (19) and (20), and where $t = t_1$. Let Δ_1 indicate the displacement y, and V_1 the velocity $\frac{dy}{dt}$ at the end of the period t_1, then

$$c_3 \sin rt_1 + c_4 \cos rt_1 = \Delta_1 - \frac{a_2 - b_2 t_1}{r^2} ,$$

$$c_3 \cos rt_1 - c_4 \sin rt_1 = \frac{V_1}{r} + \frac{b_2}{r^3} ,$$

Y-200-28

from which

$$c_3 = \sin rt_1 \left(\Delta_1 - \frac{a_2 - b_2 t_1}{r^2}\right) + \cos rt_1 \left(\frac{V_1}{r} + \frac{b_2}{r^3}\right) \qquad (23)$$

and

$$c_4 = \cos rt_1 \left(\Delta_1 - \frac{a_2 - b_2 t_1}{r^2}\right) - \sin rt_1 \left(\frac{V_1}{r} + \frac{b_2}{r^3}\right) . \qquad (23a)$$

The motion may stop either in the first or second period. Assuming that the latter is the case, the value of t, the time in the period t_2 when motion stops, can be obtained from Eq. (22),

$$\frac{dy}{dt} = r\,(c_3 \cos rt - c_4 \sin rt) - \frac{b_2}{r^2} = 0 \ , \qquad (22a)$$

or an approximate value from the relation

$$\cos rt = \frac{1}{\pm\sqrt{1 + \left(\frac{c_3}{c_4}\right)^2}} \qquad (24)$$

In general, the motion of walls reinforced in one direction will stop in the second period. On the other hand, in the case of a thin slab or a steel plate supported on four sides, the motion will generally stop in the first period. However, an investigation should be made in all cases to ascertain whether or not the motion stops during the first period. This can be accomplished by equating $\frac{dy}{dt}$ in Eq. (20) to zero and solving for t. If the smallest value of t thus obtained is less than t_1, then the motion stops in the first period and the magnitude of the final displacement is obtained from Eq. (19).

In the case of a slab supported on four sides, the derivations presented above apply, with due allowance for the changes in the resistance factor γ, and reduction factors β and β_1.

Y-200-29

Condition 2. Moment Resistance Considered.-

The relation between load and deflection for this case is shown by the broken line O A B C D of the resistance diagram in Fig. 18 A. It is to be noted that part O A B is predicted on the use of a constant moment of inertia, I_c. Obviously this condition of constant moment of inertia cannot prevail throughout this range of the deformation owing to the continued cracking of the concrete. If I_c at each stage of the deformation is modified to its actual value, line O A would tend to shift towards line O B, that is to say, for the same load there would be larger deflections than indicated by the diagram. Since it is not practicable to determine the true position of the resistance line, it may be taken as the straight line O B, using a constant value for I_c predicated on the full section of the member. In the period represented by the line O B, the relation between deflection and load is then given by

$$q_b = \frac{192\, E_c I_c}{l^4} \cdot y_b \tag{6a}$$

The purpose of the analysis is to determine the extent of maximum deflection which, as explained above, occurs when the motion in the direction of the applied pressure stops; that is when $\frac{dy}{dt} = 0$. Were the resistance-deflection and pressure-time relations continuous functions, it would be possible to determine the maximum deflection in one operation by placing the value of $\frac{dy}{dt}$ in Eq. (20) equal to zero and solving for y by Eq. (19). However, due to the discontinuity of the resistance line at points B and C, it becomes necessary to determine the time and the attained velocity at these two points before proceeding to the next stage of deformation.

Y-200-30

In addition, as in the previous case, owing to the discontinuity in the pressure-time curve, it will also be necessary to determine the deflection and the attained velocity at the time t_1.

The analysis of motion corresponding to the resistance line O B in Fig. 18 A, is the same as in the preceding case. During this period the resistance q is given by Eq. (6a). Accordingly,

$$r^2 = \frac{\beta g A}{\beta_1 W} \cdot \frac{192 E_c I_c}{h^4}$$

and the values of a_1 and b_1 remain the same as given by Eqs. (16). Substituting the value of y_{b1} for y, in Eq. (19),

$$y_{b1} = \frac{1}{r^2}\left[b_1\left(\frac{\sin r t_{b1}}{r} - t_{b1}\right) + a_1(1 - \cos r t_{b1}) \right] \quad ; \qquad (19a)$$

From this equation the value of t_{b1} is obtained by trial. The velocity, V_{b1}, at the end of this period is determined by substituting the value of t_{b1} in Eq. (20).

In the period corresponding to the resistance line B C, the resistance is constant and the equation of motion takes the following form:

$$\beta A \left[p_o\left(3 - 2\frac{t}{t_1}\right) - \frac{16 M_r}{h^2} \right] = \frac{\beta_1 W}{g}\alpha \qquad (25)$$

If the discontinuity in the pressure-time relation should occur in this period, then during the interval t_2 Eq. (25) becomes:

$$\beta A \left[p_o\left(\frac{\tau - t}{t_2}\right) - \frac{16 M_r}{h^2} \right] = \frac{\beta_1 W}{g}\alpha \qquad (25a)$$

Eqs. (25) and (25a) may be written in the general form

$$\frac{d^2y}{dt^2} = a - bt$$

in which b for the periods t_1 and t_2 is given by Eq. (16) and a is given by

$$a_1 = \frac{\beta g A}{\beta_1 W}\left(3 p_o - \frac{16 M_r}{h^2}\right) \; , \; a_2 = \frac{\beta g A}{\beta_1 W}\left(\frac{\tau}{t_2} p_o - \frac{16 M_r}{h^2}\right) \qquad (16a)$$

Y-200-31

The general solution of this ordinary differential equation is in the form

$$y = c_1 + c_2 t + \frac{a t^2}{2} - \frac{b t^3}{6} \tag{26}$$

and

$$\frac{dy}{dt} = c_2 + at - \frac{b t^2}{2} \tag{27}$$

The values of the constants c_1 and c_2 are obtained from the consideration

$$t = t_{b1} \quad , \quad y = y_{b1} \quad , \text{ and } \quad \frac{dy}{dt} = V_{b1}$$

which gives

$$c_1 = y_{b1} - c_2 t_{b1} - \frac{a t_{b1}^2}{2} + \frac{b t_{b1}^3}{6} \quad \text{and} \quad c_2 = V_{b1} - a\, t_{b1} + \frac{b t_{b1}^2}{2}$$

The values of the displacement Δ_1 and the velocity V_1 are found by substituting t_1 for t in Eqs. (26) and (27).

The time t_{c1} and the velocity V_{c1} corresponding to the end of this period, at point C on the resistance line, are obtained as in the preceding step by replacing the constants c_1 and c_2 by new constants c_3 and c_4. Thus

$$y = c_3 + c_4 t + \frac{a_2 t^2}{2} - \frac{b_2 t^3}{6} \tag{28}$$

and

$$\frac{dy}{dt} = c_4 + a_2 t - \frac{b_2 t^2}{2} \tag{29}$$

The values of c_3 and c_4, and t_{c1} and V_{c1} are obtained in the same manner as before.

In the final period of deformation, indicated by the resistance line C D, the equation of motion is

$$\frac{d^2 y}{dt^2} + r^2 y = a_2 - b_2 t \tag{15a}$$

Y-200-32

in which $$r^2 = \frac{8\beta g A S}{\beta_1 h^2 W}$$

and a_2 and b_2 are given by Eqs. (16).

The solution of Eq. (15a) is

$$y = c_5 \sin rt + c_6 \cos rt + \frac{a_2 - b_2 t}{r^2} \qquad (17a)$$

and

$$\frac{dy}{dt} = r(c_5 \cos rt - c_6 \sin rt) - \frac{b_2}{r^2} \qquad (18a)$$

Values of the constants c_5 and c_6 are determined from the condition at the beginning of this period, when

$$y = y_{c1}, \quad t = t_{c1} \quad \text{and} \quad \frac{dy}{dt} = V_{c1}$$

The maximum deformation is then found by determining the time at which $\frac{dy}{dt} = 0$ in Eq. (18a), and solving Eq. (17a) for y using the obtained value of t.

In the foregoing analysis it was assumed that the point of discontinuity of the pressure-time curve, corresponding to time t_1, would occur in the period defined by the resistance line B C in Fig. 18 A. However, it may occur in any one of the three periods. As the time in each step is determined a comparison with the time t_1 will indicate whether the pressure discontinuity occurs in that period.

It is to be noted that the motion may stop in any one of the three periods of resistance. Should it occur in the first period of resistance, indicated by line O B in Fig. 18 A, the member may be considered as over-designed; necessitating a revision of the section. If it should occur in the second period of resistance, indicated by line B C, the design may be considered conservative.

Y-200-33

For an economical design it would be desirable to so proportion the section that the motion will stop in the third period of resistance, indicated by line C D, provided that the maximum allowable deformation is not exceeded.

For an approximate analysis the resistance line O D in Fig. 18 A may be substituted for the broken line O B C D, in which case Condition I will apply.

Case II. Roof.

The pressure-time curve for this case is shown in Fig. 16. The pressure p_1, in period t_x, is given by

$$p_1 = \frac{p_0}{3}\left[3 + \frac{2}{t_x}\left(\frac{x}{v'} - t\right)\right] \qquad (12a)$$

and in the period $\tau - t_x$ by

$$p_2 = \frac{p_0}{3}\left[\frac{\tau + \frac{x}{v'} - t}{\tau - t_x}\right] \qquad (13a)$$

It will be expedient, in this case, to measure the time from the instant the shock wave reaches the section x under consideration. That is, for t = o, p_1 is equal to p_0 and Eqs. (12a) and (13a) become:

$$p_1 = \frac{p_0}{3}\left(3 - \frac{2t}{t_x}\right) \qquad (12b)$$

and

$$p_2 = \frac{p_0}{3}\left(\frac{\tau - t}{\tau - t_x}\right) \qquad (13b)$$

The analysis is carried out in the same manner as for Case I. The time t_1 now becomes t_x and t_2 becomes $\tau - t_x$; and Eqs. (12b) and (13b) replace Eqs. (12) and (13) in computing the values of the constants a and b.

Y-200-34

Since the drop in pressure is a function of t_x which, in turn, varies directly with the distance x measured from the front face of the building, the most severe condition of loading will occur over the area of the roof adjacent to the rear wall.

If the main reinforcing is parallel to the short dimension of the roof, d in Fig. 16, the orientation of the blast wave should be parallel to the long sides. The dimension x will then be taken as L. Since the roof slab will receive support from the end wall, the value of x should be taken as somewhat less than L, say, L minus 6 times the thickness of the roof slab.

It should be noted that the pressures shown in Fig. 16 are for transverse sections of the roof, and that for the unit strip used in the analysis the pressure will, in reality, be variable across the width of the strip. This variation could be neglected in the analysis and the pressures are given by Eqs. (12b) and (13b) may be assumed constant across the width of the strip.

<u>Numerical Examples.</u>

<u>Example 1</u>.- Front Wall, Moment resistant neglected: Let h = 10 ft; d = 20 ft; L = 40 ft and p_o = 10 psi or 1,440 psf. From Figs. 13 and 14, τ = 0.58 sec, and from Fig. 15, $t_1 = \frac{1}{35}$ sec.

Assuming a 6 in. thick concrete wall with one-way reinforcing of $\frac{3}{4}$"ϕ bars spaced vertically at 6 in. centers at the center of the slab, for a strip one foot wide we have:

A = 10 sq ft; W = 750 lb; A_s = 0.88 sq in and, using a yield value of 50,000 psi, S = 44,000 lb; $\beta = \frac{2}{3}$, $\beta_1 = \frac{8}{15}$ and g = 32.2 ft per sec^2.

Y-200-35

Then $r^2 = 1,889$; $a_1 = 2,318$; $b_1 = 54,096$; $\Delta_1 = 0.6359$ ft: $V_1 = 31.086$ ft per sec; $a_2 = 813$; $b_2 = 1,401$; $c_3 = 0.4512$ and $c_4 = -0.6198$. From Eq. (22a) the motion stops when sin (rt) = 0.6064 and cos (rt) = -0.7952, from which t is found to be 0.0573 sec and the final value of y is 1.18 ft which is slightly over the allowable value.

For comparison, the same member will be reviewed taking into consideration the resisting moments.

Example 2.- Front Wall, Moment resistant considered:

Assuming $E_c = 4,000,000$ psi and $f'_c = 4,000$ psi, $E_cI_c = 6,000,000$ lb ft^2 and $M_r = 9,000$ ft lb, then $y_{b1} = 0.0125$ ft; $q = 115,200y$; $r^2 = 61,824$; $a_1 = 2,318$ and $b_1 = 54,096$. Solving Eq. (19a) by trial, $t_{b1} = 0.003433$ sec and from Eq. (20), $V_{b1} = 6.7263$ ft per sec. Then $y_c = 0.4091$; $q_c = 1,440$; $a_1 = 1,546$; $b_1 = 54,096$; $c_1 = -0.0022$ and $c_2 = 1.7377$. Placing $y = y_c = 0.4091$ in Eq. (26) the value of t is found to be 0.02609 sec. Since this value of t is smaller than t_1, the point of discontinuity of the pressure-time curve occurs in the final period of deformation. From Eq. (27) $\frac{dy}{dt} = 26.6615$ at point C. The value of r^2 in the final period is 1,880 and from Eqs. (16), $a_1 = 2,318$; $b_1 = 54,096$; $a_2 = 813$ and $b_2 = 1,401$. The values of the constants, determined from the conditions at the beginning of this period, are $c_3 = 0.4450$ and $c_4 = -1.1203$. For $t = t_1$, $\Delta_1 = 0.4680$ ft and $V_1 = 23.6883$ ft per sec. The constants c_5 and c_6 are then determined from Eqs. (17 a) and (18a) using the known values of y and $\frac{dy}{dt}$ at the time t_1, from which, $c_5 = 0.2373$ and $c_6 = -0.5130$. The motion stops when $t = 0.0616$ sec and the final value of y is 0.95 ft. Since this value is less

Y-200-36

than $\frac{1}{10}$ of the span, the design is considered to be adequate.

Example 3.- Front Wall, Slab supported by heavy members on 4 edges. Let h = 10 ft; d = 20 ft; L = 40 ft and p_0 = 20 psi or 2,880 psf. We find τ = 0.49 and $t_1 = \frac{1}{35}$ sec. Assuming a $\frac{3}{16}$" steel plate 8 ft high and 4 ft wide in the front wall, A = 32 sq ft; W = 244.8 lb; S = 67,500 lb per ft (yield value of 30,000 psi is taken for plate steel); $\frac{t_2}{t_1} = 2.0$ and from Fig. 20, $\gamma = 8.8$; $\beta = 0.5$ and $\beta_1 = 0.338$. Then $r^2 = 231{,}128$; $a_1 = 53{,}790$ and $b_1 = 1{,}255{,}100$. Equating $\frac{dy}{dt}$ in Eq. (20) to zero, t = 0.00633 which is less than t_1. Hence, the motion stops in the first period and y at the end of motion is 0.43 ft. Since this value of y is only slightly larger than $\frac{1}{10}$ the short span, the design is considered as adequate.

OVER-ALL STRENGTH

In the local investigation of strength, it was assumed that the members would have adequate support and anchorage to justify the concept of deformation and the mode of failure outlined in the preceding analysis. This requirement will be met by making the reinforcing of the member continuous, or spliced for full strength, between its supports, and by providing an end anchorage capable of development of the full or ultimate strength of the reinforcing.

In addition, it was assumed that the supports would not displace during local bending of the member, and that there would be no appreciable participation in the over-all bending of the framing as a whole. Insofar as the first condition is concerned, it is an assumption on the conservative side and wholly justifiable for members framed to strong elements, such as transverse walls and floors. The other assumption will be satisfied if the framing is so arranged that under a lateral loading the deformations of frames composed of floors and transverse walls became very small in comparison with the corresponding deformations of bents composed of local elements.

In connection with the latter consideration, it is to be noted that in order to provide the necessary strength to resist strong atomic blast, the use of strong frames comprising structural floors and transverse walls becomes almost mandatory. The analysis presented here contemplates the use of this type of framing.

In conformity with the foregoing concept, floors are considered as girders, transmitting the loads from local members to the transverse walls. These loads are then carried to the foundations through the walls, either by frame or cantilever action. If the transverse walls are placed no farther apart than, say, three times the width of the floor, then the problem of strength will be primarily that of shear. Accordingly, the needed investigation may be confined to shear strength only.

There is but little information available regarding the shear strength of materials under dynamic loading. Until such data are obtained, use will be made of values deduced from static tests. For reinforced concrete, an allowable working value for shear is given by the following formula*

$$v_a = f_s\,(0.005 + r)\,, \qquad \cdots\cdots (30)$$

in which, v_a is allowable unit shearing stress, f_s is allowable stress in steel and r is the ratio of the volume of steel to the volume of concrete. In this investigation f_s may be taken as the yield value of the steel, and the unit shear is computed by dividing the total shear by the cross sectional area of the floor or wall.

The maximum shears in the floors and walls occur at the instant the shock wave strikes the front wall. In obtaining the total load, the maximum value of the reflected pressure is used.

To illustrate, consider the building shown in Fig. 21. The total maximum load, p_H, obtained from the reflected pressure is

$$P_H = 3 p_o h L$$

Assuming that this total load is carried to the foundations by the two end walls only, the corresponding unit shear in the concrete, v_c, will then be

$$v_c = \frac{3 p_o h L}{2dT}\,, \qquad \cdots\cdots (31)$$

where T is the thickness of the wall.

STABILITY

<u>Stability Against Sliding</u>.- When a shock wave hits the front

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* "Principles of Reinforced Concrete Construction," Turneaure and Maurer. John Wiley & Sons. p. 103, 1936.

face of a structure, the reflected pressures will tend to move it by bodily displacement and rotation. The tendency to displacement will be resisted first by the developed friction under the base or foundations and the passive pressures of the surrounding earth, then, as the blast wave envelopes the structure, by the additional aid of the blast pressures on top and the rear face.

To develop the relations of motion needed for this investigation, consider the simple rectangular building shown in Fig. 21. The applied force tending to move the building is the total blast pressure P_H on the front wall, and the total force resisting sliding, R, is given by

$$R = P_{H1} + R_e + c_f \cdot (W + P_V) \text{ ,} \qquad \cdots\cdots (32)$$

where R_e is the passive resistance of the earth (assumed to be constant during the motion), c_f the coefficient of friction, W the total weight of the structure, including the footings, and P_V is the total pressure on the roof. The resulting equation of motion is

$$\frac{d^2x}{dt^2} = \frac{g}{W}(P_H - R) \text{ ,} \qquad \cdots\cdots (33)$$

in which x is the distance moved by the structure. The forces P_H and P_{H1} are given by,

$$P_H = h\,L\,p_1'$$

and

$$P_{H1} = h\,L\,p_2'$$

in which p_1' and p_2' are obtained from the pressure-time curves shown in Figs. 15 and 17.

Y-200-40

The expressions for P_H, P_{H1} and P_V will vary for the various time intervals. These time intervals may be defined as follows:

t_a = Time for wave to reach rear edge of roof;

t_b = Time for wave to reach center of rear wall;

t_c = Time for pressure on front face to drop to value of p_0 ($= t_1$ in Fig. 15);

t_d = Time for wave to reach 7d;

where all times are measured from the instant the shock wave strikes the front wall. Accordingly,

$$t_a = \frac{d}{v'},$$

$$\left.\begin{aligned} t_b &= \frac{2d+h}{2v'}, \\ t_c &= \frac{4h}{v'}, \end{aligned}\right\} \text{ for } h < \frac{L}{2}$$

$$\left.\begin{aligned} t_b &= \frac{2d+L}{2v'}, \\ t_c &= \frac{2L}{v'}, \end{aligned}\right\} \text{ for } h > \frac{L}{2}$$

$$t_d = \frac{7d}{v'}.$$

The values of P_V for the various time intervals are given in Fig. 22. Here Case 1 is for $0 \leqq t \leqq t_a$; Case 2, for $t_a \leqq t \leqq t_d$; and Case 3, for $t > t_d$.

Since P_H, P_{H1} and P_V are functions of t and t^2, the general equation of motion, Eq. (23), may be expressed as

$$\frac{d^2x}{dt^2} = a_2 + b_2 t + c_2 t^2. \qquad \cdots\cdots (34)$$

The solution of this equation is

$$x = c_{2n-1} + c_{2n} t + \frac{a_2 t^2}{2} + \frac{b_2 t^3}{6} + \frac{c_2 t^4}{12}. \qquad \cdots\cdots (35)$$

Y-200-41

in which c_{2n-1} and c_{2n} are constants of integration.

Differentiating Eq. (25) we have,

$$\frac{dx}{dt} = c_{2n} + a_2 t + \frac{b_2 t^2}{2} + \frac{c_2 t^3}{3} \cdot \qquad \cdots\cdots (36)$$

If Δ_n and V_n represent the displacement and velocity, respectively, at the beginning of any period at the time t_n, then the constants of integration for the equation of motion during that period are found from the relations,

$$c_{2n} = V_n - a_2 t_n - \frac{b_2 t_n^2}{2} - \frac{c_2 t_n^3}{3} \quad , \qquad \cdots\cdots (37)$$

$$c_{2n-1} = \Delta_n - c_{2n} t_n - \frac{a_2 t_n^2}{2} - \frac{b_2 t_n^3}{6} - \frac{c_2 t_n^4}{12} . \qquad \cdots\cdots (38)$$

The motion stops when $\frac{dx}{dt} = 0$. The total distance moved may then be determined by setting the right side of Eq. (26) equal to zero, solving for t and substituting this value of t in Eq. (25).

It is to be noted that the pressures on those parts of the footing which extend beyond the wall lines are not included in the above analysis. Inclusion of these areas is not warranted except in unusual cases.

Numerical Illustration.- The building shown in Fig. 21 will be investigated for stability against sliding under a peak overpressure of 10 psi = 1440 psf, which corresponds to a distance of approximately one-half mile from center of explosion of a 20 kiloton bomb. The coefficient of friction is assumed to be 0.5, and

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the total passive resistance of the earth, 39,000 lb. From Figs. 13 and 14, $\tau = 0.58$ sec. The values of the time intervals are:

$t_a = \frac{1}{70}$ sec; $t_b = \frac{1}{56}$ sec; $t_c = \frac{1}{35}$ sec and $t_d = \frac{1}{10}$ sec. The numerical work is shown in tabular form in Table 2. It is to be noted that the motion stops in the time interval $\frac{1}{35} \leq t \leq \frac{1}{10}$. The time when the motion stops is found from Eq. (26),

$$\frac{dx}{dt} = c_8 - 98.70\ t + 713\ t^2 - 2,363\ t^3 = 0,$$

from which t = 0.0774 sec. Substituting this value of t in Eq. (25), the total sliding motion is found to be 0.10 ft, or about $1\frac{1}{4}$ in.

Stability Against Overturning.- The reflected pressures on the front face, in addition to causing the structure to slide, will tend also to rotate the structure as a unit about the rear edge of the footing. Referring to Fig. 21, the force tending to produce rotation about an axis through 0 is P_H and the forces tending to resist such motion are P_V, P_{H1}, and the total weight of the structure W. For small angles of rotation, θ, expressed in radians, the overturning moment, M_T, causing the rotation is given by,

$$M_T = \left(\frac{h}{2} + h'\right)(P_H - P_{H1}) + M_P - (d + d')P_V - \left(\frac{d}{2} + d'\right)W \quad \cdots\cdots (39)$$

in which

h' = depth of rear footing below ground level;

d' = extension of rear footing beyond rear wall and

M_P = moment of P_V about front edge of roof, (see Fig. 22).

The general equation of motion is given by

$$\frac{d^2\theta}{dt^2} = \frac{M_T}{I_m}, \quad \cdots\cdots (40)$$

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in which I_m is the mass moment of inertia of the structure about an axis through O.

As explained in the preceding section, P_H, P_{HI}, M_V and P_V are functions of t and t^2. Accordingly, Eq. (30) will take the form,

$$\frac{d^2\theta}{dt^2} = a_3 + b_3 t + c_3 t^2 \qquad \cdots\cdots (41)$$

The solution of this equation is found in the same manner as that given for sliding, Eq. (24). The maximum value of θ occurs when $\frac{d\theta}{dt}$ changes sign or when $\frac{d\theta}{dt} = 0$. The maximum uplift, δ, at the front edge is then simply

$$\delta = (d + 2d')\,\theta \qquad \cdots\cdots (42)$$

Numerical Illustration.- The building shown in Fig. 21 will now be investigated for stability against overturning under the same peak overpressure as used in the investigation for sliding, namely 10 psi. The mass moment of inertia about an axis through O is assumed to be 1.5×10^6 lb sec^2 ft. The numerical work is shown in tabular form in Table 3. It can be seen that the maximum value of θ occurs in the time interval $\frac{1}{56} \leq t \leq \frac{1}{35}$. The time when θ is a maximum is found from the relation,

$$\frac{d\theta}{dt} = c_6 - 2.215t - 44.46t^2 - 149.46t^3 = 0$$

From this equation t = 0.0205 sec and θ_{max} = 0.000337 radian. The maximum uplift at A is found to be 0.0074 ft or about $\frac{1}{10}$ in.

From the small values obtained in the above examples, it may be concluded that the structure possesses adequate stability against sliding and overturning. Obviously, the extent of displacements which may be deemed as permissible will vary with the type and functional requirements of the structure.

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CONCLUSION

The possibility of an atomic warfare has made every one protection conscious. Having survived the initial shock of implications related to the power of this new weapon, we are now able to give at least a qualified reply of assurance to the inevitable question, "can we provide protection?" The answer may be summarized as follows:

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100-22759

(a) No complete or unqualified structural protection against the A-bomb, as well as some of the conventional bombs, is feasible.

(b) Within the critical range of explosion, all buildings, regardless of type of framing, will suffer some damage, varying in severity in accordance with their proximity to ground zero.

(c) In general, the design criteria for protection against an atomic bomb of the presently known type are not as severe as those required for some of the conventional weapons of the last war. Assuming the bomb is to explode at an altitude corresponding to its maximum range of damage, for the needed protection, the changes to be introduced in conventional designs of reinforced concrete will be relatively small.

(d) By proper arrangement of framing and details, adequate over-all strength can be provided for a structure to avert collapse, even though there may occur severe local damage in the form of large plastic deformations or failures. (with)

(e) With certain assumptions of probable structural behavior, the adequacy of over-all strength of a framing and the extent of

Y-200-45

damage to local members can be satisfactorily ascertained. For this purpose, the procedure presented above may be used as a practical method of design.

As a concluding thought, it should be remembered that protective design, as a measure of defense, can only follow the progress of weapons of offense in a vicious circle of continued improvement. For what is devised now to be adequate against weapons of today, is apt to be inadequate for weapons of tomorrow. This sad reflection should serve as a sobering influence against the creation of a false sense of future security.

NOTATIONS

Part I

A_p	=	Sectional pressure of a bomb or projectile;
a'	=	A ratio, = T/D;
C	=	Weight of charge in a bomb or projectile:
c'	=	Penetration coefficient for explosion;
D	=	Impact penetration in an infinitely thick mass;
D'	=	Depth of penetration due to impact in a slab of thickness T;
D_e	=	Depth of penetration due to explosive charge of a bomb or projectile;
g	=	Acceleration due to gravity;
H	=	Height of fall of bomb;
K	=	Thickness ratio, = D'/D;
k	=	Penetration coefficient for impact;
T	=	Thickness of resisting slab;
V	=	Impact velocity;
V'	=	A function of V.

Part II

A	=	Surface area of a slab or beam;
A_s	=	Area of tensile reinforcement;
c_f	=	Coefficient of sliding friction of structure on ground;
d	=	Width or depth of a rectangular structure;
d'	=	Extension of footing outside of wall;

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E_c	=	Modulus of elasticity of **concrete.**
F	=	Total force acting to move a structure or a component;
f	=	Net force causing motion of a unit area;
f'_c	=	Ultimate compressive strength of concrete;
h	=	Height of a rectangular structure;
h'	=	Depth of footing;
I_c	=	Moment of inertia of unit width of concrete member;
I_m	=	Mass moment of inertia of a structure about an axis of rotation;
L	=	Length of a rectangular structure;
ℓ	=	Length of a one-way slab or beam;
ℓ_1	=	Length of short side of two-way slab or plate;
ℓ_2	=	Length of long side of two-way slab or plate;
M_p	=	Total moment of roof pressure about front edge of roof;
M_r	=	Resisting moment at yield stress per unit width of slab or beam;
M_T	=	Total overturning moment on a structure;
P_H	=	Total pressure on front wall;
P_{H1}	=	Total pressure on rear wall;
P_V	=	Total pressure on roof;
p	=	Unit pressure, above atmospheric, on a surface;
p_o	=	Peak pressure, above atmospheric;
q	=	Modulus of resistance;
R	=	Total resistance to motion of a structure in translation;
r	=	Ratio of volume of reinforcing steel to volume of concrete; also a constant used in the solution of equations of motion;

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S	=	Stress per unit width of resisting steel in a slab or beam;
t	=	Time measured in seconds;
t_1	=	Time for reflected pressure on front face to drop to p_o;
t'	=	Time for shock wave to travel from front face to center of rear face;
v	=	Velocity;
V_1	=	Velocity at time t_1 ;
v'	=	Velocity of shock wave;
v_a	=	Allowable unit shearing stress in concrete;
v_c	=	Unit shearing stress in concrete;
W	=	Total weight of a structure or component part;
w	=	Unit weight;
x	=	Displacement of a structure in direction of shock wave;
y	=	Displacement of center of a slab or beam relative to its supports;
y	=	Displacement of any point on a slab or beam relative to supports;
α	=	Acceleration;
β	=	Reduction factor for area;
β_1	=	Reduction factor for weight;
γ	=	Resistance factor;
Δ_1	=	Displacement at time t_1 ;
δ	=	Maximum uplift at front footing;
θ	=	Angular displacement of a structure;
τ	=	Duration of positive phase of shock wave.

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Material	$Ft.^3\ lb.^{-1}$
Limestone	5.38×10^{-3}
Concrete [1]	7.99×10^{-3}
Reinforced concrete [2]	4.76×10^{-3}
Specially-reinforced concrete [3]	2.82×10^{-3}
Stone masonry	11.72×10^{-3}
Brickwork	20.48×10^{-3}
Sandy soil	36.7×10^{-3}
Soil with vegetation	48.2×10^{-3}
Soft soil	73.2×10^{-3}

[1] Mass concrete with a crushing strength of 2,200 pounds per square inch.

[2] Normal reinforced concrete with a crushing strength of 3,200 pounds per square inch and 1.4 percent of reinforcement.

[3] Specially-reinforced concrete with a crushing strength of 5,700 pounds per square inch and 1.4 percent of reinforcement.

TABLE I. — Values of penetration coefficient (k) for various materials.

Y-200-50

TERM		TIME INTERVAL IN SECONDS			
		$0 \le t \le \frac{1}{70}$	$\frac{1}{70} \le t \le \frac{1}{56}$	$\frac{1}{56} \le t \le \frac{1}{35}$	$\frac{1}{35} \le t \le \frac{1}{10}$
$P_H \times 10^{-6}$		$1.728 - 40.32\,t$	$1.728 - 40.32\,t$	$1.728 - 40.32\,t$	$0.606 - 1.045\,t$
$P_{H1} \times 10^{-6}$		0	0	$0.396 - 0.662\,t$	$0.396 - 0.662\,t$
$P_V \times 10^{-6}$		$63.141\,t$	$1.107 - 15.402\,t + 73.382\,t^2$	$1.107 - 15.402\,t + 73.382\,t^2$	$1.107 - 15.402\,t + 73.382\,t^2$
$R \times 10^{-6}$		$0.159 + 37.885\,t$	$0.823 - 9.241\,t + 44.029\,t^2$	$1.219 - 9.903\,t + 44.029\,t^2$	$1.219 - 9.903\,t + 44.029\,t^2$
$(P_H - R) \times 10^{-6}$		$1.569 - 78.205\,t$	$0.905 - 31.079\,t - 44.029\,t^2$	$0.509 - 30.417\,t - 44.029\,t^2$	$-0.613 + 8.858\,t - 44.029\,t^2$
Equation of Acceleration	$\frac{d^2x}{dt^2} =$	$252.62 - 12591\,t$	$145.71 - 5004\,t - 7089\,t^2$	$81.95 - 4897\,t - 7089\,t^2$	$-98.70 + 1426\,t - 7089\,t^2$
Equation of Displacement	$x =$	$C_1 + C_2 t + 126.31\,t^2 - 2098.5\,t^3$	$C_3 + C_4 t + 72.855\,t^2 - 834.0\,t^3 - 590.75\,t^4$	$C_5 + C_6 t + 40.975\,t^2 - 816.17\,t^3 - 590.75\,t^4$	$C_7 + C_8 t - 49.35\,t^2 + 237.67\,t^3 - 590.75\,t^4$
Equation of Velocity	$\frac{dx}{dt} =$	$C_2 + 252.62\,t - 6295.5\,t^2$	$C_4 + 145.71\,t - 2502\,t^2 - 2363\,t^3$	$C_6 + 81.95\,t - 2448.5\,t^2 - 2363\,t^3$	$C_8 - 98.70\,t + 713\,t^2 - 2363\,t^3$
Values at beginning of period	$x =$	0	0.0197 ft.	0.0284 ft.	0.0542 ft.
	$\frac{dx}{dt} =$	0	2.3241 ft./sec.	2.5507 ft./sec.	2.1690 ft./sec.
Values of Constants in equations		$C_1 = 0$	$C_3 = -0.0036$	$C_5 = -0.0136$	$C_7 = -0.0382$
		$C_2 = 0$	$C_4 = 0.7600$	$C_6 = 1.8815$	$C_8 = 4.4621$
Values at end of period	$x =$	0.0197 ft.	0.0284 ft.	0.0542 ft.	0.0931 ft.
	$\frac{dx}{dt} =$	2.3241 ft./sec.	2.5507 ft./sec.	2.1690 ft./sec.	-0.6409 ft./sec.

TABLE 2. COMPUTATIONS FOR STABILITY AGAINST SLIDING

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TERM		TIME INTERVAL IN SECONDS		
		$0 \leq t \leq \frac{1}{70}$	$\frac{1}{70} \leq t \leq \frac{1}{56}$	$\frac{1}{56} \leq t \leq \frac{1}{35}$
$P_H \times 10^{-6}$		$1.728 - 40.32\,t$	$1.728 - 40.32\,t$	$1.728 - 40.32\,t$
$P_{H1} \times 10^{-6}$		0	0	$0.396 - 0.662\,t$
$P_V \times 10^{-6}$		$63.141\,t$	$1.107 - 15.402\,t + 73.382\,t^2$	$1.107 - 15.402\,t + 73.382\,t^2$
$M_P \times 10^{-6}$		$51029\,t^2$	$12.8 - 179.2\,t + 868.43\,t^2$	$12.8 - 179.2\,t + 868.43\,t^2$
$M_T \times 10^{-6}$		$9.896 - 1608.2\,t + 51029\,t^2$	$-0.551 - 138.0\,t - 672.59\,t^2$	$-3.323 - 133.364\,t - 672.59\,t^2$
Equation of Acceleration	$\frac{d^2\theta}{dt^2} =$	$6.597 - 1072\,t + 34019\,t^2$	$-0.367 - 92\,t - 448\,t^2$	$-2.215 - 88.9\,t - 448\,t^2$
Equation of Displacement	$\theta =$	$C_1 + C_2 t + 3.2985\,t^2 - 178.67\,t^3 + 2835\,t^4$	$C_3 + C_4 t - 0.1835\,t^2 - 15.33\,t^3 - 37.37\,t^4$	$C_5 + C_6 t - 1.1075\,t^2 - 14.82\,t^3 - 37.37\,t^4$
Equation of Velocity	$\frac{d\theta}{dt} =$	$C_2 + 6.597\,t - 536\,t^2 + 11340\,t^3$	$C_4 - 0.367\,t - 46\,t^2 - 149.5\,t^3$	$C_6 - 2.215\,t - 44.5\,t^2 - 149.5\,t^3$
Values at beginning of period	$\theta =$	0	0.000270 rad.	0.000322 rad.
	$\frac{d\theta}{dt} =$	0	0.017915 rad./sec.	0.010908 rad./sec.
Values of Constants in equations		$C_1 = 0$	$C_3 = -0.000117$	$C_5 = -0.000406$
		$C_2 = 0$	$C_4 = 0.032981$	$C_6 = 0.065490$
Values at end of period	$\theta =$	0.000270 rad.	0.000322 rad.	0.000192 rad.
	$\frac{d\theta}{dt} =$	0.017915 rad./sec.	0.010908 rad./sec.	-0.037576 rad./sec.

TABLE 3. COMPUTATIONS FOR STABILITY AGAINST OVERTURNING

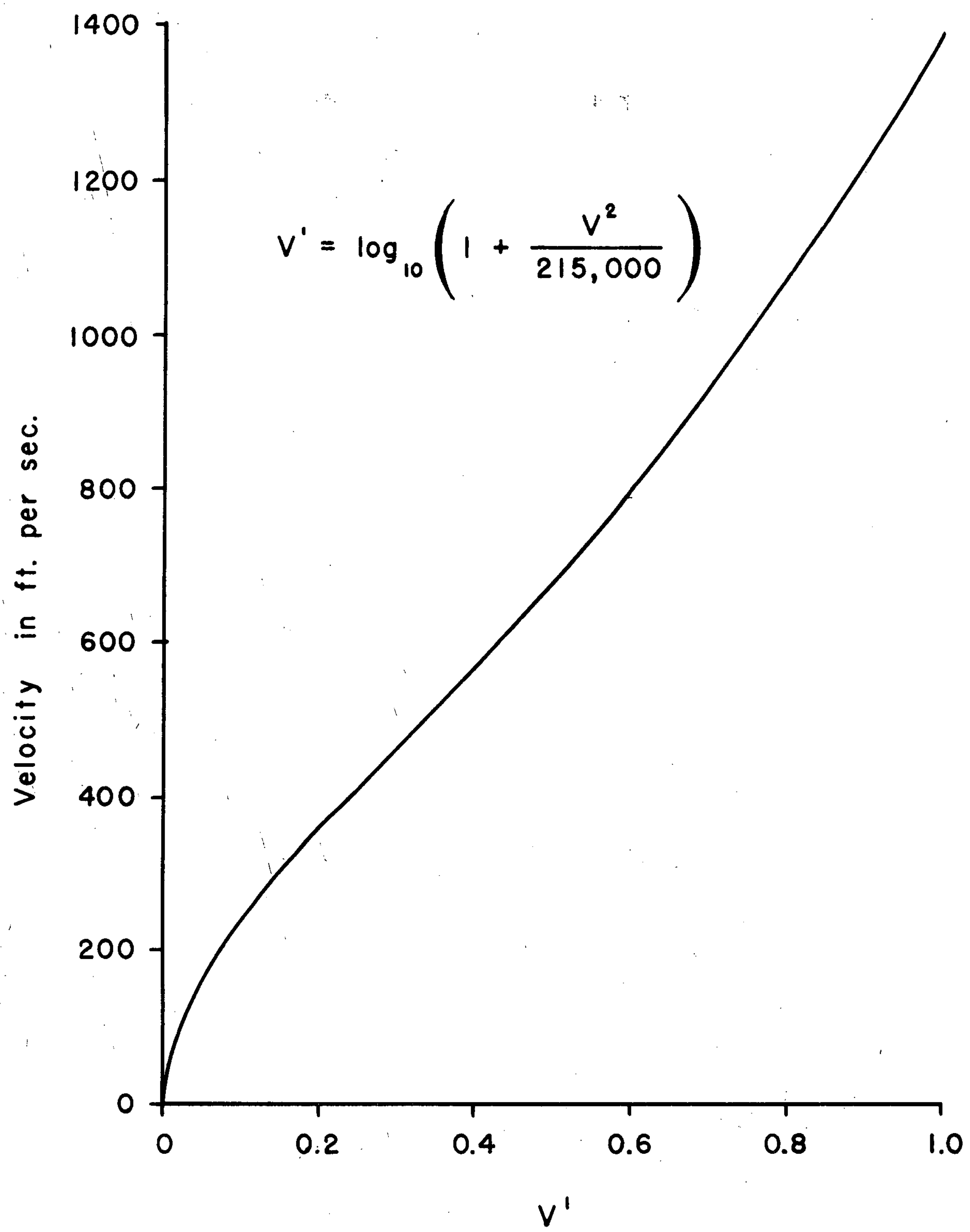


FIG. I VELOCITY FACTOR (V') FOR IMPACT PENETRATION

Y-200-53

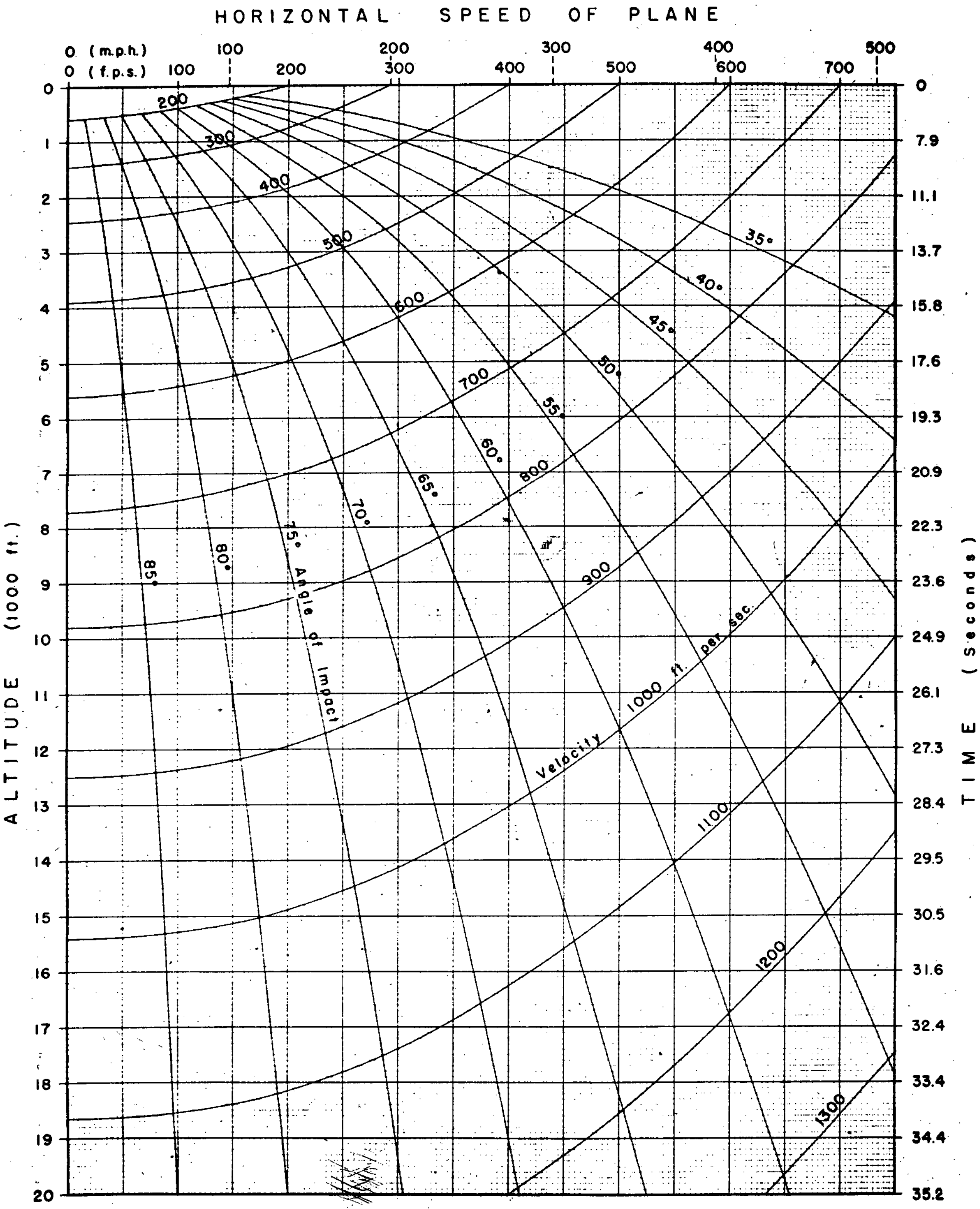


FIG.2 FLIGHT CHARACTERISTICS OF BOMBS (AIR RESISTANCE NEGLECTED)

Y-200-54

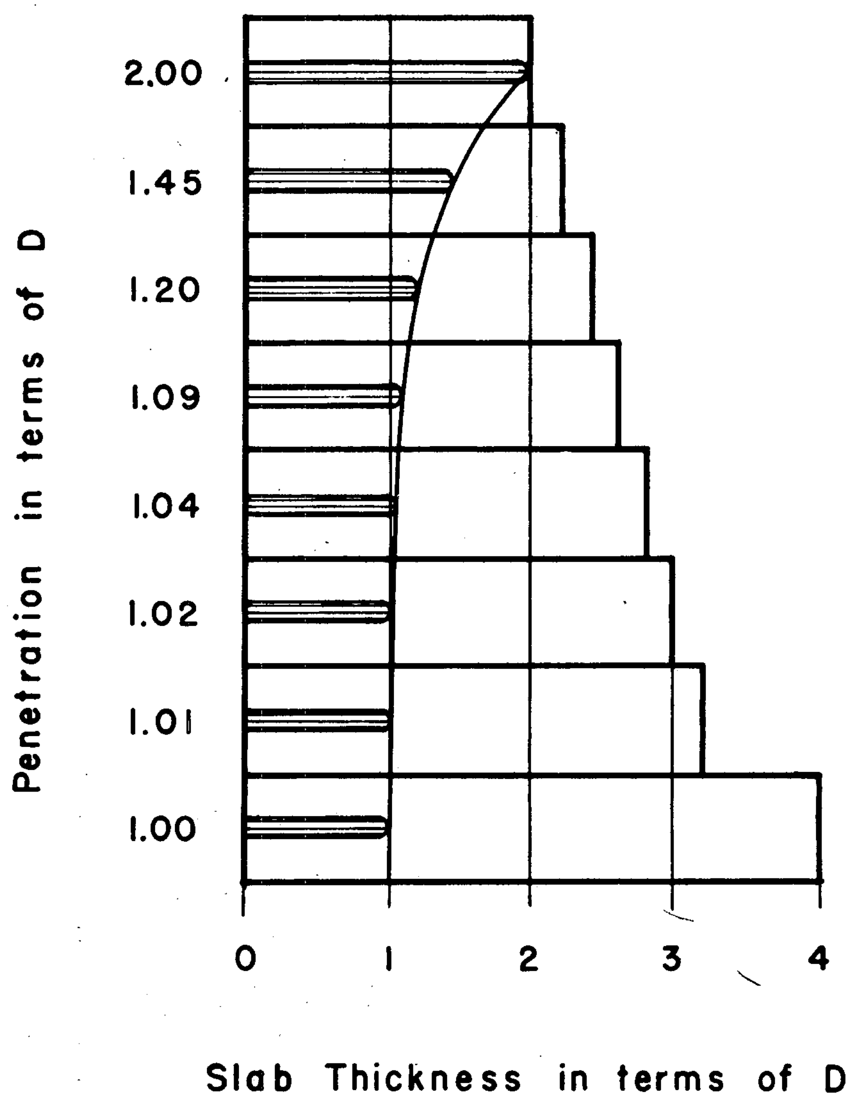


FIG. 3 DIAGRAMMATIC REPRESENTATION OF RELATION OF RELATIVE SLAB THICKNESS TO PENETRATION

Y-200-55

X-2002-56

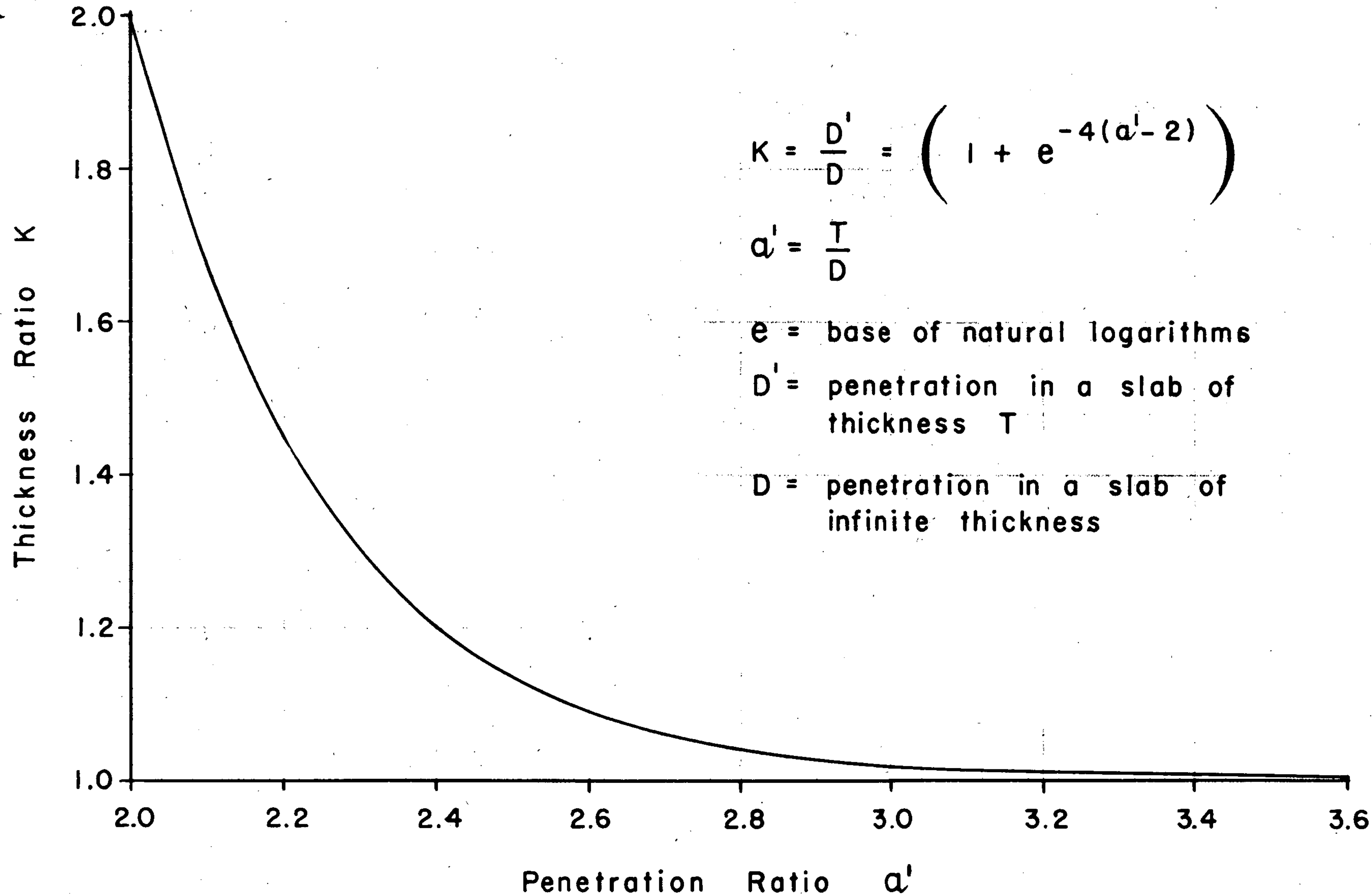
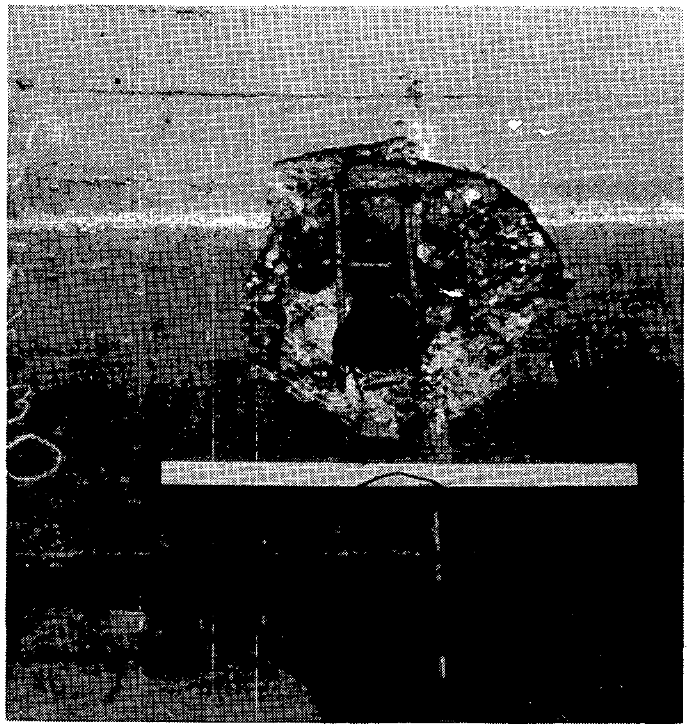
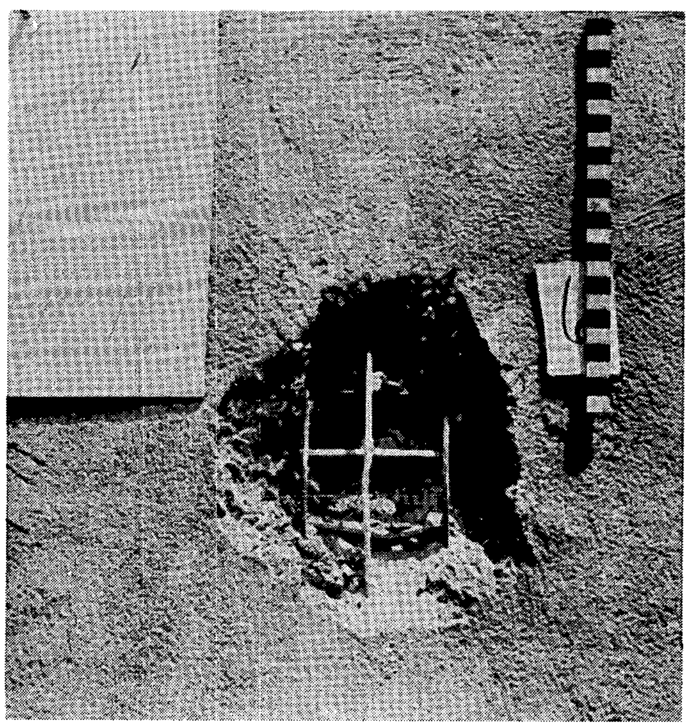


FIG. 4 RELATION OF RELATIVE SLAB THICKNESS TO PENETRATION



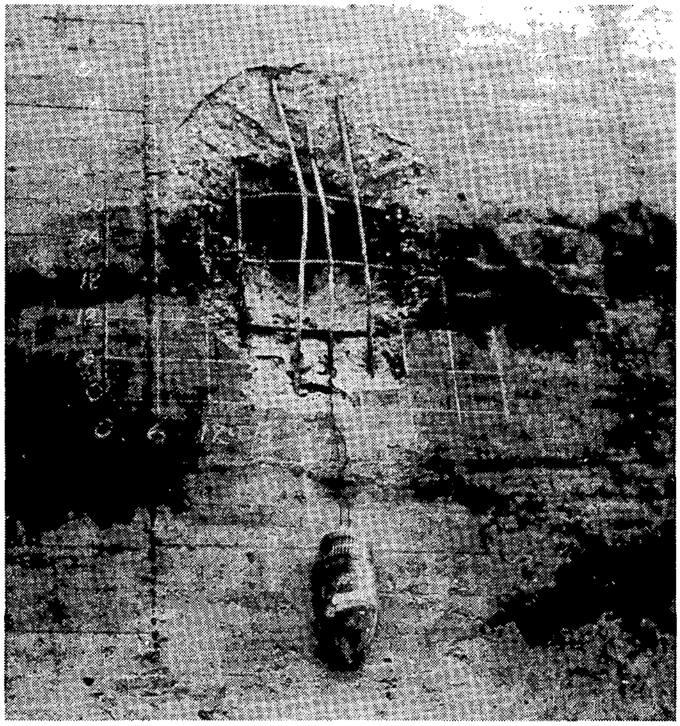
(a)



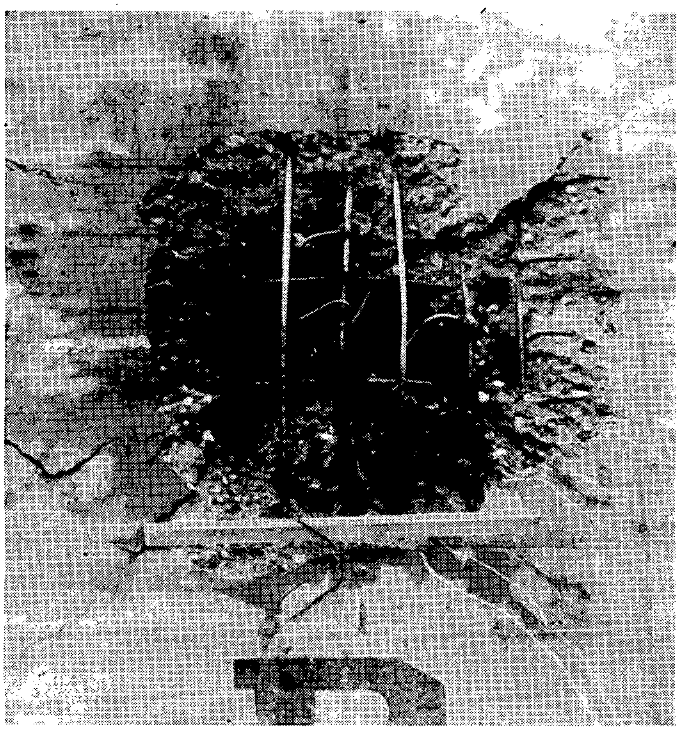
(b)

Fig. 5 Types of Damage in Impact Face of Concrete Slab
(a) Complete Penetration
(b) Nearly Complete Penetration

Y-200-57



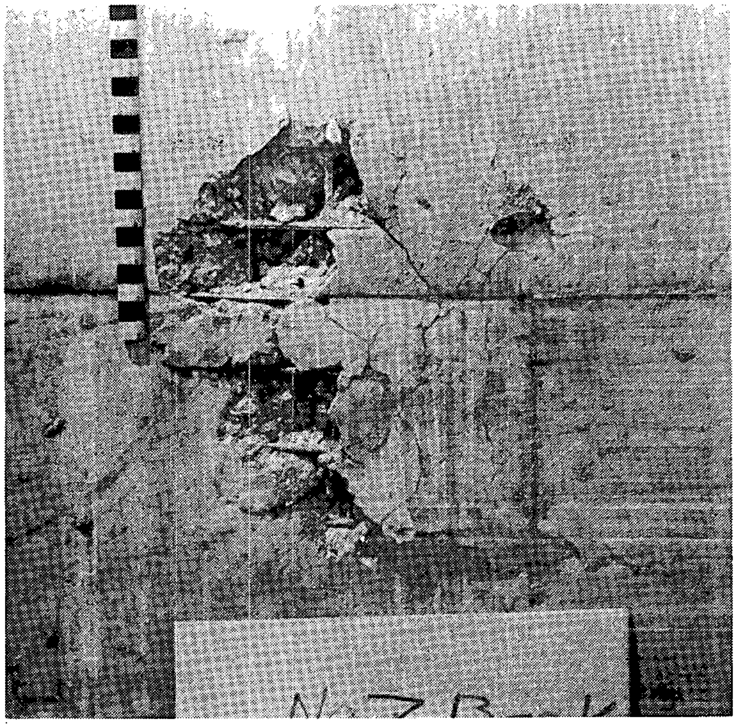
(a)



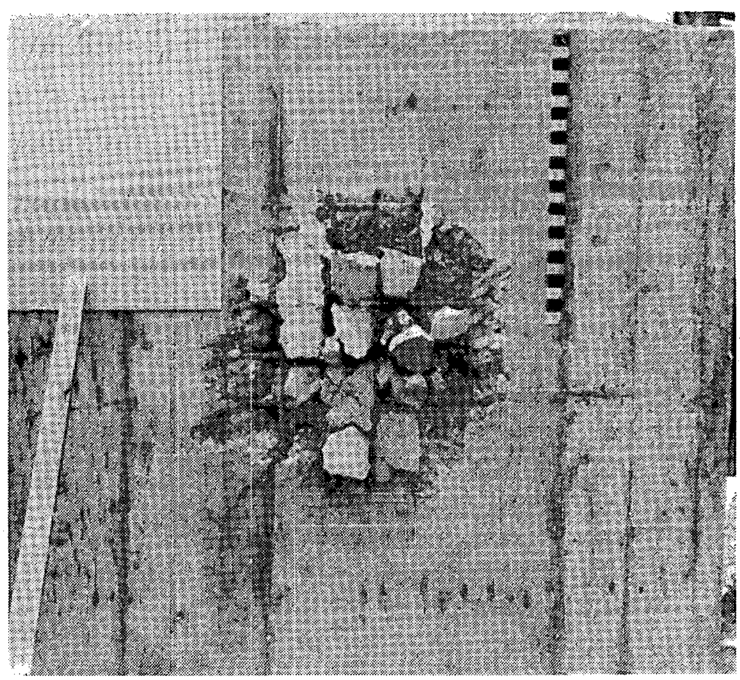
(b)

Fig. 6 Types of Damage in Impact Face of Concrete Slab
(a) Partial Penetration Due to Impact
(b) Additional Penetration Due to Charge Exploded in Crater of (a)

Y-200-58



(a)



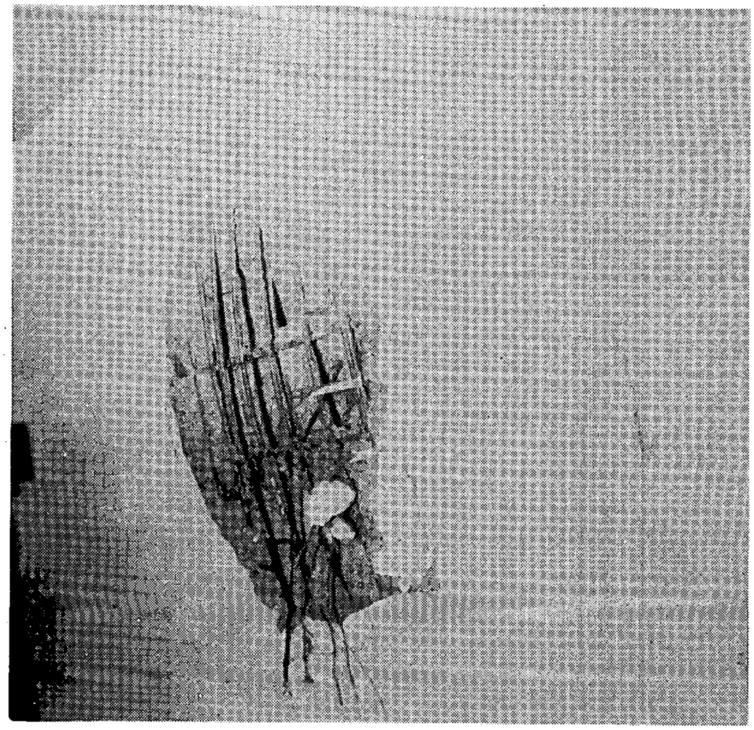
(b)

Fig. 7 Types of Damage in Rear Face of Concrete Slab

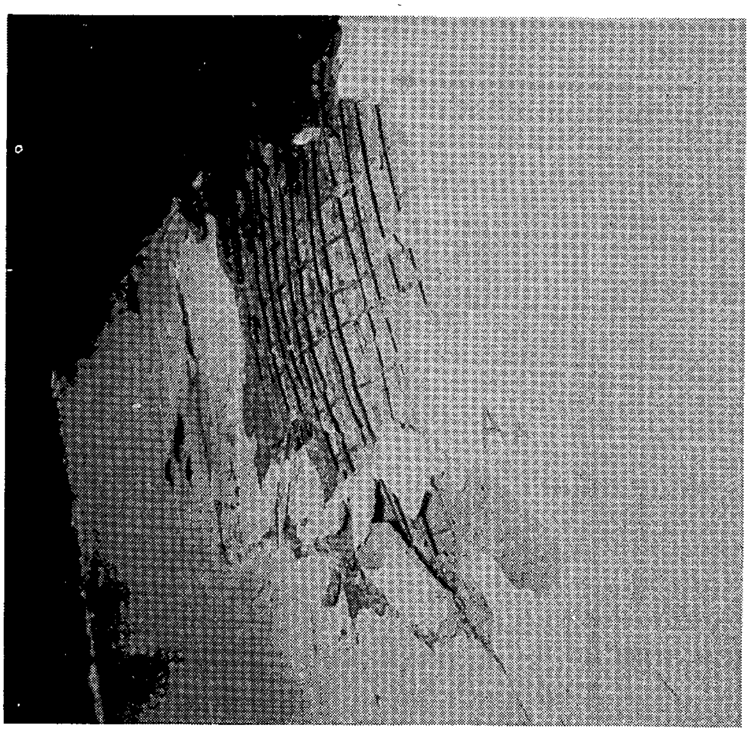
(a) Partial Penetration Due to Impact

(b) Nearly Complete Penetration Due to Impact

Y-200-59



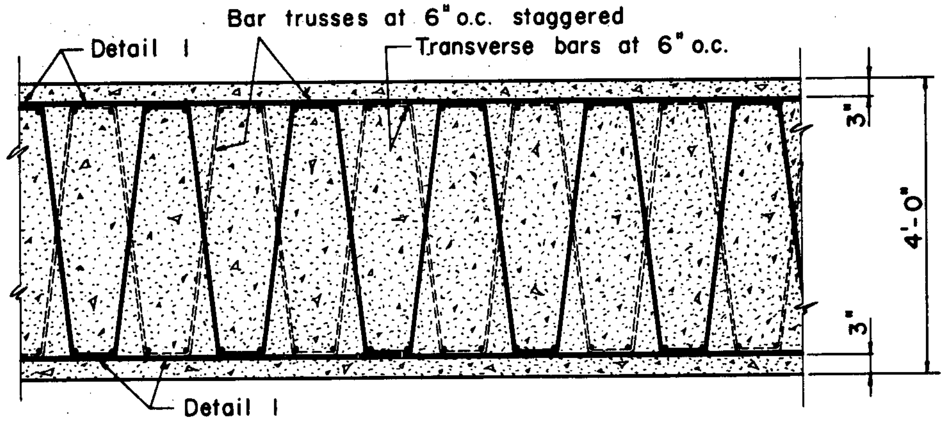
(a)



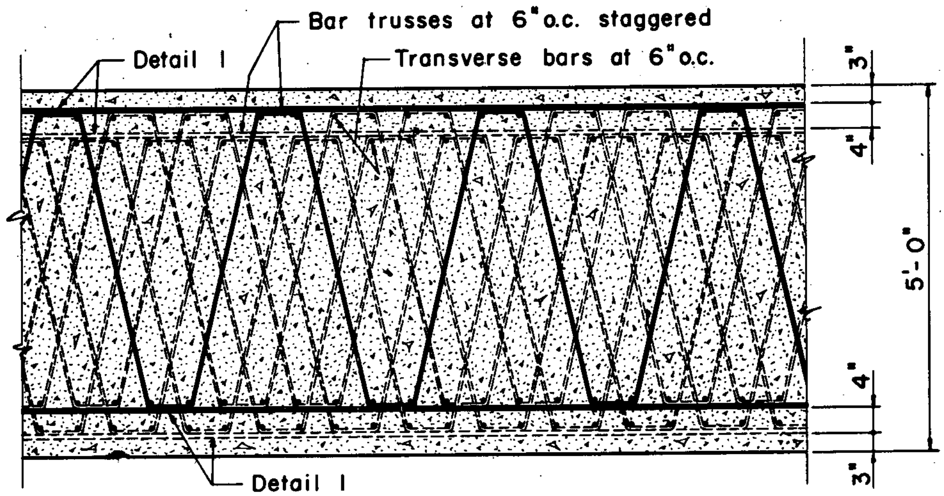
(b)

Fig. 8 Types of Damage in Rear Face of Concrete Slab
(a) Charge Exploded in Open Crater
(b) Charge Exploded in Confined Crater

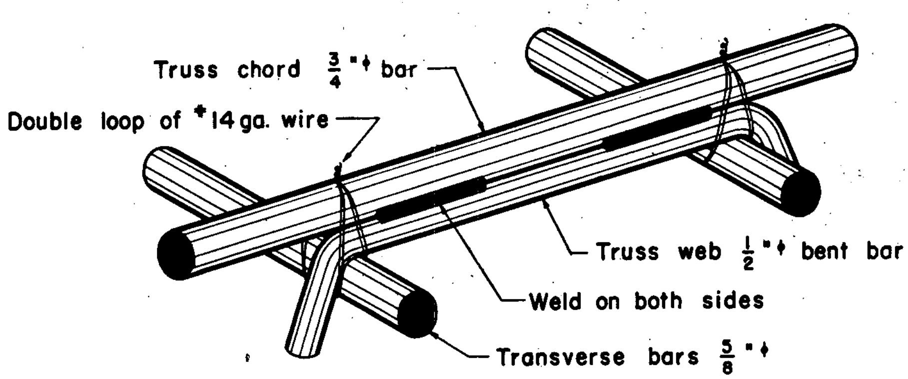
Y-200-60



TYPE A



TYPE B



TYPICAL DETAIL I

FIG. 9 ARRANGEMENT OF REINFORCING IN SLABS OF BOMB-RESISTANT STRUCTURES

Y-200-61

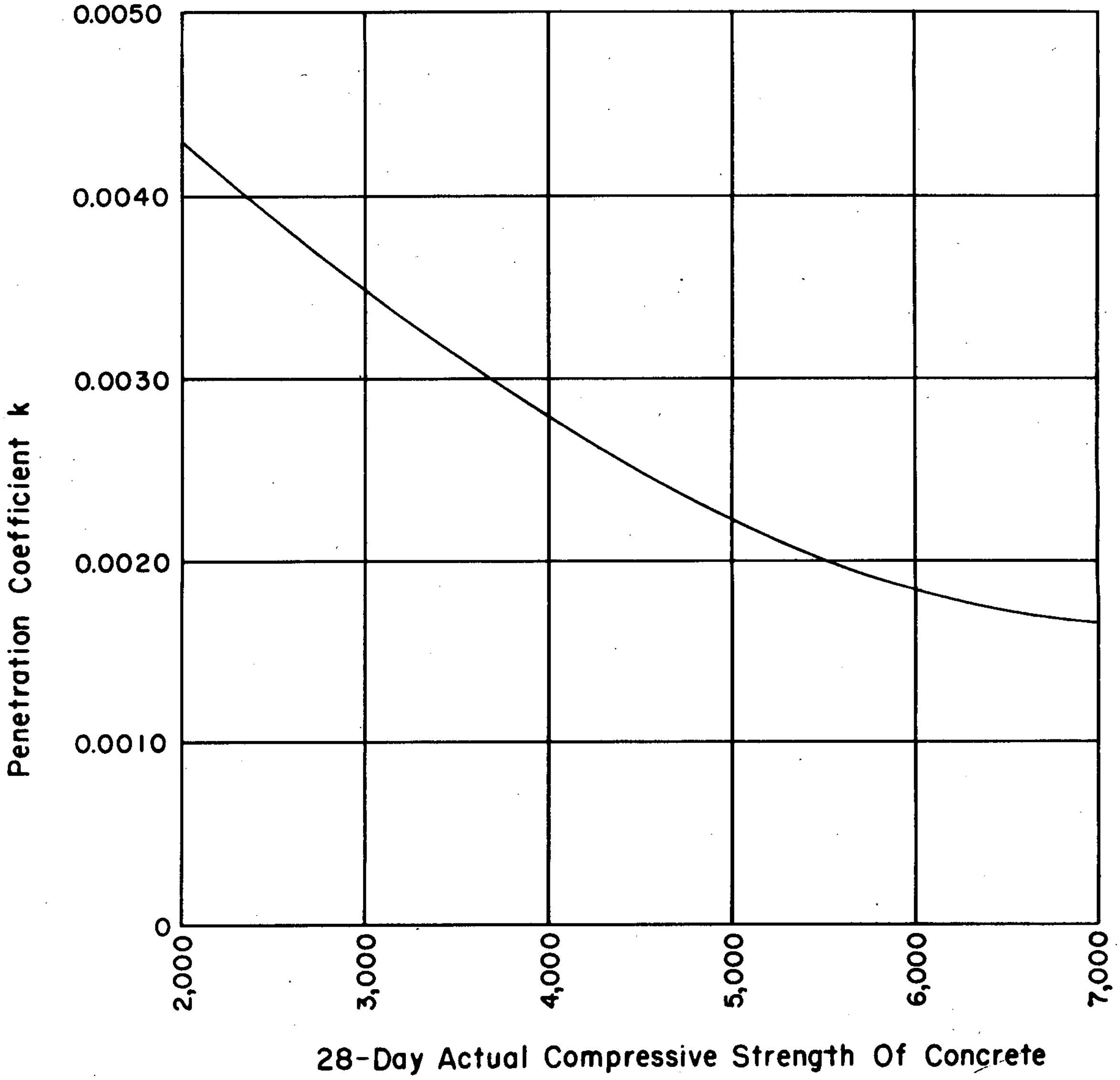


FIG.10 VALUES OF PENETRATION COEFFICIENT (k) FOR REINFORCED CONCRETE

Y-200-62

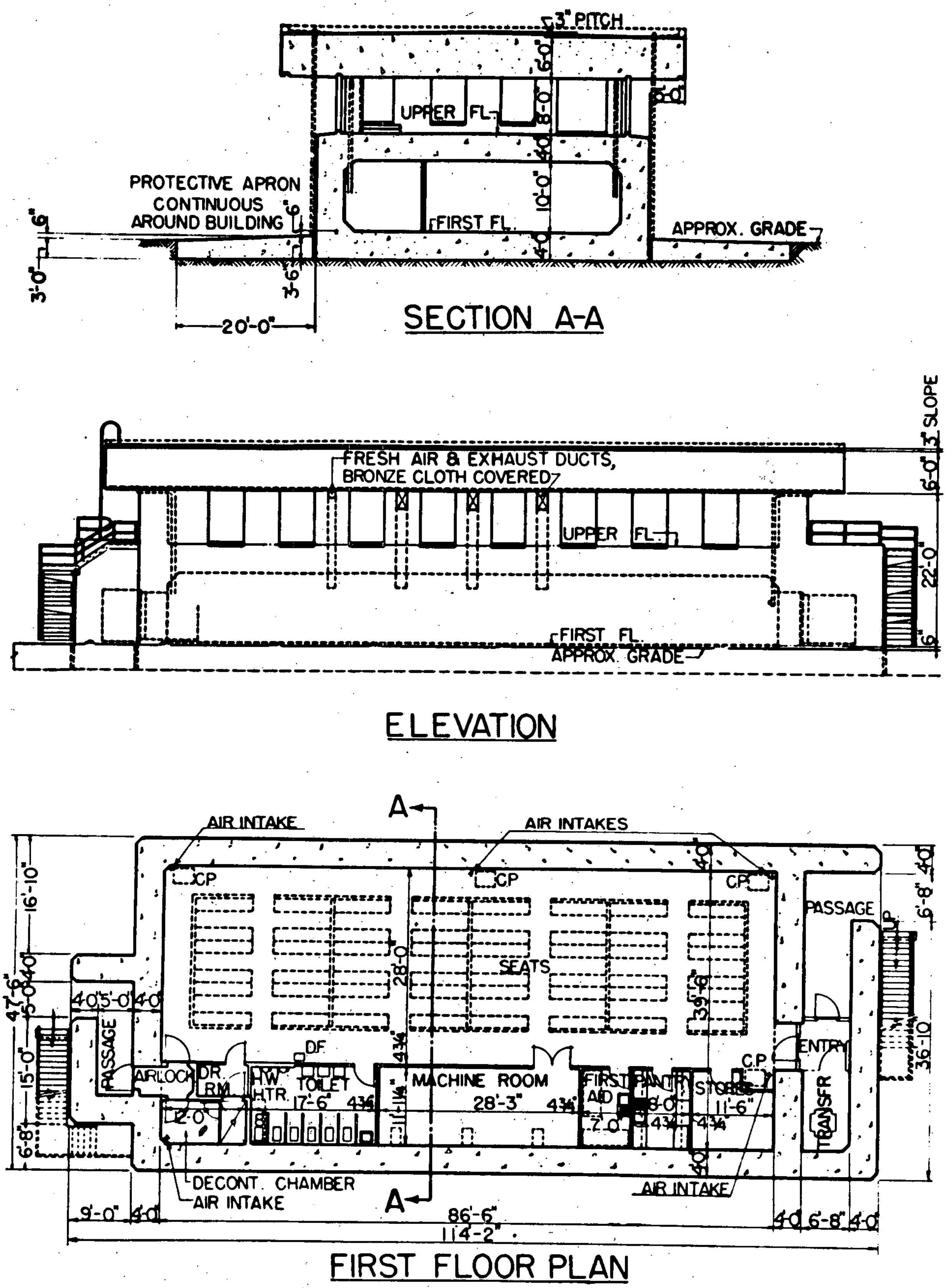


FIG. 11 ARRANGEMENT OF FRAMING
BOMB-RESISTANT PERSONNEL SHELTER

Y-200-63

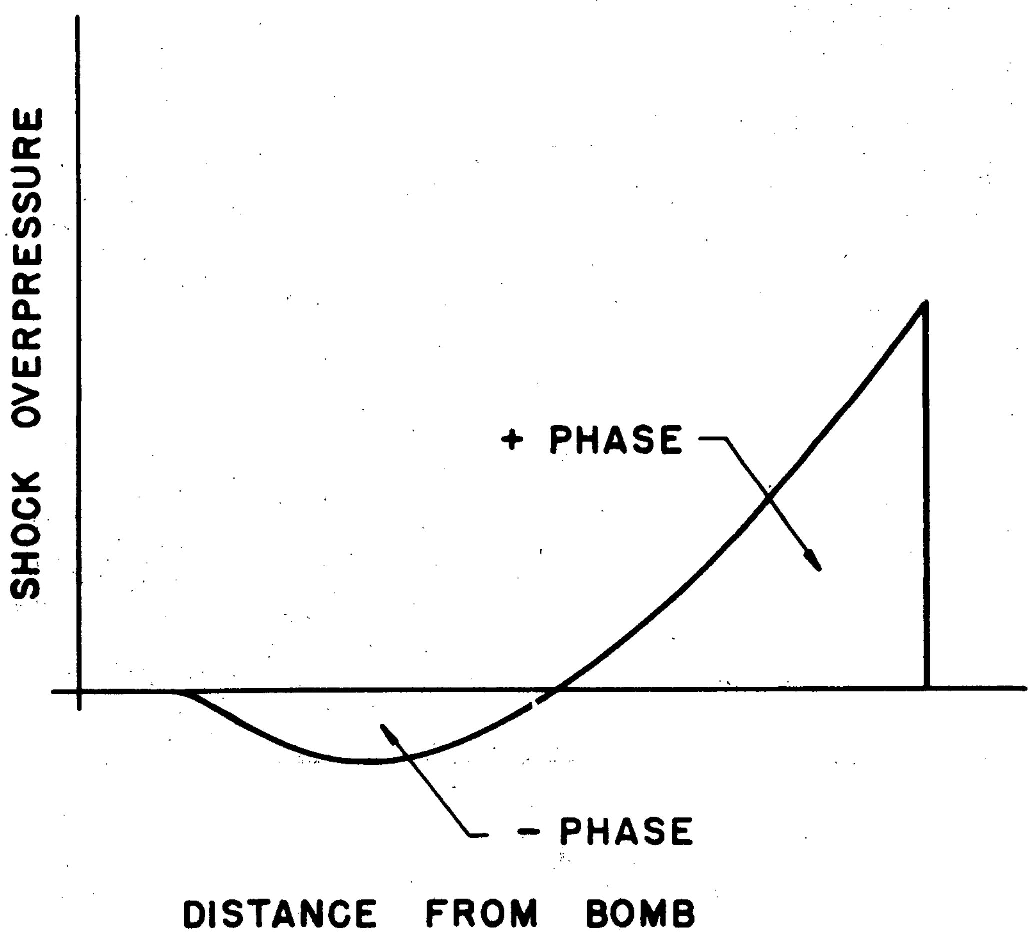


FIG. 12 PRESSURE-DISTANCE CURVE AT A GIVEN INSTANT SHOWING POSITIVE AND NEGATIVE PHASES.

Y-200-64

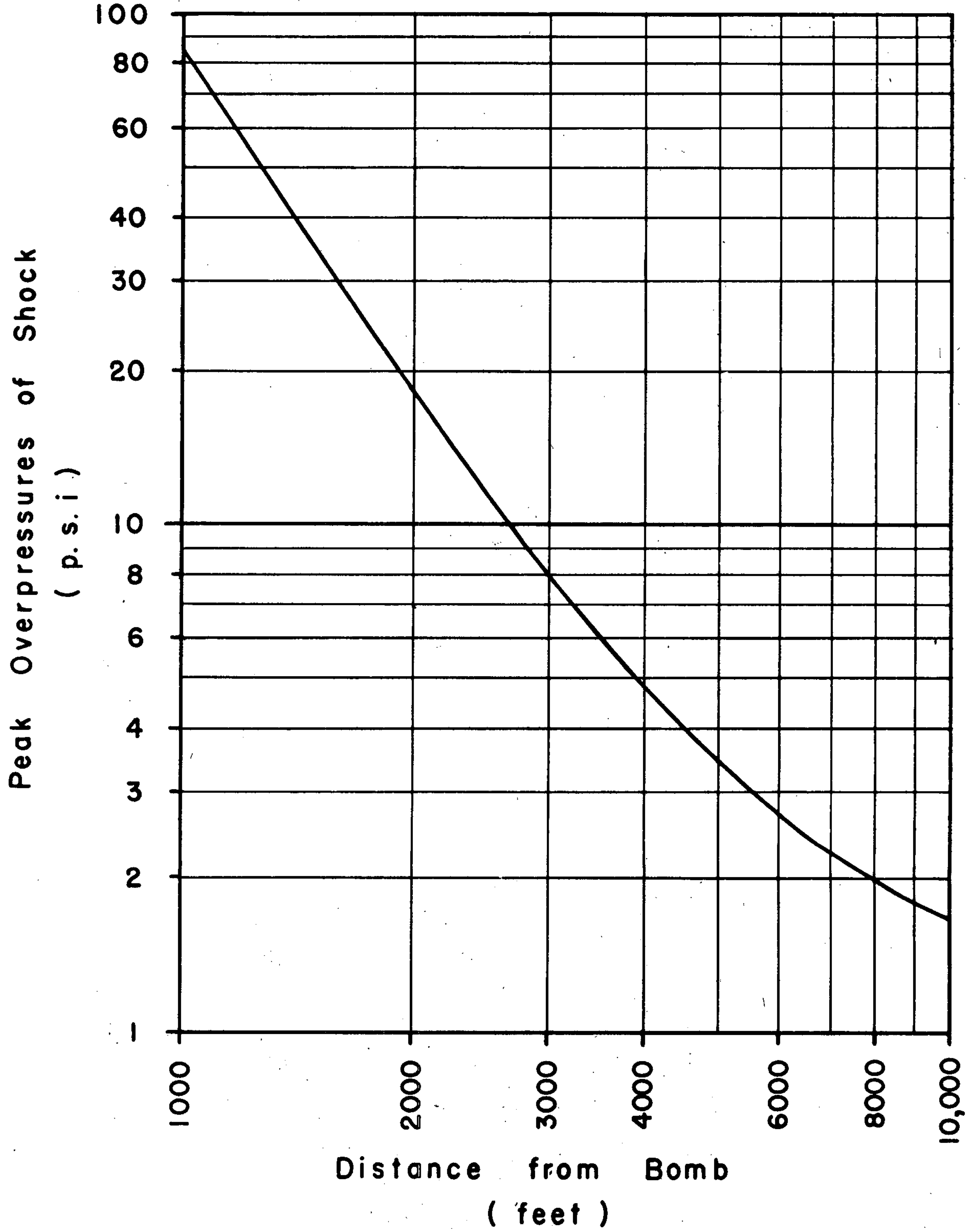


FIG. 13 PEAK OVERPRESSURE AT VARIOUS DISTANCES FROM CENTER OF EXPLOSION

Y-200-65

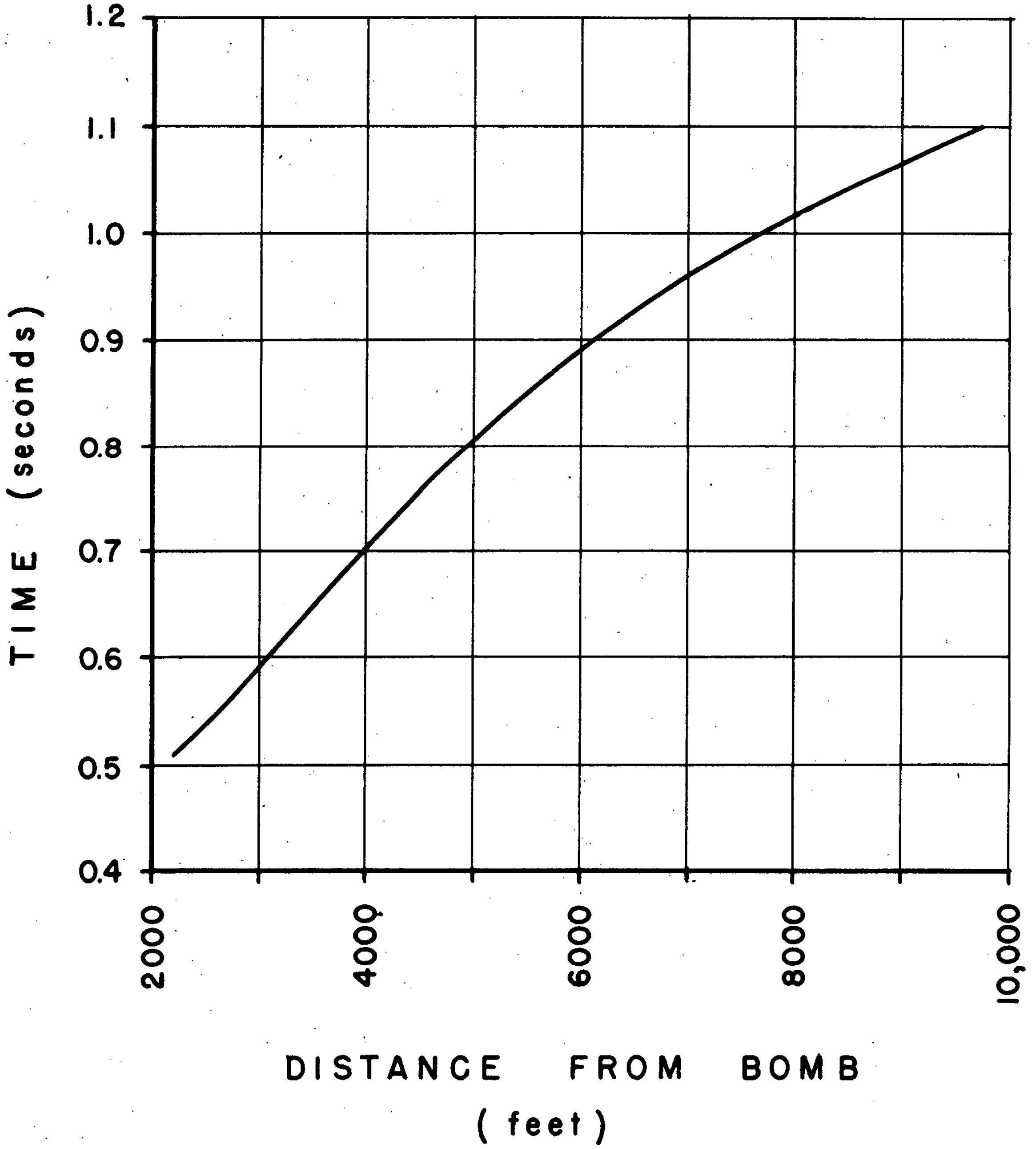
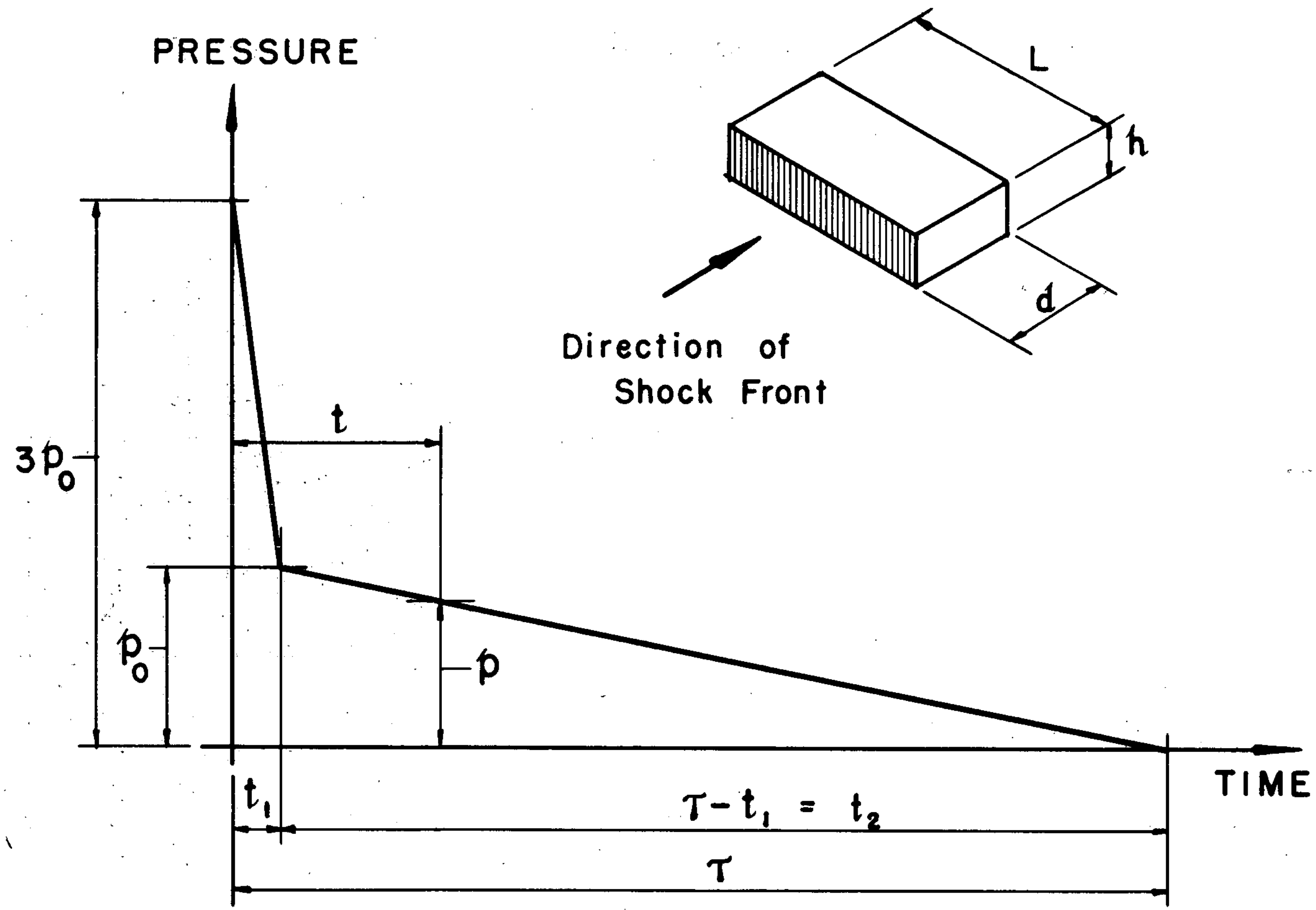


FIG.14 DURATION OF POSITIVE PHASE OF SHOCK WAVE

Y-200-66



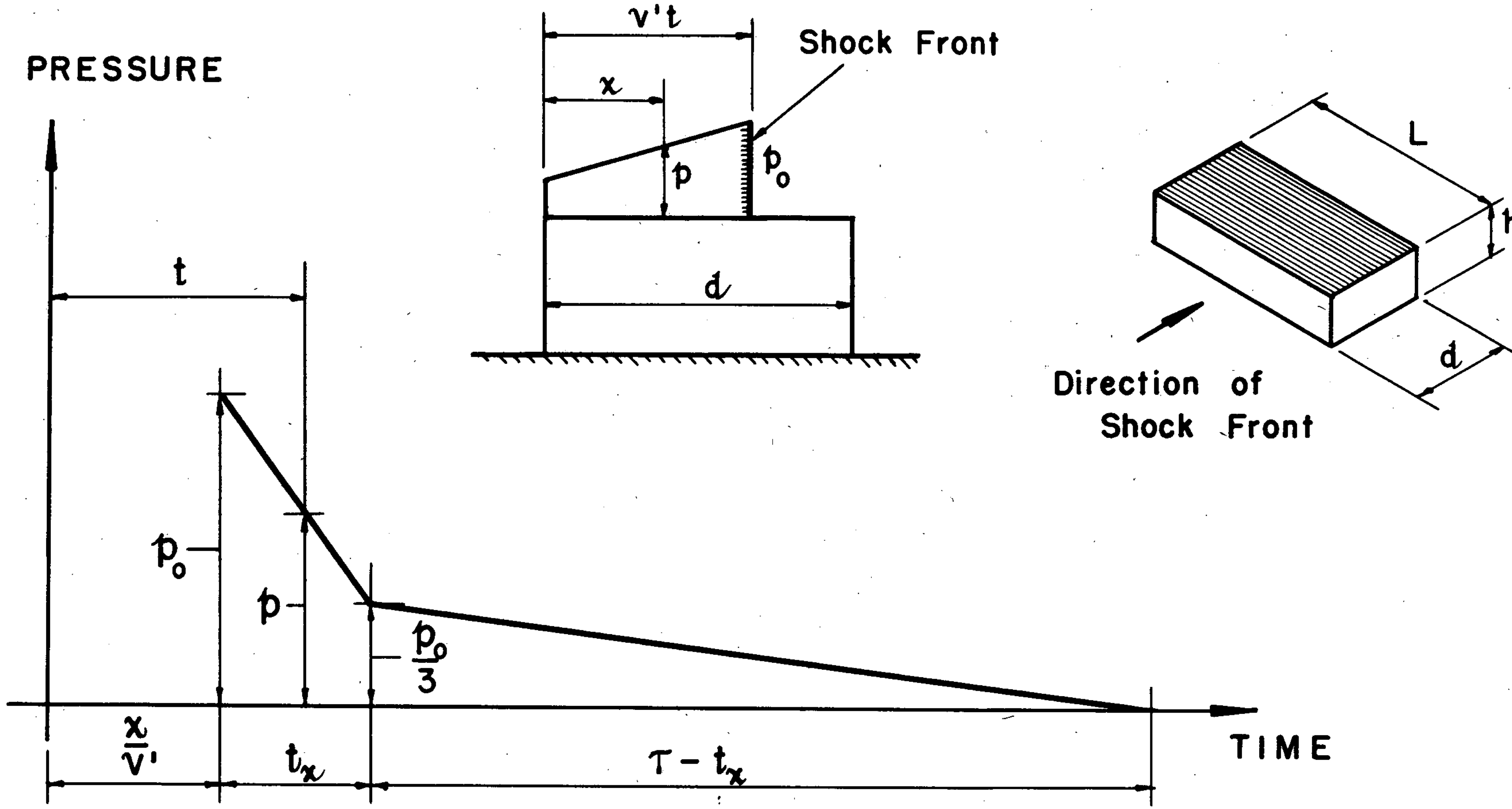
$$t_1 = \frac{4h}{v'} , \quad \text{for} \quad h \leqq \frac{L}{2}$$

$$t_1 = \frac{2L}{v'} , \quad \text{for} \quad h > \frac{L}{2}$$

FIG. 15 PRESSURE—TIME RELATION FOR FRONT FACE OF STRUCTURE

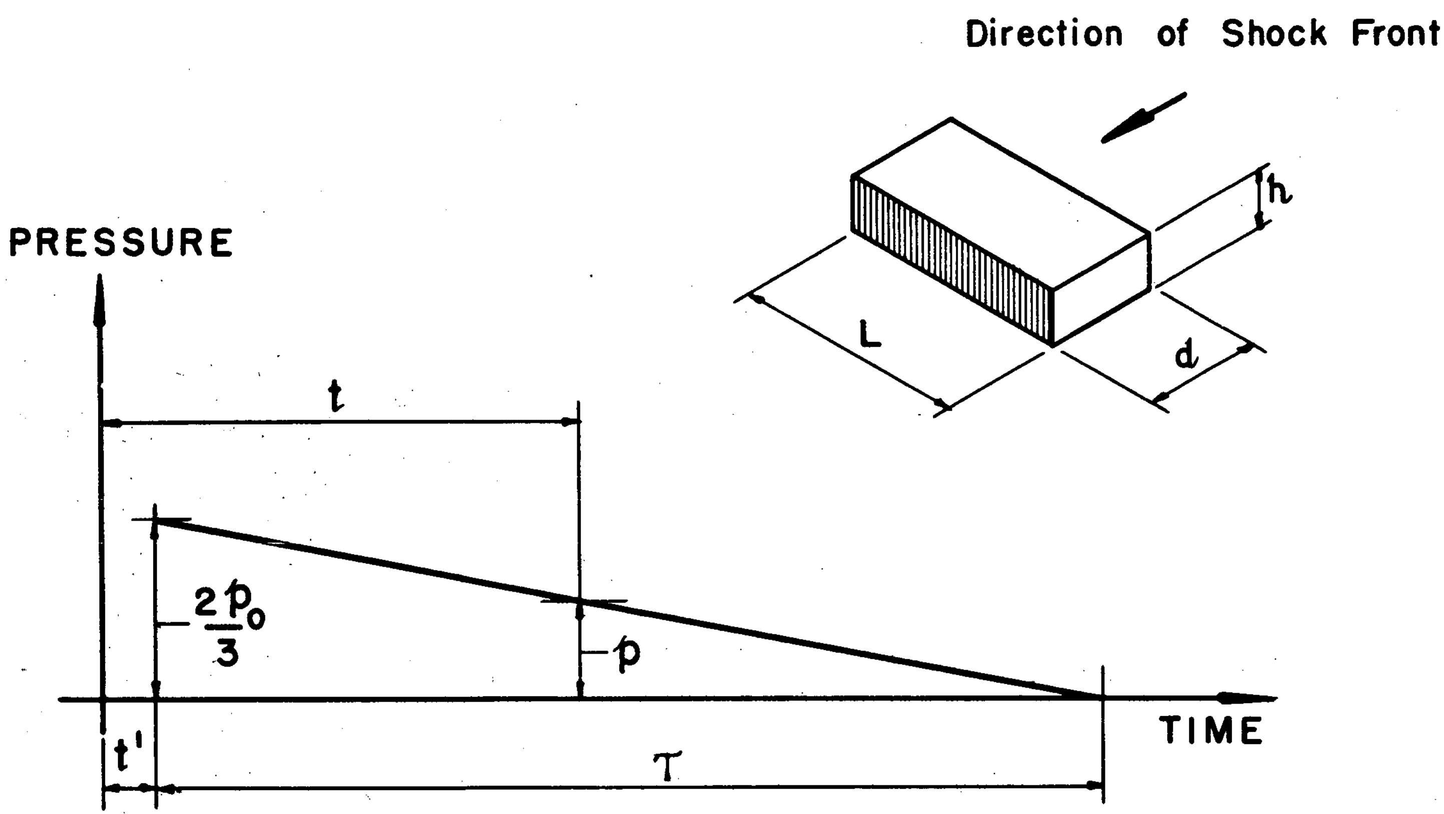
Y-200-67

K-200-68



$$t_x = \frac{6x}{v'}$$

FIG. 16 PRESSURE-TIME RELATION FOR ROOF OF STRUCTURE



$$t' = \frac{1}{v'}\left(d + \frac{h}{2}\right), \quad \text{for } h \leqq L$$

$$t' = \frac{1}{v'}\left(d + \frac{L}{2}\right), \quad \text{for } h > L$$

FIG. 17 PRESSURE–TIME RELATION FOR REAR FACE OF STRUCTURE

X-200-69

For: $0 \leqq t \leqq t_a$

$$q_a = \frac{384\, E_c I_c \cdot y_a}{l^4}$$

For: $t = t_{b1}$

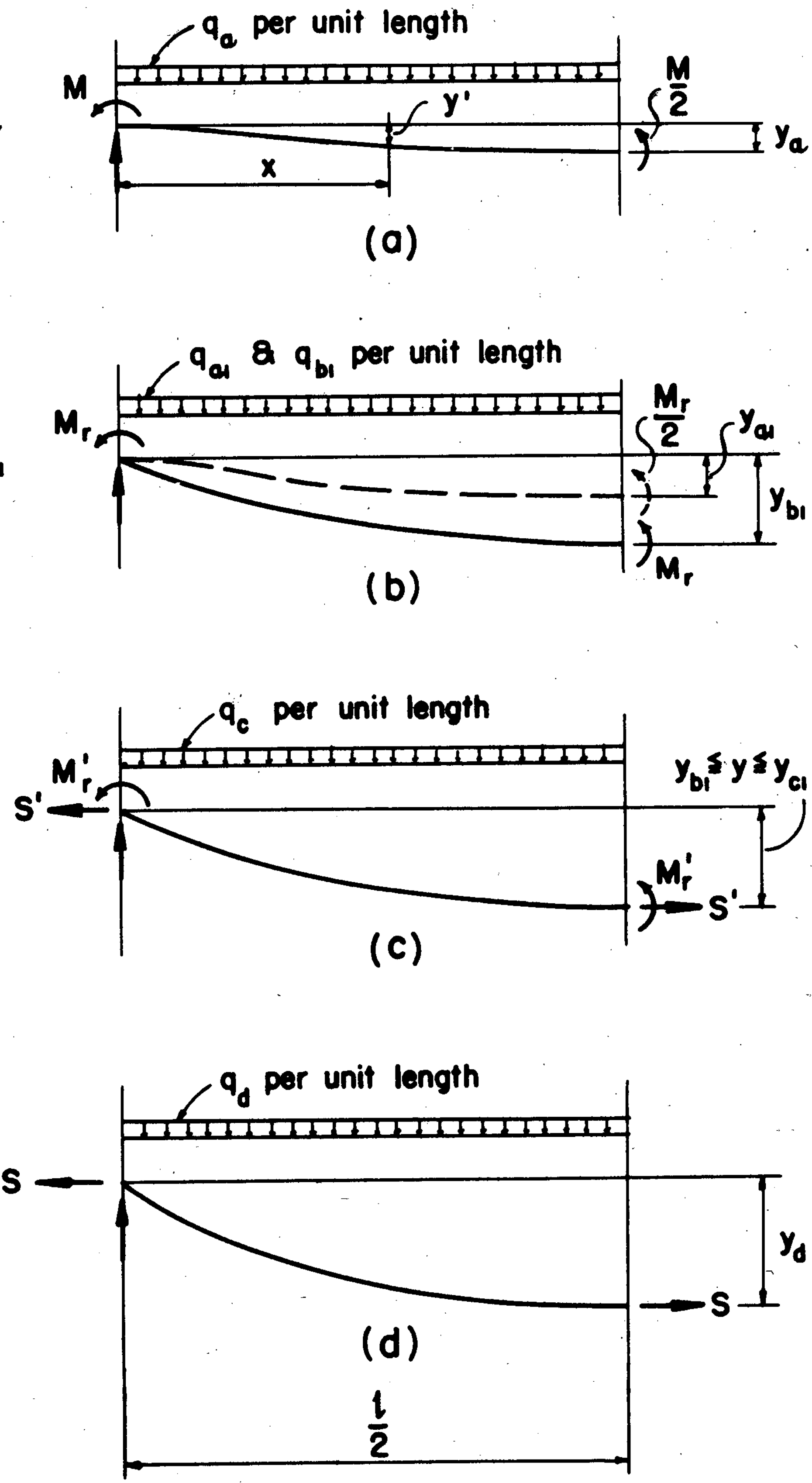
$$q_{b1} = \frac{192\, E_c I_c \cdot y_{b1}}{l^4}$$

For: $t_{b1} \leqq t \leqq t_{c1}$

$$q_c = \frac{16\, M_r}{l^2}$$

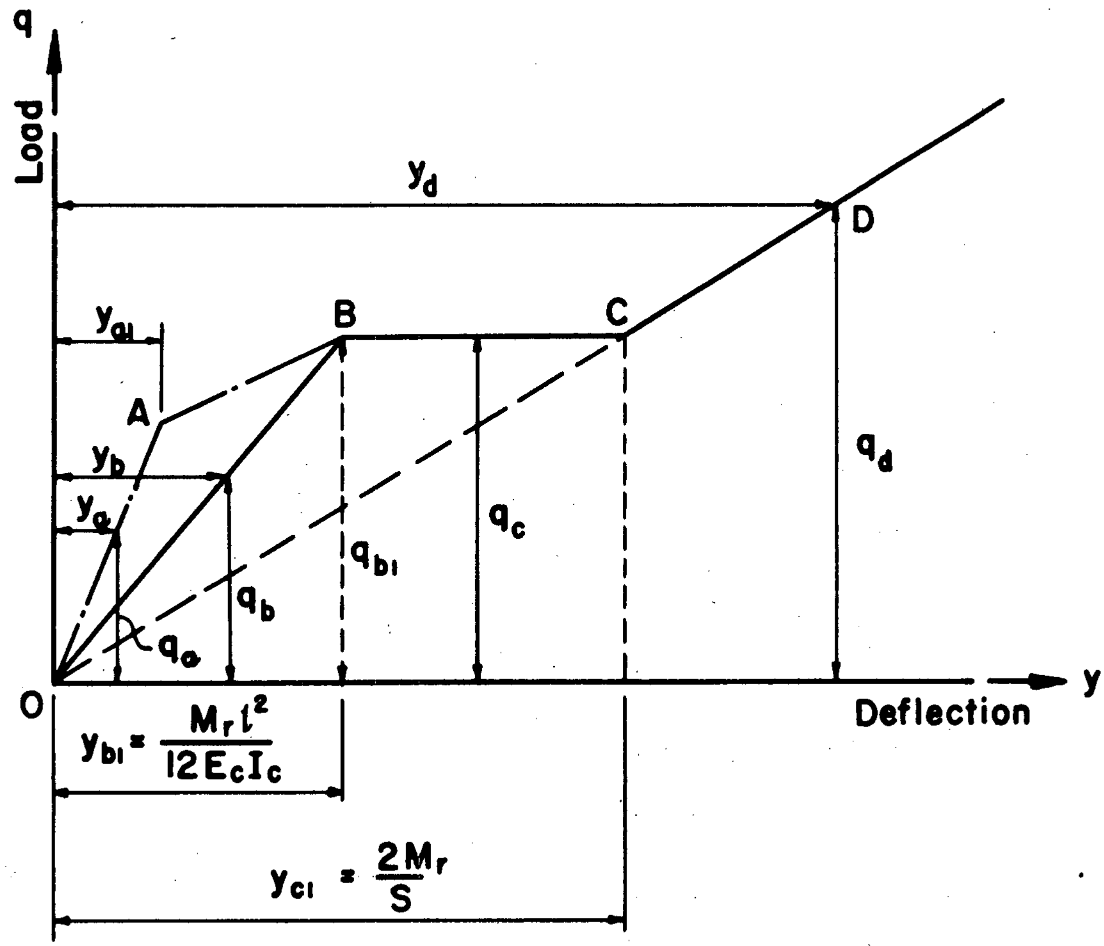
For: $t \geqq t_{c1}$

$$q_d = \frac{8\, S \cdot y_d}{l^2}$$



Y-200-70

FIG. 18 DEFORMATIONS OF A BEAM UNDER A UNIFORM LOADING



For: $0 \leq y \leq y_{b1}$

$$q_b = \frac{192\, E_c I_c \cdot y_b}{l^4}$$

For: $y_{b1} \leq y \leq y_{c1}$

$$q_c = \frac{16\, M_r}{l^2}$$

For: $y \geq y_{c1}$

$$q_d = \frac{8S \cdot y_d}{l^2}$$

FIG. 18A RESISTANCE—DEFLECTION RELATION FOR A REINFORCED CONCRETE MEMBER

Y-200-71

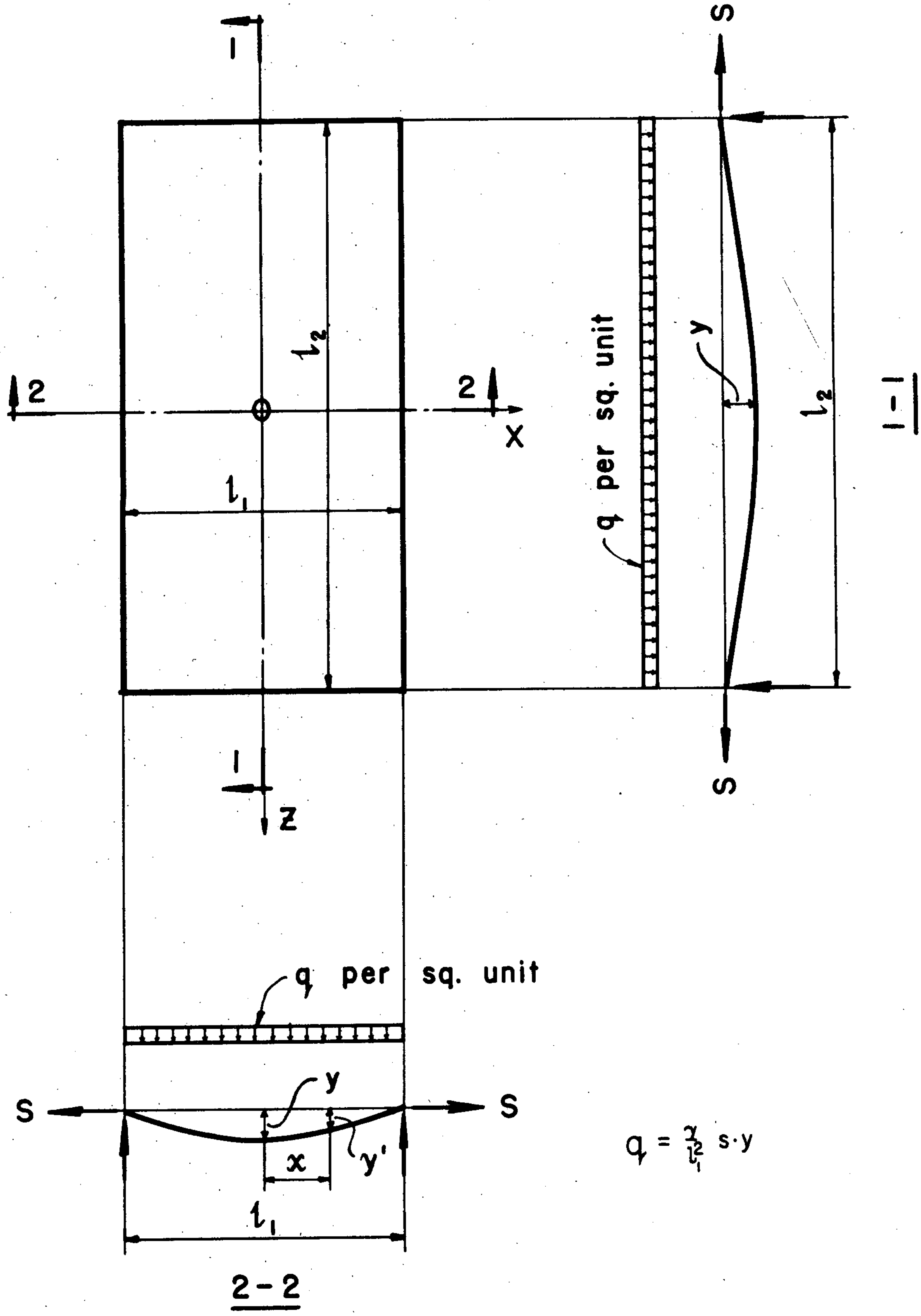


FIG. 19 PLASTIC DEFORMATION OF A TWO WAY THIN SLAB UNDER A UNIFORM LOADING (q)

Y-200-72

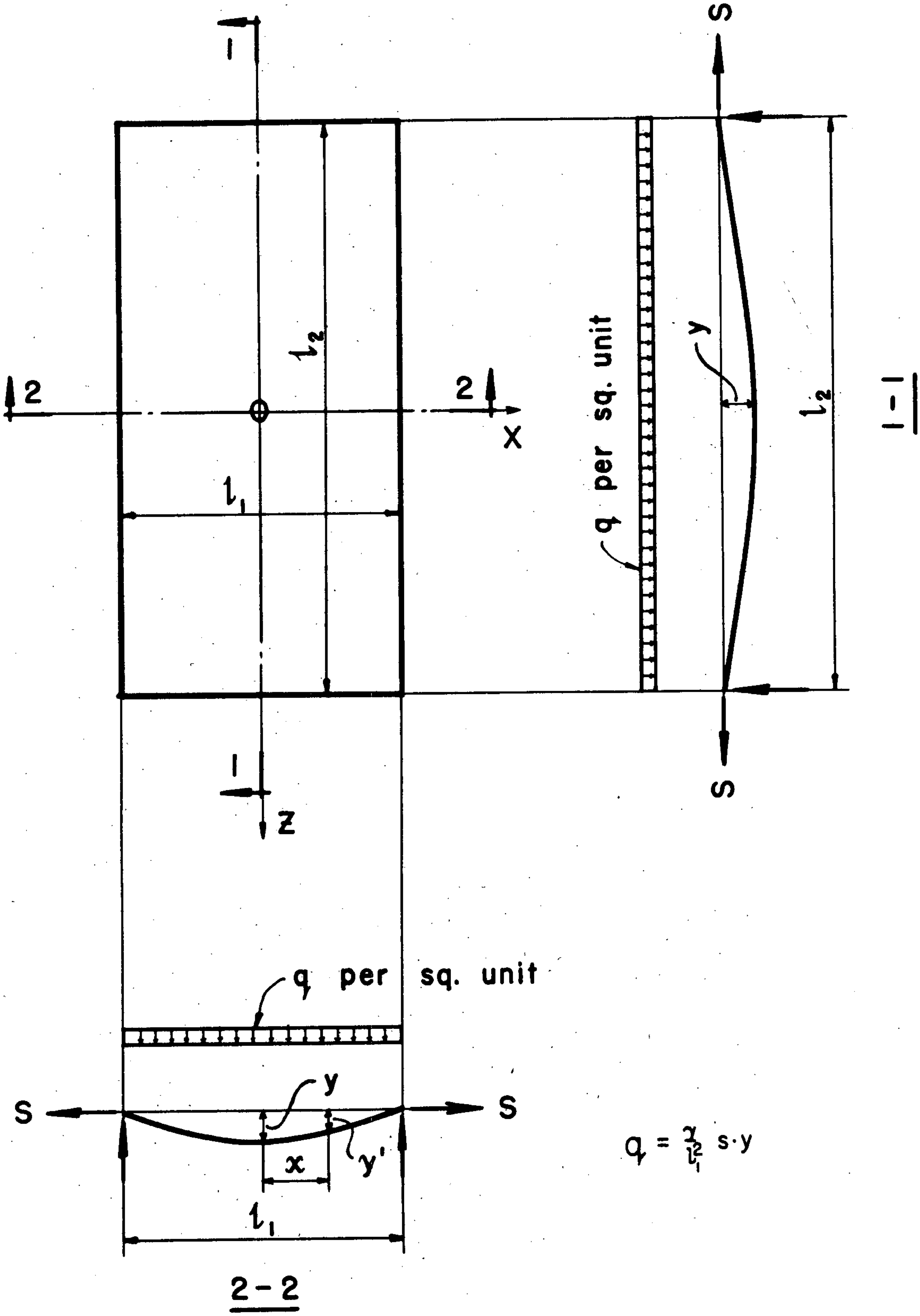
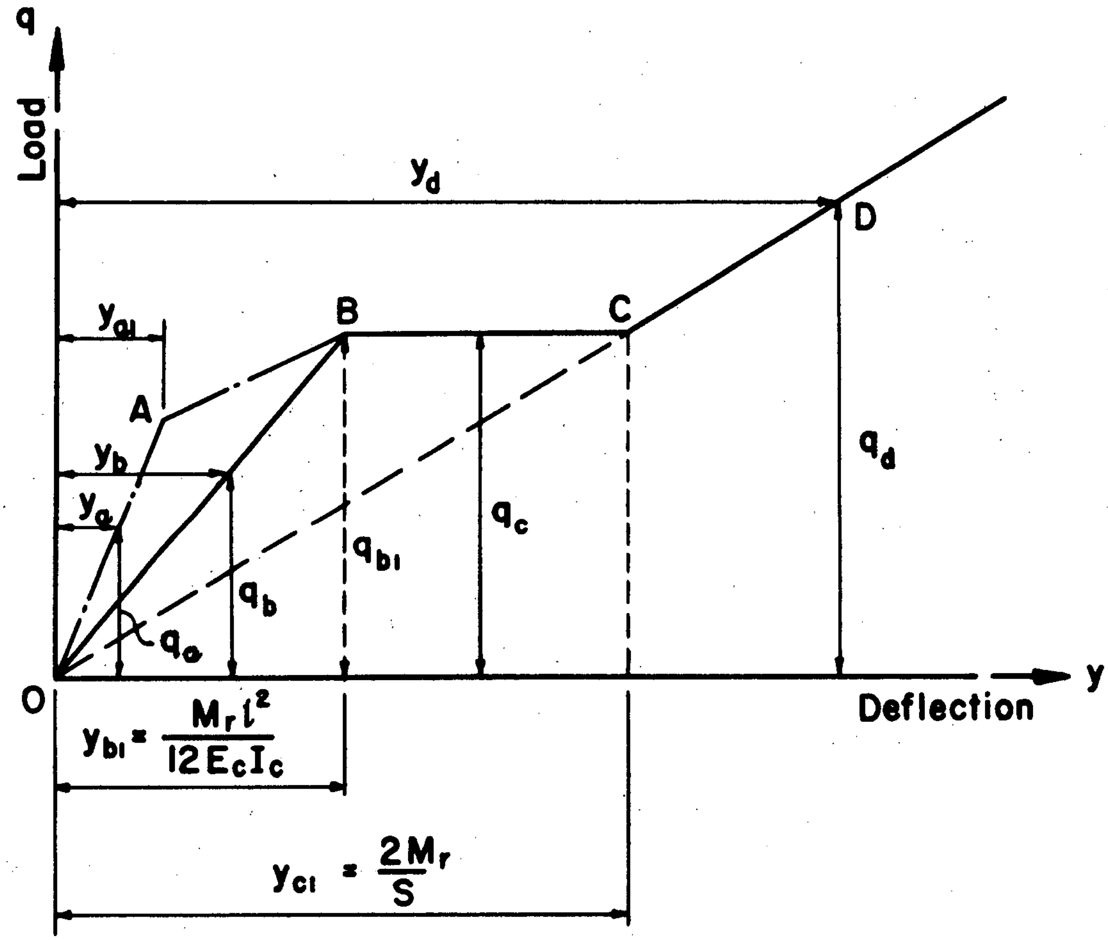


FIG. 19 PLASTIC DEFORMATION OF A TWO WAY THIN SLAB UNDER A UNIFORM LOADING (q)

Y-200-74



For: $0 \leqq y \leqq y_{b1}$

$$q_b = \frac{192\, E_c I_c \cdot y_b}{l^4}$$

For: $y_{b1} \leqq y \leqq y_{c1}$

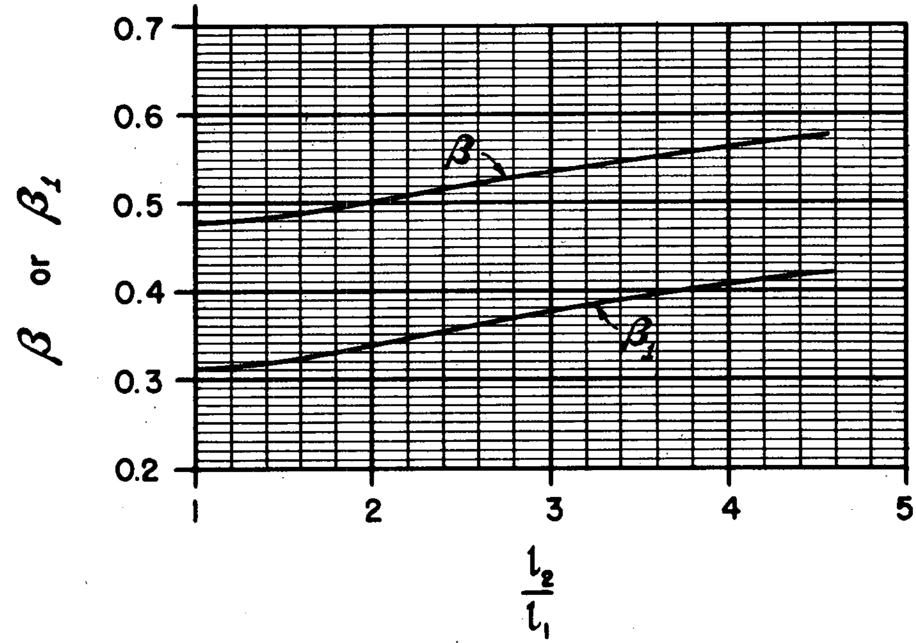
$$q_c = \frac{16\, M_r}{l^2}$$

For: $y \geqq y_{c1}$

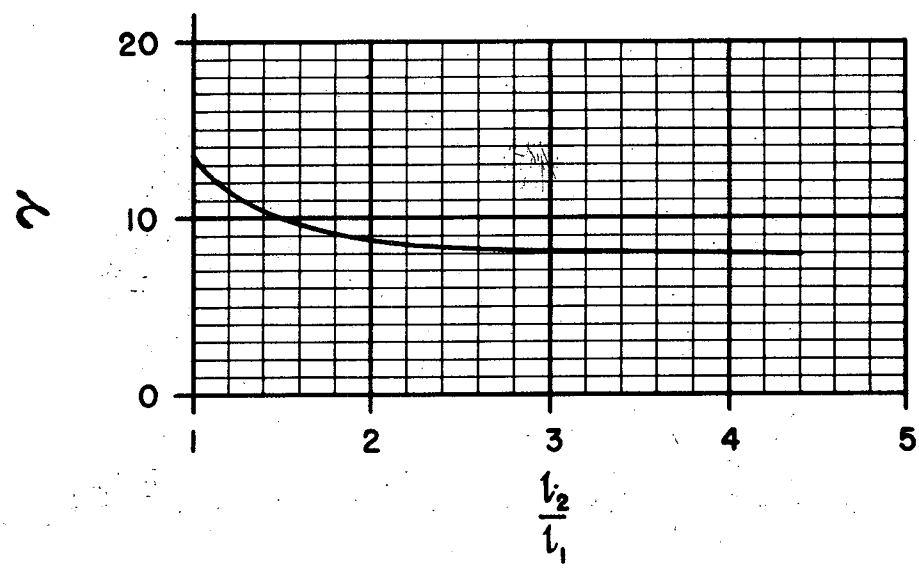
$$q_d = \frac{8S \cdot y_d}{l^2}$$

FIG. 18A RESISTANCE—DEFLECTION RELATION FOR A REINFORCED CONCRETE MEMBER

Y-200-73



(a) REDUCTION FACTORS FOR AREA AND WEIGHT



(b) RESISTANCE FACTOR

FIG. 20 REDUCTION FACTORS (β AND β_1) AND RESISTANCE FACTOR (γ) FOR TWO-WAY SLABS

Y-200-75

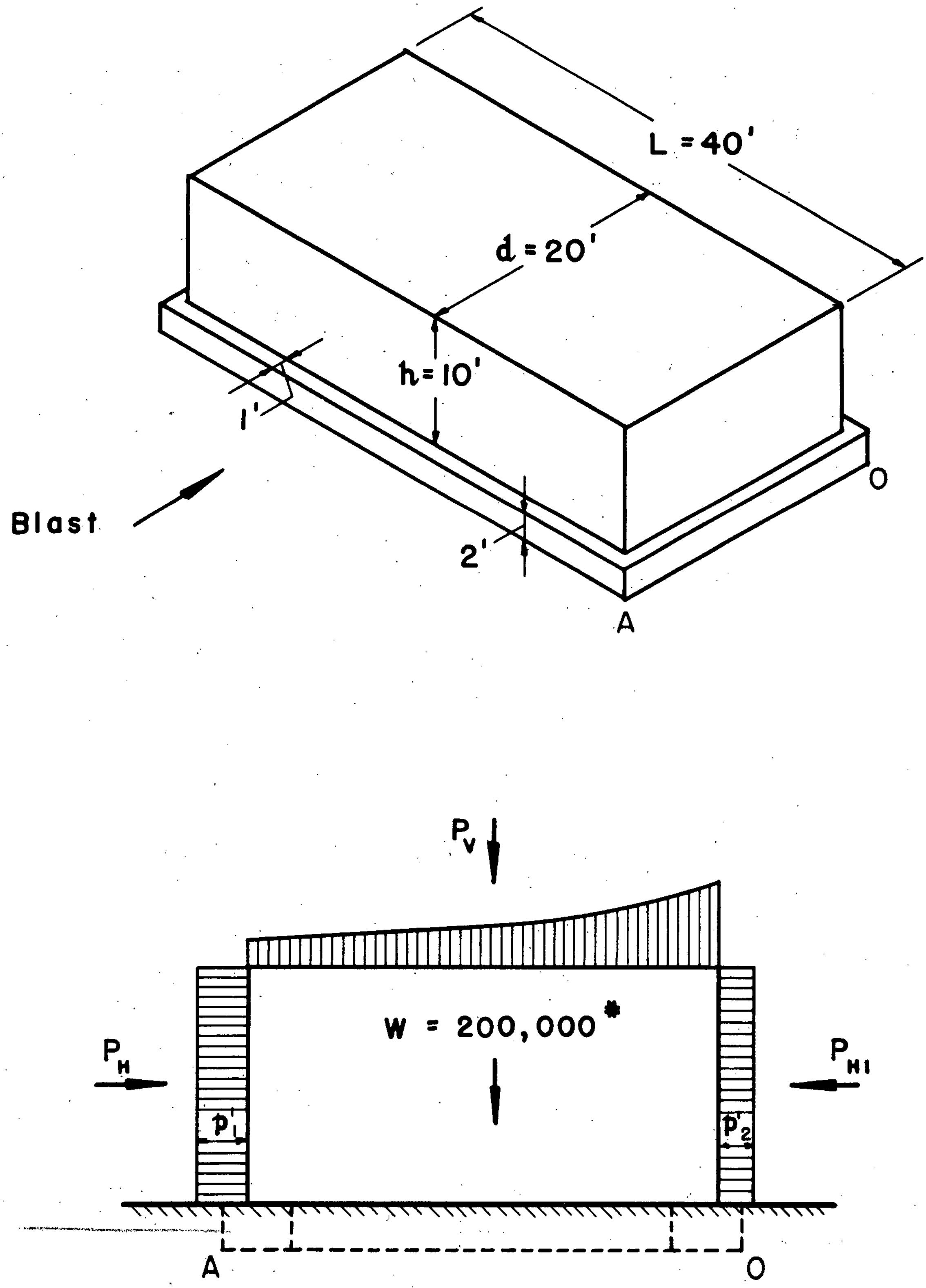
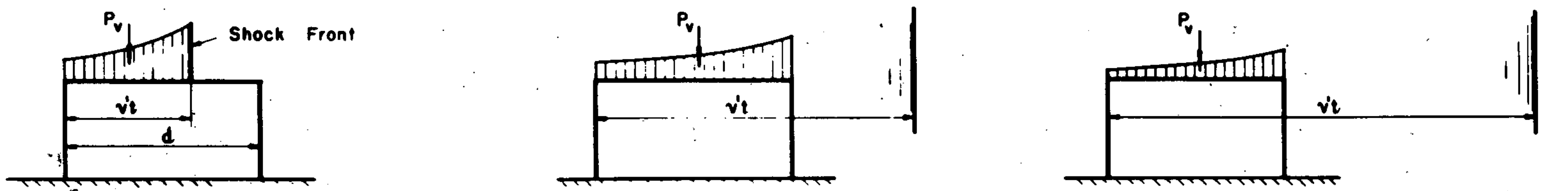


FIG. 21 TYPICAL STRUCTURE IN SHOCK WAVE

Y-200-76



$0 \leq v't \leq d$ — CASE 1

$d \leq v't \leq 7d$ — CASE 2

$v't \geq 7d$ — CASE 3

CASE 1	$P_v = Lp_o\left[\frac{17}{18}v't - \frac{7v'\tau - 6v't}{108}\log_e\left(\frac{7\tau - 6t}{7\tau}\right) - \frac{v't}{9}\log_e(7)\right]$ $P_v \approx 0.783\, Lp_o v't$ *	$M_p = Lp_o\left\{\frac{113}{252}(v't)^2 - \frac{7v'\tau - 6v't}{108}\left[\frac{v't}{7} + \frac{v'\tau}{6}\log_e\left(\frac{7\tau - 6t}{7\tau}\right)\right]\right\}$ $M_p \approx 0.452\, Lp_o (v't)^2$ *
CASE 2	$P_v = Lp_o\left[\frac{10}{9}d - \frac{v't}{6} - \frac{7v'\tau - 6v't}{108}\log_e\left(\frac{7\tau - 6t}{7\tau}\right) - \frac{v't}{9}\log_e\left(\frac{7d}{v't}\right)\right]$ $P_v \approx Lp_o d\left[0.961 - 0.191\frac{v't}{d} + 0.013\left(\frac{v't}{d}\right)^2\right]$ *	$M_p = Lp_o\left\{\frac{(v't)^2}{252} - \frac{7v'\tau - 6v't}{108}\left[\frac{v't}{7} + \frac{v'\tau}{6}\log_e\left(\frac{7\tau - 6t}{7\tau}\right)\right] + \frac{d}{9}(5d - v't)\right\}$ $M_p \approx Lp_o\left[\frac{d}{9}(5d - v't) + \frac{(v't)^2}{130}\right]$ *
CASE 3	$P_v = Lp_o\left[-\frac{d}{18} - \frac{7v'\tau - 6v't}{108}\log_e\left(\frac{v'\tau - 6d}{v'\tau}\right)\right]$ $P_v \approx Lp_o d\left[0.405 - 0.014\frac{v't}{d}\right]$ *	$M_p = Lp_o\left\{-\frac{d^2}{36} - \frac{7v'\tau - 6v't}{108}\left[d + \frac{v'\tau}{6}\log_e\left(\frac{v'\tau - 6d}{v'\tau}\right)\right]\right\}$ $M_p \approx Lp_o(d + 1.03)^2\left[0.168 - 0.0036\, v't\right]$ *

P_v = Total pressure on roof.

M_p = Moment of P_v about front edge of roof.

* Approximate values are for $v' = 1400$ ft. per second, $0.5 \leq \tau \leq 0.75$ sec., and $d \leq 50$ feet.

Shock wave moves from left to right, L = length of structure.

FIG. 22 TOTAL ROOF PRESSURES & MOMENTS OF TOTAL PRESSURES

Y-200-77